

Grover's Algorithm Using IBM's Fake Providers

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Abstract: In this work we will show Grover's algorithm using Fake Providers which are images of real quantum computers to give us an idea of their behavior in the implementation in Qiskit and we will show the fidelities with respect to the quantum simulator which gives us the result closest to the analytical one.

Keywords: Quantum Computing, quantum circuit, quantum algorithm.

1. Introduction

A significant milestone in the implementation of quantum computers was established by Bennett [1], who introduced the concept of reversible computing as a model. Furthermore, Toffoli [2] made a substantial contribution by developing a universal set of gates for this model in classical computation.

Shor's Algorithm [3] has garnered considerable interest within the academic community. It has proven capable of efficiently solving challenging problems, such as the Integer Factorization Problem and the Discrete Logarithm Problem, when executed on a quantum computer.

In a notable experiment [4], Grover's search algorithm was successfully implemented on a quantum system with four states. This represented an important step in practically demonstrating the efficiency of quantum algorithms compared to their classical counterparts.

This work is organized as follows: Section 2.1 discusses Grover's algorithm, Section 2.2 covers Fake Providers, Section 3 presents the implementation results, and Section 4 provides a brief conclusion of the work.

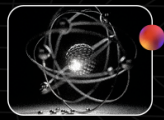
2. Preliminaries

In this section, we'll discuss Grover's Algorithm and Fake Providers. These are the essential tools we'll use to achieve the results presented in this work.

2.1. Grover's algorithm

Grover's algorithm can find the marked element with high probability using only $O(\sqrt{N})$ function evaluations, compared to classical search which would require $O(N)$ evaluations [5]. Grover's algorithm is a powerful tool in quantum computing, especially in search and optimization problems where efficiency is crucial.

The main barrier to demonstrating acceleration with Grover's algorithm is that the quadratic speedup achieved is too modest to overcome the



significant overhead of near-term quantum computers.

In Grover's algorithm, we consider a function $f: \{0, 1, \dots, N-1\} \rightarrow \{0, 1\}$ as input. In the "unstructured database" analogy, the function's domain represents the indices of a database, and $f(x) = 1$ if and only if the data pointed to by index x satisfies the search criteria. We assume that only one index, called ω , satisfies $f(x) = 1$, and our goal is to identify this index.

To access the function f , we use a subroutine, also known as an **oracle**, in the form of a unitary operator U_ω , which acts as follows:

$$U_\omega |x\rangle = -|x\rangle \text{ for } x = \omega, \text{ when, } f(x) = 1,$$

$$U_\omega |x\rangle = |x\rangle \text{ for } x \neq \omega, \text{ when, } f(x) = 0.$$

This uses the N -dimensional state space H , which is provided by a register with $n = \lceil \log_2 N \rceil$ qubits. This is often written as:

$$U_\omega |x\rangle = (-1)^{f(x)} |x\rangle. \quad (1)$$

In this algorithm, we use an N -dimensional state space H , which is provided by a register with $n = \lceil \log_2 N \rceil$ qubits. In this way, the operator U_ω acts on the quantum states represented by the qubits, applying a negative sign (-1) to the state $|x\rangle$ if $f(x) = 1$, and keeping the state $|x\rangle$ unchanged if $f(x) = 0$.

The **oracle** U_ω has its behavior defined by the equation:

$$U_\omega: |x\rangle|a\rangle = |x\rangle|a \oplus f(x)\rangle. \quad (2)$$

where $|x\rangle \in H^N$, $|a\rangle \in H^2$, and $\oplus$ represents the XOR operation.

Grover's algorithm produces the value ω with a probability of at least $1/2$ using $O(\sqrt{N})$ applications of U_ω . The **Grover matrix**, represented by G , is defined by:

$$G|x\rangle = \left(\frac{2}{N} - 1\right)|x\rangle + \frac{2}{N} \sum_{i \neq x} |i\rangle. \quad (3)$$

The **Grover evolution operator**, U_G , is given by:

$$U_G = GU_\omega \quad (4)$$

The algorithm's **initial condition** is:

$$|\psi_0\rangle = |G\rangle|-\rangle, \quad (5)$$

where $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ and

$G|x\rangle = \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} |u\rangle$. Grover's algorithm instructs us to iteratively apply U_G approximately $\lceil \frac{\pi}{4} \sqrt{N} \rceil$ times.

After these iterations, we measure the first register in the computational basis, and the result will be x with a probability greater than or equal to $1 - \frac{1}{N}$.

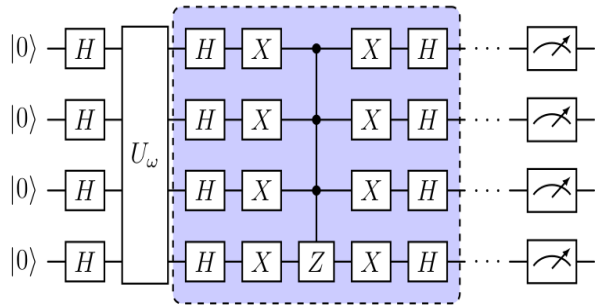
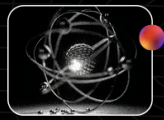


Figure 1. Grover's algorithm circuit for 4 qubits

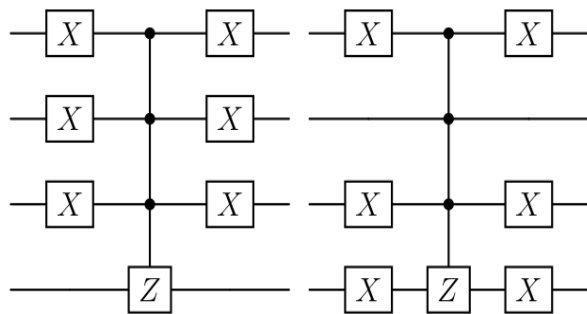


Figure 2. Circuits of the oracles of $\omega_1 = 0111$

and $\omega_2 = 1101$.

1.2. Fake providers

The *fake providers* module allows developers to create realistic simulations of IBM quantum computers without needing physical access to a real device. It does this by using "**snapshots**" of actual quantum systems, which capture information like qubit connectivity and the types of errors that can occur [6].

Imagine you're building a new program for a quantum computer. Instead of testing your program directly on a real device (which can be expensive and hard to access), you can use *fake providers* to create a simulation. This simulation will mimic the behavior of a real quantum

computer, including the common errors and inaccuracies found in these systems.

Why Use Fake Backends?

- **Testing and Development:** It lets you test and debug quantum algorithms before running them on a real device.
- **Learning:** It's an excellent tool for learning about quantum computing and understanding how errors affect quantum calculations.
- **Optimization:** It helps optimize quantum circuits before executing them on a real device.

Limitations

- **Accuracy:** Simulations might not be perfectly accurate, as the snapshots are from real systems that can change over time.
- **Complexity:** Simulating large-scale quantum systems can be computationally expensive.

While *fake providers* use snapshots of real quantum systems for its simulations, *qiskit-aer* offers a variety of simulators because it's a more general library. The *fake providers* system uses the most recent calibration results.

3. Results

We now show the results of the quantum simulation of Grover's algorithm for 4 qubits for two cases of 1 and 2 marked elements, $\omega_1 = 0111$ and $\omega_2 = 1101$. We will

compare it with 3 fake providers and finally present the fidelity of the results using the Hellinger metric $h^2 = \frac{1}{2} \sum_x (\sqrt{p_x} - \sqrt{q_x})^2$, where p is the probability distribution of the simulation and q is the quantum computer.

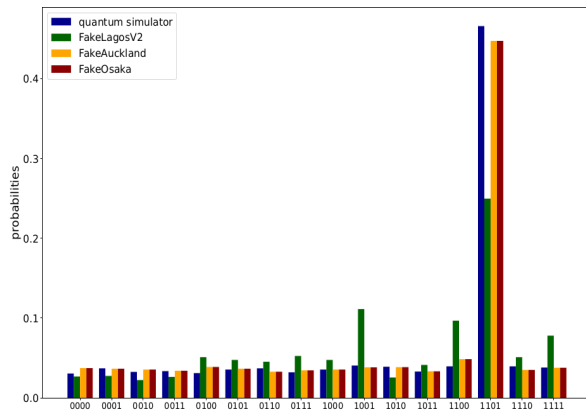
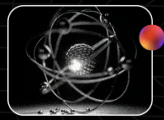


Figure 3. Probability distribution of Grover's Algorithm with 4 qubits and one marked element

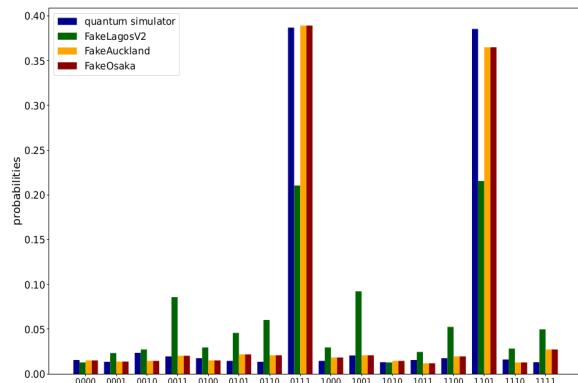


Figure 4. Probability distribution of Grover's Algorithm with 4 qubits and two marked elements

n° marked	<i>Fake LagosV2</i>	<i>Fake Auckland</i>	<i>Fake Osaka</i>
1	0.958	0.999	0.998
2	0.922	0.996	0.993

Table 1. Fidelity of Grover's algorithm with 4 qubits, using IBM's Fake Providers.

As we can see in Figure 3, *Fake Osaka* and *Fake Auckland* yield quite similar results. Notably, *Fake LagosV2*, despite having the lowest fidelity value, still provides good outcomes. This demonstrates that *Fake Providers* exhibit

behavior akin to real quantum computers. In Figure 4, we observe similar behavior, specifically for the case of two marked elements. Furthermore, Table 1 shows the fidelities of these providers when compared to the quantum simulator.

As mentioned, with these backends, it's possible to implement our quantum models and see how they behave. With these, we could observe, for example, quantum superiority in certain problems. Another useful feature would be that, since they already include the error of real quantum computers, we can use this as an exercise to reduce circuits through equivalences, improving our results and using this as a reference for future developments.

We also have to take into account the high cost of using real quantum computers today (with 10 minutes of free monthly use on IBM's QPUs), which limits research and development in this area.

4. Conclusions

With these results, we can see that Fake providers are a tool for observing the behavior of quantum computers to determine if our algorithm implementations are performing as desired.

Although these are only snapshots of real quantum computers, most of which have already been retired, they show us how these computers have evolved over the years and that the error rate is decreasing, leading to the possibility of one day having a real, logical quantum computer.

Acknowledgement

This work was entirely funded by Project QIN-AFCCT-FC-CC-2024-6-11-1, supported by QuIIN-Quantum Industrial Innovation, the



EMBRAPII CIMATEC Competence Center in Quantum Technologies. This funding was secured through MCTI Call number 053/2023, established with EMBRAPII.

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