Monetary Policy and Liquidity Management in a Model of Endogenous Network Formation for the Interbank Market

Luiz Carpizo Timo Hiller

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Abstract

This paper develops a tractable endogenous network formation model of the interbank market. Due to liquidity shocks, banks face a trade-off between investing their resources in a liquid asset and a high-yield illiquid asset. The interbank market is modeled as a network. A link extended by one bank to another is interpreted as a credit line that the former bank can use to cover liquidity outflows. The central bank, by means of its standing facilities, lends resources to banks that are short in liquidity and borrows from institutions with liquidity surpluses at predetermined rates. These rates establish a corridor in which the interbank rate must lie. In this setting, we characterize the unique equilibrium of banks' liquidity holdings for any network. We then endogenize the network, via banks' decision of credit lines, and provide a sharp equilibrium characterization: every equilibrium network is a complete core-periphery graph. This characterization is consistent with empirically observed networks. Moreover, we introduce a trade-off for central banks when choosing the corridor rate: a narrower corridor implies more precise targeting of the interbank rate, which is important for the conduct of monetary policy. However, if we account for banks' linking decisions, this may lead to an equilibrium with a sparser network, where total liquidity holdings are higher, incurring an implicit cost since these funds could be invested in the more productive illiquid asset instead. We then provide conditions such that the central bank does not find optimal to minimize the interbank rate variance and study how it should react during moments of financial crises.

Keywords: Interbank market, Endogenous Networks, Corridor Rate

JEL Codes: E58, D85, G21

1 Introduction

Recent work shows that banks' liquidity management is one of the main transmission channels of monetary policy (see, for example, a recent paper by Bianchi and Bigio (2021)). When choosing to hold more liquidity, banks have to forgo more productive investment opportunities. Monetary policy decisions, in turn, affect the trade-off between investments in liquid and illiquid assets and thus have an impact on the real economy. The existence of an interbank market also affects banks' portfolio decisions: by allowing banks to trade liquid assets with each other, it reduces each bank's marginal benefit of directly holding liquid assets. Understanding how banks decide the composition of their portfolios in the presence of an active interbank market is, therefore, important for the conduct of monetary policy.

In this paper, we develop a tractable endogenous network formation model for the interbank market, where (directed) links are interpreted as interbank credit lines. Banks solve a liquidity management problem, making simultaneous decisions regarding liquidity holdings and credit lines. Our equilibrium analysis shows that equilibrium networks display core-periphery types of structures, which is in accordance with empirically observed networks. Our model unveils a novel trade-off that central banks operating a corridor system face when choosing the corridor width: a narrower corridor implies more precise targeting of the interbank rate, which is important for monetary policy conduction. However, by letting banks review not only their allocation to liquid assets but also their interbank credit lines, we show that a narrower corridor reduces banks' incentives to borrow in the interbank market. This, in turn, can lead to excessive levels of liquidity holdings, which incurs an implicit cost since these funds could be invested in more productive (illiquid) assets instead. In this setting, we demonstrate that once accounting for the cost of holding (unproductive) liquid assets, it is not optimal for the central bank to choose the corridor width that minimizes the interbank rate variance.[1] This provides a new rationale for a strictly positive spread between

[1]Bindseil (2016) states that the choice of the corridor width reflects the trade-off between control over the interbank rate and interbank market activity. This trade-off is directly related to ours. In the model developed in this paper, total liquidity is lower in equilibria with more interbank activity.

standing facilities rates, even when the central bank's main goal is to precisely target the short-term interbank rate.[2] Lastly, we conduct comparative statics analysis regarding the optimal corridor width and show how failing to account for how interbank credit lines are established (and altered) when setting the corridor rate may lead to suboptimal policy. This can help explain, for example, why some central banks during the 2009 financial crisis narrowed the corridor rate at first, but later decided to return to the pre-crisis width, stating that the narrower corridor was responsible for the crowding out of the interbank market. To the best of our knowledge, this paper is the first to study the optimal corridor rate width in a network setting.

Bindseil (2016) highlights some aspects that have been the consensus for operational frameworks among most central banks after 2007. First, central banks set as an operational target a short-term interest rate that is under their control. In most cases, this target is the overnight interbank rate. Another key aspect is that central banks offer standing facilities to absorb liquidity from and provide liquidity to commercial banks at predetermined rates. These rates establish a corridor within which the interbank rate fluctuates. A narrower corridor reduces the variance of the interbank rate, allowing the central bank to target it with higher precision.[3] We argue that a central bank, when optimally deciding the corridor width, faces a trade-off between precise targeting of the interbank rate and commercial banks' total allocation to liquid assets, which is generally higher for a wider corridor. Holding liquidity bears an implicit cost since these resources could be allocated to more productive assets instead. Consistent with this trade-off is the evidence that banks in regions such as the U.S and Europe have drastically decreased their interbank activity and increased their holding of reserves after the Great Recession.[4] Thus, an important policy question is how narrow

[2]See Berentsen and Monnet (2008), Berentsen et al. (2010) and Bindseil and Jabłecki (2011) for discussions on the optimality of a non-zero corridor.

[3]For a comprehensive analysis of how corridor systems are an effective way for the central bank to control the interbank rate see Bindseil and Jabłecki (2011), Quiros and Mendizabal (2006) and Whitesell (2006).

[4]See Kim et al. (2020) for evidence in the U.S interbank market. During this period the Federal Reserve Bank started paying interest on reserves, thus effectively establishing a corridor rate, while the European Central Bank reduced the width of its corridor rate from 200 basis points to 75 basis points as of August 2022.

should the central bank set the corridor rate.

To address this question, we model banks' decisions in an endogenous network setting. In our model, every bank faces a trade-off between investing its resources in liquid and illiquid assets, such as reserves and bonds, respectively. Liquid assets can be used to cover withdrawals from depositors and therefore provide a buffer against potentially large liquidity outflows. However, the illiquid asset is more productive, in the sense that it yields a higher expected return. If banks had perfect information regarding their liquidity outflows, they would choose to hold the minimum necessary to pay all their depositors and invest the remaining resources in the more productive asset. However, banks cannot perfectly predict withdrawals when choosing their portfolio, and therefore face a probability of being short of liquid assets. Our model features a central bank that offers two standing facilities. The lending facility allows banks with liquidity needs to borrow the necessary resources from the central bank at the so-called discount rate. The deposit facility allows banks with an excess of liquid assets to lend these to the central bank at the deposit facility rate. Banks' liquidity management involves two types of costs: the cost of having to borrow from the central bank when liquidity holdings are not sufficient, and the opportunity cost of holding excess liquidity.

The main feature of the model is an interbank market, which we model as a network. A link extended by one bank to another allows the former to borrow up to a pre-specified share of the latter's liquid assets. We refer to this share as the weight of the link. These links are interpreted as credit lines that a potential borrower must open with its counterparty before it can obtain a loan. This setup captures the fact that interbank trading is mainly characterized by stable relationships between banks.[5] We assume there is a cost of establishing a credit line since it requires sustaining personal relationships/working relations from the borrower to the counterparty. Cohen-Cole et al. (2015), for example, argue that, in the Italian

[5] For evidence on the stability of interbank relationships, see Blasques et al. (2018) for the Dutch market, Dordal i Carreras et al. (2021) for the German market, and Afonso et al. (2013) for the US market. See Müller (2006) for a similar interpretation of links as credit lines.

interbank market, operations simply consist of a phone call between banks' managers and are executed fast, which suggests a prior relationship between the parties. The linking cost can be interpreted as sustaining this relationship. Note that this setup allows for reciprocated links, i.e. two banks can both extend a link to the other, thereby establishing mutual credit lines. But it is still the case that each one must pay the cost of establishing them. The existence of this market gives rise to local strategic substitution effects between banks' own liquidity holdings. When a bank increases its investments in liquid assets, it also increases how much liquidity banks that connect to it can access through the market. This, in turn, generates incentives to allocate more of their own resources to illiquid assets.

Moreover, it is assumed that banks incur a marginal cost when lending liquid assets in the interbank market. As in Bucher et al. (2020), this cost can be associated with financial regulations and/or creditworthiness checks.[6] Banks observe how costly it is to trade in the interbank market only after credit lines are already established. When the interbank transaction cost is realized, a bank will be willing to lend to a counterparty only if the expected profit of the loan is weakly higher than the outside option, which is lending to the central bank. The central bank's deposit facility rate, thus, sets a lower bound on the interest rate that can be charged in interbank loans. In principle, the discount rate would set the upper bound. However, empirical studies provide evidence of stigma effects when borrowing from the central bank (see Armantier et al. (2015) for the U.S and Lee (2016) for the UK and the euro area).[7] As Whitesell (2006) points out, in the presence of such effects, the market rate that is equivalent to borrowing at the discount window is higher than the discount rate. We refer to this rate as the effective discount rate, which sets an upper bound on the interbank rate. This, in turn, implies that a bank that is short of own liquidity

[6]Bucher et al. (2020), argue that interbank transaction costs can be associated with asymmetric information even if the model does not explicitly account for such asymmetry (for example, private information regarding default risk). The authors argue that lenders, to avoid exposing themselves to any sort of credit risk, engage in costly creditworthiness checks, which removes any asymmetries. This, in turn, allows them to assume that all observed interbank transactions are risk-free.

[7]For a formal treatment on stigma effects, see Ennis and Weinberg (2013). To simplify the exposition, we abstract from other factors that can make banks willing to borrow at the interbank market at a rate higher than the discount rate (such as central bank's collateral requirements).

holdings will borrow from the central bank if either what it can borrow in the interbank market is not sufficient to cover its liquidity outflows, or no bank is willing to lend resources to it at a rate lower than the effective discount rate due to prohibitively high interbank transaction costs.

Banks' decisions to create interbank credit lines are mainly affected by the size of the corridor width relative to the cost of sustaining a relationship with a potential lender. A low relationship cost increases banks' incentives to invest in these relationships because the interbank market provides a liquidity source that is potentially cheaper than the central bank's lending facility. On the other hand, when the corridor is too narrow, a prohibitively high interbank transaction cost becomes more likely. In this case, the probability that a bank with liquidity needs will have to borrow resources exclusively at the central bank's lending facility, even if it established credit lines with other banks, increases. Because setting credit lines is costly, the incentive to do so in the first place reduces. Therefore, the network of interbank relationships is more connected when the linking cost is low and the corridor width is wider.[8]

Throughout this paper, we make the following two simplifying assumptions, so as to yield a tractable model. First, we adopt the common simplifying assumption that only one bank is hit by a liquidity outflow per period.[9] With this assumption, the amount that an individual bank can choose to borrow in the interbank market is fully determined by the links it extends. This follows from the fact that if a bank is hit by an adverse liquidity shock, then it knows that other banks have not been hit and, therefore, will not borrow resources through their interbank credit lines. Second, when a bank decides to borrow in the interbank market, both borrower and lender must negotiate the interest rate on the loan.

[8]Blasques et al. (2018) estimate a network model using Dutch interbank market data and show that a wider corridor increases bank's incentives to operate outside of its established relationships. In the model presented in this paper, this can be interpreted as higher incentives for a bank to establish new credit lines.

[9]This assumption has been mainly used to study the risk of contagion in interbank networks. We make use of this assumption in this paper, but we abstract from the possibility of contagion. See Upper (2011) for a survey on contagion in interbank markets, which suggests that, in this setting, contagion arising from banks' defaults is unlikely.

We will make the simplifying assumption that the borrower makes a take it or leave it offer to the lender. In this case, the borrower will offer the rate that makes the lender indifferent between lending in the interbank market and lending to the central bank. This assumption implies that a bank with an incoming link does not want to hold more liquidity only to lend to other banks, since an interbank loan yields a return equal to lending liquid assets to the central bank. Therefore, holding liquidity is valuable to a bank only because it provides a buffer against liquidity outflows.

In this setting, we first characterize the unique equilibrium of banks' liquidity holdings for any network. Then, we endogenize banks' linking decisions to characterize the equilibrium network in the network formation game. We use this to demonstrate that the central bank faces a trade-off between the variance of the interbank rate and total amount of liquidity held by the banking system when optimally deciding the corridor width. We provide sufficient conditions such that the optimal corridor width is different from the width that minimizes the variance of the interbank rate. We conclude our analysis by presenting comparative statics results regarding the optimal corridor width with respect to the model's structural parameters.

A novel contribution of this paper is to provide a micro-foundation for the linear-quadratic payoff framework analyzed in Ballester et al.(2006) in the context of banks' liquidity management problem. This framework has been commonly used to study a variety of settings due to its analytical tractability.[10] We provide a sufficient condition for the existence and uniqueness of a Nash equilibrium in the choice of liquid assets for any fixed network. This condition can be generalized to any network game where players choose a single continuous variable, payoffs are linear-quadratic and the game displays local payoff substitutability.

We then proceed by providing a sharp equilibrium characterization of the network formation game when the equilibrium is strict: every strict Nash equilibrium network is a complete

[10]See, for example, König et al. (2014) for applications on trade and interbank networks, Bramoullé et al. (2014) for crime networks and König et al. (2019) for R&D networks.

core-periphery graph.[11] This characterization is consistent with the empirical evidence on interbank market networks, which has identified core-periphery structures in many countries.[12]

Furthermore, we study the problem of a central bank whose objective is to optimally define standing facilities rates. In order to do so, we define a loss function for the central bank, which takes into account the volatility of the interbank rate and total liquidity holdings. We then show that the central bank's problem reduces to the choice of the corridor width that minimizes its loss function, where the corridor width is defined as the spread between the effective discount rate and the central bank's deposit facility rate. Note that, in this case, the minimum corridor width that the central bank can choose is not zero. Even if it sets the discount rate equal to the deposit facility rate, the effective discount rate would still be higher than the latter due to stigma effects.

Under certain restrictions on the range of the linking cost, we can derive general properties with respect to the optimal corridor width. More specifically, we assume parameter ranges such that the star network with weighted links directed to the center is the equilibrium network. This simplifies the problem because the equilibrium link weight changes continuously with respect to the corridor width, which, in turn, implies that total liquidity holdings also changes continuously.[13] Also, these restrictions guarantee that the equilibrium is unique for every size of the corridor width.

We prove that, as long as the number of peripheral banks is not too small, it is not optimal for the central bank to set the minimum corridor width. The key intuition is that for too narrow corridors, peripheral banks choose to borrow a very small share of the center's liquidity, which, in turn, increases these banks' incentives to hold a high amount of liquidity. In this case, the central bank can reduce its loss by widening the corridor. Although this

[11] Proposition 3 provides conditions such that a strict Nash equilibrium always exists. Therefore, for each Nash equilibrium that is not strict, there exists a (different) Nash equilibrium that is strict. Strict Nash equilibria are more robust to small perturbations in banks' payoffs than non-strict equilibria.

[12] The literature that has identified core-periphery type structures on empirical interbank networks includes, but is not restricted to, Boss et al. (2004) for Austria, Soramaki et al. (2010) for the UK, Craig and von Peter (2014) for Germany and Soramäki et al. (2007) for the US.

[13] A discrete change in the number of banks in the core may lead to a discontinuous change in total liquidity holdings with respect to the corridor width.

implies a less precise targeting of the interbank rate, the reduction in total liquidity is sufficiently large to outweigh the increase in volatility. This result holds for more general equilibria networks, i.e for networks with more than one bank in the core.

Finally, we conduct comparative statics analysis regarding the optimal corridor width with respect to changes in the size of interbank transaction costs and liquidity outflows. This analysis can be relevant for moments of financial distress, when such variables are likely to rise.[14] We find that these parameters have qualitatively different impacts on the optimal corridor. This result may seem counterintuitive at first. We show that this difference is related to how changes in interbank transaction costs and liquidity outflows have distinct implications for how banks in the periphery adjust their links to the center. We then proceed by presenting a numerical example that illustrates how large variations in these parameters have non-monotonic impacts on the optimal corridor width. Our results show that the decision to widen or narrow the corridor depends on the increase of transaction costs relative to liquidity outflows, as well as on the increase of these variables relative to their original size. Consistent with these findings, recent events have shown that different central banks reacted in distinctive ways to episodes of crises. The European Central Bank, for example, during the Great Recession, decided to first narrow the corridor and then widen it, while during the Covid-19 outbreak the corridor was kept unchanged. On the other hand, the FED decided to narrow the corridor in both circumstances.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the model. Section 4 presents the equilibrium characterization. Section 5 presents the central bank problem, derives general properties for the optimal corridor width and conducts comparative statics analysis. Section 6 concludes. All proofs are relegated to the appendix.

[14]For discussions on how financial crises impact interbank transaction costs and liquidity outflows, see Bindseil and Jabłecki (2011) and Bucher et al. (2020).

2 Literature Review

This paper relates to different literatures. First, it contributes to a strand of the literature on financial networks that studies overnight interbank loans. This work also adds to the literature on the optimal width of the corridor rate established by central bank's standing facilities and, more broadly, to the literature on banks' liquidity management and monetary policy.

We start by discussing our contributions to the literature on financial networks. Cohen-Cole et al. (2015) and Denbee et al. (2021) study systemic risk in a network model for the interbank market. The former paper focuses on banks' lending decisions in an integrated interbank market while abstracting from liquidity risk. Assuming banks choose lending quantities à la Cournot, the payoff functions are linear-quadratic as in Ballester et al. (2006). The latter studies banks' liquidity holdings decisions under the assumption that banks have linear-quadratic utility with respect to liquid assets. Banks' own liquidity holdings can either be local strategic substitutes or complements, depending on the value of structural parameters. Our approach differs from both of these papers. In our model, banks explicitly solve a liquidity management problem. Liquidity risk arises from adverse liquidity shocks, which generates incentives for banks to hold liquidity and also to borrow liquid resources in the interbank market. Under appropriate simplifying assumptions on the distribution of liquidity outflows and interbank transaction costs, the model yields linear-quadratic payoff functions. Therefore, we contribute to the literature by providing a micro-foundation for Ballester et al. (2006)'s framework in the context of banks' liquidity management problem, which also encompasses, to some extent, Denbee et al. (2021)'s framework (when liquidity holdings are strategic substitutes).

Closer to this paper is Anufriev et al. (2021). The authors develop a model of network formation for the interbank market. Banks enter the market to adjust their end of the day expected reserves via bilateral agreements that set the interest rate and amount of the loan between the two parties. A link is two-sided, unlike in this paper where links are one-sided, and is defined as the amount that one bank agrees to lend to another, which differs from our interpretation of credit lines. This difference is crucial for the interpretation of results on how changes in the corridor width affect the interbank market: while Anufriev et al. (2021)'s analysis focus on what one would typically observe on a single trading day, our analysis best describes the effects on the interbank market over a longer period of time. Moreover, Anufriev et al. (2021) abstracts from the trade-off banks face between liquidity and return by taking as exogenous banks' reserve holdings (in the absence of interbank borrowing or lending). By explicitly considering this trade-off, our network approach accounts for how the corridor width impacts banks' reserves holdings and how this impact is related to the network structure of interbank relationships. This allows us to set up a central bank's problem which unveils a trade-off between interbank rate volatility and excessive liquidity holdings by commercial banks.

This paper contributes to the literature on the optimal corridor width established by central bank's standing facilities. To the best of our knowledge, we are the first to study this topic in a network setting. The literature has focused on the rationales behind the optimality of non-zero corridors for central banks that have the interbank rate as their target rate. Berentsen and Monnet (2008) are the first to address this issue. In a dynamic general equilibrium model, the authors show that a non-zero corridor is optimal as long as loans from the central bank to commercial banks require (costly) collateral. Bindseil and Jabłecki (2011) develop a simple model of liquidity management with two banks. The optimal corridor width is derived from a central bank's objective function that takes into account the volatility of the interbank rate, interbank trading volume and central bank's intermediation costs. This paper makes a contribution by considering a new trade-off that the central bank faces, which is between interbank rate volatility and banks' opportunity cost of holding reserves. We show that a minimum corridor width is not optimal and, therefore, the proposed trade-off provides new insights as to why a non-minimum corridor width can be optimal.

We also relate to the literature on banks' liquidity management and monetary policy. Whitesell (2006) compares interest rate corridors regimes and reserves regimes in a model with an interbank market. In his model, banks face a trade-off between holding liquid assets and lending in the interbank market, while abstracting from illiquid investments. By modeling the market as an endogenous network, we show that a corridor regime impacts the formation of trading relationships in the interbank market, which, in turn, affects banks' liquidity holdings and interbank trading. This effect is not considered in Whitesell (2006) when deriving banks' demand for reserves. Afonso and Lagos (2015) employ a search model where a large population of atomistic banks is randomly matched to model the interbank market. Bianchi and Bigio (2021) closely follow Afonso and Lagos (2015) formulation of this market and integrate it into a dynamic general equilibrium model. The authors study the transmission and implementation of monetary policy in the context of banks' liquidity management problem. These papers are very different conceptually from this paper. They rely on search models as the modeling tool for the interbank market, while we opt for an endogenous network approach. Bianchi and Bigio (2021) and Afonso and Lagos (2015) random matching in a large population of banks implies that two banks have zero probability of being matched more than once, thus, abstracting from the empirically observed stability in interbank relationships and its core-periphery structure. By taking interbank relationships into account, our network approach explains its structure and how the endogenous formation of these relationships is affected by changes in the corridor width, which, in turn, impacts banks' liquidity holding decisions.

3 Model Description

There is a set $N = \{1, 2, \ldots, n\}$ of players, with $n \geq 3$, which we refer to as banks. Each bank i has an amount w_i of resources which it can invest in a liquid asset z_i and an illiquid asset $x_i = w_i - z_i$. The illiquid asset yields an expected return of r^x.

The central bank offers banks a deposit facility, in which banks can lend resources to the central bank at the rate $\underline{r}$, and a lending facility, known as the discount window, in which banks can borrow resources from the central bank at the so-called discount rate $\tilde{r}$. When $\tilde{r}$ is set at a penalty rate, i.e. $\tilde{r} \geq \underline{r}$, there exists a stigma cost in borrowing from the central bank at the discount window. This implies that the market rate equivalent to borrowing at the discount window is higher than the discount rate. We, thus, define the effective discount rate $\bar{r}$ as:

$$\bar{r} = \begin{cases} \tilde{r} + \delta, & \tilde{r} \geq \underline{r} \\ \tilde{r}, & \tilde{r} < \underline{r} \end{cases}$$

where $\delta > 0$ represents costs associated with stigma. Note that $\bar{r}$ is the relevant rate for a bank when borrowing from the central bank.

To prevent arbitrage opportunities the central bank sets $\bar{r} > \underline{r}$. Otherwise, banks would profit by borrowing from and then lending to the central bank. Note that this implies $\bar{r} - \underline{r} \geq \delta > 0$. We assume $\bar{r} > r^x > \underline{r}$. This assumption is necessary for banks to hold a positive amount of both liquid and illiquid assets in equilibrium.[15]

Banks can borrow liquid assets from other banks via the interbank market. If bank i extends a link $g_{i,l} \in [0, 1]$ to bank l, it pays a linking cost of $\tilde{\kappa} g_{i,l}$, where $\tilde{\kappa} > 0$, and it can choose to borrow up to a fraction $\psi g_{i,l}$ of z_l, where $\psi < 1$. We set $g_{i,i} = 0, \forall i \in N$. This prevents the possibility of a bank borrowing resources from itself. The total amount of liquid assets that bank i can access through the market, which we refer to as accessible liquidity, is equal to $\sum_{j \in N} \psi g_{i,j} z_j \equiv y_i, \forall i \in N$.[16]

Interbank rate: For a bank l to be willing to lend to bank i, the expected profit of the loan must be weakly higher than the outside option, i.e. lending to the central bank. We assume that there exists a stochastic marginal cost c in lending liquid assets via the interbank market, with cdf $H(.)$ defined over the support $[0, \bar{c}]$, with $\bar{c} \geq \bar{r} - \underline{r}$. Hence, the

[15] If $r^x \geq \bar{r}$, banks would invest all of their resources in the illiquid asset, whereas, if $r^x \leq \underline{r}$, banks would only invest in the liquid asset.

[16] See Denbee et al. (2021) for a similar definition of accessible liquidity.

interest rate of a loan from bank l to bank i, $r_{i,l}$, must satisfy:

$$r_{i,l} - c \geq \underline{r} \Leftrightarrow r_{i,l} \geq \underline{r} + c.$$

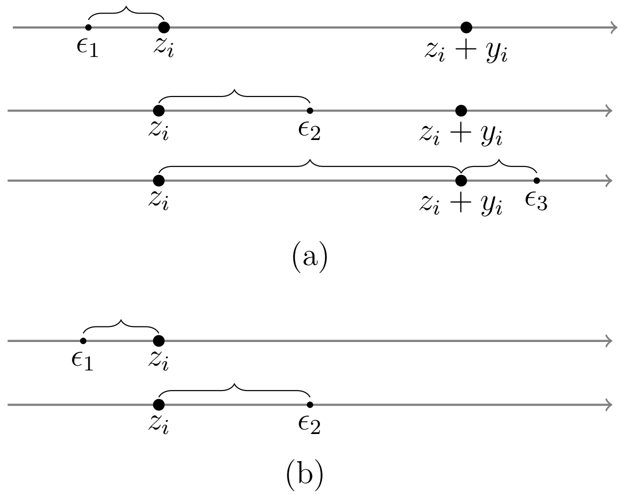
We assume that the borrower makes a take it or leave it offer to the lender. This simplifying assumption implies that a bank with an incoming link does not want to hold more liquidity only to lend to other banks since an interbank loan yields an expected return equal to lending liquid assets to the central bank. Hence, holding liquidity is valuable to a bank only because it provides a buffer against liquidity shocks. Given this assumption, bank i can borrow liquid assets from bank l in the interbank market at a rate $r_{i,l} = \underline{r} + c \equiv \hat{r}(c)$, $\forall\, i, l \in N$. Of course, bank i will only borrow resources via the market if $\hat{r}(c) \leq \bar{r} \Leftrightarrow c \leq \bar{r} - \underline{r}$, otherwise it would find it more profitable to borrow directly from the central bank at the discount window. We thus define the realized interbank rate $r(c) \equiv \underline{r} + c$, with $c \in [0, \bar{r} - \underline{r}]$. Note that the support of $r(c)$ is $[\underline{r}, \bar{r}]$.

Network: We define bank i's links as $\mathbf{g_i} = (g_{i,1}, \ldots, g_{i,n}) \in G_i = [0,1]^n$ and the network as $\mathbf{g} = (\mathbf{g}_1, \mathbf{g}_2, ..., \mathbf{g}_n)$. We say that a bank i connects to bank j if $g_{i,j} > 0$. If the link $g_{i,j} \in (0,1)$, then it is said to be weighted. The network is said to be directed, because i can connect to j, without j connecting to i. We also define the undirected network $\bar{\mathbf{g}}$, where $\bar{g}_{i,j} = \max\{g_{i,j}\,,\, g_{j,i}\} = \bar{g}_{j,i}$. The adjacency matrix G is defined as the $n \times n$ matrix in which the element in the i-th row and j-th column is equal to $g_{i,j}$.

Strategies: Each bank i has a set of strategies $S_i = [0, w_i] \times G_i$. A strategy profile $\mathbf{s_i} = (z_i, \mathbf{g_i}) \in S_i$ specifies the amount of liquid assets that bank i holds, z_i, and the share of other banks' liquid assets it can borrow, $\psi \mathbf{g_i}$. The set of strategies of all banks is $S = S_1 \times S_2 \times ... \times S_n$. A strategy profile $\mathbf{s} = (\mathbf{z}, \mathbf{g}) \in S$ then specifies each bank's own liquidity holdings, $\mathbf{z} = (z_1, z_2, ..., z_n)$, and links, $\mathbf{g}$.

Timing: One randomly chosen bank is hit by a liquidity outflow $\epsilon \geq 0$ with cdf $F(.)$ known to the bank. Agents that were not chosen are not hit by any shock. Each bank

Figure 1: Visual representation of liquidity outflow.



Notes: Panel (a) presents the relevant regions of ϵ for bank i when $c \leq \bar{r} - \underline{r}$. Panel (b) presents the relevant regions of ϵ for bank i when $c > \bar{r} - \underline{r}$.

must decide its strategy profile before a bank is chosen and before the realization of ϵ and c. Suppose bank i is hit by a liquidity outflow. Consider first the case where $c \leq \bar{r} - \underline{r}$. We then have three possible outcomes, which are represented in Panel (a) of Figure 1. If $\epsilon \leq z_i$, then bank i deposits the difference, $z_i - \epsilon$, at the central bank at the rate $\underline{r}$. Otherwise, $\epsilon > z_i$ and bank i must borrow the difference, $\epsilon - z_i$, to cover its outflows. If $\epsilon \leq z_i + y_i$, then bank i uses its credit lines to borrow the necessary amount of resources in the interbank market at the rate $r(c)$. If $\epsilon > z_i + y_i$, then the most that bank i can borrow in the market is not sufficient to cover the outflow and, thus, it borrows y_i at the rate $r(c)$ and the remaining resources, $\epsilon - (z_i + y_i)$, in the discount window at the rate $\bar{r}$. Now consider the case where $c > \bar{r} - \underline{r}$. Then there are only two possible outcomes, which are represented in panel (b) of Figure 1. If $\epsilon \leq z_i$, then the bank proceeds as in the previous case. Otherwise, $\epsilon > z_i$ and bank i borrows the necessary resources exclusively at the discount window at the rate $\bar{r}$.[17] Note that bank i does not use its credit lines since it cannot borrow in the interbank market at a rate lower than $\bar{r}$.

Payoffs: Let us break down the utility of bank i into three parts. First, the expected

[17]We adopt here the standard assumption that the central bank always has enough resources to lend to the bank with liquidity needs (see Poole (1968)).

profit from the investment in the illiquid asset is $r^x(w_i - z_i)$.

Second, if $c \leq \bar{r} - \underline{r}$, then bank i's profit is:

$$\Pi(z_i, c, \mathbf{g_i}) = \int_0^{z_i} \underline{r}(z_i - \epsilon)dF(\epsilon) - \int_{z_i}^{z_i+y_i} r(c)(\epsilon - z_i)dF(\epsilon) - \\ - \int_{z_i+y_i}^{\infty} [r(c)y_i + \bar{r}(\epsilon - (z_i + y_i))]dF(\epsilon). \quad (1)$$

If $c > \bar{r} - \underline{r}$, then bank i's profit is:

$$\Pi(z_i, c, \mathbf{g_i}) = \int_0^{z_i} \underline{r}(z_i - \epsilon)dF(\epsilon) - \int_{z_i}^{\infty} \bar{r}(\epsilon - z_i)dF(\epsilon). \quad (2)$$

The first term in (1) and (2) is the benefit from lending the excess reserves to the central bank. The following terms are the cost from borrowing the necessary resources from other banks in the interbank market (when $r(c) \leq \bar{r}$) and from the central bank. Lastly, bank i pays the total cost of linking $\sum_{j \in N} \tilde{\kappa} g_{i,j}$.

The utility of bank i can then be written as:

$$u_i(z_i, \mathbf{g_i}) = r^x(w_i - z_i) + \int_0^{\bar{c}} \Pi(z_i, c, \mathbf{g_i})dH(c) - \tilde{\kappa} \sum_{j \in N} g_{i,j}. \quad (3)$$

For the remainder of this paper we will assume that $c \sim \mathcal{U}[0, \bar{c}]$ and $\epsilon \sim \mathcal{U}[0, \zeta]$, where $0 < \zeta < \min_j w_j$. The payoff function for bank i can then be written as:[18]

$$u(z_i, \mathbf{g_i}) = \zeta\beta z_i - \frac{\gamma}{2} z_i^2 - \lambda z_i y_i + \zeta\lambda y_i - \frac{\lambda}{2} y_i^2 - \kappa \sum_{j \in N} g_{i,j}, \quad (4)$$

where $\kappa \equiv \zeta\tilde{\kappa}$, $\beta \equiv \bar{r} - r^x$, $\gamma \equiv \bar{r} - \underline{r}$, $\lambda \equiv \frac{\gamma}{\bar{c}}(\bar{r} - E[r(c)]) = \frac{\gamma^2}{2\bar{c}}$. The term $E[r(c)]$, which we refer to as expected interbank rate, is equal to $\int_0^{\gamma} \frac{r(c)}{\gamma} dc = \frac{1}{2}(\bar{r} + \underline{r})$. Our model thereby provides, in the context of banks' liquidity management problem, a micro-foundation for the linear-quadratic payoff framework. This directly implies the existence and uniquenesses of

[18]For expositional clarity, we multiplied (3) by ζ and subtracted all constant terms.

an interior equilibrium in the choice of liquidity holdings for any given network, as we show in the next section.

Note that we have omitted from (4) the region such that $z_i + y_i > \zeta$.[19] In terms of equilibrium analysis of the network formation game, the results are the same whether we consider or not this region. To obtain intuition for this, note first that a higher y_i is valuable for bank i only because it decreases the probability that i has to resort to discount window borrowing when $r(c) \leq \bar{r}$. However, if $z_i + y_i \geq \zeta$, then this probability is zero because i can cover any liquidity outflow that hits it using only its interbank credit lines. It is then straightforward that if $z_i + y_i > \zeta$, then i would profit from decreasing the weight on some of its positive links such that $z_i + y_i' = \zeta$. This would reduce the cost of linking while keeping the previously mentioned probability equal to zero. Hence, for any strategy profile $\mathbf{s_i} = (z_i, \mathbf{g_i})$ such that $z_i + y_i > \zeta$, there exists a profile $\mathbf{s_i'} = (z_i, \mathbf{g_i'})$ such that $z_i + y_i' = \zeta$ and $u(\mathbf{s_i'}) > u(\mathbf{s_i})$. Moreover, for equilibrium analysis when the network is fixed, we provide sufficient conditions in the next section such that $z_i + y_i \leq \zeta$ for every Nash equilibrium in the choice of liquid assets and any given network.

The payoff function is strictly concave in own liquidity holdings, with concavity parameter $\gamma = \left| \frac{\partial^2 u_i}{\partial z_i^2} \right|$. From bank i's perspective, an increase in liquidity holdings from a bank such that $g_{i,l} > 0$ produces a positive externality:

$$\frac{\partial u_i}{\partial z_l} = (\zeta - (z_i + y_i))\, \lambda \psi g_{i,l} > 0.$$

This increase also creates incentives for i to reduce its own allocation to liquid assets:

$$\frac{\partial^2 u_i}{\partial z_i \partial z_l} = -\lambda \psi g_{i,l} < 0.$$

Together, we say that the network game displays local positive externalities and strategic substitutability.

[19]See the Appendix for a complete derivation of the payoff function.

The best response function for bank i, given the network $\mathbf{g}$ and $\mathbf{z_{-i}}$ is:

$$z_i(\mathbf{g}, \mathbf{z_{-i}}) = \frac{\zeta\beta}{\gamma} - \frac{\lambda}{\gamma} y_i. \tag{5}$$

Substituting (5) in (4) we obtain the value function, which can be written as:

$$V(y_i, \mathbf{g}) = v(y_i) - \kappa \sum_{j \in N} g_{i,j}, \tag{6}$$

where $v(.)$ is strictly increasing and concave with respect to y_i. We refer to $v(.)$ as the gross value-function. A formal derivation of the value function is provided in the Appendix.

4 Analysis - Network Formation

In this section, we provide the equilibrium analysis of our model. First, we provide a sufficient condition such that an interior Nash equilibrium in the choice of liquidity holdings exists for any fixed network. Then, we present the formal definition of a complete core-periphery network and show that in any strict Nash equilibrium of the network formation game the equilibrium network displays a core-periphery structure. We proceed by deriving a sufficient condition such that an equilibrium exists for any linking cost value, as well as a (weaker) condition such that an equilibrium with a non-empty network exists. Lastly, we turn to the case when the equilibrium network is a star with at least one weighted link. We provide general properties that will be useful for the next section when we study how the equilibrium network adapts to changes in the corridor width, $\gamma = \bar{r} - \underline{r}$.

Our first proposition presents a sufficient condition for the existence of a unique interior Nash equilibrium in the choice of liquid assets for any fixed network.

Proposition 1. *If $\psi < \frac{\gamma}{\lambda(n-1)}$ and $\beta < \lambda$, then there exists a unique NE in the choice of $\mathbf{z}$ and the unique NE is interior for any given network $\mathbf{g}$.*

Let us build intuition for the above proposition. First, β and ψ sufficiently small guarantee that, in any NE, $z_i + y_i \leq \zeta\,, \forall i \in N$. To see this, note that a smaller ψ reduces how much liquidity a bank can access through the interbank market, y_i, while a smaller β increases banks' incentives to allocate fewer resources to liquid assets, z_i. Hence, we can consider banks' payoffs as in (4).

Next, consider, without loss of generality, bank i's payoff function. For i to choose a positive amount of liquidity holdings, its own concavity parameter must be high enough to counter effects of local substitutability. This translates into the following condition in our model, $\gamma > \sum_{j \neq i} \left| \frac{\partial^2 u_i}{\partial z_i \partial z_j} \right| = \psi\lambda \sum_{j \neq i} g_{i,j}$.[20] The term $\psi\lambda$ measures the level of this substitution effect, $\left| \frac{\partial^2 u_i}{\partial z_i \partial g_{i,j} z_j} \right|$, and $\sum_{j \neq i} g_{i,j}$ the intensity at which these effects impact bank i through the network. Since $g_{i,j} \leq 1$, $\forall i, j \in N$, the maximal value of $\sum_{j \neq i} g_{i,j}$ equals $n-1$, which yields the condition $\gamma > \psi\lambda(n-1)$.

Before presenting the following proposition, which characterizes equilibrium networks, we provide a formal definition of a complete core-periphery network for undirected networks.

Definition 1. A core-periphery network $\bar{\mathbf{g}}$ is such that there are two disjoint groups of players, the periphery $P(\bar{\mathbf{g}})$ and the core $C(\bar{\mathbf{g}})$. $P(\bar{\mathbf{g}})$ is such that players are not connected (i.e. $\forall i, j \in P(\bar{\mathbf{g}})$, $\bar{g}_{i,j} = 0$), while $C(\bar{\mathbf{g}})$ is such that players are all connected (i.e. $\forall l, k \in C(\bar{\mathbf{g}})$, $\bar{g}_{l,k} \neq 0$). A complete core-periphery network is such that every player in $P(\bar{\mathbf{g}})$ is connected to every player in $C(\bar{\mathbf{g}})$ (i.e. $\forall i \in P(\bar{\mathbf{g}})$ and $k \in C(\bar{\mathbf{g}})$, $\bar{g}_{i,k} \neq 0$).

Figure 2 displays two examples of core-periphery networks. In both cases, $C(\bar{\mathbf{g}}) = \{1, 2, 3\}$ and $P(\bar{\mathbf{g}}) = \{4, 5, 6\}$. But note that only the graph in Panel (b) is a complete core-periphery graph since, in Panel (a), bank $1 \in C(\bar{\mathbf{g}})$ and $6 \in P(\bar{\mathbf{g}})$ are not connected.

Note that Definition 1 applies only to undirected networks. The network that is formed from banks' linking decisions, $\mathbf{g}$, is directed since a bank i can extend a link to bank j, without j extending a link to i. We, thus, provide an equilibrium characterization to the undirected network $\bar{\mathbf{g}}$, which we defined in the previous section as the closure of $\mathbf{g}$ (i.e.

[20] Kolstad and Mathiesen (1987) refer to this condition as diagonal dominance when deriving sufficient conditions for the uniqueness of equilibrium in a Cournot game.

Figure 2: Two Examples of Core-Periphery Graphs.

(a) (b)

Notes: Panel (a) and Panel (b) display core-periphery networks. In both cases, $C(\bar{\mathbf{g}}) = \{1, 2, 3\}$ and $P(\bar{\mathbf{g}}) = \{4, 5, 6\}$. The graph in Panel (a) is not a complete core-periphery graph because $1 \in C(\bar{\mathbf{g}})$ and $6 \in P(\bar{\mathbf{g}})$, for example, are not connected.

$\bar{g}_{i,j} = max\{g_{i,j}\ ,\ g_{j,i}\}$).

Proposition 2. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, $\bar{\mathbf{g}}$ *is a complete core-periphery network. Furthermore, there exists a partition* $\{C(\bar{\mathbf{g}}), P(\bar{\mathbf{g}})\}$ *of* N *such that:*

- *All peripheral banks display the same level of liquidity holdings.*
- *Core banks display higher levels of liquidity holdings than peripheral banks.*

The characterization of the equilibrium network $\bar{\mathbf{g}}$ is consistent with the empirical literature that has identified core-periphery structures in several interbank markets. To provide some intuition for the result, suppose we have three banks i, k and j, with $z_i \leq z_k$. If i connects to k , then it must be that j also connects to k. To see this, suppose that j does not connect to k. Recall that $v(.)$, i.e. the gross value function, is increasing in the amount of accessible liquidity. Then, it must be the case that i connects to every bank l such that $z_l \geq z_k$ with $g_{i,l} = 1, l \neq i$. If this were not the case, then i could weakly increase its payoff by reducing its link with k and increasing by the same amount a link to a bank with a weakly higher amount of liquid assets. An analogous argument tells us that j can only connect to banks that hold an amount of liquidity strictly higher than z_k. Hence, $y_i \geq y_j + \psi g_{i,k} z_k$. That is, if bank j were to extend a link of weight $g_{i,k}$ to k, it would still have an accessible liquidity weakly lower than i. Recall that $v(.)$ is concave. So the marginal benefit of bank j creating this link is weakly higher than the marginal benefit of i, while the marginal cost, κ, is constant. Therefore, if i finds it weakly profitable to link to k, so does j and we have reached a contradiction. We thus have that if some bank k receives at least one incoming link from a bank that holds weakly less liquid than it, then every bank in the network connects to k. The characterization of $\bar{\mathbf{g}}$ then follows from the fact that if $\bar{\mathbf{g}} \neq \mathbf{0}$, then a bank that holds minimum liquidity in equilibrium always connects to all banks that hold strictly more liquidity. We provide the following numerical example of two strict Nash equilibria to illustrate Proposition 2.

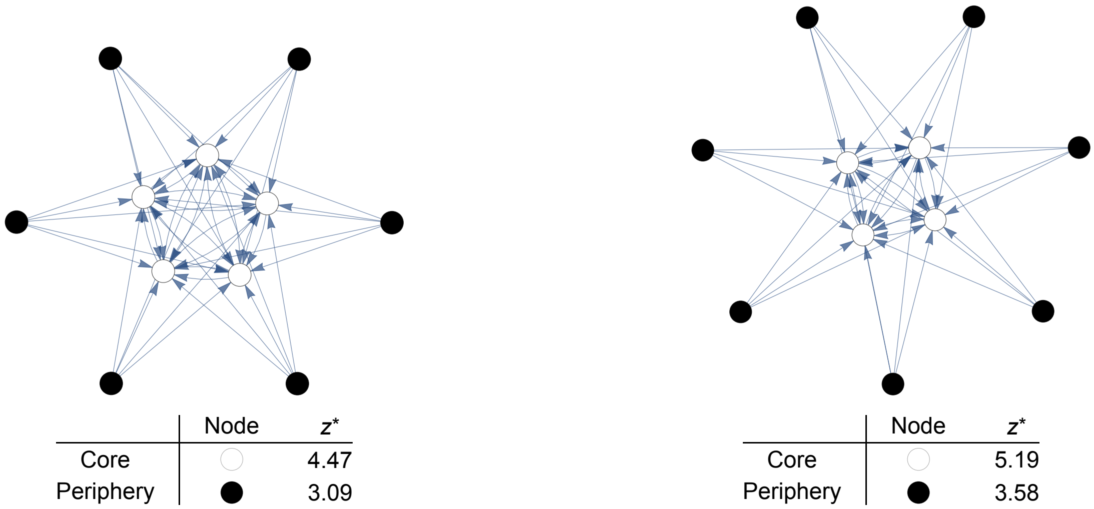
Example 1. Assume $n = 11,\ \gamma = 50,\ \beta = 5,\ \lambda = 15.625,\ \psi = 0.99,\ \kappa = 0.44,\ \zeta = 100$, where $\{\gamma, \beta, \lambda\}$ are expressed in basis points. Then there exists two different equilibrium networks: a complete core-periphery graph with 5 banks in the core and binary links only (i.e. $g_{i,j} \in \{0, 1\}, \forall i, j \in N$), as depicted in Panel (a) of Figure 3, and a complete core-periphery graph with 4 banks in the core and binary links only, as depicted in Panel (b) of Figure 3. Note that when links are all binary, all core banks display the same level of liquidity holdings. Also, we can see that, for the same set of parameter values, total liquidity is higher when the network is sparser.

Proposition 3 provides existence results for strict Nash equilibria in the choice of liquidity holdings and links. We present a sufficient condition such that an equilibrium exists for any linking cost κ. We also provide a (weaker) condition such that an equilibrium with a non-empty network exists. Let us define $\hat{\beta} \equiv \frac{\gamma\lambda^2\psi(\gamma+\lambda\psi)}{2\gamma^2(\gamma-\lambda)+\gamma\lambda^2\psi+\lambda^3\psi^2}$ and note that $\hat{\beta}$ is strictly smaller than λ.

Proposition 3. *If $\beta < \hat{\beta}$, then for any linking cost $\kappa \in \mathbb{R}_+$ there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$. If $\beta < \lambda$, then there exists κ such that a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, exists and the equilibrium network $\mathbf{g}$ is non empty, i.e. $\mathbf{g} \neq \mathbf{0}$.*

To satisfy at least one of the conditions above, it is sufficient for the return on the illiquid

Figure 3: Equilibrium Networks and Liquidity Holdings.



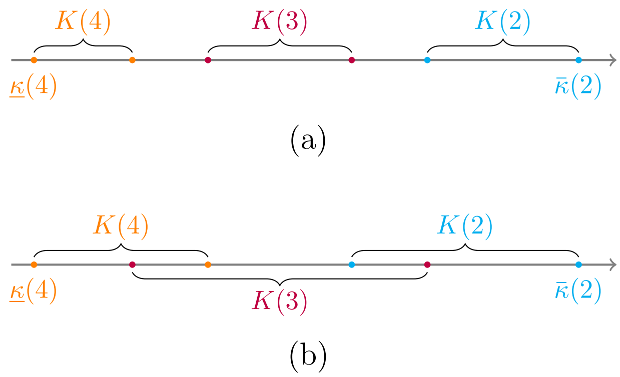
(a) (b)

Notes: Liquidity holdings are rounded to the second decimal. Panel (a) presents an equilibrium network with 5 banks in the core. Panel (b) presents an equilibrium network with 4 banks in the core. In both panels, all links are binary, i.e. $g_{i,j} \in \{0,1\}, \forall i,j \in N$.

asset r^x to be sufficiently close to $\bar{r}$ (i.e. β sufficiently close to zero). To get some intuition for Proposition 3, define K as the set of linking costs κ such that a strict NE exists. Note first that there always exists a sufficiently high linking cost such that it is never profitable for a bank to extend a link. Therefore, set K is always non-empty. However, it may be the case that, for all values of κ in K, the corresponding equilibrium always displays the empty network.

To illustrate the conditions in Proposition 3, let us consider a complete core-periphery graph with $n_C \in \{2,\dots,n-1\}$ banks in the core and binary links only, which we refer to as $\mathbf{g}(n_C)$. For $\mathbf{s} = (\mathbf{z}, \mathbf{g}(n_C))$ to be a strict NE, the linking cost must lie in between two linking thresholds $\bar{\kappa}(n_C)$ and $\underline{\kappa}(n_C)$, which are formally defined in the appendix (Lemma 8). These thresholds are such that: (i) if $\kappa \geq \underline{\kappa}(n_C)$, then a bank in the core does not find it profitable to extend a link to a peripheral bank, (ii) if $\kappa \leq \bar{\kappa}(n_C)$, then a bank in the periphery does not find it profitable to reduce the weight of the link it extends to a core bank. The condition $\beta < \lambda$ implies $\bar{\kappa}(n_C) > \underline{\kappa}(n_C), \forall n_C$, which guarantees the existence of a κ that satisfies (i)

Figure 4: Visual representation of the intervals $K(n_C)$.



Notes: Consider $n = 5$ and $n_C \in \{2, 3, 4\}$. Panel (a) displays the case when adjacent intervals have an empty intersection. Panel (b) displays the case when adjacent intervals intersect.

and (ii) simultaneously for a given n_C.

Moreover, define $K(n_C) \equiv [\underline{\kappa}(n_C), \bar{\kappa}(n_C)]$ and consider the set $\bigcup_{n_C} K(n_C) \subseteq K$. Figure 4 displays two possibilities for the union of $K(n_C)$. For simplicity, we assume $n = 5$. In Panel (a), adjacent intervals do not intersect and, therefore, the union of $K(n_C)$ is not an interval in $\mathbb{R}_+$. In Panel (b), we have that adjacent intervals always overlap and, thus, $[\underline{\kappa}(n-1), \bar{\kappa}(2)] = \bigcup_{n_C} K(n_C) \subseteq K$. When we also consider linking cost intervals such that the equilibrium network is the empty, star or complete network, then the overlapping condition guarantees that $K = \mathbb{R}_+$. In turn, the overlapping condition holds if $\beta < \hat{\beta}$.

Next, we derive properties of strict NE that display a star network with at least one weighted link. More specifically, we show that, in this case, the equilibrium is unique and the equilibrium link weight changes continuously with respect to the corridor width. These results will be useful for the next section when we study how the equilibrium network adapts to changes in the corridor width, $\gamma = \bar{r} - \underline{r}$.

Define $\mathbf{g}^s$ as the set of networks $\mathbf{g}$ that are a star network with at least one weighted link.[21] Also, consider two linking cost thresholds $\kappa_0 \equiv v'(0)\psi z^s$ and $\kappa_1 \equiv v'(\psi z^s)\,\psi z^s$, where $z^s = \frac{\zeta\beta}{\gamma}$ is the liquidity holding of the center. κ_0 and κ_1 represent the marginal benefit of a

[21] Formally, a star network $\mathbf{g}$ is such that some bank i receives an incoming link from every other bank (i.e. $\forall j \neq i$, $g_{j,i} \neq 0$) and the remaining banks do not receive any incoming links (i.e. $\forall j, k \neq i, g_{j,k} = 0$).

bank increasing the weight of the link to the center when its accessible liquidity is zero and z^s, respectively. We then have the following result.

Proposition 4. *Assume $\beta < \lambda$. Then a strict NE $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ exists for some $\mathbf{g} \in \mathbf{g}^s$ if and only if $\kappa \in (\kappa_1, \kappa_0)$. Moreover:*

- *All peripheral banks extend a weighted link to the center and the weight is the same across banks.*
- *The equilibrium link weight $g^* \in (0,1)$ satisfies $\kappa = v'(\psi g^* z^s)\,\psi z^s$, where $z^s = \frac{\zeta\beta}{\gamma}$ is the liquidity holding of the center.*
- *$\mathbf{s}$ is the unique strict NE.*

We briefly provide some intuition for the results. First, for $\mathbf{g} \in \mathbf{g}^s$ to be part of an equilibrium, the linking cost must be low enough for a peripheral bank to not find it profitable to access zero liquidity (i.e. $\kappa < \kappa_0$). But it also must be sufficiently high to prevent a peripheral bank from accessing as much liquidity as it can from the center (i.e. $\kappa > \kappa_1$).

The fact that links extended to the center are all of equal weight follows from the fact that, in any strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, any two banks i and k that are not connected have the same strategy profile in equilibrium (Lemma 3). To see why this holds, suppose $\mathbf{s_i} \neq \mathbf{s_k}$ and, without loss of generality, assume $u(\mathbf{s_i}) \geq u(\mathbf{s_k})$. Since i does not connect to k, a possible deviation for k is $\mathbf{s'_k} = \mathbf{s_i}$, which is weakly profitable for bank k.

The condition for the equilibrium weight g^* can be obtained by simply taking the derivative of the value function in (6) with respect to the link extended to the center. Note that, by the strict concavity of $v(.)$, we can uniquely solve for g^* and the solution is continuous in the parameters of the model (including the corridor width, γ).

Finally, we discuss the uniquenesses of $\mathbf{s}$. Since $v'(.)$ is strictly decreasing, condition $\kappa = v'(\psi g^* z^s)\,\psi z^s$ shows that any link to the center $g' \in [0,1]$ different from g^* cannot be part of an equilibrium. Consider then a network $\mathbf{g}'$ with at least two banks in the core. In any strict NE that displays $\mathbf{g}'$, some bank i extends at least two links. In equilibrium, banks extend at most one weighted link (Lemma 1). Hence, i extends a link of weight one to some bank j. From (5), we have that j holds weakly less liquidity than z^s. From the concavity of $v(.)$, if i were to only connect to j, it would again extend a link of weight one. Since $z_j \leq z^s$, this implies that it is optimal for a peripheral bank in a star to extend a link of weight one to the center. This contradicts the fact that peripheral banks extend a link of weight less than one to the center in the strict NE $\mathbf{s}$.

5 Policy Analysis

5.1 The Central Bank's Problem

The model developed in this paper contains two variables that are under the control of the central bank: the discount rate $\tilde{r}$ (which ultimately defines $\bar{r}$) and the deposit facility rate $\underline{r}$. These rates affect banks' liquidity holdings and credit lines decisions since they determine the cost of borrowing from the central bank when necessary and the opportunity cost of holding excess liquidity. In addition, $\bar{r}$ and $\underline{r}$ also impact the distribution of the realized interbank rate $r(c) = \underline{r} + c$, where c is the marginal lending cost for interbank loans. To see the this, recall that $r(c)$ only assumes values in $[\underline{r}, \bar{r}]$. Also, the interbank rate expected value, $E[r(c)] = \frac{1}{2}(\bar{r} + \underline{r})$, and volatility, $Var[r(c)] = \frac{1}{12}(\bar{r} - \underline{r})^2$, are defined by the standing facilities rates.

Following the design of central banks' operational frameworks in Bindseil (2016), we assume that the central bank sets a target rate r^* for the expected interbank rate $E[r(c)]$. We abstract from how the central bank sets the level of the target rate r^*. It is assumed that r^* derives from an exogenous goal (e.g. price stability). Hence, the primary goal of the central bank is to choose a pair $\{\bar{r}, \underline{r}\}$ such that:

$$r^* = \frac{\bar{r} + \underline{r}}{2}. \tag{7}$$

The following section explores how the central bank should set the optimal corridor width, $\gamma = \bar{r} - \underline{r}$, conditional on its target to the interbank rate. The central bank faces a trade-off when setting the corridor width: a narrower corridor reduces the variance of the interbank rate, allowing the central bank to target it with higher precision. However, the equilibrium network may change in response, yielding a sparser network, in which interbank activity is lower, and, thus, increasing total liquidity holdings in equilibrium. This incurs an implicit cost since these funds could be invested in the more productive illiquid asset. To study this trade-off, we define a loss function to the central bank which is a convex combination between interbank rate volatility and total liquidity holdings. The central bank chooses a pair of standing facilities rates $\{\bar{r}, \underline{r}\}$ to minimize its loss function. We show that as long as the expected return on the illiquid asset, r^x, has a positive premium with respect to the target rate, the central bank's problem can be reduced to the choice of the corridor width that minimizes the loss function.

In light of this, let us characterize the central bank's problem. The central bank controls $\bar{r}$ and $\underline{r}$, which is equivalent to choosing $\bar{r}$ and $\gamma = \bar{r} - \underline{r}$. Define the set of endogenous parameters as $\{\bar{r}, \gamma\}$, and the set of exogenous parameters as $\Phi = \{\bar{c}, \delta, r^x, \psi, \kappa, \zeta\}$. Starting from a strict NE $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, the volatility of the interbank rate is given by:

$$V(\gamma) \equiv \frac{\gamma^2}{12}. \tag{8}$$

Total liquidity is defined as:

$$Z(\bar{r}, \gamma) \equiv \sum_{i=1}^{n} z_i\left(\bar{r}, \gamma, \Phi\right). \tag{9}$$

We define a loss function to the central bank that is increasing in both the volatility of the interbank rate and total liquidity holdings. We opt for the most parsimonious specification, which is a convex combination of these terms:

$$L(\bar{r}, \gamma) = \theta V(\gamma) + (100 - \theta) Z(\bar{r}, \gamma)\,,$$

where $\theta \in (0, 100)$ is given and measures the relative importance of the precise targeting of the interbank rate for the central bank.[22]

To fully characterize the problem, let us discuss the restrictions on $\{\bar{r}, \gamma\}$. First, recall the assumption $\bar{c} \geq \bar{r} - \underline{r} \Leftrightarrow \bar{c} \geq \gamma$. This assumption guarantees that the interbank rate, $r(c)$, can assume any value in between $\underline{r}$ and $\bar{r}$. Second, we have a no arbitrage condition $\bar{r} > \underline{r}$, which implies $\bar{r} - \underline{r} \geq \delta \Leftrightarrow \gamma \geq \delta$. We also have condition (7), which can be rewritten as $r^* = \bar{r} - \frac{\gamma}{2}$. We make one last assumption that allows us to write the loss function and the constraints in terms of γ only. We assume that the expected return on the illiquid asset, r^x, has a positive premium with respect to the target rate. We can then write:

$$r^x = r^* + \eta, \tag{10}$$

with $\eta > 0$, where η can be interpreted as a liquidity premium, risk premium, or a combination of both. Replacing (10) in (7), yields the following condition:

$$\beta = \frac{\gamma}{2} - \eta. \tag{11}$$

Replacing (11) in (5) allows us to rewrite (9) as $Z(\gamma) \equiv \sum_{i=1}^{n} z_i\left(\gamma, \Phi\right)$ where $\Phi = \{\bar{c}, \delta, \eta, \psi, \kappa, \zeta\}$. We are now able to define an optimal corridor width.

Optimal corridor width: $\gamma \in \mathbb{R}_+$ such that:

$$min_\gamma \left\{L(\gamma) \,|\, \bar{c} \geq \gamma \geq \delta\right\}. \tag{12}$$

The solution to (12) is denoted as $\gamma^*\left(\Phi, \theta\right)$. When $\gamma^*\left(\Phi, \theta\right) = \delta$, we say that the minimum corridor width is optimal. For the remainder of this paper, we assume that $\delta > 2\eta$ and $8\eta > \bar{c}$.

[22]Alternatively, one could define the convex combination for $\theta \in (0, 1)$. We opt for the interval $(0, 100)$ just for expositional clarity of the numerical examples.

The first condition guarantees that for every $\gamma \in [\delta, \bar{c}]$, $\beta > 0$, and, thus, liquidity holdings are always strictly positive. The second assumption implies $\lambda > \beta$, which, by Proposition 3, assures that for some linking cost value we have a non-empty equilibrium network.

5.2 Optimal Corridor Width

This section analyzes the optimal corridor width. We first derive restrictions on the range of the linking cost, κ, such that for each corridor width the equilibrium network is a star with weighted links directed to the center. In this setting, the equilibrium is unique and total liquidity holdings decrease continuously with respect to increases in the corridor width. Our main result shows that, if the number of peripheral banks is larger than some threshold, then it is not optimal for the central bank to set the corridor to its minimum width, δ. This result follows from the fact that for corridors that are too narrow, peripheral banks access a small share of the center's liquidity and, therefore, hold large amounts of liquidity. We then provide a numerical example that illustrates how failing to account for the endogeneity of links can lead to suboptimal policy.

In order to derive general properties regarding the optimal width of the corridor rate, we restrict our analysis to sets Φ such that for every $\gamma \in [\delta, \bar{c}]$, the equilibrium network is a star network with weighted links directed to the center. This restriction in the set of exogenous parameters simplifies the problem for two reasons: first, it rules out multiplicity of equilibria, as we can see from Proposition 4. Second, it implies that $Z(\gamma)$ (and, therefore, $L(\gamma)$) is continuous with respect to γ. To see this second point, let us further analyze how the equilibrium network changes with respect to γ. From Proposition 4, the equilibrium link weight g^* must satisfy:

$$\kappa = v'\left(\psi g^* z^s\right)\psi z^s. \tag{13}$$

From the strict concavity of $v(.)$, we can uniquely solve for g^*. The solution will then be continuous in γ. Hence, $Z(\gamma)$ is continuous because the intensity of the link g^* changes continuously with γ, while the size of the core remains constant.[23]

Next, we ask what restrictions to the set Φ are necessary to ensure that, for every $\gamma \in [\delta, \bar{c}]$, the corresponding strict NE displays a star network with weighted links. From Proposition 4, we have that for each γ, it must be that:

$$\kappa \in (\kappa_1(\gamma), \kappa_0(\gamma)), \tag{14}$$

where we have made explicit that linking cost thresholds depend on the corridor width.[24] Both $\kappa_1(\gamma)$ and $\kappa_0(\gamma)$ are increasing in γ. Hence, to satisfy (14) for all γ, it is sufficient:

$$\kappa \in (\kappa_1(\bar{c}), \kappa_0(\delta)). \tag{15}$$

Finally, we need $\kappa_1(\bar{c}) < \kappa_0(\delta)$ to guarantee that, for some positive κ, (15) holds. This inequality is satisfied as long as $\sqrt{\frac{\bar{c}^2+4\eta\bar{c}-4\eta^2}{2}} < \delta$ and $\frac{2\left(\bar{c}^2-\delta^2\right)}{(\bar{c}-2\eta)^2} < \psi < 1$. Unless otherwise stated, we take Φ such that (15) holds.

We now turn to the properties of $\gamma^*(\Phi, \theta)$. Note that $L(\gamma)$ is continuous. Since $[\delta, \bar{c}]$ is compact, $\gamma^*(\Phi, \theta)$ is always non-empty. Our first proposition establishes conditions under which it is not optimal for the central bank to set the corridor width to its minimum size, δ. More specifically, it shows that, for any weight that the central bank assigns to the targeting of the interbank rate, θ, one can find a (finite) number of banks in the periphery such that it is not optimal for the central bank to minimize the interbank rate variance.

Proposition 5. *For each θ, there exists a $\bar{n} \in \mathbb{R}$ such that, for all $n > \bar{n}$, the loss function, $L(\gamma)$, is decreasing in the size of the corridor at $\gamma = \delta$. Moreover, the optimal corridor width, $\gamma^*(\Phi, \theta)$, is then strictly larger than the minimum corridor width, δ.*

[23]Changes in the equilibrium network that come from discrete changes in the size of the core can create discontinuities in $Z(\gamma)$.

[24]The exact expressions for the linking thresholds are $\kappa_0(\gamma) \equiv \frac{\zeta^2\psi}{8\bar{c}}\left(\gamma^2 - 4\eta^2\right)$ and $\kappa_1(\gamma) \equiv \frac{\zeta^2\psi}{16\bar{c}^2}\left(2\bar{c}\left(\gamma^2 - 4\eta^2\right) - \psi(2\bar{c}-\gamma)(\gamma-2\eta)^2\right)$.

This result highlights the importance of the trade-off between volatility of the interbank rate and total liquidity to the central bank's decision. $L(\gamma)$ decreasing at $\gamma = \delta$ implies that the central bank can reduce the loss function by a sufficiently small increase of the corridor width. Although this implies a higher variance of the interbank rate, there exists a marginal reduction in total liquidity that compensates for the increase in volatility. It then follows from this trade-off that the minimum corridor width is not optimal.

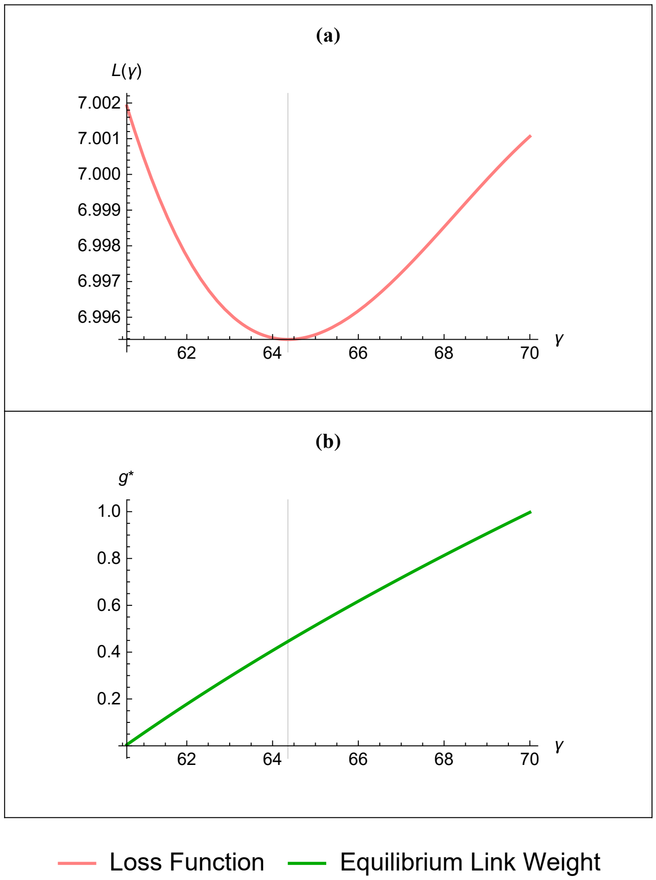
Let us discuss the conditions under which Proposition 5 holds. First, recall the assumption $\delta > 2\eta$. Note that, if to the contrary $\delta \leq 2\eta$, then the optimal corridor is the minimum corridor since $\beta < 0$ when $\gamma < 2\eta$, which, in turn, implies that total liquidity is zero in equilibrium. Hence, due to stigma costs, the proposed trade-off affects central bank's optimal decision.

Furthermore, the fact that n has to be sufficiently large is intuitive. Note that the bank at the center of the star holds $z^s = \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)$, which is strictly increasing in the size of the corridor. Therefore, for total liquidity to decrease, it must be that banks in the periphery reduce their liquidity holdings. It follows that the larger the number of banks in the periphery, the more liquidity decreases when the corridor increases. Of course, how large n must be depends on θ (See Proposition 5 in the Appendix for a formal derivation of the threshold $\bar{n}$). More precisely, the more weight the central bank puts on the interbank rate volatility, the larger must be n.

Next, we present a numerical example to illustrate Proposition 5.

Example 2. Assume $n = 3,\ \bar{c} = 70,\ \delta = 60.6,\ \eta = 8.8,\ \psi = 0.9,\ \kappa = 6.9948 * 10^{-7},\ \zeta = 0.036,\ \theta = 99.9897$, where $\{\bar{c}, \delta, \eta\}$ are expressed in basis points. Then $\gamma^*(\Phi, \theta) \approx 64.4$. Panel (a) of Figure 5 displays the loss function over the relevant range of the corridor width. As we can see, it is not optimal to set a corridor too close to δ. Panel (b) shows that this is the case because the equilibrium link weight g^* is close to zero, which implies an excessive amount of total liquidity. As we increase the corridor width, peripheral banks start strengthening their link to the center, total liquidity goes down and the loss function

Figure 5: Optimal Corridor Width: A Numerical Example.



Notes: Panel (a) presents the loss function $L(\gamma)$ over the domain $[\delta, \bar{c}]$. The loss function is multiplied by 10^4. Panel (b) presents the equilibrium link weight g^* for each $\gamma \in [\delta, \bar{c}]$. Vertical lines are drawn at $\gamma = \gamma^* (\Phi, \theta)$.

decreases up to the point that it reaches the minimum. At this point, the gain from reducing total liquidity no longer compensates for the increase in volatility that comes with a wider corridor. Note that even for θ close to 100, it is still the case that for relatively low values of n, the optimal corridor width is larger than δ.

To understand the role that the endogeneity of the network plays in Proposition 5, let us discuss how the widening of the corridor affects peripheral banks' incentives to hold liquidity. On one hand, a wider corridor implies a higher $\bar{r}$, which increases the cost of being short of liquid assets, thus, increasing peripheral banks' incentives to hold more liquidity. On the other hand, $\underline{r}$ is lower, reducing incentives to hold liquid assets. We have also seen that the bank at the center of the star increases its liquidity holdings when the corridor widens, which, due to local substitutability, reduces peripheral banks' incentives to allocate resources to liquid assets. If one were to fix the weight of the link to the center, then, for a sufficiently low fixed weight, the widening of the corridor would result in an increase in peripheral banks' liquidity holdings (See Proposition 7 in the Appendix).

Instead, if banks can optimally choose the weight of their links, then liquidity holdings decrease. This follows from the fact that a link becomes more valuable when the corridor widens since this reduces the probability that a bank with liquidity needs cannot borrow in the interbank market at a rate lower than $\bar{r}$. As a result, a bank in the periphery finds it profitable to increase the weight of its link to the center, as we can see in Panel (b) of Figure 5. This, in turn, increases its accessible liquidity, which, again, reduces incentives to hold own liquidity. This additional effect always outweighs remaining peripheral banks' incentives to hold more liquidity, if any.

Note that Proposition 5 may not hold if banks cannot revise their linking decisions since for a fixed network the trade-off in narrowing the corridor may not be present. This, in turn, implies that the central bank may set the corridor width at a suboptimal level if it does not account for how links adapt to changes in the corridor.

5.3 Comparative Statics

We conclude the analysis by providing comparative statics results regarding the optimal corridor width with respect to structural parameters. More specifically, as in Bindseil and Jabłecki (2011), we focus on changes in the size of interbank transaction costs, $\bar{c}$, and liquidity outflows, ζ. As in the previous section, we consider the restrictions in the linking cost, κ, such that the equilibrium network is a star network with weighted links directed to the center. First, we provide conditions such that the optimal corridor width is decreasing in $\bar{c}$ and increasing in ζ. The fact that the central bank reacts differently depending on which parameter changes may seem counterintuitive at first. We show that this result is related to how changes in interbank transaction costs and liquidity outflows have different implications for how banks in the periphery adjust their links to the center. Next, we provide a numerical example to illustrate that, when such conditions do not hold, sufficiently large changes in $\bar{c}$ and ζ may lead to non-monotonic changes in the optimal corridor width. This implies that the central bank should react differently to relatively small and large changes in the size of interbank transaction costs and liquidity outflows.

We start by presenting Proposition 6, which provides comparative statics results regarding the optimal corridor width. As in the previous section, we consider values for ζ and $\bar{c}$ such that, for each corridor width, the equilibrium network is a star with weighted links directed to the center. Our results show that the optimal corridor width is then increasing in the size of liquidity outflows and decreasing in the size of interbank transaction costs.

Consider a original set of exogenous parameters $\Phi = \{\bar{c}, \delta, \eta, \psi, \kappa, \zeta\}$ and the corresponding optimal corridor width $\gamma^* (\Phi, \theta)$. When ζ changes to ζ', the new set of exogenous parameters is $\Phi_{\zeta'} = \{\bar{c}, \delta, \eta, \psi, \kappa, \zeta'\}$. Analogously, we define $\Phi_{\bar{c}'}$ as the set of exogenous parameters when $\bar{c}$ changes to $\bar{c}'$. We consider sets Φ, $\Phi_{\zeta'}$ and $\Phi_{\bar{c}'}$ such that (15) holds, i.e. such that $\kappa_0 (\delta) > \kappa > \kappa_1(\bar{c})$ (and, therefore, the unique equilibrium network is a star with weighted links). We then have the following proposition.

Proposition 6. *If $\zeta' \geq \zeta$, then the optimal corridor width increases, i.e. $\gamma^*(\Phi_{\zeta'}, \theta) \geq \gamma^*(\Phi, \theta)$. If $\bar{c}' \leq \bar{c}$ and $\bar{c}'$ is not lower than the original optimal corridor width, i.e. $\bar{c}' \geq \gamma^*(\Phi, \theta)$, then the optimal corridor width increases, i.e. $\gamma^*(\Phi_{\bar{c}'}, \theta) \geq \gamma^*(\Phi, \theta)$.*

Proposition 6 follows mainly from Topkis's Theorem.[25] It shows that the optimal corridor width is increasing in ζ and decreasing in $\bar{c}$. For a decrease in the size of interbank transaction costs, note that the reduction must be such that $\gamma^*(\Phi, \theta)$ is still a feasible option for the central bank, i.e. $\gamma^*(\Phi, \theta) \in [\delta, \bar{c}']$. If this is not the case, then it is straightforward that Proposition 6 does not hold, since $\gamma^* (\Phi, \theta) > \bar{c}' \geq \gamma^* (\Phi_{\bar{c}'}, \theta)$.

[25] Topkis's Theorem cannot be directly applied to arbitrary changes in $\bar{c}$, since the constraint in (12) is decreasing in $\bar{c}$. We show in the appendix how the restriction $\bar{c}' \geq \gamma^*(\Phi, \theta)$ circumvents this issue, using the same reasoning as in Topkis's Theorem.

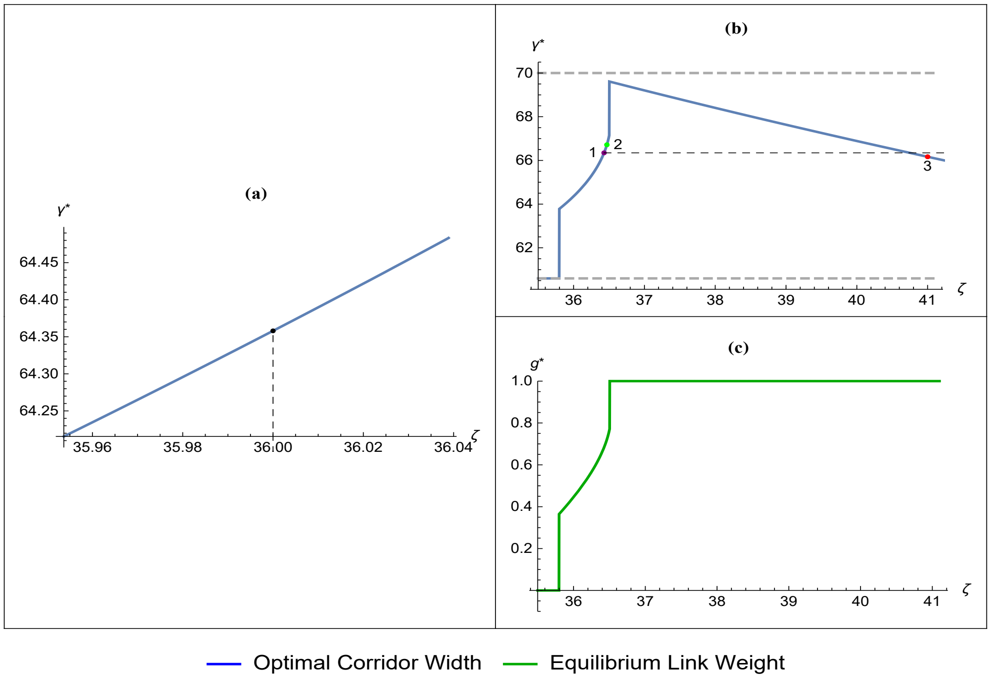
Let us discuss the intuition for why an increase in the size of liquidity outflows, ζ, has the opposite effect on the corridor width as an increase in the size of interbank transaction costs, $\bar{c}$. For that, we will assume n is large enough such that total liquidity is increasing in both ζ and $\bar{c}$. The reason why Proposition 6 holds may not be clear at first since ζ and $\bar{c}$ affect total liquidity in the same direction. However, these variables have different effects on the equilibrium link weight g^*. All else equal, when $\bar{c}$ increases, links become less valuable, since it is less likely that they will be used to cover outflows, and, therefore, g^* decreases. On the other hand, when ζ is higher, peripheral banks' incentives to increase g^* are higher due to the increased risk of being short of liquid assets, which makes it profitable to access more liquidity through the interbank market. Now note that liquidity holdings decrease more in response to an increase in the corridor width when the equilibrium link weight is higher due to local substitutability. Therefore, a higher g^* increases central bank's incentive to widen the corridor, and vice-versa.

Next, we go back to the set of parameter values of example 2 to provide a numerical example for Proposition 6. Figure 6 presents the analysis with respect to changes in the size of the liquidity outflow.[26] Panel (a) shows the optimal corridor width as a function of ζ when (15) holds. By Proposition 6, we know that the corridor gets wider for higher values of ζ. Panel (b) extends the analysis to values of ζ such that (15) does not hold. In this case, we can see that there exists a threshold value $\bar{\zeta} \approx 0.0365$, such that for $\zeta > \bar{\zeta}$, the corridor width is no longer increasing in ζ. From Panel (c), which displays, for each ζ, the corresponding equilibrium link weight g^*, we get an intuition for this result. Note that for $\zeta > \bar{\zeta}$, g^* equals one and, therefore, the equilibrium network is a star network with binary links only. Now, when ζ increases, peripheral banks cannot further increase the weight of the link they extend to the center and, thus, only adjust their liquidity holdings.

We show in Panel (b) how the existence of such a threshold $\bar{\zeta}$ implies that the central bank should react differently to relatively small and large changes in ζ. Suppose we originally

[26]For expositional clarity, we have scaled ζ up by 10^3 in the horizontal axis.

Figure 6: Comparative Statics: Changes in the Size of Liquidity Outflows.



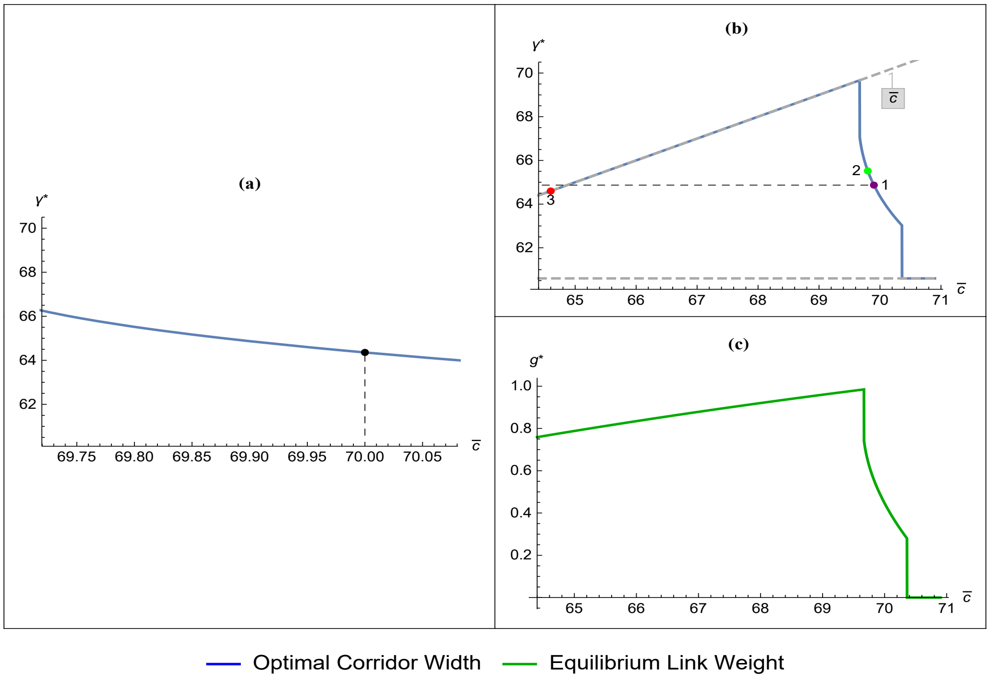
Notes: ζ has been scaled up by 10^3 in the horizontal axis. Panel (a) presents the optimal corridor width $\gamma^*(\Phi, \theta)$ as a function of ζ, when (15) holds. The black dot represents ζ and $\gamma^*(\Phi, \theta)$ as in example 2. Panel (b) extends the range of ζ to regions such that (15) does not hold. Top and bottom dashed gray lines represent, respectively, $\bar{c}$ and δ. Points 1, 2 and 3 are different combinations of ζ and $\gamma^*(\Phi, \theta)$. Panel (c) displays the equilibrium link weight for each value of ζ and corresponding optimal corridor width $\gamma^*(\Phi, \theta)$.

have a size of liquidity outflow ζ_1 and optimal corridor width γ_1^*, represented by point 1. For small increases in ζ, such as the one from point 1 to point 2, we see that the central bank reacts by widening the corridor. However, when the increase is relatively large, such as the change from point 1 to point 3, the central bank's response is to narrow the corridor.

Figure 7 displays an analogous analysis with respect to changes in the size of interbank transaction costs. One difference with respect to the previous analysis is what drives the existence of a threshold value for $\bar{c}$ such that for values below the threshold, the optimal corridor is increasing in $\bar{c}$. In Panel (c), we see that is never the case that g^* equals one. Instead, what happens for sufficiently low values of $\bar{c}$ is that the restriction $\bar{c} \geq \gamma$ is binding, as we can see in Panel (b) by the intersection of $\gamma^*(\Phi, \theta)$ and $\bar{c}$.

Understanding how the central bank should react to changes in the size of transaction

Figure 7: Comparative Statics: Changes in the Size of Interbank Transaction Costs.



Notes: Panel (a) presents the optimal corridor width $\gamma^*(\Phi,\theta)$ as a function of $\bar{c}$, when (15) holds and $\bar{c} > \gamma^*(\Phi,\theta)$. The black dot represents $\bar{c}$ and $\gamma^*(\Phi,\theta)$ as in example 2. Panel (b) extends the range of $\bar{c}$ to regions such that either (15) does not hold or $\bar{c} = \gamma^*(\Phi,\theta)$. Top and bottom dashed gray lines represent, respectively, $\bar{c}$ and δ. Points 1, 2 and 3 are different combinations of $\bar{c}$ and $\gamma^*(\Phi,\theta)$. Panel (c) displays the equilibrium link weight for each value of $\bar{c}$ and corresponding optimal corridor width $\gamma^*(\Phi,\theta)$.

costs and liquidity outflows can be important during moments of financial distress, when ζ and $\bar{c}$ are likely to rise. Our comparative statics analysis shows that the decision to widen or narrow the corridor depends on two dimensions: (i) the increase in ζ relative to $\bar{c}$, since by Proposition 6 these two increases can have different implications for how the optimal corridor width changes, and (ii) the increase in ζ and $\bar{c}$ relative to their size, given the non-monotonicity observed in Figures 6 and 7. The failure to account for one of these dimensions may lead to suboptimal policy.

6 Conclusion

In this paper, we develop a simple and tractable endogenous network formation model of the interbank market. Banks solve a liquidity management problem, making simultaneous decisions regarding liquidity holdings and interbank credit lines, which are modeled as (directed) links. When deciding optimally, banks take the corridor rate that is established by central bank's standing facilities into account.

In this setting, we define a loss function to the central bank, which is a convex combination between interbank rate volatility and total liquidity held by banks, and focus on the case when the equilibrium network is a star network with directed links to the bank at the center of the star. Our results show that, as long as the number of peripheral banks is not too small, the central bank does not find it optimal to minimize interbank rate volatility since this implies a large amount of total liquidity in the banking system. This paper, thus, provides a new rationale for a positive spread between standing facilities rates, even when the main goal of monetary policy is to control the short-term interbank rate. We also show how the central bank should optimally react to changes in the size of interbank transaction costs and liquidity outflows. We find that changes in transaction costs relative to liquidity outflows, as well as the magnitude of these changes, impact the central bank's optimal decision regarding the corridor width and failing to account for one of these dimensions can lead to suboptimal policy.

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A Appendix

A.1 Derivation of the value function

When writing the utility in (4) we only considered the region where $z_i + y_i \leq \zeta$. If we also consider $z_i + y_i > \zeta$, then the utility writes:

$$u(z_i, \mathbf{g_i}) = \begin{cases} \zeta\beta z_i - \frac{\gamma}{2}z_i^2 - \lambda z_i y_i + \zeta\lambda y_i - \frac{\lambda}{2}y_i^2 - \kappa\sum_{j\in N} g_{i,j}, & z_i + y_i \leq \zeta \\ \mu + \zeta\bar{\beta}z_i - \frac{\bar{\gamma}}{2}z_i^2 - \kappa\sum_{j\in N} g_{i,j}, & z_i + y_i > \zeta \end{cases}$$

where $\bar{\beta} \equiv \beta - \lambda$, $\bar{\gamma} \equiv \gamma - \lambda$, $\mu \equiv \frac{\zeta^2\lambda}{2}$. Note that $(\gamma - \beta) = r^x - \underline{r} > 0$ and $(\gamma - \lambda) = \frac{\gamma}{\bar{c}}\left(\bar{c} - \frac{\gamma}{2}\right) > 0$, where the inequality follows from $\bar{c} \geq \bar{r} - \underline{r} = \gamma$. Consider the best response function derived in (5). If we add y_i to both sides of the equation we get:

$$z_i + y_i = \frac{\zeta\beta}{\gamma} + \left(1 - \frac{\lambda}{\gamma}\right)y_i.$$

For the right-hand side of the above expression to be lower than ζ, y_i must satisfy:

$$y_i \leq \frac{\zeta(\gamma - \beta)}{(\gamma - \lambda)}.$$

If $y_i > \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$, then we have:

$$z_i(\mathbf{g}, \mathbf{z_{-i}}) = \frac{\zeta\, max\{\bar{\beta}, 0\}}{\bar{\gamma}}. \tag{16}$$

The value function then writes :

$$V(y_i, \mathbf{g}) = \begin{cases} \frac{(\zeta\beta)^2}{2\gamma} + \frac{\zeta(\gamma-\beta)\lambda}{\gamma}y_i - \frac{(\gamma-\lambda)\lambda}{2\gamma}y_i^2 - \kappa\sum_{j\in N} g_{i,j}, & y_i \leq \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)} \\ \mu + \frac{(\zeta\, max\{\bar{\beta},0\})^2}{2\bar{\gamma}} - \kappa\sum_{j\in N} g_{i,j}, & y_i > \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)} \end{cases}$$

Define $v(y_i) \equiv V(y_i, \mathbf{g}) - \kappa\sum_{j\in N} g_{i,j}$. One can easily check that $v(.)$ is continuous at $y_i = \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$. To show that $v(.)$ is increasing and concave, it is sufficient to show that this holds for $y_i \in \left[0, \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}\right]$ since $v(.)$ is constant for $y_i > \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$. Taking the derivative with respect to $y_i \in \left[0, \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}\right]$:

$$v'(y_i) = \frac{\zeta(\gamma-\beta)\lambda}{\gamma} - \frac{(\gamma-\lambda)\lambda}{\gamma} y_i. \tag{17}$$

It is straightforward that the expression above is strictly decreasing in $\left[0, \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}\right]$. To see that (17) is also non-negative, it is then sufficient to check that $v'\left(\frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}\right) = 0$ is non-negative.

We argue that the relevant domain for $v(.)$, in terms of network formation analysis, is $\left[0, \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}\right]$. To see this, take $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ and assume that for some $i \in N$, $\mathbf{s_i} = (z_i, \mathbf{g_i})$ is such that $y_i > \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$. We will show that $\mathbf{s_i} = (z_i, \mathbf{g_i})$ is strictly dominated by some strategy profile $\mathbf{s'_i}$ such that $y'_i = \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$. Consider a strategy profile $\mathbf{s'_i} = (z'_i, \mathbf{g'_i})$ such that $y'_i = \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$ and $\mathbf{g'_i} \leq \mathbf{g_i}$. Because $v(.)$ is constant for $y_i \geq \frac{\zeta(\gamma-\beta)}{(\gamma-\lambda)}$, $v(y_i) = v(y'_i)$. However, the total cost of linking under $\mathbf{s'_i}$ is strictly smaller than under $\mathbf{s_i}$. This implies $V(y'_i, \mathbf{g}') > V(y_i, \mathbf{g})$.

A.2 Proposition 1

If $\psi < \frac{\gamma}{\lambda(n-1)}$ and $\beta < \lambda$, then there exists a unique NE in the choice of $\mathbf{z}$ *and the unique NE is interior for any given network* $\mathbf{g}$. **Proof.** First, let us show that, in any NE, $z_i + y_i \leq \zeta\,, \forall\, i \in N$. Assume to the contrary that $z_i + y_i > \zeta$ for some $i \in N$. From (16) and $\beta < \lambda$, we have $z_i = 0$ and, therefore, $y_i > \zeta$. From (5), we can see that, in any NE, $z_j \leq \frac{\zeta\beta}{\gamma}$ for every $j \in N$. Hence, $y_i \leq \psi(n-1)\frac{\zeta\beta}{\gamma}$. From $\psi < \frac{\gamma}{\lambda(n-1)}$, we then get that $y_i < \frac{\zeta\beta}{\lambda} < \zeta$ and we have reached a contradiction. It then follows that, in any Nash equilibrium, (5) must hold $\forall i \in N$. Therefore, $\mathbf{z}$ must solve:

$$\mathbf{z} = \frac{\zeta\beta}{\gamma}\mathbf{1} - \left(\frac{\lambda\psi}{\gamma}\right) G\mathbf{z} \Rightarrow \left[\mathbf{I} + \left(\frac{\lambda\psi}{\gamma}\right) G\right] \mathbf{z} = \frac{\zeta\beta}{\gamma}\mathbf{1}, \tag{18}$$

where $\mathbf{1}$ is the n-dimensional vector of ones. A unique solution to the above system of equations exists if and only if $\left[\mathbf{I} + \left(\frac{\lambda\psi}{\gamma}\right) G\right]$ is invertible. Let $\sigma(G)$ denote G's spectrum.

Then $\left[\mathbf{I} + \left(\frac{\lambda\psi}{\gamma}\right) G\right]^{-1}$ is well-defined if and only if:

$$-\frac{\gamma}{\lambda\psi} \notin \sigma(G). \tag{19}$$

Denote by $\rho(G)$ the spectral radius of G. Then a sufficient condition for (19) is:

$$\rho(G) < \frac{\gamma}{\lambda\psi}, \tag{20}$$

which can be rewritten as $\psi < \frac{\gamma}{\lambda\rho(G)}$. Since G is non-negative, we can apply the Perron-Frobenius Theorem.[27] This Theorem provides an upper bound for the value of $\rho(G)$:

$$\rho(G) \leq \max_{i \in N} \sum_j g_{i,j}.$$

Since $g_{i,j} \leq 1$, $\forall i, j \in N$ and $g_{i,i} = 0$, $\forall i \in N$, we have $\rho(G) \leq n - 1$ for any adjacency matrix G. Hence, $\psi < \frac{\gamma}{\lambda(n-1)}$ implies that (20) holds for every G. Thus, $\left[\mathbf{I} + \left(\frac{\lambda\psi}{\gamma}\right) G\right]$ is always invertible.

Denote by $\mathbf{z}^*(G)$ the solution to (18). To show that $\mathbf{z}^*(G)$ is interior, define $\mathbf{W} = [w_1, w_2, \ldots, w_n]'$. We want to prove that, for every G, $\mathbf{0} < \mathbf{z}^*(G) < \mathbf{W}$. We first show that $\mathbf{z}^*(G) > \mathbf{0}$. Start by rewriting (18) as:

$$[\mathbf{I} + \omega G]\, \mathbf{z}^*(G) = \alpha \mathbf{1},$$

where $\alpha \equiv \frac{\zeta\beta}{\gamma} > 0$ and $\omega \equiv \frac{\lambda\psi}{\gamma}$. We have already shown that if $\omega < \frac{1}{n-1}$, then $[\mathbf{I} + \omega G]$ is invertible for any adjacency matrix G. Inverting $[\mathbf{I} + \omega G]$ yields:

$$\mathbf{z}^*(G) = \alpha\, [\mathbf{I} + \omega G]^{-1}\, \mathbf{1}.$$

[27] Perron-Frobenius Theorem provides some properties of positive square matrices. We use the property that if $A = (a_{i,j})$ is a positive $n \times n$ matrix, then $\rho(A) \leq \max_i \sum_j a_{i,j}$ holds.

The matrix $[\mathbf{I}+\omega G]^{-1}$ is well-defined and can be written as:

$$\left[\mathbf{I}-\omega G+(\omega G)^2-(\omega G)^3\ldots\right]=\sum_{k=0}^{\infty}\left[(\omega G)^{2k}-(\omega G)^{2k+1}\right].$$

We can rewrite the above infinite sum of matrices as:

$$\sum_{k=0}^{\infty}\left[(\omega G)^{2k}-(\omega G)^{2k+1}\right]=\sum_{k=0}^{\infty}(\omega G)^{2k}\left(\mathbf{I}-\omega G\right)=\left[\sum_{k=0}^{\infty}(\omega G)^{2k}\right]\left(\mathbf{I}-\omega G\right).$$

Therefore, $\mathbf{z}^*(G)$ is equal to:

$$\alpha\left[\sum_{k=0}^{\infty}(\omega G)^{2k}\right]\left(\mathbf{I}-\omega G\right)\mathbf{1}.$$

We have that $\alpha>0$. Also, $\left[\sum_{k=0}^{\infty}(\omega G)^{2k}\right]$ is a positive matrix since ωG is a positive matrix. Therefore, it is sufficient to show that the column matrix $(\mathbf{I}-\omega G)\,\mathbf{1}$ is a positive matrix. Consider then the element in the i-th row of $(\mathbf{I}-\omega G)\,\mathbf{1}$. This can be written as:

$$1-\omega\sum_{j\in N}g_{i,j}.$$

This expression is positive for every $i\in N$. To see this note that $\max_{i\in N}\sum_j g_{i,j}=(n-1)$ and recall that $\omega<\frac{1}{n-1}$. Therefore, $\mathbf{z}^*(G)>\mathbf{0}$. Now let us show that $\mathbf{z}^*(G)<\mathbf{W}$. Consider again (5). By $\mathbf{z}^*(G)>\mathbf{0}$, we have that, for every $i\in N$ and adjacency matrix G:

$$z_i^*=\frac{\zeta\beta}{\gamma}-\left(\frac{\lambda}{\gamma}\right)\psi\sum_{j\in N}g_{i,j}z_j^*\leq\frac{\zeta\beta}{\gamma}. \tag{21}$$

Note that $(\gamma-\beta)=(r^x-\underline{r})>0$, which implies $\frac{\beta}{\gamma}<1$. Therefore:

$$z_i^*\leq\frac{\zeta\beta}{\gamma}<\zeta<\min_j w_j\leq w_i,\ \forall i.$$

Hence, $\mathbf{z}^*(G)<\mathbf{W}$, which concludes the proof. ■

A.3 Proposition 2

In any strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*,* $\bar{\mathbf{g}}$ *is a complete core-periphery network. Furthermore, there exists a partition* $\{C(\bar{\mathbf{g}}), P(\bar{\mathbf{g}})\}$ *of* N *such that:*

- *All peripheral banks display the same level of liquidity holdings.*
- *Core banks display higher levels of liquidity holdings than peripheral banks.*

Proof. First, we present five lemmas, which directly imply that any strict NE network is a complete core-periphery graph.

Lemma 1. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*, every bank* $i \in N$ *has at most one link with weight in* $(0, 1)$*. Moreover, if* $g_{i,k} \in (0, 1)$*, then bank* k *holds less liquidity than every other bank to which* i *extends a link of weight one, i.e* $z_k < z_m$*,* $\forall\, m$ *such that* $g_{i,m} = 1$*.*

Proof. For the first part, assume contrary to the above statement that for some bank i with strategy profile $\mathbf{s_i} = (z_i, \mathbf{g_i})$ there exist at least two links with weights in $(0, 1)$. Take two links $g_{i,l}$ and $g_{i,k}$ and assume, without loss of generality, that $z_k \geq z_l$. Consider the deviation $\mathbf{s'_i} = (z_i, \mathbf{g'_i})$, where $\mathbf{g'_i}$ is defined as follows:

$$\begin{cases} g'_{i,j} = g_{i,j}, & \forall j \in N \setminus \{k, l\} \\ g'_{i,k} = g_{i,k} + \nu \\ g'_{i,l} = g_{i,l} - \nu \end{cases}$$

for some $\nu > 0$ such that $g'_{i,k} < 1$ and $g'_{i,l} > 0$. Bank i's liquidity holding z_i is the same under both strategies. The total cost of linking for agent i is also the same, since $\kappa \sum_{j \in N} g'_{i,j} = \kappa \left(\sum_{j \notin \{k,l\}} g_{i,j} + g_{i,k} + \nu + g_{i,l} - \nu \right) = \kappa \sum_{j \in N} g_{i,j}$. However, agent i's accessible liquidity is weakly higher under $\mathbf{s'_i}$, since $y'_i = \psi \sum_{j \in N} g'_{i,j} z_j = \psi \sum_{j \in N} g_{i,j} z_j + \psi\nu(z_k - z_l) = y_i + \psi\nu(z_k - z_l) \geq y_i$. Therefore, $\mathbf{s'_i}$ yields a weakly higher payoff than $\mathbf{s_i}$, which contradicts the fact that $\mathbf{s}$ is a strict Nash equilibrium. For the second part, assume there exists a bank m such that

$g_{i,m} = 1$ and $z_m \leq z_k$. Just as in the first part, bank i can weakly increase its payoff by increasing the weight in the link with bank k to $g'_{i,k} = g_{i,k} + \nu$ and reducing the weight in the link with bank m to $g'_{i,m} = g_{i,m} - \nu$, for some $\nu > 0$ such that $g'_{i,k} < 1$ and $g'_{i,m} > 0$, which again contradicts the fact that $\mathbf{s}$ is a strict Nash equilibrium. ■

Lemma 2. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*, if* $g_{i,l} > 0$*, then* $g_{i,k} = 1$*,* $\forall k : z_k \geq z_l$*.*

Proof. Assume contrary to the above statement that $g_{i,k} \neq 1$ for some k such that $z_k \geq z_l$. Suppose first that $g_{i,k} = 0$. If bank i deletes its link with l, then y_i decreases by $\psi g_{i,l} z_l$. Moreover, if bank i creates a link with k such that $g_{i,k} = g_{i,l} \frac{z_l}{z_k} > 0$, then y_i increases by $\psi g_{i,k} z_k = \psi g_{i,l} \frac{z_l}{z_k} z_k = \psi g_{i,l} z_l$. So y_i is unaltered. But note that $g_{i,k} \leq g_{i,l}$ since $z_l \leq z_k$. So bank i now has a weakly smaller total cost of linking. Therefore, there exists a deviation that weakly increases bank i's payoff, which contradicts the fact that $\mathbf{s}$ is a strict Nash equilibrium. Now suppose that $g_{i,k} \in (0, 1)$. If $g_{i,l} \in (0, 1)$, by Lemma 1, we have reached a contradiction since bank i cannot extend more than one weighted link. If $g_{i,l} = 1$, again by Lemma 1, we reach a contradiction, since $z_k \geq z_l$. ■

Lemma 3. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*, if* $\bar{g}_{i,l} = 0$*, then* $\mathbf{s_i} = \mathbf{s_l}$*.*

Proof. Take i and $l \in N$ such that $\bar{g}_{i,l} = 0$ and assume contrary to the above statement that $\mathbf{s_i} \neq \mathbf{s_l}$. Without loss of generality, assume $u(\mathbf{s_i}) \geq u(\mathbf{s_l})$. Since i does not connect to l (i.e. $g_{i,l} = 0$), a possible deviation for l is $\mathbf{s'_l} = \mathbf{s_i}$, which is weakly profitable for bank l. This contradicts the fact that $\mathbf{s}$ is a strict NE. ■

Lemma 4. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*, if* $\bar{g}_{i,l} > 0$ *and* $z_i \leq z_l$*, then* $g_{i,l} > 0$*.*

Proof. Take i and $l \in N$ such that $\bar{g}_{i,l} > 0$ and $z_i \leq z_l$. Note that, by (5), $z_i \leq z_l$ implies $y_i \geq y_l$. Assume contrary to the above statement that $g_{i,l} = 0$. Then it must be that $g_{l,i} > 0$. By Lemma 2, we have $g_{l,m} = 1, \forall m : z_m \geq z_i, m \neq l$. We then have:

$$y_l = \psi \sum_{j \in N} g_{l,j} z_j \geq \psi \left(g_{l,i} z_i + \sum_{\{j \neq l \,|\, z_j > z_i\}} z_j \right). \tag{22}$$

Also, by Lemma 2, we have $g_{i,m} = 0, \forall m : z_m \leq z_l$, which implies:

$$y_i = \psi \sum_{j \in N} g_{i,j} z_j \leq \psi \sum_{\{j \,|\, z_j > z_l\}} z_j. \tag{23}$$

Since $z_i \leq z_l$, $\{j \,|\, z_j > z_l\} \subseteq \{j \neq l \,|\, z_j > z_i\}$, which yields:

$$\sum_{\{j \neq l \,|\, z_j > z_i\}} z_j \geq \sum_{\{j \,|\, z_j > z_l\}} z_j. \tag{24}$$

From (22), (23) and (24), we have:

$$y_i \leq \psi \sum_{\{j \,|\, z_j > z_l\}} z_j < \psi \left(g_{l,i} z_i + \sum_{\{j \neq l \,|\, z_j > z_i\}} z_j \right) \leq y_l.$$

Thus, $y_i < y_l$ and we have reached a contradiction. ■

Lemma 5. *In any strict NE,* $\mathbf{s} = (\mathbf{z}, \mathbf{g})$*, if* $g_{i,l} > 0$ *for* i *and* l *such that* $z_i \leq z_l$*, then* $g_{k,l} \geq g_{i,l}, \forall\, k \in N \setminus \{l\}$.

Proof. Assume contrary to the above statement that $g_{i,l} > g_{k,l}$ for some i and k, with $z_i \leq z_l$. We will show that it is profitable for agent k to increase the weight of the link it extends to l from $g_{k,l}$ to $g_{i,l}$. Note that $g_{k,l} < 1$. By Lemma 2, we must have $g_{k,m} = 0, \forall m : z_m \leq z_l$. Define the set $\bar{Z}_l = \{j \in N \,| j \neq k \,\text{and}\, z_j > z_l\}$ as the set of banks different from k that hold strictly more liquidity than l (note that $i \notin \bar{Z}_l$). We then have:

$$y_k = \psi \sum_{j \in N} g_{k,j} z_j \leq \psi g_{k,l} z_l + \psi \sum_{j \in \bar{Z}_l} z_j. \tag{25}$$

Also, by Lemma 2, we have $g_{i,m} = 1, \forall m : z_m \geq z_l, m \neq i$. This yields:

$$y_i = \psi \sum_{j \in N} g_{i,j} z_j \geq \psi g_{i,l} z_l + \psi \sum_{j \in \bar{Z}_l} z_j. \tag{26}$$

Adding $\psi g_{i,l} z_l$ to both sides of (25) we obtain:

$$y_k - \psi g_{k,l} z_l + \psi g_{i,l} z_l \leq \psi g_{i,l} z_l + \psi \sum_{j \in \bar{Z}_l} z_j \leq y_i. \tag{27}$$

For $g_{i,l} > g_{k,l}$ to be part of a strict NE it must be that bank i does not find it profitable to change the weight of its link to l from $g_{i,l}$ to $g_{k,l}$, which implies:

$$v(y_i) - v(y_i - \psi g_{i,l} z_l + \psi g_{k,l} z_l) > \kappa(g_{i,l} - g_{k,l}). \tag{28}$$

From the concavity of $v(.)$ and (27) it follows that:

$$v(y_k - \psi g_{k,l} z_l + \psi g_{i,l} z_l) - v(y_k) \geq v(y_i) - v(y_i - \psi g_{i,l} z_l + \psi g_{k,l} z_l) > \kappa(g_{i,l} - g_{k,l}).$$

Hence, bank k finds it profitable to increase the weight of the link it extends to l to $g_{i,l}$. We have reached a contradiction. ■

Define the set of i's neighbors in $\bar{\mathbf{g}}$ as $N_i(\bar{\mathbf{g}}) = \{j \in N | \bar{g}_{i,j} \neq 0\}$. To show that $\bar{\mathbf{g}}$ is a complete core-periphery graph, it is sufficient to show that $\forall k \in C(\bar{\mathbf{g}})$, $N_k(\bar{\mathbf{g}}) = N \setminus \{k\}$ and $\forall i \in P(\bar{\mathbf{g}})$, $N_i(\bar{\mathbf{g}}) = C(\bar{\mathbf{g}})$.

Let us start by defining $\underline{z} = min\{z_1, \dots, z_n\}$ as the lowest amount of liquidity that is held by some bank in equilibrium. Also, define the set $\underline{Z} = \{i \in N \,|\, z_i = \underline{z}\}$ as the set of banks that have liquidity holdings equal to $\underline{z}$. Note that $\underline{Z}$ is non-empty. We then consider two different cases.

First, assume that for some i and $j \in \underline{Z}$, $g_{i,j} > 0$. By Lemma 2, $g_{i,l} > 0$, $\forall\, l \in N \setminus \{i\}$, which implies $N_i(\bar{\mathbf{g}}) = N \setminus \{i\}$. Now take $k \in N \setminus \{i\}$. Note that bank i holds weakly less liquidity than k and i extends a link to k. By Lemma 5, every other bank in the network also extends a link to k. Hence, $N_k(\bar{\mathbf{g}}) = N \setminus \{k\}$. To show that $\bar{\mathbf{g}}$ is a complete core-periphery graph, define $C(\bar{\mathbf{g}}) = N$ and $P(\bar{\mathbf{g}}) = \emptyset$.

Second, assume that for all i and $j \in \underline{Z}$, $g_{i,j} = 0$. Set $P(\bar{\mathbf{g}}) = \underline{Z}$ and $C(\bar{\mathbf{g}}) = N \setminus \underline{Z}$. If

$\underline{Z} = N$, then $\bar{\mathbf{g}}$ is the empty network, which is a complete core-periphery graph. Otherwise, note that, by construction, $\forall\, i \in P(\bar{\mathbf{g}})$ and $k \in C(\bar{\mathbf{g}})$, $\mathbf{s_i} \neq \mathbf{s_k}$. By Lemma 3, $\bar{g}_{i,k} > 0$. Therefore, $N_i(\bar{\mathbf{g}}) = C(\bar{\mathbf{g}})\,, \forall i \in P(\bar{\mathbf{g}})$. Now since $z_i < z_k$, by Lemma 4, $g_{i,k} > 0$. By Lemma 5, $N_k(\bar{\mathbf{g}}) = N \setminus \{k\}$, $\forall\, k \in C(\bar{\mathbf{g}})$. Hence, $\bar{\mathbf{g}}$ is a complete core-periphery graph.

To conclude the proof, note that by construction, all banks in $P(\bar{\mathbf{g}})$ hold the same amount of liquidity, which is strictly smaller than that of banks in $C(\bar{\mathbf{g}})$. ■

A.4 Proposition 3

If $\beta < \hat{\beta}$, then for any linking cost $\kappa \in \mathbb{R}_+$ there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$. If $\beta < \lambda$, then there exists κ such that a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, exists and the equilibrium network $\mathbf{g}$ is non empty, i.e. $\mathbf{g} \neq \mathbf{0}$.

Proof. First, recall our assumptions $\bar{r} > r^x > \underline{r}$ and $\bar{c} > \bar{r} - \underline{r}$, which imply $\beta < \gamma$ and $\lambda < \gamma$. Define $K \subseteq \mathbb{R}_+$ as the set of linking costs κ such that a strict NE exists We start by demonstrating that K is non-empty. Define $z^e = \frac{\zeta\beta}{\gamma}$ and $\kappa^e \equiv v'(0)\psi z^e = \frac{\zeta^2\beta\lambda\psi(\gamma-\beta)}{\gamma^2} > 0$. The following lemma shows that κ^e is a linking threshold such that for higher values of the linking cost κ there exists a strict NE such that the equilibrium network is empty.

Lemma 6. *For all $\kappa \geq \kappa^e$, there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, with $\mathbf{g} = \mathbf{0}$. Therefore, if $\kappa \geq \kappa^e$, then $\kappa \in K$.*

Proof. Consider the empty network (i.e. $\mathbf{g} = \mathbf{0}$). In this case, $y_i = 0$, $\forall i \in N$. From (5), we can then see that in equilibrium every bank must hold the same level of liquidity, z^e. For the empty network to be an equilibrium, no bank can weakly increase its payoff by extending a link to another bank, which yields the following condition:

$$v'(0)\psi z^e \leq \kappa.$$

Therefore, if $\kappa \in [\kappa^e, \infty)$, there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{0})$, which concludes the proof. ■

Define the non-empty set $K_1 = [\kappa^e, \infty)$. By Lemma 6, $K_1 \subseteq K$, which implies that K is non-empty.

Moreover, define $z^s \equiv z^e$ and $\bar{\kappa} \equiv v'(\psi z^s)\,\psi z^s$. We provide a condition such that for $\kappa \in (\bar{\kappa}, \kappa^e)$ there exists a strict NE in which $\mathbf{g} \neq \mathbf{0}$.

Lemma 7. *If $\beta < \lambda$, then $\kappa \in (\bar{\kappa}, \kappa^e)$ if and only if there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, in which $\mathbf{g}$ is a star network with at least one weighted link. Therefore, if $\kappa \in (\bar{\kappa}, \kappa^e)$, then $\kappa \in K$.*

Proof. Consider the star network with at least one weighted link. The bank at the center of the star is the unique core bank and, therefore, does not extend any link. Hence, z^s is the liquidity holding of the center. At least one peripheral bank extends a weighted link $g \in (0,1)$ to the center. By Lemma 3, all remaining peripheral banks also extend a link of weight g to the center. From (5), the liquidity holding of a peripheral bank is $z^p(g) \equiv \frac{\zeta\beta}{\gamma} - \frac{\lambda}{\gamma}\psi g z^s = z^s\left(1 - \frac{\lambda}{\gamma}\psi g\right)$. The liquidity peripheral banks access through the network is $y^p(g) = \psi g z^s$, while the center does not access any liquidity. From the strict concavity of $v(.)$, if a peripheral bank extends a link of weight g to the center, then g must satisfy:

$$v'(y^p(g))\psi z^s = \kappa \Leftrightarrow \kappa = \frac{\zeta^2\beta\lambda\psi\left(\gamma^2 - \beta(\gamma + g\psi(\gamma-\lambda))\right)}{\gamma^3} \equiv \kappa(g). \tag{29}$$

For the star with weighted links equal to g to be an equilibrium, the center cannot find it weakly profitable to extend a link to a bank in the periphery. Therefore:

$$v'(0)\psi z^p(g) \leq \kappa(g). \tag{30}$$

We can see that (30) holds for any $g \in (0,1)$. as long as $\lambda > \beta$:

$$\kappa(g) - v'(0)\psi z^p(g) = \frac{\zeta^2 g\beta\lambda(\lambda-\beta)\psi^2}{\gamma^2} > 0, \forall g \in (0,1)$$

Hence, $\kappa = \kappa(g)$ for some $g \in (0,1)$ if and only if there exists a strict NE such that $\mathbf{g}$ is the star with weighted links equal to g. Now we can see from (29) that $\kappa(g)$ is strictly decreasing in g. Note that $\kappa(1) = \bar{\kappa}$ and $\kappa(0) = \kappa^e$. Therefore, $\kappa \in (\bar{\kappa}, \kappa^e)$ if and only if $\kappa = \kappa(g)$ for some $g \in (0, 1)$, which concludes the proof. ■

Define $K_2 = (\bar{\kappa}, \kappa^e)$. If $\beta < \lambda$, then by Lemma 7, $K_2 \subseteq K$. Therefore, for any $\kappa \in K_2$, there exists a strict NE, $\mathbf{s} = (\mathbf{z}, \mathbf{g})$, with $\mathbf{g} \neq \mathbf{0}$.

For the remainder of the proof, assume $\beta < \hat{\beta} \equiv \frac{\gamma\lambda^2\psi(\gamma+\lambda\psi)}{2\gamma^2(\gamma-\lambda)+\gamma\lambda^2\psi+\lambda^3\psi^2}$. One can show by algebraic manipulation that $\hat{\beta} < \lambda$. Note then, that this assumption implies $\beta < \lambda$. Let us present one more lemma that, together with lemmas 6 and 7, directly implies that a strict NE exists for any non-negative κ and, therefore, $K = \mathbb{R}_+$.

Lemma 8. *For every $\kappa \in [0, \bar{\kappa}]$ there exists $n_C \in \{1, \ldots, n\}$ and a strict NE $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ such that $\mathbf{g}$ is a complete core-periphery graph with n_C banks in the core and no weighted links (i.e. $g_{i,j} \in \{0, 1\}, \forall i, j \in N$). Therefore, if $\kappa \in [0, \bar{\kappa}]$, then $\kappa \in K$.*

Proof. In any non-empty equilibrium network with no weighted links, $g_{i,j} = g_{j,i} = 1$ for every $i, j \in C(\bar{\mathbf{g}})$. Let us consider first $n_C \in \{1, \ldots, n-1\}$. Define $z_C(n_C)$ as the the level of liquidity that every core bank holds in an equilibrium with n_C core banks and no weighted links. We can explicitly solve for $z_C(n_C)$ in (5):

$$z_C(n_C) = \frac{\zeta\beta}{\gamma} - \left(\frac{\lambda}{\gamma}\right)\psi(n_C - 1)z_C(n_C) \Rightarrow z_C(n_C) = \frac{\zeta\beta}{\gamma + (n_C - 1)\lambda\psi}. \tag{31}$$

The liquidity holdings of a peripheral bank as function of n_C, $z_P(n_C)$, can be written as:

$$z_P(n_C) = \frac{\zeta\beta}{\gamma} - \left(\frac{\lambda}{\gamma}\right)\psi n_C z_C(n_C) \Rightarrow z_P(n_C) = z_C(n_C)\left(1 - \frac{\lambda\psi}{\gamma}\right). \tag{32}$$

Let $y_C(n_C) = \psi(n_C - 1)z_C(n_C)$ and $y_P(n_C) = \psi n_C z_C(n_C)$ be the liquidity accessed through the network by a core and peripheral bank, respectively, and note that $y_P > y_C$.

For every n_C, each bank extends a link to every core bank. Because core banks hold more liquidity than peripheral banks and the value function $v(.)$ is increasing, the only relevant deviations for each bank i is to either create links to peripheral banks, which increases both y_i and the total cost of linking, or reduce the weight of the existing links with the core, which decreases y_i and the total cost of linking. If for every agent these two deviations are not weakly profitable, then so is every other possible deviation. Consider first creating links to peripheral banks. Since $v(.)$ is strictly concave, the marginal benefit of an increase in y_i is higher if i is a core bank than if it is a peripheral bank, given that $y_P > y_C$. Therefore, we only need to check that the marginal benefit of creating a link for a core bank is not higher than the marginal cost, which yields the following condition:

$$v'(y_C(n_C))\psi z_P(n_C) \leq \kappa. \tag{33}$$

Consider now a bank reducing the weights of the existing links with the core. From the strict concavity of $v(.)$, we have that the marginal loss of a decrease in y_i is lower if i belongs to the periphery. Hence, we need the marginal loss of reducing a link for a peripheral bank to be higher than the marginal reduction in cost:

$$v'(y_P(n_C))\psi z_C(n_C) \geq \kappa. \tag{34}$$

Define $v'(y_C(n_C))\psi z_P(n_C) \equiv \underline{\kappa}(n_C)$ and $v'(y_P(n_C))\psi z_C(n_C) \equiv \bar{\kappa}(n_C)$. Since $v(.)$ is strictly increasing and in equilibrium every bank i chooses a positive z_i, we have $\bar{\kappa}(n_C) > 0$ and $\underline{\kappa}(n_C) > 0$, $\forall n_C$. Replacing (17) and (31), these expressions are given by:

$$\begin{aligned}\bar{\kappa}(n_C) &= \frac{\zeta^2\beta\lambda\psi(\gamma(\gamma+(n_C-1)\lambda\psi)-\beta(\gamma+\gamma n_C\psi-\lambda\psi))}{\gamma(\gamma+(n_C-1)\lambda\psi)^2}\\ \underline{\kappa}(n_C) &= \frac{\zeta^2\beta\lambda\psi(\gamma-\lambda\psi)(\gamma-\beta+(n_C-1)(\lambda-\beta)\psi)}{\gamma(\gamma+(n_C-1)\lambda\psi)^2}\end{aligned} \tag{35}$$

It is clear from (33) and (34) that, for a complete core-periphery graph with n_C banks in the core and no weighted links to be part of a strict NE for some $\kappa > 0$, we need $\bar{\kappa}(n_C) \geq \underline{\kappa}(n_C)$.

We can check from (35) that this is indeed the case:

$$\bar{\kappa}(n_C) - \underline{\kappa}(n_C) = \frac{\zeta^2 \beta \lambda \psi^2 (\lambda - \beta)}{\gamma(\gamma + (n_C - 1)\lambda\psi)} > 0.$$

Define $K(n_C) = [\underline{\kappa}(n_C), \bar{\kappa}(n_C)]$. If $\kappa \in \bigcup_{n_C=1}^{n-1} K(n_C)$, then there exists a $n_C \in \{1, \ldots, n-1\}$ and a strict NE $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ such that $\mathbf{g}$ is a complete core-periphery graph with n_C banks in the core and no weighted links. Next, we show the existence of $\underline{\kappa}$ such that $[\underline{\kappa}, \bar{\kappa}] = \bigcup_{n_C=1}^{n-1} K(n_C)$. First, we state that $\underline{\kappa}(n_C)$ and $\bar{\kappa}(n_C)$ are both decreasing in n_C. To see this, note that (31) implies $z_C(n_C)$ is decreasing in n_C. From (32), we see that this in turn implies $z_P(n_C)$ is decreasing in n_C. Since $z_C(n_C)$ and $z_P(n_C)$ decrease with n_C, it follows from the best response function that $y_C(n_C)$ and $y_P(n_C)$ are both increasing in n_C. Because $v(.)$ is concave, $v'(y_C(n_C))$ and $v'(y_P(n_C))$ are also decreasing in n_C. Hence, $v'(y_C(n_C))\psi z_P(n_C)$ and $v'(y_P(n_C))\psi z_C(n_C)$ are both decreasing in n_C. Now set $\underline{\kappa} = \underline{\kappa}(n-1)$ and note that $\bar{\kappa}(1) = \bar{\kappa}$. It is straightforward that $\bigcup_{n_C=1}^{n-1} K(n_C) \subseteq [\underline{\kappa}, \bar{\kappa}]$. For the converse to hold, a sufficient condition is:

$$K(n_C) \bigcap K(n_C - 1) \neq \emptyset, \forall n_C \in \{2, \ldots, n-1\}. \tag{36}$$

i.e. adjacent intervals overlap. Since $\underline{\kappa}(.)$ and $\bar{\kappa}(.)$ are decreasing functions, we can state condition (36) as:

$$\bar{\kappa}(n_C) \geq \underline{\kappa}(n_C - 1), \forall n_C \in \{2, \ldots, n-1\}. \tag{37}$$

To see that this condition is sufficient, take any $\kappa \in [\underline{\kappa}, \bar{\kappa}]$. If $\kappa > \underline{\kappa}(1)$, then $\kappa \in K(1) \Rightarrow \kappa \in \bigcup_{n_C=1}^{n-1} K(n_C)$. Otherwise, there exists $n'_C \in \{2, \ldots, n-1\}$ such that $\underline{\kappa}(n'_C) \leq \kappa \leq \underline{\kappa}(n'_C - 1)$. From (37), $\bar{\kappa}(n'_C) \geq \underline{\kappa}(n'_C - 1)$. Therefore, $\kappa \in K(n'_C) \Rightarrow \kappa \in \bigcup_{n_C=1}^{n-1} K(n_C)$.

Replace (35) in (37). We can then isolate β such that (37) holds if and only if:

$$\beta \leq B(n_C),\ \forall n_C, \tag{38}$$

where:

$$B(n_C) \equiv \frac{\lambda^2\psi(n_C-1)(\gamma+(n_C-2)\lambda\psi)(\gamma+(n_C-1)\lambda\psi)}{2\gamma^3+\gamma^2\lambda(3(n_C-2)\psi-2)+\lambda^3\psi^2(2n_C-3+(n_C-2)(n_C-1)^2\psi)+\gamma\lambda^2\psi(5-2n_C+(n_C-2)(2n_C-3)\psi)}$$

One can check by taking the derivative with respect to n_C that $B(n_C)$ is strictly increasing in n_C. Therefore, (38) is satisfied $\forall n_C \in \{2, \dots, n-1\}$, as long as it holds for $n_C = 2$. This yields the following condition for β:

$$\beta \leq \frac{\gamma\lambda^2\psi(\gamma+\lambda\psi)}{2\gamma^2(\gamma-\lambda)+\gamma\lambda^2\psi+\lambda^3\psi^2} \tag{39}$$

(39) holds by assumption and, therefore, $[\underline{\kappa}, \bar{\kappa}] \subseteq \bigcup_{n_C=1}^{n-1} K(n_C)$.

Furthermore, consider the case when $n_C = n$. Denote by $z^c \equiv z_C(n)$ the level of liquidity that every bank holds in the complete network and by $y^c = \psi(n-1)z^c$ the liquidity each bank accesses through the network. In the complete network, we only need to check that a bank does not find it weakly profitable to reduce the weight of the links it extends to other banks. From the strict concavity of $v(.)$, this is true as long as the marginal loss of reducing a link is weakly higher than the marginal reduction in cost:

$$v'(y^c)\psi z^c \geq \kappa.$$

Set $\kappa^c \equiv v'(y^c)\psi z^c = \frac{\zeta^2\beta\lambda\psi(\gamma-\beta+(n-1)(\lambda-\beta)\psi)}{(\gamma+(n-1)\lambda\psi)^2}$. Since $\gamma > \lambda > \beta$, we can see that κ^c is strictly positive. Next, we show that $\underline{\kappa} < \kappa^c$. Recall that we have shown $\bar{\kappa}(n_C) - \underline{\kappa}(n_C - 1) \geq 0, \forall n_C \in \{2, \dots, n-1\}$. Note that this holds for any $n_C \geq 2$. Of course, $\bar{\kappa}(.)$ and $\underline{\kappa}(.)$ only have a meaningful interpretation if $n_C \in \{2, \dots, n-1\}$. But we can set $n_C = n$ and

$\bar{\kappa}(n) - \underline{\kappa}(n-1) \geq 0$ still holds. We then have:

$$\kappa^c - \underline{\kappa} = \kappa^c - \underline{\kappa}(n-1) = (\kappa^c - \bar{\kappa}(n)) + (\bar{\kappa}(n) - \underline{\kappa}(n-1)) \geq$$

$$\kappa^c - \bar{\kappa}(n) = \frac{(\zeta\beta\psi)^2\lambda(\gamma-\lambda)}{\gamma(\gamma+(n-1)\lambda\psi)^2} > 0$$

Hence, $[0, \kappa^c] \cup [\underline{\kappa}, \bar{\kappa}] = [0, \bar{\kappa}]$, which concludes the proof. ■

In order to prove that $K = \mathbb{R}_+$, note first that $K \subseteq \mathbb{R}_+$ by definition. Next, define $K_3 = [0, \bar{\kappa}]$. By Lemma 8, $K_3 \subseteq K$. Recall that, by lemmas 6 and 7, $K_1 = [\kappa^e, \infty)$ and $K_2 = (\bar{\kappa}, \kappa^e)$ are also contained in K. Hence, $\bigcup_{i=1}^3 K_i \subseteq K$. Lastly, note that $\bigcup_{i=1}^3 K_i = \mathbb{R}_+$, which concludes the proof. ■

A.5 Proposition 4

Assume $\beta < \lambda$. Then a strict NE $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ exists for some $\mathbf{g} \in \mathbf{g}^s$ if and only if $\kappa \in (\kappa_1, \kappa_0)$. Moreover:

- *All peripheral banks extend a weighted link to the center and the weight is the same across banks.*
- *The equilibrium link weight $g^* \in (0,1)$ satisfies $\kappa = v'(\psi g^* z^s)\psi z^s$, where $z^s = \frac{\zeta\beta}{\gamma}$ is the liquidity holding of the center.*
- *$\mathbf{s}$ is the unique strict NE.*

Proof. Note that $\kappa_0 \equiv v'(0)\psi z^s = \kappa^e$ and $\kappa_1 \equiv v'(\psi z^s)\psi z^s = \bar{\kappa}$. Then it follows directly from Lemma 7 that $\mathbf{s} = (\mathbf{z}, \mathbf{g})$ is a strict NE for some $\mathbf{g} \in \mathbf{g}^s$ if and only if $\kappa \in (\kappa_1, \kappa_0)$. The fact that peripheral banks extend links with equal weights to the center follows directly from Lemma 3. Now, to obtain the condition in g^*, consider the value function of a peripheral bank when $\mathbf{g} \in \mathbf{g}^s$:

$$V(y_i, \mathbf{g}) = v(\psi g z^s) - \kappa g \tag{40}$$

Take the derivative of (40) with respect to g and set it equal to zero. This yields the following condition:

$$v'\left(\psi g^* z^s\right)\psi z^s = \kappa. \tag{41}$$

To show that $\mathbf{s}$ is unique, assume to the contrary that there exists $\mathbf{s}$' = $(\mathbf{z}', \mathbf{g}') \neq \mathbf{s}$ such that $\mathbf{s}$' is also a strict NE. Let us consider three different cases which cover all possible cases. First, assume $\mathbf{g}'$ is the star network with weighted links equal to $g' \in [0, g^*)$. Note that if $g' = 0$, then $\mathbf{g}'$ is the empty network. From (40), we must have:

$$v'\left(\psi g' z^s\right)\psi z^s \leq \kappa.$$

From the strict concavity of $v(.)$ and $g' < g^*$ it follows that $v'\left(\psi g^* z^s\right)\psi z^s < v'\left(\psi g' z^s\right)\psi z^s \leq \kappa$, which contradicts (41). Next, assume $\mathbf{g}'$ is the star network with weighted links equal to $g' \in (g^*, 1]$. Note that if $g' = 1$, then $\mathbf{g}'$ is the star network with binary links only. From (40), we must have:

$$v'\left(\psi g' z^s\right)\psi z^s \geq \kappa.$$

From the strict concavity of $v(.)$ and $g' > g^*$ it follows that $v'\left(\psi g^* z^s\right)\psi z^s > v'\left(\psi g' z^s\right)\psi z^s \geq \kappa$, which contradicts (41).

Lastly, assume $\mathbf{g}'$ is a complete core-periphery graph with at least two banks in the core. Then, there exists $i \in N$ such that i extends at least two links. By Lemma 1, at least one of these links has weight one. Take player j such that $g_{i,j} = 1$. By (5), $z_j \leq z^s$. For $g_{i,j} = 1$ to be part of a strict NE, it must be that:

$$v'(y_i)\psi z_j \geq \kappa.$$

From the strict concavity of $v(.)$, we then have $v'(\psi z_j)\psi z_j > v'(y_i)\psi z_j \geq \kappa$. If $v'(\psi z_j)\psi z_j$ is increasing in z_j, then we have a contradiction. To see this, note that this implies

$v'(\psi z^s)\,\psi z^s > \kappa$, which contradicts (41). Let us then show that $v'(\psi z_j)\psi z_j$ is indeed increasing in z_j. We start by substituting $v'(.)$ for (17) in $v'(\psi z_j)\psi z_j$ to obtain:

$$\frac{\lambda\psi z_j(\zeta(\gamma-\beta)-\psi z_j(\gamma-\lambda))}{\gamma}. \tag{42}$$

Differentiating (42) with respect to z_j then yields:

$$\frac{\lambda\psi(\zeta(\gamma-\beta)-2\psi z_j(\gamma-\lambda))}{\gamma}. \tag{43}$$

It is sufficient to check that the above expression is positive for all $z_j \in [0, z^s]$. Note that (43) is strictly decreasing in z_j since $\gamma > \lambda$. Hence, it is sufficient to check that (43) is positive for $z_j = z^s$. Replace z_j for z^s in (43). We then have:

$$\frac{\zeta\lambda\psi\left(\gamma^2-\beta(\gamma+2\gamma\psi-2\lambda\psi)\right)}{\gamma^2}.$$

One can show by algebraic manipulation that the above expression is strictly positive given $\beta < \lambda$ and $\gamma > 2\lambda = \frac{\gamma^2}{\bar{c}}$.[28] ■

A.6 Proposition 5

For each θ, there exists a $\bar{n} \in \mathbb{R}$ such that, for all $n > \bar{n}$, the loss function, $L(\gamma)$, is decreasing in the size of the corridor at $\gamma = \delta$. Moreover, the optimal corridor width, $\gamma^*(\Phi, \theta)$, is then strictly larger than the minimum corridor width, δ. **Proof.** Recall that Φ is such that $\kappa_0(\delta) > \kappa > \kappa_1(\bar{c})$ holds. We begin by deriving the functional form of $Z(\gamma) \equiv \sum_{i=1}^{n} z_i(\gamma, \Phi)$ when $\mathbf{g} \in \mathbf{g}^s$. Recall $\lambda \equiv \frac{\gamma^2}{2\bar{c}}$ and condition (11), i.e. $\beta = \frac{\gamma}{2} - \eta$. The liquidity holding of the bank at the center of the star is $z^s = \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)$. Moreover, there are $n-1$ peripheral banks that access $\psi g^* z^s$ of liquidity. Therefore, the liquidity holding of a bank in the periphery is $z^p \equiv \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)\left(1 - \frac{\gamma\psi g^*}{2\bar{c}}\right)$. By Proposition 4, g^* solves $v'(\psi g^* z^s)\,\psi z^s = \kappa$. Solving for g^*,

[28] The calculations were executed in Mathematica and are available from the author.

we obtain:

$$g^* = \frac{2\bar{c}(\zeta^2\psi(\gamma^2 - 4\eta^2) - 8\bar{c}\kappa)}{\zeta^2\psi^2(2\bar{c} - \gamma)(\gamma - 2\eta)^2}. \tag{44}$$

Condition $\kappa_0(\delta) > \kappa > \kappa_1(\bar{c})$ guarantees that $g^* \in (0,1)$, $\forall\, \gamma \in [\delta, \bar{c}]$. Replacing (44) in z^p, we get:

$$z^p = \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)\left(1 - \frac{\gamma(\zeta^2\psi(\gamma^2 - 4\eta^2) - 8\bar{c}\kappa)}{\zeta^2\psi(2\bar{c} - \gamma)(\gamma - 2\eta)^2}\right).$$

$Z(\gamma)$ then writes:

$$Z(\gamma) = z^s + (n-1)z^p = \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)\left(n - \frac{(n-1)\gamma(\zeta^2\psi(\gamma^2 - 4\eta^2) - 8\bar{c}\kappa)}{\zeta^2\psi(2\bar{c} - \gamma)(\gamma - 2\eta)^2}\right).$$

From (8) we have the functional form of the interbank rate variance $\frac{\gamma^2}{12}$. We then obtain the functional form of the loss function:

$$L(\gamma) =$$

$$\theta\frac{\gamma^2}{12} + (100-\theta)\zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)\left(n - \frac{(n-1)\gamma(\zeta^2\psi(\gamma^2 - 4\eta^2) - 8\bar{c}\kappa)}{\zeta^2\psi(2\bar{c} - \gamma)(\gamma - 2\eta)^2}\right). \tag{45}$$

Taking the derivative of $L(\gamma)$, evaluating at $\gamma = \delta$ and collecting terms with respect to n yields:

$$L'(\delta) = A_2(\Phi, \theta) - A_1(\Phi, \theta)\, n,$$

where the expressions for $A_1(\Phi,\theta)$ and $A_2(\Phi,\theta)$ are given by:

$$A_1(\Phi,\theta) \equiv \frac{\bar{c}(100-\theta)\left(\zeta^2\psi(\delta - 2\eta)^2\left(\delta^2 - 4\eta(\bar{c} - \delta)\right) + 8\delta^2\kappa(\bar{c} - \delta + \eta)\right)}{\delta^2\zeta\psi(2\bar{c} - \delta)^2(\delta - 2\eta)^2}$$

$$A_2(\Phi,\theta) \equiv \frac{\theta\left(\delta(2\bar{c} - \delta)^2 - 6\zeta(\bar{c} + \eta)\right) + 600\zeta(\bar{c} + \eta) + \frac{48(100-\theta)\bar{c}\kappa(\bar{c} - \delta + \eta)}{\zeta\psi(\delta - 2\eta)^2}}{6(2\bar{c} - \delta)^2}$$

To show the existence of $\bar{n}$ such that $L'(\delta) < 0$, it is sufficient to show that $A_1(\Phi,\theta) > 0$, $\forall\, \Phi, \theta$. From the functional form of $A_1(\Phi,\theta)$, it is sufficient to check that $\delta^2 - 4\eta(\bar{c} - \delta) > 0 \Leftrightarrow \delta^2 > 4\eta(\bar{c} - \delta)$. This, in turn, holds due to the condition that $\sqrt{\frac{\bar{c}^2 + 4\eta\bar{c} - 4\eta^2}{2}} < \delta \Leftrightarrow$

$\delta^2 > \frac{\bar{c}^2+4\eta\bar{c}-4\eta^2}{2}$. To see this, let us show that $\left(\frac{\bar{c}^2+4\eta\bar{c}-4\eta^2}{2}\right) > 4\eta(\bar{c}-\delta)$. Start by subtracting $4\eta(\bar{c}-\delta)$ from both sides of the inequality:

$$\left(\frac{\bar{c}^2+4\eta\bar{c}-4\eta^2}{2}\right) - 4\eta(\bar{c}-\delta) = \frac{1}{2}\left(\bar{c}^2 - 4\eta^2 - 4\eta(\bar{c}-2\delta)\right) > 0,$$

where the last inequality holds since $\bar{c} > \delta > 2\eta > 0$. Define $\bar{n} \equiv \frac{A_2(\Phi,\theta)}{A_1(\Phi,\theta)}$. If $n > \bar{n}$, we have $L'(\delta) < 0$. Since $L(\gamma)$ is continuously differentiable, there exists $\gamma' > \delta$ such that $L'(\gamma) < 0, \forall\, \gamma \in [\delta, \gamma')$. Hence, for some $\tilde{\gamma} \in (\delta, \gamma')$, $L(\tilde{\gamma}) < L(\delta)$, which implies $\gamma^*(\Phi,\theta) > \delta$. ■

A.7 Proposition 6

If $\zeta' \geq \zeta$, then the optimal corridor width increases, i.e. $\gamma^*(\Phi_{\zeta'},\theta) \geq \gamma^*(\Phi,\theta)$. If $\bar{c}' \leq \bar{c}$ and $\bar{c}'$ is not lower than the original optimal corridor width, i.e. $\bar{c}' \geq \gamma^*(\Phi,\theta)$, then the optimal corridor width increases, i.e. $\gamma^*(\Phi_{\bar{c}'},\theta) \geq \gamma^*(\Phi,\theta)$. **Proof.** Note first that the constraint

$\gamma \in [\delta, \bar{c}]$ does not depend on ζ. Therefore, to show the first result it is sufficient to check that $L(.)$ has decreasing differences in (γ,ζ) (Topkis's Theorem). Starting from (45), which presents the functional form of $L(\gamma)$ when $\kappa_0(\delta) > \kappa > \kappa_1(\bar{c})$, we can see that $L(\gamma)$ is twice continuously differentiable with respect to γ, ζ. Therefore, it is sufficient to check that $\frac{\partial^2 L}{\partial\gamma\,\partial\zeta} \leq 0$. Taking the cross derivative with respect to γ, ζ yields:

$$\frac{(100-\theta)\left(4n\bar{c}^2\eta - 4n\gamma\bar{c}\eta - \gamma^2(\bar{c}(n-1)-\eta)\right)}{\gamma^2(2\bar{c}-\gamma)^2}. \qquad (46)$$

One can show by algebraic manipulation that (46) is negative given $\kappa_0(\delta) > \kappa > \kappa_1(\bar{c})$.[29]

Moreover, to show the second result, consider first the constraint in (12), $\gamma \in [\delta, \bar{c}]$. The set $[\delta, \bar{c}]$ is not increasing in $\bar{c}$. Therefore, even if $L(.)$ has increasing differences in $(\gamma, \bar{c})$, we cannot state that $\gamma^*(\Phi,\theta)$ will increase for arbitrary decreases in $\bar{c}$. Let us then consider

[29] The calculations were executed in Mathematica and are available from the author.

decreases in $\bar{c}$ such that $\bar{c} \geq \bar{c}' \geq \gamma^*(\Phi, \theta)$. We write the loss function as $L(\gamma, \bar{c})$ to make it explicit that it depends on $\bar{c}$. By definition, $\gamma^*(\Phi_{\bar{c}'}, \theta) = argmin_\gamma\{L(\gamma, \bar{c}') \,|\, \bar{c}' \geq \gamma \geq \delta\}$ and $\gamma^*(\Phi, \theta) = argmin_\gamma\{L(\gamma, \bar{c}) \,|\, \bar{c} \geq \gamma \geq \delta\}$. Since $\bar{c}' \geq \gamma^*(\Phi, \theta)$, we have that $\gamma^*(\Phi, \theta) = argmin_\gamma\{L(\gamma, \bar{c}) \,|\, \bar{c}' \geq \gamma \geq \delta\}$. Notice then that we do not need to consider the impact that the reduction in $\bar{c}$ has in the constraint of (12) when comparing $\gamma^*(\Phi, \theta)$ to $\gamma^*(\Phi_{\bar{c}'}, \theta)$. Therefore, for changes in $\bar{c}$ such that $\bar{c} \geq \bar{c}' \geq \gamma^*(\Phi, \theta)$, it is sufficient to check that $L(.)$ has increasing differences in $(\gamma, \bar{c})$. To show this, it is sufficient that $\frac{\partial^2 L}{\partial\gamma\,\partial\bar{c}} \geq 0$. Taking the cross derivative with respect to $\gamma, \bar{c}$ yields:

$$\frac{(100-\theta)(n-1)\left(\zeta^2\psi(\gamma-2\eta)^2(\gamma+2\bar{c}+4\eta)+8\eta\kappa(\gamma+2\bar{c})-8\gamma^2\kappa\right)}{\zeta\psi(\gamma-2\eta)^2(2\bar{c}-\gamma)^3}. \tag{47}$$

One can show by algebraic manipulation that (47) is positive given $\kappa_0(\delta) > \kappa > \kappa_1(\bar{c})$.[30] ■

A.8 Proposition 7

Proposition 7. *Define $\hat{Z}(\gamma, g)$ and $\hat{\gamma}^*(\Phi, \theta, g)$, respectively, as total liquidity holdings and the solution to (12) when the equilibrium network* $\mathbf{g}$ *is a star with fixed links of weight g. If $g < \frac{4\eta}{\bar{c}\psi}$, then $\hat{Z}(\gamma, g)$ is strictly increasing in γ. Therefore, $\hat{\gamma}^*(\Phi, \theta, g) = \delta$, $\forall\, \Phi, \theta$.*

Proof. We begin by deriving the functional form of $\hat{Z}(\gamma, g)$. The liquidity holding of the bank at the center of the star is $\hat{z}^s = \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)$. Moreover, there are $n-1$ peripheral banks that access $\psi g \hat{z}^s$ of liquidity. Therefore, the liquidity holding of a bank in the periphery is $\hat{z}^p \equiv \zeta\left(\frac{1}{2} - \frac{\eta}{\gamma}\right)\left(1 - \frac{\gamma\psi g}{2\bar{c}}\right)$, where g is fixed. We can see that $\hat{z}^s$ is strictly increasing in γ. Let us show that this is also the case for $\hat{z}^p$. Taking the derivative of $\hat{z}^p$ with respect to γ yields:

$$\frac{\zeta\eta}{\gamma^2} - \frac{g\zeta\psi}{4\bar{c}}.$$

For the expression above to be strictly positive for all $\gamma \in [\delta, \bar{c}]$, it is sufficient to check it for

[30] The calculations were executed in Mathematica and are available from the author.

$\gamma = \bar{c}$, which yields the condition:

$$g < \frac{4\eta}{\bar{c}\psi}.$$

Hence, if $g < \frac{4\eta}{\bar{c}\psi}$, then $\hat{z}^p$ is strictly increasing in γ. This in turn implies that $\hat{Z}(\gamma, g)$ is also strictly increasing in γ. It is then straightforward that the solution that minimizes $\hat{L}(\gamma) = \theta\frac{\gamma^2}{12} + (100 - \theta)\hat{Z}(\gamma, g)$ is $\hat{\gamma}^*(\Phi, \theta, g) = \delta$ since $\hat{L}'(\gamma) > 0, \forall \gamma \in [\delta, \bar{c}]$.

■