Brazilian stocks with higher and lower price-earnings ratios: short or long-memory behaviour?

Area: Financial Econometrics

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Abstract: The main aim of this paper is to verify whether there is short or long memory behaviour in returns and volatilities and whether the behaviour is similar for two groups of companies listed on IBOVESPA: higher and lower price-earnings (P/E) ratios in December 2019. It is considered the period from 01 January, 2016 to 31 December, 2022. The fractionally integrated parameter (d) is used to check for short or long memory. In general, for returns and volatilities, the results are very similar for both groups of companies (higher P/E or lower P/E). The long-memory behaviour, when occurs, especially for volatilities, is not constant over time, transitory and disappears. Moreover, even in periods of possible above-average, as in the case of the COVID-19 pandemic, this is not possible without the investor incurs above-average risks.

Keywords: efficient markets; long memory; price-earnings ratio; Brazilian companies.

Jel classification: C22; G10; G15.

1. Introduction

Measuring and managing financial market risks are critical factors for market participants. Investors, especially traders, are aware of the potential losses that can arise due to market fluctuations. In this context, taking into account, among other issues, the different results of several studies, with regard to market efficiency or inefficiency, studying whether or not stock markets are efficient (Efficient Market Hypothesis – EMH) remains a challenging and very important task.

The main aim of this research is to verify whether the returns and volatilities of some Brazilian companies listed on the São Paulo Stock Exchange Index (IBOVESPA) exhibit short or long-memory behaviour, and

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whether the behaviour is similar, in the period from 01 January, 2016 to 31 December, 2022 (daily data). These companies were selected based on price-earnings (P/E) ratios, considering two sets of companies: higher P/E and lower P/E. The contribution of this paper is to analyse whether, taking into account the P/E ratio of December 2019, that is, before the COVID 19 pandemic, both groups of companies behaved similarly or differently after the start of the pandemic, in terms of log-range dependence. Two hypotheses are formulated: i) behaviour varies over time, especially in periods of turbulence such as the COVID-19 pandemic; and ii) there is a similar behaviour in both groups of companies. To the best of knowledge, there are so far no similar studies for the Brazilian stock market.

As stated by Lakonishok, Shleifer and Vishny (1994), for many years, researchers and investment professionals have claimed that value strategies outperform the market. Here, the idea is for buying stocks that have low prices regarding to earnings, dividends, historical prices, book assets, or other measures of value. In this context, value strategies have also gained prominence. For example, Basu (1977), Jaffe, Keim and Westerfield (1989), Chan, Hamao and Lakonishok (1991) and Fama and French (1992) demonstrate that stocks with low price-earnings (P/E) ratios earn higher returns (P/E ratio is indicator of the future investment performance of a stock); low P/E stocks will tend to outperform high P/E stocks. In this context, according to Wong (2021), the so-called behavioural financial economics is an important milestone in the development of modern theory in financial economics, because many studies have shown that there are some anomalies and paradoxes and many financial phenomena (Shiller, 2000), that traditional theories of financial economics cannot explain.

Unlike, authors as Daniel and Titman (1997) show that anomalies[3] such that BM and size only represent the preference of investors, and do not determine stock returns. To Fama (1998), even in the scenario of anomalies and paradoxes, the EMH still holds, since the anomalies documented in several studies are not persistent and disappear when the model, sample or data frequencies change. To Fama (1998), recent finance literature, such as behavioural finance, seems to produce many long-term return anomalies. Although, consistent with the efficiency market hypothesis that anomalies are the result of chance, apparent overreaction of stock prices to information is as common as underreaction. In addition, post-event continuation of pre-event abnormal returns

[3] To Woo, Mai an McAleer (2020), there are several anomalies that can affect the financial market, and which are contrary to the EMH, as for example: winner-loser effect, reversal effect; momentum effect; calendar anomalies (as instance January effect, weekend effect, and reverse weekend effect); book-to-market (BM) effect; value anomaly; size effect; disposition effect; equity premium Puzzle; herd effect and ostrich effect; bubbles; among others.

is almost as frequent as post-event reversal. Even more important, long-term return anomalies are fragile and tend to disappear.

In addition, to Malkiel (2003)[4], as long as stock markets exist, the collective judgment of investors will sometimes make mistakes. Undoubtedly, some market participants are demonstrably less than rational. As a result, pricing irregularities and even predictable patterns in stock returns can appear over time and even persist for short periods. Moreover, the market cannot be perfectly efficient, or there would be no incentive for professionals to discover the information that is reflected so quickly in market prices, a point emphasized by Grossman and Stiglitz (1980). However, Malkiel (2003, p. 80) points out that "... I suspect that the end result will not be the abandonment of the belief of many in the profession that the stock market is remarkably efficient in the use of information".

In this context, as highlighted by Lekhal and Oubani (2020), in addition to the methodological problems highlighted by Fama (1998), the controversial conclusions can be attributed to the fact that the arguments of both EMH and behavioural finance are partially valid. Thus, one can switch from one paradigm to another at any time, depending on changes in market conditions. In this way, the two paradigms can be reconciled to provide a convincing explanation of market behaviour.

In this perspective, as described by Hull and Mcgroarty (2014), alternative theories of market dynamics for EMH are emerging. These new approaches see efficiency as something the market tends to, rather than a state that automatically maintains itself at all times. One of the first alternatives refers to the Fractal Market Hypothesis (FMH), which is a theory of market behaviour proposed by Peters (1991, 1994), which aims to explain the fractal characteristics of market prices. A second alternative to EMH concerns the Heterogeneous Market Hypothesis (HMH), proposed by Dacorogna et al. (2001), which describes how prices arise from the interaction of market participants with different investment horizons. Furthermore, similar characteristics to the HMH are observed in another alternative to the EMH, called the Adaptive Market Hypothesis (AMH), proposed by Lo (2004, 2005), which combines behavioural finance concepts with the dynamics of evolution. Unlike EMH, these two alternative hypotheses of market dynamics, HMH and AMH, allow for the possibility of serial dependency, at least for some time. As described by Lo (2004), it is as if AMH could reconcile EMH with all its behavioural alternatives.

[4] Interesting describe that Malkiel (2003, p. 2) presents a different concept of Fama (1970, 1991) for market efficiency: "At first, it is important to make clear what I mean by the term "efficiency". I will use the definition of efficient financial markets as those that do not allow investors to obtain above-average returns without accepting above-average risks".

Here, to analyse the behaviour of efficiency or not of the financial market, the concept of long memory is adopted. This concept was used, in the financial context, first by Lo (1991). For Lo (1991), the presence of long memory in the returns would distance the market from the Random Walk Hypothesis (RWH). It is worth mentioning that time series with long-term memory exhibit an unusually high degree of persistence, such that observations in the remote past are non-trivially correlated with observations in the distant future, even when the time interval between the two observations increases. Besides, it is important to mention that the presence of long memory components in asset returns has important implications for many of the paradigms used in modern financial economics[5].

In this paper, the question of long-range dependency is investigated by means of the fractionally integrated parameter (d). First, in all estimates the Geweke and Porter-Hudak (GPH) estimator (Geweke and Porter-Hudak, 1983) is adopted. To ensure the robustness of the results, in some estimates, two other estimators are used, namely: Exact Local Whittle (ELW) (Shimotsu and Phillips, 2005) and Two-Step Exact Local Whittle (2SELW). In addition, since the estimated fractionally integrated parameter may vary over time, rolling estimation is adopted to capture the time-variation of $\hat{d}$.

The structure of this paper is as follows. In addition to this introduction, Section 2 provides a literature review. Section 3 presents the data and methodology. In Section 4, the results and discussion are presented. Finally, the concluding remarks are presented in Section 5.

2. Literature review

This section provides a literature review of works that aim to verify the efficiency of financial markets, highlighting the presence or absence of long memory behaviour (in asset returns and asset price volatility), some of which are specific to the Brazilian economy and others for the COVID-19 pandemic. A couple of studies specifically address the P/E ratio. To the best of knowledge, so far, no study has examined the

[5] For example, optimal consumption/savings and portfolio decisions can become extremely sensitive to the investment horizon if stock returns are long-term dependent. Problems also arise in pricing derivative securities (such as options and futures) with martingale methods, as the most commonly employed class of continuous-time stochastic processes is inconsistent with the presence of long memory (see Maheswaran (1990), Maheswaran and Sims (1990) and Sims (1984), for example). In addition, the traditional tests of the capital asset pricing model and the arbitrage pricing theory are no longer valid, since the usual forms of statistical inference do not apply to time series that exhibit such persistence. And, the conclusions of more recent tests of hypotheses of "efficient" markets or stock market rationality also depend precariously on the presence or absence of long-term memory.

efficiency or not of the Brazilian stock market considering companies with higher and lower values for the P/E ratio, and adopting a period before e after the COVID-19 pandemic, which is the main contribution of the present study to the empirical financial literature.

The available empirical evidence of long-range dependence is somewhat mixed. Long-memory evidence is found by Mandelbrot (1972), Greene and Fielitz (1977), Booth, Kaen and Koveos (1982), Niu and Wang (2013), Abbritti et al. (2016), Caporale, Gil-Alana and Plastun (2019), among others. Conversely, the following works do not find long-memory behaviour: Lo (1991), Jacobsen (1995), Berg and Lyhagen (1998), Crato and Ray (2000), Malkiel (2003), Serletis and Rosenberg (2007), Lu and Perron (2010). This controversy may be because the degree of persistence of the series can vary over time, as presented by Corazza and Malliaris (2002), Glenn (2007), Bennett and Gartenberg (2016), due to structural breaks (Charfeddine and Guégan, 2012) or even due to the models and statistical methods used (Fama, 1998).

Here it is important to say that there is also a debate between the presence of long-range dependence for returns and for volatility. According to Engle (1982) and Bollerslev (1986), among others, the volatility of financial returns may present a strong autocorrelation structure, while the returns show no memory and random-walk behaviour. For example, Crato and Ray (2000) examine the memory of future returns using a modified version of the R/S statistic developed by Lo (1991), as well as a test based on the GPH estimator of the long-memory parameter. The results demonstrate no long-memory behaviour in future returns. However, when the analysis considers volatility (squared log returns), the findings reveal overwhelming evidence of persistence, consistent with the works of Ding, Granger and Engle (1993), Bollerslev and Mikkelsen (1996), Baillie, Bollerslev and Mikkelsen (1996), and Breidt, Crato and Lima (1998). For Bhattacharya, Bhattacharya and Guhathakurta (2018), there is a consensus among the financial community that long memory is a characteristic of asset price volatility, which does not occur for asset returns.

Another relevant point to be highlighted refers to that described by Hull and McGroarty (2014): market efficiency is expected to be related to the level of economic development. The results reveal strong evidence of long memory in volatility clustering and weak evidence of long memory in returns, including for Brazil. Besides, the estimates show greater efficiency in returns and volatility for “advanced” emerging markets. As point out by Barkoulas, Baum and Travlos (2000), emerging capital markets (ECM) tend to exhibit different characteristics from those observed in developed capital markets. Biases due to market thinness and nonsynchronous trading should be expected to be more severe in the case of ECM. In addition, unlike developed capital markets which are highly efficient in terms of the speed of information reaching all traders, investors in emerging capital markets tend to react slowly and gradually to new information.

Regarding to the period of the COVID-19 pandemic, one of the world's biggest health crises in recent time, pandemic's immediate effects has been a substantial increase in volatility. Here, Vera-Valdés (2021) analyze the long-term effects of COVID-19 on the Chicago Board Options Exchange (CBOE) Volatility Index (VIX) and realized variances for several international markets. The authors consider the period from January 2018 to January 2021, for daily data. GPH and ELW estimators are adopted as methodology. The results show that volatility measures for most countries experienced increases in the degrees of memory following the pandemic. Besides, several volatility measures became nonstationary, signaling the start of a period with higher and more persistent financial volatility. Gil-Alana and Claudio-Quiroga (2020) verify the impact that the COVID-19 on three Asian stock markets: Korean SE Kospi Index, Japanese Nikkei 225 and Chinese Shanghai Shenzhen CSI 300 Index. The daily data cover the sample July 2006 to September 2020. By means of fractional integration methods, the estimates indicate that mean reversion and thus transitory effects of shocks occurred in the Nikkei 225 index. However, for the Kospi and Shanghai Shenzhen indices, this hypothesis is rejected, implying that shocks are permanent.

In the case of the Brazilian stock market, Resende and Teixeira (2002) is one of the first studies on the subject. The authors study the long-memory behaviour of the IBOVESPA index, considering sub-periods before and after the Real Stabilization. The results demonstrate the existence of short memory for both periods, despite the so-called reforms that the Brazilian economy underwent in the 1990s and, in particular, after the Real Plan. Costa e Vasconcelos (2003) analyse the possible presence of long-range correlations of the IBOVESPA, spanning over 30 years of data from January 1968 up to May 2001. The results show the existence of long-term dependence (persistence) which lasted for up to six months. In particular, the paper find that the structural reforms set off after 1990 (Collor Plan in the early 1990's and Real Plan in 1994) leaded to a more efficient stock market in Brazil.

Cajueiro and Tabak (2004), considering the stock returns of the indices of the countries of Latin America and Asia, suggest that emerging markets are becoming more efficient, except for Brazil, Philippines and Thailand. In another research, Cajueiro and Tabak (2005) employ a rolling methodology to estimate Hurst exponents for emerging markets of the Latin America and Asia, considering squared and absolute returns (United States and Japan are included for comparison purposes). The estimations show that these markets present strong long-range dependence in volatility. In addition, empirical results suggest that Asian equity markets are more efficient than those of Latin America and that the US is the most efficient country.

Cavalcante and Assaf (2005) investigate the long-memory property in returns and volatility of the Brazilian stock market from 03.01.1994 to 17.05.2002. Estimates reveal significant long-memory behaviour in the volatility measures, while there is little evidence of long memory in the returns themselves. Thus, during the study period, the Brazilian stock market had an underlying fractal structure, contrary to EMH. Ely (2011) search evidence of predictability in the Brazilian stock market, using portfolios grouped by sector and company size, taking into account the period from 1999 to 2008. The results reveal that: i) the stocks of the industrial sector are highly predictable; ii) the stocks of small companies tend to be more predictable than those of large companies; and, iii) the Brazilian stock market had an increase in efficiency from 1994 to 2008.

Chen and Metghalchi (2012) investigate the predictive power of several trading rules with different combinations of the most popular indicators in technical analysis for IBOVESPA, in the period from 05.01.1996 to 03.01.2011. The results strongly support the weak form of market efficiency for the Brazilian stock market. Carvalho, Suen and Gallo (2016) use intraday stock returns and mandatory disclosures of material facts by publicly traded companies on the BM&FBOVESPA to assess market efficiency in Brazil, considering the period from November 2012 to February 2014. The results show that relevant facts, informed by companies, in fact reveal unexpected information to investors. The speed of price response to new information and the observed magnitudes of cumulative returns indicate that market participants can benefit from profit opportunities in the minutes close to the release of material facts.

Finally, regarding to P/E ratio, as previously described, Basu (1977) verifies empirically whether the investment performance of common stocks is related to their P/E ratios. The data base represents over 1.400 industrial firms traded on the NYSE between September 1956 and August 1971. According to results, P/E ratio information is not “fully reflected” in stock prices in as rapid a manner as postulated by the semi-strong form of the efficient market hypothesis. Otherwise, it seems that disequilibria persisted in capital markets during the period studied. To Campbell and Shiller (1998), initial P/E ratios explain as much as 40 percent of the variance of future returns. Other authors also reach these results, such as Nicholson (1960) and later confirmed by Ball (1978). At the Brazilian level, Amorim and Camargos (2021), considering the period from December 2004 to June 2018, demonstrate that the price-earnings index based on IBOVESPA present a non-linear trend and mean reversion, which contrasts with the EMH. As claimed by Malkiel (2003), these findings are consistent with the views of behavioralists that investors tend to be overconfident of their ability to project high earnings growth and thus overpay for "growth" stocks (for instance, Kahneman and Riepe, 1998).

3. Data and empirical methodology

3.1. Data

The data set analyzed in this paper is the daily closing price (P_t) of the stock of 10 companies listed on the São Paulo Stock Exchange Index (IBOVESPA Index). The series used in this study are described in Table 1. The study considers the period from 01 January, 2016 to 31 December, 2022, using daily data. The data source is the Investing.com (www.investing.com). The daily returns r_t are obtained by calculating the first logarithmic difference of the stock market price P_t at day t, $r_t = log\ (P_t) - log\ (P_{t-1})$, for each time series.

As previously mentioned, this research works with two sets of companies, higher P/E and lower P/E, considering as a basis the companies listed on the IBOVESPA on the date in December 2019, a little before the COVID-19 pandemic. The P/E ratio is also reports in Table 1. It is important to highlight that companies with a negative P/E ratio were removed from the sample, as well as companies with a large percentage of daily returns equal to zero and those that were among the companies with higher or lower P/E ratios, but that do not have data for the entire analysis period.

Table 1 – Variables, unit, acronym, P/E ratio and source

Variable	Units	Ticker	P/E	Source
Higher P/E				
CVC Brasil Operadora e Agência de Viagens S.A.	Price	CVCB3	144.44	Investing.com
BRF Brasil Foods S.A.	Price	BRFS3	96.10	Investing.com
Cogna Educação S.A.	Price	COGN3	79.89	Investing.com
Ultrapar Participacões S.A.	Price	UGPA3	75.91	Investing.com
Natura &Co Holding S.A.	Price	NTCO3	67.97	Investing.com
Usinas Siderúrgicas de Minas Gerais S.A.	Price	USIM5	55.88	Investing.com
Lower P/E				
Transmissora Aliança de Energia Elétrica S.A.	Price	TAEE11	10.72	Investing.com
Petróleo Brasileiro S.A.	Price	PETR4	10.40	Investing.com
Banco do Brasil S.A.	Price	BBAS3	9.23	Investing.com
Companhia Energética do Estado de Minas Gerais S.A.	Price	CMIG4	6.43	Investing.com
Centrais Elétricas Brasileiras S.A.	Price	ELET6	4.84	Investing.com
Equatorial Energia S.A.	Price	EQTL3	1.90	Investing.com

Note: P/E ratio considers as a basis the companies listed on the IBOVESPA on the date in December 2019.
Source: Own elaboration.

According to Fama and French (1988), daily returns adopted as the sample sizes from such short return horizons are sufficiently large to generate precise estimates for the statistical tests. Bollerslev and Wright (2000) demonstrate that increasing the sample size beyond the daily frequency does not significantly improve the accuracy of the time dependency estimates. To Ray and Tsay (2000), the log transformation becomes a problem when zero or very small returns are encountered. To mitigate this problem, following Perron and Qu (2010), it is adopted $r_t + 0.001$. The conclusions remain the same if the returns with absolute magnitudes below 0.001 were eliminated. Besides, squared log returns are used as measure for volatility, as adopted by: Taylor (1986), Crato and Lima (1994), Starica and Granger (2005), Bentes et al. (2008), Hull and McGroarty (2014), among others.

Table 2 shows the basic descriptive statistics of the daily returns (first difference of the natural logarithms). For several returns, the distributions appear to be asymmetric, since there are positive and negative estimates of skewness. All returns series have heavy tails and show a strong deviation from normality (the skewness and kurtosis coefficients are all different from those of the standard normal distribution, which are 0 and 3, respectively). In addition, Jarque-Bera (JB) test rejected the null hypothesis of normality at a significance level of 5%. To Maldelbrot (1963) and Fama (1965), excess kurtosis and nonnormality are stylized facts regarding financial returns.

Table 2 – Descriptive statistics, Jarque-Bera (JB) test and unit roots tests

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Mean	0.0004	-0.0001	0.0002	0.0006	0.0010	0.0019	0.0016	0.0018	0.0017	0.0019	0.0019	0.0018
Median	0.0003	-0.0003	-0.0016	0.0010	0.0003	0.0000	0.0015	0.0023	0.0018	0.0010	0.0017	0.0018
Std. dev.	0.0393	0.0281	0.0336	0.0271	0.0309	0.0381	0.0160	0.0336	0.0275	0.0281	0.0313	0.0177
Min.	-0.4264	-0.2190	-0.2341	-0.2393	-0.2766	-0.2378	-0.1963	-0.3514	-0.2367	-0.2339	-0.2136	-0.1138
Max.	0.2816	0.1518	0.1790	0.2111	0.1595	0.3075	0.0845	0.2017	0.1592	0.1660	0.2792	0.0816
Skewness	-1.3105	-0.4880	-0.4606	-0.5486	-0.5794	0.3403	-1.2860	-2.0422	-0.7200	-0.6065	0.1702	-0.2905
Kurtosis	19.707	8.099	6.646	12.439	9.135	5.873	15.634	20.364	10.997	9.376	8.627	4.165
JB	28614	4820	3261	11291	6141	2533	18175	31230	8908	6474	5399	1282
ADF	-17.835	-26.744	-28.964	-15.589	-29.607	-28.611	-30.348	-22.380	-29.591	-29.228	-29.334	-29.542
PP	-40.871	-40.405	-42.990	-48.072	-43.395	-40.428	-43.750	-42.403	-42.604	-42.368	-41.281	-45.223
KPSS	0.5152	0.0896	0.2137	0.0653	0.4084	0.3003	0.0901	0.0947	0.1614	0.0355	0.0973	0.1252
N. obs.	1733	1733	1733	1733	1733	1733	1733	1733	1733	1733	1733	1733

Note1: 1) The normality test is the Jarque-Bera test which has a χ^2 distribution with 2 degrees of freedom under the null hypothesis of normally distributed errors. The 5% critical value is equal to 5.99; 2) ADF, PP and KPSS denote the statistics of Augmented Dickey-Fuller, Phillips-Perron and Kwiatkowski, Phillips, Schmidt and Shin unit root tests, respectively. Critical values of the ADF, PP and KPSS tests, at the 5% level of significance, are equal to −1.95, −2.86 and 0.463, respectively.
Source: Own elaboration.

Besides, Table 2 presents the results for three unit roots tests: Augmented Dickey-Fuller – ADF (Dickey and Fuller, 1981), Phillips-Perron – PP (Phillips and Perron, 1988) and Kwiatkowski-Phillips-Schmidt-Shin – KPSS (Kwiatkowski et al., 1992). Results revealed that all daily returns are stationary, i.e., the hypothesis of unit root is rejected at 5% significance level. As described by Hull and McGroarty (2014), according to Henry (2002), stationarity in a time series does not exclude the possibility of serial correlation.

3.2. Empirical methodology

3.2.1. Long memory

According to Morettin (2006), the phenomenon known as long memory (long-range dependence) originated with the works of Hurst (1951, 1957), Mandelbrot and Wallis (1968) and McLeod and Hipel (1978), in problems related to studies in the area of hydrology. Long memory models are also used in climatological research. In financial economic literature, the concept of long memory began to be studied by Granger (1980), Granger and Joyeux (1980) and Hosking (1981).

As described by Tsay (2010), the autocorrelation function (ACF) for a stationary time series decays exponentially to zero as lag increases. In the other hand, when the time series presents a unit root (i.e., it is nonstationary), the sample ACF converges to one for all fixed lags as the sample size increases (see, Chan and Wei (1988) and Tiao and Tsay (1983)). However, for some time series, the ACF slowly decays to zero at a polynomial rate as the lags increase. These processes are known to exhibit long-memory behaviour. Baillie (1996) provided an excellent review about long-range dependence in econometrics.

To Baillie (1996), a wider definition of long memory is presented as follows. Given a discrete time series process (x_t), $t = 1, \dots, T$, with autocovariance function γ_h, at lag h, in the time domain, the process presents long-range dependence if

$$\gamma_h \approx \Xi(h) h^{2d-1}, \text{ as } h \to \infty, \tag{1}$$

where $\approx$ denotes approximate equality for large h, $d \neq 0$ is the fractional long-memory parameter, $\Xi(h)$ is a slowly varying function at infinity. As a consequence, the autocorrelation function presents slow and hyperbolic decay.

In addition, in the frequency domain, considering $f(\lambda)$ as the spectral density of the long memory series (x_t), with $t = 1, \ldots, T$, at frequency λ, then,

$$f(\lambda) \sim c\lambda^{-2d}, \text{ as } \lambda \to 0, \tag{2}$$

where c is a constant. That is, $f(\lambda)$ is unbounded when the frequency is near zero (Baillie (1996), Charfeddine (2016), among others).

In this context, the following definitions regarding the behaviour of the process $\{x_t\}$ can be described: $d = 0$, short memory (or white noise); ii) $0 < d < 0.5$, stationary with long memory; iii) $0.5 \leq d < 1$, the process is mean reverting, even though it is not covariance stationary; and, iv) if $d \geq 1$, nonstationary and does not present mean reversion. In the particular case of $d = 1$, there is a non-stationary process, characterized by the presence of a unit root. Besides, the process has an anti-persistence behaviour when $d \in (-0.5;\ 0)$.

In the literature, there are several estimators of the fractionally integrated parameter (d), which can be classified as parametric and semi-parametric. The former involves the simultaneous estimation of the parameters of the so-called autoregressive fractionally integrated moving average $(ARFIMA)$ model (introduced independently by Granger and Joyeux (1980) and Hosking (1981)), generally using the maximum likelihood method; see, e.g., Dahlhaus (1989), Fox and Taqqu (1986) and Sowell (1992). In semi-parametric procedures, the estimation of the model parameters is carried out in two steps: first, the long memory parameter d is estimated, for example, through a regression model of the logarithm of the periodogram function and, subsequently, the autoregressive and moving average parameters are estimated.

One of the most popular estimators in the class of semi-parametric estimators is the Geweke and Porter-Hudak (GPH) estimator (Geweke and Porter-Hudak, 1983). In this work, this estimator is used in all estimates of the fractionally integrated parameter (d). To ensure the robustness of the results, in some estimates, two other estimators are used, namely: Exact Local Whittle (ELW) (Shimotsu and Phillips, 2005) and Two-Step Exact Local Whittle (2SELW). In addition, since the estimated fractionally integrated parameter may vary over time, rolling estimation is adopted to capture the time-variation of $\hat{d}$. In this case, the estimated parameter $\hat{d}$ is calculated one first time window, and then the sample is rolled forward one point by eliminating the first observation and adding the next one, and then recalculating the parameter $\hat{d}$. The rolling method has some shortcomings, but the technique provides a good first proxy of the time-variation of $\hat{d}$.

3.2.2. The GPH estimator (GPH)

Be $f(\lambda)$ the function defined in Equation (2). The GPH employs the periodogram function $I(\lambda_j)$ as an estimate of the spectral density function in Equation (2), at frequency $\lambda_j = \frac{2\pi j}{T}$, $j = 0,1,\dots,\left\lfloor\frac{T}{2}\right\rfloor$, where T is the sample size and $\lfloor\cdot\rfloor$ denotes the integer part function. In this case, considering frequencies close to zero, Geweke and Porter-Hudak (1983) suggest the following approximation to estimate d (see, Molinares, Reisen and Cribari-Neto (2009), Charfeddine and Guégan (2012) and Charfeddine (2016)),

$$\ln\{I(\lambda_j)\} = \beta - d ln\left\{4 sin^2\left(\frac{\lambda_j}{2}\right)\right\} + \varepsilon_j,\ j = 1,2,\dots,g, \tag{3}$$

where $\{\varepsilon_j\}$ is the white noise and g is the bandwidth ($g = T^{\alpha}$, with T equal to the number of observations), which corresponds to the number of frequencies used in the regression of Equation (3). In this paper, the following bandwidths is considered: $g = T^{0.7}$ (details of the choice of bandwidth can be consulted at Reisen (1994), Lee and Robinson (1996), Hurvich, Deo and Brodsky (1998) and Diebold and Inoue (2001)). Other bandwidths, such as $g = T^{0.5}$, $g = T^{0.6}$ and $g = T^{0.8}$, generate similar results. Under some conditions, the GPH estimator is consistent and asymptotically normally distributed. To see some asymptotic properties of the estimator, see Hurvich, Deo and Brodsky (1998) and Velasco (2000).

3.2.3. The ELW estimator (ELW)

Shimotsu and Phillips (2005) develop a semiparametric estimator namely Exact Local Whittle (ELW) estimator, to estimate the fractionally integrated parameter (d). This estimator is consistent and has the same $N(0, 1/4)$ limit distribution for all values of d if the optimization covers an interval of width less than $9/2$ and the mean (initial value) of the process is known (Shimotsu, 2010). In this context, ELW offers a good general-purpose estimation procedure for the memory parameter that is applied throughout the stationary and nonstationary regions of d. Thus, ELW is a better estimator than Local Whittle (LW; for instance, Kunch, 1987) and also GPH, which are not a good general-purpose when the value of d may take on values in the nonstationary zone beyond $3/4$ (see, Kim and Phillips (1999) and Shimotsu and Phillips (2005)).

The idea is to estimate (G, d) by minimizing the objective function

$$Q_g(G, d) = \frac{1}{g}\sum_{j=1}^{g}\left[\ln\left(G\lambda_j^{-2d}\right) + \frac{1}{G} I_{\Delta^d x_t}\left(\lambda_j\right)\right], \tag{4}$$

where $I_{\Delta^d x}\left(\lambda_j\right) = \frac{1}{2\pi T}\left|\sum_{i=1}^{T} \Delta^d x_t \exp\left(i\lambda_j t\right)\right|$ is the periodogram of $\Delta^d x_t$, $\Delta^d = (1 - B)^d$ and B is the lag operator.

Concentrating $Q_m(G, d)$ with respect to G, the estimated value for d $(\hat{d}_{ELW})$ obtained by means of this method is

$$\hat{d}_{ELW} = \underset{d\in[d_1,d_2]}{\arg\min}\, R(d), \tag{5}$$

where d_1 and d_2 are the lower and upper bounds of the admissible values of d so that $-\infty < d_1 < d_2 < \infty$. Furthermore,

$$R(d) = ln\, \hat{G}(d) - 2d\frac{1}{g}\sum_{j=1}^{g} ln\, \lambda_j \text{ and } \hat{G}(d) = \frac{1}{g}\sum_{j=1}^{g} I_{\Delta^d x_t}\left(\lambda_j\right), \tag{6}$$

where g is the truncation parameter. This paper considers $g = T^{0.7}$ (other values for g presented similar results). The ELW estimator has been shown to be consistent and asymptotically normally distributed.

3.2.4. Two-Step Exact Local Whittle (2SELW)

Shimotsu (2010) propose the Two-Step Exact Local Whittle (2SELW) to accommodate an unknown mean and a polynomial time trend. The author showed that the two-step ELW estimator, which is based on a modified ELW objective function using a tapered Local Whittle estimator in the first stage, has an $N(0, 1/4)$ asymptotic distribution for $d \in (-1/2\,, 2)$ (or $d \in (-1/2\,, 7/4)$ when the data have a polynomial trend). Also, Shimotsu (2010) demonstrated that the two-step ELW estimator inherits the desirable properties of the ELW estimator.

Following Shimotsu (2010) and Charfeddine and Khediri (2016), two-step ELW estimator $(\hat{d}_{2SELW})$ can be obtained by minimization of the objective function:

$$Q_g(G_S, d) = \frac{1}{g}\sum_{j=1}^{g}\left[\ln\left(G_S\lambda_j^{-2d}\right) + \frac{1}{G_S} I_{\Delta^d(x_t - \hat{\mu}(d))}\left(\lambda_j\right)\right]. \tag{7}$$

Then, based on Equation (7), Shimotsu (2010) defined the resulting two-step ELW estimator as

$$\hat{d}_{2SELW} = \underset{d\in[d_1,d_2]}{\arg\min}\, R_S(d), \tag{8}$$

where d_1 and d_2 are the lower and upper bounds of the admissible values of d so that $-\infty < d_1 < d_2 < \infty$.

Besides,

$$R_S(d) = ln\, \hat{G}_S(d) - 2d\frac{1}{g}\sum_{j=1}^{g} ln\, \lambda_j \text{ and } \hat{G}_S(d) = \frac{1}{g}\sum_{j=1}^{g} I_{\Delta^d(x_t-\hat{\mu}(d))}(\lambda_j), \tag{9}$$

where $I_{\Delta^d(x_t-\hat{\mu}(d))}(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{i=1}^{T} \Delta^d(x_t - \hat{\mu}(d)) \exp\,(i\lambda_j t)\right|$ is the periodogram of $\Delta^d(x_t - \hat{\mu}(d))$.

According to Shimotsu (2010), the resulting estimator encounters the difficulty in proving its global consistency for certain values of d. Thus, he applies two-step estimation to circumvent this difficulty. Let $\tilde{d}$ denote this first-stage estimator. Then, two-step ELW estimator can be expressed as

$$\hat{d}_{2SELW} = \tilde{d} - \frac{R_S'(\tilde{d})}{R_S''(\tilde{d})}, \tag{10}$$

where $R_S(d)$ is the objective function defined in Equation (9), $'$ and $''$ denote the first and second-order derivatives, respectively. Iterating and updating the estimator of Equation (10) can substantially improve its finite-sample properties.

This paper considers $g = T^{0.7}$ (other values for g presented similar results). For more details, including the estimators of the unknown mean that can be acceptable, see Shimotsu (2010).

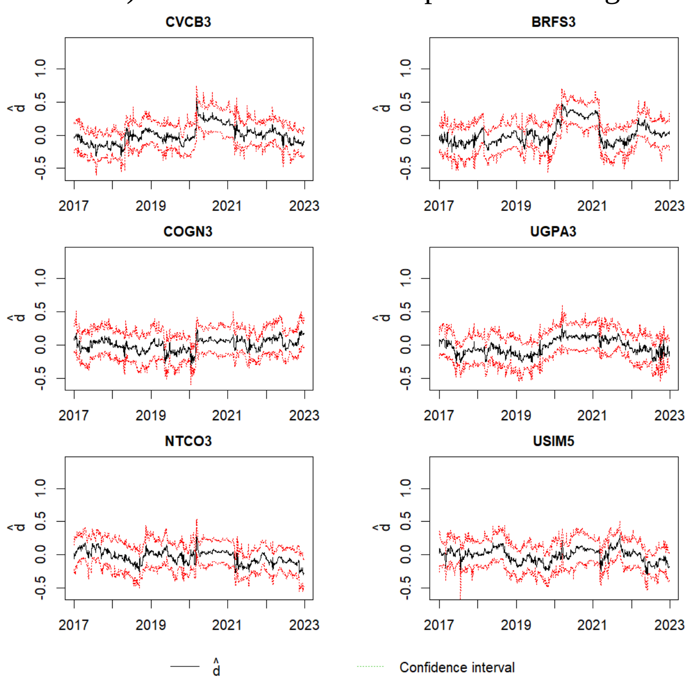
4. Results

As previously described, most of the estimates of the fractional parameter d were performed using the GPH estimator. To ensure the robustness of the results, in some cases, the estimators ELW and 2SELW also are used. Furthermore, since the fractionally integrated parameter estimated may vary over time, rolling estimation is adopted to capture the time-variation of $\hat{d}$. In this way, with the window rolling forward, it is possible to verify the evolution of the estimated fractional integration parameter. Since it is impossible to report the estimated parameter ($\hat{d}$) for each rolling window, the results are presented through graphical depiction, and tables with the annual average and general descriptive statistics. The section is divided into two subsections: estimates for returns and estimates for volatilities[6].

4.1. Estimates for returns

Firstly, Figures 1 and 2 present time-varying fractional coefficient ($\hat{d}$), for the returns of companies with higher P/E and lower P/E, respectively. Here, GPH is adopted. As can be seen, the parameter $\hat{d}$ is time varying in nature, which is unsurprising. However, in general, for both groups of companies, the estimated value of d is close to zero and the long-range dependence hypothesis can be rejected, did not find long-memory behaviour in the most of the time. Few exceptions occur for the most turbulent period of the COVID-19 pandemic, in which the possibility of long-range dependency for the returns of some companies (as instance, CVCB3 and BRFS3, from the group with higher P/E) was not rejected. However, even in these cases, the estimated parameter is in the range from 0 to 0.5 ($0 < \hat{d} < 0.5$), indicating mean reversion and transitory effects.

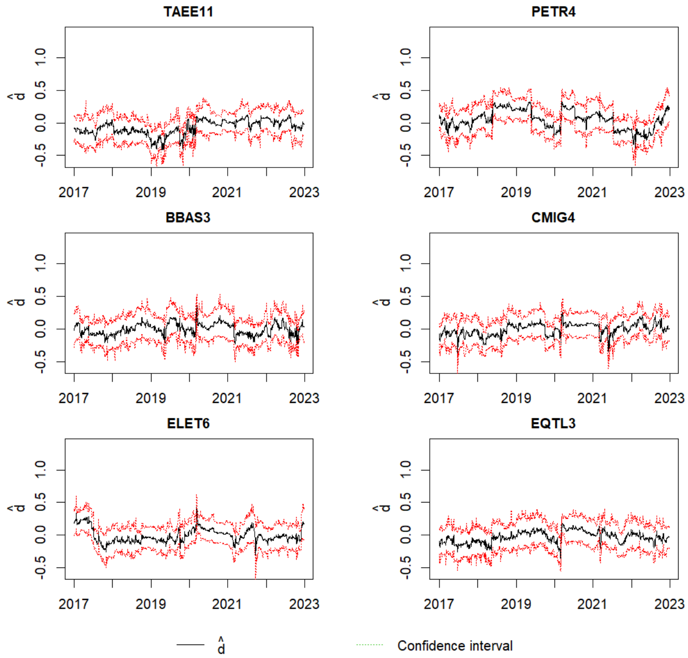
Figure 1 – Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (here, GPH is used) for the returns of companies with higher P/E



Note: 1) In the estimates of d, $g = T^{0.7}$ is considered, with T equal to the number of observations; and 2) Dashed lines (and red) corresponding to 95% confidence interval.
Source: Own elaboration.

[6] For information, Appendix A present the time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (GPH is used) for returns and volatility of IBOVESPA index.

Figure 2 – Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (here, GPH is used) for the returns of companies with lower P/E



Note: 1) In the estimates of d, $g = T^{0.7}$ is considered, with T equal to the number of observations; and 2) Dashed lines (and red) corresponding to 95% confidence interval.
Source: Own elaboration.

In addition, in the Table 3, it is demonstrated the annual average of the time-varying fractional parameter ($\hat{d}$) for returns (also for companies with higher P/E and lower P/E). It is important to say that in this case, the estimators GPH, ELW and 2SELW present similar results. As in Figures 1 and 2, the long-memory hypothesis could be rejected in most cases, both considering each year specifically and aggregating all years. Again, in the few cases where long-memory behaviour is significant, this behaviour disappears.

Furthermore, Table 4 shows the descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$), considering the whole period. The idea is complementary the analysis of long-range dependence for returns. It is observed that, although some maximum values are close to 0.5, which occurred during periods of turbulence, such as election periods and the COVID-19 pandemic, the medians are very close to zero (medians are more interesting than means due to the fact that the estimated parameter $\hat{d}$ is not normally distributed). In addition, even observing the values of the third quartile, the estimated values are still very close to zero.

In general, the results for returns, whether for companies with higher P/E or those with lower P/E, are similar to those found in the literature; see, for example, Assaf and Cavalcante (2005), Hull and McGroarty (2014) and Bhattacharya, Bhattacharya and Guhathakurta (2018). These authors do not find (or find little evidence) of long memory for return series in emerging markets, including Brazil (estimates for the IBOVESPA index). Other works, which took into account several international stock markets, also do not find evidence (or strong evidence) of long-memory behaviour, namely: Lo (1991), Jacobsen (1995), Berg and Lyhagen (1998), Crato and Ray (2000), Serletis and Rosenberg (2007), Lu and Perron (2010).

Specifically considering the period of the COVID-19 pandemic, the results of this research are in part similar to those found by some international works, at least for the returns of some companies. In other words, in some cases, there was an increase in persistence (increase in the value of the fractional parameter), but the long memory disappears. Gil-Alana and Claudio-Quiroga (2020), for example, considering three financial markets in Asia, show that for Japanese stock market (Nikkei 225 index) there is mean reversion and thus transitory effects of shocks during the pandemic.

Table 3 – Annual average of the time-varying fractional parameter ($\hat{d}$) for returns

	2017		2018		2019		2020		2021		2022		All years	
	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI
CVCB3	-0.131	(-0.34, 0.07)	-0.045	(-0.26, 0.16)	-0.039	(-0.24, 0.16)	0.201	(0.00, 0.40)	0.051	(-0.16, 0.26)	-0.03	(-0.23, -0.03)	0.001	(-0.20, 0.20)
BRFS3	-0.095	(-0.35, 0.16)	-0.045	(-0.26, 0.17)	-0.041	(-0.26, 0.17)	0.303	(0.10, 0.50)	-0.037	(-0.25, 0.18)	0.067	(-0.13, 0.07)	0.025	(-0.19, 0.24)
COGN3	0.025	(-0.2, 0.25)	-0.052	(-0.28, 0.18)	-0.061	(-0.28, 0.16)	0.032	(-0.17, 0.23)	0.017	(-0.19, 0.22)	0.075	(-0.15, 0.08)	0.006	(-0.21, 0.22)
UGPA3	-0.061	(-0.27, 0.15)	-0.089	(-0.32, 0.14)	-0.105	(-0.32, 0.11)	0.127	(-0.08, 0.33)	0.074	(-0.13, 0.27)	-0.097	(-0.32, -0.1)	-0.025	(-0.24, 0.19)
NTCO3	0.069	(-0.15, 0.28)	-0.065	(-0.30, 0.17)	-0.022	(-0.23, 0.19)	0.032	(-0.16, 0.22)	-0.097	(-0.28, 0.09)	-0.124	(-0.32, -0.12)	-0.034	(-0.24, 0.17)
USIM5	0.013	(-0.2, 0.22)	0.037	(-0.18, 0.25)	-0.111	(-0.33, 0.11)	0.039	(-0.18, 0.26)	0.044	(-0.16, 0.25)	-0.092	(-0.29, -0.09)	-0.011	(-0.22, 0.2)
TAEE11	-0.096	(-0.32, 0.13)	-0.131	(-0.33, 0.06)	-0.212	(-0.41, -0.02)	0.001	(-0.19, 0.19)	0.025	(-0.17, 0.22)	0.004	(-0.20, 0.00)	-0.068	(-0.27, 0.13)
PETR4	-0.008	(-0.22, 0.21)	0.182	(-0.02, 0.39)	0.096	(-0.09, 0.28)	0.094	(-0.12, 0.30)	-0.009	(-0.2, 0.19)	-0.071	(-0.28, -0.07)	0.048	(-0.16, 0.25)
BBAS3	-0.054	(-0.26, 0.15)	0.012	(-0.22, 0.24)	0.026	(-0.19, 0.24)	0.056	(-0.16, 0.27)	-0.061	(-0.26, 0.14)	-0.001	(-0.21, 0.00)	-0.004	(-0.22, 0.21)
CMIG4	-0.067	(-0.29, 0.15)	-0.071	(-0.28, 0.14)	0.014	(-0.18, 0.2)	0.057	(-0.13, 0.24)	-0.018	(-0.21, 0.18)	0.054	(-0.14, 0.05)	-0.005	(-0.20, 0.19)
ELET6	0.041	(-0.15, 0.24)	-0.075	(-0.26, 0.11)	-0.060	(-0.25, 0.13)	0.081	(-0.09, 0.25)	-0.020	(-0.23, 0.19)	-0.029	(-0.22, -0.03)	-0.010	(-0.20, 0.18)
EQTL3	-0.117	(-0.34, 0.11)	-0.077	(-0.29, 0.14)	0.023	(-0.20, 0.25)	0.035	(-0.16, 0.23)	-0.019	(-0.22, 0.19)	-0.04	(-0.25, -0.04)	-0.033	(-0.25, 0.18)
	2017		2018		2019		2020		2021		2022		All years	
	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI
CVCB3	-0.147	(-0.29, -0.01)	-0.033	(-0.18, 0.11)	-0.027	(-0.17, 0.11)	0.179	(0.04, 0.32)	0.029	(-0.11, 0.17)	-0.013	(-0.16, -0.01)	-0.002	(-0.14, 0.14)
BRFS3	-0.085	(-0.23, 0.06)	-0.052	(-0.19, 0.09)	-0.017	(-0.16, 0.13)	0.299	(0.16, 0.44)	0.044	(-0.1, 0.19)	0.073	(-0.07, 0.07)	0.044	(-0.1, 0.19)
COGN3	-0.033	(-0.17, 0.11)	-0.070	(-0.21, 0.07)	-0.093	(-0.24, 0.05)	0.037	(-0.11, 0.18)	-0.008	(-0.15, 0.13)	0.061	(-0.08, 0.06)	-0.018	(-0.16, 0.12)
UGPA3	-0.065	(-0.21, 0.08)	-0.047	(-0.19, 0.1)	-0.051	(-0.19, 0.09)	0.087	(-0.05, 0.23)	0.047	(-0.09, 0.19)	-0.018	(-0.16, -0.02)	-0.008	(-0.15, 0.13)
NTCO3	0.093	(-0.05, 0.23)	-0.016	(-0.16, 0.13)	-0.026	(-0.17, 0.12)	0.049	(-0.09, 0.19)	-0.083	(-0.23, 0.06)	-0.070	(-0.21, -0.07)	-0.009	(-0.15, 0.13)
USIM5	0.022	(-0.12, 0.16)	0.025	(-0.12, 0.17)	-0.082	(-0.22, 0.06)	0.035	(-0.11, 0.18)	0.069	(-0.07, 0.21)	-0.111	(-0.25, -0.11)	-0.007	(-0.15, 0.14)
TAEE11	-0.122	(-0.26, 0.02)	-0.168	(-0.31, -0.03)	-0.163	(-0.31, -0.02)	0.015	(-0.13, 0.16)	0.048	(-0.09, 0.19)	0.005	(-0.14, 0.01)	-0.064	(-0.21, 0.08)
PETR4	0.014	(-0.13, 0.16)	0.169	(0.03, 0.31)	0.103	(-0.04, 0.24)	0.103	(-0.04, 0.25)	-0.007	(-0.15, 0.13)	-0.046	(-0.19, -0.05)	0.056	(-0.09, 0.20)
BBAS3	-0.053	(-0.20, 0.09)	0.041	(-0.10, 0.18)	0.040	(-0.10, 0.18)	0.033	(-0.11, 0.18)	-0.014	(-0.16, 0.13)	0.038	(-0.10, 0.04)	0.014	(-0.13, 0.16)
CMIG4	-0.017	(-0.16, 0.13)	-0.063	(-0.20, 0.08)	0.013	(-0.13, 0.16)	0.061	(-0.08, 0.20)	0.060	(-0.08, 0.20)	0.041	(-0.10, 0.04)	0.016	(-0.13, 0.16)
ELET6	0.029	(-0.11, 0.17)	-0.093	(-0.24, 0.05)	-0.042	(-0.18, 0.10)	0.074	(-0.07, 0.22)	-0.023	(-0.17, 0.12)	-0.006	(-0.15, -0.01)	-0.010	(-0.15, 0.13)
EQTL3	-0.091	(-0.23, 0.05)	-0.040	(-0.18, 0.10)	0.032	(-0.11, 0.17)	0.037	(-0.10, 0.18)	-0.046	(-0.19, 0.1)	-0.059	(-0.20, -0.06)	-0.028	(-0.17, 0.11)
	2017		2018		2019		2020		2021		2022		All years	
	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI
CVCB3	-0.117	(-0.26, 0.03)	-0.021	(-0.16, 0.12)	-0.026	(-0.17, 0.12)	0.167	(0.03, 0.31)	0.002	(-0.14, 0.14)	-0.017	(-0.16, -0.02)	-0.002	(-0.14, 0.14)
BRFS3	-0.187	(-0.33, -0.05)	-0.061	(-0.20, 0.08)	0.037	(-0.1, 0.18)	0.310	(0.17, 0.45)	0.030	(-0.11, 0.17)	0.072	(-0.07, 0.07)	0.033	(-0.11, 0.18)
COGN3	-0.054	(-0.20, 0.09)	-0.063	(-0.20, 0.08)	-0.087	(-0.23, 0.06)	0.040	(-0.10, 0.18)	-0.014	(-0.16, 0.13)	0.038	(-0.10, 0.04)	-0.023	(-0.17, 0.12)
UGPA3	-0.075	(-0.22, 0.07)	-0.075	(-0.22, 0.07)	-0.179	(-0.32, -0.04)	0.078	(-0.06, 0.22)	0.040	(-0.1, 0.18)	0.004	(-0.14, 0.00)	-0.035	(-0.18, 0.11)
NTCO3	0.043	(-0.10, 0.18)	-0.016	(-0.16, 0.13)	-0.049	(-0.19, 0.09)	0.093	(-0.05, 0.23)	-0.238	(-0.38, -0.1)	-0.054	(-0.20, -0.05)	-0.037	(-0.18, 0.11)
USIM5	-0.088	(-0.23, 0.05)	0.089	(-0.05, 0.23)	-0.082	(-0.22, 0.06)	0.011	(-0.13, 0.15)	0.097	(-0.04, 0.24)	-0.096	(-0.24, -0.1)	-0.011	(-0.15, 0.13)
TAEE11	-0.112	(-0.25, 0.03)	-0.119	(-0.26, 0.02)	-0.114	(-0.26, 0.03)	0.016	(-0.13, 0.16)	-0.030	(-0.17, 0.11)	-0.009	(-0.15, -0.01)	-0.061	(-0.20, 0.08)
PETR4	0.000	(-0.14, 0.14)	0.163	(0.02, 0.3)	0.078	(-0.06, 0.22)	0.103	(-0.04, 0.24)	-0.055	(-0.2, 0.09)	-0.042	(-0.18, -0.04)	0.041	(-0.10, 0.18)
BBAS3	-0.081	(-0.22, 0.06)	-0.009	(-0.15, 0.13)	0.050	(-0.09, 0.19)	0.023	(-0.12, 0.17)	-0.064	(-0.21, 0.08)	0.031	(-0.11, 0.03)	-0.008	(-0.15, 0.13)
CMIG4	-0.023	(-0.16, 0.12)	-0.137	(-0.28, 0.00)	0.002	(-0.14, 0.14)	0.060	(-0.08, 0.2)	0.033	(-0.11, 0.17)	0.035	(-0.11, 0.04)	-0.005	(-0.15, 0.14)
ELET6	0.023	(-0.12, 0.17)	-0.146	(-0.29, -0.01)	-0.067	(-0.21, 0.08)	0.066	(-0.08, 0.21)	-0.122	(-0.26, 0.02)	-0.030	(-0.17, -0.03)	-0.046	(-0.19, 0.10)
EQTL3	-0.047	(-0.19, 0.10)	-0.089	(-0.23, 0.05)	0.027	(-0.11, 0.17)	-0.035	(-0.18, 0.11)	-0.089	(-0.23, 0.05)	-0.175	(-0.32, -0.18)	-0.068	(-0.21, 0.07)

Source: Own elaboration.

Table 4 – Descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$) for returns

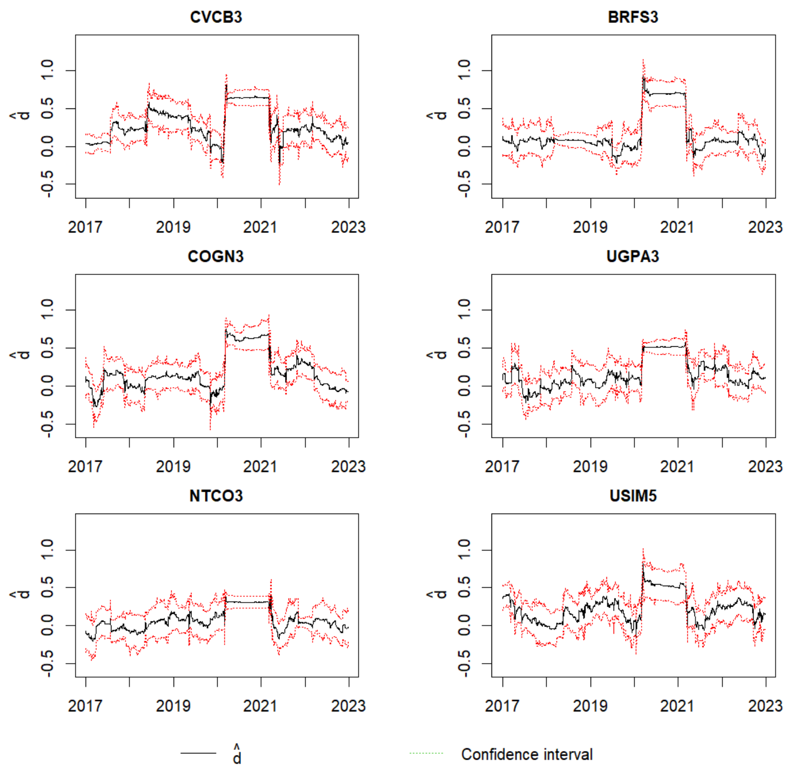
	GPH											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	-0.0112	-0.0151	0.0141	-0.0410	-0.0198	-0.0004	-0.0594	0.0554	-0.0056	0.0065	-0.0333	-0.0294
Mean	0.0012	0.0252	0.0057	-0.0251	-0.0344	-0.0115	-0.0684	0.0476	-0.0038	-0.0052	-0.0101	-0.0326
Minimum	-0.3206	-0.2633	-0.3610	-0.2885	-0.3028	-0.2940	-0.4311	-0.3910	-0.2349	-0.3849	-0.2985	-0.3644
Maximum	0.5581	0.4850	0.2373	0.3009	0.2821	0.2916	0.1270	0.3241	0.3648	0.2566	0.4271	0.1813
1. Quartile	-0.0987	-0.0931	-0.0530	-0.1100	-0.1106	-0.0803	-0.1376	-0.0713	-0.0717	-0.0715	-0.0860	-0.0961
3. Quartile	0.0667	0.0825	0.0711	0.0840	0.0342	0.0545	0.0179	0.1693	0.0562	0.0625	0.0452	0.0324
Variance	0.0182	0.0266	0.0077	0.0131	0.0094	0.0096	0.0120	0.0207	0.0074	0.0080	0.0115	0.0076
Stdev	0.1350	0.1630	0.0879	0.1144	0.0971	0.0982	0.1095	0.1438	0.0862	0.0896	0.1072	0.0873
Skewness	0.5931	0.9059	-0.3850	0.0546	-0.1389	-0.2700	-0.6767	-0.1073	0.1771	-0.4323	0.8214	-0.2106
Kurtosis	0.1111	-0.2219	0.0193	-1.0053	-0.3257	-0.2468	0.1751	-0.8395	-0.0312	0.1488	0.4486	-0.2893
JB	88.062	206.45	36.798	62.933	11.204	21.740	115.55	46.155	7.821	47.791	180.03	16.047
	ELW											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	-0.0008	0.0038	-0.0160	-0.0241	0.0013	0.0039	-0.0656	0.0320	0.0242	0.0389	-0.0222	-0.0464
Mean	-0.0021	0.0436	-0.0180	-0.0078	-0.0087	-0.0067	-0.0642	0.0561	0.0140	0.0160	-0.0104	-0.0278
Minimum	-0.2665	-0.1619	-0.1935	-0.1692	-0.2278	-0.2106	-0.2446	-0.3106	-0.2164	-0.1751	-0.2053	-0.3066
Maximum	0.5383	0.4291	0.2277	0.1831	0.2373	0.2285	0.1395	0.4296	0.3797	0.2479	0.4332	0.1866
1. Quartile	-0.0803	-0.0677	-0.0786	-0.0617	-0.0723	-0.0715	-0.1543	-0.0395	-0.0167	-0.0466	-0.0680	-0.0850
3. Quartile	0.0384	0.0957	0.0509	0.0520	0.0574	0.0525	0.0320	0.1280	0.0477	0.0786	0.0391	0.0425
Variance	0.0152	0.0206	0.0056	0.0049	0.0074	0.0069	0.0093	0.0147	0.0032	0.0052	0.0069	0.0054
Stdev	0.1232	0.1435	0.0752	0.0700	0.0862	0.0830	0.0962	0.1212	0.0569	0.0719	0.0830	0.0736
Skewness	0.4931	1.0407	0.0175	0.4175	-0.1101	-0.0858	-0.0358	0.2491	-0.0880	-0.4844	0.8801	0.1104
Kurtosis	0.0583	-0.0391	-0.7618	-0.9709	-0.5292	-0.5008	-1.5774	-0.8329	3.2550	-0.7863	1.1146	-0.6517
JB	60.550	268.69	35.697	101.22	20.117	17.134	153.90	58.015	660.89	96.165	269.723	29.051
	2SELW											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	-0.0024	0.0110	-0.0239	-0.0261	-0.0048	-0.0178	-0.0655	0.0311	0.0226	0.0290	-0.0374	-0.0490
Mean	-0.0019	0.0335	-0.0235	-0.0348	-0.0368	-0.0114	-0.0614	0.0412	-0.0083	-0.0049	-0.0458	-0.0679
Minimum	-0.4950	-0.5426	-0.2356	-0.5326	-0.5510	-0.4900	-0.5343	-0.4883	-0.5231	-0.4926	-0.4999	-0.5484
Maximum	0.4614	0.4598	0.2058	0.3463	0.4538	0.5205	0.1435	0.4103	0.4466	0.2399	0.4506	0.3042
1. Quartile	-0.0750	-0.0585	-0.0773	-0.0729	-0.0849	-0.0858	-0.1140	-0.0481	-0.0255	-0.0543	-0.0968	-0.1045
3. Quartile	0.0393	0.1011	0.0394	0.0640	0.0565	0.0502	0.0307	0.1263	0.0485	0.0711	0.0423	0.0296
Variance	0.0146	0.0345	0.0058	0.0204	0.0253	0.0212	0.0113	0.0180	0.0118	0.0121	0.0194	0.0224
Stdev	0.1206	0.1856	0.0761	0.1427	0.1591	0.1454	0.1065	0.1341	0.1088	0.1101	0.1393	0.1497
Skewness	0.1901	-0.1672	-0.1653	-1.5259	-1.2236	0.9033	-1.4374	-0.3631	-2.0656	-1.9860	-0.9352	-1.1713
Kurtosis	1.0951	1.2291	-0.5550	3.5086	2.1736	3.1783	3.6955	0.6971	5.9806	5.0964	2.2580	1.8985
JB	83.914	101.28	25.609	1343.0	665.54	830.68	1361.5	63.188	3280.3	2592.1	534.33	564.84

Source: Own elaboration.

4.2. Estimates for volatilities

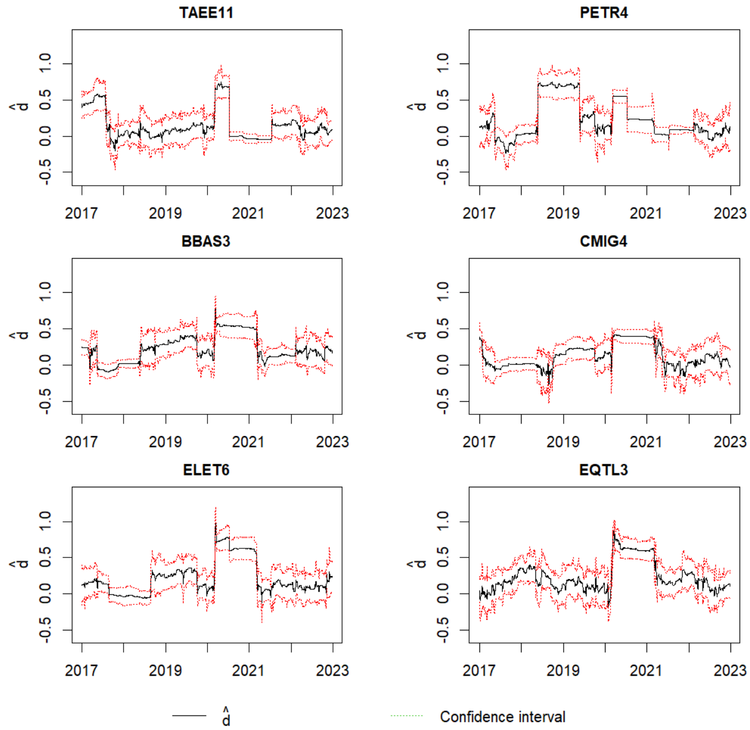
This subsection presents the results for volatilities (as previously described, here represented by square returns). In Figures 3 and 4 is possible to see the time-varying fractional coefficient ($\hat{d}$), for the volatilities of companies with higher P/E and lower P/E, respectively (GPH is used). As in the case of returns, note that the fractional parameter also varies with time. However, for volatility there is a substantial difference. In many cases, the estimated parameter is far from zero and it is statistically significant, that is, the long-range dependence hypothesis cannot be rejected. Thus, in some periods there is long-memory behavior. Specifically, these requests correspond to times of turmoil, whether internal to the Brazilian economy (presidential elections, cases of corruption or political interference in companies linked to the government, for example), or external, such as the recent crisis generated by the COVID-19 pandemic. Import to say that during COVID-19 pandemic the estimated parameter $\hat{d}$ reached significant values between 0.5 and 1 ($0.5 \leq \hat{d} < 1$), revealing periods in which volatility did not show stationary covariance, but with men reversion; the long-range dependence is transitory and disappears.

Figure 5 – Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval for volatilities of companies with higher P/E (here, GPH is used)



Note: 1) In the estimates of d, $g = T^{0.7}$ is considered, with T equal to the number of observations; and 2) Dashed lines (and red) corresponding to 95% confidence interval.
Source: Own elaboration.

Figure 6 – Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval for volatilities of companies with lower P/E (here, GPH is used)



Note: 1) In the estimates of d, $g = T^{0.7}$ is considered, with T equal to the number of observations; and 2) Dashed lines (and red) corresponding to 95% confidence interval.
Source: Own elaboration.

Table 5 presents the annual average of the time-varying fractional parameter ($\hat{d}$) for volatilities (also for companies with higher P/E and lower P/E). The results here are similar to those found for Figures 5 and 6, even using different estimators (GPH, ELX and 2SELW). Looking at the annual averages, more periods with long memory behavior are observed than in the case of returns, but, again, the highlight is the year 2020, which presented the highest estimated values ($\hat{d}$) for the fractional parameter, again, due to the COVID-19 pandemic. Still looking at Figures 3 and 4 and Table 6, it can be seen that, even with the strong increase in persistence in 2020, already in 2021, for several companies, there was a reduction in the magnitude of the value of the fractional parameter, making are even statistically insignificant. At the latest, such a reduction and non-significance occurred in the year 2022.

In Table 6 are demonstrated the descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$), for volatilities, considering the whole period. One can observe much higher median values than for the returns, and, in relation to the maximum value of the fractional parameter, for almost all companies, it presented a result higher than 0.5 ($0.5 \leq \hat{d} < 1$), again, largely due to the pandemic of the COVID-19. For some estimators, values were even greater than unity ($\hat{d} > 1$). The third quartile also showed expressive values, and they are higher than those observed for returns.

In summary, for both groups of companies, the results seem to reveal that, in periods of turbulence, whether internal or external to the Brazilian economy, volatility presents a significant long-memory behaviour, while in periods of non-turbulence, the behaviour does not exist (since the estimated parameter was not significant in most periods of non-turbulence.) Also, while for returns, only for a few companies and periods, there is a long-range dependence during the COVID-19 pandemic, in the case of volatility, the COVID-19 pandemic heightens the level of persistence. The results of this research are in line with some works that find evidence of persistence in volatility in some financial markets, especially in periods of turmoil (see Ding, Granger and Engle (1993); Bollerslev and Mikkelsen (1996); Baillie, Bollerslev and Mikkelsen (1996); Breidt, Crato and Lima (1998); Crato and Ray (2000); Caporale, Gil-Alana and Plastun (2018), among others).

As described previously, according to Engle (1982) and Bollerslev (1986), the volatility of financial returns may present a strong autocorrelation structure. To Bhattacharya, Bhattacharya and Guhathakurta (2018), there is a consensus that long memory is a characteristic of asset price volatility, which does not occur in the case of asset returns. Hull and McGroarty (2014), under study for 22 financial markets (advanced and secondary emerging markets), including Brazil, find strong evidence of long memory persistence in volatility over time. Furthermore, the authors found weak evidence of long memory in returns.

Table 5 – Annual average of the time-varying fractional parameter ($\hat{d}$) for volatilities

	2017		2018		2019		2020		2021		2022		All years	
	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI	GPH	CI
CVCB3	0.128	(-0.01, 0.27)	0.364	(0.17, 0.56)	0.241	(0.06, 0.42)	0.527	(0.42, 0.64)	0.275	(0.08, 0.47)	0.160	(-0.04, 0.16)	0.283	(0.11, 0.45)
BRFS3	0.062	(-0.14, 0.27)	0.080	(-0.03, 0.19)	-0.006	(-0.19, 0.17)	0.595	(0.42, 0.77)	0.160	(-0.03, 0.35)	0.060	(-0.12, 0.06)	0.159	(-0.01, 0.33)
COGN3	0.039	(-0.16, 0.24)	0.068	(-0.12, 0.25)	0.056	(-0.13, 0.24)	0.518	(0.36, 0.67)	0.317	(0.13, 0.51)	0.056	(-0.14, 0.06)	0.176	(-0.01, 0.36)
UGPA3	0.009	(-0.19, 0.21)	0.061	(-0.14, 0.26)	0.084	(-0.13, 0.29)	0.440	(0.34, 0.54)	0.269	(0.10, 0.44)	0.105	(-0.09, 0.11)	0.161	(-0.02, 0.34)
NTCO3	-0.045	(-0.23, 0.14)	0.007	(-0.21, 0.23)	0.076	(-0.13, 0.28)	0.279	(0.18, 0.38)	0.083	(-0.07, 0.24)	0.026	(-0.16, 0.03)	0.071	(-0.1, 0.25)
USIM5	0.166	(-0.03, 0.37)	0.106	(-0.11, 0.33)	0.222	(0.02, 0.42)	0.465	(0.26, 0.67)	0.206	(-0.01, 0.42)	0.231	(0.02, 0.23)	0.233	(0.02, 0.44)
TAEE11	0.289	(0.11, 0.47)	0.037	(-0.15, 0.23)	0.100	(-0.10, 0.30)	0.260	(0.15, 0.38)	0.055	(-0.06, 0.17)	0.090	(-0.1, 0.09)	0.139	(-0.03, 0.31)
PETR4	0.018	(-0.17, 0.20)	0.443	(0.29, 0.60)	0.397	(0.19, 0.61)	0.325	(0.18, 0.47)	0.088	(-0.01, 0.19)	0.065	(-0.12, 0.07)	0.223	(0.06, 0.39)
BBAS3	0.046	(-0.06, 0.16)	0.156	(0.01, 0.31)	0.307	(0.13, 0.48)	0.474	(0.31, 0.64)	0.190	(0.05, 0.33)	0.176	(-0.01, 0.18)	0.225	(0.07, 0.38)
CMIG4	0.025	(-0.11, 0.16)	0.004	(-0.15, 0.16)	0.186	(0.03, 0.34)	0.355	(0.25, 0.46)	0.098	(-0.08, 0.28)	0.071	(-0.14, 0.07)	0.123	(-0.03, 0.28)
ELET6	0.092	(-0.06, 0.24)	0.063	(-0.09, 0.21)	0.244	(0.06, 0.43)	0.569	(0.41, 0.72)	0.191	(0.01, 0.38)	0.111	(-0.10, 0.11)	0.212	(0.04, 0.39)
EQTL3	0.122	(-0.09, 0.33)	0.219	(0.02, 0.41)	0.118	(-0.11, 0.34)	0.537	(0.39, 0.69)	0.278	(0.09, 0.46)	0.131	(-0.08, 0.13)	0.234	(0.04, 0.43)
	2017		2018		2019		2020		2021		2022		All years	
	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI	ELW	CI
CVCB3	0.144	(0.01, 0.29)	0.349	(0.21, 0.49)	0.223	(0.08, 0.36)	0.541	(0.40, 0.68)	0.280	(0.14, 0.42)	0.144	(0.01, 0.14)	0.280	(0.14, 0.42)
BRFS3	0.035	(-0.11, 0.18)	0.077	(-0.06, 0.22)	-0.018	(-0.16, 0.12)	0.570	(0.43, 0.71)	0.167	(0.03, 0.31)	0.051	(-0.09, 0.05)	0.147	(0.01, 0.29)
COGN3	0.095	(-0.05, 0.24)	0.086	(-0.06, 0.23)	0.077	(-0.06, 0.22)	0.490	(0.35, 0.63)	0.251	(0.11, 0.39)	0.068	(-0.07, 0.07)	0.178	(0.04, 0.32)
UGPA3	0.050	(-0.09, 0.19)	0.104	(-0.04, 0.25)	0.147	(0.01, 0.29)	0.456	(0.31, 0.6)	0.269	(0.13, 0.41)	0.086	(-0.06, 0.09)	0.186	(0.04, 0.33)
NTCO3	0.005	(-0.14, 0.15)	0.073	(-0.07, 0.21)	0.092	(-0.05, 0.23)	0.285	(0.14, 0.43)	0.094	(-0.05, 0.24)	0.057	(-0.08, 0.06)	0.101	(-0.04, 0.24)
USIM5	0.156	(0.01, 0.30)	0.115	(-0.03, 0.26)	0.192	(0.05, 0.33)	0.405	(0.26, 0.55)	0.187	(0.05, 0.33)	0.230	(0.09, 0.23)	0.214	(0.07, 0.36)
TAEE11	0.272	(0.13, 0.41)	0.057	(-0.08, 0.20)	0.077	(-0.06, 0.22)	0.248	(0.11, 0.39)	0.073	(-0.07, 0.21)	0.087	(-0.05, 0.09)	0.136	(-0.01, 0.28)
PETR4	0.106	(-0.04, 0.25)	0.454	(0.31, 0.6)	0.392	(0.25, 0.53)	0.342	(0.2, 0.48)	0.110	(-0.03, 0.25)	0.053	(-0.09, 0.05)	0.243	(0.10, 0.38)
BBAS3	0.104	(-0.04, 0.25)	0.148	(0.01, 0.29)	0.277	(0.14, 0.42)	0.466	(0.32, 0.61)	0.199	(0.06, 0.34)	0.170	(0.03, 0.17)	0.227	(0.09, 0.37)
CMIG4	0.049	(-0.09, 0.19)	0.045	(-0.10, 0.19)	0.188	(0.05, 0.33)	0.348	(0.21, 0.49)	0.125	(-0.02, 0.27)	0.069	(-0.07, 0.07)	0.138	(0.01, 0.28)
ELET6	0.118	(-0.02, 0.26)	0.053	(-0.09, 0.19)	0.220	(0.08, 0.36)	0.538	(0.40, 0.68)	0.187	(0.05, 0.33)	0.068	(-0.07, 0.07)	0.198	(0.06, 0.34)
EQTL3	0.143	(0.01, 0.28)	0.168	(0.03, 0.31)	0.055	(-0.09, 0.2)	0.506	(0.36, 0.65)	0.243	(0.10, 0.38)	0.124	(-0.02, 0.12)	0.206	(0.07, 0.35)
	2017		2018		2019		2020		2021		2022		All years	
	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI	2SELW	CI
CVCB3	0.145	(0.01, 0.29)	0.349	(0.21, 0.49)	0.170	(0.03, 0.31)	0.505	(0.36, 0.65)	0.287	(0.15, 0.43)	-0.002	(-0.14, 0.00)	0.243	(0.1, 0.38)
BRFS3	0.038	(-0.10, 0.18)	0.054	(-0.09, 0.2)	-0.027	(-0.17, 0.11)	0.494	(0.35, 0.63)	0.090	(-0.05, 0.23)	0.060	(-0.08, 0.06)	0.118	(-0.02, 0.26)
COGN3	0.027	(-0.11, 0.17)	0.093	(-0.05, 0.23)	0.066	(-0.08, 0.21)	0.405	(0.26, 0.55)	0.155	(0.01, 0.30)	0.069	(-0.07, 0.07)	0.136	(-0.01, 0.28)
UGPA3	-0.011	(-0.15, 0.13)	0.097	(-0.04, 0.24)	0.135	(-0.01, 0.28)	0.416	(0.27, 0.56)	0.285	(0.14, 0.43)	0.085	(-0.06, 0.08)	0.168	(0.03, 0.31)
NTCO3	0.009	(-0.13, 0.15)	0.072	(-0.07, 0.21)	0.067	(-0.07, 0.21)	0.259	(0.12, 0.4)	0.008	(-0.13, 0.15)	0.064	(-0.08, 0.06)	0.080	(-0.06, 0.22)
USIM5	0.040	(-0.10, 0.18)	0.114	(-0.03, 0.26)	0.176	(0.03, 0.32)	0.310	(0.17, 0.45)	0.022	(-0.12, 0.16)	0.121	(-0.02, 0.12)	0.130	(-0.01, 0.27)
TAEE11	0.271	(0.13, 0.41)	0.045	(-0.10, 0.19)	0.079	(-0.06, 0.22)	0.367	(0.23, 0.51)	0.106	(-0.04, 0.25)	0.080	(-0.06, 0.08)	0.158	(0.02, 0.30)
PETR4	0.098	(-0.04, 0.24)	0.368	(0.23, 0.51)	0.340	(0.20, 0.48)	0.320	(0.18, 0.46)	0.111	(-0.03, 0.25)	-0.044	(-0.19, -0.04)	0.199	(0.06, 0.34)
BBAS3	0.108	(-0.03, 0.25)	0.142	(0.01, 0.28)	0.276	(0.13, 0.42)	0.394	(0.25, 0.54)	0.125	(-0.02, 0.27)	0.152	(0.01, 0.15)	0.200	(0.06, 0.34)
CMIG4	0.121	(-0.02, 0.26)	0.048	(-0.09, 0.19)	0.142	(0.00, 0.28)	0.338	(0.20, 0.48)	0.096	(-0.05, 0.24)	0.067	(-0.07, 0.07)	0.136	(-0.01, 0.28)
ELET6	0.114	(-0.03, 0.26)	0.050	(-0.09, 0.19)	0.186	(0.04, 0.33)	0.454	(0.31, 0.60)	0.110	(-0.03, 0.25)	0.068	(-0.07, 0.07)	0.164	(0.02, 0.31)
EQTL3	0.129	(-0.01, 0.27)	0.154	(0.01, 0.30)	-0.019	(-0.16, 0.12)	0.406	(0.26, 0.55)	0.093	(-0.05, 0.23)	0.120	(-0.02, 0.12)	0.147	(0.01, 0.29)

Source: Own elaboration.

Table 6 – Descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$) for volatilities

	GPH											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	0.2330	0.0790	0.1276	0.1035	0.0515	0.2223	0.0785	0.1245	0.1976	0.0879	0.1380	0.1715
Mean	0.2828	0.1589	0.1757	0.1614	0.0710	0.2326	0.1387	0.2229	0.2248	0.1233	0.2119	0.2343
Minimum	-0.2486	-0.2187	-0.3220	-0.2170	-0.2124	-0.1587	-0.2000	-0.2360	-0.0920	-0.2662	-0.1217	-0.1782
Maximum	0.8174	0.9348	0.7517	0.5779	0.4065	0.8229	0.7552	0.7534	0.7901	0.4258	0.9708	0.8656
1. Quartile	0.1281	0.0332	-0.0055	0.0408	-0.0209	0.0895	0.0032	0.0388	0.1161	0.0098	0.0630	0.0974
3. Quartile	0.4127	0.1057	0.2511	0.2382	0.1253	0.3211	0.1571	0.2914	0.3375	0.2262	0.2805	0.3088
Variance	0.0438	0.0652	0.0582	0.0351	0.0178	0.0321	0.0411	0.0672	0.0332	0.0253	0.0523	0.0427
Stdev	0.2092	0.2554	0.2412	0.1874	0.1335	0.1792	0.2028	0.2591	0.1822	0.1590	0.2287	0.2067
Skewness	0.4395	1.5297	0.8895	0.7500	0.5775	0.4682	1.5611	0.8706	0.3930	0.4630	1.1330	1.0350
Kurtosis	-0.6720	0.9126	-0.1192	-0.3396	-0.3976	-0.5527	1.4651	-0.5013	-0.5856	-0.7971	0.3285	0.1441
JB	75.571	632.41	197.05	146.47	92.306	73.035	738.29	203.32	59.280	92.17	325.24	267.03
	ELW											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	0.2268	0.0597	0.1173	0.1294	0.0804	0.1806	0.0747	0.1102	0.1932	0.0919	0.1366	0.1516
Mean	0.2802	0.1472	0.1779	0.1856	0.1012	0.2142	0.1359	0.2433	0.2273	0.1376	0.1976	0.2065
Minimum	-0.2497	-0.3978	-0.3965	-0.3304	-0.4071	-0.2965	-0.3436	-0.4227	-0.3618	-0.3778	-0.3061	-0.2286
Maximum	0.8628	0.9087	0.9489	1.6732	1.0362	0.7490	0.7517	1.0896	1.8146	0.7876	1.0995	1.0170
1. Quartile	0.1444	0.0237	0.0504	0.0839	0.0201	0.1006	0.0418	0.0661	0.1250	0.0293	0.0579	0.0742
3. Quartile	0.3692	0.0983	0.1757	0.2194	0.1131	0.2879	0.1434	0.2807	0.3058	0.2103	0.2352	0.2244
Variance	0.0352	0.0602	0.0395	0.0295	0.0131	0.0188	0.0274	0.0605	0.0266	0.0186	0.0428	0.0342
Stdev	0.1876	0.2454	0.1988	0.1717	0.1144	0.1371	0.1655	0.2459	0.1631	0.1365	0.2068	0.1850
Skewness	0.8162	1.5563	1.3798	1.4537	1.4257	0.7597	1.6685	1.0343	1.5469	0.7303	1.3216	1.3127
Kurtosis	-0.3830	0.9812	0.7167	4.3605	4.1250	-0.1339	1.7728	-0.3385	7.7677	0.0149	0.8799	0.7335
JB	174.11	660.86	504.36	1705.9	1561.8	144.170	886.292	272.22	4340.7	132.28	481.67	461.09
	2SELW											
	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	0.2175	0.0599	0.0988	0.1221	0.0753	0.1130	0.0827	0.1030	0.1633	0.0897	0.1125	0.1176
Mean	0.2429	0.1181	0.1358	0.1680	0.0798	0.1304	0.1581	0.1993	0.1997	0.1356	0.1639	0.1473
Minimum	-0.4858	-0.4327	-0.3217	-0.4884	-0.4984	-0.2632	-0.2369	-0.4732	-0.1330	-0.2134	-0.2082	-0.3449
Maximum	0.6682	0.9927	0.6567	0.6025	0.3950	0.4898	1.1261	0.9348	0.5761	0.6780	0.8861	0.6798
1. Quartile	0.0982	0.0182	0.0372	0.0749	0.0177	0.0625	0.0442	0.0567	0.0985	0.0237	0.0478	0.0552
3. Quartile	0.3684	0.1038	0.1455	0.2069	0.1110	0.1926	0.1491	0.2814	0.2790	0.2070	0.1923	0.1923
Variance	0.0558	0.0526	0.0384	0.0343	0.0173	0.0291	0.0427	0.0630	0.0232	0.0232	0.0360	0.0361
Stdev	0.2362	0.2293	0.1959	0.1852	0.1317	0.1707	0.2067	0.2510	0.1523	0.1524	0.1898	0.1901
Skewness	-0.2424	1.6740	1.2606	0.2701	-0.1953	0.1967	1.9611	0.5954	0.7823	0.7193	1.6017	0.7757
Kurtosis	0.4773	2.3549	1.5088	0.9847	1.7480	-0.0080	4.0714	0.6531	-0.1877	0.0208	1.8720	1.3983
JB	28.941	1040.21	536.12	78.743	199.92	9.5901	1984.23	114.71	153.86	128.36	854.60	271.23

Source: Own elaboration.

5. General considerations

The focus of this research is to verify whether there is short or long memory behaviour in returns and volatilities and whether the behaviour is similar for two groups of companies listed on IBOVESPA: higher and lower price-earnings (P/E) ratios in December 2019. It is considered the period from 01 January, 2016 to 31 December, 2022. The fractionally integrated parameter (d) is used to check if there is long-range dependence or not. Firstly, the GPH estimator is adopted in all estimates. To ensure the robustness of the results, in some estimates, two other estimators are used: ELW and 2SELW. Furthermore, since the estimated fractionally integrated parameter may vary over time, rolling estimation is used to capture the time-variation of $\hat{d}$.

The results reveal that: i) for returns, the estimates seem to reveal a behaviour of non-persistence and, even in the few cases of statistically significant persistence, as in the COVID-19 pandemic, most of the time, the long-memory is transitory and disappears; ii) for volatility, there is a substantial difference. In some cases, the estimated parameter is far from zero and it is statistically significant, that is, the long-range dependence hypothesis cannot be rejected. Then, in some periods there is long-memory behaviour. Specifically, these requests correspond to times of turmoil, whether internal to the Brazilian economy (presidential elections, cases of corruption or political interference in companies linked to the government, for example), or external, such as the recent crisis generated by the COVID-19 pandemic. During COVID-19 pandemic the estimated parameter $\hat{d}$ reached significant values between 0.5 and 1 ($0.5 \leq \hat{d} < 1$), revealing periods in which volatility did not show stationary covariance; and iii) moreover, even in periods of possible above-average returns (as described by the literature contrary to the HME), as the COVID-19 pandemic, this is not possible without the investor incurs above-average risks.

In additon, for returns e volatilities, the results are very similar for both groups of companies (higher P/E or lower P/E). The long-memory behaviour, when occurs, is not constant over time, transitory and disappears. Thus, the hypothesis of this research is confirmed. Especially for volatility, there seems to be a cyclical pattern of efficiency/inefficiency, with inefficiency arising from periods of negative turbulence (internal or external to the Brazilian economy). However, in all cases, there is a convergence towards efficiency, which can be explained, for example, by the increase in computational capacity and the speed at which information is disseminated to agents (Santos, Fávero, Brugni and Serra, 2023).

Therefore, two main considerations can be derived from the results, and it may be said that they are in line with the authors who claim that AMH can reconcile EMH with all its behavioural alternatives: i) one of the practical implications of the AMH is that the profit opportunities arise from time to time depending on the degree of market efficiency and according to market conditions. This can be seen for some periods, especially for volatility; and ii) as previously described, to Malkiel (2003), some market participants are demonstrably less than rational. Thus, pricing irregularities and even predictable patterns in stock returns can appear over time and even persist for short periods. However, to author, the end result will not be the abandonment of the belief of many in the profession that the stock market is remarkably efficient in the use of information. Furthermore, as previously described, Malkiel (2003) states that efficient financial markets are those that do not allow investors to earn above-average returns without accepting above-average risks. In this study, especially for volatility, there is great persistence in moments of negative turbulence, but with transitory persistence.

A possible extension of this study is to analyse market efficiency for other groups of companies, considering some sectors, for example, or even using other metrics to separate groups, such as the size of dividend yields. Another extension of this work could be to investigate market efficiency for different frequencies (e.g., weekly and monthly). In addition, check for possible structural changes (structural breaks) in the time-varying estimated fractional parameter ($\hat{d}$).

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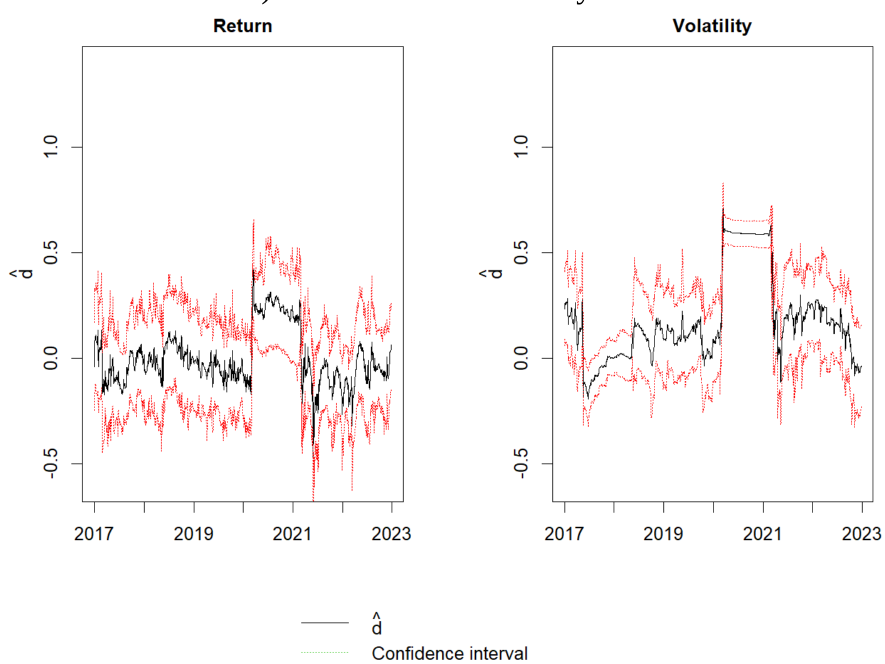
Appendix A

Figure 1A – Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (here, GPH is used) for returns and volatility of IBOVESPA



Source: Own elaboration.