Paying by waiting in line: Public sector queuing and the public sector wage gap

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Abstract

We show that public sector jobs in Brazil are characterized by price and quantity controls in the form of wages larger than those of private sector counterparts but with a limited number of employment contracts (analogous to a quota). Entrance to public sector jobs is decided according to the results of a double-blinded admission exam. The resulting combination prevents the usual price mechanism from equating the value of supplying labor to private or public sector jobs. We show that the equilibrium mechanism operates through increases in the candidate-to-vacancy ratios that reduce the likelihood of success at any attempt to access a public sector job. In our empirical analysis, we look at exams administered between 2007 and 2017. We show that the value of time spent waiting actually exceeds the average gain, dissipating all rents that would otherwise be generated by the public wage premium.

1 Introduction

Government employees make up a large portion of the labor market. Hiring, firing, promotion, and wage setting in government jobs operate in a specific manner that is usually unique to those jobs. Government jobs are usually characterized by low unemployment risk, stronger unions, and somewhat predictable career progression. Employment in the public sector is a-cyclical, meaning that it does not grow much during the boom of the business cycle, but it also does not shrink during the bust.

In developing countries, there is some evidence that public sector jobs have a wage premium. This is somewhat puzzling since the lower unemployment risk should suggest that those jobs would be typically priced at a discount in a standard Hedonic equilibrium model. In this paper, we argue that the combination of wage posting – a form of a price control with a subsidy – from the employer perspective (there is no bargaining of job conditions at the point of hiring); combined with a quantity restriction – the government sets the quantity to be demanded in a way that is independent of the actual supply of laborers to the position – yields a market that is characterized by large surpluses of labor supply to government jobs.

The assignment of workers to these jobs is set by the outcomes of admission exams. These exams are designed to test workers in some of the abilities required to perform the jobs – such as reading, writing, law, accounting, etc. These exams are graded in a double-blinded manner, and candidates are by and large ranked solely on the outcomes of these tests. Employment contracts are offered to those with the best scores in the admission exams.

We argue in this paper that the combination of characteristics of public sector jobs (externally set wages, positive wage premium, and quantity restrictions) leads to an oversupply of workers to the public sector career. Given that the entry to this career is decided according to the rankings in a high-stakes admission exam, workers spend real time and effort preparing for these exams, competing with others for the shot of supplying labor to the public sector. The entrance point of the public sector gets crowded, and the tightness (the ratio of vacancies to applicants) shrinks until the labor market reaches an equilibrium. This equilibrium is characterized by an approximate equalization between the monetary net present value of supplying labor to the private sector and the monetary net present value of supplying labor to the public sector only after paying the equilibrium price: time spent waiting in line to access the public sector jobs.

We use an unique dataset of public sector admission exams that includes data on the wages of thousands of admission exams administered in Brazil from 2007 to 2016. The dataset includes data on the educational requirements of the job, the number of vacancies, and the scores of each individual who took the test. The panel nature of the dataset allows us to see whether individuals took multiple tests over the course of a large number of years, which allows us to see how long individuals usually take to enter the public sector career.

In our empirical analysis, we estimate a series of regression models that are derived from our theoretical work. Our theory predicts that the tightness should have a unitary elasticity with the value of the job. In the simplest form of the model in which workers are risk neutral and do not value the other amenities of public sector employment, this implies that the coefficient of a regression of the logarithm of the number of candidates on the posted logarithm of the wage should be one. We find that even the simplest version of the model provides an excellent fit to the data.

Our results also allow us to calculate how much of the nominal differential between the private sector value of the career and the value of the public sector career remains after accounting for the queue at the point of entrance in the public sector. Our results show that likely all of the potential career benefits of supplying labor to the public sector are dissipated in the queue, leading the value of public sector jobs to be not that different from private sector jobs. This value dissipation process comes from workers taking advantage of obvious arbitrage opportunities in the labor market. The queue length increases for jobs that pay relatively more, making them proportionally harder to access, up to the point where value is equalized across jobs.

2 Literature Review

The existence of a wage gap between the public and private sectors is well documented in many countries.[1] As noted in Schager (1993), these gaps often feature a "double imbalance," in which the lowest wages are greater in the public sector while the highest wages are greater in the private sector. Although countries in the former Eastern bloc are prominent exceptions (Lausev 2014; Danzer 2019), in general, average wages are found to be much higher in the public sector (e.g. Bender 2003). In particular, Araujo (2020) finds that the average public sector premium in Brazil (the focus of this paper) is approximately 48% and is decreasing in educational attainment.[2]

If a country's public sector overpays for its labor (relative to comparable private sector positions) then it will face multiple problems related to resource misallocation. Quadrini and Trigari (2007) find theoretically that misallocated public sector labor can increase the volatility of both employment and output at a macro level. In Brazil, Glomm, Jung, and Tran (2009) estimate that the generosity of public pensions cost nearly 3% of GDP annually through early retirements. Albrecht, Robayo-Abril, and Vroman (2019) and Pousada and Ulyssea (2017) each calibrate models with heterogeneous agents (using Colombian and Brazilian data, respectively) and find that in each case misallocation arises from sorting based on education, skills, and risk preferences. In a similar model, using Brazilian data, Cavalcanti and Santos (2021) find that adverse sorting across sectors specifically costs 11.2% of annual output. While some effects are estimated near zero (e.g. Dos Reis and Zilberman 2013), since the public sectors of many countries are quite large (17.9 % average in OECD countries) even small deviations from efficiency in their labor markets can be of first order importance (OECD 2021).[3]

Any employer, public or private, would seem to have an incentive to minimize costs. Why then might public sector labor markets have inefficiently high wages? Some of the oldest accounts favored rent seeking by bureaucrats (Barro 1973), efficiency wages (Stiglitz 1984), or unionization (Gregory and Borland 1999) as explanations. While undoubtedly possible, each has faced both theoretical and empirical challenges.[4] More recent explanations emphasize the insurance value of a (presumably more stable) government job (Rodrik

1. See Gregory and Borland (1999) for a review
2. Evidence for negative selection bias is also found, with OLS specifications only picking up about half of this premium
3. For an overview of the macroeconomic misallocation literature, see Restuccia and Rogerson (2017), who note that government wage policy is an important source of such misallocation which can significantly slow macroeconomic growth.
4. The range of the observed gaps, the potential for Rosen (1986) compensating differentials through both soft factors such as "mission-oriented" motivation (Besley and Ghatak 2005) and fringe benefits (Danzer 2019), as well as the varying degree of unionization are all make it difficult to explain the public wage gaps through these lenses alone.

2000), search frictions (Montgomery 1991), and the potential for Roy (1951) sorting across skill levels or (Weiss 1980) ability discrimination between the public and private sectors (Depalo 2018).

If a gap exists in total compensation between the public and private sectors, then a corresponding excess of labor supply should be induced as workers attempt to capture these rents. This will be observable as a queue, or a persistent ratio of applicants per job[5] greater than one (Krueger 1988; Holzer, Katz, and Krueger 1991). Mohanty (1992) finds such a queue for unionized jobs, both public and private, in the U.S. Mengistae (1999) finds evidence of public job queues in Ethiopia and Hyder (2007) finds similar evidence in Pakistan. In a government recruiting experiment in Mexico, Bó, Finan, and Rossi (2013) are actually able to observe the increasing relationship between queue length and the posted wage for a randomized set of public jobs, confirming the theoretical prediction.[6] Finally, Mangal (2021) finds that a partial public sector job freeze in India caused a 30% increase in unemployment among likely test-takers[7] which was not associated with an increase in general human capital (suggesting that studying for public sector job applications is an economically unproductive activity).

3 Model

3.1 Sector Choice

We will assume that workers take market wages as given and choose where to supply labor. There are two sectors: public and private. Private sector wages are denoted by w_0 and public sector wages are denoted by $w_1 \equiv (1+\delta)w_0$, where $\delta > 0$ is the public sector wage premium. Workers have at their disposal only the choice of sector $S \in \{0, 1\}$. Their goal is to maximize utility:

$$S^* = \arg\max_{S \in \{0,1\}} E[SU(w_1) + (1-S)U(w_0)]$$

The solution to this problem is given by:

$$S^* = \mathbb{1}\{E[U(w_1)] > E[U(w_0)]\}$$

That is, workers will choose the public sector if, and only if, the expected utility of working in the public sector is larger than the expected utility of working in the private sector. We also assume that workers are risk-neutral and care only about the wage and employment prospects of each choice. Thus:

5. Note: this is the inverse of "tightness"

6. A 33% increase in wages led to a 26% increase in applications, which yields an elasticity estimate of 0.787, quite close to our estimates in this paper.

7. India has an exam-based public sector job allocation system that is quite similar to Brazil's

$$E[U(w_s)] = p_s w_s$$

where p_s is the probability of finding a job when choosing sector s to supply labor.

If both sectors have employment in equilibrium, then it must be the case that $p_1 < p_0$. Furthermore, the higher the public sector wage premium, the lower must be the chances of finding a job there. Note that in equilibrium $E[U(w_1)] = E[U(w_0)]$, so workers are indifferent between supplying labor to the public and of the private sectors. Using the equilibrium condition, we obtain:

$$p_1 w_1 = p_0 w_0$$

$$\frac{p_1}{p_0} = \frac{w_0}{w_0(1+\delta)}$$

$$\log(p_1) - \log(p_0) = \log(1) - \log(1+\delta)$$

$$\textit{\%Difference in employment probabilities} = -\textit{\% Difference in Wages}$$

3.2 Asset pricing

We can also calculate how many years of earnings workers are willing to sacrifice in exchange for a public-sector job. Assume that workers discount time at rate r (and thus have a discount factor $\beta = 1/(1+r)$. Assume further that private and public-sector wages are constant at, respectively, w_0 and $w_1 = (1+\delta)w_0$. Workers are paid at the end of every period, are perpetually young (live forever), and do not place any value in non-wage features of the job. Thus, the lifetime utility of taking a private-sector job is equal to:

$$V_0 = \sum_{t=1}^{\infty} \beta^t U(w_{0,t}) = \sum_{t=1}^{\infty} \beta^t w_{0,t} = \sum_{t=1}^{\infty} \beta^t w_0 = w_0 \left(\sum_{t=0}^{\infty} \beta^t - 1 \right)$$

$$= w_0 \frac{1}{1-\beta} - w_0 = w_0 + \frac{w_0}{r} - w_0 = \frac{w_0}{r}$$

Similarly:

$$V_1 = \frac{w_1}{r} = \frac{(1+\delta)w_0}{r} = (1+\delta)V_0$$

Thus, the value of a public sector job is the value of the private sector job multiplied by the public sector wage differential.

Now, the question we seek to answer is how many years of a private-sector job a worker would be willing to sacrifice to secure entrance to a public sector career. One approach to the problem is to look at the worker's willingness to pay for a public sector job, and then translate that number from dollars to time (using the worker's private sector wage as the conversion factor). A worker is willing to pay to enter the public sector career at most:

$$\textit{Willingness to pay} = V_1 - V_0 = (1+\delta)V_0 - V_0 = \delta V_0 = \delta \frac{w_0}{r}$$

We next ask how many years of work does it take a worker to obtain $\delta w_0/r$. This value is found by finding the period T such that:

$$\delta \frac{w_0}{r} = \sum_{t=1}^{T} \beta^t w_0 = w_0 \left(\sum_{t=0}^{T-1} \beta^t - 1 + \beta^T \right)$$

$$\Rightarrow \frac{\delta}{r} = \left(\frac{1-\beta^T}{1-\beta} - 1 + \beta^T \right) = \left(\frac{\beta - \beta^{T+1}}{1-\beta} \right) = \frac{1}{r}(1-\beta^T)$$

$$\Rightarrow \beta^T = (1-\delta) \Rightarrow T\log(\beta) = \log(1-\delta)$$

$$\Rightarrow -Tr \approx -\delta \Rightarrow T \approx \frac{\delta}{r}$$

Hence a worker is willing to sacrifice a number of years proportional to the public sector wage gap and inversely proportional to the discount factor.

One way to interpret this result is that if there were a literal queue to enter the public sector, and workers were to get a job there on a first-come, first-served basis, then workers would be willing to enter the queue for the public sector job as long as the length of time queuing was lower than δ/r years. In other words, whenever the queue length was lower than δ/r, workers in the private sector would start queuing and the length would increase. Conversely, whenever the queue was found to be higher than δ/r, workers at the end of the queue would stop waiting for a public job and take jobs in the private sector. As a result δ/r is the only queue length that can prevail in equilibrium.

3.3 Combining these two

Workers can, in every period, decide whether or not to supply labor to the private or public sectors. A worker can always find a job in the private sector, but can only find a job in the public sector with probability p (determined endogenously). Assume further that wages are fixed over the course of the worker's career, that there are no job separations, that workers care only about salary and that workers are perpetually young.

A worker's choice set is simply the sector in which he decides to supply labor at every period. If the worker decides to supply labor in the private sector, he gets:

$$V_0 = \frac{w_0}{r}$$

If the worker decides to supply labor in the public sector, he gets a value of:

$$V_s = p\beta V_1 + (1-p)(0 + \beta V_s)$$

$$rV_s = 0 + p(V_1 - V_s)$$

where V_s is the value of *searching* for a job in the public sector, and V_1 is the value of *obtaining* a job in the public sector. Here we use the fact that while studying/waiting to get a public sector job the worker gets no wage and that the chance that the worker finds a job in every period is given by p.

The worker's decision at every point is whether or not to move to the public sector. In equilibrium, if both sectors have positive labor supply, it must be the case that:

$$V_s = V_0$$

Now, we have the following system of equations that determine p:

$$V_s = V_0$$

$$V_1 = (1+\delta)V_0$$

$$(r+p)V_s = pV_1$$

Solving for p, we obtain:

$$(r+p)V_0 = pV_0(1+\delta) \Rightarrow r+p = p+p\delta$$

$$\Rightarrow p = \frac{r}{\delta} \tag{1}$$

At every period some workers will be lucky and find a job, while others won't. The distribution of the time it takes to get a job will be geometric, with a probability of success equal to r/δ. The expected duration of the unemployment while queuing for a public sector job is given by the inverse of the chance of success. Thus:

$$\textit{Expected unemployment duration} = E[T] = \frac{\delta}{r} \tag{2}$$

where T is the random variable that denotes the unemployment duration, or "waiting time." This expression coincides exactly with the formula for the waiting time based on the asset pricing equations. The only difference is that, here, it does not hold deterministically. Instead, it holds in expectation: Workers expect in equilibrium to wait for a public sector job *precisely* the exact amount of time they would be willing to sacrifice if they were asked to do so. Thus, the queue acts as a compensating wage differential, establishing the ex-ante equality of utilities across different career options.

3.4 The value of employment insurance

Public and private sector jobs differ not only in terms of their monetary compensation. On top of wage differentials, public jobs also have a different package of job amenities. The most salient of them is essentially guaranteed job safety.[8] Private enterprises can fire workers more or less freely, which diminishes the expected value of a private-sector job.[9]

In this section, we characterize the market value of public-sector employment insurance. This allows us to place a monetary value on the most salient job amenity that differentiates public and private sector jobs.

The setting is the same as before. However, now at every period, private-sector workers have a chance s of being separated from their jobs. When a job is destroyed, a private-sector worker moves to the pool of unemployed, where he starts searching for a job, which he finds with a probability p. The value of employment and unemployment in the private sector are then given by:

8. Public-sector workers can be fired without cause in the first 3 years on the job—which very rarely happens—and can only be fired after that period under specific circumstances as described in the law.

9. By that we mean, they might have to pay firing cost fee, but firing is an option that is still often exercised.

$$V_{e,0} = \frac{w_0}{1+r} + \frac{s}{1+r}V_{u,0} + \frac{1}{1+r}(1-s)V_{e,0}$$

$$V_{u,0} = \frac{b}{1+r} + \frac{p}{1+r}V_{e,0} + \frac{1}{1+r}(1-p)V_{u,0}$$

Thus, we have that:

$$rV_{e,0} = w_0 + s(V_{u,0} - V_{e,0})$$

$$rV_{u,0} = b + p(V_{e,0} - V_{u,0})$$

After some algebra, it can be shown that:

$$rV_{e,0} = \omega_e w_0 + (1-\omega_e)b$$

$$rV_{u,0} = \omega_u w_0 + (1-\omega_u)b$$

where $\omega_e = \frac{p+r}{p+r+s}$ – thus $(1-\omega_e) = \frac{s}{p+r+s}$ and $\omega_u = \frac{p}{p+r+s}$ – thus $(1-\omega_u) = \frac{r+s}{p+r+s}$.

These expressions show that the value of a private-sector job $V_{e,0}$ and the value of searching for a job in the private sector market $V_{u,0}$ are a weighted average of the flow value of a job in the private sector w_0 and the flow value of unemployment in the private-sector market, which is given by the value of time at home/unemployment benefits b.

The weights depend on how often the worker is expected to stay in each of the states of the world (employed and unemployed). The larger is the job separation rate s, and the lower is the job-finding rate p, the higher is the weight put on the unemployment benefit b, and the lower is the weight placed on the private sector wage w_0. In other words, the harder it is to find or to keep a job in the private sector, the higher it will be the discount associated with the wage in that job. This is perhaps easier to see when b is equal to zero. In that case, workers perceive in expectation that a private sector employment contract displays a wage times a discount factor, which marks down the wage due to the risk of unemployment. Another useful special case to note is that when r is very small relative to p and s, the weights reduce to the fraction of time the worker is expected to stay in each state since in this model the fraction of periods the worker is expected to find himself employed is given by $\frac{p}{p+s}$ and the fraction of periods that the worker is expected to find himself unemployed is given by $\frac{s}{s+p}$.

It is immediate from these results that, given knowledge of b, s, and p, one can immediately find the value of the public sector's employment insurance. One way is to look at a worker's willingness to pay for such an amenity. That is, how much would a worker be willing to spend on setting s to zero, which is essentially inducing employment insurance on the private sector job. The answer to this question is identical to the difference between the value of a private-sector job when ω_e is set to be equal to one. That difference is going to be larger when s is large and when p is small.

3.5 Accounting for differences in returns to tenure

The public sector wage differential isn't necessarily constant over the course of a worker's career. In this section we show how to account for such differences in our analysis.

Let $w_{0,t}$ be the wage a worker earns in the private sector at time t, and let $w_{1,t}$ be the wage a public sector worker earns at time t. Let g_0 be the per period growth rate of wages in the private sector and g_1 be the growth rate of wages in the public sector. Let $w_{1,1}/w_{0,1} = (1+\delta)$ be the baseline difference in wages at the beginning of the worker's career. Then:

$$V_0 = \sum \beta^t U(w_{0,t}) = \sum \beta^t w_{0,t} = \sum \beta^t w_0 (1+g_0)^t = \frac{w_0}{1 - \frac{1+g_0}{1+r}} = \frac{w_0(1+r)}{r - g_0}$$

Thus, as usual, wage growth over the course of a workers career will manifest itself as a lower "effective rate of time discount." Similarly, for workers in the private-sector, we have that:

$$V_1 = \frac{w_1(1+r)}{r - g_1}$$

As a result, the difference between the value of a job in the public sector when compared to the private sector is given by:

$$\frac{V_1}{V_0} = \frac{w_1}{w_0}\frac{r - g_0}{r - g_1}$$

$$\frac{V_1}{V_0} = (1+\delta)\frac{r - g_0}{r - g_1}$$

The equation above shows that, as long as $g_1 = g_0$, the difference between the value of a job in the public sector when compared to a job in the private sector is still given by the wage differential at the outset of the worker's career $(1+\delta)$.

However, in the general case in which the rates g_1 and g_0 differ, then the public sector premium will have one component associated with differences in level (δ) and another one associated with the difference in the slope ($(r - g_0)/(r - g_1)$).

Using a Taylor approximation, we obtain:

$$\log \frac{V_1}{V_0} = \delta + \frac{g_1 - g_0}{r}$$

Thus, the difference between the value of a public sector job is, in percentage terms, given by the difference in levels δ, plus the difference in wage growth, normalized by the worker's impatience r.

3.6 Finite Careers

Assume all workers enter the labor market at time zero, and retire at $\bar{T}$. Here, we analyse the effects of a finite career on the results we derived thus far. For simplicity, we go back to the setting with no job destruction and no wage growth over the life-cycle.

$$V_0 = \sum_{t=1}^{\bar{T}} \beta^t U(w_{o,t}) = \sum_{t=1}^{\bar{T}} \beta^t w_0 = w_0 \sum_{t=1}^{\bar{T}} \beta^t = w_0 \left[\left(\sum_{t=0}^{\bar{T}-1} \beta^t \right) + \beta^{\bar{T}} - 1 \right]$$

$$= w_0 \left[\frac{1 - \beta^{\bar{T}}}{1 - \beta} + \beta^{\bar{T}} - 1 \right] = w_0 \frac{1}{r}(1 - \beta^{\bar{T}})$$

Similarly, for the public sector job, we have:

$$V_1 = w_1 \frac{1}{r}(1 - \beta^{\bar{T}})$$

Thus,

$$V_1 = w_0(1 + \delta)\frac{1}{r}(1 - \beta^{\bar{T}})$$

This implies that:

$$\frac{V_1}{V_0} = (1 + \delta)$$

Thus, whether or not the worker's career is finite plays no role in the relative value of public versus private jobs. It is, in both cases, given by $(1 + \delta)$, the public sector wage differential. However, in absolute terms, it matters. Note that:

$$V_1 - V_0 = w_0(1 + \delta)\frac{1}{r}(1 - \beta^{\bar{T}}) - w_0 \frac{1}{r}(1 - \beta^{\bar{T}})$$

$$V_1 - V_0 = \delta \frac{1}{r}(1 - \beta^{\bar{T}})$$

It is not quite straightforward to derive how close a worker must be to the end of his career so to be not worthwhile to enter the queue to join the public sector.

It pays to simplify the model a little and assume that workers do not discount time, so $r = 0$ and $\beta = 1$. In this case, the worker lifetime utility is the product of the number of years working times the wage in the chosen sector. Thus:

$$V_1 = n_1 w_1$$

$$V_0 = n_0 w_0$$

Thus, the worker will be better off by joining the queue to enter the public sector if, and only if $V_1 > V_0$, which implies:

$$n_1 w_1 > n_0 w_0$$

Or, alternatively:

$$\frac{w_1}{w_0} > \frac{n_0}{n_1}$$

Thus, defining ζ to be the percentage decrease in the length of the worker's career associated with queuing for the public sector job, we have that:

$$\textit{Percentage increase in Wages} > \textit{Percentage Decline in career length}$$

Or, in other words, it must be the case that $\delta > \zeta$ for the worker to choose the public sector. It is immediate from this expression that workers closer to retirement will perceive a greater decline in their career lengths with the same amount of expected wait to enter the public sector. Thus, the older is the worker, *ceteris paribus*, the less likely it is that he will find investing in entering the public sector a worthwhile enterprise.

3.7 The price of anarchy

As long as public sector workers are paid at a premium, there is an economic surplus to be fought for between workers in this labor market. However, part of this surplus is lost due to the time workers spend attempting to get into the public sector. In this section we discuss how much of this surplus is lost, and what would be the workers' welfare if they could coordinate their actions. In the decentralized equilibrium, a fraction θ of workers end up in the private sector, whereas the remaining fraction $(1-\theta)$ end up in the public sector. Thus, the average expected welfare of workers is given by:

$$\textit{Expected Welfare} = \theta V_0 + (1-\theta)V_s = \theta V_0 + (1-\theta)\frac{1}{r}p(V_1 - V_s)$$

$$= \theta V_0 + (1-\theta)\frac{1}{r}p(V_1 - V_0) = \theta V_0 + (1-\theta)\frac{1}{r}p\delta V_0 = V_0$$

where in the second-to-last equation we use the fact that, in equilibrium, p is equal to $\frac{r}{\delta}$.

This implies that, in the decentralized equilibrium, the expected welfare of workers is equal to the welfare in the private sector. Thus, all the surplus associated with the presence of the public sector premium is dissipated by the congestion externalities workers impose in one another when queuing to enter the public career.

How much surplus is lost? To answer this question, we must find the maximum of workers' welfare that can be obtained in the presence of the public sector premium. One simple way to look at this is to think that workers could coordinate their actions through a chain of contracts. It is straightforward to see that the maximum welfare of workers that can be obtained is given by, where η is the share of total jobs in the public sector:

$$\textit{Centralized Welfare} = (1-\eta)V_0 + \eta V_0 + \eta\delta V_0 = (1-\eta)V_0 + \eta V_1 = (1+\eta\delta)V_0$$

Comparing workers' welfare in the decentralized equilibrium and the one that could be obtained if worker's were able to coordinate their actions, we get the price of anarchy. The decentralized equilibrium costs workers an amount that is proportional to the size of the public sector (relative to the number of workers) and to the public sector wage premium. In other words, all of the potential surplus from the public sector premium is lost due to the workers' inability to coordinate their actions to limit the queue's length.

3.8 Roy Heterogeneity - Wages

So far, we have assumed away heterogeneity in wages in both the public and the private sectors. The unique price for labor in the public sector is appropriate since the wages are set regardless of which worker happens to take the job. However, the same assumption is unappealing for the case of private jobs. Here, we study the model's equilibrium in the presence of wage heterogeneity in the private sector.

Workers are characterized by a public sector wage w_1 and a private sector wage $w_0 = we^{\epsilon}$, where ϵ is a mean-zero wage heterogeneity component in the private sector – which can be thought of as a combination of luck, human capital investments, and innate ability–, and w is a baseline private sector wage. Note that workers with ϵ greater than zero earn more than w and workers with ϵ smaller than zero are less than w. Now, let $\gamma \equiv log(w_1) - log(w)$, that is, γ measures the strength of the *average* public sector premium.

In the presence of wage heterogeneity in the private sector, not all workers will find it worthwhile to search or wait for a job in the public sector. It is then useful to consider the fraction of workers for which the public sector is worth considering, depending on the size of the queue. In the presence of equal probabilities of finding a job in both sectors, that is, in the absence of a queue, a worker will choose the public sector if, and only if:

$$\log(w_1) > \log(w_0)$$

$$\log(w_1) > \log(w) + \epsilon$$

$$\epsilon < \gamma$$

Thus, the fraction of workers that would choose the public sector in the absence of a queue is given by $Pr[\epsilon < \gamma] = F_e(\gamma)$. Under the assumption that unobserved heterogeneity in log-wages in the private sector is normally distributed with mean zero and variance σ^2, we have that $Pr[\epsilon < \gamma] = \Phi(\frac{\gamma}{\sigma})$. Thus, a worker will consider joining the public sector only if his employment prospects in the private sector are below a threshold. Moreover, we can see that there are two key components that determine the fraction of workers that might choose to enter the public sector: The first is the average public sector premium γ, and the second is the size of wage heterogeneity σ. The higher wage heterogeneity in the private sector, the lower the fraction of workers that may choose to enter the public sector.

In the discussion that follows, we assume that the mass of workers for which this is the case is larger than the number of jobs in the public sector, so that in equilibrium there is rationing of jobs in the public sector.

Now, in the presence of a queue, workers need to consider the trade-off between finding a job immediately in the private sector, versus wasting time but finding a better job in the public sector.

Before we look at equilibrium objects, it is worthwhile to consider the worker's willingness to wait for a public sector job. Recall from our previous discussion that the worker's willingness to wait for a public sector job is the number of years a worker is willing to queue to secure employment in the public sector. In the presence of wage heterogeneity, this object is going to be different for different workers. The worker's willingness to wait for a public sector job is the value of T_i that solves:

$$\beta^{T_i}(1+\gamma)\frac{w}{r} = \frac{w_i}{r}$$

$$T_i log(\beta) + \gamma + log(w) = log(w) + \epsilon_i$$

$$T_i = \frac{\gamma - \epsilon_i}{r}$$

Thus, a worker's idiosyncratic willingness to wait for a public sector job is increasing in the average public sector premium, decreasing in the worker's impatience, and decreasing in his private sector earnings potential ϵ. Note that the expression above is identical to the willingness to wait we derive in the absence of wage heterogeneity in the private sector, except for the ϵ_i term. Thus, it follows that the expected willingness to wait across the population of all workers is:

$$E[T_i] = \frac{\gamma}{r},$$

which is identical to the expression we obtained before.

A worker will decide to enter the queue for a public sector job if, and only if, the expected wait for a public sector job is lower than $\frac{\gamma - \epsilon_i}{r}$. That is, if the expected wait for a public sector job is lower than the maximum wait that he would consider acceptable to secure public employment.

The equilibrium in this model is characterized by the following conditions: (i) Given the (equilibrium) probability of finding a job in the public sector p and each worker's wage heterogeneity component ϵ, all workers choose the sector that yields the highest expected utility. Given the number of jobs in the public sector and the number of workers that choose the public sector, the probability of finding a job in the public sector is equal to p.

Now, we are going find the expressions that characterize the equilibrium. It is useful to note that if, in equilibrium, a worker with a value of ϵ equal to ϵ^* is going to the private sector, then *all* workers with $\epsilon_i > \epsilon^*$

must also choose the private sector. Thus, it is sufficient to search for the marginal worker, the worker with the value of ϵ that makes him indifferent between the two sector choices. The condition that characterizes such worker is:

$$E[U(S=1)] = E[U(S=0)]$$

$$pw_1 = w_0 e^{\epsilon^*} \Rightarrow pw_1 = we^{\epsilon^*}$$

$$\Rightarrow p(1+\gamma) = e^{\epsilon^*} \Rightarrow \log(p) + \gamma = \epsilon^*$$

The expression above relates the equilibrium employment probability in the public sector p with the public sector premium γ. We can remove the endogenous value of p by expressing the employment probability p with the ratio of the number of jobs in the public sector η and the number of workers that, in equilibrium, attempt to get these jobs $n_1 = \Phi(\frac{\epsilon^*}{\sigma})$. Thus, we obtain:

$$\frac{\eta}{n_1}(1+\gamma) = e^{\epsilon^*}$$

$$\eta(1+\gamma) = e^{\epsilon^*} n_1$$

$$\eta(1+\gamma) = e^{\epsilon^*} \Phi(\frac{\epsilon^*}{\sigma})$$

The expression above implicitly characterizes the equilibrium value of ϵ^*. Although there is no closed-form solution for it,[10] it can be easily seen that there is a unique value of ϵ^* for which the equation above holds. This happens because the left hand side is a constant function of ϵ^*, whereas the right hand side is a monotonically increasing function of ϵ^* which tends to zero as ϵ^* goes to minus infinity and tends to plus infinity when ϵ^* goes to infinity. Thus, the RHS crosses $\eta(1+\gamma)$ only once.

The equilibrium value of waiting time characterizes the willingness to wait for the marginal worker, the worker for which labor market potential in the public sector and on the private sector coincide. As a result,

10. One way to arrive at a closed form expression for ϵ^* is assume that the distribution of ϵ is Gumbel with parameter α. Then, $F(\epsilon^*) = Pr[\epsilon < \epsilon^*] = e^{e^{-\frac{\epsilon^*}{\beta}}}$. Assuming also that private sector wages are given by $w_0 = we^{e^{\epsilon}}$, we find that the equilibrium value of ϵ^* associated with the marginal worker is given by:

$$\epsilon^* = \frac{\beta}{\beta - 1} \log \log \eta(1+\gamma)$$

the equilibrium waiting time is a lower bound on the willingness to wait for the infra-marginal workers that end up self-selecting to the public sector. Moreover, note also that in the homogeneous version of the model all rents are dissipated and, as a result, all workers end up with the same utility regardless of the sector that they work at. Here, only the marginal worker ends up with the same utility in both sectors. Thus, most workers in the public sector still manage to earn a economic rent from the public sector premium, although a fraction of the (potential) public sector employment rent is dissipated by the rationing of jobs (and corresponding queue) to enter the public sector.

3.9 Roy Heterogeneity - Skill

Now suppose that there is a second dimension of heterogeneity - "skill." Suppose also that public sector jobs have different wages and possibly different threshold scores that are required to secure admission to them. Skill influences how successful an individual is likely to be on an admissions exam, but does not necessarily determine it. Individuals will therefore choose to compete for public sector exams in order to maximize their expected utility, given that their expected waiting time will depend on both their skill and the wage premia, i.e.

$$\max_{\delta} \beta^{T(u,\delta)}(1+\delta)\frac{w_{00}}{r}$$

where u indicates skill level and w_{00} is a reference private sector wage, a normalization. Test taking skills are also valued in the private sector markets, since some of the productive capacities that make a worker excel at a high-stakes test might also be used in some sectors of production. The problem above simply states that the choice of which exam to take must be made optimally. That is, when the public sector offers a multiplicity of different wages for different positions, individuals must choose not only whether or not to attempt to enter the public sector but also which particular position in the public sector career to aim for.

The function T has two arguments: It depends on skill level u. This captures the fact that there is heterogeneity in terms of test-taking abilities and that will affect how fast a worker can reach the scores associated with successfully securing a public sector job. Individuals with a lower u will take more time to access any job with a given δ; individuals with a higher u will have to wait less to reach any job with a fixed level of δ. The function T also depends on δ since different δ will have different equilibrium score thresholds that would be required to ensure admission to a particular job. In other words, T is decreasing in skill level u and increasing in δ.

The first order condition for the optimal choice of δ is

$$\frac{\partial T}{\partial \delta} ln(1+r) + \frac{1}{1+\delta} = 0$$

$$\frac{\partial T}{\partial \delta} = \frac{1}{(1+\delta) ln(1+r)} \approx \frac{1}{(1+\delta) r}$$

A simple function that satisfies this condition is $T(u,\delta) = -(\theta_0 u\delta - \theta_1 \delta^2/2) + ln\big((1+\delta)r\big)/r + m(u)$ for some function $m(u)$ and constants θ_0, θ_1 such that $\theta_0 u = \theta_1 \delta^*(u)$ at the optimum. This is a reduced-form representation of the relationship between time-to-access different jobs and the pair of wages associated with the job and skill level of the worker. The key condition that is required for sorting in equilibrium is the interaction term. It ensures that the costs of accessing jobs with higher wages rise for all workers regardless of skill, but it rises slower for high-skilled workers. This can be micro-founded from different concavities in the production technology of test scores of high and low-skilled workers, but for our purposes, we just need to state that T rises with delta for all u, but it has a smaller derivative with respect to delta for higher values of u. In equilibrium workers are indifferent between sectors so it must be the case that

$$\frac{w_0(u)}{r} = \beta^{T(u,\delta^*(u))} \big(1+\delta^*(u)\big) \frac{w_{00}}{r}$$

i.e., that expected net present values are equal. Using the fact that skill is positively associated with the private sector wage, $w_0(u)$, according to some function $g(u)$, we have

$$\frac{w_{00}\big(1+g(u)\big)}{r} = \beta^{T(u,\delta^*(u))} \big(1+\delta^*(u)\big) \frac{w_{00}}{r}$$

$$g(u) = -rT(u,\delta^*(u)) + \delta^*(u)$$

$$T(u,\delta^*(u)) = \frac{\delta^*(u) - g(u)}{r} = \frac{\theta_0 u}{\theta_1 r} - \frac{g(u)}{r}$$

Waiting times can therefore be any function satisfying these conditions. Despite not pinning down exact functional forms, they nevertheless ensure 1) individual rationality in the choice of exam and 2) no arbitrage remaining for any skill level. Note that the observed relationship between waiting times and wages is a lower envelope of the waiting time functions for each skill level. In equilibrium, individuals with higher skill will aim for public sector jobs with higher nominal wage $\delta^*(u) = \frac{\theta_0}{\theta_1} u$.[11] Despite accessing a job with higher

11. Interestingly, $\theta_0/\theta_1 - g'(u)$ is difference in the marginal returns to greater test-taking skills in the private sector, relative to the (entrance to the) public sector. It measures how much more, or less, the private sector values the abilities that makes an individual capable of securing high test scores (such as dedication, organization, memorization, mathematical reasoning,

pay in equilibrium, neither individuals with high or low skill will are able to obtain any rents, since the competition with others of similar skill level ensures that the waiting times for them will be such that the benefits of trying to secure admission to the jobs that they find the best to aim for is still such that the value of a private sector career is just as large.[12]

4 Costs of Admission

Thus far in our analysis of the optimal waiting times, we have only considered the opportunity costs of time that could have been used productively on the private sector. In reality, workers lose more than just foregone private-sector wages when they decide to search for a public sector job.

Public sector jobs admit workers based on the results of an admission exam. As a result, a significant part of the time that workers are queuing for a public sector job, they are deliberately practicing for an admission exam. Often, they pay for training classes specializing in preparing workers for these admission exams. Now, we are going to incorporate these costs, the costs of exam training and preparation, in our analysis.

Assume that during the period that the worker is queuing for a public sector job, he needs to pay a cost $\tilde{c}$ to obtain specialized instruction. Our goal is to derive the equilibrium waiting times, as a function of the public sector premium δ, the rate of time discounting r, and the costs of training $\tilde{c}$.

$$V_0 = \frac{w_0}{r}$$

$$V_1 = -\sum^{T-1} \beta^t \tilde{c} + \sum \beta^{T+t} w_1$$

In equilibrium, $V_1 = V_0$ and thus:

$$\frac{w_0}{r} = -\frac{\tilde{c}(1-\beta^{T-1})}{r} + \beta^T \frac{w_1}{r} \Rightarrow 1 = -c(1-\beta^{T-1}) + \beta^T (1+\delta)$$

language skills, etc.).

12. Note that no other public sector job is as good as $\delta^*(u)$, the optimal public sector job that the individual of skill level u might consider in equilibrium, and not even $\delta^*(u)$ yields any rent in equilibrium. That is, if an individual tries to overshoot to a better job he will end up taking too much time to access it relative to the pay, and if he tries to undershoot, he will obtain access faster than those who enter that job in equilibrium, but it will cost him relatively more than then since the same force that led you to be skilled enough to access the job in faster-than equilibrium time makes you pay a opportunity costs that is larger since that skill is somewhat priced in the private sector market as well. That is, his private sector wage $g(u)$ is high enough that the benefits of waiting to enter a job that pays less than δ^* are too low relative to the cost (waiting time) to access it.

$$\Rightarrow ln(1+c) = ln(\beta^T(1+\delta+\frac{c}{\beta})) \Rightarrow c = -Tr + \delta + \frac{c}{\beta}$$

$$\Rightarrow Tr = +\delta + c(\frac{1}{\beta} - 1) \Rightarrow T = \frac{\delta}{r} - c \tag{3}$$

Thus, the costs of training will reduce the time the worker is willing to wait in line for a public sector job. The exact form of this relationship is given by the equation 1 above. It is also insightful to look at equation one in a different lens. Note that:

$$T + c = \frac{\delta}{r}$$

The right-hand side is the monetary value of the gains to obtain a job in the public sector. This is, still, precisely how much a worker would be willing to pay – either by sacrificing his time through waiting or by sacrificing his resources through out-of-pocket payments, bribes, or any device to ensure entrance to the public sector.

What our result shows is that the worker will be paying in equilibrium exactly what the public sector job is worth. Before, he would do that by waiting in line. Now that there is a second component to the costs of entrance to the public sector, the *total* amount of resources that the worker will dispose to enter the public sector will still be $\frac{\delta}{r}$. However, a part of it will be going towards the training agencies – out of pocket payments –, and a part of it will be the worker will pay with his time. The sum of the costs paid with time and with money will add to the value of the public sector job.

There are a couple of implications of this result. The theoretical one is that we can, just as before, obtain estimates of the willingness to spend time trying to enter the public sector as long as we know both δ and r. Now, some part of this time will be spent actually waiting in line, and others will be spent paying for training resources. The fraction that is spent on training resources is going to depend on both the private sector wage w_0 and the costs of training.

This result has also implications for how to empirically estimate and test this theory. Moving c to the other side of the equation above, we have that:

$$T = \frac{c}{\delta} - c = \frac{\delta}{r} - \frac{\tilde{c}}{w_0}$$

Thus, in the presence of training costs, we will have an extra term in the equation that determines waiting times. Waiting times will be decreasing in the costs of training and increasing in the private sector wage. The reason that they are increasing in the private sector wages – which is a counterintuitive property at first glance – is that higher private sector wages implies that a lower number of hours must be spent to acquire the resources to finance the exam preparation ($\tilde{c}$), leaving a larger number of hours that can be spent waiting in line.

The equations above also show that in the presence of non-trivial admission training costs, there will be a unit elasticity of *total* waiting times and the public sector premium and (minus) the interest rate. When one look at the fraction of time which is spent out of the labor market waiting to get in, which is T and not $T + c$, then the unit elasticity result is gone. The elasticity will, however, still be close to one the smaller is c relative to the magnitude of δ over r.

5 Data and Empirical Methods

To test this theory, we collected data from two of the largest non-profit foundations which conduct entrance exams on behalf of the Brazilian government. All exams from 2007 to 2017 administered by these companies are represented, including such items as exam dates, individual IDs, test scores, and the final ranking. As described in Araujo (2020), this information was merged with manually collected public notices (*editais*) of public sector exams which provide additional job-specific contextual data.

Summary statistics for exams are displayed in Table 1. Note that there is wide variation in both wages and competitors per exam, with the maxima both orders of magnitude greater than the respective means. There are more exams requiring a college degree than a high school or no degree and jobs are concentrated at the state or municipal level of government. Figure 1 panel (a) depicts the geographic dispersion of jobs in the dataset across Brazil. These are clustered in the north and south-central, particularly in the state of Sao Paulo (SP).[13]

Table 2 shows summary statistics at the test-taker level. Here we can see that on average, test-takers are 31, have taken roughly two exams across two different years, and are slightly more likely to be male than female. Note that the average number of competitors is much higher than in Table 1, indicating significant concentration of exam competition. The jobs requiring a high school degree and those at the state level also seem to be the most popular. In panel (b) of Figure 1, we can see that, despite the clustering in location of the jobs, applicants for these jobs are distributed all across Brazil. This distribution compares favorably to the unconditional distribution of population across Brazil in 2014, depicted in panel (c).

Tables 3 through 5 show summary statistics conditional on wage terciles within each educational level. The number of competitors, top exam scores, and average years of testing are all clearly increasing with wages for non-degree jobs. Only the number of competitors and years of testing monotonically increase with

13. Exam-administration organizations are generally regionally-focused, which is the case here

wages for high school jobs. College jobs feature an inverse-U relationship with wages and all key metrics. This pattern is likely a consequence of heterogeneity in test-taking skills, where sorting across particular skill sets is more pronounced for higher educated individuals, who also, presumably, have greater skills and thus better options in the private sector.

Our theory predicts that if we could regress the log of the waiting time on the log of the appropriate wage differential for each public sector job, we would get a coefficient of exactly one. Instead, what we observe is the number of competitors, the number of posted vacancies, the posted wage, and the frequency with which each exam type appears over time. Since waiting time should be proportional to tightness, v/c, multiplied by the inverse of the rate of arrival of exams, after taking logs and rearranging, we get that:

$$\ln C = \ln(w_1) - \ln(w_0) + \ln V + \ln \lambda - \ln r \tag{4}$$

That is, we expect the number of test-takers to be increasing in the percentage difference between the posted wage for the position and the private sector wage, increasing in the number of slots, increasing in the arrival rate of tests (or decreasing in the time gap between exams for the same position), and decreasing in the interest rate. Importantly we expect, up to approximation errors, that these relationships have elasticities that are not too far away from one.

We do not observe the private sector wage directly, although we show below that there are several different ways to deal with that problem. Looking across pairs of exams undertaken by the same individuals, we can difference out the unobserved private sector wage. Looking at the difference in the number of test-takers across different exams for the same individual, we have that:

$$\Delta \ln C = \Delta \ln(w_1) + \Delta \ln V + \Delta \ln \lambda$$

That is, across exams in which we can argue that individuals have similar counterfactual private-sector wages, the term $\ln w_0$ is differenced out and can be ignored (or alternatively, absorbed by the constant).

In general, the term $\ln w_0$ can be treated as an unobservable, individual-specific, fixed-effect. In doing so, we arrive at:

$$\ln C = \ln(w_1) + \ln V + \ln \lambda - \ln r + \alpha_i \tag{5}$$

We, therefore run regressions of the following form:

$$\ln C_{ity} = \beta_0 + \beta_1 \ln W_{ity} + \beta_2 \ln V_{ity} + g(Freq_{ity}) + \mu_i + \lambda_y + \epsilon_{ity} \tag{6}$$

where i indicates individuals, t indicates exam periods, and y indicates calendar years. The function $g(\cdot)$ acts to correct for the possibility that exam timing might not be an i.i.d. process, which we approximate with a second order global polynomial. In the simplest setting in which each each job has the same inter-arrival rate of admission exams, then the number of other test takers is proportional to both the expected number of tests required to be admitted to that job, and the expected time required to be admitted to that job. When jobs with different wages also differ in how frequent admission exams are, then there is a possible important source of confounding. Two jobs might have the same number of test takers, but if one has a test frequency that is half of the other, then the individuals that attempt to access the former will take, on average, twice as long to be able to when compared to the latter. Individual fixed effects control for private wages and idiosyncratic test-prep costs.[14]

Table 1: Summary Stats: Exams

	Mean	SD	Min	Max
Year of exam	2012	2.7	2007	2016
Number test-takers	1,513	9,486	1	150,337
Hourly wage	18	14	2.2	155
Job requirements				
Education				
College	0.51	0.5		
High school	0.32	0.47		
Elementary	0.15	0.36		
Other				
Professional cert.	0.34	0.47		
Related experience	0.14	0.35		
Level of government				
Federal	0.034	0.18		
State	0.45	0.5		
Municipal	0.51	0.5		
Observations	7,066			

6 Results

Table 6 displays the main results of our empirical analysis. In the first column, we estimate our model without accounting for frequency or fixed effects (i.e. a random effects model). In the subsequent columns, we first add individual fixed-effect, then we add controls for the frequency of exams, then finally we add year fixed-effects. Individual fixed effects are appropriate here because the theory does not require that all jobs have a tightness-to-wage ratio that is constant; it instead requires that this is true for all jobs for which the worker is willing to take the test. Thus, the within-individual variation in the size of the

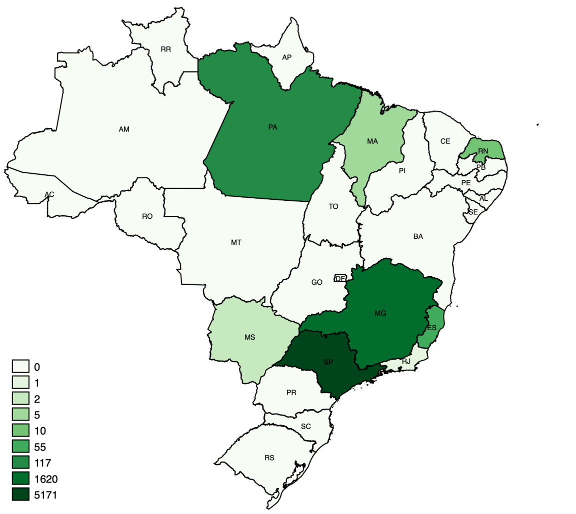
14. Multiple test-taking is extremely common, giving us a lot of within-individual variation

Table 2: Summary Stats: Test-Takers

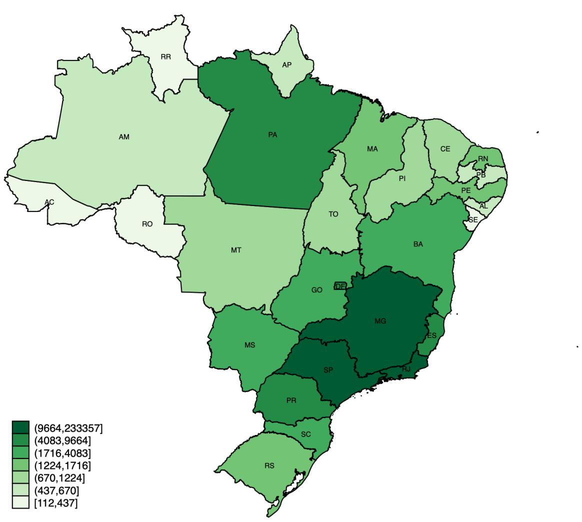
	Mean	SD	Min	Max
Year of exam	2012	2.8	2007	2016
Age	31	9.6	3	79
Num. tests taken	1.8	1.5	1	46
Num. years tested	2.1	2.1	1	10
Competitors	32,688	40,184	1	150,337
Final grade	40	26	0.088	488
Hourly wage	16	13	2.2	155
Classified	0.29	0.46		
Female	0.45	0.5		
Job requirements				
Education				
College	0.27	0.44		
High school	0.59	0.49		
Elementary	0.15	0.35		
Other				
Professional	0.097	0.3		
Relevant experience	0.037	0.19		
Level of government				
Federal	0.014	0.12		
State	0.68	0.47		
Municipal	0.31	0.46		
Observations	3,635,014			

Candidates who score above a job-specific threshold are considered "classified" and are invited for a second round of testing. This may be a demonstration of skills (e.g. driving test) or a second written exam. In either case, the final grade reported is either the sum of both exams if classified or the grade on the first exam if not.

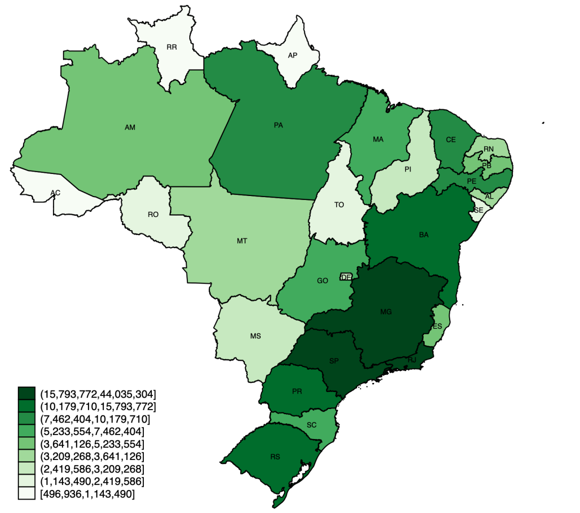
Figure 1: Spatial Distributions



(a) Location of Jobs Available



(b) Location of Test-Takers



(c) Population of Brazil, 2014

Table 3: Summary Statistics by Wage Tercile - Elementary

	1^{st}	2^{nd}	3^{rd}
Competitors	5,430.76	9,174.04	16,128.83
	(5,087.30)	(9,752.71)	(20,176.15)
Tightness	0.03	0.02	0.01
	(0.06)	(0.05)	(0.02)
Score threshold	108.34	93.51	89.12
	(51.64)	(26.76)	(31.83)
Score threshold (adjusted)	9.17	11.04	15.00
	(3.15)	(2.46)	(3.45)
Max. score	117.29	99.41	94.68
	(54.21)	(31.06)	(28.91)
Max. score (adjusted)	10.52	12.19	17.27
	(3.18)	(3.13)	(3.75)
Average years testing	2.76	2.95	3.34
	(0.83)	(0.91)	(0.67)
Observations	178,430	155,130	280,345

"Tightness" defined as the number of jobs advertised for a given exam divided by the number of competitors. This is necessarily an underestimate of the true tightness since the number of jobs actually awarded will be at least the number advertised but possibly more. Adjusted scores account for the facts that 1) each exam has an idiosyncratic grading scale and 2) reported scores are a mixture of two different exam scores for those candidates who are "classified". Assuming that scores are i.i.d. normally distributed, the mean and standard deviation of each test can be inferred to generate the normalized scores reported as "adjusted".

Table 4: Summary Statistics by Wage Tercile - High School

	1st	2nd	3rd
Competitors	4,463.58	23,169.92	44,254.64
	(4,970.28)	(24,743.32)	(43,021.64)
Tightness	0.03	0.02	0.01
	(0.05)	(0.03)	(0.01)
Score threshold	59.92	84.91	41.65
	(17.92)	(23.41)	(41.03)
Score threshold (adjusted)	6.88	10.46	9.09
	(2.21)	(2.61)	(4.13)
Max. score	73.99	96.57	56.63
	(18.85)	(25.05)	(43.30)
Max. score (adjusted)	11.25	13.37	11.84
	(2.77)	(3.12)	(5.14)
Average years testing	3.03	3.38	3.55
	(0.77)	(0.88)	(0.86)
Observations	481,869	824,656	2,357,497

"Tightness" defined as the number of jobs advertised for a given exam divided by the number of competitors. This is necessarily an underestimate of the true tightness since the number of jobs actually awarded will be at least the number advertised but possibly more. Adjusted scores account for the facts that 1) each exam has an idiosyncratic grading scale and 2) reported scores are a mixture of two different exam scores for those candidates who are "classified". Assuming that scores are i.i.d. normally distributed, the mean and standard deviation of each test can be inferred to generate the normalized scores reported as "adjusted".

Table 5: Summary Statistics by Wage Tercile - College

	1^{st}	2^{nd}	3^{rd}
Competitors	2,771.60	18,082.16	5,252.43
	(3,291.53)	(26,213.11)	(5,800.29)
Tightness	0.03	0.01	0.02
	(0.04)	(0.04)	(0.10)
Score threshold	80.39	100.25	88.15
	(24.72)	(39.87)	(82.72)
Score threshold (adjusted)	10.52	12.44	12.18
	(2.15)	(2.43)	(6.04)
Max. score	89.60	106.57	100.02
	(30.71)	(40.95)	(108.93)
Max. score (adjusted)	12.87	14.21	15.33
	(3.00)	(2.63)	(11.07)
Average years testing	3.47	3.55	3.30
	(1.02)	(0.89)	(1.09)
Observations	395,928	601,571	377,484

"Tightness" defined as the number of jobs advertised for a given exam divided by the number of competitors. This is necessarily an underestimate of the true tightness since the number of jobs actually awarded will be at least the number advertised but possibly more. Adjusted scores account for the facts that 1) each exam has an idiosyncratic grading scale and 2) reported scores are a mixture of two different exam scores for those candidates who are "classified". Assuming that scores are i.i.d. normally distributed, the mean and standard deviation of each test can be inferred to generate the normalized scores reported as "adjusted".

competition and its relationship with the wage is more informative of the appropriateness of our model than the between-individual variation.[15] The control for test frequency is also justified by our model.

Lastly, we add year fixed-effects. Year fixed-effects are not necessarily required if one believes that the environment is stationary. But they are advisable particularly if different jobs had different wage trajectories over time in unexpected ways. Workers cannot arbitrage back in time (although arbitrage forward, by waiting to begin their test preparation investments should be feasible). Thus whether year fixed effects should be introduced depends on whether workers can accurately forecast the wages that are going to be prevailing in the future. Adding year fixed-effects restricts our variation to be only within-year.

In general, we find strong support for the theory. In column (1), our simplest specification, we find that the elasticity is estimated to be 1.03 (0.0010) In column (2), we add individual fixed-effects and obtain an elasticity of 0.977 (0.0016). In column (3), we add controls for the frequency of admission exams and find the elasticity to be 0.94 (0.0016). In column (4) we add year fixed-effects. We find an estimated elasticity of 1.1 (0.0017), our preferred estimate. This specification is overlaid on a binscatter by education level in Figure 2, which visually indicates an excellent fit to the data. In all specifications the estimated elasticity is very close to one.[16]

In Table 7, we investigate how much of the public sector wage differential is lost in the queue. In equilibrium, one should expect the elasticity to be one, and thus, all of the wage differentials should be perfectly offset by longer waiting times. We find that, in our preferred specification, waiting to enter the public sector career dissipates the entire public sector pay differential, plus an additional amount, which varies depending on the discount rates used (less than 10% of the federal minimum wage in Brazil, or about \$1.00 USD/hour). The resulting net present value of the career is just below, but very close to the value of the careers found in private-sector positions.[17]

Next, in Table 8 we calculate the expected number of exam attempts it takes to secure a public sector job across wage quintiles using the properties of the geometric distribution (i.e. assuming a constant probability of success). We also calculate the expected waiting times in Table 9 and the variance in waiting times in Table 10 using similar calculations. As expected, in general, our estimates imply more tests and longer waiting times (*much*[18] longer, for the highest wages) on average are required to access public sector jobs as

15. Note, however, that if one restricts the heterogeneity across workers, then both the within and the between variation became equally useful. Given that there is substantially more information across individuals, there is a real trade-off between bias and precision at play here.

16. The theory also predicts that the elasticity of log vacancies should be one. The government must hire at least the number of people as there were jobs initially advertised, but they often hire more. Likely, then, the consistently low coefficient on log vacancies is due to measurement error with respect to the "true," or realized, number of vacancies.

17. This is potentially explained by unobserved amenity values for public sector jobs

18. Since no one is literally waiting millennia to get into these jobs, clearly, the geometric distribution assumption is factually incorrect. That said, these waiting time estimates give a good sense of just how much competition there is for the public sector in some cases.

the wage increases. This pattern is not exact however, especially for jobs requiring a college degree, which is similar to the pattern observed in the college summary statistics. As before, skill sorting is the likely confounder here.

Finally, to get a different sense of how well our theory fits our data, in Table 11, we alternately restrict the coefficients on wages and/or vacancies to be their theoretical values (i.e. exactly 1). This is, in a sense, the reverse of the analysis in Table 6. Instead of asking how close our coefficents are to their theoretically expected values, we now set them to these values and ask how well this describes the observed pattern. As it turns out, these constrained regressions fit quite well. In column (1) we reproduce our preferred specification from Table 6, noting the RMSE value of 1.6263. In column (2) we restrict log wages and get an RMSE of 1.6265 (a 0.012% increase from baseline). In column (3) we restrict log vacancies and get RMSE of 1.6979 (a 4.403% increase from baseline). In column (4) we restrict both coefficients and get RMSE 1.6991 (a 4.476% increase from baseline). Especially for our primary explanatory variable, log wages, our theory performs remarkably well.

Table 6: Log Competitors vs. Log Wages

	(1) RE	(2) FE	(3) FE+freq.	(4) 2xFE+freq.
Log hourly wage	1.03***	0.977***	0.94***	1.1***
	(0.000954)	(0.00158)	(0.00155)	(0.00173)
Log vacancies	0.63***	0.627***	0.625***	0.621***
	(0.000245)	(0.000391)	(0.000384)	(0.000374)
High school	-0.057***	-0.236***	0.356***	-0.293***
	(0.00163)	(0.00249)	(0.00319)	(0.0038)
College	-1.8***	-1.96***	-1.02***	-1.9***
	(0.00206)	(0.00324)	(0.00446)	(0.00546)
Test frequency			-0.00936***	0.0000989*
			(0.000036)	(0.0000485)
Test frequency^2			0.000012***	2.42e-07***
			(5.37e-08)	(6.73e-08)
Ind. FE		X	X	X
Year FE				X
Observations	5,584,412	5,584,412	5,584,412	5,584,412
$R^2{}_{within}$	0.60	0.60	0.62	0.66
$R^2{}_{between}$	0.64	0.64	0.66	0.69
$R^2{}_{overall}$	0.64	0.64	0.66	0.69

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure 2: Binscatter Regression Overlay

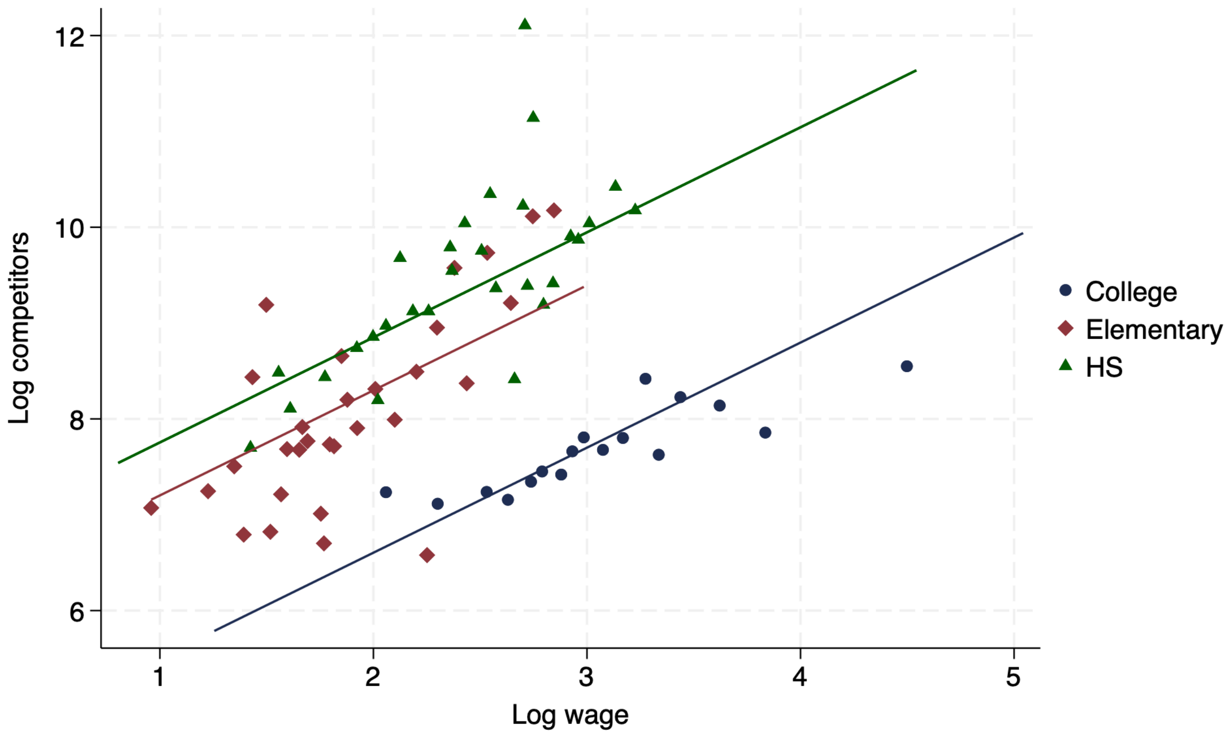


Table 7: Net Public Sector Rents Retained

r =	0.025	0.05	0.075	0.10
Raw fraction	-0.165	-0.122	-0.099	-0.0824
	(0.0034)	(0.0024)	(0.0019)	(0.0016)
% of min. wage (hourly)	-8.10%	-5.99%	-4.86%	-4.04%
	(0.1668)	(0.1177)	(0.0932)	(0.0785)
USD (hourly)	-$0.85	-$0.63	-$0.51	-$0.42
	(0.0175)	(0.0123)	(0.0098)	(0.0082)

Calculated under the assumption that the fraction, θ, of the total public sector rent multiple, δ, remaining after being dissipated in line is determined by the power ϵ, estimated in the main regression: $\frac{\delta-\theta}{r} = (\frac{\delta}{r})^{\epsilon}$. Standard errors calculated via delta method. Minimum wage as of 2012: BRL20.39. 2012 USD/BRL exchange rate: 0.5137

Table 8: Expected Attempts by Wage and Edu.

	Elementary	High School	College
w_{low}	41.59	64.41	19.41
	(91.64)	(146.89)	(15.23)
$w_{med.low}$	114.2	111.7	56.31
	(175.15)	(220.86)	(71.46)
w_{medium}	538.6	8,407.6	74.18
	(443.59)	(26,444.58)	(88.00)
$w_{med.high}$	836.8	418.3	89.99
	(702.08)	(326.01)	(132.66)
w_{high}		1,087.0	72.01
		(1,416.00)	(155.29)
Observations	931	1,776	3,355

Assuming an exponential distribution of attempts, the mean is the inverse of the probability of success, i.e. c/v

Table 9: Expected Wait Times by Wage and Edu.

	Elementary	High School	College
w_{low}	0.597	0.807	8.734
	(1.32)	(1.84)	(6.85)
$w_{med.low}$	3.211	1.217	2.293
	(4.93)	(2.41)	(2.91)
w_{medium}	179.5	177.2	0.706
	(147.86)	(557.38)	(0.84)
$w_{med.high}$	1,882.8	15.24	0.746
	(1,579.68)	(11.88)	(1.10)
w_{high}		2,445.8	0.477
		(3,186.00)	(1.03)
Observations	931	1,776	3,355

Measured in years. Assuming an exponential distribution of attempts, the mean waiting time is the inverse of the probability of success divided by the frequency of exams, i.e. $(c/v) \cdot (1/freq.)$

Table 10: Variance of Wait Times by Wage and Edu.

	Elementary	High School	College
w_{low}	2.084	4.036	120.8
	(18.07)	(48.90)	(200.75)
$w_{med.low}$	34.49	7.264	13.69
	(144.48)	(69.35)	(41.01)
w_{medium}	53,285.6	341,113.6	1.198
	(76,700.56)	(1.59e+06)	(3.50)
$w_{med.high}$	5,416,590.3	372.9	1.767
	(5.96e+06)	(630.81)	(9.38)
w_{high}		13,594,777.5	1.286
		(2.56e+07)	(11.62)
Observations	931	1,776	3,355

Measured in years. Assuming an exponential distribution of attempts, the variance of waiting time is the inverse of the probability of success divided by the frequency of exams squared, i.e. $[(c/v) \cdot (1/freq.)]^2$

Table 11: Constrained Competitor vs. Wage Regressions

	(1) 2xFE+freq.	(2) Wage Const.	(3) Vac. Const.	(4) Both Const.
Log hourly wage	1.1***	1	1.23***	1
	(0.00318)		(0.00331)	
Log vacancies	0.621***	0.619***	1	1
	(0.000688)	(0.000686)		
High school	-0.293***	-0.247***	-0.775***	-0.67***
	(0.00699)	(0.00682)	(0.00724)	(0.00708)
College	-1.9***	-1.81***	-2.31***	-2.1***
	(0.01)	(0.00962)	(0.0105)	(0.01)
Test frequency	0.0000989	-0.000267**	0.0000986	-0.000774***
	(0.0000893)	(0.0000885)	(0.0000932)	(0.0000924)
Test frequency^2	2.42e-07	8.23e-07***	-3.01e-07*	1.08e-06***
	(1.24e-07)	(1.22e-07)	(1.29e-07)	(1.28e-07)
Ind. FE	X	X	X	X
Year FE	X	X	X	X
RMSE	1.6263	1.6265	1.6979	1.6991
Observations	5,584,412	5,584,412	5,584,412	5,584,412

Regressions compare model fit with constraints on the coefficients of log hourly wage, log vacancies, or both. When a constraint is active, that coefficient is fixed at 1.

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

7 Conclusion

In conclusion, we develop and test a model of queuing for public sector jobs in Brazil. The effective price controls, wage subsidies, and quotas imposed by the test-based public sector job system in Brazil generate the potential for large rents to be captured by its workers. Instead, our model implies that public sector workers actually capture none of these rents. The opportunity cost of the entry process – study time, training fees, and multiple test attempts in the queue – exactly offset these potential gains such that in equilibrium, the welfare of workers is approximately equalized across sectors.

Our empirical analysis bears out this prediction. Our best estimate suggests an average elasticity of approximately 1.1 between wages and applicants for public sector jobs. Since this is above 1, this implies that, after paying the entry cost, public sector workers are worse off in terms of observables[19] than their private sector counterparts. Given the years that applicants spend attempting to enter this sector, this should have been no surprise. Nevertheless, it is somewhat counter intuitive based on the discussion surrounding these jobs in popular media.

We thus arrive at practical advice for the aspiring government worker in Brazil. As in almost every other context, here too, there is no free lunch. Accounting for the total benefits *and* the total costs, the value of jobs in either sector is exactly the same despite the large nominal wage differential. Those with comparative advantages or with unique personal situations should, of course, continue to take these factors into account. But for the vast majority of workers, the only choice available is essentially one of timing: lower wages in the private sector now or higher wages in the public sector (potentially) much later.

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19. Accounting for unobserved amenities, we expect that they are *exactly* equal

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A Alternative Specifications

Table 12: Log Competitors vs. Log Wages

	(1) RE	(2) FE	(3) FE+freq.	(4) 2xFE+freq.	(5) 2xFE+freq.
Log hourly wage					
× Elementary	1.4***	1.45***	1.46***	1.85***	
	(0.00266)	(0.00406)	(0.00398)	(0.00396)	
× High school	1.25***	1.14***	1.09***	1.26***	
	(0.00119)	(0.00194)	(0.00191)	(0.00205)	
× College	0.364***	0.446***	0.429***	0.539***	
	(0.00172)	(0.00268)	(0.00263)	(0.00259)	
Constrained					1.1***
					(0.00173)
Log vacancies	0.637***	0.637***	0.635***	0.636***	0.621***
	(0.000242)	(0.000388)	(0.000381)	(0.00037)	(0.000374)
High school	0.0949***	0.335***	1.05***	0.833***	-0.293***
	(0.00609)	(0.00942)	(0.00953)	(0.00918)	(0.0038)
College	0.982***	0.665***	1.64***	1.34***	-1.9***
	(0.00766)	(0.0121)	(0.0122)	(0.0118)	(0.00546)
Test frequency			-0.00962***	0.000138**	0.0000989*
			(0.0000355)	(0.0000475)	(0.0000485)
Test frequency^2			0.0000125***	3.86e-07***	2.42e-07***
			(5.29e-08)	(6.58e-08)	(6.73e-08)
Ind. FE		X	X	X	X
Year FE				X	X
Observations	5,584,412	5,584,412	5,584,412	5,584,412	5,584,412
$R^2{}_{within}$	0.61	0.61	0.63	0.67	0.66
$R^2{}_{between}$	0.66	0.65	0.67	0.70	0.69
$R^2{}_{overall}$	0.65	0.65	0.67	0.71	0.70

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: Log Competitors vs. Log Wages

	(1) RE	(2) FE	(3) FE+freq.	(4) 2xFE+freq.	(5) 2xFE+freq. (cons.)
Log hourly wage					
× Elementary	0.839***	0.832***	0.837***	1.03***	
	(0.00402)	(0.00608)	(0.00601)	(0.0061)	
× High school	1.09***	0.968***	0.912***	0.935***	
	(0.00181)	(0.00291)	(0.00289)	(0.00315)	
× College	0.46***	0.675***	0.654***	0.73***	
	(0.0026)	(0.004)	(0.00395)	(0.00397)	
Constrained					0.89***
					(0.00261)
High school	0.0333***	0.133***	0.976***	0.713***	0.516***
	(0.00924)	(0.0142)	(0.0145)	(0.0142)	(0.00571)
College	-0.285***	-0.818***	0.295***	-0.392***	-1.19***
	(0.0116)	(0.018)	(0.0184)	(0.0181)	(0.00823)
Test frequency			-0.0117***	-0.000609***	-0.000254***
			(0.0000536)	(0.0000733)	(0.0000734)
Test frequency^2			0.0000155***	2.05e-06***	1.51e-06***
			(7.98e-08)	(1.01e-07)	(1.02e-07)
Ind. FE		X	X	X	X
Year FE				X	X
Observations	5,657,585	5,657,585	5,657,585	5,657,585	5,657,585
$R^2{}_{within}$	0.14	0.14	0.16	0.23	0.23
$R^2{}_{between}$	0.22	0.21	0.24	0.26	0.26
$R^2{}_{overall}$	0.21	0.21	0.23	0.27	0.27

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Log Competitors vs. Log Wages

	(1) RE	(2) FE	(3) FE+freq.	(4) 2xFE+freq.
Log hourly wage	0.888***	0.872***	0.829***	0.89***
	(0.00143)	(0.00234)	(0.00232)	(0.00261)
High school	0.637***	0.453***	1.16***	0.516***
	(0.0024)	(0.00362)	(0.00471)	(0.00571)
College	-1.52***	-1.35***	-0.274***	-1.19***
	(0.00307)	(0.00475)	(0.00662)	(0.00823)
Test frequency			-0.0115***	-0.000254***
			(0.0000536)	(0.0000734)
Test frequency^2			0.0000153***	1.51e-06***
			(7.98e-08)	(1.02e-07)
Ind. FE		X	X	X
Year FE				X
Observations	5,657,585	5,657,585	5,657,585	5,657,585
$R^2{}_{within}$	0.14	0.14	0.16	0.23
$R^2{}_{between}$	0.21	0.21	0.23	0.26
$R^2{}_{overall}$	0.21	0.20	0.23	0.27

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

B Measurement error

Assuming that individuals make decisions based on actual values of v and δ, but we observe error-ridden measures of v, we have that

$$y = \beta_\delta \delta + \beta_v v + \epsilon$$

where y is the number of candidates. Our goal is to analyse the effects of regressing y on δ and $(v + u)$, where u is random noise, independent of all other variables in the model. To begin, let's remember how the bivariate ols expression works.

$$\beta = \Sigma^{-1} Cov(y, X),$$

where $X = (\delta, v)$, and Σ is the covariance matrix of X.

Performing the matrix inversion, we obtain that:

$$\beta_\delta = \frac{Cov(y, \delta) - Cov(\delta, v)Cov(y, v)}{Var(\delta)Var(v) - Cov(\delta, v)Cov(\delta, v)}$$

In the special case that δ is uncorrelated with v we obtain the usual univariate regression. It is useful to re-write this expression emphasizing the relationship between δ and v. Let θ be the coefficient of a regression of v on δ, and note that the R squared of such regression is given by $Var(\delta)$ times θ squared. Thus, after some algebra, we have that:

$$\beta_\delta = \frac{\beta_\delta(1 - R^2_{v,\delta})}{1 - R^2_{v,\delta}}$$

When v is measured with error, the same steps yields the following result for the coefficient on δ.

$$\beta_{me} = \frac{A}{B}$$

where the numerator A is

$$A = Var[v+u]Cov(y,\delta) - Cov(\delta, v+u)Cov(y, v+u)$$

$$B = Var[v+u]Var[\delta] - Cov(\delta, v+u)Cov(\delta, v+u)$$

After some algebra, both of them can be written as:

$$A = Var[v+u](\beta_\delta Var[\delta] + \beta_v Cov(\delta, v)) - Cov(\delta, v)(\beta_\delta Cov(\delta, v) + \beta_v Var[v])$$

and

$$B = Var[\delta](Var[v+u](1 - R^2_{v+u,\delta})$$

After some more tedious algebra, we get:

$$\frac{A}{B} = \frac{\beta_\delta(1 - R^2_{v+u,\delta}) - \beta_v \theta_{\delta,v}(1 - \frac{Var[v]}{Var[v+u]})}{1 - R^2_{v+u,\delta}}$$

Note that if measured v is reliable (that is, variance of v+u is close to variance of v) then the last term in the numerator is zero and the ols coefficient coincides with the desired parameter, β_δ.