Trading choices*

Lucas B. Dyskant
Barclays Corporate & Investment Bank

André C. Silva
Nova SBE

Bruno Sultanum
University of Essex

January 2025

Abstract

We propose a model to study the choice between three ways of trading in over-the-counter asset markets: principal inventory trades, agency risk-free trades, or all-to-all (A2A) trades. Principal and agency trades occur through dealers. A2A trades occur without dealers, as direct customer-customer transactions. We show the possibility of multiple equilibria. Equilibrium changes may lead to substantial changes in dealer intermediation. Increased agency trading is compatible with increased activity in A2A markets. The model explains three facts about corporate bond markets following the Dodd-Frank Act: (1) an increase in A2A and agency trades, (2) longer trade execution, and (3) a decrease in measured illiquidity.

JEL classification: D53, G12, G18, G28.

Keywords: over-the-counter markets, intermediation costs, liquidity, corporate bond markets, financial market regulations, post-2008 regulations, Volcker rule.

March 5, 2025. Updates in this link.

*The views in this paper are those of the authors and not necessarily the views of the Federal Reserve Bank of Richmond, the Federal Reserve System, or Barclays Corporate & Investment Bank. Dyskant participated in this work in his personal capacity and not as part of his employment with Barclays. We benefited from discussions with Paco Buera, Briana Chang, Veronica Guerrieri, Zhiguo He, Yesol Huh, Ben Lester, Dmitry Orlov, Sebastien Plante, Erwan Quintin, Andreas Rapp, Philip Reny, Roberto Robatto, Sang Seo, Cathy Zhang and participants at seminars and conferences at the Wisconsin School of Business, Miami Business School, Purdue University, the IMF, the Board, the Philadelphia Fed, and the St. Louis Fed. Silva thanks the hospitality of the Kenneth C. Griffin Department of Economics at the University of Chicago, where part of this article was written. This work was partially funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2019, UIDB/00124/2020 and Social Sciences DataLab, PINFRA/22209/2016), POR Lisboa and POR Norte (Social Sciences DataLab, PINFRA/22209/2016).

1 Introduction

We introduce a model to study the composition of trades in over-the-counter markets. We consider three trading mechanisms: principal inventory trades, agency risk-free trades, and all-to-all (A2A) customer-customer trades. In the model, principal and agency trades are both executed through dealers, while in A2A trades, customers trade directly with other customers. Trading choices made by customers and dealers influence the frequency of each trading mechanism. Transaction fees, search costs, and other parameters affect these choices. The model helps analyze how regulatory and technological changes impact financial markets, particularly over-the-counter markets such as corporate bond markets.

In the model, dealers face different intermediation costs when providing on-the-spot liquidity using their inventory in principal trades and when facilitating trading by matching customers in agency trades. Customers can also avoid trading with dealers and trade directly with other customers in A2A markets. A customer finds a dealer or another customer according to a search and matching model with elements of Lagos and Rocheteau (2009) and Hugonnier, Lester, and Weill (2022).

When the intermediation costs of principal trade increase relative to the intermediation costs of other trading mechanisms, customers accept to wait longer until they are matched with another customer with the assistance of the dealer, or to wait until they find another customer in A2A markets. The intermediation cost creates a segmentation of the corporate bond market into A2A trades and customer-dealer trades subdivided into inventory trades and agency trades. Agency trades represent customer liquidity provision as described by Choi et al. (2024). In agency and A2A trades, customers pay low intermediation fees, but face high search costs. They might pay a smaller price for the asset (or sell for a higher price), but they take longer to trade. In principal inventory trades, customers are willing to pay high intermediation costs for immediacy.

We find that an increase in intermediation costs can lead to multiple equilibria. Equilibrium multiplicity arises because customers need to predict the probability of matching with a dealer or another customer based on their trading choice. They cannot coordinate their search. For the same parameters, one equilibrium may feature a small agency market while another may feature a large agency market. The benefits of coordination increase as intermediation costs rise. We show that small intermediation costs rule out equilibrium multiplicity. However, an increase in principal or agency intermediation fees raises the likelihood of multiple equilibria.

We show that we can have different market configurations. We can have a large principal market and small agency and A2A trades, a common configuration before 2010; and a smaller principal market but larger agency market, as we have today. An increase in agency trade does not necessarily imply an increase in A2A trades. Therefore, we can have a stable fraction of A2A trades together with a faster increase in agency trade, as has been observed since the introduction of A2A trading platforms. In addition to intermediation costs, the markets can move according to search-intensity, bargaining, and other parameters. We interpret search and bargaining parameters changes as being approximately the result of technological changes whereas the intermediation costs and being approximately the result of regulatory changes.

An ingredient of the model is the ability of investors to choose to participate in customer-dealer or customer-customer matches. In the spirit of Guerrieri et al. (2010), agents direct their search in financial markets. An increase in dealer costs makes customers look for other customers to trade, which represents customer liquidity provision intermediated by dealers. For a large enough increase in dealer costs, this change in composition decreases the aggregate bid-ask spread in equilibrium. Standard indicators of illiquidity rely on observed transactions. Therefore, a decrease in the aggregate bid-ask spread implies an improvement in standard indicators of liquidity. The model then generates a change in the structure of the corporate bond market together with improvement of indicators of market illiquidity.

The model allows us to propose a new measure of illiquidity. The measure obtained from the model takes into account equilibrium prices, search frictions, and the fraction of the market that engages in inventory trade or customer liquidity provision.

After the 2008 financial crisis, several regulations were enacted with the objective of avoiding future financial crises. In particular, the Dodd-Frank Wall Street Reform and Consumer Protection Act in 2010. To some extent, the Dodd-Frank Act and similar regulations in other countries accomplished their goal, as there was not a comparable financial crisis after 2008. However, the focus of academics, practitioners and government officials turned to how such regulations affect financial markets in normal times, when the economy is not under distress. There are indications that the form in which trades take place in over-the-counter markets has changed after the regulations were put in place.[1]

The improvement in the traditional measures of market liquidity after the 2008 financial crisis has been documented by Bessembinder et al. (2018). We confirm this finding with recent data for the BPW and Amihud liquidity measures (respectively after Bao, Pan, and Wang 2011 and Amihud 2002). The improvement in measured market liquidity could suggest that regulations had a minor impact on financial market liquidity. However, Bessembinder et al. (2018) and Choi et al. (2024) report a decrease in dealer trade frequency. Especially, Choi et al. (2024) indicate a change in the composition of the provision of liquidity. The provision of liquidity has increasingly been made by customers rather than dealers. In practice, there is a perceived movement of customers from principal trade to A2A trade or agency trade.[2]

The model is able to explain changes on the corporate bond market after the Dodd-Frank Act. The Act includes the Volcker rule, which prohibits institutions from trading that uses the inventory of assets purchased earlier with the intention of profiting from a higher sale price (proprietary trade). In the context of the model, we interpret the introduction of the Volcker rule as an increase in costs of principal trade. According to the model, the increase in principal cost implies (1) the increase in agency trades or A2A trades; (2) longer trade executions; and (3) the decrease in measured illiquidity, even though market participants indicate more difficulty in trading. These predictions are in line with the observations on the corporate bond market after the post-2008 regulations.

The model builds on Duffie, Gârleanu, and Pedersen (2005), Lagos and Rocheteau (2009) (LR) and Hugonnier, Lester, and Weill (2022) (HLW). Depending on the parameters, it implies LR or HLW as particular cases. Time is continuous. There is an asset that pays dividends over time. There are two types of traders: customers and dealers. Dealers have access to a competitive interdealer market where the asset is traded at an equilibrium price. Customers trade with dealers or with other customers. There are search frictions when customers search

[1] During a 2015 congressional hearing, for example, Rep. French Hill questioned the Federal Reserve Chair at the time, Janet Yellen, on whether regulations were to blame for the deterioration of liquidity on different bond markets. Yellen replied: “I am not ruling out the possibility that regulations could play a role here, it is simply we have not been able to understand through a lot of different factors and we need to look at it more to sort out just what is going on and what the different influences are, but I am not ruling that out.”

[2] We obtain similar findings on the importance of illiquidity for the yield spreads as Li and Yu (2023) and Wu (2023), which worked in independent papers. Our empirical results are different in some aspects (such as our focus on the BPW and Amihud measures) and they complement their findings. However, our focus is on the model to explain the movements in the corporate bond market. We discuss in more detail the liquidity premium in Dyskant et al. (2025).

for a trade counterparty. These search frictions are different for finding another dealer or customer.

Each asset has a stochastic maturity date and a stochastic issue opportunity. Customers are heterogeneous in the valuation of the asset. The heterogeneity in asset valuation implies gains to trade. Assets trade hands over time. We characterize the stationary distribution of asset holdings as well as the equilibrium bid-ask spreads. We show that the empirical behavior of market illiquidity can be rationalized by the model when we interpret the Dodd-Frank act as causing an increase in intermediation costs.

The increase in the intermediation costs of dealers increases the equilibrium bid-ask spread. Customers that do not have the asset but have a high valuation of it still trade with dealers. They pay a high ask price because they want to find a trade counterparty fast. Similarly, customers that have the asset but have a low valuation of it accept a lower bid price from dealers to sell the asset fast. On the other hand, customers with intermediary valuations of the asset avoid trading with dealers. They wait to be matched with other customers to avoid the surcharge in the form of large bid-ask spreads. Empirically, Choi et al. (2024) find that matched customers in fact pay lower spreads. We show that an increase in trade costs increases the number of customer-customer trades. As a result, the measured bid-ask spread decreases, as well as the measured illiquidity measures.

2 Model

2.1 Environment

We model over-the-counter markets as markets in which agents take decisions under search frictions. First search models applied to OTC markets include Gehrig (1993), Spulber (1996), and Rust and Hall (2003).[3] Our model builds more directly on Duffie et al. (2005).

Agents, time, goods and assets There are two types of agents in a continuum: a measure one of customers and an infinite measure of dealers. Time is continuous and infinite. There is a unique good and an endogenous supply $s \geq 0$ of assets. A unit of the asset pays a unit flow of dividends in the form of goods, which cannot be traded. The agent holding an asset

[3] See also Cimon and Garriott (2019), Saar et al. (2023), An and Zheng (2022).

consumes its dividends. All agents are infinitely-lived, discount the future at rate $r > 0$, and have transferable utility. Customers can hold either zero or one unit of the asset. We refer to customers holding an asset as owners, and to those not holding an asset as non-owners. Dealers can hold discrete amounts i of the asset, $i \in \{-1, 0, 1\}$. We interpreted a negative dealer position $i < 0$ as either the dealer borrowing the asset to sell, a common practice for short selling, or holding a position below the target for its private account.[4]

Preferences Customers are heterogeneous in the utility ν that they derive from consuming the dividend flow of the asset. We refer to ν as the customer utility type. Types are fixed over time, common knowledge, independent across customers, and initially drawn from a given cumulative distribution F. The distribution F has support $\mathbb{R}$, and a continuous density $f > 0$. We assume that $\int \nu^2 f(\nu) d\nu < \infty$ and that there is no free disposal of assets. The assumption that the distribution of types has unbounded support is convenient because we do not have to consider corner solutions. However, we can obtain our main results with a bounded support $[\underline{\nu}, \bar{\nu}]$ if we assume that the density is sufficiently low at the extremes. Similarly, the assumption of no free disposal can be replaced with the assumption that the measure of agents with $\nu < 0$ is sufficiently low. Dealers do not derive utility from holding assets. They hold inventory only to profit from intermediation. They pay a flow cost $c^l \geq 0$ when they take a long position in bonds, that is, hold $i = 1$, and pay a flow cost $c^s \geq 0$ when they take a short position in bonds, $i = -1$.

Decentralized market There is an OTC asset market in which customers choose to search for another customer or for a dealer. Customers who find a dealer then choose between agency and principal trade. Figure 1 depicts customers' sequence of actions.

When searching for another customer, the customer finds one with Poisson arrival rate $\lambda_C/2 > 0$. This implies a meeting rate of λ_C (the two customers search for each other at $\lambda_C/2 > 0$). Customers searching for customers do not find customers searching for dealers. We interpret customer-to-customer trades as performed in all-to-all trade platforms, which

[4] We could alternatively model dealers holding inventory in order to sell the bond; a form of "inventory-in-advance" constraint proposed by Cohen et al. (2024). Both versions imply similar results because we can pick the cost of shorting the bond to match the cost of holding inventory. We keep the current version where dealers can take a negative position because it simplifies the analysis.

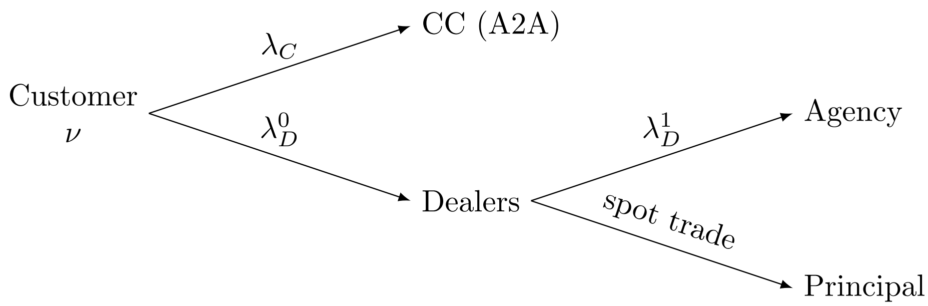


Figure 1: Customers' sequence of actions. A customer of type ν chooses to search for another customer in the customer-customer market (all-to-all trade platform), or to search for a dealer. When meeting a dealer, customers choose between agency and principal trade.

allow customers to search for a counterpart without the participation of dealers.

When searching for a dealer, the customer finds one with Poisson arrival rate $\lambda_D^0 > 0$. After the customer and the dealer meet, they can trade in two ways: risky principal trade or agency trade. In a risky principal trade, the dealer trades with the customer on its own account and then joins an interdealer market to rebalance its portfolio. In an agency or risk-free trade, the dealer joins the interdealer market on behalf of the customer and completes the transaction afterwards. An agency trade takes longer to complete because the dealer has to search in the interdealer market. In a risky principal trade, the dealer uses its own account and completes the trade on the spot. To simplify the model and notation, we assume dealers exit the economy once they intermediate a transaction. In the case of a principal trade, the dealer exits the economy after rebalancing their portfolio. In the case of an agency trade, the dealer exits the economy after completing the trade on behalf of the customer.

The interdealer market Following Lagos and Rocheteau (2009), dealers access a competitive market with Poisson arrival rate $\lambda_D^1 > 0$, where the endogenous asset price is p. Dealers pay an intermediation cost $\tau \geq 0$ when trading on behalf of customers in an agency trade. That is, the net revenue of selling an asset in an agency trade is $p - \tau$, and the cost of buying an asset in an agency trade is $p + \tau$.

Asset supply Asset issuance and maturity determine the supply of assets. Customers issue new assets at no cost with Poisson arrival rate $\eta > 0$, and an asset matures with Poisson arrival rate $\mu > 0$. The asset disappears from a portfolio and from the economy at maturity.

In an agency trade, if the customer state changes because of asset maturity or issuance, the match ends. Dealers do not issue assets, and assets in their possession do not mature. This assumption simplifies the model, but is not important for the qualitative results. Asset issuance and maturity in our model follows Bethune et al. (2022) and implies that a steady state with positive trade emerges without introducing time-varying types.

Bargaining We use Nash bargaining to model trade. In a customer-dealer trade, the customer bargaining power is $\theta_D \in [0,1]$. In a customer-customer trade, the buyer bargaining power is $\theta_C^n \in [0,1]$ and the seller bargaining power is $\theta_C^o = 1 - \theta_C^n$. We assume the following relation between the search and bargaining parameters.

Assumption 1. *Dealers' search technology is sufficiently better than customers' search technology. That is,* $\frac{\lambda_D^0 \lambda_D^1 \theta_D}{r+\mu+\eta+\lambda_D^0+\lambda_D^1} > \lambda_C \max\{\theta_C^o, \theta_C^n\}$.

Assumption 1 implies that customers are better off searching for dealers when dealers face no principal trade cost ($c^s = c^l = 0$), or no agency trade cost ($\tau = 0$).

2.2 Value functions and reservation value

We first specify the value function of customers as functions of the search choices. When searching for dealers, the value function is $V_D^o(\nu)$ for owners, and $V_D^n(\nu)$ for non-owners. When searching for customers, the value function is $V_C^o(\nu)$ for owners, and $V_C^n(\nu)$ for non-owners. The value function yields the maximum between the two trading choices,

$$V^o(\nu) = \max\{V_D^o(\nu), V_C^o(\nu)\}, \text{ for owners,} \tag{1}$$

$$V^n(\nu) = \max\{V_D^n(\nu), V_C^n(\nu)\}, \text{ for non-owners.} \tag{2}$$

The value function of customers waiting in an agency trade with a dealer is $\tilde{V}_D^o(\nu)$ for owners, and $\tilde{V}_D^n(\nu)$ for non-owners. Finally, the reservation value of customers, $\Delta(\nu) = V^o(\nu) - V^n(\nu)$, is the compensation that makes them indifferent between owning an asset or not.

The value function of a dealer with asset position i is W^i. The reservation value of a dealer is $W^l = W^1 - W^0$ when buying an asset, $W^s = W^0 - W^{-1}$ when selling it, and W^a when in an agency trade. We anticipate the following results. Dealers' likelihood of serving a customer is zero since there is an infinite measure of them, implying that $W^0 = 0$. Moreover, dealers exit the economy after an agency trade, losing the option to serve future customers and implying that dealers' reservation value in an agency trade is $W^a = W^0 = 0$.

The distribution of customer types is $\Phi^o(\nu)$ for owners, and $\Phi^n(\nu)$ for non-owners. As each owner holds exactly one unit of the asset, the measure of assets is $s = \int d\Phi^o$. Let $\Pi = \{\pi_{-1}, \pi_0^o, \pi_0^n, \pi_1\}$ denote the inventory distribution among matched dealers. π_{-1} is the measure of dealers matched with a customer and who sold an asset in a principal trade, π_0^o and π_0^n are the measures of dealers in an agency trade matched to owners and non-owners, and π_1 is the measure of dealers matched with a customer and who bought an asset in a principal trade. Remember that owners try to sell an asset and non-owners try to buy an asset. Let $\pi_0 = \pi_0^o + \pi_0^n$ be the measure of dealers matched with customers waiting to access the interdealer market to trade on behalf of the customer in an agency trade.[5]

Let the sets Ω_D^o and Ω_C^o represent the customer owners that search for dealers and that search for other customers. Analogously, Ω_D^n and Ω_C^n represent the set of customer non-owners that search for dealers and that search for other customer. $\{\Omega_D^o, \Omega_C^o\}$ and $\{\Omega_D^n, \Omega_C^n\}$ form two partitions of $\mathbb{R}$. We denote the search partitions of customers by $\mathcal{P} = \{\Omega_D^o, \Omega_C^o, \Omega_D^n, \Omega_C^n\}$. We can further partition the sets Ω_D^o and Ω_D^n into partitions of principal trade and agency trade, $\{\Omega_D^{o,p}, \Omega_D^{o,a}\}$ and $\{\Omega_D^{n,p}, \Omega_D^{n,a}\}$, where the superscripts p and a denote principal trade or agency trade. Denote the trade-mode partition by $\mathcal{P}_D = \{\Omega_D^{o,p}, \Omega_D^{o,a}, \Omega_D^{n,p}, \Omega_D^{n,a}\}$.

Searching for dealers The value function of a type-ν owner searching for a dealer is

$$rV_D^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_D^0\theta_D \max\Big\{\underbrace{W^l - \Delta(\nu)}_{\substack{\text{Principal}\\ \text{trade}}}, \underbrace{\tilde{V}_D^o(\nu) - V_D^o(\nu)}_{\substack{\text{Agency}\\ \text{trade}}}, 0\Big\}. \tag{3}$$

The first and second terms in the right-hand side are the utility flow of holding the asset and the expected loss of reservation value in case of asset maturity. The third term, $\lambda_D^0\theta_D \max\{W^l - \Delta(\nu), \tilde{V}_D^o(\nu) - V_D^o(\nu), 0\}$, is the profit of an owner when meeting a dealer. The pair owner-dealer has three options: they can trade in a principal trade so that the dealer purchases the asset immediately, they can wait to access the interdealer market in an agency trade, or they can

[5] The distribution function could change over time in general, but we focus on steady-state equilibria and omit time subscripts.

decide not to trade. The gains from trade are respectively $W^l - \Delta(\nu)$, $\tilde{V}^o_D(\nu) - V^o_D(\nu)$, and zero. The owner keeps a share θ_D of the gains from trade.

The value function of a type-ν owner matched with a dealer in an agency trade is

$$r\tilde{V}^o_D(\nu) = \nu - \mu[\tilde{V}^o_D(\nu) - V^n(\nu)] + \lambda^1_D \max\{p - \tau - [\tilde{V}^o_D(\nu) - V^n(\nu)], 0\}. \tag{4}$$

The expected loss in case of maturity, in the second term, now takes into account the fact that the customer has already been matched. The third term is the profit of an owner when selling the asset in the interdealer market.

The value function of a type-ν non-owner searching for dealers is

$$rV^n_D(\nu) = \eta\Delta(\nu) + \lambda^0_D\theta_D \max\Big\{\underbrace{\Delta(\nu) - W^s}_{\substack{\text{Principal}\\ \text{trade}}}, \underbrace{\tilde{V}^n_D(\nu) - V^n_D(\nu)}_{\substack{\text{Agency}\\ \text{trade}}}, 0\Big\}. \tag{5}$$

The first term is the expected gain in reservation value in case of asset issuance. The second term is the profit of a non-owner when meeting a dealer. The pair owner-dealer have three options: the non-owner can buy the asset from the dealer in a principal trade, the non-owner can wait the dealer to access the interdealer market for an agency trade, or they can decide not to trade. The gains from trade are respectively $\Delta(\nu) - W^s$, $\tilde{V}^n_D(\nu) - V^n_D(\nu)$, and zero. The non-owner keeps a share θ_D of the gains from trade.

The value function of a non-owner of type ν matched with a dealer in an agency trade is

$$r\tilde{V}^n_D(\nu) = \eta[V^o(\nu) - \tilde{V}^n_D(\nu)] + \lambda^1_D \max\{V^o(\nu) - \tilde{V}^n_D(\nu) - (p + \tau), 0\}. \tag{6}$$

The expected gain in an asset issuance takes into account that the non-owner is matched. The second term is the profit of a non-owner when buying an asset in the interdealer market.

Value functions for the dealers The value function of a dealer holding long and short positions are given respectively by

$$rW^1 = rW^l = -c^l + \lambda^1_D[p - W^l], \tag{7}$$

$$rW^{-1} = -rW^s = -c^s + \lambda^1_D[W^s - p]. \tag{8}$$

The dealer long on an asset pays the flow cost c^l, has an expected loss in case of asset maturity, and sells the asset at the price p in the interdealer market. The interdealer market can be accessed at the rate λ_D. Similarly, the dealer short on an asset pays the flow cost c^s, has an expected gain in case of asset issuance, and buys the asset at the price p in the interdealer market.[6]

Customers searching for customers The value function of an owner of type ν searching for a non-owner is

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C \theta_C^o \int_{\Omega_C^n} [\Delta(\tilde{\nu}) - \Delta(\nu)] \mathbb{1}_{\{\Delta(\tilde{\nu}) > \Delta(\nu)\}} d\Phi^n(\tilde{\nu}). \tag{9}$$

The first term of the value function is the utility flow of holding the asset. The second term is the expected loss of the reservation value in case of asset maturity. The third term is the expected profits of an owner when meeting a non-owner. When trading with a non-owner of type $\tilde{\nu}$, an owner of type ν sells the asset if the reservation value of the counterparty, $\Delta(\tilde{\nu})$, is higher than the reservation value of the owner, $\Delta(\nu)$. The gains from trade are $\Delta(\tilde{\nu}) - \Delta(\nu)$ and the owner keeps a share θ_C^o of it. We obtain the expected value of the gains from trade by integrating it in $\tilde{\nu}$ over Ω_C^n using the distribution of non-owners $\Phi^n(\tilde{\nu})$.

The value function of a non-owner of type ν searching for an owner is

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C \theta_C^n \int_{\Omega_C^o} [\Delta(\nu) - \Delta(\tilde{\nu})] \mathbb{1}_{\{\Delta(\nu) > \Delta(\tilde{\nu})\}} d\Phi^o(\tilde{\nu}). \tag{10}$$

The first term of the value function is the expected gain of reservation value in case of asset issuance. The second term is the expected profit of a non-owner when searching for an owner. A non-owner of type ν buys the asset from an owner of type $\tilde{\nu}$ if the reservation value of the non-owner is higher than the reservation value of the owner. The non-owner keeps a share θ_C^n of the gains from trade, $\Delta(\nu) - \Delta(\tilde{\nu})$. The expected gains from trade are obtained by integrating it in $\tilde{\nu}$ over Ω_C^o using the distribution of owners $\Phi^o(\tilde{\nu})$.

[6] These expressions account for the fact that $W^0 = 0$, that is, the reservation value of a dealer holding a position of zero is equal to zero, as the likelihood of a particular dealer meeting a customer is zero.

Value functions, reservation value and the optimal searching choice The value functions V^o and V^n of a customer of type ν satisfy

$$V^o(\nu) = \max\{V^o_D(\nu), V^o_C(\nu)\} \quad \text{and} \quad V^n(\nu) = \max\{V^n_D(\nu), V^n_C(\nu)\}, \tag{11}$$

and the reservation value function satisfies

$$\Delta(\nu) = V^o(\nu) - V^n(\nu). \tag{12}$$

We characterize the search partition $\mathcal{P} = \{\Omega^o_D, \Omega^o_C, \Omega^n_D, \Omega^n_C\}$ in the following way. For Ω^o_D, an owner searches for a dealer if it yields higher value than searching for a non-owner. In the same way, for Ω^n_D, a non-owner searches for a dealer if it yields higher value then searching for an owner. We have

$$\Omega^o_D = \{\nu \in \mathbb{R};\ V^o_D(\nu) \geq V^o_C(\nu)\} \quad \text{and} \quad \Omega^n_D = \{\nu \in \mathbb{R};\ V^n_D(\nu) \geq V^n_C(\nu)\}. \tag{13}$$

Analogously,

$$\Omega^o_C = \{\nu \in \mathbb{R};\ V^o_C(\nu) > V^o_D(\nu)\} \quad \text{and} \quad \Omega^n_C = \{\nu \in \mathbb{R};\ V^n_C(\nu) > V^n_D(\nu)\}. \tag{14}$$

Similarly, we characterize the search partition $\mathcal{P}_D = \{\Omega^{o,p}_D, \Omega^{o,a}_D, \Omega^{n,p}_D, \Omega^{n,a}_D\}$ in the following way. An owner-dealer pair or a non-owner-dealer use a principal trade if it yields higher value than waiting in an agency trade. Then for principal trade we have

$$\begin{aligned}
\Omega^{o,p}_D &= \left\{\nu \in \Omega^o_D;\ W^l - \Delta(\nu) \geq \tilde{V}^o_D(\nu) - V^o_D(\nu) \text{ and } W^l - \Delta(\nu) \geq 0\right\}, \\
\Omega^{n,p}_D &= \left\{\nu \in \Omega^n_D;\ \Delta(\nu) - W^s \geq \tilde{V}^n_D(\nu) - V^n_D(\nu) \text{ and } \Delta(\nu) - W^s \geq 0\right\}.
\end{aligned} \tag{15}$$

Analogously, for agency trade we have

$$\begin{aligned}
\Omega^{o,a}_D &= \left\{\nu \in \Omega^o_D;\ \tilde{V}^o_D(\nu) - V^o_D(\nu) > \max\{W^l - \Delta(\nu), 0\}\right\} \\
\Omega^{n,a}_D &= \left\{\nu \in \Omega^n_D;\ \tilde{V}^n_D(\nu) - V^n_D(\nu) > \max\{\Delta(\nu) - W^s, 0\}\right\}.
\end{aligned} \tag{16}$$

Customers search for dealers when indifferent between searching for a dealer or other customers (eq. 13). Customers-dealers use principal trade when indifferent between agency or principal trade (eq. 15). There are no inactive customers. For the equilibrium class that we consider, these assumptions are without loss of generality because there is a measure zero of customers that are indifferent in equilibrium.

2.3 Interdealer market clearing

The interdealer market clears when the measure of dealers selling an asset is equal to the measure of dealers buying an asset. Dealers can sell or buy assets either to rebalance their account or on an agency trade. The measure π_1 of dealers holding an asset satisfies

$$\dot{\pi}_1 = -\lambda_D^1 \pi_1 + \lambda_D^0 \int_{-\infty}^{\infty} \mathbb{1}_{\{\nu \in \Omega_D^{o,p}\}} d\Phi^o(\nu) = 0. \tag{17}$$

The measure π_{-1} of dealers short on an asset satisfies

$$\dot{\pi}_{-1} = -\lambda_D^1 \pi_{-1} + \lambda_D^0 \int_{-\infty}^{\infty} \mathbb{1}_{\{\nu \in \Omega_D^{n,p}\}} d\Phi^n(\nu) = 0. \tag{18}$$

The measure $\pi_0^o(\nu)$ of dealers with an owner type $\tilde{\nu} \leq \nu$ in an agency trade satisfies

$$\dot{\pi}_0^o(\nu) = -\left(\mu + \lambda_D^1\right) \pi_0^o(\nu) + \lambda_D^0 \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu} \in \Omega_D^{o,a}\}} d[\Phi^o(\tilde{\nu}) - \pi_0^o(\tilde{\nu})] = 0, \tag{19}$$

and the measure π_0^n of dealers with a non-owner type $\tilde{\nu} \leq \nu$ in an agency trade satisfies

$$\dot{\pi}_0^n(\nu) = -\left(\eta + \lambda_D^1\right) \pi_0^n(\nu) + \lambda_D^0 \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu} \in \Omega_D^{n,a}\}} d[\Phi^n(\tilde{\nu}) - \pi_0^n(\tilde{\nu})] = 0. \tag{20}$$

The interdealer market clears if

$$\lambda_D^1 \left(\pi_1 + \bar{\pi}_0^o\right) = \lambda_D^1 \left(\pi_{-1} + \bar{\pi}_0^n\right), \tag{21}$$

where $\bar{\pi}_0^o = \lim_{\nu \to \infty} \pi_0^o(\nu)$ and $\bar{\pi}_0^n = \lim_{\nu \to \infty} \pi_0^n(\nu)$. The interdealer market clears if the measure of dealers selling an asset on principal or agency trade equals the measure of dealers buying an asset on principal or agency trade.

2.4 The distribution of assets among customers

The cumulative distribution of owners and non-owners are given respectively by Φ^o and Φ^n. The change over time of the distribution of owners Φ^o satisfies

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \int_{-\infty}^{\nu}\left[\lambda_D^0 \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^{o,p}\}} + \frac{\lambda_D^0\lambda_D^1 \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^{o,a}\}}}{\mu+\lambda_D^0+\lambda_D^1}\right] d\Phi^o(\tilde{\nu}) \\
&+ \int_{-\infty}^{\nu}\left[\lambda_D^0 \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^{n,p}\}} + \frac{\lambda_D^0\lambda_D^1 \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^{n,a}\}}}{\eta+\lambda_D^0+\lambda_D^1}\right] d\Phi^n(\tilde{\nu}) \\
&- \lambda_C \int_{-\infty}^{\nu}\int_{\nu}^{\infty} \mathbb{1}_{\{\tilde{\nu}\in\Omega_C^o \hat{\nu}\in\Omega_C^n,\Delta(\hat{\nu})>\Delta(\tilde{\nu})\}} d\Phi^n(\hat{\nu})d\Phi^o(\tilde{\nu}), \\
&+ \lambda_C \int_{-\infty}^{\nu}\int_{\nu}^{\infty} \mathbb{1}_{\{\tilde{\nu}\in\Omega_C^o,\hat{\nu}\in\Omega_C^n,\Delta(\tilde{\nu})>\Delta(\hat{\nu})\}} d\Phi^o(\hat{\nu})d\Phi^n(\tilde{\nu}) \qquad (22)\end{aligned}$$

The first term on the right-hand side of (22) accounts for the inflow of owners that issue an asset and the second term accounts for the outflow of owners because of asset maturity. The third and fourth terms account for owners searching for dealers. The third term for the outflow of owners with type below ν searching for dealers and that sell their asset, and the fourth for the inflow of non-owners with type below ν searching for dealers and that buy an asset. The fifth and sixth terms account for customers searching for other customers. The fifth term for the outflow of owners with type below ν searching for other customers, which sell their asset to non-owners of type above ν, and the sixth term for the inflow of non-owners with type below ν searching for other customers, which buy an asset from owners of type above ν. In an steady-state equilibrium, $\dot{\Phi}^o(\nu) = 0$ for all ν.

As the measure of customers F is exogenous, the measures of owners and non-owners satisfy the equilibrium condition

$$\Phi^o(\nu) + \Phi^n(\nu) = F(\nu). \qquad (23)$$

All assets in the economy are held by owners or dealers. The stock of assets held by owners is

$$s = \int_{-\infty}^{\infty} d\Phi^o(\nu) = \Phi^o(\infty). \qquad (24)$$

2.5 Equilibrium

We define a stationary equilibrium in the following way.

Definition 1. *An equilibrium is a family of value functions, reservations value, price, distributions and partitions,* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *satisfying equations* (3)–(24).

An equilibrium, even in steady state, can be a complicated object. To simplify it further, let $\Omega_C = \Omega_C^o = \Omega_C^n = (\nu_l, \nu_h)$ and $\Omega_D = \Omega_D^o = \Omega_D^n = (-\infty, \nu_l] \cup [\nu_h, \infty)$. Notice that we can have $\Omega_C^o = \Omega_C^n$ as we have some customers of type ν holding the asset and other customers of the same type that do not hold the asset. Moreover, let $\Omega_D^{o,a} = (\nu_l^a, \nu_l]$, $\Omega_D^{n,a} = [\nu_h, \nu_h^a)$, $\Omega_D^{o,p} = (-\infty, \nu_l^a]$, and $\Omega_D^{n,p} = [\nu_h^a, \infty)$. Define the following class of equilibrium.

Definition 2. *An equilibrium* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *is regular if*

- $\Omega_C = \Omega_C^o = \Omega_C^n = (\nu_l, \nu_h)$,
- $\Omega_D = \Omega_D^o = \Omega_D^n = (-\infty, \nu_l] \cup [\nu_h, \infty)$,
- $\Omega_D^{o,a} = (\nu_l^a, \nu_l]$, $\Omega_D^{n,a} = [\nu_h, \nu_h^a)$, $\Omega_D^{o,p} = (-\infty, \nu_l^a]$, *and* $\Omega_D^{n,p} = [\nu_h^a, \infty)$

for some $\nu_l^a, \nu_l, \nu_h, \nu_h^a \in \mathbb{R}$ *satisfying* $\nu_l^a \leq \nu_l \leq \nu_h \leq \nu_h^a$ *with* $\nu_l < \nu_h$ *inequality if* $c^l, c^s, \tau > 0$, *and the reservation value* $\Delta(\nu)$ *is continuous and strictly increasing.*

Figure 2 illustrates the partitions of a regular equilibrium. The motivation to look for an equilibrium with the characteristics of a regular equilibrium is the following. Customers with type close to each other, in $\Omega_C = (\nu_l, \nu_h)$, choose to trade among themselves to avoid the costs associated with trade with a dealer, τ and c, and because they do not gain much from trading. Customers with moderate types, outside $\Omega_C = (\nu_l, \nu_h)$ but neither in $\Omega_D^{o,p} = (-\infty, \nu_l^a]$ nor in $\Omega_D^{n,p} = [\nu_h^a, \infty)$, are willing to cover higher dealer cost for an agency trade, but not the cost associate with a principal trade. Only customers with extreme types, in $\Omega_D^{o,p} = (-\infty, \nu_l^a]$ or $\Omega_D^{n,p} = [\nu_h^a, \infty)$, are in a hurry to trade and they are willing to cover higher dealer cost for a principal trade.

We also impose $\nu_l < \nu_h$ when the intermediation cost is strictly positive. The reason is that we can always build an equilibrium where customers do not search for customers because they expect other customers to do the same. In this case, the probability of finding a customer

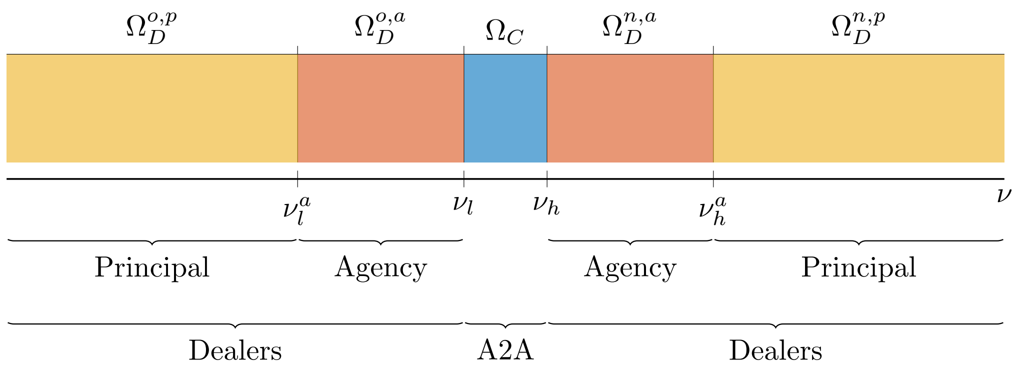


Figure 2: Partitions $\mathcal{P}$ and $\mathcal{P}_D$ in a regular equilibrium.

is zero so customers may as well search for dealers. The assumption that $\nu_l < \nu_h$ if $c^l, c^s, \tau > 0$ rules out equilibria built on this sort of weak inequality. In the next section, we characterize a regular equilibrium and provide conditions that it exists.

3 Model solution: two limiting cases

To understand the results of the model, it is helpful to examine two limiting cases: one with high principal trade costs, where all dealer intermediation occurs through agency trades, and another with high agency trade costs, where all dealer intermediation occurs through principal trades.

3.1 Agency trade only

Let principal trade costs, c^l and c^s, be sufficiently high so that dealers perform agency trades only. From equations (7) and (8), this implies $W^l = -\infty$, $W^s = \infty$ and, $\Omega_D^{o,p} = \Omega_D^{n,p} = \emptyset$. The threshold types for the agency trade are then $\nu_l^a = -\infty$ and $\nu_h^a = \infty$, and dealers engage exclusively in agency risk-free trades.

A regular equilibrium has then two blocks. Given ν_l and ν_h, customers $\nu \in \Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$ perform dealer-agency trades whereas customers $\nu \in \Omega_C = (\nu_l, \nu_h)$ trade bilaterally with other customers in all-to-all trades. We can solve these two blocks separately using the tools developed in Lagos and Rocheteau (2009) and Hugonnier et al. (2022). The challenge is to characterize ν_l and ν_h that are consistent with the equilibrium equations (13) and (14). We need to find ν_l and ν_h such that customers searching for dealers, with $\nu \leq \nu_l$ or $\nu \geq \nu_h$, are not better off searching for other customers. Analogously, customers searching for customers with intermediary types $\nu_l < \nu < \nu_h$ are not better off searching for dealers.

3.1.1 Dealer-agency block

The reservation value of a type-ν customer searching for a dealer is $\Delta(\nu) = V_D^o(\nu) - V_D^n(\nu)$. The value functions, $V_D^o(\nu)$ and $V_D^n(\nu)$, of a type-ν customer searching for a dealer when holding and not holding an asset are stated in equations (3)–(6). It is useful to define

$$\lambda_D^{al} = \frac{\lambda_D^0 \lambda_D^1}{r + \mu + \lambda_D^0 \theta_D + \lambda_D^1} \quad \text{and} \quad \lambda_D^{as} = \frac{\lambda_D^0 \lambda_D^1}{r + \eta + \lambda_D^0 \theta_D + \lambda_D^1}. \tag{25}$$

Solving the four equations to isolate $\Delta(\nu)$ yields the following lemma.

Lemma 1. *Consider a regular equilibrium* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *and the set of utility types* $\Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$. *Then, the reservation value* $\Delta(\nu)$ *satisfies*

$$\Delta(\nu) = \begin{cases} \sigma_D^{al}[\nu + \lambda_D^{al}\theta_D(p - \tau)], & \nu \leq \nu_l, \\ \sigma_D^{as}[\nu + \lambda_D^{as}\theta_D(p + \tau)], & \nu \geq \nu_h, \end{cases} \tag{26}$$

where

$$\sigma_D^{al} = \frac{1}{r + \mu + \eta + \lambda_D^{al}\theta_D} \quad \textit{and} \quad \sigma_D^{as} = \frac{1}{r + \mu + \eta + \lambda_D^{as}\theta_D}. \tag{27}$$

Moreover, $\Delta(\nu_l) \leq p - \tau$ *and* $\Delta(\nu_h) \geq p + \tau$.

The derivative of the reservation value with respect to ν is given by σ_D^{al}, when $\nu \leq \nu_l$, and σ_D^{as}, when $\nu \geq \nu_h$. As HLW, we can interpret σ_D^{al} and σ_D^{as} as the local surplus at ν. It captures the trade surplus generated if the asset of an agent type ν is transferred to an agent type $\nu + d\nu$. For all $\nu \notin (\nu_l, \nu_h)$, the local surplus depends only on whether $\nu \leq \nu_l$ or $\nu \geq \nu_h$. That is because all customers type $\nu \leq \nu_l$ want to sell the asset and face the same price after bargaining and intermediation costs. As a result, the trade surplus is constant in this region. Similarly, all customers type $\nu \geq \nu_h$ want to buy the asset and face the same price after bargaining and intermediation costs.

The reason the trade surplus is different in the two regions is that customers type $\nu \leq \nu_l$

holding the asset to sell face the risk of the asset depreciating before it is sold in an agency trade, so they discount at the rate λ_D^{al}. Customers type $\nu \geq \nu_h$ waiting to buy an asset may have an issuance opportunity before it purchases the asset in an agency trade, so they discount at the rate λ_D^{as}.

In addition to the interpretation above, σ_D^{al} and σ_D^{as} are indicators of market friction. To see this, suppose that there are no search friction for dealer intermediation. In the model, that means $\lambda_D^0, \lambda_D^1 \to \infty$. In this case, $\sigma_D^{al} = \sigma_D^{as} = 0$ and so $\Delta(\nu)$ is constant in ν at $p - \tau$ or $p + \tau$. The reservation value is given by the competitive price plus or minus the intermediation cost, and there is no change in reservation value associated with search frictions. Higher values of σ_D^{al} and σ_D^{as} are then associated with higher search frictions. We later discuss an analogous measure of search frictions for the customer-customer market.

To establish the distributions of owners and non-owners Φ^o and Φ^n, we need to determine the distribution of dealers in agency trade, π_0^o and π_0^n. From Lemma 1, owners $\nu \leq \nu_l$ always sell in the interdealer market in agency trades and non-owners $\nu \geq \nu_h$ always buy. Then, equations (19) and (20) can be written as

$$-\left(\mu + \lambda_D^1\right) \pi_0^o(\nu) + \lambda_D^0 \left[\Phi^o(\nu) - \pi_0^o(\nu)\right] = 0,\ \nu \leq \nu_l,$$

$$-\left(\eta + \lambda_D^1\right) \pi_0^n(\nu) + \lambda_D^0 \left[\Phi^n(\nu) - \Phi^n(\nu_h) - \pi_0^n(\nu)\right] = 0,\ \nu \geq \nu_h.$$

Moreover, non-owners $\nu \leq \nu_l$ are inactive. It does not compensate for them searching for other customers because owners with reservation value below $\Delta(\nu)$ are not active in the customer-customer market. It does not compensate for them buying an asset in agency because $\nu \leq \nu_l$ implies that $\Delta(\nu) \leq p - \tau$, as established in lemma 1. The argument is analogous for owners of type $\nu \geq \nu_h$. Therefore, non-owners of type $\nu \leq \nu_l$ and owners of type $\nu \geq \nu_h$ are inactive. This leads to the following result.

Lemma 2 (Agency, dealer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n,

Π, $\mathcal{P}, \mathcal{P}_D\}$ *is such that the measures of owners and non-owners in agency trade satisfy*

$$\pi_0^o(\nu) = \begin{cases} \dfrac{\lambda_D^0 \Phi^o(\nu)}{\mu + \lambda_D^0 + \lambda_D^1}, & \nu \leq \nu_l, \\ \dfrac{\lambda_D^0 \Phi^o(\nu_l)}{\mu + \lambda_D^0 + \lambda_D^1}, & \nu > \nu_l, \end{cases} \tag{28}$$

$$\pi_0^n(\nu) = \begin{cases} 0, & \nu \leq \nu_l \\ \dfrac{\lambda_D^0 \left[\Phi^n(\nu) - \Phi^n(\nu_h)\right]}{\eta + \lambda_D^0 + \lambda_D^1}, & \nu \geq \nu_h. \end{cases} \tag{29}$$

Moreover, since there is no principal trade, $\pi_1 = \pi_{-1} = 0$.

We can now establish the distribution of owners and non-owners, Φ^o and Φ^n.

Lemma 3 (Distributions, dealer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n, Π, $\mathcal{P}, \mathcal{P}_D\}$ *is such that the cumulative distribution of owners satisfies*

$$\Phi^o(\nu) = \begin{cases} \dfrac{\eta F(\nu)}{\mu + \eta + \tilde{\lambda}_D^{al}}, & \nu \leq \nu_l, \\ \dfrac{\eta}{\mu + \eta} - \dfrac{(\eta + \tilde{\lambda}_D^{as})[1 - F(\nu)]}{\mu + \eta + \tilde{\lambda}_D^{as}}, & \nu \geq \nu_h, \end{cases} \tag{30}$$

where $\tilde{\lambda}_D^{al} = \frac{\lambda_D^0 \lambda_D^1}{\mu + \lambda_D^0 + \lambda_D^1}$, $\tilde{\lambda}_D^{as} = \frac{\lambda_D^0 \lambda_D^1}{\eta + \lambda_D^0 + \lambda_D^1}$, *and* $\Phi^o(\nu) + \Phi^n(\nu) = F(\nu)$*. Moreover,* ν_l *and* ν_h *satisfy*

$$\frac{\eta \tilde{\lambda}_D^{al} F(\nu_l)}{\mu + \eta + \tilde{\lambda}_D^{al}} = \frac{\mu \tilde{\lambda}_D^{as}[1 - F(\nu_h)]}{\mu + \eta + \tilde{\lambda}_D^{as}} \quad \text{and} \quad \tilde{\lambda}_D^{al} \Phi^o(\nu_l) + \tilde{\lambda}_D^{as} \Phi^n(\nu_h) = \frac{\tilde{\lambda}_D^{as} \mu}{\mu + \eta}. \tag{31}$$

Define the separating utility type ν_s as the value of ν such that

$$\frac{\eta \tilde{\lambda}_D^{al} F(\nu_s)}{\mu + \eta + \tilde{\lambda}_D^{al}} = \frac{\mu \tilde{\lambda}_D^{as}[1 - F(\nu_s)]}{\mu + \eta + \tilde{\lambda}_D^{as}} \;\Rightarrow\; \nu_s = F^{-1}\left[\frac{\mu \tilde{\lambda}_D^{as}(\mu + \eta + \tilde{\lambda}_D^{al})}{\eta \tilde{\lambda}_D^{al}(\mu + \eta + \tilde{\lambda}_D^{as}) + \mu \tilde{\lambda}_D^{as}(\mu + \eta + \tilde{\lambda}_D^{al})}\right]. \tag{32}$$

If $\tau = 0$, then $\nu_l = \nu_h = \nu_s$ and all customers trade with dealers. In this case, the marginal type ν_s separates the market into owners $\nu < \nu_s$ who want to sell the asset and non-owners $\nu \geq \nu_s$ who want to buy the asset.

3.1.2 Customer-customer block

For customers searching for other customers, the value functions and reservation values are obtained in the following way. For the value functions, from equations (9) and (10), the value function of a type $\nu \in \Omega_C$ customer holding an asset satisfies

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Delta(\tilde{\nu}) - \Delta(\nu)]d\Phi^n(\tilde{\nu}), \text{ and} \tag{33}$$

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Delta(\nu) - \Delta(\tilde{\nu})]d\Phi^o(\tilde{\nu}). \tag{34}$$

Combined with the definition of reservation value, $\Delta(\nu) = V_C^o(\nu) - V_C^n(\nu)$, given in equation (12), the equations above imply the following lemma.

Lemma 4 (Reservation value, customer-customer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n, Π, $\mathcal{P}$, $\mathcal{P}_D\}$ *satisfies*

$$\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu} \tag{35}$$

for all $\nu \in (\nu_l, \nu_h)$*, and*

$$\sigma_C(\nu) = \frac{1}{r + \mu + \eta + \lambda_C\Big\{\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]\Big\}}. \tag{36}$$

for almost all $\nu \in (\nu_l, \nu_h)$*.*

The trade surplus between a seller ν and buyer $\nu + d\nu$ is approximately equal to $\sigma_C(\nu)d\nu$, which agrees with the interpretation discussed in HLW of $\sigma_C(\nu)d\nu$ as the local surplus. The function $\sigma_C(\nu)$ discounts the additional utility $d\nu$ by the discount rate r, the likelihood that the asset will mature μ, the loss in the likelihood of issuing an asset η, and the loss in option value from either meeting another buyer with higher valuation $\lambda_C\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big]$, or finding another seller with lower valuation, $\lambda\theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]$.

We now turn to the distributions Φ^o, Φ^n among customers searching other customers.

Lemma 5 (Distributions, customer-customer market)**.** *A regular equilibrium* $\{V^o$*,* V^n*,* Δ*,* p*,*

Φ^o, Φ^n, ν_l, $\nu_h\}$ *is such that the cumulative distribution of owners satisfies*

$$\begin{aligned}\tilde{\Phi}^o(\nu) &= F(\nu) - F(\nu_l) - \tilde{\Phi}^n(\nu) \\ &= -\tfrac{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]}{2\lambda_C} + \tfrac{\sqrt{\{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]\}^2+4\lambda_C\eta\big[F(\nu)-F(\nu_l)\big]}}{2\lambda_C},\end{aligned} \tag{37}$$

$\nu \in (\nu_l, \nu_h)$*, where* $\tilde{\Phi}^o(\nu) \equiv \Phi^o(\nu) - \Phi^o(\nu_l)$ *and* $\tilde{\Phi}^n(\nu) \equiv \Phi^n(\nu) - \Phi^n(\nu_l)$*, and*

$$s_C \equiv \Phi^o(\nu_h) - \Phi^o(\nu_l) = \frac{\eta}{\mu+\eta}\big[F(\nu_h) - F(\nu_l)\big]. \tag{38}$$

Figure 3 shows a representation for the reservation value as function of ν. Lemma 1 implies that $\Delta(\nu)$ is linear for $\nu \leq \nu_l$ and $\nu \geq \nu_h$. Moreover, $\Delta(\nu_l) \leq p - \tau$ and $\Delta(\nu_h) \geq p + \tau$. That is, owners with $\nu \leq \nu_l$ choose sell to dealers and non-owners with $\nu \geq \nu_h$ choose to buy from dealers. Lemmas 4 and 5 imply the nonlinear shape of Δ in (ν_l, ν_h). Customers that trade with dealers have $\nu \leq \nu_l$. Customers that trade with other customers have $\nu \in (\nu_l, \nu_h)$.

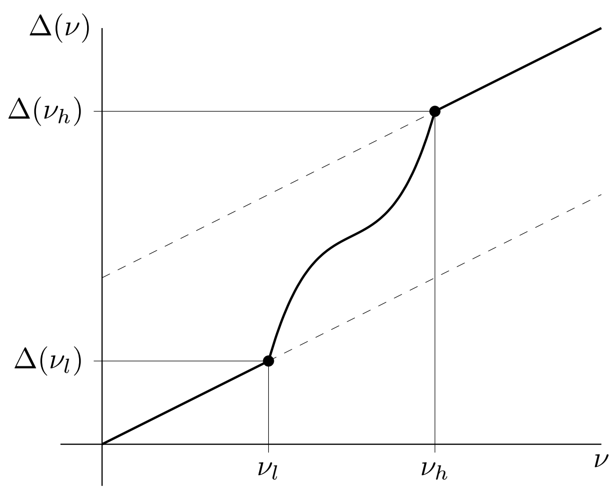


Figure 3: Reservation value as function of the customer type, $\Delta(\nu)$. Customers that trade with dealers have $\nu \leq \nu_l$. Customers that trade with other customers have $\nu \in (\nu_l, \nu_h)$.

3.1.3 Characterization

The results in sections 3.1.1 and 3.1.2 establish necessary conditions for the equilibrium objects V^o, V^n, Δ, p, Φ^o, Φ^n, s, ν_l and ν_h. The equilibrium objects can all be written as functions of ν_l and ν_h. We now provide necessary conditions on ν_l and ν_h and show that, together with equations (5)–(38), these conditions are also sufficient for a regular equilibrium. These results provide a full characterization of the equilibrium.

Lemma 6. *A regular equilibrium* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *satisfies*

$$2\tau\theta_D = \int_{\nu_l}^{\nu_h} w(\nu)\frac{\sigma_C(\nu)-\sigma_D^{as}}{\lambda_D^{as}\sigma_D^{as}} + [1-w(\nu)]\,\frac{\sigma_C(\nu)-\sigma_D^{al}}{\lambda_D^{al}\sigma_D^{al}}d\nu, \tag{39}$$

where $w(\nu) = \frac{\theta_C^o\left[\Phi^n(\nu_h)-\Phi^n(\nu)\right]}{\theta_C^o\left[\Phi^n(\nu_h)-\Phi^n(\nu)\right]+\theta_C^n\left[\Phi^o(\nu)-\Phi^o(\nu_l)\right]}$. *Moreover,*

$$\begin{aligned} p &= \Delta(\nu_l) + \tau + \frac{\lambda_C\theta_C^o}{\lambda_D^{al}\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}\sigma_C(\tilde\nu)d\tilde\nu d\Phi^n(\nu) \\ &= \Delta(\nu_h) - \tau - \frac{\lambda_C\theta_C^n}{\lambda_D^{as}\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h}\sigma_C(\tilde\nu)d\tilde\nu d\Phi^o(\nu). \end{aligned} \tag{40}$$

Lemmas 3 to 6 establish necessary conditions that are satisfied in all regular equilibria. In the proposition below, we show that these conditions are not only necessary but sufficient. Therefore they fully characterize a regular equilibrium.

Proposition 1. *If a family* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *is a regular equilibrium, it satisfies equations* (26)–(40)*. Reversely, if* $\{\Delta, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *satisfies equations* (26)–(40)*, then* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *is a regular equilibrium where the value functions* W^l*,* W^s*,* V^o *and* V^n *are constructed using equations* (3)–(11)*.*

3.2 Principal trade only

Let now agency trade cost τ be sufficiently high so that dealers undertake principal trades only. From equations (7) and (8), this implies that the threshold types for the agency trade are $\nu_l^a = \nu_l$ and $\nu_h^a = \nu_h$. Dealers and customers engage exclusively in principal trades and the sets of agency trades disappears, $\Omega_D^{o,a} = \Omega_D^{n,a} = \emptyset$.

3.2.1 Dealer-principal trades

The dealer reservation value can be obtained from equations (7) and (8). It yields

$$W^l = \frac{\lambda_D^1(p-\tilde c^l)}{r+\lambda_D^1} \quad \text{and} \quad W^s = \frac{\lambda_D^1(p+\tilde c^s)}{r+\lambda_D^1}, \tag{41}$$

where $\tilde{c}^l = \frac{c^l}{\lambda_D^1}$ and $\tilde{c}^s = \frac{c^s}{\lambda_D^1}$. The reservation value of a type-ν customer searching for a dealer is $\Delta(\nu) = V_D^o(\nu) - V_D^n(\nu)$. The value functions, $V_D^o(\nu)$ and $V_D^n(\nu)$, of a type-ν customer searching for a dealer when holding and not holding an asset are stated in equations (3)–(6). It is useful to define

$$\lambda_D^p = \frac{\lambda_D^0 \lambda_D^1}{r + \lambda_D^1}. \tag{42}$$

Solving the four equations to isolate $\Delta(\nu)$ yields the following lemma.

Lemma 7. *Consider a regular equilibrium* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *and the set of utility types* $\Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$*. Then, the reservation value* $\Delta(\nu)$ *satisfies*

$$\Delta(\nu) = \begin{cases} \sigma_D^p[\nu + \lambda_D^p \theta_D (p - \tilde{c}^l)], & \nu \leq \nu_l, \\ \sigma_D^p[\nu + \lambda_D^p \theta_D (p + \tilde{c}^s)], & \nu \geq \nu_h, \end{cases} \tag{43}$$

where

$$\sigma_D^p = \frac{1}{r + \mu + \eta + \lambda_D^p \theta_D}. \tag{44}$$

Moreover, $\Delta(\nu_l) \leq p - \tau$ *and* $\Delta(\nu_h) \geq p + \tau$*.*

The derivative of the reservation value with respect to ν is given by σ_D^p, when $\nu \leq \nu_l$, and σ_D^p, when $\nu \geq \nu_h$. The interpretation of σ_D^p and σ_D^p are analogous to the interpretation of σ_D^{al} and σ_D^{as} discussed in Section 3.1.

Before establishing the distributions of owners and non-owners Φ^o and Φ^n, we establish the distribution of dealers holdings, π_1 and π_{-1}. From Lemma 1, owners of type $\nu \leq \nu_l$ always sell in the interdealer market in principal trades, whereas non-owners of type $\nu \geq \nu_h$ always buy. Equations (17) and (18) can be rewritten as

$$-\lambda_D^1 \pi_1 + \lambda_D^0 \Phi^o(\nu_l) = 0 \tag{45}$$

for $\nu \leq \nu_l$, and

$$-\lambda_D^1 \pi_{-1} + \lambda_D^0 [1 - s - \Phi^n(\nu_h)] = 0 \tag{46}$$

for $\nu \geq \nu_h$, where $s = \Phi^o(\infty)$ is asset holdings of owners. This leads to the following result.

Lemma 8 (Principal, dealer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n,

Π, $\mathcal{P}, \mathcal{P}_D\}$ *is such that the distribution of dealers' holdings satisfies*

$$\pi_1 = \frac{\lambda_D^0 \Phi^o(\nu_l)}{\lambda_D^1} \qquad \textit{and} \qquad \pi_{-1} = \frac{\lambda_D^0[1 - s - \Phi^n(\nu_h)]}{\lambda_D^1}. \tag{47}$$

Moreover, since there is no agency trade, $\pi_0^n(\nu) = \pi_0^n(\nu) = 0$ *for all* ν.

We can now establish the distribution of owners and non-owners, Φ^o and Φ^n.

Lemma 9 (Distributions, dealer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n, Π, $\mathcal{P}, \mathcal{P}_D\}$ *is such that the cumulative distribution of owners satisfies*

$$\Phi^o(\nu) = \begin{cases} \dfrac{\eta F(\nu)}{\mu + \eta + \lambda_D^0}, & \nu \leq \nu_l, \\ \dfrac{\eta}{\mu + \eta} - \frac{(\eta + \lambda_D^0)[1 - F(\nu)]}{\mu + \eta + \lambda_D^0} & \nu \geq \nu_h, \end{cases} \tag{48}$$

where $\Phi^o(\nu) + \Phi^n(\nu) = F(\nu)$. *Moreover,* ν_l *and* ν_h *satisfy*

$$\eta F(\nu_l) = \mu[1 - F(\nu_h)] \quad \textit{and} \quad \Phi^o(\nu_l) + \Phi^n(\nu_h) = \frac{\mu}{\mu + \eta}. \tag{49}$$

Analogously to the previous section, define the separating utility type ν_s as

$$\eta F(\nu_s) = \mu[1 - F(\nu_s)] \Longrightarrow \nu_s = F^{-1}\left(\frac{\mu}{\eta + \mu}\right). \tag{50}$$

If $c^l = c^s = 0$, then $\nu_l = \nu_h = \nu_s$ and all customers trade with dealers. In this case, ν_s is the marginal type and separates the dealer-customer market into owners type $\nu < \nu_s$ who want to sell the asset, and non-owners type $\nu \geq \nu_s$ that want to buy the asset.

3.2.2 Customer-customer block

The customer-customer block of the model is the same whether dealers perform agency or principal trades. For this reason, in this section we only repeat the results obtained in Section 3.1.2 when studying the limit case with only agency trade.

Lemma 10 (Reservation value, customer-customer market)**.** *A regular equilibrium* $\{V^o$, V^n,

Δ, W^l, W^s, p, Φ^o, Φ^n, Π, $\mathcal{P}$, $\mathcal{P}_D\}$ *satisfies*

$$\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu} \tag{51}$$

for all $\nu \in (\nu_l, \nu_h)$*, and*

$$\sigma_C(\nu) = \frac{1}{r + \mu + \eta + \lambda_C \Big\{ \theta_C^o \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \theta_C^n \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] \Big\}}. \tag{52}$$

for almost all $\nu \in (\nu_l, \nu_h)$*.*

Lemma 11 (Distributions, customer-customer market)**.** *A regular equilibrium* $\{V^o$, V^n, Δ, p, Φ^o, Φ^n, ν_l, $\nu_h\}$ *is such that the cumulative distribution of owners satisfies*

$$\begin{aligned} \tilde{\Phi}^o(\nu) &= F(\nu) - F(\nu_l) - \tilde{\Phi}^n(\nu) \\ &= -\tfrac{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]}{2\lambda_C} + \tfrac{\sqrt{\{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]\}^2 + 4\lambda_C \eta \big[F(\nu)-F(\nu_l)\big]}}{2\lambda_C}, \end{aligned} \tag{53}$$

$\nu \in (\nu_l, \nu_h)$*, where* $\tilde{\Phi}^o(\nu) \equiv \Phi^o(\nu) - \Phi^o(\nu_l)$ *and* $\tilde{\Phi}^n(\nu) \equiv \Phi^n(\nu) - \Phi^n(\nu_l)$*, and*

$$s_C \equiv \Phi^o(\nu_h) - \Phi^o(\nu_l) = \frac{\eta}{\mu + \eta} \big[F(\nu_h) - F(\nu_l)\big]. \tag{54}$$

3.2.3 Characterization

The results in sections 3.2.1 and 3.2.2 establish necessary conditions for the equilibrium objects V^o, V^n, Δ, p, Φ^o, Φ^n, s, ν_l and ν_h. The equilibrium objects can all be written as functions of ν_l and ν_h. We now provide necessary conditions on ν_l and ν_h and show that, together with equations (53)–(54), these conditions are also sufficient for a regular equilibria. These results provide a full characterization of the equilibrium.

Lemma 12. *A regular equilibrium* $\{V^o$, V^n, Δ, W^l, W^s, p, Φ^o, Φ^n, Π, $\mathcal{P}$, $\mathcal{P}_D\}$ *satisfies*

$$\lambda_D^p \theta_D (\tilde{c}^l + \tilde{c}^s) = \int_{\nu_l}^{\nu_h} \frac{\sigma_C(\nu) - \sigma_D^p}{\sigma_D^p} d\nu. \tag{55}$$

Moreover,

$$p = \frac{\lambda_D^0}{\lambda_D^p}\Delta(\nu_l) + \tilde{c}^l + \frac{\lambda_C \theta_C^o}{\lambda_D^p \theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^n(\nu)$$
$$= \frac{\lambda_D^0}{\lambda_D^p}\Delta(\nu_h) - \tilde{c}^s - \frac{\lambda_C \theta_C^o}{\lambda_D^p \theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^o(\nu). \tag{56}$$

Lemmas 9 to 12 establish necessary conditions that are satisfied in all regular equilibria. In the proposition below, we show that these conditions are not only necessary but sufficient. Therefore they fully characterize a regular equilibrium.

Proposition 2. *If a family* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *is a regular equilibrium, it satisfies equations* (41)–(56)*. Reversely, if* $\{\Delta, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *satisfies equations* (41)–(56)*, then* $\{V^o, V^n, \Delta, W^l, W^s, p, \Phi^o, \Phi^n, \Pi, \mathcal{P}, \mathcal{P}_D\}$ *is a regular equilibrium where the value functions* V^o *and* V^n *are constructed using equations* (3)–(11)*.*

4 Principal vs agency trade

The decision to engage in agency or principal trading is based on the costs associated with each form of intermediation, the speed of trade, bargaining power, and customer valuation. Proposition 3 characterizes the regions where customers and dealers engage in principal or agency trade.

Proposition 3. *Consider a regular equilibrium. Owners type* $\nu \le \nu_l$ *sell in principal trade when meeting a dealer if and only if*

$$\underbrace{p\theta_D(\sigma_D^p\lambda_D^p - \sigma_D^{al}\lambda_D^{al})}_{\substack{\text{Gain from payment} \\ \text{in fast execution}}} - \underbrace{\theta_D\sigma_D^p\lambda_D^p\tilde{c}^l}_{\substack{\text{Principal} \\ \text{trade cost}}} \ge \underbrace{(\sigma_D^{al} - \sigma_D^p)\nu}_{\substack{\text{Gain from value} \\ \text{in slow execution}}} - \underbrace{\theta_D\sigma_D^{al}\lambda_D^{al}\tau}_{\substack{\text{Agency} \\ \text{trade cost}}}. \tag{57}$$

Similarly, non-owners type $\nu \ge \nu_h$ *buy in principal trade when meeting a dealer if and only if*

$$\underbrace{(\sigma_D^{as} - \sigma_D^p)\nu}_{\substack{\text{Gain from value} \\ \text{in fast execution}}} - \underbrace{\theta_D\sigma_D^p\lambda_D^p\tilde{c}^s}_{\substack{\text{Cost of} \\ \text{principal trade}}} \ge \underbrace{p\theta_D(\sigma_D^p\lambda_D^p - \sigma_D^{as}\lambda_D^{as})}_{\substack{\text{Gain from payment} \\ \text{in slow execution}}} - \underbrace{\theta_D\sigma_D^{as}\lambda_D^{as}\tau}_{\substack{\text{Cost of} \\ \text{agency trade}}}. \tag{58}$$

Moreover, equation (57) *holds with equality at* $\nu = \nu_l^a$ *if* $\nu_l^a < \nu_l$*, and equation* (58) *holds with* *equality at* $\nu = \nu_h^a$ *if* $\nu_h^a > \nu_h$.

Equations (57) and (58) provide a framework to characterize the decision between principal or agency trade. In equation (57), owners gain by selling assets fast because they receive the payment earlier with its faster execution. As they own the asset, they gain by selling assets in agency trade because they hold the asset for longer with its slower execution. This effect is more relevant as ν increases. Owners compare these gains net of the costs of principal and agency trade. As $\sigma_D^{al} > \sigma_D^p$, smaller values of ν makes this equation more like to be satisfied and so principal trade more likely. In (58), similarly, non-owners gain with principal trade because they will own the asset faster. They gain with agency trade because they delay payment given its slower execution. Higher values of ν will make principal trade more likely. We have agency trade for intermediary values of ν and principal trade for extreme values of ν.

Proposition 4. *Consider a regular equilibrium. The cumulative distribution of owners of types* $\nu \leq \nu_l$ *satisfies*

$$\Phi^o(\nu) = \begin{cases} \dfrac{\eta F(\nu)}{\mu + \eta + \lambda_D^0}, & \nu \leq \nu_l^a, \\ \dfrac{\eta F(\nu_l^a)}{\mu + \eta + \lambda_D^0} + \dfrac{\eta [F(\nu) - F(\nu_l^a)]}{\mu + \eta + \tilde{\lambda}_D^{al}}, & \nu_l^a < \nu \leq \nu_l; \end{cases} \tag{59}$$

and the cumulative distribution of non-owners of types $\nu \geq \nu_h$ *satisfies*

$$\Phi^n(\infty) - \Phi^n(\nu) = \begin{cases} \dfrac{\mu[1 - F(\nu)]}{\mu + \eta + \lambda_D^0}, & \nu \geq \nu_h^a, \\ \dfrac{\mu[1 - F(\nu_h^a)]}{\mu + \eta + \lambda_D^0} + \dfrac{\mu[F(\nu) - F(\nu_h)]}{\mu + \eta + \tilde{\lambda}_D^{as}}, & \nu_h^a > \nu \geq \nu_h, \end{cases} \tag{60}$$

where $\Phi^n(\infty) = 1 - s = \frac{\eta}{\mu+\eta}$. *Moreover, the market clearing condition implies*

$$\underbrace{\frac{\lambda_D^0 \eta F(\nu_l^a)}{\mu + \eta + \lambda_D^0}}_{\substack{\textit{Supply from} \\ \textit{principal}}} + \underbrace{\frac{\tilde{\lambda}_D^{al} \eta [F(\nu_l) - F(\nu_l^a)]}{\mu + \eta + \tilde{\lambda}_D^{al}}}_{\substack{\textit{Supply from} \\ \textit{agency}}} = \underbrace{\frac{\lambda_D^0 \mu [1 - F(\nu_h^a)]}{\mu + \eta + \lambda_D^0}}_{\substack{\textit{Demand from} \\ \textit{principal}}} + \underbrace{\frac{\tilde{\lambda}_D^{as} \mu [F(\nu_h^a) - F(\nu_h)]}{\mu + \eta + \tilde{\lambda}_D^{as}}}_{\substack{\textit{Demand from} \\ \textit{agency}}}. \tag{61}$$

Proposition 4 describes the distribution of owners and non-owners who trade with dealers, that is, with types $\nu \leq \nu_l$ and $\nu \geq \nu_h$. Owners $\nu \leq \nu_l^a$ choose principal trade, with measure determined by the arrival of dealer meetings, λ_D^0. Owners $\nu \in (\nu_l^a, \nu_l]$ choose agency trade, with measure determined by the arrival of dealer meetings combined with the arrival rate of access to the inter-dealer market $\tilde{\lambda}_D^{al}$. Similarly,non-owners $\nu \geq \nu_h^a$ choose principal trade, with measure determined by λ_D^0, and non-owners $\nu \in [\nu_h, \nu_h^a)$ choose agency trade, with measure determined by $\tilde{\lambda}_D^{as}$. The market clearing condition in (61) equates supply and demand of bonds in the inter-dealer market, constituted by dealers carrying a position after a principal trade and by customers in agency trade.

5 Equilibrium multiplicity

The results above imply a procedure for solving for an equilibrium. The procedure involves the determination of the equilibrium values of ν_l, ν_l^a, and p. To facilitate the exposition, consider the symmetric case, for which $\lambda_D^{as} = \lambda_D^{al} \equiv \lambda_D^a$, and so $\sigma_D^{al} = \sigma_D^{as} \equiv \sigma_D^a$, and $\tilde{c}^l = \tilde{c}^s \equiv \tilde{c}$.

According to lemmas 3 and 6, ν_h can be expressed as functions of $\nu_l \in (-\infty, \nu_s]$. Therefore, define the functions $G,\ H : (-\infty, \nu_s] \to \mathbb{R}$ as

$$G(\nu_l) = \frac{1}{2}\int_{\nu_l}^{g(\nu_l)} \frac{\sigma_C(\nu) - \sigma_D^a}{\lambda_D^a \theta_D \sigma_D^a} d\nu, \tag{62}$$

$$H(\nu_l) = \frac{1}{2}\int_{\nu_l}^{h(\nu_l)} \frac{\sigma_C(\nu) - \sigma_D^p}{\lambda_D^p \theta_D \sigma_D^p} d\nu, . \tag{63}$$

$g(\nu_l)$ yields the value of ν_h such that the measure of dealers in agency trade that want to sell is equal to the measure of those that want to buy, and analogously for $h(\nu_l)$ for principal trade. G and H are such that $G(\nu_s) = H(\nu_s) = 0$, and positive for ν_l sufficiently small.

Suppose that the parameters are such all trade with dealers is made with agency. According to lemma 6, an equilibrium ν_l solves

$$\tau = G(\nu_l). \tag{64}$$

After obtaining ν_l, we can obtain the other equilibrium variables through proposition 3. Given the equilibrium price p, define the principal-agency condition line $q(\nu)$ by

$$q(\nu) = \frac{\sigma_D^p \lambda_D^p - \sigma_D^a \lambda_D^a}{\sigma_D^p} p + \frac{\sigma_D^a \lambda_D^a}{\sigma_D^p} \tau - \frac{\sigma_D^a - \sigma_D^p}{\theta_D \sigma_D^p} \nu. \tag{65}$$

The value of $q(\nu_l)$ is equal to the relative gain from principal trade relative to agency trade, not taking into account the cost of principal trade. It is a linear function of ν because investors evaluate the asset linearly and because the interdealer price p is obtained competitively. As $\sigma_D^a > \sigma_D^p$, the slope of q is negative. The gain of principal trade declines as the valuation of investor increases. Owners gain less by selling the asset fast as ν increases.

According to proposition 3, owners $\nu \leq \nu_l$ sell in principal trade if and only if

$$\lambda_D^p \tilde{c} \leq q(\nu). \tag{66}$$

If there is a value ν_l^a that satisfies this equation with equality such that $\nu_l^a < \nu_l$ then we have found an equilibrium ν_l^a for which principal and agency trades coexist equilibrium. Agents $\nu < \nu_l^a$ engage in principal trade, agents $\nu_l^a \leq \nu < \nu_l$ engage in agency trade and agents $\nu_l \leq \nu \leq \nu_s$ engage in all-to-all trade with other customers. There are analogous thresholds ν_h, ν_h^a for of non-owners with $\nu \geq \nu_s$ that want to buy assets. Agents select themselves into different forms of trade.

If ν_l^a that satisfies (66) is such that $\nu_l^a > \nu_l$ then there is no agency trade. This situation is more likely if the cost of principal trade $\tilde{c}$ is low relative to the cost of agency trade τ. There is principal trade only in this case, and we use equation (63) to determine the equilibrium value of ν_l^a.

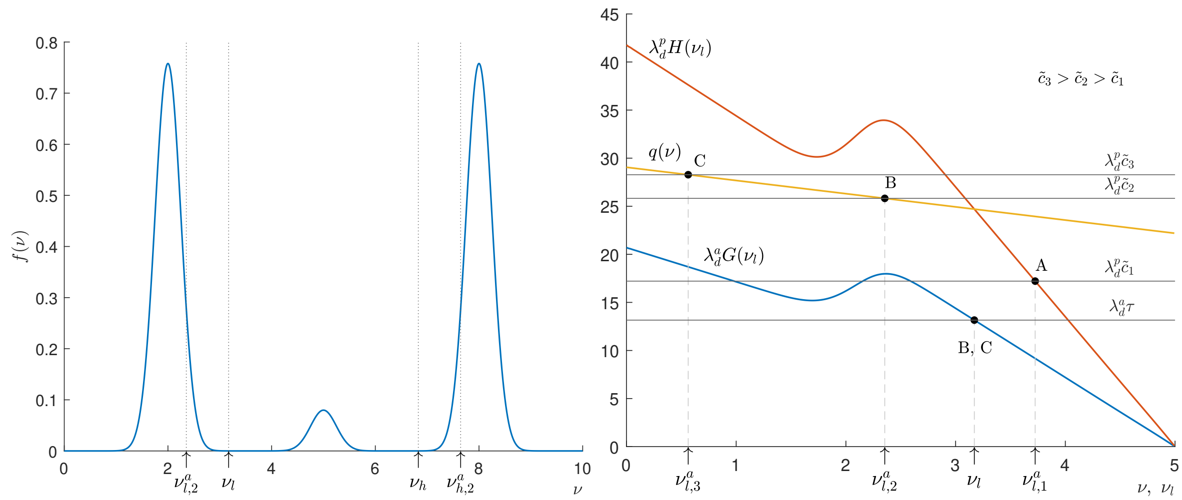
The equilibrium might not be unique. For small τ, equation (64) implies ν_l close its highest possible value, ν_s, and unique. However, depending on the distribution of investor types, we can have multiple equilibria as agents decide how to trade depending on their expectations for the number of agents on the other side of trade.

Figures 4 and 5 show how to obtain the equilibrium. They show cases of unique and multiple equilibria. The meeting rates are such that it is faster to find a dealer than finding another customer in A2A markets, $\lambda_C = 1$ and $\lambda_0 = 5$, and that it is easier to find a customer with the help of the dealer, $\lambda_1 = 3 > \lambda_C$. Panel 4a shows the distribution of investors for $\nu \in [0, 10]$. There is a concentration of investors with low and high types.[7]

Panels 4b-5b show different equilibrium patterns for low, median, and high *tau*, and different values of $\tilde{c}$. We have unique equilibrium for low τ. We have multiple equilibrium

[7]The distribution is a combination of three normal distributions $N(\mu, \sigma)$, $f(\nu) = 0.475N(2, 0.25) + 0.05N(5, 0.25) + 0.475N(8, 0.25)$. Other parameters are $r = 0.05$, $\mu = \eta = 0.15$, and $\theta_D = \theta_C^n = \theta_C^o = 0.5$.

in the agency or principal markets for median and high τ depending on the value of $\tilde{c}$. The panels show the functions G and H multiplied by λ_D^a λ_D^p so that they can be shown in the same graph and compared with the agency and principal costs. We explain each case below.[8]



(a) Density $f(\nu)$ and thresholds of equilibrium B

(b) Unique equilibrium for low τ

Figure 4: Low agency cost τ and an increase in principal flow cost $\tilde{c}$ (right). Equilibrium A, with low $\tilde{c}$, has no agency trade as $\nu_l < \nu_l^a$. Equilibria B and C have coexistence of principal and agency trade. Agency trade increases as the cost of principal trade $\tilde{c}$ increases. The market composition changes.

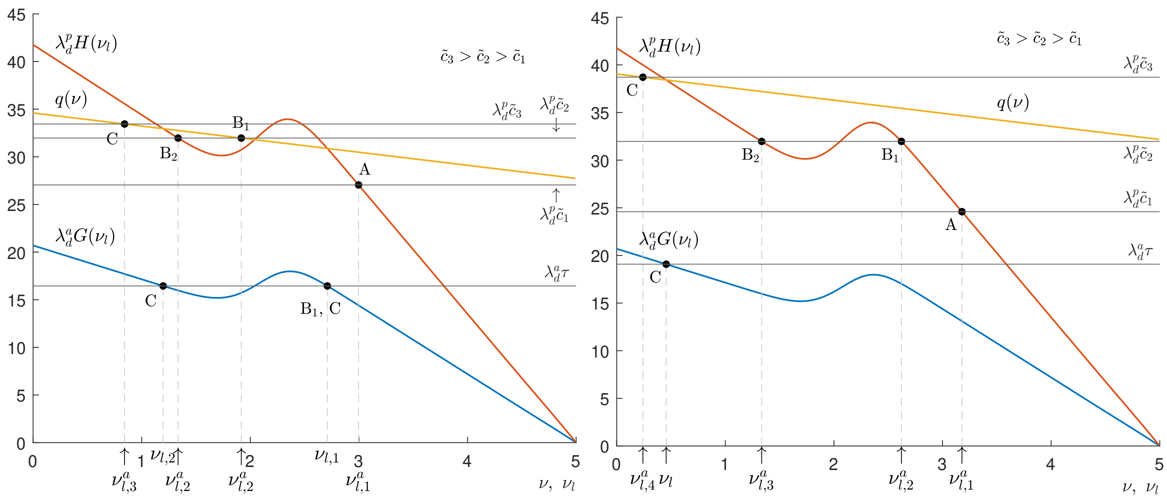
Panel 4b shows the case with low τ. Given τ and equation (64), an equilibrium ν_l is given by the intersection of $\lambda_D^a G$ and $\lambda_D^a \tau$. For $\tilde{c} = \tilde{c}_1$, the value of ν_l^a implied by the principal-agency condition line q is such that $\nu_l^a > \nu_l$. Therefore, the pair ν_l with this ν_l^a cannot be an equilibrium. The equilibrium is given by $\nu_{l,1}^a$, for which $\lambda_D^p H$ and $\lambda_D^p \tilde{c}_1$ intersect. This is equilibrium A. We have unique equilibrium with principal trade only.

As $\tilde{c}$ increases in panel 4b, we move to equilibria B and C. For these equilibria, $\nu_l^a < \nu_l$, where ν_l^a is such that $\lambda_D^p \tilde{c} = q(\nu_l^a)$. Agency trade coexists with principal trade in each case. Panel 4a shows the values of ν_l^a, ν_l, ν_h and ν_h^a of equilibrium B. We principal trade for $\nu \leq \nu_l^a$, agency trade in $(\nu_l^a, \nu_l]$, A2A trade in (ν_l, ν_h), and again agency and principal trade for $[\nu_h, \nu_h^a)$ and $\nu \geq \nu_h^a$. As the cost of principal trade increases, the market for agency trade increases.

Panel 5a shows the case with median τ. We have multiple equilibrium in this case for agency trade, as $\lambda_D^a \tau$ crosses $\lambda_D^a G$ in multiple points. The thresholds $\nu_{l,1}$, $\nu_{l,2}$ determine

[8]The panels show $\nu \in (-\infty, \nu_s]$ and they determine ν_l and ν_l^a. There are symmetrical panels for $\nu \in [\nu_s, \infty)$ to determine ν_h and ν_h^a.

stable equilibria and we concentrate on these values (we discuss stability later in this section). Equilibrium A occurs with low principal cost $\tilde{c}_1$ and has unique equilibrium with principal trade only, similar to panel 5b.



(a) Multiple equilibrium for median τ

(b) Multiple equilibrium for high τ

Figure 5: Multiple equilibrium with median and for high τ. Left: Cost $\tilde{c}_2$ implies two equilibria, one with principal and agency (B_1) and one with principal only (B_2). Cost $\tilde{c}_3$ has principal and two possible equilibria with agency trade. Right: Equilibrium B has multiple equilibria with principal only (no agency trade). The interaction between parameters imply different market structures. Principal trade can shrink abruptly in the case of an increase in $\tilde{c}$.

With $\tilde{c}_2$ in panel 5a, we have multiple equilibrium with agency or principal only. If the agency market is large, with $\nu_{l,1}$ at equilibrium B_1, then the principal-agency condition line q and $\tilde{c}_2$ determines $\nu_l^a < \nu_{l,1}$, which is the equilibrium threshold for the principal market. The market has agency and principal trade in coexistence (in addition to the A2A trade for $\nu \in (\nu_{l,1}, \nu_{h,1})$). If the agency market is small, with $\nu_{l,2}$, then q and $\tilde{c}_2$ would determine $\nu_l^a < \nu_{l,1}$, which rules out $\nu_{l,2}$ as equilibrium. The equilibrium $\nu_{l,2}^a$ is then determined by $\lambda_D^p H$ and q, at B_2, and has principal only. The market has two possible equilibria: B_1 with agency and principal, and B_2 with principal only. A change from B_1 to B_2 implies less revenues for dealers either from principal trade or from facilitating agency trades. It implies an abrupt loss of business for dealers.

As $\tilde{c}$ increases to $\tilde{c}_3$ in panel 5a we have multiple equilibria for the agency market and unique equilibrium for the principal market, C. q and $\tilde{c}_3$ determine $\nu_{l,3}^a$, which is smaller than $\nu_{l,2}$ and $\nu_{l,1}$. These two values are then equilibrium thresholds for the agency market and $\nu_{l,3}^a$

is an equilibrium threshold for the principal market. If the equilibrium changes from $\nu_{l,1}$ and $\nu_{l,2}$ then the set of principal trades would not change, but the set of agency trades facilitated by dealers would shrink from $(\nu^a_{l,3}, \nu l, 1]$ to $(\nu^a_{l,3}, \nu l, 2]$. More investors would coordinate in the A2A market. Dealers would lose a substantial amount of brokerage fees.

Panel 5b shows multiple equilibria in the principal market. τ is high and consequently the agency market is small. We have a unique ν_l. On the other hand, as $\tilde{c}$ increases, we can have unique equilibrium with principal only (A), multiple equilibria with principal only (B_1, B_2), and unique equilibrium with principal and agency (C). For equilibria B_1, B_2, the principal-agency condition line q with $\lambda^p_D \tilde{c}_2$ implies $\nu^a_l > \nu_l$ which rules out the agency market. The equilibrium ν^a_l is then determined by $\lambda^p_D H$ and $\lambda^p_D \tilde{c}_2$, which implies two stable values for ν^a_l. We then have principal only but the volume of principal trades can decrease abruptly for A2A if the equilibrium changes from $\nu^a_{l,2}$ to $\nu^a_{l,3}$.

Multiple equilibria arise because G or H may not be monotone. The non-monotonicity occurs because more customers search for customers in A2A if they are convinced that others will follow this strategy. When they do so, the probability of matching is higher and the gain of searching in A2A increases. As the figures above show, multiple equilibrium happens when intermediation costs are sufficiently high.

Proposition 5. *For a given $\tilde{c}$, there exists $\bar{\tau} > 0$ such that a regular equilibrium is unique for $\tau \in [0, \bar{\tau})$. Analogously, for a given τ, there exists a $\bar{\tilde{c}}$ such that a regular equilibrium is unique for $\tilde{c} \in [0, \bar{\tilde{c}})$.*

The reinforcement of searching when others search (strategic complementarity) is not strong enough to generate multiplicity when intermediation costs are small. Assumption 1 implies that the technology of dealers is superior to A2A net of trading costs. If τ is small, no matter how many customers search for customers, it is still preferable to search for dealers. Multiplicity happens only if τ is large enough so that the measure of customers searching for customers affects the decision on the trading mode.

About stability, we argue that an equilibrium threshold ν is stable when it is determined in a region where G or H are decreasing. We focus next on the argument for G, as is the same for H.

An interpretation of G is that it is a proxy for the expected difference in valuation of an owner with ν_l and a non-owner with ν_h, both using A2A trade. A large G implies that a non-owner might need to pay a substantial amount to buy the asset. If $G(\nu_l) > \tau$, it is better to switch from A2A to dealer agency trade. A buyer might pay τ in a DC trade, but the total payment would still be smaller than the expected price to pay in A2A trade. A switch from A2A to dealer-agency implies an increase in ν_l and a smaller interval (ν_l, ν_h).

$G(\nu_l)$ decreases with ν_l if the valuation of agents that engage in A2A trades gets closer to each other as ν_l increases. This is the case in panel 5a for equilibria B_1 and C. An increase in τ decreases the gain of agency trades. The equilibrium ν_l would decrease and the set of A2A trades would increase. Similarly, an increase in λ_C makes A2A trades more effective. It implies a downward shift in G. For a constant τ, it would decrease ν_l and increase the set of dealer agency trades.

There is a point in between equilibria B_1 and C, however, that determines an additional equilibrium. G intersects with τ, but G is increasing in this region. This region includes the higher density of utility types shown in panel 5a. An increase in τ would increase the equilibrium ν_l. Dealer agency trades would increase with τ. Similarly, an increase in λ_C, would shift G downward and decrease the set of A2A trades.

These counterfactual effects are related with the instability of equilibrium for intermediary value of ν_l. For this equilibrium, suppose that a small set of agents to the left of ν_l switch their decisions from dealer agency trade to A2A trade. The set of agents in A2A would increase to $(\nu_l - \epsilon, \nu_h')$, where $\nu_h' = g(\nu_l - \epsilon)$. We would then have $G(\nu_l - \epsilon) < G(\nu_l) < \tau_2$, which implies that it is beneficial for an agent to the left of $\nu_l - \epsilon$ also to switch from dealer-agency to A2A. All agents to the left would behave in the same way, which would increase further the set of agents in CC trades, until the equilibrium with $\nu_{l,2}$ in C reached.

The equilibrium is stable for B_1 and C. A switch of a small set of agents to the left of $\nu_{l,2}$ from dealers to A2A would increase G. It would be better to return to trade with dealers. The same reasoning can be applied to a switch from A2A to dealers to the right of $\nu_{l,2}$ and also to the other stable equilibrium at $\nu_{l,1}$.

A stable equilibrium is therefore associated with a region where G is decreasing; and an unstable equilibrium with a region where G is increasing. In regions where G is decreasing, small perturbations in A2A or in dealer agency trades would make agents return to their previous decisions on the counterparty. In regions where G is increasing, such perturbations would make agents switch the trading counterparty permanently toward a new equilibrium. We then focus on regions where G is decreasing and, therefore, have stable equilibria.

6 Market Composition and Liquidity

The US corporate bond market is experiencing a shift in how trades are executed. The model of dealers holding large inventories and engaging in principal trade is expanding towards agency trades, where dealers primarily act as intermediaries between buyers and sellers, and all-to-all trades, where customers access a broader range of counterparties directly. Several sources have documented this transition. Choi et al. (2024) provide evidence for the shift towards agency trades using TRACE data, documenting a significant increase in the fraction of agency trades in corporate bonds since 2011, and Kargar et al. (2023a) provide evidence for the shift towards all-to-all trades using MarketAxess data, documenting that the platform represented approximately 21% of the total trade volume in corporate bonds by the third quarter of 2022. It is also evident in the declining net position of primary dealers in corporate debt instruments, shown in Figure 6. The median monthly net position of primary dealers in corporate debt instruments was over \$100 billion before 2011, decreased to less than \$40 billion between 2013 and 2020, and has recently reached below \$20 billion.

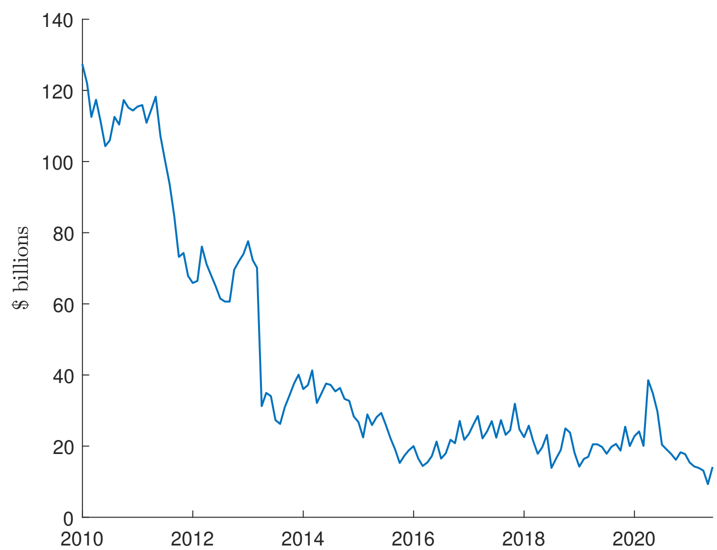


Figure 6: Primary dealers median net monthly position in corporate debt. Data: see appendix.

Market participants in the US corporate bond market attribute this shift largely to two factors. The 2010 Dodd-Frank Act, which increased capital requirements for banks, and the establishment of electronic bond markets, which made it easier for dealers to perform agency trading and fostered the growth of all-to-all trading platforms.

These factors exert competing effects on market efficiency and liquidity. Financial regulations increase dealers' holding costs, leading to longer execution time, wider bid-ask spreads, and lower trade volume. The rise of electronic trading and all-to-all platforms increases access to counterparties, leading to shorter execution times, tighter bid-ask spreads, and higher trade volume. We use our model to understand the impact of these changes. Specifically, we study the impact of changes in the model parameters governing trade speed and cost on dealers' net position, bid-ask spread and turnover.

6.1 Dealers' asset holdings

A key indicator of dealer activity and market structure is the *net assets holdings* of dealers. In our model, dealers can hold long (+1) or short (-1) positions in the asset after engaging in principal trades. Dealers involved in agency trades are pure intermediaries and do not contribute to asset holdings. Therefore, the dealers' net asset holdings is

$$H_D = \pi_1 - \pi_{-1} \tag{67}$$

where π_1 is the measure of dealers with a long position after buying the asset from a customer in a principal trade, and π_{-1} is the measure of dealers with a short position after selling the asset to a customer in a principal trade. A positive value of H_D implies that dealers hold more assets in inventory than they have shorted, and a negative value of H_D implies that dealers have shorted more assets than they hold in inventory.

Net asset holdings (67) capture the overall directional exposure of dealers, but it is the *gross position* that provides a measure of the total scale of dealer involvement in principal trading. We define dealers' gross position as

$$G_D = \pi_1 + \pi_{-1} \tag{68}$$

where π_1 and π_{-1} retain their definitions from equation (67). In contrast to net holdings, the gross position is insensitive to the balance between buying and selling by dealers; it only reflects the magnitude of their principal trading activity. A higher value of G_D indicates a greater overall dealer activity to provide immediacy through principal trades.

Proposition 6. *Consider a region with a unique regular equilibrium in which agency trade is active for both buying and selling bonds. Then,*

(a) Dealers' net position is decreasing in c^l and increasing in c^s.

(b) Dealers' gross position is decreasing in c^l and c^s.

(c) Dealers' net and gross positions do not depend on λ_C.

Consider a region with a unique regular equilibrium in which agency trade is inactive. Then,

(d) Dealers' net position equals zero.

(e) Dealers' gross position is decreasing in c^l and c^s.

(f) Dealers' gross position is decreasing in λ_C.

When the cost for dealers to hold long positions (c^l) increases, principal trades where customers sell to dealers become less attractive. To see this, note that the set of customers selling to dealers in principal trades are those of type $\nu \leq \nu_l^a$. From Proposition 3, we have

$$\underbrace{p\theta_D(\sigma_D^p\lambda_D^p - \sigma_D^{al}\lambda_D^{al})}_{\substack{\text{Gain from payment} \\ \text{in fast execution}}} - \underbrace{\theta_D\sigma_D^p\lambda_D^p\tilde{c}^l}_{\substack{\text{Principal} \\ \text{trade cost}}} = \underbrace{(\sigma_D^{al} - \sigma_D^p)\nu_l^a}_{\substack{\text{Gain from value} \\ \text{in slow execution}}} - \underbrace{\theta_D\sigma_D^{al}\lambda_D^{al}\tau}_{\substack{\text{Agency} \\ \text{trade cost}}},$$

where $\tilde{c}^l = \frac{c^l}{\lambda_D^1}$. An increase in c^l reduces the net benefit of fast trade execution. As a result, ν_l^a has to decrease to account for the marginal types that strictly prefer agency trade after the change. This shift leads to a decrease in π_1 and, consequentially, a decrease in H_D and G_D. The effect of an increase in c^s is analogous. An increase in c^s reduces the net benefit of fast trade execution. As a result, ν_h^a has to increase and π_{-1} decreases. This decrease, in turn, causes a decrease in gross holdings G_D. The effect on net holdings is the opposite since it is defined as $H_D = \pi_1 - \pi_{-1}$. So a decrease in π_{-1} leads to an increase in H_D.

6.2 Bid-ask spread

Bid-Ask Spread in Principal Trade The bid-ask spread in principal trade is the average difference in principal trade between how much customers pay when buying an asset and how much customers receive when selling an asset. That is,

$$\mathrm{BA}_P = p_{D,P}^{buy} - p_{D,P}^{sell} \tag{69}$$

where $p_{D,P}^{buy}$ and $p_{D,P}^{sell}$ are the average price paid and received by customers in principal trades. In a regular equilibrium, these are

$$\begin{aligned}
p_{D,P}^{sell} &= \frac{\int_{-\infty}^{\nu_l^a}[\theta_D W^l + (1-\theta_D)\Delta(\nu)]d\Phi^o(\nu)}{\Phi^o(\nu_l^a)} = \frac{\int_{-\infty}^{\nu_l^a}\left[\theta_D \frac{\lambda_D^1[p-\tilde{c}^l]}{r+\lambda_D^1} + (1-\theta_D)\Delta(\nu)\right]d\Phi^o(\nu)}{\Phi^o(\nu_l^a)} \\
&= (1-\theta_D)\mathbb{E}[\Delta(\nu)|\nu \leq \nu_l^a] + \theta_D \frac{\lambda_D^1[p-\tilde{c}^l]}{r+\lambda_D^1}, \qquad \text{and} \qquad (70) \\
p_{D,P}^{buy} &= \frac{\int_{\nu_h^a}^{\infty}[\theta_D W^s + (1-\theta_D)\Delta(\nu)]d\Phi^n(\nu)}{\Phi^n(\infty)-\Phi^n(\nu_h^a)} = \frac{\int_{\nu_h^a}^{\infty}\left[\theta_D \frac{\lambda_D^1(p+\tilde{c}^s)}{r+\lambda_D^1} + (1-\theta_D)\Delta(\nu)\right]d\Phi^n(\nu)}{\Phi^n(\infty)-\Phi^n(\nu_h^a)} \\
&= (1-\theta_D)\mathbb{E}[\Delta(\nu)|\nu \geq \nu_h^a] + \theta_D \frac{\lambda_D^1(p+\tilde{c}^s)}{r+\lambda_D^1}. \qquad (71)
\end{aligned}$$

Substituting (70) and (71) into (69), we get

$$\mathrm{BA}_P = (1-\theta_D)\left\{\mathbb{E}[\Delta(\nu)|\nu \geq \nu_h^a] - \mathbb{E}[\Delta(\nu)|\nu \leq \nu_l^a]\right\} + \theta_D \frac{c^s + c^l}{r+\lambda_D^1}, \tag{72}$$

where we used that $\tilde{c}^s = \frac{c^s}{\lambda_D^1}$ and $\tilde{c}^l = \frac{c^l}{\lambda_D^1}$.

Bid-Ask Spread in Agency Trade The bid-ask spread in agency trade, denoted by BA_A, is defined analogously to the principal trade bid-ask spread. It is the average difference between how much customers pay when buying an asset and how much they receive when selling an asset in agency trades. That is,

$$\mathrm{BA}_A = p_{D,A}^{buy} - p_{D,A}^{sell} \tag{73}$$

where $p_{D,A}^{buy}$ and $p_{D,A}^{sell}$ are the average prices paid and received by customers.

In an agency trade, the dealer acts as an intermediary, connecting an asset owner with a buyer in the interdealer market. When we write the value function of owners we assume that the owner pays the dealer a fraction $1 - \theta^D$ of the gains from this service. In practice, however, the payment should be made at the time of the trade — not when the customer meets the dealer. That is, the dealer charges a fee, denoted by $f^o(\nu)$, to the owner of type ν so the final price in an agency trade is $p - \tau - f^o(\nu)$. The same applies to non-woners when buying an asset. In this case, the final price in an agency trade is $p + \tau + f^n(\nu)$. The fees $f^o(\nu)$ and $f^n(\nu)$ are formally defined implicitly by

$$\theta_D[\tilde{V}_D^o(\nu) - V_D^o(\nu)] = \tilde{\tilde{V}}_D^o(\nu) - V_D^o(\nu) \quad \text{and} \quad \theta_D[\tilde{V}_D^n(\nu) - V_D^n(\nu)] = \tilde{\tilde{V}}_D^n(\nu) - V_D^n(\nu)$$

where

$$\begin{aligned} r\tilde{\tilde{V}}_D^o(\nu) &= \nu - \mu[\tilde{\tilde{V}}_D^o(\nu) - V^n(\nu)] + \lambda_D^1 \max\{p - \tau - f^o(\nu) - [\tilde{\tilde{V}}_D^o(\nu) - V^n(\nu)], 0\}, \\ r\tilde{\tilde{V}}_D^n(\nu) &= \eta[V^o(\nu) - \tilde{\tilde{V}}_D^n(\nu)] + \lambda_D^1 \max\{V^o(\nu) - \tilde{\tilde{V}}_D^n(\nu) - (p + \tau + f^n(\nu)), 0\}. \end{aligned}$$

We then obtain that, in a regular equilibrium, the fee paid to the dealer upon successful execution of an agency trade are

$$f^o(\nu^o) = (1 - \theta_D^o)[p - \tau - \Delta(\nu^o)] \quad \text{and} \quad f^n(\nu^n) = (1 - \theta_D^n)[\Delta(\nu^n) - p - \tau] \tag{74}$$

by an owner of type $\nu^o \in \Omega_D^{o,a}$ and non-owner of type $\nu^n \in \Omega_D^{n,a}$, where

$$\theta_D^o = \frac{\theta_D(\lambda_D^0 + r + \mu + \lambda_D^1)}{r + \mu + \lambda_D^0\theta_D + \lambda_D^1} \quad \text{and} \quad \theta_D^n = \frac{\theta_D(\lambda_D^0 + r + \eta + \lambda_D^1)}{r + \eta + \lambda_D^0\theta_D + \lambda_D^1}.$$

We are now in a position to compute $p_{D,A}^{sell}$ and $p_{D,A}^{buy}$. In a regular equilibrium, these are

$$p_{D,A}^{sell} = \frac{\int_{\nu_l^a}^{\nu_l}[p - \tau - f^o(\nu)]d\Phi^o(\nu)}{\Phi^o(\nu_l) - \Phi^o(\nu_l^a)} = \theta_D^o(p - \tau) + (1 - \theta_D^o)\mathbb{E}\big[\Delta(\nu)|\nu_l^a \le \nu \le \nu_l\big], \tag{75}$$

$$p_{D,A}^{buy} = \frac{\int_{\nu_h}^{\nu_h^a}[p + \tau + f^n(\nu)]d\Phi^n(\nu)}{\Phi^n(\nu_h^a) - \Phi^n(\nu_h)} = \theta_D^n(p + \tau) + (1 - \theta_D^n)\mathbb{E}\big[\Delta(\nu)|\nu_h \le \nu \le \nu_h^a\big]. \tag{76}$$

Substituting (75) and (76) into (73), we get

$$\begin{aligned}\mathrm{BA}_A &= (\theta_D^n - \theta_D^o)p + (\theta_D^n + \theta_D^o)\tau \\ &\quad + (1-\theta_D^n)\mathbb{E}[\Delta(\nu)|\nu_h \le \nu \le \nu_h^a] - (1-\theta_D^o)\mathbb{E}[\Delta(\nu)|\nu_l^a \le \nu \le \nu_l]. \end{aligned} \tag{77}$$

Agency *vs.* Principal Trade

Proposition 7 (Agency vs. Principal Trade Prices)**.** *Consider a region with a unique regular equilibrium in which agency trade is active for both buying and selling bonds. Then the following relationships holds.*

(a) The average price customers pay to buy an asset in an agency trade, $p^{buy}_{D,A}$, is lower than the average price they pay in a principal trade, $p^{buy}_{D,P}$.

(b) The average price customers receive to sell an asset in an agency trade, $p^{sell}_{D,A}$, is higher than the average price they receive in a principal trade, $p^{sell}_{D,P}$.

(c) The bid-ask spread charged by dealers in agency trades, BA_A*, is smaller than the bid-ask spread charged by dealers in principal trades,* BA_P*.*

Proposition 7 highlights the trade-off between trade speed and cost present in agency and principal trading. Parts (a) and (b) show that agency trades offer better prices: buyers pay a lower average price in agency trades ($p^{buy}_{D,A} < p^{buy}_{D,P}$), while sellers receive a higher average price ($p^{sell}_{D,A} > p^{sell}_{D,P}$). Principal trades offer immediacy — customers can trade with the dealer upon a meeting — but this convenience comes at a price premium for buyers or a discount for sellers. Agency trades, while slower due to the search process, offer better prices because the dealer acts purely as an intermediary, bearing no inventory risk. Part (c) of the proposition formalizes this price difference as a smaller bid-ask spread in agency trades ($\mathrm{BA}_A < \mathrm{BA}_P$).

Dealers Bid-Ask Spread Having established the differences between agency and principal trade prices, we now turn to the overall bid-ask spread in the market, which reflects the combined effect of both trading mechanisms. We define the average dealer bid-ask spread, denoted by BA, as the difference between the average price paid by customers when buying an asset and the average price received by customers when selling an asset, considering *all* dealer-intermediated trades (both principal and agency). Formally,

$$p_D^{sell} = \frac{\lambda_D^0 \Phi^o(\nu_l^a) p_{D,P}^{sell} + \lambda_D^1[\pi_0^o(\nu) - \pi_0^o(\nu_l^a)] p_{D,A}^{sell}}{\lambda_D^0 \Phi^o(\nu_l^a) + \lambda_D^1[\pi_0^o(\nu) - \pi_0^o(\nu_l^a)]} \tag{78}$$

$$p_D^{buy} = \frac{\lambda_D^0 [1 - s - \Phi^n(\nu_h^a)] p_{D,P}^{buy} + \lambda_D^1[\pi_0^n(\nu_h^a) - \pi_0^n(\nu)] p_{D,A}^{buy}}{\lambda_D^0 [1 - s - \Phi^n(\nu_h^a)] + \lambda_D^1[\pi_0^n(\nu_h^a) - \pi_0^n(\nu)]}. \tag{79}$$

Taking the difference between (78) and (79) we get

$$\begin{aligned} \text{BA} = {} & p_D^{buy}(\theta_D^n - \theta_D^o)p + (\theta_D^n + \theta_D^o)\tau \\ & + (1 - \theta_D^n)\mathbb{E}[\Delta(\nu)|\nu_h \le \nu \le \nu_h^a] - (1 - \theta_D^o)\mathbb{E}\left[\Delta(\nu)|\nu_l^a \le \nu \le \nu_l\right]. \end{aligned} \tag{80}$$

where π_1^{buy} represents the measure of dealers buying the asset in principal trade, π_{-1}^{sell} the measure of dealers selling in principal trade. This measure is a weighted average, as it reflects the relative frequencies of principal, agency, and all-to-all (A2A) trades. The numerator sums the total value paid or received by customers across all trade types, and the denominator sums the total number of trades of each type. The bid-ask spread BA provides a comprehensive measure of transaction costs. It encompasses the pricing impact of dealer immediacy provision in principal trades and dealer-facilitated matching in agency trades, as well as direct customer-to-customer trading in A2A trades.

6.3 Turnover

Turnover is the ratio of trade volume to the outstanding bond amount. High turnover is associated with greater liquidity. It suggests that it is easy and cheap to find counterparties. In our model, turnover is expressed as

$$\begin{aligned} T = {} & \frac{\lambda_D^1 \left(\pi_1 + \bar{\pi}_0^o\right) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu})\, d\Phi^o(\nu)}{\pi_1 - \pi_{-1} + \int d\Phi^o(\nu)} \\ = {} & \frac{2\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \, d\Phi^n(\tilde{\nu})\, d\Phi^o(\nu)}{\eta/(\mu + \eta)}. \end{aligned} \tag{81}$$

The volume of assets sold by customers to dealers is $\lambda_D \Phi^o(\nu_l)$ and the volume of assets bought by customers from dealers is $\lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)]$. As market clearing implies that $\lambda_D \Phi^o(\nu_l) = \lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)]$, we have that the total volume of bonds traded between customers and dealers is $2\lambda_D \Phi^o(\nu_l)$. The total volume of bonds traded between customers is $\lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)$. Finally, the amount of bonds outstanding is $s = \int d\Phi^o(\nu) = \frac{\eta}{\mu+\eta}$.

Empirically, turnover in the corporate bond market has declined since the 2008 financial crisis, mirroring the trend observed in dealer inventories and consistent with the model's predictions when dealer intermediation costs increase. Proposition 8 provides theoretical support. It shows that turnover is always decreasing in the the intermediation cost τ in a neighborhood of $\tau = 0$.

Proposition 8. *Turnover is always decreasing in τ in a neighborhood of $\tau = 0$.*

In conclusion, the changing composition of the corporate bond market, with the increased prevalence of customer-to-customer trades, has profound implications for market liquidity. While traditional measures based on transaction data might suggest improvements in liquidity, a more comprehensive analysis that accounts for search costs and the evolving roles of dealers and customers reveals a more complex reality. The model presented in this paper provides a valuable framework for understanding these intricate dynamics and underscores the importance of considering factors beyond observed transaction prices and volumes when evaluating market liquidity, especially in the context of evolving market structures and regulatory landscapes.

7 Observed changes in the corporate bond market

7.1 Trade composition and perception of illiquidity

Our first observation is the change in the composition of trades and increased perception of illiquidity in the corporate bond market. As documented by Choi et al. (2024), after the regulations that followed the 2008 financial crisis, it is more common to find trades for which customers are matched with other customers instead of trades for which dealers use their inventory to provide liquidity. Dealers facilitate both forms of trade. However, when customers are matched with other customers, customers provide liquidity. In this case, the dealer does not use its inventory of bonds. It is a customer that provides liquidity to other customers either as a seller or a buyer.[9]

[9] See Dyskant et al. (2025) for more empirical evidence on the corporate bond market and for changes in the liquidity premium.

The changes in the composition of trades have been connected with the enactment of regulations that affect depository institutions, such as banks, with access to the Federal Reserve as a lender of last resort or to FDIC insurance (Adrian et al. 2017, Bao et al. 2018, Choi et al. 2024; Duffie 2012 pointed out some risks of the new regulations). Especially, the Volcker rule prohibits banks from engaging in proprietary trading, that is, trading that uses the inventory of assets purchased earlier with the intention of profiting from a higher sale price. The Volcker rule is part of the Dodd-Frank Wall Street Reform and Consumer Protection Act. The Dodd-Frank act was enacted in July 2012. The Volcker rule was put into effect in July 2015 after a period of transition. The objective of the Volcker rule is to limit risk taking of protected institutions. However, as the rule prohibits proprietary trade, it decreases incentives of maintaining an inventory of assets.

Choi et al. (2024) classify trades as being the result of a match customer-customer (DC-DC), a match customer-dealer intermediated by an interdealer (DC-ID), and inventory trades. They focus on over-the-counter trades in the corporate bond market. The classification is made with TRACE data, using dealer identifiers, counterparty pair types, and the time record of the trades. DC-DC and DC-ID trades have matches identified within a period of 15 minutes. Inventory trades are not matched with the opposing side, which implies that the asset is held after the trade as inventory. Since 2011, they find an increase in the fraction of customer-customer trades.

Customer-customer trades require a longer search and matching process for the trade execution. Suppose that a customer contacts a dealer to sell an asset. This customer demands liquidity. Conventionally, the dealer would buy the asset and provide liquidity. Instead, especially for trades equal to or larger than 1 million dollars, it is now more frequent that the dealer uses its relationships with other clients to find a customer willing to purchase the asset. The second customer found by the dealer provides liquidity. This process takes time. Kargar et al. (2021) examine corporate bond market liquidity and trade composition during COVID. They find an increase in the costs of customer-dealer trades and a shift of customers toward slower customer-customer trades.[10]

[10]However, they do not investigate the effects of regulations and only imply that the decrease in the liquidity provision of dealers during the crisis might have been a consequence of regulatory restrictions. Their model is also silent about the connection between regulations and measures of liquidity.

An evidence of the value of the provision of liquidity is that customers are compensated for it (Choi et al. 2024). Customers that provide liquidity by buying the asset pay smaller, even negative spreads. The same happens for customers that provide liquidity by selling the asset. Giannetti et al. (2023) find that bond mutual funds engage more frequently in liquidity provision since 2015, and that the performance of funds with strategies of liquidity provision has improved. Rapp and Waibel (2023) show that regulatory costs are associated with the use of the client network for the provision of liquidity. As we discuss below, the decrease in spreads charged to customers implies a decrease in bid-ask spreads and an improvement of liquidity measures, even though these DC-DC trades could take longer to be executed.

There is evidence that the search process can be costly. Transactions datasets such as TRACE contain only the final outcome of successful transactions. It is then not possible to measure the duration of the whole search process. Using data from electronic platforms, Kargar et al. (2023b) find that a substantial number of requests for quote are not promptly fulfilled. If the quote is not fulfilled initially, it takes on average from 2 to 3 days for a trade to be finalized. Another indicator of the need to match customers is the advent of electronic platforms to facilitate matching between trade counterparties (Hendershott et al. 2021).

The Volcker rule does not allow proprietary trading, but allows trading to facilitate transactions that were driven by customers. The law recognizes the role of dealers in the functioning of markets. Dealers cannot transact in a way intended to make profits based on the increase in the price of the asset, but they can profit from bid-ask spreads. As a result, a change in the market structure, with more frequent customer liquidity provision, would imply higher transaction costs for those trades that are executed with the inventory of dealers.

In fact, Choi et al. (2024) find that inventory trades have a transaction cost 60% higher than before the financial crisis. According to the classification above, inventory trades do not require a match of another dealer or customer to be executed. These trades are faster to be finalized. Therefore, the higher transaction cost reflects a higher premium on immediacy after the change in the market structure.

Early evidence that the new regulations affected markets was shown by Adrian et al. (2017) and Bao et al. (2018). Adrian et al. (2017) found that the ability to intermediate customer trades of affected institutions decreased. Bao et al. (2018) note that dealers affected by the

Volcker rule have been the main liquidity providers. They found that the illiquidity of bonds in time of stress has increased after the Volcker rule. As stated above, there was an increase in the fraction of liquidity provided by customers, but only with a more costly matching procedure. The increase in illiquidity during stress events can be explained by a change in the structure of markets toward costly matching.

There is therefore evidence that the structure of the corporate bond market has changed toward the prearrangement of trades between customers. This prearrangement is made to save on inventory of securities. The complete trade from the first contact until the final transaction becomes costly and protracted.

The changes in market structure, however, are not fully captured by standard measures of illiquidity. These measures do not take into account the time for the arrangement of matches. They use recorded prices at the final moment of the trade. We next discuss the behavior of the illiquidity measures over time.

7.2 Illiquidity measures

Our second observation is the improvement of the illiquidity measures since 2008. This improvement is surprising given the changes in the market structure, as discussed above. Trade in over-the-counter markets has moved toward prearranged matching of customers instead of a faster trade using existing dealer inventory. As these trades take longer to be executed, it is surprising to observe an improvement of measured illiquidity. As we argue later, the source of the difference is the fact that these measures use observed trading records. We later offer an alternative measure of illiquidity implied by the model in section 2.

We discuss the behavior of two measures of illiquidity: the γ measure, proposed by BPW, and the Amihud measure, proposed by (Amihud, 2002). Figure 7 shows the evolution of the measures over time.

The γ measure (BPW) is given by the covariance of subsequent price changes. The γ_i measure for bond i is defined as

$$\gamma_i = -\mathrm{Cov}(\Delta p_{it}, \Delta p_{it+1}), \tag{82}$$

where $\Delta p_{it} = p_{it} - p_{it-1}$ and p_{it} is the logarithm of the clean price P_{it} of bond i on trade

t. The clean price is the bond price minus accrued interest since the last coupon payment. We require a bond to have at least ten pairs of consecutive annualized-returns to estimate γ_i. The objective of the measure is to extract a transitory component from observed prices. This transitory component is interpreted as the impact of illiquidity, as efficient markets with no trading frictions imply uncorrelated returns.

In addition to γ, we estimate the Amihud measure (Amihud 2002). The Amihud measure for each bond is given by the average of absolute returns divided by the volume of trades,

$$\mathrm{AMD}_{id} = \frac{1}{N_{id}} \sum_{j=1}^{N_{id}} \frac{|r_{ij}|}{V_{id}}, \tag{83}$$

where N_{id} is the number of available returns r_{ij} of bond i on day d, and V_{id} is the volume of trade of bond i on day d in millions of dollars. We require at least two trades on each day to estimate AMD_{id}.

High Amihud measure implies high price change per unit of volume, that is, high impact or order flow. Liquid markets should not show large changes in price relative to volume. Therefore, a high Amihud measure is interpreted as lack of market liquidity. Table 1 shows the correlations between γ, Amihud and other variables.[11]

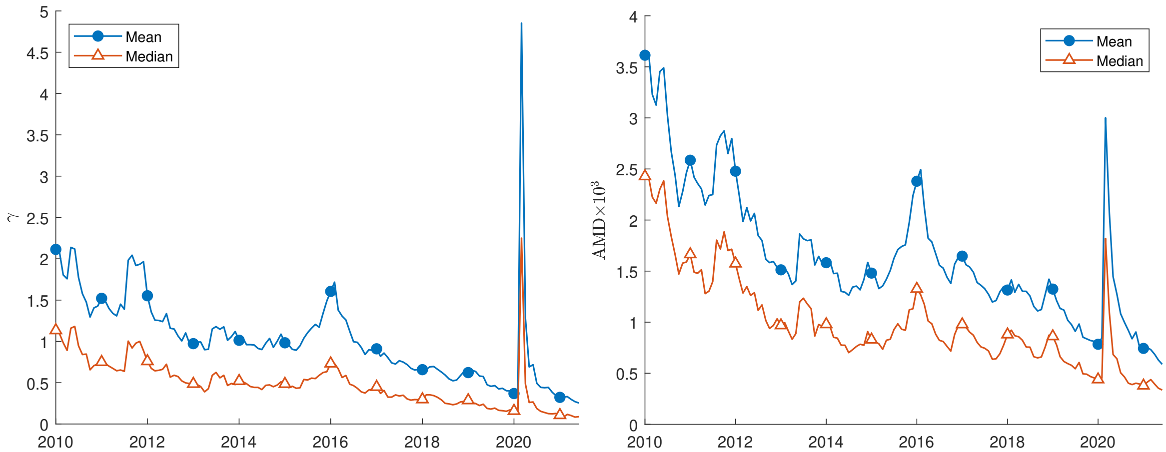
Table 1: Correlations between illiquidity measures and other variables

	γ	AMD	Spread	CDS	Volume	Frequency	Maturity	Age	Turnover	ZTD
γ	1.00									
AMD	.466	1.00								
Spread	.385	.444	1.00							
CDS	.290	.347	.816	1.00						
Volume	−.002	−.055	.040	.056	1.00					
Frequency	.047	.196	.146	.140	.420	1.00				
Maturity	.163	.149	.092	−.026	.097	−.052	1.00			
Age	.017	.109	.079	.056	−.202	−.001	−.075	1.00		
Turnover	.013	−.008	.125	.127	.588	.303	.110	−.209	1.00	
ZTD	−.051	−.198	−.080	−.088	−.199	−.356	.084	.016	−.034	1.00

Correlations between our main illiquidity measures, γ and AMD, and other commonly-used liquidity metrics, the spread, and the CDS. Data description in appendix B. Spread is the corporate bond yield spread with respect to the US Treasury with the same maturity (appendix B). Maturity is the issue's time to maturity. Maturity and age are calculated in years at the last business day of each month. Turnover is the traded volume divided by the amount outstanding. ZTD is the percentage of zero-trading days.

[11] Additional measures of illiquidity are given, among others, by Mahanti et al. (2008) and Dick-Nielsen et al. (2012). Mahanti et al. present a liquidity measure based on the accessibility of the issues. Dick-Nielsen et al. introduce a measure computed by an average of different illiquidity measures.

We define a measure of aggregate market level illiquidity over time by taking the median, mean or volume-weighted average of bond measures in each cross-section. Figure 7 shows the aggregate measures for the corporate bond market γ and AMD over time. γ and AMD increase when liquidity worsens. Both illiquidity measures strongly increased during the financial crisis. After the crisis, liquidity gradually improved. The covid shock was large but brief and did not affect the trend.



(a) γ

(b) AMD

Figure 7: Illiquidity measures γ and AMD. The decline in the illiquidity measures indicate improvement in market liquidity. The covid period shows a peak, but it does not affect the trend.

According to our first point, liquidity provision by dealers has been replaced by customer liquidity provision. At the same time, however, illiquidity indicators declined. Figure 6 shows the net position of Primary Dealers in corporate debt instruments and illiquidity over time. It shows the decrease in inventories together with a decrease in γ. We explain this paradox with the model of section 2.

Also used as a measure of liquidity, turnover has declined after the 2008 financial crisis. We calculate daily turnover for an individual bond by dividing the amount traded in each day by the amount outstanding at the end of the corresponding month. We then define the monthly turnover measure for an individual bond by the median of its daily turnover. Figure 8 shows the median turnover of all bonds as an aggregate measure (the behavior looks similar for the mean turnover). The figure also shows the moving average of 12 months.

The turnover rate decreased after January 2010. The 12-month average decreased from

12.4% in January 2010 to 7.8% in August 2017. This decline is consistent with the decrease in inventories as discussed above. The decline in turnover is consistent with our results in proposition 8, which states that an increase in intermediation costs, such as the one induced by the Dodd-Frank regulations, leads to a decrease in turnover.

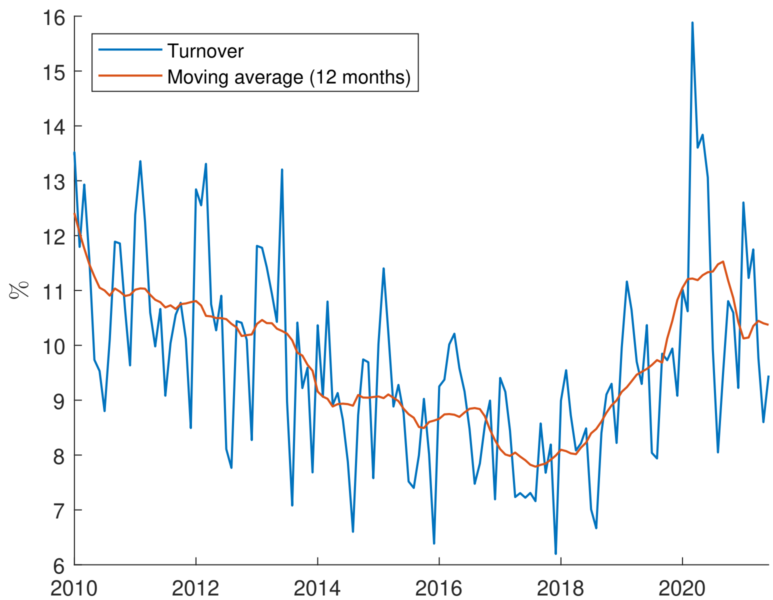


Figure 8: Monthly turnover (median from daily values). Turnover has decreased since 2010.

8 Conclusions

We propose a model to explain the composition of trades in financial markets, especially in over-the-counter markets such as the corporate bond market. It has been identified a larger fraction of trades for which dealers do not need to maintain asset inventory. In these trades, customers provide liquidity to other customers. Although these trades take longer to be executed, standard measures of liquidity show an improvement of liquidity since 2010. The model explains these at first sight conflicting observations.

The model combines Lagos and Rocheteau (2009) and Hugonnier, Lester, and Weill (2022). Lagos and Rocheteau study trades between customers and dealers. Hugonnier, Lester, and Weill study trades between customers and customers. We combine the two models to include the decision of a customer to trade with dealers or with other customers. Both models study decisions on OTC markets with search frictions, as in Duffie et al. (2005).

We interpret the regulations in Dodd-Frank, which include increased capital requirements, increased reporting requirements, and increased restrictions on trading activities, as an increase in the intermediation cost parameters of the model. The regulations made it more expensive for dealers to provide liquidity. We then examine the equilibrium outcomes from the model.

When intermediation costs of dealers increase, customers seek liquidity from other customers, which increases A2A and agency trades and decreases principal trades. In the context of the model, the measure of customers in A2A and agency markets increases. The average bid-ask spread, which considers final transaction prices, decreases. However, the average trade becomes more costly. A measure of illiquidity based on final prices would imply a decrease in illiquidity, as we find empirically.

The model allows us to propose a new measure of illiquidity. This measure takes into account the distortions caused by the search frictions, as well as the bargaining power, number of customers engaged in principal, agency, and A2A trades, and other variables. An increase in agency and A2A trades increases the value of this comprehensive measure of illiquidity.

The model implies the possibility of multiple equilibria. Depending on the intermediation cost parameters, there can be equilibria with a large or small number of trades undertaken by dealers. This is so because the decision to direct search on one market or the other depends on the expected number of agents that engage in the same activity.

The 2008 financial crisis generated a strong response in financial regulations. Our results indicate a way to connect the changes in regulations with changes in the structure of financial markets. Especially, in the structure of financial markets based on over-the-counter trades such as the corporate bond market.

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A Proofs

Proof of Lemma 1

Proof. First note that we can rewrite $\tilde{V}_D^o(\nu)$ in equation (4) as

$$\begin{aligned} r\tilde{V}_D^o(\nu) &= \nu - \mu[\tilde{V}_D^o(\nu) - V^n(\nu)] + \lambda_D^1 \max\{p - \tau - [\tilde{V}_D^o(\nu) - V^n(\nu)], 0\} \\ &= \nu - \mu[\tilde{V}_D^o(\nu) - V_D^o(\nu) + \Delta(\nu)] + \lambda_D^1 \max\{p - \tau - [\tilde{V}_D^o(\nu) - V_D^o(\nu) + \Delta(\nu)], 0\}. \end{aligned}$$

Take the difference between the equation above and (3) to obtain

$$\tilde{V}^o_D(\nu) - V^o_D(\nu) = \frac{\lambda^1_D \max\{p - \tau - [\tilde{V}^o_D(\nu) - V^n(\nu)], 0\} - \lambda^0_D \theta_D \max\left\{\tilde{V}^o_D(\nu) - V^o_D(\nu), 0\right\}}{r + \mu}.$$

The above equation implies that $\tilde{V}^o_D(\nu) - V^o_D(\nu) \geq 0$. Otherwise, we would have

$$\tilde{V}^o_D(\nu) - V^o_D(\nu) = \frac{\lambda^1_D \max\{p - \tau - [\tilde{V}^o_D(\nu) - V^n(\nu)], 0\}}{r + \mu} < 0,$$

which is a contradiction. Since $\tilde{V}^o_D(\nu) - V^o_D(\nu) \geq 0$, we can rewrite the difference as

$$\tilde{V}^o_D(\nu) - V^o_D(\nu) = \frac{\lambda^1_D \max\{p - \tau - \Delta(\nu), \tilde{V}^o_D(\nu) - V^o_D(\nu)\}}{r + \mu + \lambda^0_D \theta_D + \lambda^1_D}.$$

Finally, if $\max\{p - \tau - \Delta(\nu), \tilde{V}^o_D(\nu) - V^o_D(\nu)\} = \tilde{V}^o_D(\nu) - V^o_D(\nu)$, then the above equation implies that $\tilde{V}^o_D(\nu) - V^o_D(\nu)$. As a result, we can conclude that

$$\tilde{V}^o_D(\nu) - V^o_D(\nu) = \frac{\lambda^1_D \max\{p - \tau - \Delta(\nu), 0\}}{r + \mu + \lambda^0_D \theta_D + \lambda^1_D}.$$

After replacing $\tilde{V}^o_D(\nu) - V^o_D(\nu)$ above in the equation (3) we obtain

$$rV^o_D(\nu) = \nu - \mu\Delta(\nu) + \lambda^l_D \theta_D \max\{p - \tau - \Delta(\nu), 0\}, \tag{84}$$

where $\lambda^l_D = \frac{\lambda^0_D \lambda^1_D}{r+\mu+\lambda^0_D\theta_D+\lambda^1_D}$. Analogously,

$$rV^n_D(\nu) = \eta\Delta(\nu) + \lambda^s_D \theta_D \max\{\Delta(\nu) - (p + \tau), 0\}, \tag{85}$$

where $\lambda^s_D = \frac{\lambda^0_D \lambda^1_D}{r+\eta+\lambda^0_D\theta_D+\lambda^1_D}$. Take the difference between equations (84) and (85) to obtain

$$r\Delta(\nu) = \nu - \mu\Delta(\nu) - \eta\Delta(\nu) + \lambda^{al}_D \theta_D \max\{p - \tau - \Delta(\nu), 0\} - \lambda^{as}_D \theta_D \max\{\Delta(\nu) - p - \tau, 0\}, \tag{86}$$

which implies

$$\Delta(\nu) = \frac{\nu + \lambda^{al}_D \theta_D \max\{p - \tau, \Delta(\nu)\} + \lambda^{as}_D \theta_D \min\{p + \tau, \Delta(\nu)\}}{r + \nu + \mu + [\lambda^{al}_D + \lambda^{as}_D]\theta_D} \tag{87}$$

for all types $\nu \in (-\infty, \nu_l] \cup [\nu_h, \infty)$. Equation (87) is associated with a functional operator satisfying all Blackwell's conditions for a contraction. Then, by the contraction mapping theorem, there is a unique function Δ satisfying the equation (87). Also note that if $\tau = 0$, then the results follow directly from equation (87). So we focus on the case with $\tau > 0$.

As Δ is strictly increasing and continuous, we must have that $\Delta(\nu_l) \leq p - \tau$. To see this, notice first that, if $p - \tau < \Delta(\nu_l) < p + \tau$, then the customer would not trade with a dealer because of transaction costs, as the reservation value of a potential seller is higher than the highest bid price of a dealer, $p - \tau$, and the reservation value of a potential buyer is smaller than the lowest ask price of a dealer, $p + \tau$. The last terms in equations (3) and (5) would be zero. Therefore, searching for a dealer is equivalent to be inactive. In this case, the customer would be better off searching for customers type $\nu \in (\nu_l, \nu_h)$ to obtain a share of the gains from trade. This implies that $\nu_l \notin \Omega_D$, which is a contradiction. Implicit in this argument is the fact that the densities of Φ^o and Φ^n are bounded away from zero in the set (ν_l, ν_h) because of issuance and maturity (see proof of Lemma 3), and $\nu_l \neq \nu_h$ (which holds by assumption on a regular equilibrium with $\tau > 0$).

Moreover, if $\Delta(\nu_l) \geq p + \tau$, then either $p - \tau < \Delta(\nu) < p + \tau$ for some customer type $\nu \in \Omega_D$ or $\Delta(\nu) \geq p + \tau$ for all customer type $\nu \in \Omega_D$. The first cannot hold because again it would imply $\nu \notin \Omega_D$. The second would be inconsistent with interdealer market clearing because all customers searching for a dealer would want to buy assets as their reservation value would be greater than or equal to the highest ask price.

Therefore, we must have $\Delta(\nu_l) \leq p - \tau$. An analogous argument applies for ν_h in the opposite direction. That is, $\Delta(\nu_h) \geq p + \tau$. With $\Delta(\nu_l) \leq p - \tau$ and $\Delta(\nu_h) \geq p + \tau$, we can solve for the max relations in equation (87), which then implies equation (26). ■

Proof of Lemma 2

Proof. The proof of this result can be found in the text. Specifically, equations (19) and (20) can written as

$$-\left(\mu + \lambda_D^1\right) \pi_0^o(\nu) + \lambda_D^0 \left[\Phi^o(\nu) - \pi_0^o(\nu)\right] = 0$$

for $\nu \leq \nu_l$, and

$$-\left(\eta + \lambda_D^1\right)\pi_0^n(\nu) + \lambda_D^0\left[\Phi^n(\nu) - \Phi^n(\nu_h) - \pi_0^n(\nu)\right] = 0.$$

for $\nu \geq \nu_h$. Moreover, non-owners of type $\nu \leq \nu_l$ and owners of type $\nu \geq \nu_h$ are inactive, which leads to the result stated in Lemma 2. ■

Proof of Lemma 3

Proof. First note that equation (22) implies

$$\dot{\Phi}^o(\infty) = \eta\Phi^n(\infty) - \mu\Phi^o(\infty) \tag{88}$$

as, when ν goes to infinity, both the inflow and outflow from trading goes to zero. Then, from $\dot{\Phi}^o(\infty) = 0$ and equation (23), we have that

$$\eta[F(\infty) - \Phi^o(\infty)] - \mu\Phi^o(\infty) = 0 \iff \Phi^o(\infty) = \frac{\eta}{\mu + \eta}, \tag{89}$$

which characterizes the total supply of assets $s = \Phi^o(\infty)$, equal, by definition, to the measure of owners. This also establishes that the measure of non-owners is given by

$$\Phi^n(\infty) = F(\infty) - \Phi^o(\infty) = 1 - \Phi^o(\infty) \implies \Phi^n(\infty) = \frac{\mu}{\mu + \eta}. \tag{90}$$

And by definition we have that $1 - s = \Phi^n(\infty)$. Consider now the case $\nu \leq \nu_l$. According the law of motion for Φ^o, given by equation (22), we have

$$\dot{\Phi}^o(\nu) = \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D^1\pi_0^o(\nu) = \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \tilde{\lambda}_D^{al}\Phi^o(\nu), \tag{91}$$

as no-owners with $\tilde{\nu} \leq \nu \leq \nu_l$ will neither purchase the asset in agency or from other customers. Substituting $\Phi^n(\nu) = F(\nu) - \Phi^o(\nu)$ and setting $\dot{\Phi}^o(\nu) = 0$ implies

$$\Phi^o(\nu) = \frac{\eta F(\nu)}{\mu + \eta + \tilde{\lambda}_D^{al}}, \quad \nu \leq \nu_l. \tag{92}$$

Consider now the case $\nu \geq \nu_h$. In this case, it is useful to work with the measure of non-owners of type above ν, $\Phi^n(\infty) - \Phi^n(\nu)$. Using equations (23) and (22), we have

$$
\begin{aligned}
0 &= \dot{\Phi}^n(\infty) - \dot{\Phi}^n(\nu) && (93)\\
&= -\eta[\Phi^n(\infty) - \Phi^n(\nu)] - \lambda_D^1[\pi_0^n(\infty) - \pi_0^n(\nu)] + \mu[\Phi^o(\infty) - \Phi^o(\nu)] && (94)\\
&= -\eta[\Phi^n(\infty) - \Phi^n(\nu)] - \tilde{\lambda}_D^{as}[\Phi^n(\infty) - \Phi^n(\nu)] + \mu[\Phi^o(\infty) - \Phi^o(\nu)] && (95)\\
&= -\eta[1 - s - F(\nu) + \Phi^o(\nu)] - \tilde{\lambda}_D^{as}[1 - s - F(\nu) + \Phi^o(\nu)] + \mu[s - \Phi^o(\nu)] && (96)\\
&= -(\eta + \lambda_D)[1 - F(\nu)] + (\mu + \eta + \tilde{\lambda}_D^{as})[s - \Phi^o(\nu)] && (97)
\end{aligned}
$$

$$
\Longrightarrow s - \Phi^o(\nu) = \frac{(\eta + \tilde{\lambda}_D^{as})[1 - F(\nu)]}{\mu + \eta + \tilde{\lambda}_D^{as}}. \tag{98}
$$

As $s = \frac{\eta}{\mu+\eta}$, we have

$$
\Phi^o(\nu) = \frac{\eta}{\mu + \eta} - \frac{(\eta + \tilde{\lambda}_D^{as})[1 - F(\nu)]}{\mu + \eta + \tilde{\lambda}_D^{as}}, \quad \nu \geq \nu_h. \tag{99}
$$

Now let us show that $\frac{\eta\tilde{\lambda}_D^{al}F(\nu)}{\mu+\eta+\tilde{\lambda}_D^{al}} = \frac{\mu\tilde{\lambda}_D^{as}[1-F(\nu)]}{\mu+\eta+\tilde{\lambda}_D^{as}}$. According to the market clearing condition (21) and Lemmas 1 and 2,

$$
\lambda_D^1 \pi_0^o(\infty) = \lambda_D^1 \pi_0^n(\infty) \implies \tilde{\lambda}_D^{al}\Phi^o(\nu_l) = \tilde{\lambda}_D^{as}\left[\Phi^n(\infty) - \Phi^n(\nu_h)\right]. \tag{100}
$$

We know that $\Phi^o(\nu_l) = \frac{\eta F(\nu_l)}{\mu+\eta+\tilde{\lambda}_D^{al}}$. Moreover,

$$
\begin{aligned}
\Phi^n(\infty) - \Phi^n(\nu_h) &= F(\infty) - \Phi^o(\infty) - [F(\nu_h) - \Phi^o(\nu_h)]\\
&= 1 - s - \left[F(\nu_h) - \frac{\eta}{\mu + \eta} + \frac{(\eta + \tilde{\lambda}_D^{as})[1 - F(\nu_h)]}{\mu + \eta + \tilde{\lambda}_D^{as}}\right] = \frac{\mu[1 - F(\nu_h)]}{\mu + \eta + \tilde{\lambda}_D^{as}}. \qquad (101)
\end{aligned}
$$

Thus, $\frac{\eta\tilde{\lambda}_D^{al}F(\nu)}{\mu+\eta+\tilde{\lambda}_D^{al}} = \frac{\mu\tilde{\lambda}_D^{as}[1-F(\nu)]}{\mu+\eta+\tilde{\lambda}_D^{as}}$. Finally, the result that $\tilde{\lambda}_D^{al}\Phi^o(\nu_l) = \frac{\mu\tilde{\lambda}_D^{as}}{\mu+\eta} - \tilde{\lambda}_D^{as}\Phi^n(\nu_h)$ comes from equation (100) and the fact that $\Phi^n(\infty) = F(\infty) - \Phi^o(\infty) = 1 - \frac{\eta}{\mu+\eta} = \frac{\mu}{\mu+\eta}$. ■

Proof of Lemma 4

Proof. By taking the difference between equations (33) and (34), we know that the reservation value satisfies

$$\Delta(\nu) = \frac{\nu + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Delta(\tilde{\nu}) - \Delta(\nu)]d\Phi^n(\tilde{\nu}) - \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Delta(\nu) - \Delta(\tilde{\nu})]d\Phi^o(\tilde{\nu})}{r + \mu + \eta}. \tag{102}$$

Moreover, because Δ is continuous and monotone, equation (102) implies that Δ is Lipschitz continuous in the interval (ν_l, ν_h). To see this, note that we can rearrange equation (102) to show that

$$\left|\frac{\Delta(\nu + t) - \Delta(\nu)}{t}\right| \leq \left|\frac{1 + 2\lambda_C \sup_x f(x)[\Delta(\nu_h) - \Delta(\nu_l)]}{r + \mu + \eta}\right|$$

for all ν and $\nu + t$ in the interval (ν_l, ν_h), where f is the density of the distribution F. Given that Δ is Lipschitz continuous in the interval (ν_l, ν_h), Δ is differentiable almost everywhere in the interval (ν_l, ν_h) and satisfies $\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu}$, where $\sigma_C(\nu)$ denote the derivative of Δ. Using this result, take the derivative on both sides of equation (102) to obtain

$$\sigma_C(\nu) = \frac{1 - \lambda_C \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\nu)]\sigma_C(\nu) - \lambda_C \theta_C^n[\Phi^o(\nu) - \Phi^o(\nu_l)]\sigma_C(\nu)}{r + \mu + \eta}. \tag{103}$$

We then obtain $\sigma_C(\nu)$ by rearranging the equation above. ■

Proof of Lemma 5

Proof. We have $\dot{\tilde{\Phi}}^o(\nu) = \dot{\Phi}^o(\nu) - \dot{\Phi}^o(\nu_l)$. From equation (22), we have

$$\begin{aligned}
\dot{\tilde{\Phi}}^o(\nu) &= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C \int_{\nu_l}^{\nu} \int_{\nu}^{\nu_h} d\Phi^n(\hat{\nu})d\Phi^o(\tilde{\nu}) \\
&= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C\tilde{\Phi}^o(\nu)\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \\
&= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C\tilde{\Phi}^o(\nu)\big[F(\nu_h) - F(\nu)\big] + \lambda_C\tilde{\Phi}^o(\nu)\big[\tilde{\Phi}^o(\nu_h) - \tilde{\Phi}^o(\nu)\big] \\
&= \eta\big[F(\nu) - F(\nu_l)\big] - \tilde{\Phi}^o(\nu)\Big\{\mu + \eta + \lambda_C\big[F(\nu_h) - F(\nu) - \tilde{\Phi}^o(\nu_h)\big]\Big\} - \lambda_C\tilde{\Phi}^o(\nu)^2. \quad (104)
\end{aligned}$$

We can then solve the quadratic equation above with $\dot{\tilde{\Phi}}^o(\nu) = 0$ to obtain equation (37). Equation (38) is obtained by the solution of the quadratic equation for $\nu = \nu_h$. ■

Proof of Lemma 6

Proof. In a regular equilibrium, customers of type ν_l are indifferent between searching for dealers or customers. The reason is that equations (3)–(10) and the continuity of Δ imply the continuity of V_C^o, V_C^n, V_D^o and V_D^n. As a result,

$$\begin{aligned} rV_C^o(\nu_l) &= \nu_l - \mu\Delta(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^o[\Delta(\nu) - \Delta(\nu_l)]d\Phi^n(\nu) \\ &= \nu_l - \mu\Delta(\nu_l) + \lambda_C\theta_C^o \int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^n(\nu) \\ &= \nu_l - \mu\Delta(\nu_l) + \lambda_D^l\theta_D[p - \tau - \Delta(\nu_l)] = rV_D^o(\nu_l). \end{aligned} \tag{105}$$

Which implies that

$$p = \Delta(\nu_l) + \tau + \frac{\lambda_C\theta_C^o}{\lambda_D^l\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^n(\nu). \tag{106}$$

And similarly,

$$\begin{aligned} rV_C^n(\nu_h) &= \eta\Delta(\nu_h) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^n[\Delta(\nu_h) - \Delta(\nu)]d\Phi^o(\nu) \\ &= \eta\Delta(\nu_h) + \lambda_C\theta_C^n \int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu) \\ &= \eta\Delta(\nu_h) + \lambda_D^s\theta_D[\Delta(\nu_h) - p - \tau] = rV_D^n(\nu_h). \end{aligned} \tag{107}$$

Which implies that

$$p = \Delta(\nu_h) - \tau - \frac{\lambda_C\theta_C^n}{\lambda_D^s\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu). \tag{108}$$

Equalizing the above two price equations and using lemma 4 we obtain

$$\begin{aligned} 2\tau = \lambda_D\theta_D &\int_{\nu_l}^{\nu_h} \sigma_C(\nu)d\nu \\ &- \frac{\lambda_C\theta_C^n}{\lambda_D^s\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu) - \frac{\lambda_C\theta_C^o}{\lambda_D^l\theta_D}\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^n(\nu). \end{aligned} \tag{109}$$

Applying integration by parts in the last two terms we obtain

$$\begin{aligned}
2\tau &= \int_{\nu_l}^{\nu_h} \sigma_C(\nu) d\nu \\
&- \int_{\nu_l}^{\nu_h} \left\{ \frac{\lambda_C \theta_C^n}{\lambda_D^s \theta_D} \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] + \frac{\lambda_C \theta_C^o}{\lambda_D^l \theta_D} \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \right\} \sigma_C(\nu) d\nu \\
&= \int_{\nu_l}^{\nu_h} \left\{ 1 - \frac{\lambda_C \theta_C^n}{\lambda_D^s \theta_D} \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] - \frac{\lambda_C \theta_C^o}{\lambda_D^l \theta_D} \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \right\} \sigma_C(\nu) d\nu. \qquad (110)
\end{aligned}$$

Define $w(\nu) = \frac{\theta_C^o \left[\Phi^n(\nu_h) - \Phi^n(\nu)\right]}{\theta_C^o \left[\Phi^n(\nu_h) - \Phi^n(\nu)\right] + \theta_C^n \left[\Phi^o(\nu) - \Phi^o(\nu_l)\right]}$. Then we can rewrite the above equation as

$$\begin{aligned}
2\tau = &\int_{\nu_l}^{\nu_h} \frac{w(\nu) \left\{ \lambda_D^s \theta_D - \lambda_C \theta_C^n \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] - \lambda_C \theta_C^o \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \right\}}{\lambda_D^s \theta_D} \sigma_C(\nu) d\nu \\
+ &\int_{\nu_l}^{\nu_h} \frac{[1 - w(\nu)] \left\{ \lambda_D^l \theta_D - \lambda_C \theta_C^n \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] - \lambda_C \theta_C^o \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \right\}}{\lambda_D^l \theta_D} \sigma_C(\nu) d\nu.
\end{aligned} \qquad (111)$$

From the definition of $\sigma_C(\nu)$, we have $\lambda_C \theta_C^o \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \lambda_C \theta_C^n \big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] = \frac{1}{\sigma_C(\nu)} - (r + \mu + \eta)$. Substituting above and rearranging it implies

$$2\tau\theta_D = \int_{\nu_l}^{\nu_h} w(\nu) \frac{\sigma_C(\nu) - \sigma_D^{as}}{\lambda_D^{as} \sigma_D^{as}} + [1 - w(\nu)] \frac{\sigma_C(\nu) - \sigma_D^{al}}{\lambda_D^{al} \sigma_D^{al}} d\nu, \qquad (112)$$

where we used that $r + \mu + \eta + \lambda_D^{al} \theta_D = 1/\sigma_D^{al}$ and $r + \mu + \eta + \lambda_D^{as} \theta_D = 1/\sigma_D^{as}$. This concludes the proof. ■

Proof of Proposition 1

Proof. The necessity of equations (26)–(40) are established in Lemmas 3–6. So let us focus on the sufficiency. Consider a family $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ satisfying equations (26)–(40) and value functions V^o and V^n constructed using equations (3)–(11) given the family $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$. Let us show that the family $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ is a regular equilibrium—that is, it satisfies equations (3)–(23) and definition 2.

Equations (3)–(11)**:** These equations are satisfied by the construction of V^o and V^n.

Equations (13)–(14)**:** First let us show that $V_D^o(\nu) \geq V_C^o(\nu)$ for all $\nu \leq \nu_l$.

$$V_D^o(\nu) \geq V_C^o(\nu) \iff \lambda_D\theta_D[(p-\tau)-\Delta(\nu)] \geq \lambda_C\theta_C^o \int_{\nu_l}^{\nu^h} [\Delta(\tilde{\nu})-\Delta(\nu)]d\Phi^n(\tilde{\nu})$$
$$\iff (p-\tau)-\Delta(\nu) \geq \tfrac{\lambda_C\theta_C^o}{\lambda_D\theta_D} \int_{\nu_l}^{\nu^h} [\Delta(\tilde{\nu})-\Delta(\nu_l)]d\Phi^n(\tilde{\nu})$$
$$+\tfrac{\lambda_C\theta_C^o}{\lambda_D\theta_D}[\Phi^n(\nu_h)-\Phi^n(\nu_l)][\Delta(\nu_l)-\Delta(\nu)].$$

From equation (40) we know that $\frac{\lambda_C\theta_C^o}{\lambda_D\theta_D}\int_{\nu_l}^{\nu^h}[\Delta(\tilde{\nu})-\Delta(\nu_l)]d\Phi^n(\tilde{\nu}) = (p-\tau)-\Delta(\nu_l)$, therefore

$$V_D^o(\nu) \geq V_C^o(\nu) \iff \Delta(\nu_l)-\Delta(\nu) \geq \frac{\lambda_C\theta_C^o}{\lambda_D\theta_D}[\Phi^n(\nu_h)-\Phi^n(\nu_l)][\Delta(\nu_l)-\Delta(\nu)].$$

Assumption 1 implies that $\frac{\lambda_C\theta_C^o}{\lambda_D\theta_D} \in (0,1)$. From (48), we have that $\Phi^n(\nu_h)-\Phi^n(\nu_l) = \frac{\mu[F(\nu_h)-F(\nu_l)]}{\mu+\eta} \in [0,1)$. Using (26) in Lemma 1, we have that

$$V_D^o(\nu) \geq V_C^o(\nu) \iff \nu_l-\nu \geq \frac{\lambda_C\theta_C^o}{\lambda_D\theta_D}[\Phi^n(\nu_h)-\Phi^n(\nu_l)][\nu_l-\nu].$$

We can then see that $V_D^o(\nu) \geq V_C^o(\nu)$ holds. Moreover, it holds with strictly inequality for all $\nu < \nu_l$. The proofs that $V_D^n(\nu) \geq V_C^n(\nu)$ for all $\nu \leq \nu_l$; $V_D^o(\nu) \leq V_C^o(\nu)$ for all $\nu \in (\nu_l,\nu_h)$; and $V_D^n(\nu) \leq V_C^n(\nu)$ for all $\nu \in (\nu_l,\nu_h)$ are analogous.

Equation (12)**:** Let us start with $\nu \leq \nu_l$. In this case we have that $V^o(\nu) = V_D^o(\nu)$ and $V^n(\nu) = V_D^n(\nu)$ based on equation (13). Then, from equations (3) and (5) we have that

$$\begin{aligned} V^o(\nu)-V^n(\nu) &= \frac{\nu+\lambda_D\theta_D(p-\tau)-(\mu+\eta+\lambda_D\theta_D)\Delta(\nu)}{r} \\ &= \frac{(r+\mu+\eta+\lambda_D\theta_D)\Delta(\nu)-(\mu+\eta+\lambda_D\theta_D)\Delta(\nu)}{r} = \Delta(\nu). \end{aligned}$$

The result for $\nu \geq \nu_h$ is analogous. For $\nu \in (\nu_l,\nu_h)$ we have that

$$r[V^o(\nu)-V^n(\nu)] = \nu-(\mu+\eta)\Delta(\nu)+\lambda_C\int_{\nu}^{\nu_h}\theta_C^o[\Delta(\tilde{\nu})-\Delta(\nu)]d\Phi^n(\tilde{\nu})$$
$$-\lambda_C\int_{\nu_l}^{\nu}\theta_C^n[\Delta(\nu)-\Delta(\tilde{\nu})]d\Phi^o(\tilde{\nu})$$

Replacing equation (35) and applying integration by parts we get

$$\begin{aligned}
r[V^o(\nu) - V^n(\nu)] &= \nu - (\mu+\eta)\Delta(\nu_l) - (\mu+\eta)\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&+ \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} - \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Phi^o(\tilde{\nu}) - \Phi^o(\nu_l)]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu - (\mu+\eta)\Delta(\nu_l) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&- \int_{\nu_l}^{\nu} \{\mu + \eta + \lambda_C\theta_C^n[\Phi^o(\tilde{\nu}) - \Phi^o(\nu_l)]\}\, \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu - (\mu+\eta)\Delta(\nu_l) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&- \nu + \nu_l - \int_{\nu_l}^{\nu} \{r + \lambda_C\theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\}\, \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu_l - (r+\mu+\eta)\Delta(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} + r\Delta(\nu).
\end{aligned}$$

Now we can replace equation (40) to obtain

$$\begin{aligned}
r[V^o(\nu) - V^n(\nu)] &= \nu_l - (r+\mu+\eta)\Delta(\nu_l) + \lambda_C\theta_C^o[p - \tau - \Delta(\nu_l)] + r\Delta(\nu) \\
&= \nu_l + \lambda_C\theta_C^o(p-\tau) - (r+\mu+\eta+\lambda_C\theta_C^o)\Delta(\nu_l) + r\Delta(\nu) = r\Delta(\nu),
\end{aligned}$$

where the last equality we obtained using equation (26) applied to $\Delta(\nu_l)$.

Equation (21)**:** The left-hand side of Equation (21) is given by

$$\lambda_D \int_{\Omega_D^o} \mathbb{1}_{\{\Delta(\nu) < p - \tau\}} d\Phi^o(\nu) = \lambda_D \int_{-\infty}^{\nu_l} d\Phi^o(\nu) = \lambda_D \Phi^o(\nu_l).$$

The right-hand side is

$$\lambda_D \int_{\Omega_D^n} \mathbb{1}_{\{\Delta(\nu) > p + \tau\}} d\Phi^n(\nu) = \lambda_D \int_{\nu_h}^{\infty} d\Phi^n(\nu) = \lambda_D \left[\Phi^n(\infty) - \Phi^n(\nu_h)\right].$$

Therefore, we have market clearing if, and only if, $\Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h)$. This equation holds because, from the second equation of (30), $\Phi^o(\infty) = \frac{\eta}{\mu+\eta} \Longrightarrow \Phi^n(\infty) = 1 - \Phi^o(\infty) = \frac{\mu}{\mu+\eta}$, and, from equation (31), $\frac{\mu}{\mu+\eta} - \Phi^n(\nu_h) = \Phi^o(\nu_l)$.

Equation (22): First, consider $\nu \leq \nu_l$. Then, equation (22) is given by

$$\dot{\Phi}^o(\nu) = \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu) = \eta F(\nu) - (\eta - \mu - \lambda_D)\Phi^o(\nu).$$

Equation (30) states that $\Phi^o(\nu) = \frac{\eta F(\nu)}{\eta-\mu-\lambda_D}$. Thus, $\dot{\Phi}^o(\nu) = \eta F(\nu) - \eta F(\nu) = 0$. Consider now $\nu \geq \nu_h$. Then, equation (22) is given by

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu_l) + \lambda_D[\Phi^n(\nu) - \Phi^n(\nu_h)] \\ &= (\eta + \lambda_D)F(\nu) - (\mu + \eta + \lambda_D)\Phi^o(\nu) - \lambda_D[\Phi^o(\nu_l) + \Phi^n(\nu_h)].\end{aligned}$$

Using the second equation of (30) and (31), we then have

$$\begin{aligned}\dot{\Phi}^o(\nu) &= (\eta + \lambda_D)F(\nu) + (\eta + \lambda_D)[1 - F(\nu)] - \frac{\eta(\mu + \eta + \lambda_D)}{\mu + \eta} - \frac{\lambda_D\mu}{\mu + \eta} \\ &= \eta + \lambda_D - \frac{\eta(\mu + \eta) + \lambda_D(\mu + \eta)}{\mu + \eta} = \eta + \lambda_D - (\eta + \lambda_D) = 0.\end{aligned}$$

Finally, let us consider $\nu \in (\nu_l, \nu_h)$. In this case we have

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu_l) - \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][\Phi^n(\nu_h) - \Phi^n(\nu)] \\ &= \eta[\Phi^n(\nu) - \Phi^n(\nu_l)] - \mu[\Phi^o(\nu) - \Phi^0(\nu_l)] + \eta\Phi^n(\nu_l) - \mu\Phi^o(\nu_l) - \lambda_D\Phi^o(\nu_l) \\ &- \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][F(\nu_h) - F(\nu)] + \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][\Phi^o(\nu_h) - \Phi^o(\nu)].\end{aligned}$$

We have shown that $\eta\Phi^n(\nu_l) - \mu\Phi^o(\nu_l) - \lambda_D\Phi^o(\nu_l) = 0$ when considering the case $\nu \leq \nu_l$. By using this result and the notation $\tilde{\Phi}^o(\nu) = \Phi^o(\nu) - \Phi^0(\nu_l)$ and $s_C = \tilde{\Phi}^o(\nu_h)$, we obtain

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta[F(\nu) - F(\nu_l)] - (\mu + \eta)\tilde{\Phi}^o(\nu) \\ &- \lambda_C\tilde{\Phi}^o(\nu)[F(\nu_h) - F(\nu)] + \lambda_C\tilde{\Phi}^o(\nu)\tilde{\Phi}^o(\nu_h) - \lambda_C\tilde{\Phi}^o(\nu)^2 \\ &= \eta[F(\nu) - F(\nu_l)] - (\mu + \eta)\tilde{\Phi}^o(\nu) \\ &- \tilde{\Phi}^o(\nu)\left\{\mu + \eta + \lambda_C[F(\nu_h) - F(\nu) - s_C]\right\} - \lambda_C\tilde{\Phi}^o(\nu)^2.\end{aligned}$$

The distribution $\tilde{\Phi}^o(\nu)$, as defined in equation (37), is the positive root of the equation above. Therefore, $\dot{\Phi}^o(\nu) = 0$.

Equation (23): This is directly stated in equations (30) and (37).

We showed that the family $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ is an equilibrium. That is, that it satisfies equations (3)–(23). It is easy to see that it must also be a regular equilibrium because equation (40) implies that $\nu_l \leq \nu_h$ with strict inequality if $\tau > 0$, and equations (26) and (35) imply that Δ is continuous and strictly increasing. ■

Proof of Proposition 5

Proof. First note that equation (40) is necessarily satisfied by all regular equilibrium. Therefore, it suffices to show that in a neighborhood of $\tau = 0$, there is a unique pair (ν_l, ν_h) satisfying equation (40) for all τ in this neighborhood. Equation (40) can be rewritten as

$$G(\nu_l) = \frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\left[\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]d\nu = \tau. \tag{113}$$

When $\tau = 0$, then $G(\nu_l) = \tau$ implies that $\nu_l = g(\nu_l) = \nu_h = \nu_s$. That is because $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1$ is bounded away from zero. To see this notice that

$$\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1 > 0 \Leftrightarrow \frac{r+\mu+\eta+\lambda_D\theta_D}{r+\mu+\eta+\lambda_C\left\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu)]+\theta_C^n[\Phi^o(\nu)-\Phi^o(\nu_l)]\right\}} > 1 \tag{114}$$

$$\Leftrightarrow \ \lambda_D\theta_D > \lambda_C\left\{\theta_C^o\left[\Phi^n(\nu_h) - \Phi^n(\nu)\right] + \theta_C^n\left[\Phi^o(\nu) - \Phi^o(\nu_l)\right]\right\}. \tag{115}$$

But note that $\lambda_C\Big\{\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]\Big\} < \lambda_C\max\{\theta_C^o, \theta_C^n\} < \lambda_D\theta_D$, which implies that $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1$ is bounded away from zero. As a result, we can only have $G(\nu_l) = 0$ if the limits in the integral are the same. Then, since $G(\cdot)$ is continuous, it suffices to show that it is strictly monotone in a neighborhood $(\bar{\nu}_l, \nu_s]$. Note also that $G(\cdot)$ is differentiable and that

$$G'(\nu_l) = \frac{1}{2\lambda_D\theta_D}\left\{g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D} - 1\right] - \left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]\right\}$$
$$+ \frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\frac{1}{\sigma_D}\left[\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l} + g'(\nu_l)\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}\right]d\nu. \tag{116}$$

The first term on the right-hand is negative, as $g'(\nu_l) = -\frac{\eta f(\nu_l)}{\mu f(g(\nu_l))}$, and $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1$ is bounded away from zero. Moreover, using the definition of σ_C, we can bound it above by

$$-\frac{1}{2\lambda_D\theta_D}\left[\frac{r+\mu+\eta+\lambda_D\theta_D}{r+\mu+\eta+\lambda_C\max\{\theta_D^o,\theta_D^n\}}-1\right]. \tag{117}$$

Therefore, to establish that $G'(\nu_l)<0$ in a neighborhood of $(\bar{\nu}_l,\nu_s]$ we just have to show that the second term converges to zero when $\nu_l\nearrow\nu_s$. As $g(\nu_l)\to\nu_s$ when $\nu_l\nearrow\nu_s$, it suffices to show that the terms inside the integral, $\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l}$ and $\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}$, are bounded. We can write the first term as

$$\begin{aligned}
\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l} &= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\left\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu)]+\theta_C^n[\Phi^o(\nu)-\Phi^o(\nu_l)]\right\}}{\partial\nu_l} && (118)\\
&= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\left\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu_l)+\Phi^n(\nu_l)-\Phi^n(\nu)]+\theta_C^n\tilde{\Phi}^o(\nu)\right\}}{\partial\nu_l} && (119)\\
&= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\left\{\theta_C^o\frac{\mu[F(\nu_h)-F(\nu_l)]}{\mu+\eta}-\theta_C^o[F(\nu)-F(\nu_l)]+\tilde{\Phi}^o(\nu)\right\}}{\partial\nu_l} && (120)\\
&= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\left\{\theta_C^o\frac{\eta f(\nu_l)}{\mu+\eta}+\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l}\right\}. && (121)
\end{aligned}$$

The first term in parenthesis is bounded. The second term is obtained by applying the implicit function theorem to equation (104) and it yields

$$\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l}=-\frac{\eta f(\nu_l)\left[1+\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\mu+\eta}\right]}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}, \tag{122}$$

which is also bounded. Similarly, for $\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}$,

$$\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}=-\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\left\{\theta_C^o\frac{\mu[F(\nu_h)-F(\nu_l)]}{\mu+\eta}-\theta_C^o[F(\nu)-F(\nu_l)]+\tilde{\Phi}^o(\nu)\right\}}{\partial\nu_h} \tag{123}$$

$$=-\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\left\{\theta_C^o\frac{\mu f(\nu_h)}{\mu+\eta}+\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_h}\right\}. \tag{124}$$

Again, the first term in parenthesis is bounded. The second term is

$$\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_h}=-\frac{\mu f(\nu_h)\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\mu+\eta}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)},$$

which is also bounded. Therefore, G is strictly monotone in a neighborhood $(\bar{\nu}_l, \nu_s]$ with $G'(\nu) < 0$ in this neighborhood. If we then define the neighborhood $[0, \bar{\tau})$, where $\bar{\tau} = G(\bar{\nu}_l)$, we can conclude that there is a unique regular equilibrium for any $\tau \in [0, \bar{\tau})$. ■

Proof of Lemmas 7, 8, 9, 10, 11 and 12

Proof. The proof of Lemmas 7–12 are analogous to the proof of Lemmas 1–6. ■

Proof of Proposition 8

Proof. We know that there exists neighborhood $[0, \bar{\tau})$ of $\tau = 0$ that regular equilibrium is unique. Moreover, because $G(\nu_s) = 0$, we must have $G'(\nu_l) \leq 0$ for any $\nu_l < \nu_s$ with an unique equilibrium in a neighborhood around it. To see this note that if $G'(\nu_l) > 0$ for an ν_l with $G(\nu_l) = \tau$, then there exists $\nu_l' > \nu_l$ such that $G(\nu_l') > G(\nu_l)$. But then $G(\nu_l') > G(\nu_l) \geq G(0)$ and we can conclude by continuity that there must be another ν_l'' with $G(\nu_l'') = \tau$. This is a contradiction since we started assuming that there is an unique regular equilibrium at ν_l.

Consider then any (τ^0, τ^1) and (ν_l^0, ν_l^1) such that all $\tau \in (\tau^0, \tau^1)$ is associated with a unique regular equilibrium at some $\nu_l(\tau) \in (\nu_l^0, \nu_l^1)$. Since these regular equilibrium are characterized by $G(\nu_l) = \tau$ and $G'(\nu_l) \leq 0$, we must then have that $\nu_l(\tau)$ is decreasing in τ for all $\tau \in (\tau^0, \tau^1)$. Therefore, to obtain that the turnover is decreasing in τ in the neighborhood (τ^0, τ^1), it suffices to show that it is increasing in ν_l.

From equation (81) we have that

$$T = \frac{2\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}{\frac{\eta}{\mu+\eta}} = \frac{2\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \tilde{\Phi}^o(\nu) d\tilde{\Phi}^n(\nu)}{\frac{\eta}{\mu+\eta}}.$$

From equation (30) we have $\Phi^o(\nu_l) = \frac{\eta F(\nu_l)}{\mu+\eta+\lambda_D}$, which is increasing in ν_l with its derivative given by $\frac{\partial \Phi^o(\nu_l)}{\partial \nu_l} = \frac{\eta f(\nu_l)}{\mu+\eta+\lambda_D}$. For the second term we have that

$$\begin{aligned}\frac{\partial \int_{\nu_l}^{\nu_h} \tilde{\Phi}^o(\nu) d\tilde{\Phi}^n(\nu)}{\partial \nu_l} &= -\cancel{\tilde{\Phi}^o(\nu_l)\phi^n(\nu_l)} + g'(\nu_l)\tilde{\Phi}^o(\nu_h)\phi^n(\nu_h) \\ &+ \int_{\nu_l}^{\nu_h} \frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\phi^n(\nu) + \frac{d\phi^n(\nu)}{d\nu_l}\tilde{\Phi}^o(\nu) d\nu.\end{aligned}$$

Again for the first term inside the integral

$$\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l} = \frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l} + g'(\nu_l)\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_h} = \frac{-\eta f(\nu_l)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}.$$

Now we can apply the implicit function theorem to equation (104) and obtain that

$$\phi^o(\nu) = \frac{[\eta+\lambda_C\tilde{\Phi}^o(\nu)]f(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}.$$

Thus,

$$\begin{aligned}\frac{d\phi^o(\nu)}{d\nu_l} &= -\frac{2\lambda_C\eta f(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\left[\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)\right]^2}\\
&+\lambda_C f(\nu)\frac{\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\left[\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)\right]}{\left[\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)\right]^2}\\
&-\lambda_C f(\nu)\frac{2\lambda_C\tilde{\Phi}^o(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\left[\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)\right]^2}\\
&= \frac{\lambda_C f(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}-2\lambda_C\phi^o(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} = \frac{\lambda_C[\phi^n(\nu)-\phi^o(\nu)]\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}.\end{aligned}$$

Also note that from the above we obtain $\phi^n(\nu)$ and $\frac{d\phi^n(\nu)}{d\nu_l}$ from the identity $f(\nu) = \phi^o(\nu) + \phi^n(\nu)$. Then we obtain that

$$\begin{aligned}\phi^n(\nu) &= f(\nu) - \frac{[\eta+\lambda_C\tilde{\Phi}^o(\nu)]f(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}\\
&= \frac{\left[\{\mu+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\}+\lambda_C\tilde{\Phi}^o(\nu)\right]f(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}.\end{aligned}$$

Now we can write that

$$\begin{aligned}
\frac{\partial \int_{\nu_l}^{\nu_h} \tilde{\Phi}^o(\nu) d\tilde{\Phi}^n(\nu)}{\partial \nu_l} &= g'(\nu_l)\tilde{\Phi}^o(\nu_h)\phi^n(\nu_h) + \int_{\nu_l}^{\nu_h} \frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\phi^n(\nu) + \frac{d\phi^n(\nu)}{d\nu_l}\tilde{\Phi}^o(\nu) d\nu \\
&= -\tilde{\Phi}^o(\nu_h)\frac{\eta f(\nu_l)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} \\
&\quad - \eta f(\nu_l)\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu.
\end{aligned}$$

With this equation in hand we now can show that $\frac{dT}{d\nu_l} \geq 0$, which is equivalent to show that

$$\frac{2\lambda_D \eta f(\nu_l)}{\mu+\eta+\lambda_D} \geq$$

$$\frac{\lambda_C\tilde{\Phi}^o(\nu_h)\eta f(\nu_l)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C \eta f(\nu_l)\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu,$$

which happens if, and only if,

$$\frac{2\lambda_D}{\mu+\eta+\lambda_D} \geq$$

$$\frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C \int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu.$$

Note that

$$\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu) = \mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^o(\nu)+\tilde{\Phi}^o(\nu)\right].$$

Then we have that

$$\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu)+\frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]}}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^o(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu = \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu$$

$$- \int_{\nu_l}^{\nu_h} \frac{\phi^o(\nu)\left\{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]\right\}-\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]\right\}^2} d\nu$$

$$= \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu - \int_{\nu_l}^{\nu_h} \frac{d}{d\nu}\left[\frac{\tilde{\Phi}^o(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]}\right] d\nu$$

$$= \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu - \frac{\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)}.$$

So, again, it suffices to show that

$$\frac{2\lambda_D}{\mu+\eta+\lambda_D} \geq \frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)}$$

$$+\lambda_C \int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu)+\frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu =$$

$$\frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C\int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]} d\nu - \frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)}$$

$$\Leftrightarrow \frac{2\lambda_D}{\mu+\eta+\lambda_D} \geq \int_{\nu_l}^{\nu_h} \frac{\lambda_C f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu$$

$$\Leftrightarrow \int_{\nu_l}^{\nu_h} \left[\frac{2\lambda_D}{\mu+\eta+\lambda_D} - \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]}\right] dF(\nu) \geq 0.$$

First note that the above inequality holds in a neighborhood of $\tau = 0$ because it implies that $\nu_l \approx \nu_h$. Moreover, if $\lambda_C \leq \lambda_D \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\mu+\eta)$, we must have

$$\frac{2\lambda_D}{\mu+\eta+\lambda_D} \geq \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]}.$$

We can see this by comparing the two functions $\frac{2x}{\mu+\eta+x}$ and $\frac{ax}{\mu+\eta+bx}$, where $a = F(\nu_h) - F(\nu_l)$ and $b = \tilde{\Phi}^n(\nu_h) - \tilde{\Phi}^n(\nu) + \tilde{\Phi}^o(\nu)$. Both functions are strictly increasing in x, and equal zero at $x = 0$. Moreover, the derivative of the first function is strictly greater than the derivative of the second one for $x \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\mu+\eta)$, which implies that

$$\frac{2\lambda_D}{\mu+\eta+\lambda_D} > \frac{\lambda_D[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_D[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]} \geq \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]},$$

which concludes the proof. ■

Proof of Proposition ??

Proof. The bid-ask spread, defined in (80), is

$$\mathrm{BA} = \frac{2\lambda_D \Phi^o(\nu_l)\tau + \lambda_D(1-\theta_D)\frac{\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)}{r+\mu+\eta+\lambda_D\theta_D}}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}. \tag{125}$$

To see the first part, note that, for $\tau = 0$, $\nu_l = \nu_h = \nu_s$, and the equilibrium is unique. So, we can take the derivative of BA in (**??**) with respect to τ evaluated at $\tau = 0$ and show that it has to be strictly positive. Given that all the functions are differentiable, we must have this derivative strictly positive in a neighborhood of $\tau = 0$.

We have that

$$\frac{d\,\mathrm{BA}}{d\tau} = \frac{\partial\,\mathrm{BA}}{\partial\tau} + \frac{\partial\,\mathrm{BA}}{\partial\nu_l} \times \frac{\partial\nu_l}{\partial\tau}. \tag{126}$$

For the first term in the right-hand side, we have

$$\frac{\partial\,\mathrm{BA}}{\partial\tau} = \frac{2\lambda_D \Phi^o(\nu_l)}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)},$$

which implies that $\frac{\partial\,\mathrm{BA}}{\partial\tau} = 2$ when evaluated at $\tau = 0$ since in this case $\nu_l = \nu_h = \nu_s$. For the second term we have that

$$\left.\frac{\partial\,\mathrm{BA}}{\partial\nu_l}\right|_{\substack{\tau=0,\\ \nu_l=\nu_s}} = \lambda_D(1-\theta_D)\frac{\frac{d}{d\nu_l}\left[\frac{\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)}{r+\mu+\eta+\lambda_D\theta_D}\right]\lambda_D\Phi^o(\nu_l)}{\left[\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)\right]^2} \tag{127}$$

$$- \lambda_D(1-\theta_D)\frac{\frac{\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)}{r+\mu+\eta+\lambda_D\theta_D}\frac{d}{d\nu_l}\left[\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)\right]}{\left[\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)\right]^2}. \tag{128}$$

Note that

$$\frac{d}{d\nu_l}\left[\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)\right]=\frac{d}{d\nu_l}\left[\int_{\nu_h}^{\infty}\frac{\nu\mu f(\nu)}{\mu+\eta+\lambda_D}d\nu-\int_{-\infty}^{\nu_l}\frac{\nu\eta f(\nu)}{\mu+\eta+\lambda_D}d\nu\right]$$
$$=\frac{-\nu_h\mu f(\nu_h)g'(\nu_l)-\nu_l\eta f(\nu_l)}{\mu+\eta+\lambda_D}=\frac{[\nu_h-\nu_l]\eta f(\nu_l)}{\mu+\eta+\lambda_D},$$

which is zero when evaluated at $\nu_l=\nu_h=\nu_s$. Moreover,

$$\frac{d}{d\nu_l}\left[\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)\right]$$

is positive in a neighborhood of $\tau=0$ since this is basically turnover, which we showed in the previous proof that is increasing in ν_l in a neighborhood of $\tau=0$. Thus, $\frac{\partial\,\mathrm{BA}}{\partial\nu_l}\Big|_{\substack{\tau=0,\\ \nu_l=\nu_s}}\leq 0$. We also have shown that $\frac{\partial\nu_l}{\partial\tau}<0$ in a neighborhood of $\tau=0$. Therefore, we have that

$$\frac{d\,\mathrm{BA}}{d\tau}\Big|_{\substack{\tau=0,\\ \nu_l=\nu_s}}=\underbrace{\frac{\partial\,\mathrm{BA}}{\partial\tau}\Big|_{\substack{\tau=0,\\ \nu_l=\nu_s}}}=2+\underbrace{\frac{\partial\,\mathrm{BA}}{\partial\nu_l}\Big|_{\substack{\tau=0,\\ \nu_l=\nu_s}}}_{\leq 0}\times\underbrace{\frac{\partial\nu_l}{\partial\tau}\Big|_{\substack{\tau=0,\\ \nu_l=\nu_s}}}_{\leq 0}>0.$$

This proves the first part of the proposition. Namely, that the bid-ask spread is increasing in τ in a neighborhood of $\tau=0$.

To show the second part of the proposition, it suffices to show that BA converges to zero as τ converges to infinity. First lets us show that ν_l converges to $-\infty$ when τ converges to infinity. In equilibrium we must have that

$$G(\nu_l)=\frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\frac{\sigma_C(\nu;\nu_l,\nu_h)-\sigma_D}{\sigma_D}d\nu=\tau. \tag{129}$$

The term $\frac{\sigma_C(\nu;\nu_l,\nu_h)-\sigma_D}{\sigma_D}$ is bounded below by $\frac{\lambda_D\theta_D-\lambda_C\max\{\theta_C^n,\theta_C^o\}}{\mu+\eta+\lambda_D\theta_D}$, and above by $\frac{\lambda_D\theta_D}{\mu+\eta+\lambda_D\theta_D}$. Therefore, as τ converges to infinity, in order to obtain an equilibrium we must have ν_l converging to $-\infty$, and $\nu_h=g(\nu_l)$ converging to ∞.

Consider now the formula for the bid-ask spread,

$$\mathrm{BA}=\frac{2\lambda_D\Phi^o(\nu_l)\tau+\lambda_D(1-\theta_D)\frac{\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)}{r+\mu+\eta+\lambda_D\theta_D}}{\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)}. \tag{130}$$

We can see that $\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)$ converge to zero since $\int_{-\infty}^{\infty} \nu^2 f(\nu) d\nu$ is bounded. To show that $\Phi^o(\nu_l)\tau = \frac{\eta F(\nu_l)\tau}{\mu+\eta+\lambda_D}$ converges to zero is not as simple because $F(\nu_l)$ converges to zero and τ converges to infinity. But note that

$$\lim_{\tau\nearrow\infty} F(\nu_l)\tau = \lim_{\tau\nearrow\infty} \frac{\tau}{1/F(\nu_l)} = \lim_{\tau\nearrow\infty} \frac{1}{f(\nu_l)\frac{d\nu_l}{d\tau}/F(\nu_l)^2} = \lim_{\nu_l\searrow-\infty} \frac{G'(\nu_l)F(\nu_l)^2}{f(\nu_l)}.$$

And, as we have established in the of Proposition 5,

$$\begin{aligned} G'(\nu_l) = {} & \frac{1}{2\lambda_D\theta_D}\left\{g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D} - 1\right] - \left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]\right\} \\ & + \frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)} \frac{1}{\sigma_D}\left[\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l} + g'(\nu_l)\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}\right] d\nu. \end{aligned}$$

So we only need to show that each of the terms above, when multiplied by $\frac{F(\nu_l)^2}{f(\nu_l)}$, converges to zero as ν_l converges to $-\infty$. Let us start showing that $\frac{F(\nu_l)^2}{f(\nu_l)}$ converges to zero. Note that

$$0 \le \lim_{\nu_l\searrow-\infty} -\nu_l F(\nu_l) = \lim_{\nu_l\searrow-\infty} \int_{-\infty}^{\nu_l} |\nu_l| f(\nu) d\nu \le \lim_{\nu_l\searrow-\infty} \int_{-\infty}^{\nu_l} |\nu| f(\nu) d\nu = 0,$$

where the equality in the end comes from the fact that $\int \nu^2 f(\nu) d\nu$ is finite. Therefore we can conclude that $\lim_{\nu_l\searrow-\infty} \nu_l F(\nu_l) = 0$. But then

$$0 = \lim_{\nu_l\searrow-\infty} -\nu_l F(\nu_l) = \lim_{\nu_l\searrow-\infty} \frac{-\nu_l}{1/F(\nu_l)} \overset{\text{L'Hôpital}}{=} \lim_{\nu_l\searrow-\infty} \frac{-1}{-f(\nu_l)/F(\nu_l)^2} = \lim_{\nu_l\searrow-\infty} \frac{F(\nu_l)^2}{f(\nu_l)}.$$

Now let us look the individual terms of $G'(\nu_l)$. We have that

$$\lim_{\nu_l\searrow-\infty} \frac{F(\nu_l)^2}{f(\nu_l)}\left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right] = 0$$

because $\left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]$ is bounded. We have that

$$\lim_{\nu_l\searrow-\infty} \frac{F(\nu_l)^2}{f(\nu_l)} g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D} - 1\right] = \lim_{\nu_h\nearrow-\infty} \frac{[1-F(\nu_h)]^2}{f(\nu_h)}\left[\frac{\sigma_C(\nu_h;g^{-1}(\nu_h),\nu_h)}{\sigma_D} - 1\right] = 0$$

because again $\left[\frac{\sigma_C(\nu_h;g^{-1}(\nu_h),\nu_h)}{\sigma_D} - 1\right]$ is bounded and we can show that $\lim_{\nu_h\nearrow-\infty} \frac{[1-F(\nu_h)]^2}{f(\nu_h)} = 0$

in the same way that we showed that $\lim_{\nu_l \searrow -\infty} \frac{F(\nu_l)^2}{f(\nu_l)} = 0$. Moreover,

$$0 \leq \int_{\nu_l}^{g(\nu_l)} \frac{\partial \sigma_C(\nu; \nu_l, g(\nu_l))}{\partial \nu_l} d\nu = \int_{\nu_l}^{g(\nu_l)} \sigma_C(\nu; \nu_l, g(\nu_l))^2 \lambda_C \left\{ \theta_C^o \frac{\eta f(\nu_l)}{\mu + \eta} - \frac{\partial \tilde{\Phi}^o(\nu)}{\partial \nu_l} \right\} d\nu$$

$$= \int_{\nu_l}^{g(\nu_l)} \left\{ \theta_C^o \frac{\lambda_C \eta \sigma_C(\nu;\nu_l,g(\nu_l))^2}{\mu+\eta} + \frac{\lambda_C \eta \sigma_C(\nu;\nu_l,g(\nu_l))^2 \left[1 + \frac{\lambda_C \tilde{\Phi}^o(\nu)}{\mu+\eta}\right]}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} \right\} f(\nu_l) d\nu$$

$$\leq \int_{\nu_l}^{g(\nu_l)} \left\{ \theta_C^o \frac{\lambda_C \eta \sigma_C(\nu;\nu_l,g(\nu_l))^2}{\mu+\eta} + \frac{\lambda_C \eta \sigma_C(\nu;\nu_l,g(\nu_l))^2 \left[1 + \frac{\lambda_C \tilde{\Phi}^o(\nu)}{\mu+\eta}\right]}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} \right\} f(\nu) d\nu$$

which is bounded because the term in brackets is bounded and $\int_{\nu_l}^{g(\nu_l)} f(\nu) d\nu \leq 1$. Therefore, we have that

$$\lim_{\nu_l \searrow -\infty} \frac{F(\nu_l)^2}{f(\nu_l)} \int_{\nu_l}^{g(\nu_l)} \frac{\partial \sigma_C(\nu; \nu_l, g(\nu_l))}{\partial \nu_l} d\nu = 0.$$

The proof that

$$\lim_{\nu_l \searrow -\infty} \frac{F(\nu_l)^2}{f(\nu_l)} \int_{\nu_l}^{g(\nu_l)} g'(\nu_l) \frac{\partial \sigma_C(\nu; \nu_l, g(\nu_l))}{\partial \nu_h} d\nu = 0.$$

is analogous. With that, we can conclude that $\lim_{\tau \nearrow \infty} \mathrm{BA} = 0$, which implies the second part of Proposition **??**, and concludes the proof. ■

Proof of Proposition (premium)

Proof. First note that, due to the assumed symmetry in the parameters, we have that $p^A(\tau) = \sigma_D \bar{\nu}^A$ and $p^B(\tau + d\tau) = \sigma_D \bar{\nu}^A$. Therefore, the liquidity premium can be written as

We have shown in the proof of Proposition **??** that $\frac{d\,\mathrm{BA}}{d\tau} > 0$ in a neighborhood of $\tau = 0$, which implies that $\mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) > 0$ and $LP(\tau) < \frac{\bar{\nu}^A - \bar{\nu}^B}{\bar{L}^B - \bar{L}^A}$.

We have also shown that $\lim_{\tau \to \infty} \mathrm{BA} = 0$. As $\mathrm{BA}(\tau) > 0$ in a neighborhood of $\tau = 0$, then $\frac{d\,\mathrm{BA}}{d\tau} < 0$ for some τ, which implies that $\mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) < 0$ and $LP(\tau) > \frac{\bar{\nu}^A - \bar{\nu}^B}{\bar{L}^B - \bar{L}^A}$. Moreover, as $\lim_{\tau \to \infty} \mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) = \lim_{\tau \to \infty} \mathrm{BA}(\tau + d\tau) - \lim_{\tau \to \infty} \mathrm{BA}(\tau) = 0$, we have that $\lim_{\tau \to \infty} LP(\tau) = \frac{\bar{\nu}^A - \bar{\nu}^B}{\bar{L}^B - \bar{L}^A}$. This concludes the proof. ■

B Data

We use corporate bonds transactions data from the TRACE Enhanced (ETRACE) database from January 2005 to June 2021. This initial data set provides us with a total of 171,140,493 trades as well as with 283,250 uniquely-identifiable bonds.[12]

We use a procedure based in Dick-Nielsen (2009) and Dick-Nielsen (2014) to filter out errors, cancellations, reversals and double counting as well as transactions missing individual CUSIP identification. We subsequently drop trades missing yield information and trades that are either on a when-issued basis, in a non-secondary Market, with a special condition, automatic give-ups, or in equity-linked notes.[13]

To avoid having many bonds in our sample that trade only momentarily, we add the following two conditions: (1) the bond must have existed in ETRACE for at least one complete year; and (2) the bond must have traded at least 75% of its relevant trading days (BPW and Anderson and Stulz 2017). Bonds must also have sufficient trades to satisfy the conditions, as defined in the following section, necessary to calculate their individual illiquidity measures. Having applied all these trade-based criteria, we are left with 55,753,160 transactions in 5,410 unique issues.

We use Bloomberg to collect bond information on issuance and maturity dates, provisions, coupons, currency denomination, amount outstanding, and ratings. We use the amount outstanding of each issue at the last business day of each month. A bond is defined as investment grade if its rating is greater than or equal to BBB– from S&P and Fitch or Baa3 from Moody's. We first use the rating from Standard & Poor's; if this rating is unavailable, we use the rating from Fitch; and if this rating is unavailable, we use the rating from Moody's.[14]

[12] The Trade Reporting and Compliance Engine (TRACE) is the "FINRA-developed vehicle that facilitates the mandatory reporting of over-the-counter secondary market transactions in eligible fixed income securities." The bond transactions report was implemented in different phases. It started with Phase I, on July 2002, for investment grade bonds and with issue size greater than or equal to $1 bi, and it continued later with the requirements expanded in Phase II in 2003. The complete implementation occurred in 2005, with Phase III. The report of corporate bond transactions is mandatory for all broker-dealers FINRA members. Therefore, Phase III virtually contains complete coverage of all public transactions. For consistency of the selection into the dataset, our dataset focus on Phase III. The Enhanced TRACE differs from the Standard TRACE in that it discloses more detailed information in individual transactions, e.g., actual trade size.

[13] To remove any potentially erroneous trades still remaining in the database, we also add a price filter for trades with prices deviating more than 25% from the daily average. This procedure cleans only about 0.1% of the trades.

[14] Although we use a different order based on data availability, this process is similar to Dick-Nielsen et al. (2012).

We exclude trades that took place outside the range of issuance and maturity dates of an issue, and bonds for which the outstanding amount at the last business day of that specific month was zero. Defaulting bonds are eliminated from the sample for as long as they are considered in default, and so are bonds with missing information. We only keep in our sample callable or non-provisional, fixed-rate bonds issued in the US. Callable bonds comprise a significant portion of our sample. Removing these bonds would negatively impact the quality of our results. Instead, we control our results for callability by introducing a dummy to our model. At this stage, our sample consists of 45,026,565 trades in 4,255 individual bonds.

We calculate the individual yield spread as the difference between the yield of the corporate bond and the yield of the government bond with the same maturity, as in BPW. The constant maturity yield curve is obtained from the Federal Reserve Bank of St. Louis FRED dataset. We use linear interpolation to calculate the yield of the government bond matching the exact maturity of the corporate bond. The monthly cross-sectional yield spread of a corporate bond is then calculated as the average daily spread in the month.

We use the Eikon dataset to collect each issuer's daily 5-year Credit Default Swap (CDS) quotes, which we use to proxy for the issuer's credit risk. Our measure of credit risk for each monthly cross-section is the average of the issuer's end-of-day CDS spreads. As this data is sufficiently large for the bonds in our database from December 2007, we redefine our sample period to begin in December 2007. We use stock prices to calculate the annualized equity return volatility of each issuer. Bonds missing CDS and equity volatility data are excluded from our dataset. We collect the daily stock prices of the issuers from CRSP.

Our final bond sample consists of 32,435,392 trades in 3,073 unique issues, which are distributed over a period of 115 months starting from December 2007. In total, we have 139,168 combinations of bond-month observations. The number of observations varies between monthly cross-sections depending on, among other things, newly-issued and matured bonds, trade frequency, and issues satisfying our selection criteria in the observed cross-section. Our final sample is predominantly composed by investment grade bonds.

We separate our sample period in three time intervals characterized by different macroeconomic conditions: (1) the financial crisis period from December 2007 to December 2009; (2) the post-crisis period with historically low interest rates from January 2010 to November 2015;

and (3) the monetary tightening period from December 2015 to June 2017. Table 2 presents a summary of our data together with the illiquidity measures described in the next section.

Table 2: Summary statistics

	Complete Period Dec '07–June '17			Crisis Dec '07–Dec '09			Post-Crisis Jan '10–Nov '15			Monetary Tightening Dec '15–June '17		
Observations	139,168			17,165			91,424			30,579		
Investment Grade	87%			86%			87%			87%		
N. of Bonds	3,073			1,134			2,688			1,905		
Callable	33%			33%			33%			31%		
N. of Firms	416			227			388			310		
N. of Trades	32,435,392			6,078,875			20,192,250			6,164,267		
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
γ	2.225	1.044	3.931	5.776	2.623	11.648	1.284	.639	1.842	1.067	.480	1.586
AMD ($\times 10^3$)	2.933	1.755	3.945	6.497	3.789	9.603	2.003	1.255	2.386	1.717	.947	2.325
Spread	2.163	1.543	2.179	4.160	2.856	4.634	1.598	1.202	1.387	1.646	1.090	1.909
CDS ($\times 10^{-2}$)	1.674	1.063	2.008	2.712	1.529	3.837	1.385	.972	1.375	1.390	.789	1.969

This table reports a summary of our sample variables together with a summary of the main variables calculated. The observations are the bond-month combinations. The mean, median and standard deviation are the time-series averages of the respective cross-sectional measures within each sub-period. Spread is the corporate bond yield spread detailed in section B. γ and AMD are the illiquidity measures detailed in section B.

C Additional empirical results

C.1 The determinants of bond illiquidity

Given the importance of bond illiquidity for yield spreads, we now study the determinants of illiquidity for individual bonds. We regress illiquidity on characteristics such as CDS, time to maturity, volume, issuance size, callability, and issuer's credit rating. We estimate pooled OLS regressions with two-dimensional clustered standard errors. Results are in table 3. In summary, our results indicate that the main determinants of illiquidity of a particular bond are credit risk and time to maturity.

We find that credit risk and time to maturity are the most important characteristics of a bond for illiquidity. Illiquidity is positively correlated to credit risk and time to maturity. When the CDS of an issuer widens 100 basis points, γ of its bonds increases .901. This increase corresponds to about 40% of the average γ (table 2). One additional year in time to maturity increases γ by .165, which corresponds to 7% of the average γ.

Volume and frequency show contrasting results. Bonds with greater volume are more

Table 3: γ and AMD on bond characteristics

	γ			AMD		
CDS	.817		.901	.697		.719
	[6.75]		[6.50]	[7.89]		[6.93]
Maturity		.161	.165		.128	.139
		[11.02]	[11.84]		[7.87]	[9.29]
Age			−.002			.117
			[−.12]			[5.42]
Coupon			.078			.079
			[2.14]			[2.17]
Volume			−.750			−2.01
			[−3.78]			[−7.32]
Frequency			.510			3.24
			[2.15]			[12.82]
ln(Issuance Size)			−.787			−1.17
			[−8.31]			[−8.59]
EqVol.			.481			.308
			[1.38]			[1.70]
IG			2.18			2.19
			[5.05]			[6.78]
Call			−.318			−.053
			[−3.08]			[−.50]
Constant	.526	.666	2.27	1.39	1.57	5.01
	[4.26]	[6.15]	[3.28]	[10.17]	[13.06]	[5.94]
Adj.R^2	.089	.032	.142	.111	.035	.226
Obs.	139,168	139,168	139,168	139,168	139,168	139,168

Bond-level illiquidity measures regressed on bond characteristics. We run a pooled OLS regression with standard-errors clustered by bond and month. T-statistics in square brackets. γ and AMD are the illiquidity measures detailed in section B. AMD is multiplied by 10^3. Maturity is the issue's time to maturity. Maturity and age are calculated in years at the last business-day of each month. Volume is calculated as the total \$ amount traded $\times 10^{-11}$ and frequency is in thousands of trades. Issuance size is in \$ millions. EqVol. is the issuer's annualized equity return volatility. IG is 1 if the bond is Investment Grade and 0 if otherwise. Call is 1 if the bond is callable and 0 if otherwise.

liquid, but so are bonds that trade less frequently. Bonds with larger average size per trade are potentially more liquid. Callable bonds are more liquid for both γ and AMD.

The results for the credit rating and coupon are surprising. We find that investment-grade bonds have substantially higher γ's. We interpret that the effect of credit quality is captured more strongly by the CDS than by the credit rating. This effect occurs because high-yield bonds have on average significantly higher CDS spreads than investment-grade bonds. Coupon has a non-intuitive positive slope, but it is small are not highly significant.

The results for γ and AMD in general indicate the same direction. An exception is for age, negative and not significant for γ, but positive and strongly significant for AMD. A positive coefficient for age indicates a market preference for on-the-run securities as opposed to older, off-the-run issues from the same firm. This finding is consistent with the on-the-run and off-the-run spread (for example, Krishnamurthy 2002).