Forecasting the yield curve: the role of additional and time-varying decay parameters, conditional heteroscedasticity, and macro-economic factors

João F. Caldeira
Dept. Economics, Universidade Federal de Santa Catarina, Florianòpolis (Brazil)
ORCID: 0000-0002-0723-0461

Werley C. Cordeiro
Dept. Economics, Universidade Federal de Santa Catarina, Florianòpolis (Brazil)
ORCID: 0000-0001-8394-0417

Esther Ruiz*
Dept. Statistics, Universidad Carlos III de Madrid, Getafe (Spain)
ORCID: 0000-0002-5944-9449

Andrè A.P. Santos
Dept. Quantitative Methods, CUNEF, Madrid (Spain)
ORCID: 0000-0001-6134-8930

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Abstract

In this paper, we analyse the forecasting performance of several parametric extensions of the popular Dynamic Nelson-Siegel model for the yield curve. We focus on the role of additional and time-varying decay parameters, conditional heteroscedasticity and macroeconomic variables. We also consider the role of several popular restrictions on the dynamics of the factors. Using a novel end-of-month continuously compounded Treasury yields on US zero-coupon bonds and frequentist estimation based on the extended Kalman filter, we show that a second decay parameter does not have any role in obtaining better forecasts. Also, in concordance with the preferred habitat theory, we show that, the best forecasting model depends on the maturity. For short maturities, the best performance is obtained in a heteroscedastic model with time-varying decay parameter. However, for long maturities, neither the time-varying decay nor the heteroscedasticity plays any role and the best fit is obtained in the basic DNS model with the shape of the yield curve depending on macroeconomic activity. Consequently, models for the yield

*Corresponding author. E-mail: ortega@est-econ.uc3m.es

curve should incorporate some short of non-linearity depending on the maturity. Furthermore, assuming non-stationary factors is helpful in forecasting at long horizons.

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JEL codes: C53; E43; G17.

1 Introduction

The yield curve of government bonds, i.e. the relationship between interest rates and time to maturity, is often regarded as a benchmark due to its high liquidity and low credit risk. Accurate forecasts of the term structure of interest rates or yield curve are relevant for investors and policy makers when pricing interest rate contingent assets, constructing asset and bond portfolios, managing financial risk or making economic policy decisions; see Hodges & Schaefer (1977) and Ronn (1987) for early references. Recently, these forecasts have also become important in the context of new tools of unconventional monetary policy as yield curve control or forward guidance; see, for example, Kuttner (2018) and Bernanke (2020). Consequently, during the last two decades, a large number of works has been devoted to developing alternative methodologies for modelling the term structure. These methodologies can be classified into three main alternative approaches.

First, several models focus on fitting the term structure at a point in time to ensure that no-arbitrage possibilities exist. These models are usually estimated using regression-based procedures; see the recent work by Golinski & Spencer (2021) and the references therein. In many empirical studies, it is suggested that imposing no-arbitrage conditions does not generally lead to more accurate predictions; see, for example, Joslin *et al.* (2011).

Second, many authors estimate the term structure using affine models, originally characterized by Duffie & Kan (1996), which allow multiple state variables to drive interest rates with bond yields being linear functions of these variables; see Duffee & Stanton (2012) for a comparison of alternative estimators of affine models, including Kalman filter-based estimators. Recent estimation techniques of affine models for the term structure rely on Bayesian procedures, which are computationally demanding; see, for example, Carriero *et al.* (2021) for a recent Bayesian estimator of the canonical affine term structure model, in its equivalent but computationally more stable representation of Joslin *et al.* (2011).

Finally, motivated by the rational expectation theory, Nelson & Siegel (1987) express spot interest rates in terms of forward rates and propose a three factor model for the yield curve, with the factors representing the level, slope and curvature of the curve. As a result of this interpretation of the factors, the factor loadings are heavily parametrized depending on a single exponential decay rate parameter; see Coroneo *et al.* (2011) and

Krippner (2012) for the connection between the DNS model and affine and arbitrage-free models, respectively. Diebold & Li (2006) propose modelling the dynamic evolution of the yield curve by extending the three factor model to allow for time-varying factors. The resulting model is the so-called Dynamic Nelson-Siegel (DNS) model, which is a major workhorse among academics and it is also widely used in the financial community; see, for example, Diebold *et al.* (2008), Yu & Salyards (2009), Christensen *et al.* (2011), Laurini & Hotta (2014) and Caldeira *et al.* (2016), for some few selected empirical applications, and BIS (2005) and ECB (2018) for its implementation by practitioners. In spite of its popularity, the Achilles heal of the DNS model lies in its poor forecasting performance, with forecasts that hardly beat those obtained by a random walk model. As far as we are concerned, Christensen *et al.* (2011) and Coroneo *et al.* (2016) are among the few that conclude that the DNS has a good forecasting performance improving the random walk predictions. Two main reasons have been put forward to explain this poor performance. First, the specification of the model and, in particular, the restrictions imposed on the factor loadings, may not hold in practice; see Jungbacker *et al.* (2014) and Carriero *et al.* (2021). Alternatively, there is also a literature suggesting that the performance of DNS type models has deteriorated in the post global financial crisis due to the low variability of interest rates during the zero-lower-bound interest rate constraints period from 2008 to 2012; see, for example, Diebold & Rudebusch (2013) and Altavilla *et al.* (2017).

The main contribution of this paper is the empirical analysis of the forecasting performance of a very general and flexible specification of the DNS model. In particular, we consider three extensions of the original specification, which could flexibilice the model and mitigate the adverse effects of potential misspecification. First, early on Svensson (1994) proposes a four-factor version of the Nelson & Siegel (1987) model; see Koopman *et al.* (2010), Almeida *et al.* (2018) and Swanson *et al.* (2020) for applications with four factors. Second, other two important extensions of the DNS model are due to Koopman *et al.* (2010), who extend it in two directions. On the one hand, they propose allowing the decay parameter to evolve over time; see Laurini & Hotta (2010), Hevia *et al.* (2015) and Swanson *et al.* (2020), for other proposals in which the decay parameter is allowed to change over time. On the other hand, Koopman *et al.* (2010) propose representing the overall volatility by a conditionally heteroscedastic GARCH model. Allowing for the evolution of volatility seems to be an important characteristic of the yield curve.[1] Caldeira *et al.* (2010) and Laurini & Caldeira (2016) also allow for time-variation in both the decay parameter and volatilities. Finally, an useful and popular extension of the DNS model considered in this paper, is the inclusion of macroeconomic variables to explain the yield curve; see Gürkaynak & Wright (2012) and Morley (2016) for surveys on the relationships between the yield curve and the macro-economy. Diebold *et al.* (2006) propose augmenting the specification of the yield curve by adding macroeconomic variables to explain the evolution of the level, slope and curvature factors; see Bianchi *et al.* (2009), and... for applications.

In order to disentangle the role played by the four extensions described above in predicting the yield curve, we consider a DNS model with two time-varying decay parameters, macroeconomic variables, and conditional heteroscedasticity, which is fitted to a novel data set of end-of-month continuously compounded Treasury yields on US zero-coupon bonds. Estimation of the extended DNS model is carried out using the Extended Kalman filter. Several important conclusions are obtained from this analysis. First, we show that the second decay rate does not have any role in obtaining a better forecasting performance of the factor model. Second, we show that the best specification depends on the maturity. For short maturities, the best performance is obtained in a heteroscedastic DNS model with time-varying decay. However, for large maturities, the simplest homoscedastic model with constant decay performs better if the shape of the yield curve depends on economic activity. Finally, we also show that the maturity for which the model switches is larger the larger the prediction horizon. For example, when the predictions are obtained one-step-ahead, the model switches for maturities larger than 21 months. However, one predicting one-year ahead, the model switches for maturities larger than 36 months. These results suggest that the yield curve model should incorporate some short of non-linearity depending on the maturity.

The outline of the paper is as follows. Section 2 describes the DNS model and its extensions considered in this paper. In Section 3, the model is fitted to a data set of end-of-month continuously compounded Treasury yields on US zero-coupon bonds. In

[1] Note that this extension could also be relevant for density forecasts of the yield curve; see, for example, Carriero *et al.* (2021) and Shin & Zhang (2017). Density forecasting of interest rates is important for derivatives pricing and risk management.

Section ??, we fit the switching model. Finally, Section 5 concludes.

2 Extensions of the Dynamic Nelson-Siegel model

In this section, we describe the DNS model for the yield curve as well as some of its more popular extensions. We also describe estimation of the model parameters.

2.1 Dynamic Nelson-Siegel model

The DNS model, originally proposed by Diebold & Li (2006) to represent the term structure of interest rates, is given by

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \varepsilon_{it}, \quad (1)$$

$$\beta_{t+1} = \mu + \Phi\left(\beta_t - \mu\right) + \eta_t, \quad (2)$$

where $y_t(\tau_i)$ is the yield of a security with maturity τ_i, $i = 1, \ldots, N$, observed at time t, for $t = 1, \ldots, T$, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ is the vector of factors, which represent the level (β_1), slope (β_2), and curvature (β_3) of the yield curve, and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is an $N \times N$ Gaussian white noise vector with diagonal covariance matrix Σ_ε; see, for example, Diebold & Li (2006), Diebold *et al.* (2006), Koopman *et al.* (2010), Exterkate *et al.* (2013) and Jungbacker *et al.* (2014) for the diagonality of Σ_ε. Finally, η_t is a Gaussian white noise vector, which is independent of ε_t for all lags and leads and has full covariance matrix Σ_η. The λ parameter is a strictly positive decay parameter that governs the decay of interest rates when maturity increases. Small (large) values of λ produce a slow (fast) decay. The decay parameter, λ, also governs where the loading on the curvature, β_{3t}, achieves its maximum. Finally, μ is a 3×1 vector of constant parameters and Φ is a 3×3 matrix of autoregressive parameters that govern the dynamics of the factors. The matrix Φ satisfies the stationarity conditions.[2]

Two main characteristics explain the popularity of the DNS model to explain the yield curve. First of all, the model can be easily estimated using standard estimation techniques,

[2] Alternatively, several authors propose modelling the term structure using factor models with the loadings represented by polynomial splines; see, for example, Bowsher & Meeks (2008), Koopman & van der Wel (2013) and Jungbacker *et al.* (2014).

avoiding the heavy computations often associated to Bayesian estimation procedures. Second, the factors have meaningful economic interpretations and can embody different aspects of monetary policy. First, the limit of the yield when maturity increases is β_{1t}, which embodies any effects of monetary policy that simultaneously shift all interest rates. Moreover, β_{2t} is related with conventional monetary policy, which typically affects short rates more than long rates, thereby changing the so-called term spread of the yield curve. Finally, an increase in β_{3t} increases medium-term yields and have little effect on short and long interest rates. It is related with unconventional monetary shocks, such as forward guidance or monetary policy announcements. Also note that some linear combinations of the factors are also of interest. For example, $\beta_{1t} + \beta_{2t}$ represents the instantaneous yield while $\beta_{3t} - \beta_{1t}$ represents changes in long-run expectations or risk premium that do not result in parallel shifts in the term structure.

As mentioned above, one of the advantages of the DNS model is its easy estimation with alternative estimators of its parameters available. First, Diebold & Li (2006) propose fixing the exponential decay rate, λ, to a constant chosen by the researcher and estimating the other parameters in the model by a simple two-step estimation procedure. In particular, they propose fixing $\lambda = 0.0609$, with the curvature having its larger impact on the 30-month maturity bond; see Bianchi *et al.* (2009), Swanson & Williams (2014), van Dijk *et al.* (2014), Byrne *et al.* (2017) and Almeida *et al.* (2018) for implementations using this value. If there are sufficient interest rates with different maturities at each period of time, in the first step, the factors, β_t, are estimated by Ordinary Least Squares (OLS) at each time period t. Assuming that Φ is diagonal, in the second step, univariate AR(1) models are fitted to each of the estimated factors; see, for example, Swanson & Williams (2014), van Dijk *et al.* (2014), Byrne *et al.* (2017), Almeida *et al.* (2018) and Inoue & Rossi (2021) for implementations.

Second, following Diebold *et al.* (2006), the DNS model can be viewed as a dynamic factor model (DFMs) with restricted factor loadings as follows

$$y_t = \Lambda\beta_t + \varepsilon_t, \tag{3}$$

where $y_t = (y_t(\tau_1), ..., y_t(\tau_N))'$ and Λ is an $N \times 3$ matrix of factor loadings, with its (i, j)

element given by[3]

$$\Lambda_{ij} = \begin{cases} 1, & j = 1 \\ \dfrac{1 - \exp(-\lambda\tau_i)}{\lambda\tau_i}, & j = 2 \\ \dfrac{1 - \exp(-\lambda\tau_i)}{\lambda\tau_i} - \exp(-\lambda\tau_i), & j = 3. \end{cases} \quad (4)$$

If the model parameters were known, the Kalman filter and smoothing (KFS) algorithms can be implemented to extract the factors.[4] In practice, the parameters, including the decay parameter, λ, can be estimated by Quasi Maximum Likelihood (QML), using the prediction error decomposition of the Gaussian likelihood. For the optimization of the likelihood, the start-up parameters of Φ can be obtained using the two-step OLS estimator described above, all variances can be initialized at 1.0 and $\lambda = 0.0609$; see Exterkate *et al.* (2013) and Joslin *et al.* (2013) for implementations. Once the parameters are estimated, the filter can be used to obtain one-step-ahead predictions of the underlying level, slope and curvature, and consequently, of the yields. Furthermore, h-step-ahead predictions can be obtained by running the prediction equations of the filter alone.

Next, we describe several popular extensions of the DNS model and how the Kalman filter methodology can be updated to each of them. In particular, we consider additional and time-varying discount parameters, the inclusion of macroeconomic variables and of conditional heteroscedasticity.

2.2 Additional and time-varying decay parameters

Svensson (1994) proposes extending the DNS model in (1) and (2) by including an additional decay parameter, which allows the yield curve to have more flexible shapes with

[3]Other authors have proposed alternative specifications of the loadings. For example, Bowsher & Meeks (2008) and Almeida *et al.* (2018) propose using polynomial basis instead of exponential functions, while Jungbacker *et al.* (2014) do not impose a fixed structure of the loadings further than some smoothing conditions.

[4]The filter can be initialized using the unconditional mean and covariance matrix of β_t, namely, $b_{1|0} = E[\beta_t] = \mu$ and $P_{1|0} = E[\beta_t\beta_t'] = \Sigma_\beta$, with the latter being the solution of $\Sigma_\beta - \Phi\Sigma_\beta\Phi = \Sigma_\eta$, which can be solved using the properties of the vectorization operator; see Christensen & van der Wel (2019) for details.

two humps. The so-called Dynamic-Nelson-Siegel-Svensson (DNSS) model is given by

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_1\tau_i}}{\lambda_1\tau_i}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_1\tau_i}}{\lambda_1\tau_i} - e^{-\lambda_1\tau_i}\right) + \beta_{4t}\left(\frac{1 - e^{-\lambda_2\tau_i}}{\lambda_2\tau_i} - e^{-\lambda_2\tau_i}\right) + \varepsilon_{it}, \tag{5}$$

where $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t})'$ is defined as in (2) and λ_1 and λ_2 are both strictly positive and distinct to avoid multicollinearity.

Gürkaynak *et al.* (2007) show that the yield curve often needs two humps, one at short maturities associated with monetary policy expectations and another at long maturities to capture convexity effects; see also the results by Almeida *et al.* (2018) and Swanson *et al.* (2020). The second hump in the DNSS model is difficult to identify without imposing additional restrictions; see, for example, the empirical results in Gürkaynak *et al.* (2007). Consequently, several authors propose restrictions to guarantee that the two humps are far apart; see, for example, Ferstl & Hayden (2010), Pedersen & Swanson (2019), Sasongko *et al.* (2019) and Walstrøm *et al.* (2022). However, it is important to note that Walstrøm *et al.* (2022) conclude that the restrictions can be disadvantageous when using the yield curve for monetary policy decisions.

The estimation procedures described above for the DNS model can be easily adapted to estimate the DNSS model.[5]

Koopman *et al.* (2010) propose making the yield curve more flexible by allowing the decay parameter, λ, to be time-varying. To guarantee that $\lambda > 0$, they specify a model for $\log(\lambda_t)$, assuming that it follows an AR(1) model. The resulting state space representation is given by

$$y_t = \Lambda(\lambda_t)B_t + \varepsilon_t, \tag{6}$$

$$B_t = \Phi B_{t-1} + \eta_t, \tag{7}$$

where $B_t = (\beta_{1t} - \mu_1, \beta_{2t} - \mu_2, \beta_{3t} - \mu_3, log(\lambda_t) - \mu_4)$ and $\Lambda(\lambda_t)$ is an $N \times 4$ matrix with the elements of its first three columns defined as in (3) with λ substituted by λ_t and the fourth column being a column of zeros. The model with time-varying decay parameter is

[5]Walstrøm *et al.* (2022) propose a non-linear LS method.

denoted as DNS-TVL. Note that the DNSS model with two decay parameters can also be extended to allow for both parameters to be time-varying. In this case, the model will be called DNSS-TVL while if only the first (second) decaying parameter is time-varying, the model is denoted as DNSS-TVL1 (DNSS-TVL2).

Since the measurement equation of the DNS-TVL model is non-linear, estimation of its parameters can no longer rely on the Kalman filter. To overcome this problem, estimation can be carried out using the extended Kalman filter (EKF), which accounts for non-linearities by using, at each time t, a first-order Taylor expansion around the current state estimate; see Jazwinski (1970), who demonstrates that the EKF is particularly effective in dealing with this type of non-linearity. Denote by $H(B_t) = \Lambda_1(\lambda_t)\beta_{1t} + \Lambda_2(\lambda_t)\beta_{2t} + \Lambda_3(\lambda_t)\beta_{3t}$ with $\Lambda_k(\lambda_t)$ being the k-th column of $\Lambda(\lambda_t)$, and by $b_{t|t-1} = \left(b_{1,t|t-1}, b_{2,t|t-1}, b_{3,t|t-1}, l_{t|t-1}\right)'$, the KF one-step-ahead predictions of B_t. The measurement equation in (6) can be linearised around $b_{t|t-1}$ by approximating $H(B_t)$ as follows

$$H(B_t) \approx H\left(b_{t|t-1}\right) + \dot{H}\left(b_{t|t-1}\right)\left(B_t - b_{t|t-1}\right), \tag{8}$$

where $\dot{H}(B_t) = \frac{\partial H(B_t)}{\partial B_t} = [\iota_N, \Lambda_2(\lambda_t), \Lambda_3(\lambda_t), A_t]$, with ι_N being a $N\times 1$ vector of ones and $A_t = \lambda_t \frac{\partial H(B_t)}{\partial \lambda_t}$ with its i'th element given by $\frac{\exp(-\tau_i\lambda_t)}{\tau_i\lambda_t}[(\beta_2+\beta_3)(\lambda_t\tau_i + 1 - \exp(\lambda_t\tau_i)) + \lambda_t^2\tau_i^2\beta_{3t}]$.

The Kalman filter is then run with the following measurement equation[6]

$$y_t = \beta_{1t} + \Lambda_2\left(l_{t|t-1}\right)\beta_{2t} + \Lambda_3\left(l_{t|t-1}\right)\beta_{3t} + A_t\left(log\lambda_t - l_{t|t-1}\right) + \varepsilon_t. \tag{9}$$

2.3 Adding macroeconomic variables

Several authors conclude that macroeconomic variables are significant for explaining bond yield dynamics. Central banks around the world use interest rates as their main monetary policy instrument responding to macroeconomic variables such as inflation or output. The link between the yield curve and macroeconomic aggregates may also exists in the reverse direction due to economic agents responding to changes in interest rates.

Consequently, the DNS model in (1) and (2) has also been extended by assuming that

[6] Alternatively, Gimeno & Nave (2018) propose a genetic algorithm for the estimation of the parameters of the DNSS-TVL model.

the level, slope and curvature of the yield curve, depend on the macroeconomic and financial activity, which could be represented by the inclusion of key macro-finance indicators in the equation that governs the dynamic evolution of β_t. For example, Diebold *et al.* (2006) find strong in-sample evidence in the US in favour of causal linkages between manufacturing capacity utilization, monthly average of the federal funds rate, and 12-month percent change in the price deflator for personal consumption expenditures, and future yield curve dynamics. Byrne *et al.* (2017) also find evidence in the US of a better forecasting performance when including macroeconomic variables in the DNS model, among them the Federal Fund Rate, CPI inflation and Industrial Production while Bianchi *et al.* (2009) use de-trended output, annualized monthly inflation and the policy interest rate to explain the term structure of interest rates in the UK.

Instead of using a set of specific macroeconomic variables to explain the factors of the yield curve, the macroeconomic information is often summarized extracting diffusion indexes from large sets of economic variables using Principal Components (PC); see Pedersen & Swanson (2019) for a survey on recent empirical findings regarding the out-of-sample forecast usefulness of including diffusion indexes in DNS type models.[7] Favero *et al.* (2012) favour the forecast performance of factor-augmented DNS (FA-DNS) models when compared with a large number of alternatives. They conclude that macroeconomic information is more useful at longer forecast horizons and longer maturities. Exterkate *et al.* (2013) conclude that FA-DNS models perform well in relatively volatile periods with reductions of 20%-30% in mean square forecast errors (MSFEs) when compared with the simplest DNS model. Swanson & Williams (2014) also observe decreasing sensitivity, beginning in late 2011, of medium-term interest rates to macroeconomic news. It is important to note that Swanson & Xiong (2018) and Pedersen & Swanson (2019) point out that the usefulness of diffusion indexes is crucially dependent upon whether real-time-data are used or not. When real-time data are used, pure DNS models based only on historical

[7]Exterkate *et al.* (2013) consider alternative procedures for the extraction of the macroeconomic factors and rank PC diffusion indexes second best after Partial Least Squares factor extraction. Pedersen & Swanson (2019) also survey procedures using targeted prediction, in which the variables used in the construction of the diffusion indexes are pre-selected using methods based on Machine Learning (ML). They note that, in periods when interest rates are more volatile, ML techniques may have much to offer. Swanson *et al.* (2020) also propose factor extraction based on the ML and Elastic net procedures proposed by Bai & Ng (2008). Alternatively, Coroneo *et al.* (2016) propose fitting a Dynamic Factor Model treating macroeconomic factors as unobservable components that are extracted simultaneously with the traditional yield curve factors.

information on interest rates, deliver forecasts with smaller MSFEs. However, when data are not real-time, diffusion indexes always have marginal forecasting content for interest rates.

Denote by f_t the $r \times 1$ vector of PC factors at time t extracted from a large set of macroeconomic variables. The vector β_t in (2) is substituted by

$$\begin{bmatrix} \beta_{t+1} - \mu \\ f_{t+1} \end{bmatrix} = \Phi \begin{bmatrix} \beta_t - \mu \\ f_t \end{bmatrix} + \eta_t, \tag{10}$$

where Φ is a $(3+r) \times (3+r)$ matrix of parameters allowing interrelations between the shape of the yield curve and the macro-financial factors. Finally, η_t is a $(3+r) \times 1$ Gaussian white noise vector with covariance matrix Σ_η.

The structure of the matrix Φ has information about the possibility of different characteristics of the yield curve being related to different macroeconomic aspects; see, for example, Diebold *et al.* (2006) and Exterkate *et al.* (2013).

2.4 Conditional heteroscedasticity

In the DNS in (1) and (2), the diagonal covariance matrix of ε_t is assumed to be constant over time. To allow for conditional heteroscedasticity in the yields, Koopman *et al.* (2010) propose modelling ε_t with a common pattern of evolving variances as follows

$$\varepsilon_t = \Gamma \varepsilon_t^* + \varepsilon_t^\dagger, \tag{11}$$

where Γ is an $N \times 1$ vector of constants, $\varepsilon_t^\dagger$ is an $N \times 1$ Gaussian white noise vector with diagonal covariance $\Sigma_\varepsilon^\dagger$, and ε_t^* is given by a conditionally normal GARCH(1,1) model with its conditional variance given by[8]

$$h_t = \gamma_0 + \gamma_1 \varepsilon_{t-1}^{*2} + \gamma_2 h_{t-1}, \tag{12}$$

[8] Alternatively, Bianchi *et al.* (2009) and Byrne *et al.* (2017) allow for stochastic volatility in the DNS model.

where, following Koopman *et al.* (2010), identification is achieved by fixing $\gamma_0 = 10^{-4}$ and the other parameters satisfy the usual positivity and stationarity conditions, namely, $\gamma_1, \gamma_2 \geq 0$ and $\gamma_1 + \gamma_2 < 1$, respectively.[9] Furthermore, the initial conditional variance is given by the marginal variance, $h_1 = \gamma_0(1 - \gamma_1 - \gamma_2)^{-1}$.

The DNS model with conditionally heteroscedastic errors is denoted as DNS-GARCH. The volatility of each yield is related to a common conditional variance in (12) that can be interpreted as the volatility of an underlying bond market portfolio; see Engle & Ng (1993).

The DNS model with time-varying volatility can be rewritten as a state-space model as follows

$$y_t = \begin{bmatrix} \Lambda & \Gamma \end{bmatrix} \begin{bmatrix} \beta_t \\ \varepsilon_t^* \end{bmatrix} + \varepsilon_t^\dagger, \tag{13}$$

$$\begin{bmatrix} \beta_{t+1} - \mu \\ \varepsilon_{t+1}^* \end{bmatrix} = \begin{bmatrix} \Phi & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_t - \mu \\ \varepsilon_t^* \end{bmatrix} + \omega_t, \tag{14}$$

where $\omega_t = \begin{bmatrix} \eta_t, \varepsilon_{t+1}^* \end{bmatrix}$ has covariance matrix $\Sigma_\omega = \begin{bmatrix} \Sigma_\eta & 0 \\ 0 & \gamma_0 + \gamma_1 \varepsilon_t^{*2} + \gamma_2 h_t \end{bmatrix}$.

In order to run the Kalman filter in model (13)-(14), Harvey *et al.* (1992) propose to substitute the last term in the diagonal of Σ_ω by

$$\hat{h}_{t+1|t} = \gamma_0 + \gamma_1 E\left[\varepsilon_{t|t}^{*2} + P_{t|t}^{\varepsilon}\right] + \gamma_2 \hat{h}_{t|t-1}, t = 1, \ldots, T, \tag{15}$$

where $\hat{\varepsilon}_{t|t}^*$ is the last element of the filtered state and $P_{t|t}^{\varepsilon^*}$ is its variance, both given by the Kalman filter.[10]

[9]Note that $E\left(\varepsilon_t \varepsilon_t'\right) = \Gamma\left(\frac{\gamma_0}{1-\gamma_1-\gamma_2}\right)\Gamma' + \Sigma_\varepsilon^\dagger$. Therefore, identification is usually achieved by assyming either that $\Gamma\Gamma' = I$ or by fixing γ_0 to any known constant.

[10]Hansen (n.d.) implements the same methodology in the context of a no-arbitrage yield curve with time-varying conditional variation.

3 Empirical forecasts of yields

In this section, the extended DNSS-GARCH-TVL-FULL-Macro model is fitted to end-of-month continuously compounded yields on US zero-coupon bonds. We also fit all admissible specifications described in Table 1, which summarizes the extensions of the DNS model considered in this paper together with the acronyms used for each of them. The estimated models are then use to obtain out-of-sample forecasts.

Table 1: **Description of model specifications of the yield curve**

Acronym	Number of factors	Volatility	Decay parameter λ_{kt}
DNS	3	constant	constant
DNS-Macro	3	constant	constant
DNS-GARCH	3	time-varying	constant
DNS-GARCH-Macro	3	time-varying	constant
DNS-TVL	3	constant	time-varying λ_{1t}
DNS-TVL-Macro	3	constant	constant
DNS-GARCH-TVL	3	time-varying	time-varying λ_{1t}
DNS-GARCH-TVL-Macro	3	time-varying	time-varying λ_{1t}
DNSS	4	constant	constant
DNSS-GARCH	4	time-varying	constant
DNSS-TVL1	4	constant	time-varying λ_{1t}
DNSS-TVL2	4	constant	time-varying λ_{2t}
DNSS-GARCH-TVL1	4	time-varying	time-varying λ_{1t}
DNSS-GARCH-TVL2	4	time-varying	time-varying λ_{2t}
DNSS-GARCH-TVL-FULL-Macro	4	time-varying	time-varying λ_{1t} and λ_{2t}

3.1 Data

We analyse the novel zero-coupon Treasury yield curve data set constructed by Liu & Wu (2021).[11] Yield curves, plotted in Figure 1, are available at the end-of-month from January 1972 through December 2019 for US securities with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months, with $N = 17$ maturities and $T = 576$ monthly observations. Table 2 reports some descriptive statistics of the time

[11]The data is publicly available in the Journal of Financial Economics Data Archive, as part of their supplementary material.

series of annual yields for each maturity. We can observe that, as expected, the mean of the interest rates increases with the maturity. However, the standard deviation is smaller. This reduction is due to the fact that while the minimum yield increases with maturity, the maximum decreases. When looking at the dynamic dependence of yields, we can observe that there is some evidence of non-stationarity. Table 2 also reports descriptive statistics for time series of some proxies of the level, slope and curvature. In particular, as proposed by Diebold & Li (2006), the proxy for the level is the highest maturity bond, i.e. the bond for 120 months, while that for the slope is the difference between the bond of 120 months and the bond of 3 months. Finally, the proxy for curvature is two times the bond of 24 months minus the sum of bond of 3 months and bond of 120 months. The sample autocorrelations reported in Table 2 show that the dependence of the level, slope and curvature is characterized by large persistence.

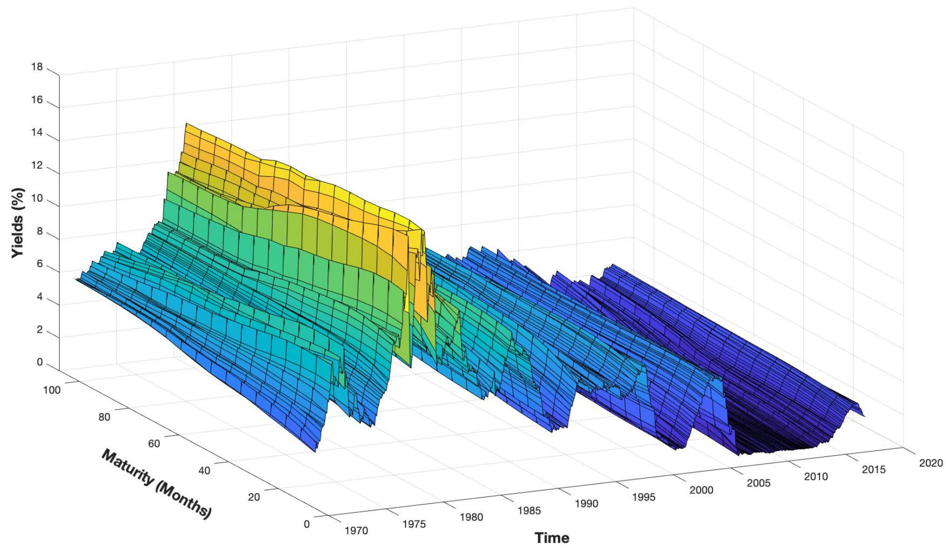
Table 2: **Descriptive statistics**

The table reports summary statistics for U.S. treasury yields from January 1972 through December 2019. Maturity is measured in months. For each maturity we show mean, standard deviation (Std. dev.), minimum, maximum, and three autocorrelation coefficients, 1 month [$\hat{\rho}(1)$], 6 months [$\hat{\rho}(6)$], and 12 months [$\hat{\rho}(12)$]. The proxies for level is the highest maturity bond (120 months), for slope, the difference between the bond of 120 months and the bond of 3 months, and for curvature, two times the bond of 24 months minus the sum of bond of 3 months and bond of 120 months.

Maturities	Mean	Std.dev.	Min.	Max.	ρ_1	ρ_6	ρ_{12}	Skewness	Kurtosis
3	4.722	3.522	0.020	16.170	0.988	0.928	0.865	0.610	3.210
6	4.873	3.561	0.040	16.210	0.989	0.934	0.874	0.569	3.073
9	4.985	3.567	0.070	16.180	0.990	0.937	0.880	0.524	2.946
12	5.072	3.559	0.100	16.030	0.990	0.940	0.886	0.483	2.844
15	5.147	3.550	0.130	15.950	0.991	0.943	0.890	0.455	2.781
18	5.216	3.544	0.160	15.960	0.991	0.945	0.895	0.441	2.754
21	5.274	3.530	0.180	15.900	0.991	0.947	0.898	0.429	2.729
24	5.321	3.501	0.200	15.660	0.991	0.948	0.900	0.410	2.685
30	5.415	3.448	0.240	15.510	0.991	0.950	0.905	0.383	2.629
36	5.518	3.411	0.320	15.550	0.992	0.952	0.907	0.387	2.643
48	5.699	3.329	0.470	15.420	0.992	0.953	0.910	0.388	2.627
60	5.834	3.237	0.640	15.010	0.992	0.953	0.912	0.384	2.600
72	5.971	3.183	0.820	14.990	0.992	0.955	0.913	0.413	2.619
84	6.070	3.116	1.000	14.960	0.992	0.954	0.911	0.433	2.663
96	6.157	3.061	1.210	14.900	0.992	0.955	0.913	0.445	2.677
108	6.229	3.008	1.410	14.810	0.993	0.955	0.913	0.459	2.705
120 (Level)	6.285	2.932	1.500	14.780	0.992	0.952	0.908	0.444	2.722
Slope	1.564	1.417	-4.280	4.340	0.942	0.713	0.476	-0.632	3.445
Curvature	-0.365	0.969	-2.680	3.080	0.921	0.746	0.631	-0.250	2.931

Finally, the macroeconomic diffusion indexes used to explain the shape of the yield

Figure 1: Monthly US Treasury yield curves from January 1972 to December 2019



curve are extracted using PC from the FRED-MD data base, which contain real-time data observed monthly over 130 variables, covering output and income, labour market, prices, and interest rates variables; see McCracken & Ng (2016) for a description. The number of factors extracted is three; see Exterkate *et al.* (2013), Pedersen & Swanson (2019) and Swanson *et al.* (2020), who also consider three factors extracted from the same dataset.

3.2 Out-of-sample forecasts of interest rates

The data set of US yields described above is divided into an initial in-sample period from January 1972 to December 1993, with $T = 264$ observations, used to estimate the parameters of the extended DNSS-GARCH-TVL-Macro model, and an out-of-sample period, from January 1994 to December 2019, with $R = 312$ observations. Note that the out-of-sample period includes the zero-lower-bound interest rates constraints period from 2008 to 2012. We obtain pseudo-real-time h-step-ahead forecasts of the interest rates at different maturities, for $h = 1, 3, 6, 12$, using the prediction equations of the Kalman filter without using the updating equations. The Kalman filter equations are run with the model parameters substituted by parameters estimated as described above, using a rolling window estimation scheme. We also fit the restricted versions, including the simplest DNS, and obtain forecasts with each of them.

For each maturity, τ_i, and forecast horizon, h, the out-of-sample forecasts of $y_{T+h}(\tau_i)$

obtained at time T are denoted by $y_{T+h|T}(\tau_i)$. They assessed by the relative size of their root mean square forecast errors (RMSFE), which is calculated as follows

$$\mathrm{RMSFE}(h, \tau_i) = \sqrt{\frac{1}{R-h+1}\sum_{r=0}^{R-h}[\hat{y}_{T+r+h|T+r}(\tau_i) - y_{T+r+h}(\tau_i)]^2}. \tag{16}$$

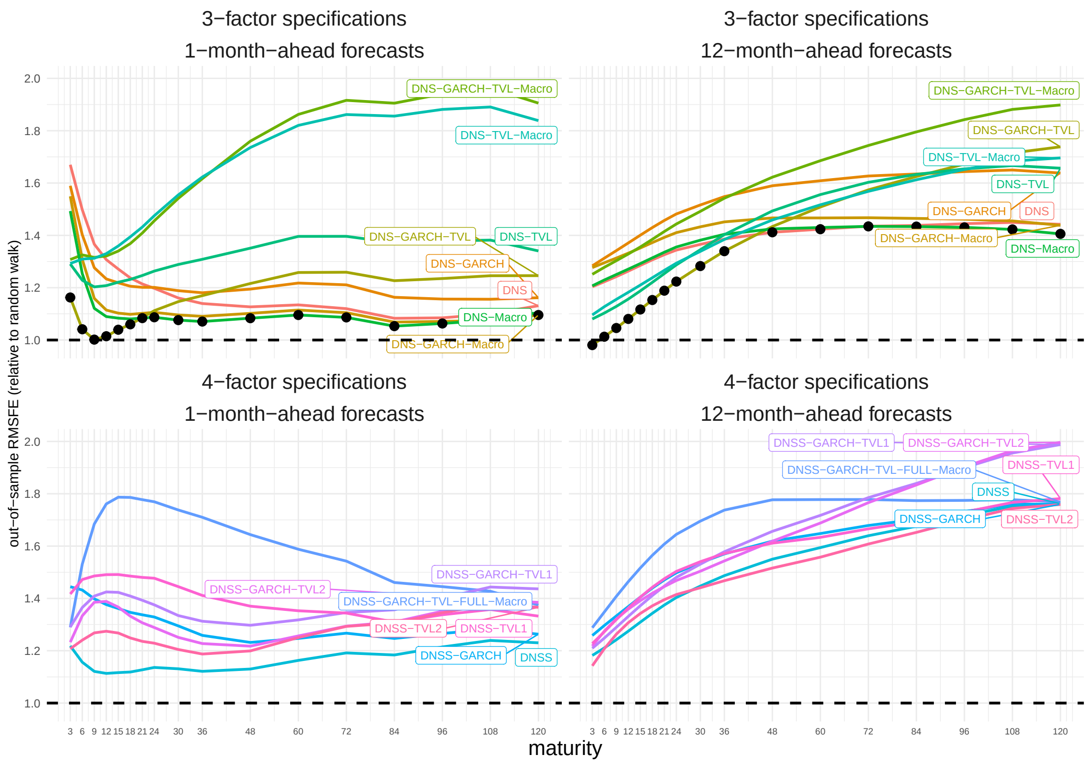
Table 3: **RMSFE of selected models**

							1-step-ahead forecasts										[b]
	3	6	9	12	15	18	21	24	30	36	48	60	72	84	96	108	120 [t]
RW	0.043	0.037	0.039	0.043	0.047	0.051	0.054	0.057	0.062	0.065	0.070	0.071	0.072	0.071	0.069	0.067	0.067
DNS	0.072	0.055	0.053	0.056	0.060	0.063	0.066	0.068	0.072	0.074	0.078	0.080	0.080	0.077	0.075	0.073	0.076
DNS-Macro	0.064	0.046	0.043	0.047	0.051	0.055	0.059	0.062	0.067	0.069	0.075	0.077	0.078	0.075	0.073	0.072	0.074
DNS-GARCH-TVL	0.050	0.038	0.039	0.044	0.049	0.054	0.059	0.063	0.071	0.076	0.085	0.089	0.090	0.087	0.085	0.083	0.084
							3-step-ahead forecasts										[b]
	3	6	9	12	15	18	21	24	30	36	48	60	72	84	96	108	120 [t]
RW	0.179	0.183	0.195	0.206	0.214	0.220	0.226	0.231	0.240	0.239	0.239	0.231	0.217	0.204	0.196	0.190	0.183
DNS	0.260	0.256	0.262	0.271	0.279	0.282	0.287	0.290	0.293	0.288	0.284	0.276	0.260	0.242	0.232	0.223	1.177
DNS-Macro	0.231	0.226	0.234	0.245	0.254	0.260	0.267	0.273	0.279	0.276	0.274	0.266	0.250	0.233	0.223	0.214	1.123
DNS-GARCH-TVL	0.180	0.183	0.196	0.212	0.228	0.240	0.252	0.264	0.281	0.288	0.302	0.307	0.299	0.287	0.281	0.275	1.461
							6-step-ahead forecasts										[b]
	3	6	9	12	15	18	21	24	30	36	48	60	72	84	96	108	120 [t]
RW	0.521	0.520	0.526	0.531	0.533	0.530	0.528	0.525	0.521	0.506	0.491	0.473	0.442	0.411	0.394	0.377	0.358
DNS	0.686	0.685	0.691	0.698	0.700	0.696	0.692	0.688	0.677	0.654	0.629	0.607	0.569	0.530	0.505	0.482	0.458
DNS-Macro	0.636	0.638	0.649	0.658	0.662	0.662	0.662	0.661	0.654	0.634	0.610	0.585	0.545	0.505	0.480	0.455	0.428
DNS-GARCH-TVL	0.509	0.522	0.543	0.564	0.583	0.595	0.607	0.618	0.633	0.634	0.646	0.651	0.633	0.608	0.594	0.580	0.560
							12-step-ahead forecasts										[b]
	3	6	9	12	15	18	21	24	30	36	48	60	72	84	96	108	120 [t]
RW	1.523	1.497	1.461	1.416	1.363	1.304	1.246	1.192	1.103	1.014	0.896	0.824	0.756	0.700	0.655	0.618	0.588
DNS	1.832	1.831	1.816	1.789	1.752	1.703	1.652	1.602	1.506	1.405	1.266	1.173	1.085	1.006	0.946	0.896	0.848
DNS-Macro	1.838	1.841	1.827	1.800	1.762	1.713	1.664	1.616	1.523	1.423	1.277	1.179	1.085	1.003	0.937	0.879	0.827
DNS-GARCH-TVL	1.493	1.516	1.528	1.531	1.523	1.503	1.481	1.458	1.414	1.358	1.285	1.243	1.191	1.137	1.095	1.060	1.022

Figure 2 plots the RMSFEs of the interest rate forecasts obtained by each of the models considered, which are described in Table 1, relative to those of the forecasts obtained by the benchmark random walk model, for $h = 1$ (first column) and 12 (second column), as functions of the maturities.

In order to investigate the role of the second discount parameter, λ_2, the first and second rows of Figure 2 plot the relative RMSFEs of the models with three and four factors, respectively. We can observe that, regardless of h, the RMSFEs of yield forecasts obtained by the DNSS models are larger than those obtained by the models with just one discount factor. Note that the lack of forecasting power of the second discount is even smaller when looking at twelve-step-ahead predictions than for one-step-ahead predictions, when the DNSS model generates predictions with slightly larger RMSFEs than those of the best competitors. To understand the role of the second discount in forecasting the yield curve, Figure 3 plots the curvature estimated by the simplest DNS

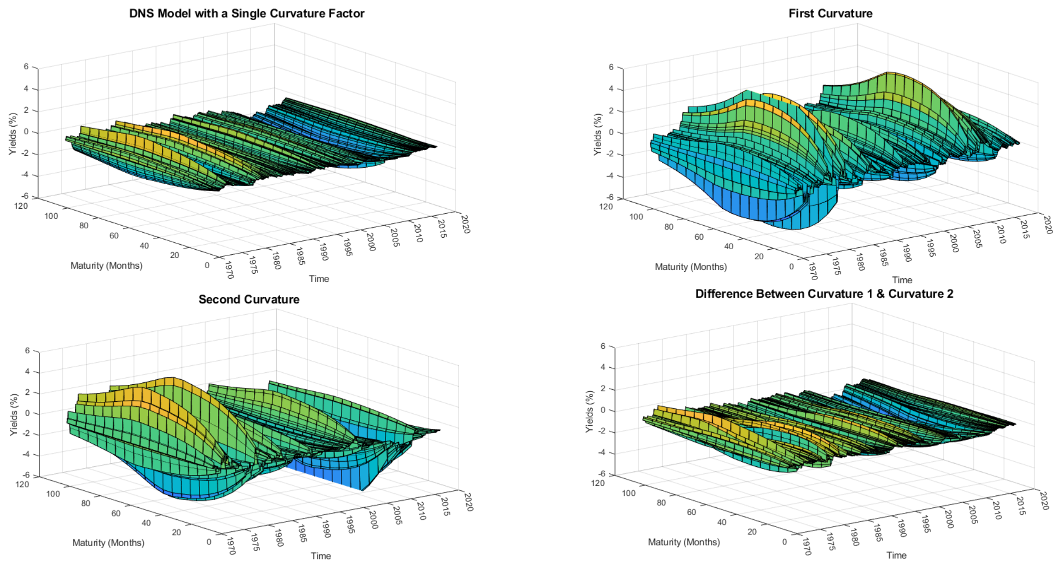
Figure 2: RMSFEs of one-step-ahead (left column) and twelve-step-ahead (right column) of in-sample yield predictions obtained with restricted versions of the DNS-GARCH-TVL-Macro model (first row) and FA-DNSS-TVL-GARCH model (second row). The RMSFEs are relative to predictions obtained with the random walk model.



model, when only one discount is included, together with the two curvatures estimated by the DNSS model, with two-discounts, together with the difference between these two latter curvatures. Comparing the curvature estimated by the DNS model with the difference between the curvatures estimated by the DNSS model, we can observe that both are very similar, explaining why, in practice the forecasting power of the second discount is very mild. Our results are in concordance with those by Walstrøm *et al.* (2022), who analyse daily market prices of Treasury instruments with maturities up to 30 years, and also conclude that one of the two curvatures of the DNSS model is superfluous due to confounding effects. Diebold & Li (2006) also fit an extended model with four and five factors and conclude that this extension provides negligible improvement in model fit; see also Dahlquist & Svensson (1996) and Almeida *et al.* (2018), who fit more complicate shapes to the yields and also conclude that the second discount parameter does not contribute to the predictive power of the DNS model.

Given the superior forecast performance of models with just one discount parameter, we focus on those models in the following more detailed analysis. The second conclusion

Figure 3: Curvature estimated by the DNS model (top left panel) and curvatures estimated by the DNSS model together with their difference (bottom right panel).

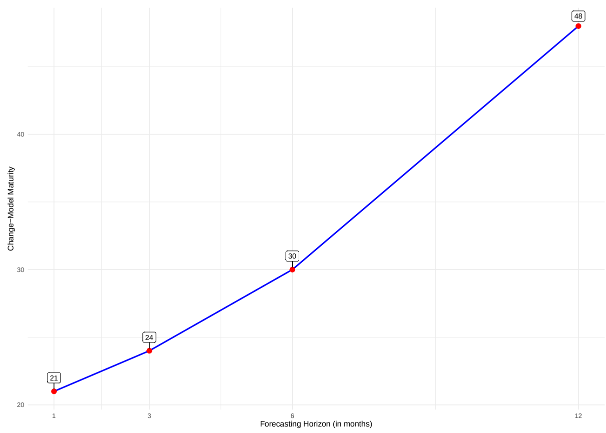


in this paper can be obtained from the first row of Figure 2, which shows that the model with best forecasting performance depends on the maturity. For short maturities, the forecasts with minimum RMSFEs are obtained by the model proposed by Koopman *et al.* (2010), DNS-GARCH-TVL, in which the discount parameter is allowed to change over time and there is conditional heteroscedasticity; in their empirical application, Koopman *et al.* (2010) also conclude that conditional heteroscedasticity and time-varying discounts have superior forecast performance and Jungbacker *et al.* (2014) observe that volatility tends to be lower for the yields of bonds with a longer time to maturity. However, for long maturities, the best performance is obtained when the discount parameter is constant and there is conditional homoscedasticity but allowing the shape of the yield curve to depend on macroeconomic conditions, i.e. DNS-macro model; see Favero *et al.* (2012), who also conclude that macroeconomic variables have forecasting power at longer maturities.

It seems that, in order to represent adequately the yield curve, one should use different models depending on whether short or long maturities are being forecast. None of the alternatives systematically outperform this segmented specification. This result is in concordance with the existence of a segmented yield curve with yields of different maturities being affected by different risk factors. In particular, volatility is a risk factor for short maturities while macroeconomic variables are the risk factor for yields with long maturities. This segmentation is postulated by the preferred habitat theory of the term structure of interest rates, according to which each investor may demand bonds of specific maturities. For instance, pension funds may prefer long-term bonds while speculators may chose short-term bonds. Arbitrageurs may also participate in the market aiming to maximize a mean-variance utility function and, consequently, choosing bonds with any maturity. By doing so, arbitrageurs guarantee some smoothness among yields with different maturities; see Modigliani & Sutch (1966) for the preferred habitat theory. Almeida *et al.* (2018) also conclude that introducing segmentation in term structure models consistently improves long-horizon forecasts. However, in their model the segmentation is not able to identify different risk factors for yields with short and long maturities.

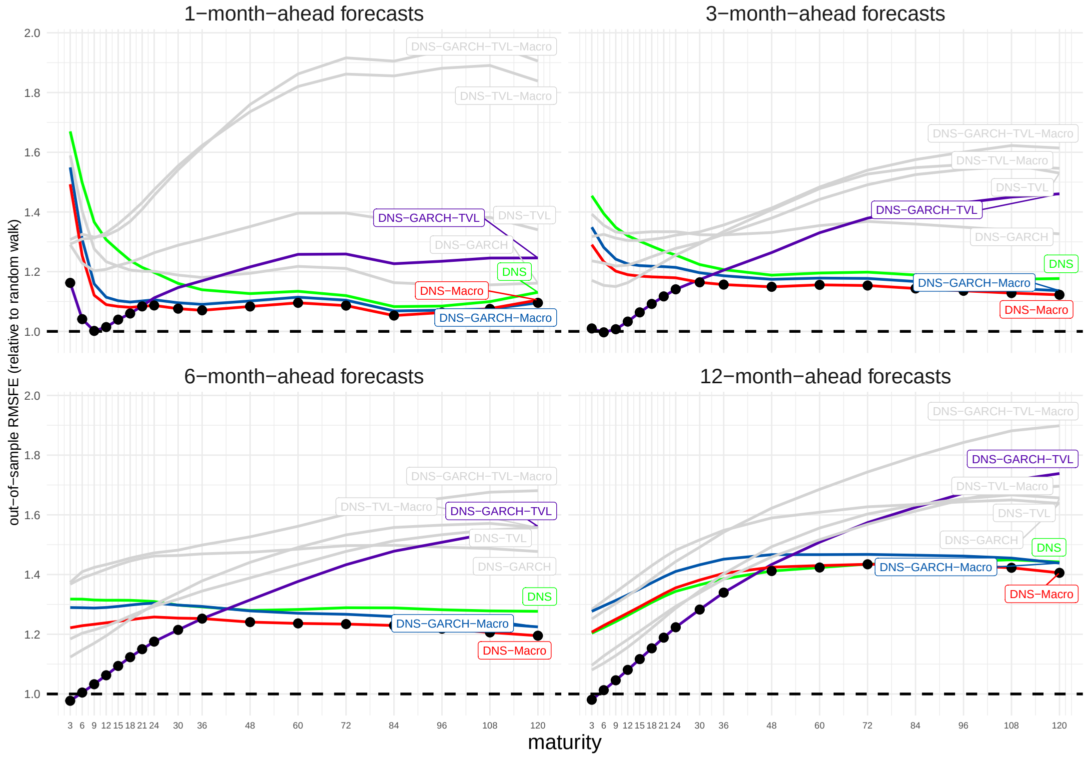
Third, Figure 2 suggests that the maturity for which the best forecasting model switches from the DNS-GARCH-TVL to the DNS-Macro depends on the forecast horizon. When $h = 1$, the model with best forecasts is DNS-GARCH-TVL for maturities smaller than or equal to 21 months. However, when $h = 12$, the maturity increases up to 48 months. Figure 5 plots the relative RMSFEs for different forecasts horizons, namely, $h = 1, 3, 6$ and 12. We can observe that when $h = 3$, the maturity for the model switch is 24 months while when $h = 6$, it is 30 months; see Figure 4 that plots the maturity as a function of the forecast horizon.

Figure 4: Maturity switching



Fourth, we can observe that, although the DNS forecasts hardly beat the random walk forecasts, they are not too far when $h = 1$. In this case, the DNS is still an alternative to be considered due to its interpretability. However, when $h = 12$, the increase in the RMSFE can be as large as 22.5% when the DNS models are used to forecast long maturities as compared to using the random walk.

Figure 5: RMSFEs of one-step-ahead (left column) and twelve-step-ahead (right column) of in-sample yield predictions obtained with restricted versions of the FA-DNS-TVL-GARCH model (first row) and FA-DNSS-TVL-GARCH model (second row). The RMSFEs are relative to forecasts obtained with the random walk model.



4 Restricted versions: non-stationary dynamics of factors and discount parameters

In the previous section, two models emerge as having the best forecasting performance, namely, the DNS-GARCH-TVL model for short maturities and the DNS-Macro model for long maturities. To avoid problems of overfitting and numerical issues associated with estimating the large number of parameters in Φ, the specification of these two models can be simplified imposing restrictions on the dynamics of the factors and/or discount parameter. In this section, we will consider three restrictions often assumed in empirical analysis of the yield curve based on the DNS framework.

First, several authors observe that the factors display little cross-correlation, so that Φ can be assumed to be diagonal; see Exterkate *et al.* (2013) and van Dijk *et al.* (2014) for this assumption. Second, the results in Diebold & Li (2006) and Diebold *et al.* (2006) suggest that β_{1t} and β_{2t} may have a unit root while β_{3t} does not; see also Bowsher & Meeks (2008) and Almeida *et al.* (2018) for non-stationary models for the factors and Jungbacker *et al.* (2014) for large persistence of the factors and diagonality of the autoregressive matrix. These results may be the consequence of non-stationarity of interest rates; see Hall *et al.* (1992) and Bauer & Rudebush (2020) . Christensen & Rudebush (2012) point out that due to this persistence, stationary AR models fitted to the factors may suffer from substantial small-sample bias with estimates implying much less persistence than the true process. Similarly, several authors suggest that the persistence of the log-discount parameter can be large and propose modelling it using a random walk; see, for example, Koopman *et al.* (2010).

Table 4: **Description of restricted model specifications**

c

Acronym	Restriction imposed
DNS-GARCH-TVL-RW	DNS-GARCH-TVL model with decay parameter λ_1 with ran
DNS-RW-GARCH-Q-DIAG	DNS-GARCH model with level, slope, and curvature factors
DNS-RW-GARCH-TVL-RW	DNS-GARCH-TVL with model with level, slope, and curvatu
DNS-RW-GARCH-TVL-RW-Q-DIAG	DNS-GARCH-TVL with level, slope, and curvature factors a

Figure 6: RMSFEs of one-step-ahead (left column) and twelve-step-ahead (right column) of in-sample yield predictions obtained with restricted versions of the DNS-TVL-GARCH-Macro model (first row) and FA-DNSS-TVL-GARCH model (second row). The RMSFEs are relative to predictions obtained with the random walk model.

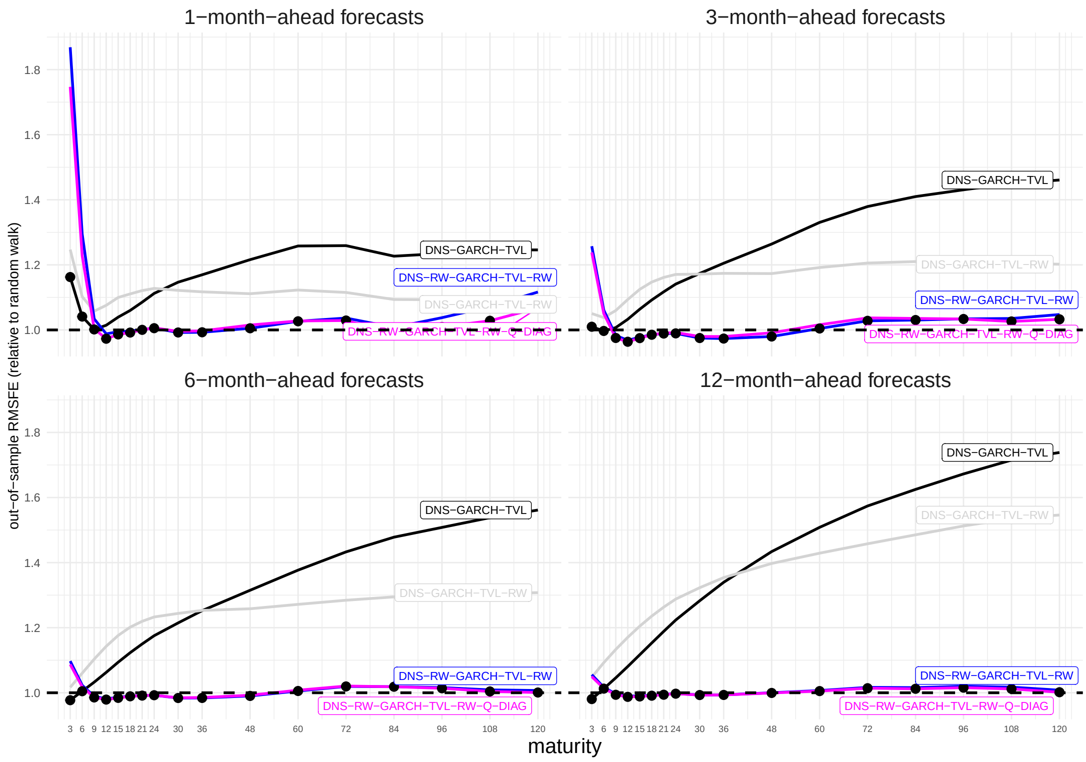
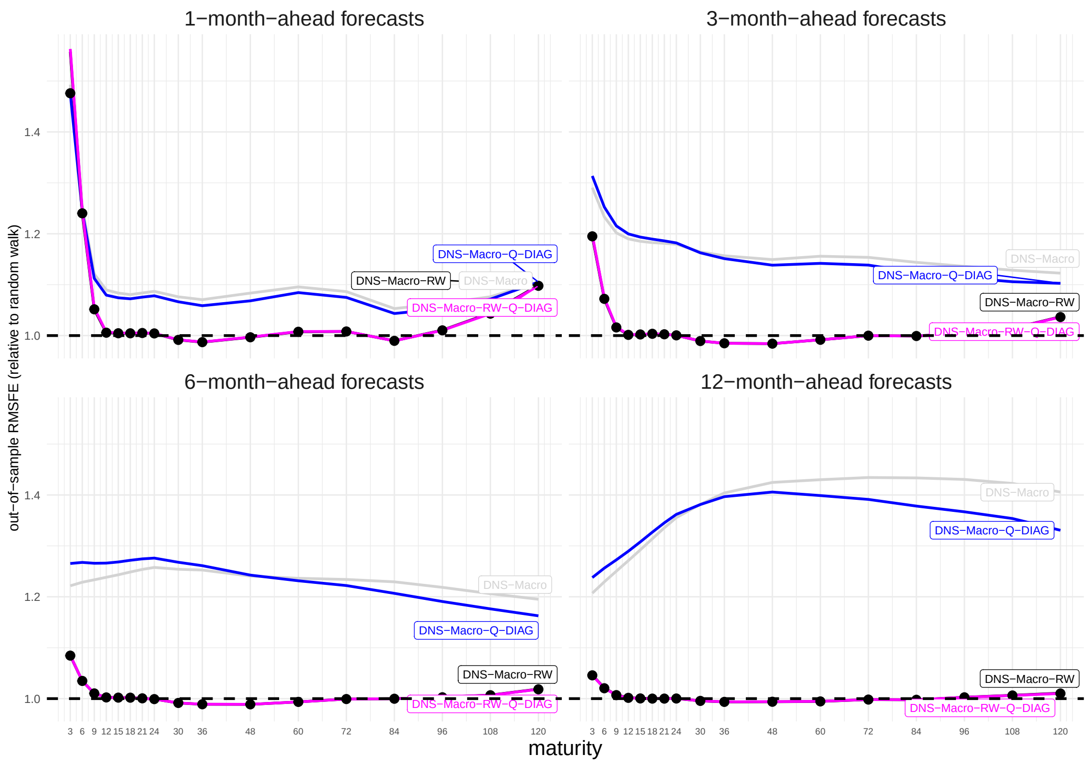


Figure 7: RMSFEs of one-step-ahead (left column) and twelve-step-ahead (right column) of in-sample yield predictions obtained with restricted versions of the DNS-TVL-GARCH-Macro model (first row) and FA-DNSS-TVL-GARCH model (second row). The RMSFEs are relative to predictions obtained with the random walk model.



We can see that when $h = 12$, the DNS-GARCH-TVL model with the factors and the logarithmic discount parameters modelled as independent random walks is best. However, for the smallest forecast horizons, these restrictions are not appropriate for the first maturity ($h = 1, 3, 6$) and for the first two ($h = 1$). In this case, the only restriction that helps is assuming that the logarithmic discount parameter is a random walk.

5 Conclusions

Further: Forecast uncertainty density forecasts Partial Least Squares Segmented yield curve

Statements and declarations

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cle. The datasets analysed during the current study are available from the corresponding author on reasonable request.

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