Quantifying Systemic Risk in Cryptocurrency Markets: A High-Frequency Approach

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Abstract We conduct a comparative analysis of the original $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$ and modified $\mathrm{CoVaR}_{\alpha,\beta}(Y|X)$ measures of Conditional Value-at-Risk (CoVaR) using high-frequency returns of Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Solana (SOL), and Binance Coin (BNB) at 5-minute intervals. Additionally, we employ the Kolmogorov-Smirnov (KS) bootstrapping test to assess potential interdependencies among cryptocurrency returns. Our results indicate that, on average, estimates derived from $\mathrm{CoVaR}_{\alpha,\beta}(Y|X)$ tend to surpass those from $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$, with superior performance in the backtesting analysis. Moreover, the Kolmogorov-Smirnov (KS) test underscores a notable degree of interconnectedness within cryptocurrency returns.

Keywords: Conditional Value-at-Risk (CoVaR), Cryptocurrencies, Backtesting.
JEL Code: C58, G17, G32

1. Introduction

According to Waltz et al. (2022), the financial system and its institutions have been shown to be volatile, fragile, and interconnected through various developments and crises in the past two decades, including the 2009 financial crisis. Reboredo and Ugolini (2015) highlights the utility of systemic risk measures in assessing the impact of financial distress in one asset on others, which has led to increased attention in recent financial literature (Reboredo and Ugolini, 2016; Zhang, 2015; Liu et al., 2019; Karimalis and Nomikos, 2018; Jin, 2018).

There are numerous methods available for estimating systemic risk within a market. Some of these include the use of Systemic Expected Shortfall (SES) as proposed by Acharya et al. (2017), which quantifies systemic risk, and the focus on Marginal Expected Shortfall (MES) as discussed by Brownlees et al. (2012). Additionally, Segoviano and Goodhart (2009) presents a method where the financial sector is represented by a portfolio of individual firms to estimate a multivariate density tail measure adjusted with empirical data from each institution, offering insights into systemic risk measures.

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However, one of the most commonly used approaches, as indicated by Waltz et al. (2022), is the model proposed by Adrian and Brunnermeier (2016) known as Conditional Value-at-Risk (CoVaR), which extends the Value-at-Risk (VaR) measure to a conditional setting. According to Girardi and Ergün (2013), CoVaR effectively captures the propensity for financial distress in one asset to be correlated with distress in another asset.

One asset class where these measures are especially relevant is cryptocurrencies. In the last decade, as highlighted by Waltz et al. (2022), the cryptocurrency market has attracted the attention of investors and policymakers. Investors embrace, gradually, the idea of these digital currencies, while the policymakers warn of their volatility, which raises the question of whether they will ever become stable and viable mainstream currencies (Velde, 2013; Gandal et al., 2018; Lo and Wang, 2014). However, this asset class is characterized by greater volatility and tail risk than traditional assets (Chaim and Laurini, 2018; Borri, 2019).

Below are descriptions of several studies that have estimated CoVaR in the cryptocurrency market. Borri (2019) explored the extent to which cryptocurrencies are exposed to each other's tail risk using a model based on quantile regressions proposed by Adrian and Brunnermeier (2016). Their findings indicate high correlations among these markets in the tails of the distribution.

Bruhn and Ernst (2022) employed a GARCH-EVT (Extreme Value Theory) approach to analyze the financial risk of individual cryptocurrencies and a portfolio of the cryptocurrency market. They also utilized a t-Student Copula to aggregate individual market risks to investigate potential diversification effects. The results suggest that Bitcoin is the most stable cryptocurrency, while others exhibit higher volatilities in their prices.

Furthermore, Rehman et al. (2020) employed both time-invariant and time-varying copula models to capture the co-movement between Bitcoin and Islamic equity indices under extreme market conditions. Their results indicate time invariance with symmetrical tail dependence for all Islamic indices paired with Bitcoin, except for DJIUK, DJIJP, and DJICA. They also observed asymmetry between downside and upside ΔCoVaR, suggesting implications for investors with varying risk preferences.

In their study, Hanif et al. (2022) applied a time-varying copula model to analyze systemic risk dynamics in cryptocurrency markets. They investigated eight cryptocurrencies (Monero, Bitcoin, Dash, Litecoin, Stellar, XRP, Ethereum, and Nem) alongside global/regional equity markets (world, Americas, Europe, and Asia Pacific). Their findings revealed that Nem and Ethereum exhibit the most significant upside and downside CoVaR spillovers on the world equity index. Conversely, the largest downside CoVaR spillovers from the world equity index to cryptocurrencies were observed with Nem and Stellar, while the largest upside CoVaR spillovers were identified with Ethereum and Nem.

This paper focuses on a modified version of Conditional Value-at-Risk (CoVaR), as proposed by Girardi and Ergün (2013), which defines financial distress as the return of the institution being at most at the VaR measure, in contrast to the exact VaR definition by Adrian and Brunnermeier (2016). We employ this modified CoVaR framework to estimate systemic risk among major cryptocurrencies based on market capitalization, including Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Solano (SOL), and Binance Coin (BNB), at a 5-minute frequency.

For comparative analysis, we also estimate CoVaR using the original model proposed by Adrian and Brunnermeier (2016), conducting backtests on both models. Furthermore, we utilize the Kolmogorov-Smirnov (KS) test to assess the impact between the quantiles of conditional and unconditional asset returns.

According to Mainik and Schaanning (2014), $X \leqslant \mathrm{VaR}_{\alpha}(X)$ corresponds to all possible outcomes for X if X is stressed, while $X = \mathrm{VaR}_{\alpha}(X)$ is the case when selecting only the most "benign" of them. This change in the model allows to consider more severe distress moments and, also, backtesting the CoVaR measure using standard tests. The model present by Girardi and Ergün (2013) is calculated using Copulas, while the Adrian and Brunnermeier (2016) model are estimated with quantile regressions (see Koenker, 2005 or Koenker et al., 2017).

Zhang (2015) highlights the importance of estimating the high-dimensional joint distribution for effective risk management, given the dynamic co-movement among risk issuers. In this regard, Copula methods emerge as strong candidates. As noted by Reboredo and Ugolini (2016), employing Copula methods offers greater flexibility in modeling compared to quantile regressions. This flexibility stems from Copulas' ability to accommodate heterogeneity in marginal distributions and to account for specific features such as volatility asymmetries, conditional heteroskedasticity, and leverage effects.

Our results indicate that CoVaR estimates from the Girardi and Ergün (2013) model are generally higher, on average, compared to those from the Adrian and Brunnermeier (2016) model. Additionally, the VaR results for both methods demonstrate higher values on average than their respective conditional measures, indicating positive interdependence in the cryptocurrency market. Through backtesting analysis, the Girardi and Ergün (2013) model exhibits a lower percentage of violations compared to the Adrian and Brunnermeier (2016) model. This indicates a better fit, in the sense of being more conservative, of VaR and CoVaR in the former model.

We also found that the interconnections among cryptocurrencies are relevant. The Kolmogorov-Smirnov (KS) bootstrapping test reveals that returns of one cryptocurrency significantly influence the returns of another cryptocurrency across all possible combinations. This finding suggests a high level of interconnections among cryptocurrencies.

The study underscores the significance of accurately measuring systemic risk for regulatory and risk management purposes, particularly in the rapidly growing and volatile cryptocurrency market. The use of Girardi and Ergün (2013) model, which is better suited for measuring systemic risk, can lead to more precise estimates, offering practical implications for investors in terms of portfolio allocation and risk management, as well as for policymakers during both crisis and non-crisis periods. The main motivation for this study is to find better methods to estimate the systemic risk present in financial markets and especially in the cryptocurrency market, which has high volatility, using a high-frequency database.

The paper is organized as follows: Section 2 formally defines the methodology implemented. Section 3 describes the data we use in the empirical part of this study and Section 4 presents the empirical results and reports the results of Kolmogorov-Smirnov (KS) test. Section 5 reports the results of the backtesting. Section 6 concludes.

2. Methodology

2.1 Conditional Value at Risk

Adrian and Brunnermeier (2016) highlights that Value-at-Risk (VaR), commonly used in financial markets, primarily assesses risk at the individual asset level, neglecting systemic risk implications. Borri (2019) emphasize that the CoVaR measure, in contrast, enables the estimation of an asset's exposure to tail-risk of another asset. Formally, CoVaR represents a risk measure conditioned on an adverse shock, with risk defined by the VaR measure.

Following Adrian and Brunnermeier (2016), the CoVaR measure is computed as the Value-at-Risk (VaR) of an asset y assuming that asset x is exactly at its VaR level, that is, $X = \text{VaR}_\alpha(\text{X})$. However, Girardi and Ergün (2013) propose an alternative way for calculate the Conditional Value-at-Risk (CoVaR), which assumes that the Value-at-Risk (VaR) of an asset y is conditioned on $X \leqslant \text{VaR}_\alpha(\text{X})$, that is, asset x is at most at its VaR level. Girardi and Ergün (2013) highlight that this modification allows compatibility of CoVaR estimation with non-parametric methods.

The Adrian and Brunnermeier (2016) definition and the alternative definition proposed by Girardi and Ergün (2013) are denoted, respectively, as follows

$$\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X) = \mathrm{VaR}_{\beta}(Y|X = \mathrm{VaR}_{\alpha}(X)), \tag{1}$$

$$\mathrm{CoVaR}_{\alpha,\beta}(Y|X) = \mathrm{VaR}_{\beta}(Y|X \leqslant \mathrm{VaR}_{\alpha}(X)). \tag{2}$$

Girardi and Ergün (2013) say that the conditioning on $X_t \leqslant \mathrm{VaR}_{q,t}$ represents a more general case of financial turmoil for asset y, which allows for higher losses, that is, beyond $\mathrm{VaR}_{q,t}$. Mainik and Schaanning (2014) also show that conditioning on $X_t \leqslant \mathrm{VaR}_{q,t}$ gives a much better response to dependence between the assets, and with positive probabilities, the fitting and backtesting of the model are statistically more advantageous.

Another important point is the estimation method: while the Adrian and Brunnermeier (2016) method is estimated using quantile regressions, Girardi and Ergün (2013) propose estimating its version of CoVaR using the bivariate distribution of assets using Copula representations. More details on this will be provided in the following sections.

2.2 Copula Functions

For Zhang (2015), Copula is a powerful method for dealing with high-dimensional joint cummulative distribution functions (CDFs). The Sklar's theorem (Trivedi et al., 2007) defines an m-dimensional copula as a function C which is defined from the unit m-cube $[0, 1]^m$ to the unit interval [0, 1] satisfying the following conditions

1. $C(1,...,1,a_n,1,...,1) = a_n$ for $n \leqslant m$ and a_n in [0, 1];
2. $C(a_1,...,a_m) = 0$ if $a_n = 0$ for $n \leqslant m$;
3. C is m-increasing.

and thus the function C is also a distribution function.

The copula function is a method to construct a multivariate distribution function combining the marginal distributions using the copula function, and in this way we have a flexible form to construct multivariate distributions with arbitrary marginal distributions and dependence structures, parameterized using the copula. The construction of the multivariate density is given by:

$$\mathrm{F}(y_1,...,y_m) = \mathrm{C}(\mathrm{F}_1(y_1),...,\mathrm{F}_m(y_m);\theta), \tag{3}$$

where $F_i(y_i)$ is the marginal distribution of the i-th element for the multivariate distribution and Θ denotes a parameter vector of the copula called the dependence parameter, controlling the dependence between the marginals functions. See Nelsen (2006) for examples of copula functions and other properties.

The flexibility of the model, as highlighted by Reboredo and Ugolini (2015), compared to parametric bivariate functions, is the major advantage of copula functions. This greater flexibility occurs because copulas allow for separate modeling of the marginals and the dependence structure.

2.3 Volatility Model

We have some stylized facts in financial time series as volatility clusters, fat tails and, also, leverage effects (Waltz et al., 2022; Zhang et al., 2018; Phillip et al., 2018). The returns in cryptocurrencies market also are characterized by this facts (Chaim and Laurini, 2018). A common approach to model this effects is to use conditional volatility models with possible dependence in the mean.

In order to capture these dynamics, we follow Reboredo and Ugolini (2016), which assumes that the asset price returns, y_t, have time-varying mean, μ_t, is given by an Autoregressive Moving-Average (ARMA) model:

$$y_t = \mu_t + \varepsilon_t, \tag{4}$$

$$\mu_t = \phi_0 + \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{h=1}^{q} \varphi_j \varepsilon_{t-h}, \tag{5}$$

denoting ϕ_0, ϕ_j and φ_j, respectively, as a intercept parameter, the autoregressive (AR) and moving average (MA) parameter vectors. The term $\varepsilon_t = \sigma_t z_t$ is a stochastic variable with unit variance and zero mean.

We assume that the dynamics of variance of the ε_t is given by a Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH) model proposed by Glosten et al. (1993) as follow

$$\sigma_t^2 = \omega + \sum_{k=1}^{r} \beta_k \sigma_{t-k}^2 + \sum_{h=1}^{m} \alpha_h \varepsilon_{t-h}^2 + \sum_{h=1}^{m} \lambda_h 1_{t-h} \varepsilon_{t-h}^2, \tag{6}$$

where ω is an intercept, β represents the GARCH parameters, α denotes the autoregressive conditional heteroskedasticity (ARCH) parameters, and the parameter 1_{t-h} equals one if $\varepsilon_{t-h} < 0$ and zero otherwise. Reboredo and

Ugolini (2016) emphasize that the TGARCH model is useful because the parameter λ captures asymmetry effects. Specifically, for $\lambda > 0$, negative past shocks have a greater impact on variance than positive shocks.

As cryptocurrencies present a strong indication of skewness and, also, non-normality (Vieira and Laurini, 2023), we choose the skewed-t density distribution of Fernández and Steel (1998), which parameters η and υ denoting, respectively, the skewness and shape as the of the distribution of the innovations for the returns. An interesting observation is that when the parameter of symmetry, denoted by η, equals one and the degrees of freedom (shape), denoted by υ, tend to infinity, the skewed-t density distribution converges to the Gaussian density. Furthermore, when $\eta = 1$ and υ is finite, the distribution converges to the symmetric Student-t distribution, and thus we assume a flexible distribution for the conditional returns.

We employ a partially automated procedure that chooses the most suitable model for each series based on an Akaike information criterion. Table 1 shows the models chosen.

Table 1
Model Selection

Series	Selected Model	Intercept in Mean
BTC	ARIMA(1, 0, 1) - GJR GARCH(1, 1)	No
ETH	ARIMA(1, 0, 1) - GJR GARCH(1, 1)	Yes
XRP	ARIMA(1, 0, 1) - GJR GARCH(1, 1)	Yes
SOL	ARIMA(1, 0, 1) - GJR GARCH(1, 1)	Yes
BNB	ARIMA(0, 0, 1) - GJR GARCH(1, 1)	No

Note: This table reports the models that were selected using the Akaike information criterion for the cryptocurrencies BTC, ETH, XRP, SOL and BNB.

2.4 Quantile and Conditional Quantile Estimations

Building on the selected univariate models, according to Reboredo and Ugolini (2016), the estimation of unconditional quantiles used in VaR estimation for the returns of assets occurs as follows.

$$\mathrm{VaR}_{\beta,t}(X) = \mathrm{q}_{\beta,t}^{x_t} = \mu_t + \sigma_t \mathrm{F}_{\upsilon,\eta}^{-1}(\beta), \tag{7}$$

where $\mathrm{F}_{\upsilon,\eta}^{-1}(\beta)$ is the β-quantile of the skewed Student-t distribution and μ_t, σ_t, υ and η are obtained from estimated models in equations (5) and (6).

Reboredo and Ugolini (2016) point out that in analysis of financial risk, the Value-at-Risk (VaR) is calculated for low values of α, normally for 5% and 1%. These significance levels are widely acknowledged and commonly accepted as standard in academic literature (Müller et al., 2022; Trucíos, 2019; Ardia et al., 2019; Müller and Righi, 2018).

Following Reboredo and Ugolini (2016), the α-quantile of an asset return distribution for a given β-quantile of another asset return given by $\mathrm{P}(y_t \leqslant \mathrm{q}^{y_t|x_t}_{\alpha,\beta,t} | x_t \leqslant \mathrm{q}^{x_t}_{\beta,t}) = \alpha$ can be calculated as

$$\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t} = \mathrm{F}^{-1}_{y_t|x_t \leqslant \mathrm{q}^{x_t}_{\beta,t}}(\alpha), \tag{8}$$

where $\mathrm{F}^{-1}_{y_t|x_t \leqslant \mathrm{q}^{x_t}_{\beta,t}}(\alpha)$ denotes the inverse distribution of y_t conditional on $x_t \leqslant \mathrm{q}^{x_t}_{\beta,t}$.

To estimate the conditional quantiles for assets' returns distribution, we can employ copula functions for the estimation of the joint distribution function:

$$\mathrm{P}(y_t \leqslant \mathrm{q}^{y_t|x_t}_{\alpha,\beta,t} | x_t \leqslant \mathrm{q}^{x_t}_{\beta,t}) = \frac{\mathrm{F}_{y_t x_t}(\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t}, \mathrm{q}^{x_t}_{\beta,t})}{\mathrm{F}_{x_t}(\mathrm{q}^{x_t}_{\beta,t})} = \alpha. \tag{9}$$

Using the Sklar (1959) Theorem, we can express the joint distribution function, that is, equation (9), in terms of a Copula function (C), where $\mathrm{C}(F_X(x), F_Y(y)) = \mathrm{F}_{XY}(x,y)$ and $\mathrm{F}_{x_t}(\mathrm{q}^{x_t}_{\beta,t}) = \beta$. Therefore, the equation (9) can be rewritten as

$$\mathrm{C}(\mathrm{F}_{y_t}(\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t}), \beta) = \alpha\beta. \tag{10}$$

Inverting the Copula function (Equation (10)) for given values of α and β, we obtain an estimated value for $\mathrm{F}_{y_t}(\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t})$, that is, $\hat{\mathrm{F}}_{y_t}(\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t})$. Then, as in Reboredo and Ugolini (2016), by inverting the marginal distribution function of y_t, the conditional quantile is obtained as

$$\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t} = \mathrm{F}^{-1}_{y_t}(\hat{\mathrm{F}}_{y_t}(\mathrm{q}^{y_t|x_t}_{\alpha,\beta,t})). \tag{11}$$

In this work we use a t-copula to construct the bivariate distributions. As discussed by Demarta and McNeil (2005), the t-copula (Embrechts et al., 2001; Fang and Fang, 2002), embodies the underlying dependence structure found within a multivariate t distribution. This model has garnered significant attention recently, particularly in the modeling multivariate financial return data. Recent studies, (e.g., Breymann et al., 2003), demonstrated that the empirical fit of the t-copula generally surpasses that of the Gaussian copula, which represents the dependence structure of the multivariate normal distribution. One rationale for this superiority lies in the t-copula's ability to more accurately capture the phenomenon of dependent extreme values, commonly observed in financial return data.

Following Karimalis and Nomikos (2018), we rescale the $\text{CoVaR}_{\alpha,\beta,t}$ measure using the fitted conditional mean $\mu_{s,t}$ and the standard deviation $\sigma_{s,t}$ of $\text{R}_{y,t}$, which are obtained, respectively, from the estimated models in equations (5) and (6). The $\text{CoVaR}_{\alpha,\beta,t}$ is calculated as follows

$$\text{CoVaR}_{\alpha,\beta,t}(Y|X) = \mu_{s,t} + \sigma_{s,t}\text{q}^{y_t|x_t}_{\alpha,\beta,t}. \tag{12}$$

3. Data Description

This section describes the database, which is made up of intraday data sampled at 5-minute frequency, on asset returns starting on 2020-08-11 and ending on 2023-10-31, for the following cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Solana (SOL) and Binance Coin (BNB) [1]. An important note is that due to the unavailability of cryptocurrencies measured in US dollars, we chose USDT as the conversion currency for our data. Tether (USDT) is a cryptocurrency (stablecoin) backed by the US dollar, and thus serves as a USD price reference for the cryptocurrency market.

Table 2 summarizes the descriptive statistics of the chosen assets and Figure 1 shows the evolution of the returns of the cryptocurrencies. The descriptive statistics support the perception of the high tail risk associated with cryptocurrencies, due to the high kurtosis values observed, and also the notable asymmetry in returns.

4. Empirical Results

First, we will analyze the VaR and CoVaR estimates for the Girardi and Ergün (2013) and Adrian and Brunnermeier (2016) models, as well as the results of Kolmogorov–Smirnov (KS) bootstrapping test for the equality between conditional and unconditional assets return quantiles. Next, the backtesting results for both models will be discussed. In the following sections, we analyze only the results estimated with 5% quantile. The results for the 1% quantile are displayed in A.

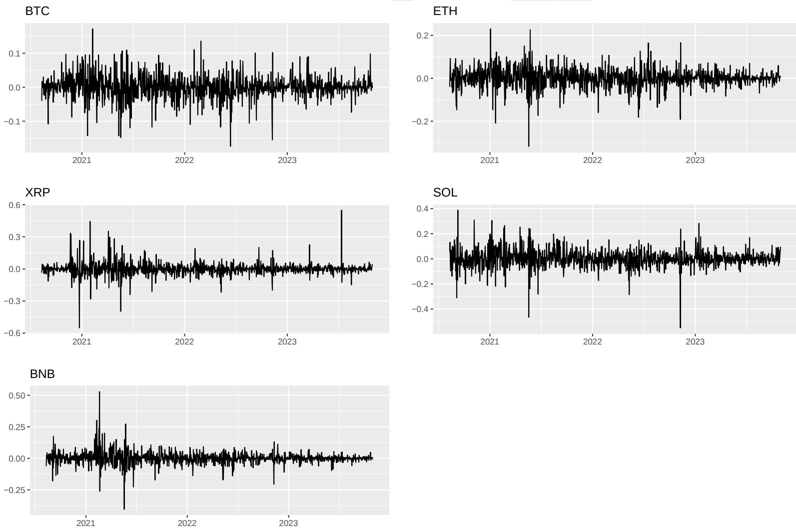
[1] Data on cryptocurrency returns was sourced from binance.com

Table 2
Descriptive Statistics for cryptocurrencies

	BTC	ETH	XRP	SOL	BNB
Mean (%)	0.000	0.000	0.000	0.001	0.001
Std (%)	0.223	0.284	0.391	0.464	0.299
Skew	-0.424	-0.285	-1.647	0.367	-0.314
Kurt	94.264	152.597	186.114	63.907	90.605
Min.	-10.415	-15.901	-24.587	-18.222	-11.831
Quantile 5%	-0.294	-0.379	-0.470	-0.624	-0.395
Median	0.000	0.000	0.000	0.000	0.000
Quantile 95%	0.296	0.382	0.463	0.620	0.389
Max.	8.987	16.463	20.105	18.032	14.333

Note: This table reports mean, standard deviation, skewness, kurtosis, minimum, quantile of 5%, median, quantile of 95% and maximum for the 5-minute returns on Bitcoin, Ethereum, Ripple, Solana and Binance Coin during the period of 2020-08-11 to 2023-10-31.

Figure 1
Evolution of daily returns of the analyzed cryptocurrencies.



The results are displayed in Tables 3 and 4 for the Girardi and Ergün (2013) model version, and Tables 5 and 6 for the Adrian and Brunnermeier (2016) model. Note that the averages of CoVaR, in the Girardi and Ergün (2013) model, are, in general, greater than the averages of the Adrian and Brunnermeier (2016) version. However, the values of minimum, skewness and kurtosis are more "conservative" in the Girardi and Ergün (2013) model than in the Adrian and Brunnermeier (2016) model.

Comparing these two approaches, Mainik and Schaanning (2014) found out that the CoVaR from Girardi and Ergün (2013) model, $\text{CoVaR}_{\alpha,\beta}(Y|X)$, presents properties of continuity and monotonicity with respect to the dependence parameter.

According to Girardi and Ergün (2013), the interpretation and properties of the risk measure appear to be more meaningful and practical when considering the event that $R_t^j \leqslant VaR_{q,t}^j$. Furthermore, unlike the Adrian and Brunnermeier (2016) model, the time-varying correlation implementation of the GARCH model allows the $\text{CoVaR}_{\alpha,\beta}(Y|X)$ of asset y to have a time-varying exposure to its VaR. This property makes it possible to detect and incorporate the variation in links between cryptocurrencies into systemic risk analyses.

Analyzing the univariate VaR resuls, we found out that, on average, they exhibit higher values than their respective conditional measures. For Waltz et al. (2022), which also found these results when analyze BTC, ETH, LTC, XMR, and XRP, this reflect the positive interdependence in the cryptocurrency market, in other words, the Conditional Value-at-Risk measure are driven by a similar dynamic as the Value-at-Risk. The results for the 1% quantile in A show that for quantiles lower than 5%, the results are analogous.

Table 3

VaR estimates - $X \leqslant \textbf{VaR}_\alpha(X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
BTC	-10.270	-0.357	-0.294	-0.036	0.277	-4.921	76.432
ETH	-16.110	-0.441	-0.366	-0.057	0.346	-6.525	147.701
XRP	-19.716	-0.602	-0.436	-0.135	0.571	-6.418	93.950
SOL	-14.594	-0.767	-0.620	-0.178	0.560	-4.282	39.134
BNB	-14.497	-0.466	-0.364	-0.090	0.394	-5.500	79.609

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of $\text{VaR}_\alpha(X)$ results using the (Girardi and Ergün, 2013) method. The α-quantile is 5%.

4.1 Testing the Effects of the Cryptocurrencies Quantiles on Other Assets Quantiles

Following Reboredo and Ugolini (2016), we estimated the impact between quantile assets returns obtained using the Girardi and Ergün (2013) model by testing the hypothesis of equality between conditional and unconditional assets return quantiles. The hypotheses can be summarized as follows:

Table 4
Conditional Value-at-Risk - CoVaR$_{\alpha,\beta}(Y|X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
ETH|BTC	-36.238	-0.970	-0.803	0.414	0.765	-6.560	150.145
XRP|BTC	-45.098	-1.323	-0.959	0.034	1.262	-6.571	100.203
SOL|BTC	-36.757	-1.717	-1.382	-0.025	1.272	-4.366	41.398
BNB|BTC	-34.429	-1.088	-0.850	1.013	0.926	-5.513	79.967
BTC|ETH	-20.470	-0.763	-0.627	0.377	0.592	-4.852	73.820
XRP|ETH	-42.213	-1.286	-0.933	-0.156	1.223	-6.527	98.170
SOL|ETH	-36.881	-1.755	-1.415	-0.370	1.293	-4.327	40.393
BNB|ETH	-32.216	-1.070	-0.837	-0.083	0.909	-5.466	78.037
BTC|XRP	-24.719	-0.794	-0.647	9.778	0.668	-4.434	67.321
ETH|XRP	-35.199	-0.982	-0.808	9.908	0.814	-5.884	122.974
SOL|XRP	-35.345	-1.538	-1.230	9.672	1.195	-4.374	43.312
BNB|XRP	-29.905	-0.997	-0.777	9.523	0.890	-5.125	69.807
BTC|SOL	-21.938	-0.782	-0.640	13.117	0.667	-3.959	55.179
ETH|SOL	-37.250	-1.017	-0.838	12.768	0.842	-5.742	120.816
XRP|SOL	-41.195	-1.166	-0.854	11.166	1.152	-6.166	91.079
BNB|SOL	-31.625	-0.972	-0.757	12.505	0.872	-4.927	66.304
BTC|BNB	-22.786	-0.828	-0.680	-0.085	0.640	-4.902	75.571
ETH|BNB	-36.597	-1.036	-0.860	-0.135	0.811	-6.492	145.746
XRP|BNB	-41.785	-1.263	-0.916	-0.288	1.195	-6.562	99.143
SOL|BNB	-31.150	-1.624	-1.313	-0.376	1.185	-4.263	38.800

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of CoVaR$_{\alpha,\beta}(Y|X)$ results using the (Girardi and Ergün, 2013) method. The α-quantile and β-quantile are both 5%. For the estimation, we use a t-copula.

Table 5
VaR estimates - $X =$ VaR$_\alpha(X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
BTC	-90.709	-0.286	-0.219	-0.178	0.591	-91.707	11460.269
ETH	-143.817	-0.370	-0.284	-0.228	0.900	-108.768	15064.272
XRP	-149.170	-0.470	-0.335	-0.276	1.180	-64.251	6109.092
SOL	-93.465	-0.615	-0.474	-0.385	0.894	-45.464	3411.802
BNB	-120.949	-0.384	-0.284	-0.231	0.830	-81.826	9738.395

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of VaR$_\alpha(X)$ results using the (Adrian and Brunnermeier, 2016) method. The α-quantile is 5%.

Table 6
Conditional Value-at-Risk - CoVaR$^{=}_{\alpha,\beta}(Y|X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
ETH|BTC	-159.124	-0.494	-0.385	-0.314	1.020	-97.628	12666.994
XRP|BTC	-180.109	-0.618	-0.461	-0.383	1.354	-71.447	7433.097
SOL|BTC	-156.754	-0.799	-0.620	-0.493	1.249	-62.880	6096.773
BNB|BTC	-144.988	-0.503	-0.384	-0.312	0.993	-86.279	10464.905
BTC|ETH	-136.418	-0.392	-0.306	-0.250	0.860	-104.256	14074.276
XRP|ETH	-210.091	-0.627	-0.469	-0.387	1.474	-81.879	9515.874
SOL|ETH	-195.317	-0.813	-0.628	-0.498	1.423	-75.077	8291.573
BNB|ETH	-170.887	-0.509	-0.388	-0.315	1.109	-97.060	12660.733
BTC|XRP	-113.596	-0.389	-0.293	-0.242	0.832	-74.462	8014.125
ETH|XRP	-169.951	-0.504	-0.379	-0.312	1.171	-85.350	10213.489
SOL|XRP	-150.766	-0.787	-0.596	-0.484	1.307	-51.795	4313.732
BNB|XRP	-150.776	-0.506	-0.372	-0.308	1.088	-73.162	8047.682
BTC|SOL	-83.570	-0.393	-0.306	-0.244	0.634	-68.517	7058.297
ETH|SOL	-129.924	-0.507	-0.392	-0.311	0.918	-80.582	9320.901
XRP|SOL	-145.516	-0.623	-0.459	-0.372	1.231	-55.210	4708.639
BNB|SOL	-116.989	-0.512	-0.384	-0.306	0.893	-64.307	6492.203
BTC|BNB	-113.762	-0.402	-0.310	-0.255	0.777	-85.750	10393.353
ETH|BNB	-165.164	-0.516	-0.395	-0.325	1.084	-93.478	11966.246
XRP|BNB	-187.359	-0.632	-0.463	-0.386	1.385	-69.801	7377.552
SOL|BNB	-169.063	-0.824	-0.626	-0.506	1.340	-59.554	5686.241

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of the CoVaR$^{=}_{\alpha,\beta}(Y|X)$ results using the (Adrian and Brunnermeier, 2016) method. We use a rolling window of 60-minute realized volatility of BTC, ETH, XRP, SOL and BNB as covariates in this model. The α-quantile and the β-quantile are both 5%.

$$\begin{aligned} \mathrm{H}_0 &: \mathrm{q}^{y_t}_{\alpha,t} = \mathrm{q}^{y_t|x_t}_{\alpha,\beta,t}, \\ \mathrm{H}_1 &: \mathrm{q}^{y_t}_{\alpha,t} \neq \mathrm{q}^{y_t|x_t}_{\alpha,\beta,t}. \end{aligned}$$

The null hypothesis will be rejected if the conditional and unconditional quantiles are distinguishable, that is, if changes in the returns of the asset x have an impact on the returns of the asset y.

The Kolmogorov–Smirnov (KS) bootstrapping test, introduced by Abadie (2002), can be useful for test these hypotheses. According to Abadie (2002), this test measures the difference between two quantile functions, without considering any underlying distribution function. The test can be computed as follows:

$$\mathrm{KS}_{mn} = \left(\frac{mn}{m+n}\right)^{\frac{1}{2}} \sup_x |\mathrm{F}_m(x) - \mathrm{G}_n(x)|, \tag{13}$$

where $F_m(x)$ and $G_n(x)$ are, respectively, the cumulative conditional and unconditional quantile distribution for the asset y, and n and m denote the size of the two samples.

Through the Kolmogorov-Smirnov (KS) test, the interconnections between the assets in a determinate market can be tested at the quantile levels. Note that this is an important feature, since more accurate results on risk measures, as highlighted by Walther et al. (2019), is what traders and investors are looking for in portfolio analysis and risk management.

The Kolmogorov-Smirnov (KS) bootstrapping test results are displayed in Table 7. The greatest differences between the quantile functions, as indicated by the KS test, are exhibited by SOL|ETH, SOL|BTC and XRP|BTC, whose values are, respectively, 0.560, 0.546 and 0.521. On the other hand, the combinations XRP|SOL, BNB|SOL and BTC|SOL show the smallest differences between the quantile functions, which are, respectively, 0.421, 0.436 and 0.448.

However, the results indicate that the differences between the unconditional and conditional returns of all cryptocurrencies are statistically significant at the 1% level. These results indicated the presence of spillover effects in the 5% quantile, i.e., a dependence in the lower tail among BTC, ETH, XRP, SOL and BNB. These high levels of connections between assets on the cryptocurrencies market are in agreement with the literature (Koutmos, 2018; Katsiampa et al., 2019; Katsiampa, 2019, Akhtaruzzaman et al., 2022, among others).

The results for the 1% quantile (A) are still statistically significant at the 1% level. However, the greatest values are exhibited by SOL|ETH, SOL|BTC and BNB|BTC, whose values are, respectively, 0.680, 0.662 and 0.642. While, the smallest results are 0.526 for XRP|SOL, 0.532 for BTC|ETH and, 0.559 for BNB|SOL and ETH|BTC.

5. Backtesting

For Karimalis and Nomikos (2018), if a risk model is well-specified, the proportion of exceedances should be approximately equal to the confidence level and these exceedances must occur independently. Girardi and Ergün (2013) point out that, with the change in definition of the systemic risk measure, we can estimate CoVaR similarly to VaR estimates for periods in which $R_t^x \leqslant VaR_{q,t}^x$ using Christoffersen (1998) and Kupiec (1995) tests.

As in Girardi and Ergün (2013), the backtesting is computed using hit sequence functions. First, for a sample with N observations where $t = 1,\ldots,N$, we compare the past ex-ante VaR forecasts with the ex-post losses for each asset x. The hit sequence function of violations is denoted as follows.

Table 7
Results of the Kolmogorov-Smirnov (KS) Bootstrapping Test

	Test Statistics
ETH|BTC	0.483***
XRP|BTC	0.521***
SOL|BTC	0.546***
BNB|BTC	0.517***
BTC|ETH	0.460***
XRP|ETH	0.507***
SOL|ETH	0.560***
BNB|ETH	0.510***
BTC|XRP	0.462***
ETH|XRP	0.476***
SOL|XRP	0.465***
BNB|XRP	0.459***
BTC|SOL	0.448***
ETH|SOL	0.486***
XRP|SOL	0.421***
BNB|SOL	0.436***
BTC|BNB	0.503***
ETH|BNB	0.518***
XRP|BNB	0.501***
SOL|BNB	0.519***

Note: This table shows the test statistics obtained by Kolmogorov-Smirnov (KS) bootstrapping test for the (Girardi and Ergün, 2013) model version. The α-quantile and the β-quantile are both 5%. The p-values which are calculated with standard errors computed by bootstrap and are represented for $^{***}p < 0.01$ $^{**}p < 0.05$ and $^{*}p < 0.10$.

$$I^x_{t+1} = \begin{cases} 1 \text{ if } R^x_{t+1} \leqslant VaR^x_{q,t+1} \\ 0 \text{ if } R^x_{t+1} > VaR^x_{q,t+1}, \end{cases} \tag{14}$$

According to Girardi and Ergün (2013), if the loss of the asset x on that day was larger than its estimated predicted VaR for that day, the hit sequence (I^x_{t+1}) returns a 1 and, otherwise, the hit function returns zero.

Using those days in which $R^x_{t+1} \leqslant VaR^x_{q,t+1}$, that is, when institution x is in financial distress as a sub-sample, a second hit sequence function of violations can be computed comparing the past ex-ante CoVaR forecasts with the past ex-post losses of the asset x as follows:

$$I_{t+1}^{y|x} = \begin{cases} 1 \text{ if } R_{t+1}^{y} \leqslant CoVaR_{q,t+1}^{y|x} \\ 0 \text{ if } R_{t+1}^{y} > CoVaR_{q,t+1}^{y|x}, \end{cases} \tag{15}$$

As in Girardi and Ergün (2013), the second hit sequence, $I_{t+1}^{y|x}$, has the number of observations equal to the number of violations of the first hit sequence. The hit function $I_{t+1}^{y|x}$ returns 1 if the asset x was in financial distress and the loss of the asset y on that day was greater than its predicted measure $CoVaR_{q,t+1}^{y|x}$ and the function returns zero otherwise.

To analyze the percentage of violations in the Girardi and Ergün (2013) model, we also estimate the Adrian and Brunnermeier (2016) model, which is the most popular CoVaR method, and compare the results of backtesting the VaR and CoVaR estimates between these models. Tables 8 and 9 display the results for, respectively, VaR and CoVaR backtesting.

First, the results of the VaR backtesting. It should be noted that, for all cryptocurrencies, the results of the Adrian and Brunnermeier (2016) model are very close to the α quantile chosen, which is $\alpha = 5\%$. These results are to be expected, since these versions of the model are estimated with quantile regressions and, for the chosen α quantile, we expect to see returns below VaR^{i} on 100α percent of the period. However, for the Girardi and Ergün (2013) model, which is estimate using copula, the results are even lower than the Adrian and Brunnermeier (2016) model: 2.649% for BTC, 2.879% for ETH, 2.260% for XRP, 2.487% for SOL and 2.594% for BNB.

Now, we analyze the CoVaR results. Note that, for all the possible combinations between the analyzed cryptocurrencies, Girardi and Ergün (2013)´s CoVaR has less violations when compared with Adrian and Brunnermeier (2016)´s CoVaR. A possible explanation for this comes from Mainik and Schaanning (2014), which highlighted the fact that, by definition, the CoVaR of Girardi and Ergün (2013) uses $X \leqslant \mathrm{VaR}_{\alpha}(\mathrm{X})$ as financial distress of X, this way the violation rate expected for $\mathrm{CoVaR}_{\alpha,\beta}$, when X is in financial distress is equal to $1-\beta$. While, by construction, the Adrian and Brunnermeier (2016)´s $\mathrm{CoVaR}_{\alpha,\beta}^{=}$ uses $X = \mathrm{VaR}_{\alpha}(\mathrm{X})$ as financial distress of X. This assumption makes the violation rate $1-\beta$ under the most benign scenario.

Another point is highlighted by Girardi and Ergün (2013), who argue that $\mathrm{CoVaR}_{\alpha,\beta}^{=}$ depends on how well the approximation of $F_{Y|X=VaR_q(X)}$ in $F_{Y|X=x}$ is performed. Mainik and Schaanning (2014) shows that, even for very basic models, this approximation fails, resulting in underestimates of the contagion effect from X to Y. Mainik and Schaanning (2014) add that this underestimation of $\mathrm{CoVaR}_{\alpha,\beta}^{=}$ is greater in markets with strong correlation and therefore high systemic risk, which is the case with the cryptocurrency market. These problems explain the higher violation rates for $\mathrm{CoVaR}^{=}_{\alpha,\beta}$ in Table 9.

The results reinforce the better accuracy of the Girardi and Ergün (2013) model even for the univariate risk measure (VaR) when compared to the quantile regression approach used in the Adrian and Brunnermeier (2016) method. Another interesting found is that for BTC|ETH (6.215%), BTC|XRP (6.431%), BTC|SOL (5.890%) and BTC|BNB (6.194%), the CoVaR of Girardi and Ergün (2013) has a percentage of violations approximately equal to the 5% quantile. These results show how accurate this method can be for analyzing BTC conditional on one of these assets in a portfolio construction case, for example. Again, a similar behavior can be observed in the 1% quantile results. See A.

Table 8
Backtesting Results (%) for Value-at-Risk

	$X \leqslant \mathrm{VaR}_{\alpha}(X)$	$X = \mathrm{VaR}_{\alpha}(X)$
BTC	2.649	4.999
ETH	2.879	4.999
XRP	2.260	4.999
SOL	2.487	4.999
BNB	2.594	4.999

Note: This table reports the violations for the VaR estimates for the $\alpha = 5\%$.

6. Final Remarks

For Katsiampa et al. (2019), as the cryptocurrency market evolves, it is important to develop our understanding of the behavior of the connection between cryptocurrencies. Therefore, in this article, we estimate the Conditional Value-at-Risk (CoVaR) and Value-at-Risk (VaR) using a 5-minute frequency for BTC, ETH, XRP, SOL and BNB. To do this, we compared the $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$ of Adrian and Brunnermeier (2016) and the modified $\mathrm{CoVaR}_{\alpha,\beta}(Y|X)$ of Girardi and Ergün (2013) measures of Conditional VaR for measuring systemic risk.

We found that, for the Girardi and Ergün (2013) model, the average $\mathrm{CoVaR}_{\alpha,\beta}(Y|X)$ estimate is generally higher than the average $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$ of Adrian and Brunnermeier (2016). In addition, the VaR results for both methods show higher values on average than their respective conditional measures, which reflects the positive interdependence in the cryptocurrency market.

The Kolmogorov-Smirnov (KS) bootstrapping test, which tests whether the returns of asset y are affected by the returns of asset x, was carried out for estimates of the Girardi and Ergün (2013) model. The results show that the

Table 9
Backtesting Results (%) for Conditional Value-at-Risk

	$\text{CoVaR}_{\alpha,\beta}(Y\|X)$	$\text{CoVaR}^{=}_{\alpha,\beta}(Y\|X)$
ETH|BTC	10.825	38.268
XRP|BTC	17.447	39.190
SOL|BTC	27.692	53.922
BNB|BTC	9.710	32.148
BTC|ETH	6.215	21.835
XRP|ETH	17.169	38.902
SOL|ETH	27.487	55.095
BNB|ETH	9.559	33.219
BTC|XRP	6.431	18.763
ETH|XRP	10.574	31.181
SOL|XRP	25.879	48.083
BNB|XRP	9.959	29.533
BTC|SOL	5.890	16.724
ETH|SOL	9.441	28.255
XRP|SOL	15.378	32.053
BNB|SOL	8.657	26.548
BTC|BNB	6.194	19.553
ETH|BNB	10.043	34.617
XRP|BNB	16.067	38.138
SOL|BNB	25.108	51.920

Note: This table compares the backtesting results for the two models studied, $\text{CoVaR}_{\alpha,\beta}(Y|X)$ and $\text{CoVaR}^{=}_{\alpha,\beta}(Y|X)$. The α-quantile and the β-quantile are both 5%. The results are present in violation percentage.

influence of the returns of one cryptocurrency on the returns of another cryptocurrency occurs in all possible combinations in our paper, which indicates a high level of connections in these cryptocurrencies.

Finally, we compared the estimates of the Girardi and Ergün (2013) and Adrian and Brunnermeier (2016) models with a backtesting procedure. The results of Girardi and Ergün (2013) show lower percentage of violations than Adrian and Brunnermeier (2016), which indicates a more conservative fit of VaR and CoVaR in this model. This is because Girardi and Ergün (2013) assume $X \leqslant \text{VaR}_{\alpha}(X)$ as financial distress in X, while Adrian and Brunnermeier (2016) assume $X = \text{VaR}_{\alpha}(X)$. The assumption $X = \text{VaR}_{\alpha}(X)$ leads the model to capture only the most benign scenario and therefore underestimates $\text{CoVaR}_{\alpha,\beta}(Y|X)$.

As pointed out by Girardi and Ergün (2013) and Waltz et al. (2022), measuring systemic risk has great importance for regulatory and risk management purposes. Therefore, it is important to understand the difference between systemic risk measures in the literature in order to implement more accurate methods. The cryptocurrency market, which is growing rapidly and known for its high volatility, has particularly caught the interest of regulators and researchers.

In this vein, our analyses show that using the Girardi and Ergün (2013) method, which is a more appropriate model for measuring systemic risk, can lead to better estimates in a market with a high level of lower tail connectivity. We hope that our paper will have important practical implications for investors, both for portfolio allocation and risk management, and also for policymakers during periods of crisis and non-crisis.

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A. Results for $\alpha = \beta = 1\%$

Table A1
VaR estimates - $X \leqslant$ VaR$_\alpha(X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
BTC	-16.714	-0.597	-0.491	-0.061	0.462	-4.909	75.923
ETH	-26.050	-0.728	-0.604	-0.094	0.570	-6.505	146.515
XRP	-33.156	-1.015	-0.736	-0.229	0.960	-6.476	95.974
SOL	-24.169	-1.276	-1.032	-0.296	0.931	-4.270	38.890
BNB	-23.201	-0.758	-0.594	-0.148	0.641	-5.491	79.208

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of VaR$_\alpha(X)$ results using the (Girardi and Ergün, 2013) method. The α-quantile is 1%.

Table A2
Conditional Value-at-Risk - CoVaR$_{\alpha,\beta}(Y|X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
ETH|BTC	-67.333	-1.868	-1.549	-0.230	1.466	-6.521	147.497
XRP|BTC	-92.230	-2.770	-2.010	-0.613	2.625	-6.552	99.084
SOL|BTC	-71.396	-3.559	-2.874	-0.812	2.607	-4.296	39.607
BNB|BTC	-71.230	-2.326	-1.820	-0.316	1.969	-5.494	79.305
BTC|ETH	-38.864	-1.469	-1.208	-0.101	1.137	-4.874	74.531
XRP|ETH	-85.956	-2.629	-1.907	-0.567	2.490	-6.539	98.384
SOL|ETH	-73.360	-3.695	-2.985	-0.820	2.702	-4.281	39.220
BNB|ETH	-67.162	-2.246	-1.758	-0.411	1.901	-5.475	78.493
BTC|XRP	-46.027	-1.664	-1.362	8.380	1.314	-4.751	71.367
ETH|XRP	-70.738	-2.008	-1.660	8.777	1.596	-6.311	138.023
SOL|XRP	-61.658	-3.021	-2.433	8.204	2.245	-4.313	40.309
BNB|XRP	-61.399	-2.057	-1.606	8.339	1.763	-5.373	75.404
BTC|SOL	-43.533	-1.621	-1.333	12.675	1.283	-4.603	67.561
ETH|SOL	-76.176	-2.140	-1.773	12.085	1.697	-6.272	137.688
XRP|SOL	-75.638	-2.290	-1.668	9.574	2.189	-6.406	95.405
BNB|SOL	-59.659	-1.966	-1.537	11.725	1.686	-5.309	74.252
BTC|BNB	-48.166	-1.770	-1.455	-0.181	1.369	-4.898	75.388
ETH|BNB	-76.024	-2.174	-1.804	-0.283	1.703	-6.483	145.216
XRP|BNB	-85.534	-2.606	-1.891	-0.594	2.465	-6.564	99.135
SOL|BNB	-62.378	-3.284	-2.656	-0.760	2.395	-4.258	38.672

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of CoVaR$_{\alpha,\beta}(Y|X)$ results using the (Girardi and Ergün, 2013) method. The α-quantile and β-quantile are both 1%. For the estimation, we use a t-copula.

Table A3
VaR estimates - $X = \mathrm{VaR}_\alpha(X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
BTC	-162.216	-0.523	-0.405	-0.332	1.054	-92.003	11526.415
ETH	-236.245	-0.663	-0.518	-0.424	1.484	-107.254	14729.768
XRP	-266.932	-0.840	-0.596	-0.491	2.135	-62.960	5876.553
SOL	-166.043	-1.081	-0.835	-0.679	1.565	-46.333	3537.974
BNB	-222.918	-0.676	-0.496	-0.401	1.514	-84.093	10173.977

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of $\mathrm{VaR}_\alpha(X)$ results using the (Adrian and Brunnermeier, 2016) method. The α-quantile is 1%.

Table A4
Conditional Value-at-Risk - $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$

	Min. (%)	Mean (%)	Median (%)	Max. (%)	Std (%)	Skew	Kurt
ETH|BTC	-280.156	-0.899	-0.705	-0.580	1.798	-96.674	12470.133
XRP|BTC	-341.064	-1.125	-0.829	-0.678	2.557	-71.866	7516.273
SOL|BTC	-276.271	-1.427	-1.105	-0.878	2.228	-61.465	5860.719
BNB|BTC	-267.533	-0.896	-0.678	-0.548	1.827	-86.732	10561.200
BTC|ETH	-240.884	-0.714	-0.560	-0.460	1.521	-103.742	13962.998
XRP|ETH	-383.832	-1.140	-0.845	-0.691	2.713	-80.584	9254.589
SOL|ETH	-333.260	-1.451	-1.121	-0.891	2.476	-71.858	7703.520
BNB|ETH	-308.149	-0.907	-0.688	-0.556	2.004	-96.394	12528.701
BTC|XRP	-201.791	-0.690	-0.518	-0.429	1.479	-74.346	7990.070
ETH|XRP	-292.309	-0.869	-0.656	-0.542	2.010	-85.901	10323.397
SOL|XRP	-263.297	-1.366	-1.030	-0.834	2.283	-51.689	4307.614
BNB|XRP	-266.810	-0.861	-0.630	-0.516	1.898	-75.520	8499.403
BTC|SOL	-151.386	-0.710	-0.554	-0.443	1.138	-69.490	7226.869
ETH|SOL	-218.973	-0.900	-0.705	-0.568	1.551	-80.092	9226.616
XRP|SOL	-272.386	-1.109	-0.811	-0.655	2.264	-57.038	5003.169
BNB|SOL	-210.454	-0.887	-0.666	-0.533	1.578	-66.840	6935.078
BTC|BNB	-215.656	-0.725	-0.552	-0.452	1.464	-87.118	10663.704
ETH|BNB	-304.858	-0.914	-0.693	-0.565	1.996	-94.094	12090.496
XRP|BNB	-361.645	-1.136	-0.817	-0.668	2.634	-72.173	7822.938
SOL|BNB	-310.097	-1.442	-1.085	-0.866	2.433	-61.136	5938.128

Note: This table reports the minimum (%), mean (%), median (%), maximum (%), standard deviation (%), skewness and kurtosis of the $\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X)$ results using the (Adrian and Brunnermeier, 2016) method. We use a rolling window of 60-minute realized volatility of BTC, ETH, XRP, SOL and BNB as covariates in this model. The α-quantile and the β-quantile are both 1%.

Table A5
Results of the Kolmogorov-Smirnov (KS) Bootstrapping Test

	Test Statistics
ETH|BTC	0.559***
XRP|BTC	0.633***
SOL|BTC	0.662***
BNB|BTC	0.642***
BTC|ETH	0.532***
XRP|ETH	0.609***
SOL|ETH	0.680***
BNB|ETH	0.628***
BTC|XRP	0.580***
ETH|XRP	0.585***
SOL|XRP	0.573***
BNB|XRP	0.584***
BTC|SOL	0.566***
ETH|SOL	0.609***
XRP|SOL	0.526***
BNB|SOL	0.559***
BTC|BNB	0.614***
ETH|BNB	0.623***
XRP|BNB	0.606***
SOL|BNB	0.624***

Note: This table shows the test statistics obtained by Kolmogorov-Smirnov (KS) bootstrapping test for the (Girardi and Ergün, 2013) model version. The α-quantile and the β-quantile are both 1%. The p-values which are calculated with standard errors computed by bootstrap and are represented for $^{***}p < 0.01$ $^{**}p < 0.05$ and $^{*}p < 0.10$.

Table A6
Backtesting Results (%) for Value-at-Risk

	$X \leqslant \text{VaR}_\alpha(X)$	$X = \text{VaR}_\alpha(X)$
BTC	0.574	1.000
ETH	0.633	0.999
XRP	0.430	1.000
SOL	0.457	1.000
BNB	0.513	1.000

Note: This table reports the violations for the VaR estimates for the $\alpha = 1\%$.

Table A7
Backtesting Results (%) for Conditional Value-at-Risk

	$\text{CoVaR}_{\alpha,\beta}(Y\|X)$	$\text{CoVaR}^{=}_{\alpha,\beta}(Y\|X)$
ETH|BTC	6.944	30.428
XRP|BTC	11.368	32.024
SOL|BTC	15.792	45.229
BNB|BTC	5.195	22.629
BTC|ETH	3.922	16.849
XRP|ETH	11.438	32.486
SOL|ETH	15.453	47.975
BNB|ETH	4.855	24.002
BTC|XRP	4.605	14.560
ETH|XRP	7.973	24.926
SOL|XRP	16.838	42.233
BNB|XRP	6.667	22.357
BTC|SOL	4.199	12.050
ETH|SOL	7.041	21.707
XRP|SOL	11.111	25.960
BNB|SOL	6.395	18.163
BTC|BNB	4.378	15.657
ETH|BNB	7.258	26.736
XRP|BNB	11.924	31.994
SOL|BNB	15.438	43.456

Note: This table compares the backtesting results for the two models studied, $\text{CoVaR}_{\alpha,\beta}(Y|X)$ and $\text{CoVaR}^{=}_{\alpha,\beta}(Y|X)$. The α-quantile and the β-quantile are both 1%. The results are present in violation percentage.