Comparison between Prestressed Slab Analysis Methods According to NBR 6118 and ACI 318

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Summary

Among the most widespread methods for designing flat slabs, the Equivalent Frame Method (EFM), the Grid Analogy Method, and the Finite Element Method (FEM) stand out. EFM has been widely used in the USA since the 1970s for prestressed flat slabs, with its normative prescriptions dictated by ACI 318. However, in Brazil, there are severe criticisms within the technical community regarding the quality of its results, especially in cases where the columns are misaligned, and the span between columns varies significantly in the same direction. Since the 2014 revision of NBR 6118, the permission to use this method for prestressed flat slabs was removed from its text.

Given this reality, this work compares the different methodologies adopted by Brazilian (FEM) and American (EFM) standards. Thus, it is expected to contribute to the technical field by providing understanding and positioning designers regarding the different approaches these standards take concerning the most widespread methods.

1 INTRODUCTION

The use of prestressed slabs has consolidated in Brazil in recent decades, with several factors contributing to its success. Among them, some highlights include architectural freedom due to the reduction or absence of beams, increased ceiling height, and easier and faster execution.

Plate Theory governs the concepts of slab structural analysis. However, given the difficulties in using analytical solutions for plate problems and the availability of computational programs for structural engineering, approximate numerical solutions have become the mainstream method for analyzing flat slabs.

Among the most widespread methods, the Equivalent Frame Method (EFM), the Grid Analogy Method, and the Finite Element Method (FEM) stand out.

This article was developed based on the research and analyses conducted between 2015 and 2017, as detailed in Nobre's 2017 master's dissertation, which was defended in 2017 under the provisions of the ACI-318 and NBR-6118 standards, both in their 2014 versions. Since then, ACI-318 has been updated to the 2019 version, and NBR-6118 to the 2023 version. However, even in the current versions of these standards, the subject addressed in this research has remained unchanged, and therefore, it remains equally relevant.

2 SLAB CALCULATION METHODS

2.1 Equivalent Frame Method (EFM)

The Equivalent Frame Method is an approximate method that calculates slabs by subdividing them into different frame lines following the column alignments. The same procedure is repeated in both directions to reinforcement design (both passive and active).

Although commercial software already implements the method, it is simple to apply and can be performed manually. However, there are criticisms in the technical community regarding its accuracy, especially in cases of misaligned supports. This scenario motivated this research.

2.2 Grid Analogy Method

The Grid Analogy Method consists of subdividing a slab panel (plate) into several continuous strips, transverse to each other, which are treated as beams (bar elements) with equivalent stiffness to their respective slab sections, forming a grid.

This method gained popularity with the advent of computational analysis and has since maintained its place in Brazilian design offices through commercial software such as TQS-CAD.

2.3 Finite Element Method (FEM)

The Finite Element Method discretizes a structure into multiple sufficiently small elements connected by nodes. The structural analysis is thus reduced to a nodal analysis, leading to significant simplification and reduced data processing costs.

Since its emergence, FEM has been increasingly accepted as the most precise and reliable method, simulating reality with high precision at structural nodes.

With computing advancements, large FEM models can now be processed on simple computers, and the trend is for FEM to become the most widespread and reliable method for structural analysis.

3 REGULATORY PROVISIONS FOR THE EQUIVALENT FRAME METHOD

3.1 American Standard: ACI 318

According to the American standard, each frame consists of a column line (or other support) and a slab-beam strip (Fig.1), laterally delimited by the axis of the panels between supports.

A distinguishing feature of the American standard compared to the Brazilian standard is the method adopted for connecting the support and the slab beam, which must be made through torsion elements. These torsion elements (Fig.2) consist of slab-beam sections transverse to the frame axis and lateral to the support, transferring forces through torsion rather than direct contact between the slab-beam parallel to the frame axis and the support's transverse face.

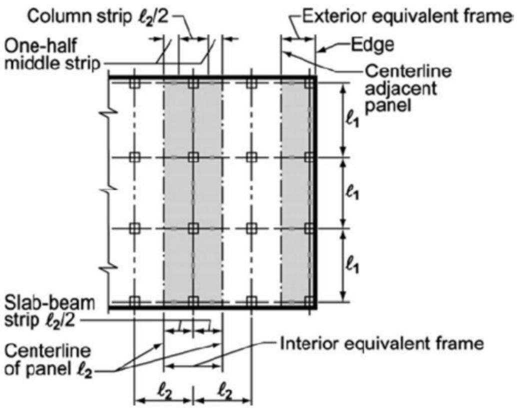


Fig. 1 Strip definition by ACI-318

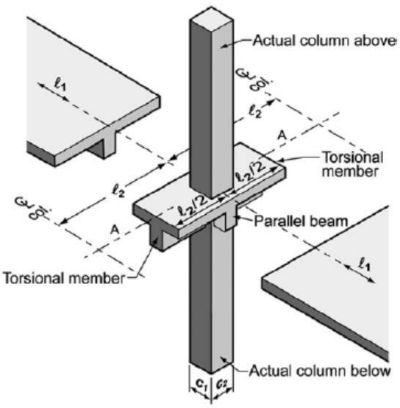


Fig. 2 Torsional member by ACI-318

3.2 Brazilian Standard: NBR 6118 since 2014

The version before 2014 of the Brazilian standards recommended that the equivalent frame method be only used when the columns were arranged orthogonally and with spans of slight variation (Fig.3). However, in the 2014 version of NBR 6118, item 14.7.8 states that this method can only be used for reinforced concrete slabs, excluding prestressed slabs. Below is the transcribed excerpt from the standard in free translation:

"The structural analysis of flat and waffle slabs must be done using an appropriate numerical procedure, such as finite differences, finite elements, and boundary elements. In the case of reinforced concrete slabs, where the columns are arranged in orthogonal rows regularly and with spans of little variation, the calculation of internal forces can be performed using an approximate elastic process with redistribution, which consists of adopting, in each direction, multiple frames to obtain the required internal forces."

Thus, there is no longer a regulatory prescription for using EFM in prestressed slabs, which is the subject of this research.

For comparison purposes, the guidelines for this method for reinforced concrete slabs consist of a structure that must be defined as multiple equivalent frames, each following the lines of their respective supports in both transverse and longitudinal directions. Each frame consists of a line of columns (or another type of support) and a slab-beam strip, laterally delimited by the axis of the panels between supports.

4 CASE STUDY

The case study analyzed in this work consists of a building with misaligned and non-orthogonal layout of columns, precisely the scenario where the equivalent frame method (EFM) is not recommended. A comparison will be made between the results obtained using EFM and the finite element method (FEM).

The analyzed floor consists of a 22 cm thick post-tensioned flat slab, with spans ranging from 5 to 6.5 meters between columns. The floor geometry is shown in Figure 3, and the frame definition is in Figure 4. The support line B1 was chosen for analysis.

Between column P21 and the left cantilever, there is no column; the support is provided by the slab itself, which rests on the orthogonal span (X direction) between columns P22 and P23. For the purposes of the equivalent frame modeling, a fictitious column was inserted in the model to simulate this support.

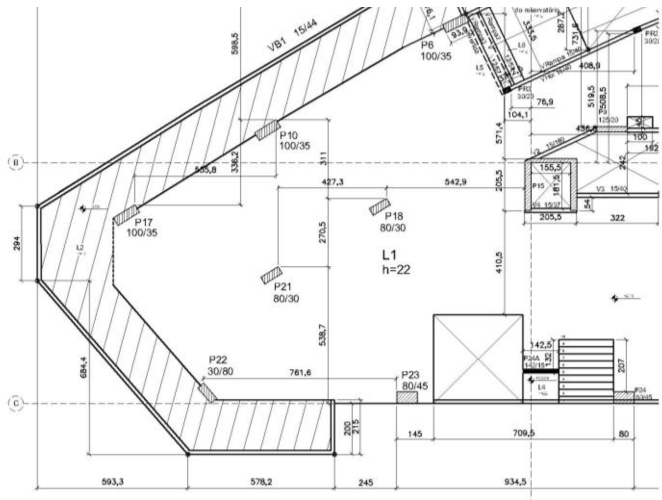


Fig. 3 Structural design

Fig. 4 Support lines definition and support line B1 (analysed)

There is no column between span 1 and the left cantilever; the support is provided by the slab itself, which is stiffer in the orthogonal span (X direction) between columns P22 and P23. For the purposes of the equivalent frame modeling, a fictitious column was inserted in the model to simulate this support (Figure 5).

4.1 Equivalent Frame Method (EFM)

For the EFM analysis, the commercial software ADAPT-PT was used. The B1 frame, selected for study and already modeled, is shown in Figure 5. The numbering reference for the spans and cantilevers is based on the perspective of an observer looking at the support line from right to left.

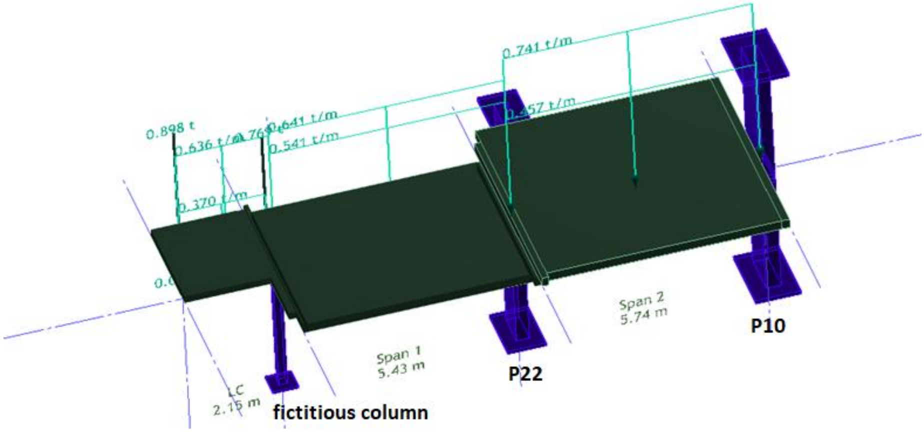


Fig. 5 Support line B1

Table 1 presents the normal stress analysis results for the frequent load combination under service conditions. Table 2 shows the deflections at the points of maximum deformation in each span for the frequent load combination.

The results related to the right cantilever were obtained from the analysis of support line B, to which this cantilever was associated in the definition of the support lines. Both support line B and all the others are thoroughly analyzed in Nobre’s (2017) master's dissertation.

Table 1 – Envelope of normal stresses in service – EFM.

Span	Maximum tensile stress (MPA)		Maximum compressive stress (Mpa)	
	Top	Bottom	Top	Bottom
Left Cantilever	-	-	-0,85	-2,70

Span 1	-	-	-1,15	-1,50
2º column	-	-	-1,74	-0,75
Span 2	-	0,05	-2,15	-
Right Cantilever	0,50	-	-	-4,00

Table 2 – Summary of Deflections (Immediate and Long-Term) – EFM.

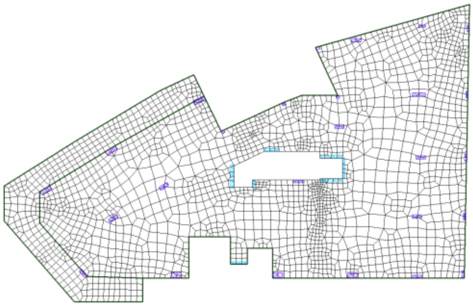
Span	deflection (mm) t = immediate	deflection (mm) t = long-term
Left Cantilever	2,7	7,0
Span 1	-0,6	-2,0
Span 1	0,9	2,0
Right Cantilever	4,3	11,0

4.2 Finite Element Method (FEM)

Two commercial software programs were used for the FEM analysis: SAP2000 and FLOOR-PRO.

4.2.1 Finite Element Method (FEM) using FLOOR-PRO

This program is designed for post-tensioned slab calculations, offering several modeling conveniences. Figure 6 shows a discretized slab, and Figure 7 shows the tributary strips in the vertical direction.



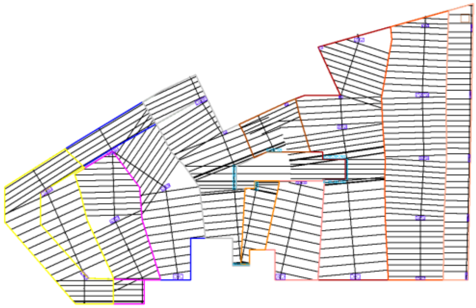


Fig. 6 Discretized Slab (FLOOR-PRO) Fig. 7 Tributary Strips in the Vertical Direction

Table 3 indicates the envelope of normal stresses in service conditions, and Table 4 presents the deflection summary (Immediate and Long-term).

Table 3 – Envelope of Normal Stresses in Service Conditions – FLOOR-PRO (MEF)

Span	Maximum tensile stress (MPa)		Maximum compressive stress (MPa)	
	Top	Bottom	Top	Bottom
Left Cantilever	-	-	-2,80	-2,35
Span 1	-	-	-0,50	-2,65
2º column	-	-	-0,87	-2,37
Span 2	-	-	-0,10	-4,00
Right Cantilever	+0,10	-	-	-4,90

Table 4 – Summary of Deflections (Immediate and Long-term) – FLOOR-PRO (MEF).

Span	deflection (mm) t = immediate	deflection (mm) t = long-term
Left Cantilever	3,5	8,4
Span 1	-0,5	0,0
Span 2	-1,0	-3,0
Right Cantilever	3,2	5,0

4.2.2 Finite Element Method (FEM) using SAP2000

SAP2000 is a widely accepted market software used to validate FLOOR-PRO results. However, modeling is considerably more complex since SAP2000 is not specifically designed for post-tensioned slabs. The "section cuts" tool must be used to obtain stress results, making the process more laborious.

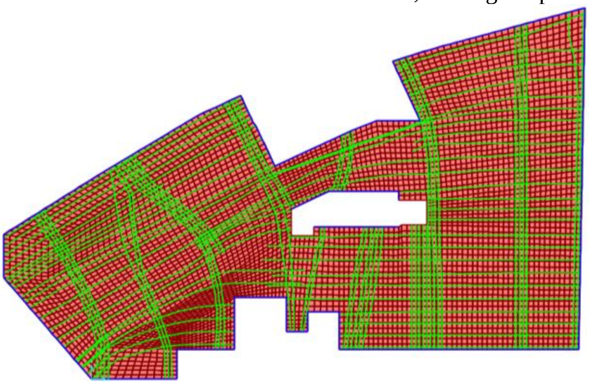


Fig. 8 Discretized slab (SAP2000) with tendons

Table 5 – Envelope of normal stresses in service conditions – SAP2000 (FEM)

Span	Maximum tensile stress (MPa)		Maximum compressive stress (MPa)	
	Top	Bottom	Top	Bottom
Left Cantilever	-	-	-2.71	-3.09
Span 1	-	-	-0.49	-2.32
2º column	-	-	-1.60	-1.43
Span 2	-	-	-0.21	-3.54
Right Cantilever	+1.37	-	-	-6.16

Table 6 – Summary of deflections (Immediate and Long-Term) – SAP2000 (FEM).

Span	deflection (mm), t = immediate	deflection (mm), t = long-term
Left Cantilever	3.0	6.9
Span 1	-0.5	-1.2
Span 2	-1.0	-2.0
Right Cantilever	2.0	6.0

5 ANALYSIS OF RESULTS

The normal stresses and total deflections are compared. The results summarize a part of the study conducted by Nobre (2017).

Table 7 summarizes the comparative normal stresses under service conditions for the frequent load combination.

The analysis considers frequent loads applied uniformly across the structure without alternating spans.

The table includes a percentage difference. dif.1 compares the calculated stresses from different methods to EFM (reference). Stresses to admissible values of 2.96 MPa (tension) and 15.75 MPa (compression), determined by the ACI318 expression for uncracked situations.

$$f_t \leq 0{,}5\sqrt{f'_c} \tag{1}$$

Table 7 – Summary of normal stresses in service conditions – maximum compression

		Max Compression			
		TOP		BOTTOM	
	METHOD	(MPa)	dif.1	(MPa)	dif.1
Span 2	EFM (ADAPT-PT)	-2.15		0.00	
	FEM (SAP2000)	-0.21	-90%	-3.54	-
	FEM (FLOOR-PRO)	-0.10	-95%	-4.00	-

(1) dif.1 = [(x - EFM) / MPE], while x is the result of other methods

The normal stress values under the frequent load combination indicate good convergence between analysis methods. However, when using EFM results as a reference (diff.1), some differences appear between the methods.

For example, at the left cantilever of frame B1, the EFM results differ substantially from the other methods. This discrepancy arises from using a fictitious column (Figure 5) to support this section instead of a stiffer slab span in the opposite direction. Because this is rigid support, it generates higher negative bending moments, leading to lower top slab compression.

At the second support, SAP2000 (FEM) yielded slightly higher stresses than FLOOR-PRO (FEM). This trend generally holds for supports, while results are similar in mid-span sections. This difference is due to stress concentration effects in SAP2000, which appear around the slab-to-column connection node. SAP2000 users should employ device modeling to mitigate these stress concentrations, connecting slab nodes with rigid bar elements.

In contrast, FLOOR-PRO models columns as elements with their actual dimensions, naturally eliminating these stress concentrations.

The maximum percentage differences in the frequent load combination occur in the 2nd span of strip B1. This is because, in the EFM, this frame was modeled with a second-type support on the right, meaning there is no continuity with the right cantilever. This distinct configuration of the structural system caused higher-intensity positive bending moments, leading to the development of tensile stresses in the lower fibers of this span.

5.2 Comparison of Deflections

Table 8 summarizes the deflections of frame B1. The column dif. (%) represents the percentage difference between deflections calculated by different methods compared to EFM, using deflection limits (e_{lim}) as a reference. According to ACI 318, the deflection limits (e_{lim}) in spans must equal L/240, and in cantilevers, equal L/120, where L is the span length in question. Positive deflection values indicate displacements in the gravitational direction.

Table 8 – Summary of relative deflection in service

		t = 0			t = long-term		
	METHOD	(mm)	dif.1 (%)	L / x	(mm)	dif.1 (%)	L / x
Span 2 e_{lim} = 23.9 mm L/x > 250	EFM	0.9		6378	2		2870
	FEM (SAP2000)	-1.0	-211%	5740	-2	-200%	2870
	FEM (FLOOR-PRO)	-1.0	-211%	5740	-3	-250%	1913

(1) dif.1 = | (x - EFM) / EFM |, while x is other methods results

The results indicate good convergence between methods, with all deflections within the ACI-318 limits.

In the second span of frame B1, EFM produced a deflection in the opposite direction to the other methods. This discrepancy results from modeling frame B1 with a simply supported right end, excluding the right cantilever. Consequently, the cantilever deflection was obtained from frame B, where it was modeled.

6 CONCLUSION

This study evaluated two calculation methods for post-tensioned slabs using a real residential building structure as a case study.

To better understand the structural behavior of the post-tensioned floor and verify EFM results, a comparative analysis was conducted using FEM with SAP2000 and FLOOR-PRO.

The analysis results showed good convergence, with small difference at the results of the design. Despite the irregular slab geometry, which contradicts practical recommendations for using EFM, the results remained satisfactory compared to FEM outputs.

SAP2000 and FLOOR-PRO produced similar results. However, using SAP2000 proved significantly more complex and labor-intensive than FLOOR-PRO. As a specialized post-tensioned slab software, FLOOR-PRO automatically calculates stresses and forces for the desired combinations, reducing the time required to define section cuts ("section cuts") in SAP2000.

Span deflections did not show significant differences.

Both calculation methods provided reliable results. In general, EFM demonstrated a more conservative tendency than FEM, which validates its results, considering FEM offers more precise calculations.

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