Optimal Taxation under Evasion and Monopolistic Competition*

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Abstract

We show that in a economy with tax evasion and monopolistic competition, the uniform commodity taxation is not necessarily optimal. Instead, the tax authority should levy uniform effective taxes and impose lower nominal taxes on sectors which can most evade taxes. Moreover, the tax rate should be inversely proportional to the elasticity of evasion similarly as the classical Ramsey taxation. In addition, optimal taxes on intermediate goods are still zero. We also simulate the equilibrium and explore how the choice of different tax regimes affect the economy.

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1 Introduction

Tax evasion and avoidance are serious problems in many countries. According to Internal Revenue Service estimates, in 2010 tax evasion cost the US federal government nearly 2,3% of GDP (US$ 458 billion). In Latin America economies, tax evasion accounts for a larger share of GDP, 4.6% for Mexico and 9.1% for Brazil.

In an economy without tax evasion and under certain conditions (e.g., separable preferences), Atkinson and Stiglitz (1972) show that similar goods should be taxed at similar rate. This is the classical result on uniform commodity taxation[1] which has also been advocated by policy makers since it is a simple and straightforward tax design which should decrease compliance costs - see the Meade (1978) Report. Another important policy lesson in the optimal taxation literature is that transactions between firms should not be taxed, and therefore taxing intermediate goods is not optimal since it distorts the allocation of factors of production between intermediate and final goods decreasing production efficiency (cf., Diamond and Mirrlees, 1971). However, are these policy lessons robust in economies in which the tax authority cannot perfectly monitor economic activities? Should the government tax sectors with different levels of informal activities in the same way? In the presence of tax evasion, should the tax authority rely on intermediate goods taxation? Those are questions addressed in this paper.

As Shaw et al. (2010) argue in the Mirlees Review, most of modern optimal tax theory abstract from administrative, compliance costs as well as evasion. In this paper we investigate the optimal tax design in the presence of tax evasion and when evasion varies by sectors of economic activity. We follow the Ramsey tradition (cf., Ramsey, 1927; Lucas and Stokey, 1983), which determines the optimal tax structure to minimizes economic distortions. Cremer and Gahvari (1993) is the closest paper to ours. They consider an economy with endogenous tax evasion[2] and investigate the optimal commodity taxation problem. They show that under evasion, the uniform taxation is no longer an optimal prescription. We differ from them in the following two main features. First, we consider an economy with intermediate goods and monopolistic competition[3] as in Dixit and Stiglitz (1977). This allows us to investigate whether or not intermediate goods should be taxed in the presence of tax evasion, while they mainly focus on final goods taxation. In addition, we provide clear results on the kind of tax rate differentiation by different sector characteristics (e.g., size, productivity and evasion), which can be easily checked in the data. The concealment technology is given and varies with sectoral features, such as size, productivity and the tax rate.[4]

[1] Regarding distributional issues uniform commodity taxation can also be advocated when there are different instruments for redistribution, such as cash transfers to the poor.

[2] See also Allingham and Sandmo (1972)

[3] See also Cremer and Thisse (1994)

[4] See Rauch (1991), Amaral and Quintin (2006), Antunes and Cavalcanti (2007) for models in which informal activities arise endoge-

Evasion may lead to important departs from traditionally recommended tax policies. For example, one of the most accepted results on optimal taxation is the less distortive aspect of direct taxation. Still, in several countries, the share of tax revenue coming from indirect taxation can be relatively high. One explanation might be the difficulty to raise tax revenue directly in comparison to indirect taxes. Boadway et al. (1994) shows that if different taxes have different evasion characteristics, some optimal tax mix of direct and indirect taxation emerges naturally. Cavalcanti and Villamil (2003) show that in the presence of tax evasion the optimal inflation tax is positive and increasing with the size of the informal economy. Emran and Stiglitz (2005) show that a tax reform which eliminate trade taxes and compensate it with a value-added tax might decrease welfare when a large informal sector is present.

The following lessons are learnt. First of all, the introduction of different sectors, monopolistic competition and intermediate goods per se do not change the main optimal taxation prescriptions, i.e., in our framework and under no evasion the uniform commodity taxation is optimal and the government should not rely on intermediate goods taxation. However, in the presence of tax evasion, uniform commodity taxation is no longer an optimal policy. Instead, the tax authority should levy uniform effective taxes and impose lower taxes on sectors which can most evade taxes. In addition and surprisingly, optimal taxes on intermediate goods are still zero, inducing intermediate firms not to worry about concealing revenues.

This paper is divided into four additional sections besides this introduction. Section 2 presents the economic environment. Section 3 solves the Ramsey problem using the primal approach and derives the main taxation lessons. Section 4 implements the model numerically and Section 5 contains concluding remarks

2 The model

Consider an one-period real economy with $N \times S$ productive sectors. There are $N \times (S+1)+1$ commodities in this economy: N consumption goods, S intermediate goods for each final consumption good and the labor input. There is one representative agent, endowed with one unit of productive time that can be used as leisure or labor in the production of intermediate goods. Government levies sales tax on firms in order to raise an exogenously defined amount of resources, to be disposable.

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nously.

2.1 Representative household

Preferences are defined over consumption $\{C_n\}_{n=1}^{N}$ and the disutility from labor, l. We assume that preferences can be represented by a utility function $u : \Re_+^N \times [0,1] \rightarrow \Re$, given by:

$$U\left(C_1, C_2, \ldots, C_N, l\right) \tag{1}$$

Function $U(\cdot,\cdot,\ldots,\cdot)$ satisfies standard properties. For instance, it is twice continuously differentiable in all arguments, and it is strictly concave in all consumption good, and strictly convex in the disutility from labor. We also assume that preferences are homothetic over the consumption goods, i.e.:

Assumption 1. *There are functions $H : \Re_+^N \rightarrow \Re$, homogenous of degree k, and $F : \Re \times [0,1] \rightarrow \Re$ such that:*

$$U\left(C_1, C_2, \ldots, C_N, l\right) = F\left(H\left(C_1, C_2, \ldots, C_N\right), l\right) \tag{2}$$

The representative household owns firms and chooses $\{C_n, l\}_{n=1}^{N}$ in order to maximize (1) subject to:

$$\sum_{n=1}^{N} P_n C_n \leq wl + \sum_{n=1}^{N}\sum_{s=1}^{S} \pi_{n,s}\left(w, \xi^{n,s}, A_{n,s}\right) \tag{3}$$

2.2 Consumption Goods

In each sector, $n = 1, \ldots, N$, there is a continuum of firms of measure one. Let Y_n and $\{d_{n,s}\}_{s=1}^{S}$ be output produced and intermediate goods used, respectively, by the representative firm in sector n. The technology employed to produce each consumption good is represented by the following CES production function:

$$Y_n = Z_n \left(\sum_{s=1}^{S} d_{n,s}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}, n = 1, \ldots, N \tag{4}$$

where Z_n is a productive factor and θ is the elasticity of substitution. Government levies a tax τ_n on revenue in each sector to finance its spending. However, firms may evade taxes. In such a case, let $\phi^n = \phi^n\left(\tau_n, Z_n\right)$ be the fraction of firm n 's revenue that is declared and hence taxed by the fiscal authority. We assume that ϕ^n is decreasing in the tax rate in sector n, i.e., $\phi_1^n < 0$, but, it is increasing in the productivity of sector $n, \phi_2^n > 0$. Therefore, firms in high productive sectors have lower probability to evade taxes[5].

Consumption good producers are price takers and maximize profits. Let the price of consumption good n be P_n and let $p_{n,s}$ be the price of intermediate good $\{n,s\}$. Let Π_n denotes profits of firms in sector n. The profit maximization problem of each firm is:

$$\Pi_n\left(P_n, \tau_n, \{p_{n,s}\}_{s=1}^{S}\right) = \max_{\{d_{n,s}\}_{s=1}^{S}} \left(1 - \tau_n \phi^n\right) Z_n P_n \left(\sum_{s=1}^{S} d_{n,s}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} - \sum_{s=1}^{S} d_{n,s} p_{n,s} \tag{5}$$

[5] Following De Soto et al. (1989), Antunes and Cavalcanti (2007) show that since loan contracts are not well enforced in the informal sector, informal entrepreneurs scale down their size and productivity.

Under perfect competition ($\Pi_n = 0$), we can show that in equilibrium must hold:

$$\left(1 - \tau_n \phi^n\right) Z_n P_n = \left(\sum_{s=1}^{S} p_{n,s}^{1-\theta} \right)^{\frac{1}{\theta}} \tag{6}$$

Also, the demand of firm in sector n for the intermediate good $\bar{s}$ is:

$$d_{n,\bar{s}} = \frac{\left(\sum_{s=1}^{S} p_{n,s}^{1-\theta} \right)^{\frac{\theta}{1-\theta}} Y_n}{p_{n,\bar{s}}^{\theta}} = \frac{\Omega_n Y_n}{p_{n,\bar{s}}^{\theta}}, \Omega_n = \left(\sum_{s=1}^{S} p_{n,s}^{1-\theta} \right)^{\frac{\theta}{1-\theta}} \tag{7}$$

Since firms operate under constant return to scale technology and zero profit condition, Y_n is entirely determined by the demand for good n, i.e., C_n.

2.3 Intermediate Goods

Each intermediate firm s in sector n has monopoly rights in its production of $y_{n,s}$, uses $h_{n,s}$ units of labor and faces demand function given by equation (7)

The technology employed to produce each intermediate good is represented by the following production function:

$$y_{n,s} = A_{n,s} h_{n,s}^{\beta}, \quad n = 1, 2, \ldots, N; s = 1, 2, \ldots, S. \tag{8}$$

$A_{n,s}$ is a labor productive factor associated to firm s in sector n and is assumed to be positive. Also $\beta \in (0, 1)$ corresponds to the elasticity of output with respect to labor.

Governments levies a tax ξ_n on revenue of each firm that supply to sector n to finance spending. Firms can, however, evade taxes. Let $\delta^{n,s} = \delta^{n,s}(\xi_n, A_{n,s})$ be the fraction of firm s 's in sector n revenue that is declared and hence taxed by the fiscal authority. Just as in the case of final goods firm, we assume that $\delta^{n,s}$ is decreasing in the tax rate for every intermediate firm supplying to sector n, $\delta_1^{n,s} < 0$, but it is increasing in the productivity of firm s in sector n, $\delta_2^{n,s} > 0$. The profit maximization problem of each intermediate good firm is:

$$\begin{aligned} \pi_{n,s}(w, \xi_n, A_{n,s}) &= \max_{h_{n,s}} \left\{ p_{n,s} \left(1 - \xi_n \delta^{n,s}\right) y_{n,s} - w h_{n,s} \right\} \\ \text{s.t. } y_{n,s} &= \frac{\Omega_n Y_n}{p_{n,s}^{\theta}} \\ \text{and } y_{n,s} &= A_{n,s} h_{n,s}^{\beta} \end{aligned} \tag{9}$$

Or

$$\pi_{n,s}(w, \xi_n, A_{n,s}) = \max_{h_{n,s}} \left\{ (\Omega_n Y_n)^{\frac{1}{\theta}} (1 - \xi_n \delta^{n,s}) A_{n,s}^{\frac{\theta-1}{\theta}} h_{n,s}^{\frac{\theta\beta-\beta}{\theta}-1} - w h_{n,s} \right\} \tag{10}$$

The associated marginal condition[6] for each firm is:

$$\left(\frac{\theta\beta - \beta}{\theta} \right) (\Omega_n Y_n)^{\frac{1}{\theta}} (1 - \xi_n \delta^{n,s}) A_{n,s}^{\frac{\theta-1}{\theta}} h_{n,s}^{\frac{\theta\beta-\beta}{\theta}-1} = w \tag{11}$$

[6]As in Dixit-Stiglitz (1977), we assume S is sufficiently large to the point that price changes of a single intermediate good do not affect general price index.

Hence firm's profit in optimal is:

$$\pi_{n,s}\left(w,\xi^{n,s},A_{n,s}\right)=\left(\frac{\theta}{\theta\beta-\beta}\right)wh_{n,s}-wh_{n,s}=\left(\frac{\theta-\theta\beta+\beta}{\theta\beta-\beta}\right)wh_{n,s} \tag{12}$$

2.4 Government Budget Constraint

The government consumes a basket $\{G_n\}_{n=1}^{N}$ of final goods and its budget constraint is given by:

$$\sum_{n=1}^{N}\tau_n\phi^n P_n Y_n+\sum_{n=1}^{N}\sum_{s=1}^{S}\xi^n\delta^{n,s}p_{n,s}y_{n,s}=\sum_{n=1}^{N}P_nG_n \tag{13}$$

2.5 Equilibrium

In competitive equilibrium firms producing final goods and households are price takers. Households maximize their utility subject to their budget constraint, firms maximize profits given their technology, labor and all goods markets clear and the government budget constraint is satisfied. Marketing clearing conditions are:

$$\begin{aligned} C_n+G_n=Z_n\left[\sum_{s=1}^{S}\left(A_{n,s}l_{n,s}^{\beta}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}&,n=1,2,\ldots,N;\\ h_{n,s}=l_{n,s},n=1,2,\ldots,N \text{ and } s=1,2,\ldots,S;&\\ \sum_{n=1}^{N}\sum_{s=1}^{S}l_{n,s}=l.& \end{aligned} \tag{14}$$

3 Ramsey Problem

Following the tradition of Ramsey (1927) we study the problem of choosing the best allocations that are consistent with the competitive equilibrium. The implementability conditions are: (i) the resource constraint for each sector; and (ii) the implementability constraint, given by:

$$\sum_{n=1}^{N}U_nC_n=-U_l l(1+\rho) \tag{15}$$

where $\rho=\frac{\theta-\theta\beta+\beta}{\theta\beta-\beta}$. Equation (15) corresponds to the household's budget constraint in which we substitute prices and taxes by quantities consistent to the competitive equilibrium. The Ramsey problem (see Lucas and Stokey, 1983) is to choose $\{C_n,l_{n,s}\}_{n=1,s=1}^{N,S}$ to maximize (1) subject to the resource constraint for each sector (equation (14)) and the implementability constraint (15).

Proposition 1. *Suppose that there is no tax evasion (i.e., $\phi^n\left(\tau_n,Z_n\right)=\delta^{n,s}\left(\xi_n,A_{n,s}\right)=1$ for all $\{n,s\}$). The Ramsey allocation is decentralized with an uniform effective tax rate policy on sectors, i.e., $\left(1-\tau_i\right)\left(1-\xi_i\right)=\left(1-\tau_j\right)\left(1-\xi_j\right)$ for every i,j in $1,2,\ldots,N$.*

Proof. Let λ_j and ψ be the Lagrange multipliers for the resource constraint of sector j, and for the implementability condition (15). The first-order condition of the Ramsey Problem with respect to C_j is

$$\lambda_j H_j = [F_H + \psi (F_{HH} kH + F_H k + l(1+\rho) F_{lH})] \quad j = 1, 2 \ldots, N. \tag{16}$$

Hence:

$$\begin{aligned} \frac{\lambda_i}{\lambda_j} &= \frac{F_H H_i}{F_H H_j} \\ &= \frac{U_i}{U_j} \end{aligned} \tag{17}$$

for any $i, j \in 1, 2 \ldots, N$. The first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{\lambda_j}{\lambda_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{18}$$

Therefore

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{19}$$

In the competitive equilibrium we have that

$$\frac{U_j}{U_i} = \frac{(1-\tau_i)(1-\xi_i) Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{(1-\tau_j)(1-\xi_j) Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{20}$$

The last two equations imply that the Ramsey allocation can be decentralized with an uniform effective tax policy, i.e., $(1-\tau_i)(1-\xi_i) = (1-\tau_j)(1-\xi_j)$. ■

As in Atkinson and Stiglitz (1972), Proposition 1 states that under no tax evasion the optimal policy is to tax all goods uniformly. However, note that uniformity does not apply to nominal tax rates, but to effective tax rate. Conversely, as shown in proposition 2, the presence of tax evasion solely in intermediate goods market compels tax authority to equalize nominal tax rates.

Proposition 2. *Suppose that only intermediate firms practice some tax evasion,* $\{\phi^n(\tau_n, Z_n) = 1\}_n = 1^N$ *and* $\{\delta^{n,s}(\xi_n, A_{n,s}) < 1\}_{n,s=1}^{N,S}$. *Also, assume that* $A_{j,r} \neq A_{j,\tilde{r}}$, *for some* $\{j, r\}$ *and* $\{j, \tilde{r}\}$. *The Ramsey allocation is decentralized with an uniform tax rate policy on final goods,* $\tau_i = \tau_j$, *and no tax on intermediate goods,* $\xi_i = \xi_j = 0$

Proof. Let λ_j and ψ be the Lagrange multipliers for the resource constraint of sector j, and for the implementability condition (15). The first-order conditions of the Ramsey Problem with respect to C_i and C_j jointly with the first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}} y_{i,v}^{\frac{\theta-1}{\theta}} l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}} y_{j,r}^{\frac{\theta-1}{\theta}} l_{j,r}^{-1}} \tag{21}$$

In the competitive equilibrium we have that

$$\frac{U_j}{U_i} = \frac{(1-\tau_i)\left(1-\xi_i\delta^{i,v}\right)Y_i^{\frac{1}{\theta}}y_{i,v}^{\frac{\theta-1}{\theta}}l_{i,v}^{-1}}{(1-\tau_j)\left(1-\xi_j\delta^{j,r}\right)Y_j^{\frac{1}{\theta}}y_{j,r}^{\frac{\theta-1}{\theta}}l_{j,r}^{-1}} \tag{22}$$

The last two equations imply that

$$(1-\tau_j)\left(1-\xi_j\delta^{j,r}\right) = (1-\tau_j)\left(1-\xi_j\delta^{j,\bar{r}}\right) \tag{23}$$

Since $A_{j,r} \neq A_{j,\bar{r}}$, it follows that $\xi_j = 0$. Together first order conditions and (22) imply that the Ramsey allocation can be decentralized only through an uniform tax rate on final goods firms, i.e., $\tau_i = \tau_j$. ■

The presence of tax evasion on intermediate goods market but not on final goods market throw away the possibility of achieving a decentralized Ramsey allocation through the taxation of intermediate firms' revenue. So, under such scenario, tax reforms should try not only to significantly reduce nominal tax rates on intermediate firms but also to equate nominal tax rates on final goods.

Nevertheless, as shown below, under generalized tax evasion, the prescription to uniformly tax commodity is no longer valid.

Proposition 3. *Suppose that every firm can practice some tax evasion, $\{\phi^n(\tau_n, Z_n) < 1\}_{n=1}^{N}$ and $\{\delta^{n,s}(\xi_n, A_{n,s}) < 1\}_{n,s=1}^{N,S}$. Also, assume that $Z_i > Z_j$, for some i and j, and $A_{j,r} \neq A_{j,\bar{r}}$, for some $\{j,r\}$ and $\{j,\tilde{r}\}$. The Ramsey allocation is decentralized with no taxation on intermediate goods, $\xi_i = \xi_j = 0$ and a non-uniform tax rate policy on final good firms, $\tau_i \neq \tau_j$. Moreover, i. If $\left\{f^j(\tau, A_j) = \phi^j(\tau, A_j)\tau\right\}_{j=1}^{N}$ and $\left\{f'^j(\tau) < 0\right\}_{j=1}^{N}$, then $\tau_j < \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$. ii. If $\left\{f^j(\tau) = \phi^j(\tau)\tau\right\}_{j=1}^{N}$ and $\left\{f'^j(\tau) > 0\right\}_{j=1}^{N}$, then $\tau_j > \tau_i$ for $\phi^j(\tau) < \phi^i(\tau)$.*

Proof. Just like previous proofs, let λ_j and ψ be the Lagrange multipliers for the resource constraint of sector j, and for the implementability condition (15). Once again, the first-order conditions of the Ramsey Problem with respect to C_i and C_j jointly with the first-order conditions with respect to $l_{i,v}$ and $l_{j,r}$ imply that

$$\frac{U_j}{U_i} = \frac{Y_i^{\frac{1}{\theta}}y_{i,v}^{\frac{\theta-1}{\theta}}l_{i,v}^{-1}}{Y_j^{\frac{1}{\theta}}y_{j,r}^{\frac{\theta-1}{\theta}}l_{j,r}^{-1}} \tag{24}$$

In competitive equilibrium

$$\frac{U_j}{U_i} = \frac{\left(1-\tau_i\phi^i\right)\left(1-\xi_i\delta^{i,v}\right)Y_i^{\frac{1}{\theta}}y_{i,v}^{\frac{\theta-1}{\theta}}l_{i,v}^{-1}}{\left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right)Y_j^{\frac{1}{\theta}}y_{j,r}^{\frac{\theta-1}{\theta}}l_{j,r}^{-1}} \tag{25}$$

Together, equations (24) and (25) imply:

$$\left(1-\tau_i\phi^i\right)\left(1-\xi_i\delta^{i,v}\right) = \left(1-\tau_j\phi^j\right)\left(1-\xi_j\delta^{j,r}\right) \tag{26}$$

For $\{j, r\}$ and $\{j, \tilde{r}\}$ we have $\xi_j \delta^{j,\tilde{r}} = \xi_j \delta^{j,r}$. Since $A_{j,r} \neq A_{j,\tilde{r}}$ it follows that $\xi_j = 0$ Together with (26) it implies

$$\tau_i \phi^i = \tau_j \phi^j$$

If $\left\{f'^j(\tau) < 0\right\}_{j=1}^N$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j < \tau_i$. Similarly if $\left\{f'^j(\tau) > 0\right\}_{j=1}^N$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j > \tau_i$. ■

Item (i) (and (ii)) suggests that if the effective tax rate, $\phi(\tau)\tau$, is decreasing (increasing) with the tax rate τ, then it is optimal to tax heavier the sector with smaller (larger) tax evasion.

3.1 Alternative Tax Policy on Intermediate Goods

Suppose now that, instead of applying the same tax rate (ξ_n) on every firm that supplies to sector n, the government applies taxes (ξ_s) conditional on firm type s. It can be imagined as a tax policy in which the authority taxes accordingly to the origination of the goods instead of to their destination.

Proposition 4. *Suppose that no firm can evade tax,* $\{\phi^n(\tau_n, Z_n) = 1\}_{n=1}^N$ *and* $\{\delta^{n,s}(\xi_s, A_{n,s}) = 1\}_{n,s=1}^{N,S}$. *The Ramsey allocation is decentralized with*

1. *a uniform tax rate policy on final good firms,* $\tau_i = \tau_j$ *for any* $i, j \in \{1, 2, \ldots, N\}$.

2. *a uniform tax rate policy on intermediate good firms,* $\xi_r = \xi_v$ *for any* $r, v \in \{1, 2, \ldots, S\}$.

Proof. From previous proofs we know that from Ramsey and competitive equilibria we must have

$$(1-\tau_i)(1-\xi_v) = (1-\tau_j)(1-\xi_r) \text{ for any } i, j \in \{1, 2, \ldots, N\}; \text{ and } v, r \in \{1, 2, \ldots, S\} \tag{27}$$

It is easy to show that $\tau_i = \tau_j$ for any $i, j \in \{1, 2, \ldots, N\}$ and $\xi_r = \xi_v$ for any $r, v \in \{1, 2, \ldots, S\}$. ■

Proposition 5. *Suppose that only intermediate firms can evade tax,* $\{\phi^n(\tau_n, Z_n) = 1\}_{n=1}^N$ *and* $\{\delta^{n,s}(\xi_s, A_{n,s}) < 1\}_{n,s=1}^{N,S}$. *Also, assume that there exist* $A_{j,r}, A_{j,v}, A_{i,r}, A_{i,v}$, *for some* $\{j, i\} \in \{1, 2, \ldots, N\}$ *and* $\{r, v\} \in \{1, 2, \ldots, S\}$, *such that* $\frac{\delta^{j,r}(\xi_r, A_{j,r})}{\delta_{r,}(\xi_r, A_{j,v})} \neq \frac{\delta^{4,r}(\xi_r, A_{i,r})}{\delta^{ir,v}(\xi_v, A_{i,v})}$. *Once again, the Ramsey allocation is decentralized with an uniform tax rate policy on final goods,* $\tau_i = \tau_j$, *and no tax on intermediate goods,* $\xi_r = \xi_v = 0$.

Proof. It follows from previous proofs that in Ramsay allocation we must have:

$$(1-\tau_i)\left(1-\xi_v \delta^{i,v}(\xi_v, A_{i,v})\right) = (1-\tau_j)\left(1-\xi_r \delta^{j,r}(\xi_r, A_{j,r})\right) \tag{28}$$

Suppose that tax authority sets $\xi_r \neq 0$ and $\xi_v \neq 0$. For any two sectors $j, i \in \{1, 2, \ldots, N\}$ it would require:

$$\frac{\xi_r}{\xi_v} = \frac{\delta^{j,r}(\xi_r, A_{j,r})}{\delta^{j,v}(\xi_v, A_{j,v})} = \frac{\delta^{i,r}(\xi_r, A_{i,r})}{\delta^{i,v}(\xi_v, A_{i,v})}.$$

It is easy to prove that it is not possible simultaneously to have $\xi_r = 0$ and $\xi_v \neq 0$. Since $\xi_r = \xi_v = 0$, it implies that $\tau_i = \tau_j$. ■

Proposition 6. *Suppose that every firm can practice some tax evasion,* $\{\phi^n(\tau_n, Z_n) < 1\}_{n=1}^{N}$ *and* $\{\delta^{n,s}(\xi_s, A_{n,s})$ $1\}_{n,s=1.}^{N,S}$ *Also, assume that* $Z_i > Z_j$*, for some* i *and* j*, and there exist* $A_{j,r}, A_{j,v}, A_{i,r}, A_{i,v}$*, for some* $\{j, i\} \in \{1, 2, \ldots, N\}$ *and* $\{r, v\} \in \{1, 2, \ldots, S\}$*, such that* $\frac{\delta^{j,r}(\xi_r, A_{j,r})}{\delta, v}(\xi_r, A_{j,v}) \neq \frac{\delta^{(,r}(\xi_r, A_{i,r})}{\delta^{r,}(\xi_v, A_{i,v})}$. *The Ramsey allocation is decentralized with no taxation on intermediate goods,* $\xi_i = \xi_j = 0$ *and a non-uniform tax rate policy on final good firms,* $\tau_i \neq \tau_j$*. Moreover,*

1. *If* $\left\{f^j(\tau, A_j) = \phi^j(\tau, A_j)\tau\right\}_{j=1}^{N}$ *and* $\left\{f'^j(\tau) < 0\right\}_{j=1}^{N}$*, then* $\tau_j < \tau_i$ *for* $\phi^j(\tau) < \phi^i(\tau)$*.*
2. *If* $\left\{f^j(\tau) = \phi^j(\tau)\tau\right\}_{j=1}^{N}$ *and* $\left\{f'^j(\tau) > 0\right\}_{j=1}^{N}$*, then* $\tau_j > \tau_i$ *for* $\phi^j(\tau) < \phi^i(\tau)$*.*

Proof. Just similar to previous proofs, in Ramsey allocations must hold:

$$\left(1 - \tau_i\phi^i(\tau_i, Z_i)\right)\left(1 - \xi_v\delta^{i,v}(\xi_v, A_{i,v})\right) = \left(1 - \tau_j\phi^j(\tau_j, Z_j)\right)\left(1 - \xi_r\delta^{j,r}(\xi_r, A_{j,r})\right) \tag{29}$$

It follows from Proposition 5's proof that $\xi_s = 0$ for every $s \in \{1, 2, \ldots, S\}$. Which means:

$$\tau_i\phi^i(\tau_i, Z_i) = \tau_j\phi^j(\tau_j, Z_j)$$

If $\left\{f'^j(\tau) < 0\right\}_{j=1}^{N}$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j < \tau_i$. Similarly if $\left\{f'^j(\tau) > 0\right\}_{j=1}^{N}$ and $\phi^j(\tau) < \phi^i(\tau)$, then $\tau_j > \tau_i$.

Item 1 (and 2) suggests that if the effective tax rate, $\phi(\tau)\tau$, is decreasing (increasing) with the tax rate τ, then it is optimal to tax heavier the sector with smaller (larger) tax evasion. ■

4 Numerical Implementation

In this section we the model numerically and perform comparative statics. We evaluate the effects of different tax policies on GDP and welfare, while keeping the same level of government expenditures.

4.1 Parameterization

First, we choose the utility function to be

$$\mathcal{U} = \eta\sum_{n=1}^{N}\gamma_n\log C_n + (1-\eta)\log(1-l)$$

We set $\eta = 0.5$ so that in equilibrium the labor supply is in the interval $[1/3, 1/2]$. Parameter β represents the labor income share over GDP and we set $\beta = 0.45$. We use reference values from the literature for the elasticity of substitution. Oberfield and Raval (2014), Redding and Weinstein (2018) and Hobijn and Nechio (2019) find values ranging from 0.75 to 3.22. We set it to $\theta = 1.5$. For the baseline economy tax rates, we use a uniform tax of 20% across sectors, both for final goods and intermediate goods firms. The sectoral values for productivity parameters, share of declared revenues and share of household's income used in each consumption goods are displayed in Table 1. The values for the elasticities of evasion with respect to the tax rates (displayed in the first column) were estimated in Albuquerque et al (2018). The authors used a natural experiment occurred in Brazil to estimate the elasticity for several sectors of the economy. Also, due to the difficulty to observe the levels of evasion for each sector of the economy, we choose a functional form for the fraction of declared revenues. In particular, we are using $\phi_i = (1 - \tau_i)^{\frac{\bar{Z}}{Z_i}}$, where $\bar{Z}$ represents the average productivity. Note that ϕ is an increasing function of τ and a decreasing function of Z.

Sector	Parameter			
	ϵ	γ	A	ϕ
Agriculture	-0.260529	0.036496	1.000000	0.345317
Extractive Activities	-0.164795	0.001217	13.881679	0.926263
Manufacturing	-0.153139	0.352798	2.399491	0.642022
Construction	-0.357528	0.001217	1.356234	0.456575
Commerce	-0.222271	0.194647	1.419847	0.472897
Transportation	-0.190024	0.048662	1.489822	0.489827
Information services	-0.124356	0.048662	9.358779	0.892602
Financial services	-0.079127	0.097324	10.653944	0.905016
Other services	-0.296588	0.218978	1.325700	0.448404

Table 1: Sectoral parameters

4.2 Counterfactuals

The optimal tax rule as described in Proposition 3 states that $\tau_i \phi^i = \tau_j \phi^j$ and that the intermediate goods are not taxed. So if we change sector's i tax rate from τ_i to $\tau_i' = \tau_i + \Delta\tau_i$, the corresponding level of declared revenue by firm i will be

$$\phi_i' = \phi_i\Big(1 + \varepsilon_i \frac{\Delta\tau_i}{\tau_i}\Big),$$

where ε_i is the elasticity of ϕ_i with respect to τ_i. So in the optimal we must have

$$(\tau_i + \Delta\tau_i)\phi_i\Big(1 + \varepsilon_i \frac{\Delta\tau_i}{\tau_i}\Big) = (\tau_j + \Delta\tau_j)\phi_j\Big(1 + \varepsilon_j \frac{\Delta\tau_j}{\tau_j}\Big), \text{ for all } i, j.$$

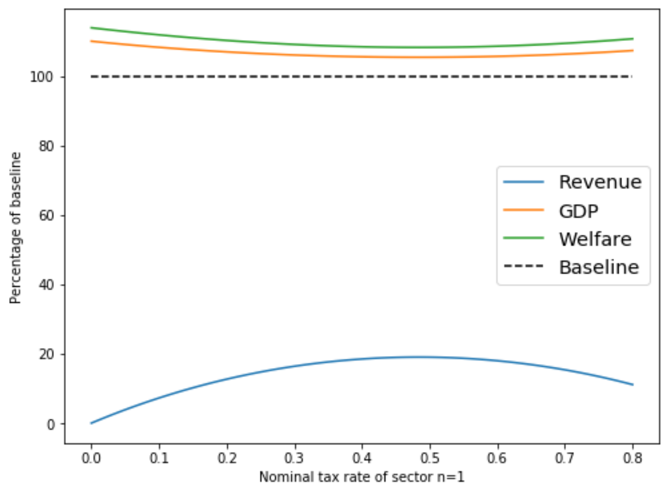

Figure 1: Optimal tax policy

Note that this means that there is a degree of freedom in the choice of tax levels. That is, we can choose how much to change in the nominal tax rate of sector i, and the remaining taxes are determined. This policy is shown in Figure 1, for different levels of nominal taxes chosen for sector 1 (Agriculture). Note that we can choose this sector to change the nominal tax rate without loss of generality. The vertical axis measures the counterfactual variables as percentages of the baseline variables, so the dotted line represents the baseline economy. Note that for all levels of final goods taxes, we have an increase both in GDP and welfare, although the government's revenues fall sharply, which means that the government should look for other sources in order to finance its spending's. At the maximum of the revenues, which is about 20% of the baseline, we have an increase of 5% of GDP and 8% of welfare, while the nominal tax for sector 1 is 48% but the effective tax is only about 10%.

Despite the benefits of the optimal tax policy reflected in figure 1, the magnitude of the losses in revenues suggest that this policy is unfeasible. Since these losses come from cutting intermediate goods taxes, we

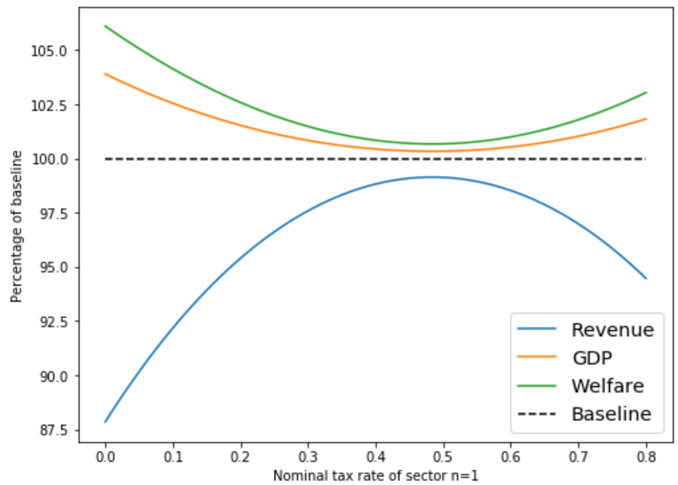

Figure 2: Optimal tax policy on final goods. Intermediate goods taxes are unchanged

implement alternative policies that change this prescription. First, we keep the intermediate goods taxes unchanged, this is, a 20% nominal tax rate, while we change the final goods taxes as we did before. This is shown in Figure 2. Note that in this policy there are still losses in the tax revenues, but they are sgnificantly smaller. At the maximum point the revenues are 99.13% of the baseline,while there is a 0.32% increase in GDP and 0.66% in welfare. Again the nominal tax levels are 48% for sector 1 (Agriculture) and the effective tax is about 10%.

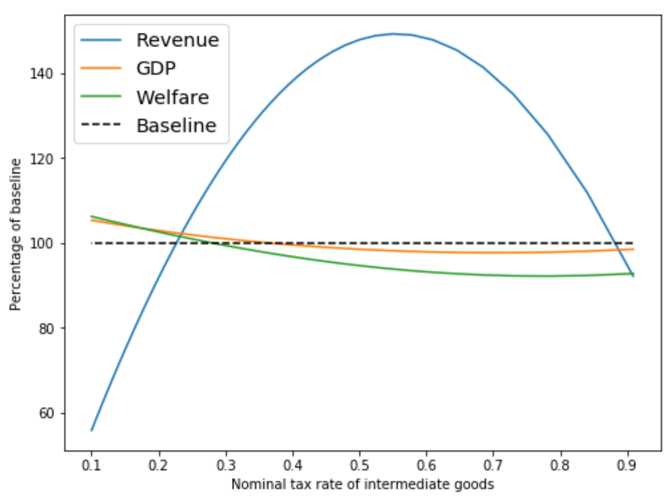

Figure 3: Optimal tax policy on final goods. Intermediate goods taxes are unchanged

Second, we impose a policy in which we keep the effective tax on (final goods) sector 1 unchanged, and we vary the intermediate goods taxes. Note that in this policy the nominal taxes of the sectors are modified accordingly to first part of proposition 3. This policy is shown in Figure 3. Note that there is a range of intermediate goods taxes in which we have a Pareto improvement: GDP and welfare increase while the government is able to finance its consumption. At the point that the revenue is constant relative to the baseline, we have a 2.26% increase in GDP and 1.55% of welfare.

5 Concluding Remarks

Classic tax evasion literature pay little or no attention to developing countries context of endemic tax evasion and how firms intrinsic characteristics would affect tax evasion. Assuming that levying commodities are necessary, we exploit whether the opportunity to tax circumvention would affect traditional policy recommendations on commodities and intermediate goods taxation. In other words, we assess whether homogeneous tax rates on commodities and zero tax rate on intermediate goods rules still applies. We also evaluate whether different collection systems would modify such scenarios.

In order to do so, we followed Ramsey (1927) approach to optimal taxation and, under a monopolistic competition model (Dixit and Stiglitz, 1977), allowed for the possibility of indirect tax evasion by firm in both final and intermediate goods sectors. Concealment technology was set in such a way that firm innate features, such as productivity, affects the probability of detection.

Our results show when there is no evasion, uniform nominal tax rates recommendation still applies to commodities taxation. However, in the presence of tax evasion, uniform nominal taxes rates are no longer an optimal policy. We derived optimal taxation conditions that dictates that tax authority should levy homogeneous effective taxes rates (and heterogeneous nominal tax rates) on final consumption goods. In addition, optimal taxes on intermediate goods are still zero even when intermediate firms are allowed to conceal revenues. Finally, optimal tax rules do not depend on which collection systems the tax authority sets up.

Optimal taxation rules under generalized tax evasion suggest that effective taxes rates, rather then nominal taxes rates, should be uniform. However it would require the government to know in advance how every final good evasion respond to taxes, i.e., $\phi^j\left(\tau_j, Z_j\right)$ for every j. As firms characteristics evolve along lifetime, taxes rates would have to adapt regularly to obey to derived rule. Since such policy can not be realistic implemented, government could rely on feasible convergent approaches. For example, could carefully choose a set of final goods to which it is be possible to diminish tax rates and simultaneously augment tax compliance. For such a set it would be wise to charge alleviated tax rates.

This paper did not addressed the question of how the presence of more complex utility functions structures would affect results. Also, it would be of great utility to explore how ϕ would endogenously emerge and to implement a more sophisticated strategy for the estimation of the parameters.

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