A Novel Approach for Simulating Loss Distribution and Optimal Capital Requirement

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Abstract

In this study we estimate different measures to understand how much systemic risk each bank brings to the Brazilian market and propose a bank run model that accounts for idiosyncratic probability of default of banks and a systemic risk process in which additional defaults occur through different channels of contagion. Through our approach, we estimate the loss distribution, the probability of default of the deposit insurance agency, and simulate the optimal capital adequacy ratio of the Brazilian banking system.

JEL-Codes: E44, E58, G01, G21, G28

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1 Introduction

The recent global financial crisis has emphasized the importance of policies that improve the overall stability of the financial system and was one of the clearest illustrations in history about systemic risk, in which banks and credit play a particularly important role (De Bandt and Hartmann, 2019). However, although there is a vast literature and financial interest on this topic, there is a broad recognition among macroeconomists and policymakers about the lack of a clear understanding of system-wide risk channels and how much systemic risk each financial institution (FI)[1] brings to the overall economy (Christiano et al., 2018; Fidrmuc and Lind, 2020). Also, because of the criticality of this theme, there is also a growing consensus among policy makers that a macroprudential approach to regulation and supervision should be adopted to ensure a sound global financial system (Hannoun, 2010; Borio, 2011; Galati and Moessner, 2013).

Understanding how each FI affects systemic risk through its idiosyncratic characteristics or its connections with the rest of the economy is key to the effective action by both central bank and policy makers. In that sense, a very important package of macroprudential regulation in response to the global financial crisis is the so-called Basel III. Basel III is a comprehensive set of reform measures in banking regulation, supervision and risk management proposed in 2010. It was developed by the Basel Committee on Banking Supervision (BCBS), at the Bank for International Settlements (BIS), to strengthen the banking sector and achieve financial stability by increasing bank liquidity and decreasing bank leverage. One of the core metrics created for this purpose is the requirement of a minimum capital adequacy ratio[2] that banks must maintain to operate in the market, which is an important tool to absorbs unexpected losses without requiring the bank to cease its operations (BCBS, 2011).

[1]We used here the term bank and financial institution as similar, although one can understand bank as a subset of the financial services sector considering that non-banking companies comes also under the category of financial institution. In this view, a bank is a financial institution that can accept deposits into various savings and demand deposit accounts, services that a non-banking financial institutions (investment banks, leasing companies, insurance companies, investment funds, finance firms and others) cannot offer. For more information, see Hagendorff (2019).

[2]The capital adequacy ratio is calculated by adding Tier 1 and Tier 2 capital and dividing them by the risk-weighted assets (RWA). Tier 1 capital is the core capital of a bank and is formed by the Common Equity Tier 1 (CET1) and Additional Tier 1 (AT1), which includes equity capital and disclosed reserves. This type of capital, also described as going-concern capital, can absorb losses without requiring the bank to cease its operations. On the other hand, Tier 2 capital, also described as gone-concern capital, has a lower standard than Tier 1 and is used to absorb losses in the event of a liquidation.

Nevertheless, financial regulator often face conflicting objectives. One of the trade-off faced by them in establishing the minimum capital requirement (CR) with the purpose to strengthen the financial system and prevent systemic crisis is that a safer system could generate greater banking concentration and an increase in the cost of capital, harming the efficiency of financial intermediation and leading to a social cost that can outweigh its benefit. The reason is that banks may try to meet higher capital requirements by either reducing assets (decreasing, therefore, the loan supply) or increasing the loan interest rate, which leads to a reduction the demand for loans. Due to this and other factors, the design of the financial system's protection mechanism must be finely adjusted to maximize social welfare (Alexandre et al., 2022).

There are also other regulatory mechanisms designed to mitigate systemic risk. In addition to capital requirements and capital buffers[3] proposed by Basel III, the Financial Stability Board (FSB) developed in 2018 the bail-in, a new resolution framework to resolve distressed banks in an orderly manner without resorting to taxpayer money as in a bailout policy. Bail-in prescribes *ex-ante* resolution planning without the use of public funds for solvency support, forcing shareholders and creditors to share the burden of losses (FSB, 2018, 2021). These and other mechanisms in ongoing development reveal the importance of this issue for policy makers and central banks.

Regarding the Brazilian context, the implementation of the Basel III Accord occurred more rigorously than that practiced internationally. While the agreement prescribed a minimum capital requirement of 8%, the Central Bank of Brazil (BCB) established a minimum of 11% to operate in the market, in addition to specific capital buffers required. Furthermore, other complementary aspects of Brazilian regulation make the banking market robust and well capitalized in such a way that few bankruptcies are observed (Liberman et al., 2018). In terms of magnitude, from 2000 to September 2022, *Fundo Garantidor de Créditos* (FGC), the Brazilian deposit insurance, recorded only 20 extrajudicial settlements or interventions made by the BCB on the banking market (8.2%), while the Federal Deposit Insurance Corporation (FDIC) registered 563 interventions (11.9%) in USA[4].

[3]Basel III introduces two capital buffers that financial institutions must hold in addition to other minimum capital requirements: the capital conservation buffer and the countercyclical capital buffer. The capital conservation buffer was introduced to ensure that banks have an additional layer of usable capital that can be drawn down when losses are incurred. The countercyclical capital buffer (CCyB) aims to protect the banking sector from periods of excess aggregate credit growth that have often been associated with the build-up of system-wide risks.

[4]For more information, see FGC for the list of banking failures in Brazil and FDIC for the list of banking

One of the consequences of this more robust and well-capitalized Brazilian financial system is the high historical banking concentration. According to BCB (2021a), since 2015 the Concentration Ratio of the Five Largest (RC5) Brazilian banks is greater than 65% for the different sets of banking segments[5] and accounting aggregates (assets, deposits, and credit operations), with an overall average of 79.2% in 2018, 78.6% in 2019, 75.5% in 2020 and 74.2% in 2021. This drop in baking concentration in the last years can be attributed to the loss of participation of the public banks (47.6% in 2019 to 43.5% in 2021), especially BNDES, and the advancement of cooperatives (4.3% in 2019 to 6.1% in 2021). Nevertheless, this high concentration resulting from a regulatory framework and market specificity hinders the greater occurrence of bank failures in the country.

In that sense, a natural question that arises is what could be the regulatory easing that would increase banking competition, financial intermediation, and social welfare, while maintaining a secure financial system. To explore this question, we estimate different measures to understand how much systemic risk each bank brings to the market and propose a bank run model that simulate the loss distribution (LD) and the optimal capital requirement (CR) of the Brazilian banking system. This paper begins by estimating the following measures: (i) Systemic Expected Shortfall (SES), (ii) Systemic Risk (SRISK), and (iii) Conditional Value-at-Risk (CoVar) (Adrian and Brunnermeier, 2016; Acharya et al., 2010, 2017; Engle, 2018).

The second part of this paper uses a similar theoretical framework in which portfolio risk is calculated in banking organizations in order to simulate the optimal CR for the Brazilian financial system. We also adopted a heterogeneous CR regime in which we have different CRs for each bank depending on the prudential segment as in Alexandre et al. (2022). Banks can also hold different levels of capital adequacy ratio (CAR) based on their own strategy and balance structure. Through this analysis, we used granular balance sheet information to estimate the PD of each FI, the amount of capital expected in times of crisis (SRISK), the banking segmentation clusters, and the net stable funding ratio to capture bank runs due to panic and liquidity shortages, respectively, and the interbank

failures in the USA. The percentages were calculated considering the financial institutions insured by the deposit insurance in each country in September 2022.

[5]The BCB (2021a) considers three levels of aggregation to calculate the concentration of RC5: banking and non-banking segment ($b1 + b2 + b3 + b4 + n1$), banking segment ($b1 + b2$) and commercial banking segment ($b1$). The business model category ($b1$) includes commercial banks, multiples with a commercial portfolio and savings banks; ($b2$), multiple banks without a commercial portfolio and investment banks; ($b3$), credit unions; ($b4$), development banks; and ($n1$), non-bank credit institutions.

network to estimate the financial contagion in the banking system.

This work is divided into 5 sections, being the first one this introduction. Section 2 presents the theoretical framework of all our analyses to estimate the systemic risk of each bank and the optimal capital requirement for the Brazilian financial system. Section 3 presents the data used in our models. Section 4 presents the results and discussion of our results and Section 5 presents the final remarks of this paper.

2 Theoretical Framework

This section presents the theoretical framework used to estimate each financial institution's contribution to systemic risk. In addition, it also presents the bank run model that considers different channels of contagion used to calculate the loss distribution and the optimal capital requirement for the Brazilian financial market.

2.1 Systemic Risk Measures

Systemic risk refers to the risk of a financial crisis or market failure that affects the stability of the financial system and has widespread impacts on the economy as a whole. This type of risk is of particular concern in financial systems due to the interconnectedness of financial institutions, which can amplify the effects of individual failures and propagate shocks throughout the system as a negative domino effect. In recent years, the concept of systemic risk has gained increasing attention from academics and policy makers, as the global financial crisis highlighted the potential consequences of systemic failures (Brunnermeier and Oehmke, 2013; Adrian and Brunnermeier, 2016; De Bandt and Hartmann, 2019).

Measuring the systemic risk of the financial system is, to some extent, related to measuring banking risk. This makes risk measures at the bank level a natural starting point for thinking about systemic risk. The purpose of these measures is to reduce a large amount of data to a single meaningful statistic that summarizes the risk of each financial institution. Over the last two decades, especially after the implementation of Basel II bank regulations and the global financial crisis, a large literature has explored and created metrics to measure and capture systemic risk such as: (i) Systemic Expected Shortfall (SES), (ii) Systemic Risk (SRISK), (iii) Conditional Value-at-Risk (CoVar), and many others[6] (Brunnermeier and Oehmke, 2013; Adrian and Brunnermeier, 2016; Acharya et al., 2010, 2017; Brownlees and Engle, 2017; Engle, 2018).

Because we are essentially dealing with market data, we begin by defining some components that these measures have in common. Consider N financial institutions and let r_t^i be the log return of the daily stock price of bank i at time t, $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Also, let r_t^m be the log return of the daily market index to which all banks participate, which in our case is captured by the Bovespa index (Ibovespa)[7]. Then, the bank and market return processes are given by 1.

$$r_t^i = \mu^i + \varepsilon_t^i \qquad \text{and} \qquad r_t^m = \mu^m + \varepsilon_t^m \tag{1}$$

In which μ is the expected return and ε_t is a zero-mean white noise. Although serially uncorrelated, the series ε_t does not need to be serially independent and can present conditional heteroskedasticity (Zakoian, 1994). Thus, to model this time-varying volatility, we used the the GJR-GARCH (p, q) model (Glosten et al., 1993) that allows shock asymmetry through γ and assumes a specific parametric form of $\varepsilon_t = \sigma_t z_t$ for this conditional heteroskedasticity, where z_t is a standard Gaussian and the volatility σ_t is given by 2.

$$\sigma_t^2 = \omega + \sum_{k=1}^{p} (\alpha_k + \gamma_k I_{t-k}) \varepsilon_{t-k}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{2}$$

Where

$$I_{t-k} := \begin{cases} 0 & \text{if } r_{t-k} \geq \mu, \\ 1 & \text{otherwise} \end{cases} \tag{3}$$

Following Brownlees and Engle (2017) and Engle (2018), we used GJR-GARCH (1,1) for all models and also found that this specification best fits our data. Furthermore, all parameters $(\mu, \omega, \alpha, \gamma, \beta)$ were simultaneously estimated by maximizing the log likelihood

[6] Bisias et al. (2012) categorize and contrast more than 30 systemic risk measures. For more information on the extensive literature on systemic risk and its connection with the current regulatory debate, see also Jobst and Gray (2013), Benoit et al. (2017) and Silva et al. (2017).

[7] Ibovespa is the main performance indicator of the stocks traded in B3 and lists major companies in the Brazilian capital market. It was created in 1968 and, over the last 50 years, has set a benchmark for investors around the world. Ibovespa is reassessed every four months and is the result of a theoretical portfolio of stocks. It is composed of stocks and units of companies listed on B3 that meet the criteria described in its methodology, accounting for about 80% of the number of trades and the financial volume of our capital markets. For more information, see B3 (2022).

and the best model was selected based on the Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC). In addition, we use the strictly positive restriction on all parameters beside μ. The assumption that z_t is Gaussian does not imply that the returns are Gaussian. Although their conditional distribution is Gaussian, their unconditional distribution presents excess of kurtosis (fat tails). However, if the true distribution is different, the Quasi-Maximum Likelihood (QML) estimator is still consistent (Glosten et al., 1993).

Despite the assumption that returns are serially uncorrelated, returns may present contemporaneous correlation. In other words, from equation 1, defining a vector of zero-mean white noise as $\varepsilon_t = r_t - \mu$, $\sum_t := E_{t-1}[(r_t - \mu)(r_t - \mu)']$ may not be a diagonal matrix. Moreover, this contemporaneous variance may be time-varying, depending on past information. Therefore, the correlation between each bank i and the market index is captured by the GARCH-DCC model (Engle, 2002, 2009) and is estimated in two steps.

The first step of the GARCH-DCC model accounts for the conditional heteroskedasticity. It consists of estimating the conditional volatility σ_t^i using a GARCH (1,1) model (Engle, 1982; Bollerslev, 1986) for each one of the N bank series of returns r_t^i. Let $\boldsymbol{D}_t$ be the diagonal matrix with these conditional volatilities, that is, $\boldsymbol{D}_t^{i,i} = \sigma_t^i$ and, if $i \neq j$, $\boldsymbol{D}_t^{i,j} = 0$. Then the standardized residuals with unit conditional volatility, $\boldsymbol{\nu}_t$, are given by Equation 4 and the Bollerslev (1990)'s constant conditional correlation (CCC) estimator, $\overline{\boldsymbol{R}}$, is given by Equation 5.

$$\boldsymbol{\nu}_t := \boldsymbol{D}_t^{-1}(r_t - \mu) \tag{4}$$

$$\overline{\boldsymbol{R}} := \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{\nu}_t \boldsymbol{\nu}_t' \tag{5}$$

The second and final step consists in generalizing Bollerslev (1990)'s CCC to capture dynamics in the correlation, giving origins to the dynamic conditional correlation (DCC). Assuming the standardized residuals are jointly Gaussian and let $\boldsymbol{Q}_t^{i,j}$ be the correlation between r_t^i and r_t^j at time t, the DCC correlations are given by Equation 6.

$$\boldsymbol{Q}_t = \overline{\boldsymbol{R}} + \sum_{k=1}^{p} \boldsymbol{\alpha}_k(\boldsymbol{\nu}_{t-k}\boldsymbol{\nu}_{t-k}' - \overline{\boldsymbol{R}}) + \sum_{j=1}^{q} \boldsymbol{\beta}_j(\boldsymbol{Q}_{t-j} - \overline{\boldsymbol{R}}) \tag{6}$$

Where both parameters, $\boldsymbol{\alpha}, \boldsymbol{\beta} > 0$, are simultaneously estimated by maximizing the log likelihood and the best model was selected based on the BIC and AIC information criterion. Once these common components of volatility and correlation are defined, the following subsections presents the concept and definition of the main metrics of systemic risk used in this paper.

2.1.1 Systemic Expected Shortfall

The systemic expected shortfall (SES) proposed by Acharya et al. (2010, 2017) measures the expected capital shortfall of a bank conditional on a substantial reduction in the capitalization of the banking system and also provides a ranking for systemically risky banks. The theoretical approach of this model considers that the aggregate capital shortfall of the financial sector imposes a negative externality on the real economy. Thus, in order to estimate the capital shortfall of the financial sector, the first step is to estimate the marginal expected shortfall (MES) of a bank. MES is the expected short-term equity loss of a financial institution conditional on the market taking a loss greater than its Value-at-Risk (VaR) at $\alpha\%$. Taking into account the parameters established in Section 2.1, the MES is given by 7.

$$MES_{it} = E_{t-1}(r_t^i | r_t^m < C) \tag{7}$$

Where $C = q^\alpha(r_t^m)$ is a threshold corresponding to the tail risk in the market at time t. Note that the definition of Acharya et al. (2010, 2017) considers the market return r_t^m as the value-weighted average of all bank returns in the market, that is, $r_t^m = \sum_{i=1}^N w_t^i r_t^i$, where w_t^i denotes the relative market capitalization of the bank i. However, although it would be possible to reconstruct the r_t^m through each r_t^i, we opt to use the main benchmark of the Brazilian stock exchange, Ibovespa, as our r_t^m for purposes of comparability of results with the literature.

Also, define the expected shortfall (ES) of the market as the expected loss in the index conditional on this loss being greater than C, i.e., $ES_t = E_{t-1}(r_t^m | r_t^m < C)$. Thus, note that MES of one bank is the derivative, or the marginal effect, of the market's ES with respect to the bank's market share (or capitalization). Therefore, the MES of a bank in this case can be interpreted as reflecting its contribution and participation to overall systemic risk. The higher the MES, the higher is the individual contribution of the bank to the risk of the financial system. However, it is still possible to define the same statistic even if the observed bank does not participate in the market index. Rather than a measure of how a particular bank's risk adds to the market risk, the MES in this other case should be viewed as a measure of the sensitivity (or resilience) of the bank's stock price to exceptionally bad market events.

Once the ES and MES are defined, the SES extends the MES and corresponds to the amount that a bank's equity drops below its target level, defined as a fraction k of assets, in case of a systemic crisis when the aggregate capital is less than a fraction k of the aggregate assets. Formally, the SES is given by 8.

$$\frac{SES_{it}}{W_{it}} = kL_{it} - 1 - E_{t-1}\Big(r_t^i \mid \sum_{i=1}^{N} W_{it} < k\sum_{i=1}^{N} A_{it}\Big) \tag{8}$$

Where A_{it} denotes the total assets, W_{it} the market capitalization or market value of the equity, and $L_{it} = A_{it}/W_{it}$ the leverage. In particular, in this work we set the prudential capital fraction k according to the prudential segment of each Brazilian bank. Acharya et al. (2010, 2017) show that the conditional expectation term can be expressed as an increasing linear function of MES, given by Equation 9.

$$SES_{it} = (kL_{it} - 1 + \theta MES_{it} + \Delta_i)W_{it} \tag{9}$$

In which θ and Δ_i are constant terms.

2.1.2 Systemic Risk

Taking into account the significant negative externalities that undercapitalization of large financial institutions has on the real economy, Brownlees and Engle (2017) proposed a systemic risk metric called SRISK to measure the capital shortfall of a bank conditional on a severe market decline[8]. Although this contribution is related to the SES measure proposed by Acharya et al. (2010, 2017), the authors argue that SRISK does not require structural assumptions or observation of the realization of a systemic crisis for estimation, making this a viable *ex-ante* measure with higher predictive power than SES. Furthermore, the authors argue that aggregate SRISK also provides early warning signals of distress in indicators of real activity.

The SRISK calculation is analogous to the stress tests that are applied to financial

[8]For an early report on SRISK, see Acharya et al. (2010, 2012) and Engle (2016).

institutions but only uses publicly available information. The capital shortfall is the variable introduced to measure the distress of a financial institution, which can be defined as the difference between the regulatory capital reserves that the bank needs to hold and the bank's equity. Formally, the capital shortfall of a bank i in time t is given by 10.

$$CS_{it} = kA_{it} - W_{it} = k(D_{it} + W_{it}) - W_{it} \tag{10}$$

Where D_{it} is the book value of the debt and the other parameters were previously defined in Section 2.1.1. The capital shortfall can be understood as the negative of the working capital of the bank. When the capital shortfall is negative, that is, the bank has a capital surplus, the bank functions properly. However, when this metric is positive, the bank experiences distress.

Because the interest is to predict the capital shortfall of a bank in the case of a systemic event, Brownlees and Engle (2017) uses the same concept of a market decline below a threshold C proposed by Acharya et al. (2010, 2017). Thus, the definition of SRISK as the expected capital shortfall conditional on a systemic event is given by 11.

$$\begin{aligned} SRISK_{it} &= E_t(CS_{it+h}|r^m_{t+1:t+h} < C) \\ &= kE_t(D_{it+h}|r^m_{t+1:t+h} < C) - (1-k)E_t(W_{it+h}|r^m_{t+1:t+h} < C) \end{aligned} \tag{11}$$

In which $\{r^m_{t+1:t+h} < C\}$ is the systemic event with probability α, $r^m_{t+1:t+h}$ is the multiperiod arithmetic market return between periods $t+1$ and $t+h$, and $SRISK_{it} \geq 0$. In order to compute this expectation, the authors assume that, in the case of a systemic event, debt cannot be renegotiated, which implies that $E_t(D_{it+h}|r^m_{t+1:t+h} < C) = D_{it}$. Finally, using this assumption in Equation 11 results in the final Equation 12 for SRISK.

$$SRISK_{it} = \max\{0 \ ; \ kD_{it} - (1-k)W_{it}(1 - LRMES_{it})\} \tag{12}$$

Where $LRMES_{it} = -E_t(r^i_{t+1:t+h}|r^m_{t+1:t+h} < C)$ is the Long Run MES, that is, the expectation of the bank equity multiperiod arithmetic return conditional on the systemic event, in which $r^i_{t+1:t+h}$ is the multiperiod arithmetic return of bank equity between periods $t+1$ and $t+h$. In other words, when a stress scenario occurs, the equity decreases by a rate called the LRMES. Note from Equation 12 that SRISK is higher for banks that are larger, more leveraged, and with higher sensitivity to market declines.

To estimate LRMES, Engle (2018) proposed a direct approach in which $LRMES_{it} = 1 - \exp\left[(\widetilde{\beta}_t^i)\log(1-\theta)\right]$, where $\widetilde{\beta}_t^i$ is the nested dynamic conditional beta (DCB) and θ is the expected drop in the market during a financial distress event[9]. The initial DCB approach described by Engle (2016) considers that beta is the product of a correlation between the bank return and the market return, $\rho_t^{i,m}$, times the standard deviation of the bank return, σ_{it}, and market return, σ_{mt}. Because all of these three values are potentially time-varying, the author estimates them through GJR-GARCH and GARCH-DCC models. Formally, the DCB estimates of beta is given by 13.

$$\widehat{\beta}_t^i = \rho_t^{i,m}\left[\frac{\sigma_{it}}{\sigma_{mt}}\right] \tag{13}$$

We also follow Engle (2018) by building an artificial model that nests both the constant beta and DCB through Equation 14.

$$r_t^i = (\phi_1 + \phi_2\widehat{\beta}_t^i)r_t^m + \varepsilon_t^i \tag{14}$$

The estimates of both coefficients are made by assuming a MA(1) GJR-GARCH error term to construct a weighted least squares (WLS) model[10]. We would expect $\phi_2 = 0$ for a constant beta or $\phi_1 = 0$ for DCB, but because both hypotheses can be rejected, it is preferable to consider some combination of constant and time-varying beta (Engle, 2018). Then, the nested DCB used to estimate LRMES and SRISK is given by 15.

$$\widetilde{\beta}_t^i = (\widehat{\phi}_1 + \widehat{\phi}_2\widehat{\beta}_t^i) \tag{15}$$

2.1.3 Conditional Value-at-Risk

The conditional value-at-risk (CoVaR) is a concept proposed by Adrian and Brunnermeier (2016) and represents the $p\%$ quantile of market return using the distribution conditional on the event that a particular bank return equals its VaR at $q\%$ and the preceding state variable are M. In other words, it is a measure of how sensitive the overall market is to a decline in a particular financial institution. First, the definition of VaR of

[9]Works like Acharya et al. (2012) and Engle (2018) consider $\theta = 40\%$ over 6 months for the US market, a measure that is further discussed in Section 2.2.2

[10]The weights are the inverse of the variance of the MA(1) GJR-GARCH model of ε_t^i (Engle, 2016).

bank i with probability q, such as 5%, is given by 16.

$$P(r_t^i < -VaR_{i,t}^q \mid M_{t-1}) = q \tag{16}$$

The CoVaR of the banking system in quantile p when a particular bank i has a market decline equal to its $VaR_{i,t}^q$ is given by 17.

$$P(r_t^m < -CoVaR_{m|i,t}^{p,q} \mid r_t^i = -VaR_{i,t}^q, M_{t-1}) = p \tag{17}$$

Adrian and Brunnermeier (2016) define the risk contribution of the bank i to the overall market as the incremental change in its risk relative to its median state, that is, $\Delta CoVaR_{m|i,t}^q = CoVaR_{m|i,t}^{q,q} - CoVaR_{m|i,t}^{q,0.5}$. In addition, they also proposed the use of quantile regression for its efficiency and simplicity in estimating CoVaR, evaluating the estimated equation with the independent variables of interest, which is given by Equation 18 and Equation 19.

$$r_t^m = \alpha_i^q + \beta_i^q r_t^i + \varepsilon_t \tag{18}$$

$$CoVaR_{m|i,t}^q = \widehat{\alpha}_i^q + \widehat{\beta}_i^q VaR_{i,t}^q \tag{19}$$

Therefore, the $\Delta CoVaR_{m|i,t}^q$ is then given by Equation 20.

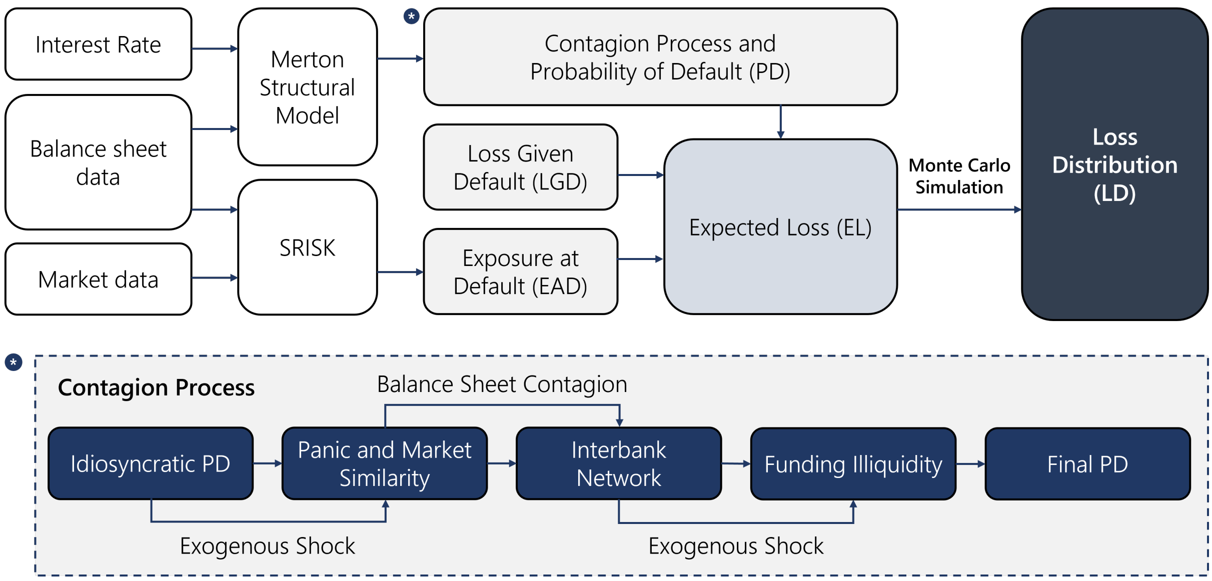
$$\Delta CoVaR_{m|i,t}^q = \widehat{\beta}_i^q (VaR_{i,t}^q - VaR_{i,t}^{0.5}) \tag{20}$$

2.2 Bank Run Model

In this section, we present the main contribution of our paper by proposing a bank run model that accounts for single, or idiosyncratic, probability of default of banks based on their balance sheet structure and a systemic risk process in which additional defaults occur through different channels of contagion. The two fundamental channels of contagion considered by the literature of financial stability and systemic risk are: (i) the exposure channel and (ii) the informational channel (Greenwood et al., 2015; Paltalidis et al., 2015; Hurd, 2016; Souza et al., 2016; De Bandt and Hartmann, 2019; Jackson and Pernoud, 2021; Radev, 2022). These fundamental channels can work in conjunction as well as independently, and we model them through three processes: (i) panic due to deposit withdrawals and market similarity, (ii) interbank network and (iii) funding illiquidity.

Several works in the literature focus on the banking contagion process through one of these three channels[11], but few studies have shown the significant role they all play together in understanding the full effect of contagion (Paltalidis et al., 2015; Glasserman and Young, 2016; Anderson et al., 2019; Jackson and Pernoud, 2021). It is with this last understanding that we propose our model. Our approach follows a similar theoretical framework in which portfolio risk is calculated in banking organizations and how banking losses are estimated in deposit insurance schemes (Lehar, 2005; Gupton et al., 2007; De Lisa et al., 2011; Bellini, 2017; O'Keefe and Ufier, 2017; Parrado-Martínez et al., 2019; Matt and Andrade, 2019; Fernández-Aguado et al., 2022). Section 2.2.1 presents the model used to calculate the idiosyncratic probability of default and Section 2.2.2 introduces the model that accounts for different channels of contagion. The diagram of the structure and steps of our model is shown in Figure 1.

Figure 1: Diagram of the structure and steps of the Bank Run model.



* Note that the contagion process amplifies the idiosyncratic probability of default given by the Merton (1974)'s structural model through the mechanisms and shocks described in Section 2.2.2. Furthermore, the LGD parameter is assumed to be equal to one for all Brazilian banks, as mentioned in Section 2.2.3.

[11] See Martin (2006), Gorton (2010), Greenwood et al. (2015), Robatto (2019) and Gertler et al. (2019) for panic due to deposit withdrawals and market similarity, Acemoglu et al. (2015), Cabrales et al. (2017) and Alexandre et al. (2022) for the interbank network and Ferrara et al. (2019) and Ardekani et al. (2020) for funding illiquidity.

2.2.1 Probability of Default

We used the Merton (1974)'s structural model to estimate each financial institution's idiosyncratic PD[12], which models credit risk using the Contingent Claim Analysis[13] (Souza et al., 2015, 2016; Guerra et al., 2016; Coccorese and Santucci, 2019; da Rosa München, 2022). It is a structural model because it provides the relationship between the debt and the value of the bank. The intuition of this approach is to consider the bank's assets as the underlying asset of a European call option, with strike price equal to its obligations and time to maturity T. Thus, if the bank defaults, the equity holders receive nothing because the bank does not have enough resources to repay its obligations. Otherwise, if it does not default, the equity holders receive the difference between the values of assets and liabilities.

Although Merton (1974)'s model establishes that a default occurs when the bank's assets (granted loans) are lower than its obligations (deposits received), in practice, however, it is possible to continue operating with a negative equity[14]. This is due to contract breakdown or liquidity scarcity problems when the bank needs to sell assets or due to debt renegotiation (Guerra et al., 2016). In order to address these characteristics, the literature proposes a threshold called distress barrier (DB) as a trigger for default, defined as a proportion of the face value of the debt. As a result, the bank defaults if its asset value falls below its DB, being computed using accounting data based on the KMV model and given by equation 21 (Crosbie and Bohn, 2003).

$$DB_{it} = STD_{it} + \alpha_{it} LTD_{it} \tag{21}$$

[12]There are works that use approaches such as CAMELS (Capital adequacy, Asset quality, Management, Earnings, Liquidity, Sensitivity to market risk) to estimate the PD using balance sheet variables (Valahzaghard and Bahrami, 2013; Calabrese and Giudici, 2015; Rosa and Gartner, 2018; Parrado-Martínez et al., 2019). Although this technique allows the use of more granular and specific bank variables, it performs better when there is a large number of observable bank defaults to estimate a robust logit model, which is not the case for the Brazilian economy. Even with strategies used to increase the default variable, such as considering interventions by supervisors, capital below the minimum required or mergers motivated by financial difficulties (Vazquez and Federico, 2015), criteria that have an inherent subjective aspect, we may not necessarily reach a sufficient number of defaults in certain economies to estimate a robust model.

[13]Contingent Claim Analysis (CCA) is a generalization of the option pricing theory presented in Black and Scholes (1973) to the analysis of the corporate capital structure. The general definition of a contingent claim is any asset whose future payoff is contingent on the outcome of an uncertain event. Therefore, CCA can be used to analyze how the value of the contingent claim changes as the value of the firm changes over time. For more information, see Jobst and Gray (2013).

[14]11 banks operated with negative equity in the Brazilian market between March 2000 and September 2022 (BCB, 2022a).

In which STD_{it} and LTD_{it} represent the sort-term (maturity ≤ 1 year) and long-term (maturity above 1 year) liabilities, respectively, and $0 \leq \alpha_{it} \leq 1$ is a parameter that proxies the share of long-term liabilities of a bank subjected to early redemption in case of stress. Due to the unavailability of time to maturity data of total liabilities for the Brazilian case, following Souza et al. (2016), we assume that they are predominantly short-term debts ($STD_{it} = 0.7$) with a significant long-term debt ($LTD_{it} = 0.3$) share. In general, the literature suggests $\alpha_{it} = 0.5$ if $LTD_{it}/STD_{it} < 1.5$, which gives $DB_{it} = 0.85TL_{it}$ (Crosbie and Bohn, 2003; Souza et al., 2015; Guerra et al., 2016; Coccorese and Santucci, 2019).

Using these definitions on the Black and Scholes (1973)'s model, the option's payoff received by the equity holder is given by 22.

$$E_{it} = max(A_{it}\, \mathcal{N}(d_{1it}) - DB_{it}\, e^{-r_t T} \mathcal{N}(d_{2it}), 0) \tag{22}$$

Where A_{it} is the asset value, r_t is the risk-free interest rate, $\mathcal{N}(.)$ is the cumulative normal distribution function,

$$d_{1it} = \frac{ln(\frac{A_{it}}{DB_{it}}) + (r_t + \frac{\sigma^2_{Ait}}{2})T}{\sigma_{Ait}\sqrt{T}} \quad \text{and}$$

$$d_{2it} = d_{1it} - \sigma_{Ait}\sqrt{T} = \frac{ln(\frac{A_{it}}{DB_{it}}) + (r_t - \frac{\sigma^2_{Ait}}{2})T}{\sigma_{Ait}\sqrt{T}},$$

in which σ_{Ait} denotes the volatility of the assets.

The time to maturity T, usually assumed to be 1 year, is the horizon for which the PD_{it} is computed. This one-year time horizon is consistent with i) the usual assets' classification into short-term and long-term liabilities that are required by the model, ii) the expected time for banks to adapt to capital increases and iii) the stress test exercises conducted by the regulatory authority BCBS (2010, 2011); O'Keefe and Ufier (2017).

To calculate each PD_{it}, two important assumptions are made. The first is that the bank's asset values are log-normally distributed (Crouhy et al., 2000; Lehar, 2005; Guerra et al., 2016; Souza et al., 2016; da Rosa München, 2022). The second is that investors are risk neutral, that is, the demand rate of return is the risk-free rate of return r_t, which is lower than that required by risk-averse investors. This assumption results in conservative (higher) PD_{it} estimates. Thus, the idiosyncratic PD_{it} of a FI in a time horizon T, calculated in $t = 0$, is given by 23.

$$
\begin{aligned}
PD_{it} &= P(DB_{it} \geq A_{it}) \\
&= P(\ln DB_{it} \geq \ln A_{it}) \\
&= \mathcal{N}(-d_{2it}) \\
&= \mathcal{N}\left[-\frac{ln(\frac{A_{it}}{DB_{it}}) + (r_t - \frac{\sigma^2_{Ait}}{2})T}{\sigma_{Ait}\sqrt{T}} \right]
\end{aligned} \tag{23}
$$

Note that the probability of default is the area under the default barrier, that is, a fraction of total liabilities. Also, note that the negative of d_{2it} can also be used to compute the distance to distress ($D2D$) for a risk neutral environment, which is the distance of the bank's asset value to the distress barrier in $t = 0$, measured in assets value' standard deviations.

2.2.2 Contagion

The following subsections present our approach for the two fundamental channels discussed in the literature, which are (i) the exposure channel and (ii) the informational channel (Greenwood et al., 2015; Paltalidis et al., 2015; Hurd, 2016; Souza et al., 2016; De Bandt and Hartmann, 2019; Jackson and Pernoud, 2021; Radev, 2022). As in Diamond and Dybvig (1983), the runs we consider are runs in the entire banking system and not on a single bank. In addition, as in Abergel et al. (2013), we do not consider the possibility of partial default in our model[15].

We begin by establishing three channels that are used to incorporate the contagion effect in the event of a bank default: (i) panic due to deposit withdrawals and market similarity, (ii) interbank network, and (iii) funding illiquidity. Note that these channels are only activated after a single bank defaults, although they do not necessarily generate a banking contagion process if the rest of the system is resilient[16]. The generation of a banking contagion process or a systemic risk event occurs when the default of an individual bank or multiple idiosyncratic defaults deteriorates the other banks in the system through

[15]In a model with partial default, represented by a default level on liabilities, it is necessary to introduce a different mechanism in which the non-defaulting banks have to sell part of their assets in order to satisfy some solvability ratio constraints. For more information, see Eisenberg and Noe (2001).

[16]Note that a run on an individual bank may not have aggregate effects if depositors simply shuffle their funds from one bank to the others in the system (Gertler et al., 2019). We capture this dynamic in our model when the shocks of the contagion process in the PD of banks that do not idiosyncratically default do not generate additional defaults.

these channels to the point of generating additional defaults.

An important component to be defined in the bank run simulation is the deterioration of the probability of default when there is a contagion process or financial crisis. Engle (2018) estimates a 40% deterioration over 6 months in PD when there is a financial distress event for the US market and Greenwood et al. (2015) estimated a 28% deterioration over 18 months in the market capitalization of European banks after the financial crisis. Through our simulations, we estimate a PD for the Brazilian financial system using the Merton (1974)'s structural model described in 2.2.1 and found a deterioration of 36% over 1 year during the 2008 financial crisis[17]. Therefore, for the first and third channels that do not depend on the interbank network, considering that the latter occurs through the bank balance sheets, we use a deterioration of 36% over the PD when they are activated.

2.2.2.1 Panic

The extensive literature on banking panics, beginning with Diamond and Dybvig (1983), presents an important theme of how sudden withdrawals triggered by the expectation of defaults can force the bank to liquidate many of its assets at a loss and, ultimately, fail. The losses worsen the conditions of banks, including those that are financially strong, reinforcing the flight to liquidity and making the process self-fulfilling. In this scenario, it is the forced liquidation at fire sale prices during a run that pushes these banks into bankruptcy, categorizing a situation of short-term illiquidity (Martin, 2006; Gorton, 2010; Greenwood et al., 2015; Kiss et al., 2018; Allen et al., 2019; Robatto, 2019; Anderson et al., 2019; Gertler et al., 2019).

Note that during this forced short-term liquidation, there is also a contagion effect on asset prices for other banks that hold similar assets or portfolios to those that suffer from these fire sales, leading to an undermining of its balance sheets. If the loss of an affected bank is so severe that it is unable to meet its minimum capital requirement, then the bank will have to sell some of its assets with a haircut, increasing the downward spiral in market prices (Nier et al., 2007; Cont et al., 2013; Huang et al., 2013; Glasserman and Young, 2016; Caccioli et al., 2018; Pichler et al., 2021).

Considering that panic behavior is fundamentally a problem of depositor expectation

[17] We construct a weighted PD considering the value of total deposits of each FI from December 2000 to September 2022. The 36% deterioration over 1 year in the weighted PD considers the annual average PD growth between June 2009 and June 2008.

and liquidity risk and has the potential to affect the overall stability of similar banks within the industry, this issue can be addressed from the complete information on the connection that each depositor and bank has with all other banks in the system (Brown et al., 2016; Anginer and Demirgüç-Kunt, 2019; Jackson and Pernoud, 2021). In that sense, after one bank idiosyncratically defaults, it is possible to anticipate the majority of banks that will be susceptible to the panic effect and the magnitude of this illiquidity shock from the cross-information of depositors and assets in those banks. However, because this is a private and confidential information, we choose to model this behavior using a cluster approach based on balance sheets and market data. Thus, through this process, we are still able to simulate a short-term panic shock in similar banks within the industry.

We choose to use the K-means clustering (Lloyd, 1957; Jancey, 1966; MacQueen, 1967) to estimate the clusters in the banking system. K-means clustering is an unsupervised learning algorithm for finding K pre-specified clusters and cluster centers (i.e. centroid) in a set of unlabeled data. Let $C_1, \ldots, C_K$ be the sets containing the indices of the observations in each banking cluster. These sets satisfy both (i) $C_1 \cup C_2 \cup \cdots \cup C_K = \{1, \ldots, n\}$ and (ii) $C_k \cap C_{k'} = \emptyset$ for all $k \neq k'$. Thus, (i) means that each observation belongs to at least one of the K clusters and (ii) means that the clusters are non-overlapping, i.e., no observations belong to more than one cluster (Hastie et al., 2009).

For instance, if the ith observation is in the kth cluster, then $i \in C_k$. Thus, a good clustering is one for which the within-cluster variation, $W(C_k)$, is as small as possible. Therefore, the objective of the K-means algorithm is to solve 24.

$$\min_{C_1,\ldots,C_K} \left\{ \sum_{k=1}^{K} W(C_k) \right\} \tag{24}$$

In order to solve 24, we first need to define the specification of the within-cluster variation. The most common choice to define this concept in the literature is the Hartigan and Wong (1979)'s algorithm, which defines the total within-cluster variation as the sum of squared Euclidean distances between items and the corresponding centroid. This specification is given by 25.

$$W(C_k) = \sum_{x_i \in C_k} (x_i - \mu_k)^2 \tag{25}$$

Where x_i denotes a data point belonging to the cluster C_k and μ_k is the mean value of the points assigned to the cluster C_k. The algorithm that solves the optimization problem when combining equations 24 and 25 can be decomposed into two major steps: (i) initially assigns a random number, from 1 to K, to each of the observations and (ii) iterates until the cluster assignments stop changing. In the second step, (a) for each of the K clusters, the algorithm computes the cluster centroid and (b) assigns each observation to the cluster whose centroid is the closest in terms of Euclidean distance. This algorithm is used iteratively until the local optimum is reached. (Hastie et al., 2009).

Because the K-means technique requires a pre-specification of the expected number of clusters, there are a variety of other direct and statistical methods used to define the within-cluster variation in order to find the optimal number of clusters. The two most common are the silhouette and the gap statistics. The silhouette statistic (Kaufman and Rousseeuw, 1990) measures how well an observation is clustered and estimates the average distance between clusters. A high average silhouette width indicates a good clustering and the optimal number of clusters K is the one that maximizes the average silhouette over a range of possible values for K. For observation i, let $a(i)$ be the average distance to other points in its cluster, and $b(i)$ the dissimilarity between i and its closest clusters to which it does not belong. Then, the silhouette statistics, $s(i)$, is defined by 26.

$$s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}} \tag{26}$$

On the other hand, the gap statistic (Tibshirani et al., 2001) compares the total within-cluster variation for different values of K with their expected values under the null reference distribution of the data. The estimate of the optimal clusters will be the value that maximizes the gap statistic, in which $\log(W_k)$ falls the farthest below this reference curve. Thus, considering $E_n[\log(W_k)]$ the expectation under a sample of size n, the gap statistic is defined by 27.

$$\text{Gap}_n(k) = E_n[\log(W_k)] - \log(W_k) \tag{27}$$

Once the clustering technique and the optimal number of clusters have been defined, we select some market and systemic risk variables used to distinguish these clusters and simulate panic behavior in the Brazilian banking system. It is important, however, that these clusters have an internal consistency with the prudential segmentation and the business model category established by the BCB (2022a). The variables used for the clustering are: (i) total deposits over total assets, (ii) loan, lease and other credit operations by risk level over total assets, (iii) risk-weighted assets (RWA), (iv) SRISK, and (v) ΔCoVaR. The dynamics of the banking panic process simulation is such that, once one of the banks within a cluster idiosyncratically defaults, the other banks in the cluster suffer a 36% shock in their probability of default, as mentioned in Section 2.2.2.

2.2.2.2 Interbank Network

Besides the panic effect, several works in the literature have demonstrated the important role that interbank networks play in the systemic risk contagion process, suggesting that the probability of default cascades is increasing in the size of interbank exposures (Anand et al., 2015; Bardoscia et al., 2015; Acemoglu et al., 2015; Silva et al., 2016; Anand et al., 2018; Anderson et al., 2019; Ferrara et al., 2019). However, it is important to note that network interconnectedness can be understood by both correlated portfolios, through common asset holdings among banks, and counterparty risk, through direct bilateral exposures between banks (Nier et al., 2007; Cont et al., 2013; Huang et al., 2013; Glasserman and Young, 2016; Pichler et al., 2021; Jackson and Pernoud, 2021). In that sense, we model these two phenomena by (i) considering market and asset correlation in the first short-term contagion channel described in Section 2.2.2.1 and (ii) considering the interbank market as a second mid-term contagion channel, which is addressed in this section.

In the interbank market, banks lend to each other at an interbank interest rate. In Brazil, the interbank deposit, DI, is traded exclusively among financial institutions and is a private fixed income instrument that assists in closing the cash of banks, as an instrument for raising funds or applying surplus resources. These securities, also called CDI, have high liquidity, no incidence of taxes on profitability and carry a very low risk, usually associated to the soundness of the banks that participate in the market. The negotiation between the banks generates the DI rate, reference for most of the fixed income securities offered to the investor. This interbank interest rate is the main benchmark of the market and is obtained by calculating the weighted average of the rates of the prefixed, extragroup (different conglomerates), and one-day transactions between financial institutions. Furthermore, it is important to note that, in the interbank market, banks can also trade one-day repurchase agreement contracts backed by federal securities with similar purpose of interbank deposit, but shifting the counterparty risk for sovereign risk (B3, 2022).

In addition to the functions of monetary, regulatory and supervisory authority exercised by the Central Bank of Brazil, it is also responsible for controlling and monitoring the liquidity of the banking system. Consistent with their role as lender of last resort (LOLR), the provision of liquidity support contributes for the credibility of domestic currency and to the financial system's stability. The BCB's liquidity facility (LFL) comprises the discount window lending operations based on non-government issued securities with financial institutions that hold a reserve or settlement account at BCB. The LFL's operations can be a short-term standing facility (LLI), normally intraday and overnight, or a long-term facility (LLT), through discretionary approval and enhanced operational process. The loans are secured against high-quality assets as collateral, which may not present immediate liquidity (BCB, 2021c).

Considering that the complete information about the Brazilian interbank network is private and only known by the Central Bank of Brazil, we reconstruct the interbank network based on the following main properties: (i) density, (ii) average degree, and (iii) assortativity. Among the several methods that can be used to estimate the matrix of bilateral exposures[18], we used an adapted version of the minimum density (MD) method proposed by Anand et al. (2015) because it offered the best match with the known properties of the Brazilian interbank network during March 2010 and September 2015[19] , which is: (i) density between [0.03,0.07], (ii) average degree between [4.6,7.8] for all banks or [21,26] for large banks, and (iii) assortativity between [-0.31, -0.54] (Castro Miranda et al., 2014; BCB, 2016; Souza et al., 2016; Silva et al., 2016; Anand et al., 2018; Alexandre et al., 2022).

The minimum density method proposed by Anand et al. (2015) is a heuristic procedure for allocating links that combines elements from information theory with economic incentives to produce networks that preserve the realistic characteristics of interbank activity. The authors argue that the MD approach is suitable for sparse networks, which is appropriate for the Brazilian financial market, and is able to reconstruct it by minimizing its cost of linkages. Let c be the fixed cost of establishing a link, $\mathbb{X} \in [0, \infty)^{N \times N}$ the matrix of bilateral aggregated exposure values, X_{ij} the unknown exposure of bank i to

[18]See, for instance, Anand et al. (2018) for a comprehensive survey of different estimation methods.

[19]The last release of the assortativity and degree metric of the Brazilian interbank network made by the BCB was in April 2016 with data until September 2015. All other data come from works that use the true Brazilian interbank network. For more information, see BCB (2016).

bank j and N the number of banks. The total observable interbank assets of bank i are $A_i = \sum_{j=1}^{N} X_{ij}$ and the total observable liabilities of bank j are $L_j = \sum_{i=1}^{N} X_{ij}$. Then, the MD problem is given by 28.

$$\min_{\mathbb{X}} c \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbf{1}[X_{ij} > 0] \qquad \text{s.t.} \tag{28}$$

$$\sum_{j=1}^{N} X_{ij} = A_i \quad \forall i = 1, 2, \ldots, N$$

$$\sum_{i=1}^{N} X_{ij} = L_j \quad \forall j = 1, 2, \ldots, N$$

$$X_{ij} \geq 0 \quad \forall i, j$$

Where the integer function $\mathbf{1}$ equals one only if bank i lends to bank j, and zero otherwise. This problem is equivalent to finding the network with lowest average degree, i.e., the lowest number of edges, under given constraints.

However, because 28 is computationally expensive to solve, the authors proposed a smooth value function, $V(\mathbb{X})$, which is high whenever the network $\mathbb{X}$ has a few links and satisfies the asset and liability constraints. First, the authors soften the constraints by assigning penalties for deviations from the marginal of each bank, which is given by 29.

$$AD_i \equiv \left(A_i - \sum_{j=1}^{N} X_{ij} \right) \qquad \text{and} \qquad LD_j \equiv \left(L_j - \sum_{i=1}^{N} X_{ij} \right) \tag{29}$$

In which LD_j measures bank j's current deficit, i.e., how much its bilateral borrowing falls short of the total amount it needs to raise, L_j, and AD_i is measures bank i's current surplus. When they are introduced into the objective function 28, the problem becomes a maximization given by 30.

$$V(\mathbb{X}) = -c \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbf{1}[X_{ij} > 0] - \sum_{i=1}^{N} (\alpha_i^2 AD_i) - \sum_{j=1}^{N} (\delta_j^2 LD_j) \tag{30}$$

Where α_i are the weights for assets deviations and δ_j are the weights for liabilities deviations. Note that sparse networks $\mathbb{X}$ that minimize marginal deviations are more efficient and achieve higher values in the objective function $V(\mathbb{X})$.

In addition to being sparse, interbank networks are typically disassortative, i.e. small banks seek to match their lending and borrowing needs through relationships with larger banks that are well placed to satisfy those needs. Such behavior contributes to sparsity, since most small banks can satisfy their needs with a single large counterparty. The authors capture this information through the set of probabilities $Q \equiv \{Q_{ij}\}$ for the relative relationships between i and j. The probability that i lends to j increases if either i is a large lender to a small borrower j or i is a small lender to a larger borrower j. This process is given by 31.

$$Q_{ij} \propto \max\left\{\frac{AD_i}{LD_j}, \frac{LD_j}{AD_i}\right\} \tag{31}$$

To ensure that the most likely network solutions are disassortative, the authors propose a probability distribution, $P(\mathbb{X})$, that should be close to the prior Q. This mechanism is given by 32.

$$\max_{P} \sum_{\mathbb{X}} P(\mathbb{X})V(\mathbb{X}) + \theta R(P \parallel Q) \tag{32}$$

Where the scaling parameter θ emphasizes the weight placed on finding solutions with characteristics similar to the prior matrix Q and $R(P \parallel Q) = \sum_{\mathbb{X}} P(\mathbb{X}) \log(P(\mathbb{X})/Q(\mathbb{X}))$ is the relative entropy between P and Q. The solution to this problem can be obtained from the first-order conditions given by 33, stating that a candidate $\mathbb{X}$ has a higher likelihood of being chosen than the prior Q specifies if the departure from Q raises the value of the maximization problem given by 30.

$$P(\mathbb{X}) \propto Q(\mathbb{X})e^{\theta V(\mathbb{X})} \tag{33}$$

Note that while the prior Q codifies the probabilities for picking links, there are no restrictions to the values one should allocate to selected links. However, because each bank has a maximum exposure limit of 25% of its Tier 1 capital that it can have with another bank in the Brazilian financial market since the publication of Resolution 4.667 in July 2018 (BCB, 2018a), we adapted the MD method proposed by Anand et al. (2015) by including one more step in the iteration process. After the end of each iteration of the MD procedure, we checked whether the solution exceeded the maximum exposure limit of each bank's assets in relation to its Tier 1 capital. Only in cases where the solution have exceeded is that we limit the asset's exposure of each bank to 25%, and then we update the simulated network until the total interbank market volume has been allocated. The optimal solution is the one that satisfies all these restrictions and produces topological features that match the moments of the Brazilian interbank network.

In terms of the topological features, the three most important are: (i) density, (ii) average degree, and (iii) assortativity[20]. Density is the number of undirected links as a percentage of the total number of links (excluding self-loops), which can also be seen as the sparsity of a network. The degree, or valency, is a strictly local measure that corresponds to the number of counterparties each bank connects in the financial network. Thus, degree can be interpreted as a proxy of bank's portfolio diversification inside the financial network. Since this is a bank-level network measurement, it is common to report the average value of all participating banks (Souza et al., 2016; Anand et al., 2018).

Assortativity is a global measure of the network that presents the correlation between the number of counterparties of pairs of banks that have operations with each other. If this measure is positive, it means that, in the assessed network, banks with many counterparties usually carry out operations with banks that have many counterparties. On the other hand, a negative measure, denoted disassortativity, reveals the predominance of financial operations between pairs of banks with different total numbers of financial operations. Usually, large banks, which have many operations and act as money centers, interconnect with small banks, which have few operations and act as investors or borrowers in the interbank. Furthermore, financial networks with negative assortativity show the existence of core-periphery structures. In Brazil, the nucleus is mainly composed of the group of large banks, whose members are highly connected to other members of the core and also intermediate operations between members of the periphery (Souza et al., 2016; Silva et al., 2016; Anand et al., 2018).

Once the interbank network is reconstructed considering all these properties, we proceed to the estimation of the impact of shock scenarios considering possible additional contagion effects and the measurement of the systemic importance of a bank in the network. Thus, we will be able to infer how the dynamics of a bank's default impact the whole system. Among the different approaches proposed by the literature for this purpose, we choose the well-known DebtRank algorithm (Battiston et al., 2012; Bardoscia et al., 2015) for its

[20]For more information on topological information of a interbank network, see Silva et al. (2016).

effective shock propagation dynamics (Souza et al., 2016; Silva et al., 2017; Poledna et al., 2021).

The DebtRank algorithm first proposed by Battiston et al. (2012) is inspired by the feedback centrality measure[21] and assumes that losses are linearly propagated between connected banks. Suppose a network of mutually exposed banks in which each of these banks has assets and liabilities, among which a fraction is related to the counterparties within the network, and a capital buffer. If a bank suffer assets losses greater than its capital buffer, it becomes insolvent and will not be able to honor any of its short-term liabilities, a scenario in which the bank defaults. On the other hand, if the losses are lower than its capital buffer, suppose 90%, the bank will be in distress and will not pay its creditors a proportional part (10%) of its liabilities, which characterizes a stress measure. In the first case, the creditors of the default bank, in turn, will suffer losses and undergo through the same dynamics. This feedback process continues until the whole system converges.

Formally, the original DebtRank method models the interbank market as a direct network $\mathcal{G} = \langle \mathcal{B}, \mathcal{E} \rangle$, in which the banks compose the vertex $\mathcal{B}$ and the exposures between them compose the set of edges $\mathcal{E}$. Again, these links are represented by a weighted adjacency matrix $\mathbb{X}$, where the (i, j)th entry, X_{ij}, represents the amount bank i lends to bank j, i.e., the exposure of bank i to bank j. In a similar notation, the total value of the asset invested by i in funding activities is $A_i = \sum_{j \in \mathcal{B}} X_{ij}$ and the relative economic value of bank i is given by $\varphi_i = A_i / \sum_{j \in \mathcal{B}} A_j$, $\varphi_i \in [0, 1]$, which is the fraction of i's assets with respect to the total assets in the interbank market.

Also, each bank i has a capital buffer against shocks, E_i, which is represented by the Common Equity Tier 1 (CET1)[22]. If $E_i \leq \gamma$, where $\gamma > 0$, the bank defaults. If vertex j

[21]Feedback centrality measures are those in which the centrality of a node, or vertex, depends recursively on the centrality of its neighbors. The recursiveness criterion effectively forces the centrality of each node to depend on the entire network structure through feedforward/feedback mechanism. In this sense, the original DebtRank of Battiston et al. (2012) is not formally a feedback centrality measure because it does not propagate a second- and high-order round of impacts that come from cycles or multiple routes in the network. These components are incremented in the DebtRank version of Bardoscia et al. (2015).

[22]Although Battiston et al. (2012) uses the Tier 1 (the sum of the Common Equity Tier 1 and the Additional Tier 1) as the capital buffer against shocks, we used only the CET1 as it is the component of highest quality capital and can absorb losses immediately as they occur (BCBS, 2011). CET1 comprises the core capital of a bank and consists mostly of issued equity and retained earnings, being used by investors to assess a bank's solvency. Even considering the capacity to absorb losses of some AT1 instruments, which includes noncumulative, nonredeemable preferred stock and related surplus and qualifying minority interest, they have a lower quality compared to the CET1 instruments, especially in periods of financial distress (Ramirez, 2017; Couaillier, 2021).

defaults, all of the neighbors i will suffer losses corresponding to their exposure towards j, given by X_{ij}. When $X_{ij} > E_i$, then vertex i defaults. The local impact of j on i is $W_{ij} = \min(1, V_{ij})$, where $V_{ij} = X_{ij}/E_i$ is the bank's stress level, so that if i's losses exceed its capital, the local impact is 1. Intermediate values within the interval $(0, 1)$ for W_{ij} lead i into distress, but not into default.

The presence of cycles in the network inflates the computed impacts by counting the local impact of a node on another more than once. To avoid the distortion caused by this double-counting, the original DebtRank algorithm evaluates the additional stress caused by some initial shock using a dynamic system, allowing only a single impact propagation per each node. It maintains two state variables for each bank $i \in \mathcal{B}$: (i) $h_i(t) \in [0, 1]$ and (ii) $s_i(t) \in \{U, D, I\}$. $h_i(t)$ is the stress level of i and $s_i(t)$ is a categorical variable that denotes the state of i. U, D, and I stand for undistressed, distressed, and inactive, respectively. The update rules of the dynamic system are given by 34 and 35.

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j \in \mathcal{D}(t)} W_{ij} h_j(t-1)\right\} \tag{34}$$

$$s_i(t) = \begin{cases} D, & \text{if } h_i(t) > 0 \text{ and } s_i(t-1) \neq I, \\ I, & \text{if } s_i(t-1) = D, \\ s_t(t-1), & \text{otherwise} \end{cases} \tag{35}$$

In which $t \geq 0$ and $\mathcal{D}(t) = \{j \in \mathcal{B} \mid s_j(t-1) = D\}$. Note that the sum of 34 occurs only for those distressed banks in the previous iteration. However, once distressed, they become inactive in the next iteration due to 35. Thus, they are not able to propagate further stress. Observe that the algorithm must converge for a sufficiently large number of steps $T \gg 1$ due to the min(.) operator, which places upper bounds on the bank's stress levels, and the non-decreasing property of $h_i(t)$, derived from the non-negative entries of the vulnerability matrix V_{ij}. We compute the resulting DebtRank due to the initial shock scenario $h(0)$ using equation 36.

$$DR(h(0)) = \sum_{i \in \mathcal{B}} \Big[h_i(T) - h_i(0)\Big] \varphi_i \tag{36}$$

Note that, by removing the initial stress $h(0)$ from the DebtRank computation in 36, it conveys the notion of additional stress given an initial shock scenario. However, the great drawback of this formulation is that it prevents banks from diffusing second- and high-order rounds of stress. This means that, once a vertex propagates stress, it will not be able to propagate additional stress due to other subsequent impacts that it receives, which can lead to a severe underestimation of the stress levels of banks.

In order to overcome the limitations of the original DebtRank algorithm, Bardoscia et al. (2015) proposes an improvement that still accounts for cycles or multiple routes in the vulnerability network and therefore prevents stress double-counting by using stress differentials between one iteration and another. As a result, at each iteration, banks are only allowed to propagate the stress increase that they received from the previous iteration. Using this mechanism, financial stress is never double-counted because differentials are always innovations from one iteration to another. Again, once a bank default at time t, it no longer propagates financial stress during the dynamic process for $t + k$, $k > 0$. Therefore, substituting the stress levels in 34 by stress differentials results in equation 37.

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j \in \mathcal{B}} W_{ij} \Delta h_j(t-1)\right\} \tag{37}$$

Where $\Delta h_j(t-1) = h_j(t-1) - h_j(t-2)$ is the stress differential of the bank j in the previous iteration $t - 1$ and $h(t) = 0 \; \forall \; t < 0$. In the beginning of the iteration process, $h(0)$ is an ex-ante input that denotes the initial stress scenario, or the list of shocks based on the bank's probability of default. Thus, we then compute the DebtRank value of an initial shock scenario in 36 using the convergent stress values of 37.

There are two important differences between the improved DebtRank of Bardoscia et al. (2015) compared to the original formulation of Battiston et al. (2012). First, the sum index in 37 runs through all the banks, such that there is no need to maintain states in the dynamic system. Second, instead of only one propagation immediately after the shock has been received, they could propagate shocks until all connected banks in the network default, which makes a formally feedback centrality measure. The dynamics now reaches global equilibrium only when the direct and indirect neighborhoods of each bank are considered, taking into account multiple routes and network cycles when establishing the final stress levels of banks.

2.2.2.3 Funding Illiquidity

It is well known in the literature, especially after the global financial crisis, as discussed in this work, that banks suffer from liquidity and funding risk and the importance of these channels for the process of contagion and systemic risk. Bank asset and liability structures proved to be highly vulnerable to deposit runs, market shocks and breakdowns in funding markets (Acemoglu et al., 2015; Paltalidis et al., 2015; Venkat and Baird, 2016; Ferrara et al., 2019; Wen et al., 2023).

However, it is important to note the difference between short-term and long-term liquidity risk and to distinguish their respective roles in the financial contagion process. Short-term liquidity risk (less than one year) can arise from various sources, such as panic behavior and asset fire sales, as mentioned in Section 2.2.2.1, and the direct exposures in the interbank network, as mentioned in Section 2.2.2.2. On the other hand, long-term liquidity risk (greather than one year) is related to sufficiently stable sources of funding or the inability of banks to raise funds when needed, such that longer-term liabilities are assumed to be more stable than short-term liabilities to mitigate the risk of future funding stress (BCBS, 2014; Venkat and Baird, 2016; Ardekani et al., 2020; Wieser, 2022).

Some mechanisms have been created to prevent and mitigate all the risks addressed so far. The design of a deposit insurance scheme (DIS), for instance, constitutes an integral part of the financial safety net provided to the banking system and is intended to prevent runs on individual banks by depositor. If the DIS is credible and depositors expect that they will receive their money back from the insurance fund, regardless of what other depositors do or whether they are last in line for reimbursement, then they no longer have incentives to run and withdraw their funds. In the event of bank failure, it also limits losses to depositors and reduces the risk that a run on one bank might undermine confidence in others through contagion effects. Thus, the existence of a credible DIS contributes to the reduction of the funding cost, especially long-term, of banks (Diamond and Dybvig, 1983; Allen et al., 2011; Anginer and Demirgüç-Kunt, 2019; Freixas and Parigi, 2019).

The most fundamental deposit insurance scheme is the paybox mandate in which the deposit insurer is only responsible for the reimbursement of insured deposits. Most countries with an established DIS have improved its legal and operational characteristics over time, usually by expanding the mandate and powers and strengthening the role of the DIS within the financial safety net. (Ognjenovic, 2017; Kerlin, 2017). In Brazil, for instance, the DIS is a privately held company and exercises a paybox plus mandate, in which the additional resolution function (e.g., financial support) is also attributed to the deposit insurer (BCB, 2018b).

Regarding the Brazilian case, the situation that a private deposit insurer has limited resources raises the question about the DI's ability to withstand a strong systemic risk event. In a scenario where all available fund resources are consumed, even with the framework of lender of last resort played by the central bank in order to provide a potential source of liquidity for banks, the uncertainty about the credibility of the entire system in this case makes it difficult for financial institutions with poor long-term liquidity to raise short-term funding in the market, a combination that further increases their probability of default (Allen et al., 2011; Vazquez and Federico, 2015; Diamond and Kashyap, 2016; Ognjenovic, 2017; Kerlin, 2017; Ebrahimi Kahou and Lehar, 2017; Bouwman, 2019).

The Basel Committee on Banking Supervision also created other mechanisms in Basel III to reduce short-term liquidity risk and long-term financing risk. The two proposed quantitative liquidity standards are the Liquidity Cover Ratio (LCR) and the Net Stable Funding Ratio (NSFR). LCR reflects short-term liquidity soundness and requires banks to hold sufficient high-quality liquid assets (HQLA) to offset the net cash outflows in a liquidity stress scenario over 30 days. On the other hand, NSFR requires a minimum amount of available stable funding (ASF) relative to the required stable funding (RSF) over a one-year horizon. Both liquidity ratios have a minimum regulatory of 100%. Note that the implementation of LCR encourages a substitution from long-term illiquid assets to short-term liquid assets, which consequently eases bank runs. Furthermore, under NSFR the bank needs to finance illiquid assets with long-term funding, which can alter the bank's incentive to use less runnable deposits (BCBS, 2013, 2014; Diamond and Kashyap, 2016; Ebrahimi Kahou and Lehar, 2017).

As mentioned previously, considering that the different aspects of short-term liquidity risk are covered in Section 2.2.2.1 and Section 2.2.2.2, we address the long-term liquidity risk through the NSFR concept in all scenarios where the available resources from DI are consumed[23]. The NSFR equation is given by 38.

[23]In those scenarios where banks suffer from the contagion channels of panic behavior and interbank network, but the DI still has enough resources to maintain the credibility and confidence in the financial system, then it can be argued that the remaining banks will not suffer from a long-term liquidity shock.

$$NSFR = \frac{ASF}{RSF} \tag{38}$$

Where available stable funding (ASF) is defined as the portion of capital and liabilities expected to be reliable over the time horizon and the required stable funding (RSF) is a function of the liquidity characteristics and residual maturities of the various assets held by the bank, as well as those of its off-balance sheet exposures (Chiaramonte et al., 2013; BCBS, 2014).

Therefore, in order to comply with a minimum regulatory of 100% for NSFR, banks can either increase their ASF or reduce their RSF. A natural option to increase ASF is to increase the proportion of long-term funding in the whole portfolio or to increase the Common Equity Tier 1 (CET1), which is the sum of Common Equity Tier 1 and Additional Tier 1. On the other hand, a natural option to reduce RSF is to shrink its balance sheet by changing the composition of its investments and loans or to change its assets to such combination that would result in a lower weight factor.

Considering that the NSFR is subject to national discretion to reflect jurisdiction-specific conditions, the Central Bank of Brazil defined the ILE (structural liquidity ratio) as its equivalent concept that has been in effect since October 2018 through the Resolution CMN 4.616 for banks in the S1 prudential segmentation (BCB, 2015, 2022c). However, because the construction of the ILE requires private and confidential data from each bank, we follow Takeuti (2020) to create a proxy for our NSFR[24]. Tables 1 and 2 detail each balance sheet data from BCB (2022a) used to calculate the proxy's for ASF and RSF.

[24]The two works that proposes a proxy for ILE in Brazil, for the best of our knowledge, are Cardoso et al. (2019) and Takeuti (2020), but we evaluate that the work of Takeuti (2020) is more accurate in terms of the concept of NSFR.

Table 1: Balance sheets accounts used for calculating the ASF proxy.

Factor	Composition	Description
1.00	[60000002]	Equity
1.00	[70000009]	Gross Revenues
1.00	[80000006]	Gross Expenses
0.90	[41100000]	Demand Deposits
0.90	[41200003]	Saving Deposits
0.90	[41500002]	Time Deposits
0.60	[43000005]	Mortgage, real estate and others
0.60	[46000002]	Onlending
0.50	[42000006]	Repurchase Agreements

Table 2: Balance sheets accounts used for calculating the RSF proxy.

Factor	Composition	Description
1.00	[19000008]	Other Assets
1.00	[14000003]	Interbank Transactions
1.00	[15000002]	Interbranches Transactions
1.00	[20000004]	Fixed Assets
1.00	[23000001]	(-) Leased Assests
1.00	[18000009]	Other Receivables
0.85	[16000001]	Loans
0.65	[23000001]	Leased Fixed Assets
0.65	[17000000]	Leases
0.40	[13000004]	Securities and Derivatives

Once the ASF and RSF metrics have been calculated to determine the NSFR proxy for each bank, we proceed to the simulation of the funding illiquidity risk. The dynamics of this third and final channel of contagion in the model is such that, in any scenario where the available resources from DI are consumed through the reimbursement of covered deposits from banks that default for idiosyncratic reasons or through the first two channels described in Section 2.2.2.1 and Section 2.2.2.2, the remaining banks with NSFR less than one experience an additional 36% shock to their probability of default, as described in Section 2.2.2.

2.2.3 Loss Distribution

As previous mentioned in Section 2.2, our approach follows a similar theoretical framework in which portfolio risk is calculated in banking organizations and how banking losses are estimated in deposit insurance schemes (Lehar, 2005; Gupton et al., 2007; De Lisa et al., 2011; Bellini, 2017; O'Keefe and Ufier, 2017; Parrado-Martínez et al., 2019; Matt and Andrade, 2019; Fernández-Aguado et al., 2022). Thus, the last part of our model consists in estimating the loss distribution (LD) of the banking system taking into account the probability of default (PD), the loss given default (LGD), and the exposure at default (EAD). To this end, we first define the expected loss (EL) of a given bank i through equation 39.

$$EL_{it} = PD_{it}^{c} \times LGD_{it} \times EAD_{it} \tag{39}$$

In which $PD_{it}^{c} \in [0, 1]$ is the final probability of default after the contagion process over the idiosyncratic PD_{it} defined in Section 2.2.1 through equation 23, $EAD_{it} \in [0, \infty)$ is given by SRISK in Section 2.1.2, and $LGD_{it} \in [0, 1]$ is assumed to be one for all banks in the Brazilian banking system[25].

The construction of the economy's LD_t is the result of a Monte Carlo simulation over the EL_t^s for each scenario s through the bank run model, where $s = 1, \ldots, \mathcal{S}$ and $t = 1, \ldots, T$. The value of $\mathcal{S}$ must be large enough to achieve convergence, which in our case is equal to 100.000. Let N be the number of banks, $\mathbf{\Phi}^{-1}(PD^c) := [\Phi^{-1}(PD_{1t}^c), \ldots, \Phi^{-1}(PD_{Nt}^c)]$ the vector of the probit function, or the inverse of the standard normal cumulative distribution (quantile) function, of each bank's PD, and $\mathcal{Z}^s := (z_1^s, \ldots, z_N^s) \sim \mathcal{N}(0, 1)$ a vector of random variables for $\mathcal{S}$ different scenarios. Under the assumption of normality over bank asset values (Crouhy et al., 2000; Lehar, 2005; De Lisa et al., 2011; Guerra et al., 2016; Souza et al., 2016; Bellini, 2017; O'Keefe and Ufier, 2017; da Rosa München, 2022), the indicator function that generates the vector of bank's default $\mathcal{P}_t^s$ used to estimate the EL_t^s

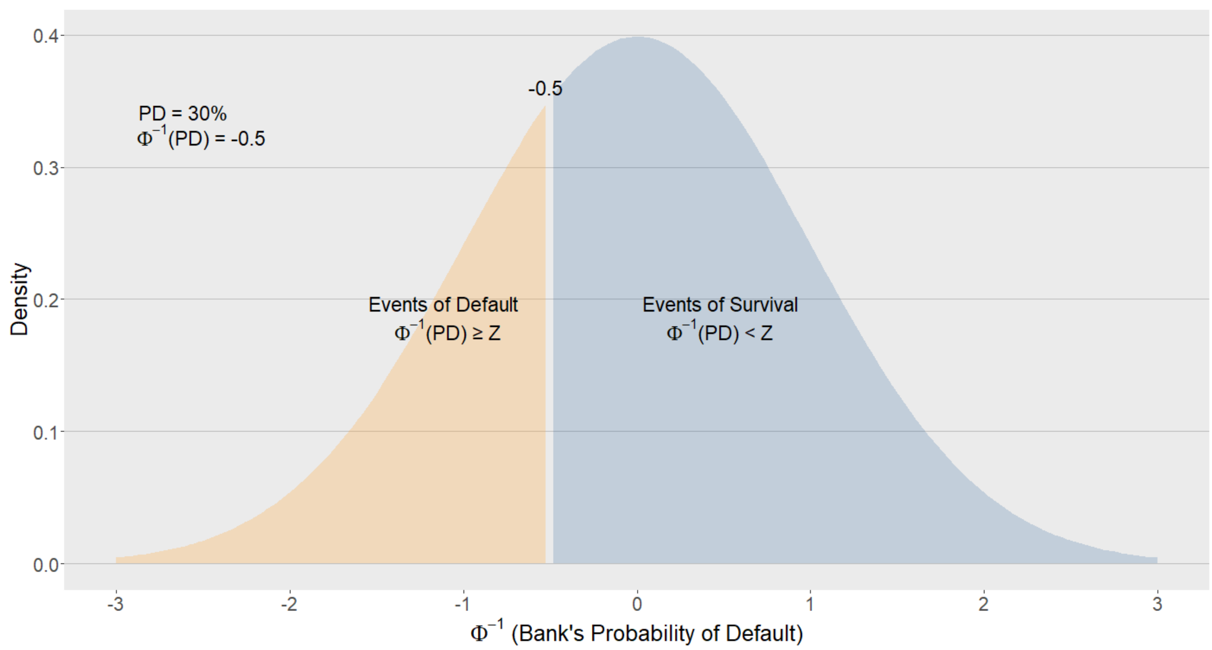
[25]The concept of loss given default can also be understood as the proportion of non-recovery assets, i.e., $LGD_{it} = 1 - RR_{it}$, in which RR_{it} is the recovery rate. Note that in this framework the LGD is absorbed by the central bank or the deposit insurer of the economy. The assumption of absence of recovery on a one-year horizon when a bank default in Brazil can be argued considering the following aspects: (i) the high historical interest rate in Brazil diminishes the present value of futures recoveries, (ii) during the process of extrajudicial settlements there are mostly poor quality and illiquid assets left, which increases the time in line for recoveries, and (iii) in periods of financial distress it is even more difficult to liquidate assets without incurring great losses due to fire sales and market conditions. Therefore, considering that we are modeling extreme distress events, the combination of all these factors further reduces the present value of recoveries on a one-year horizon, which underlies the proxy of one for the LGD of the Brazilian banking system. However, it is important to mention that if one would like to consider asset recoveries in the event of bank default, we can use the Merton (1974)'s model framework to calculate $LGD_{it} = 1 - (1 - \varphi)\left[\frac{A_{it}}{DB_{it}} \frac{\mathcal{N}(-d_1)}{\mathcal{N}(-d_2)}\right] \exp(r_t T)$, in which φ represents administrative costs (Guerra et al., 2016).

for each scenario s is given by 40.

$$\mathcal{P}_t^s := \begin{cases} 1, & \text{if } z_i^s \leq \Phi^{-1}(PD_{it}^c) \quad \forall i = 1, \ldots, N, \\ 0, & \text{otherwise} \end{cases} \tag{40}$$

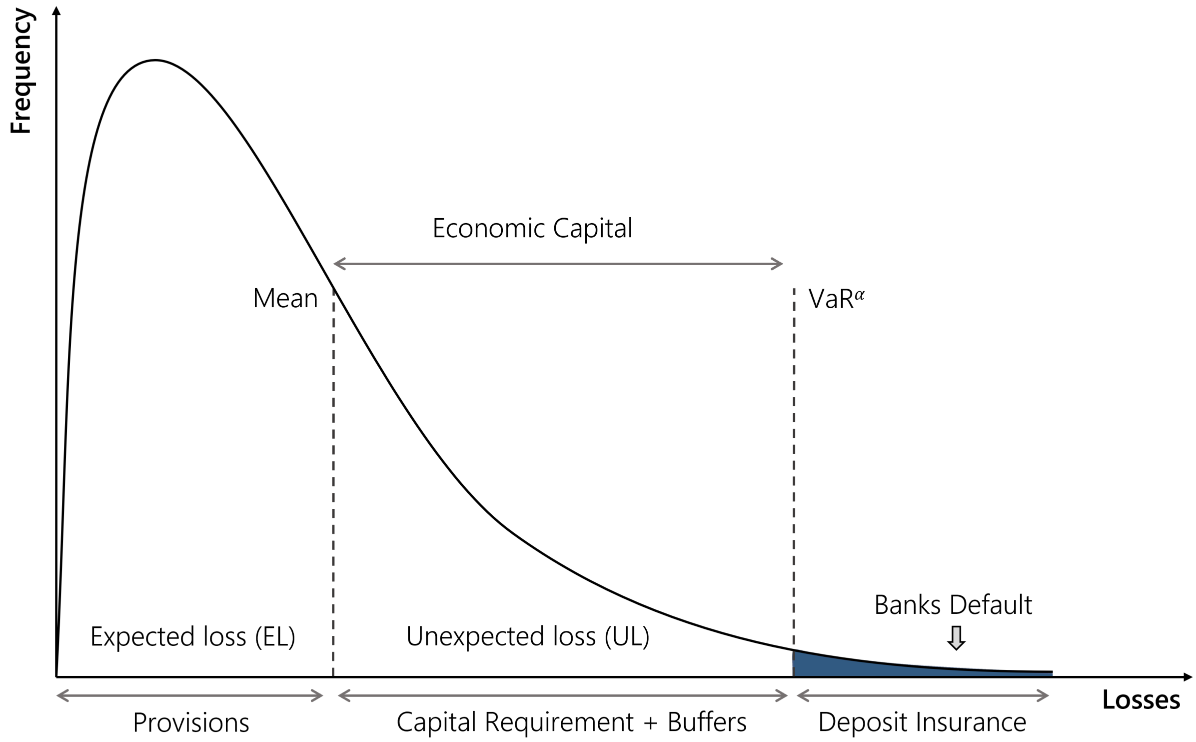
In this framework, the vector $\mathcal{Z}^s$ can be understood as the vector of initial shocks that gives origins to the idiosyncratic default PD_{it}, which will have a different combination of initial defaults for each s. After this initial shock, all three contagion processes described in Section 2.2 have their own dynamic according to the established criteria. Therefore, the final vector EL_t^s of size $1 \times \mathcal{S}$ has different values for each s because the shock vector $\mathcal{Z}^s$ is also different for each s. For example, consider that bank i has $PD_{it} = 30\%$ and its associated probit function $\Phi^{-1}(PD_{it}) = -0.5$. If $z_i^1 = -0.6$ and $z_i^2 = -0.4$, both randomly drawn from a normal distribution through Monte Carlo simulations, then bank i in time t will idiosyncratically default in scenario 1 and will not idiosyncratically default in scenario 2. However, if bank i receives one shock of 36% during the contagion process, which gives $PD_{it}^c = 41\%$ and $\Phi^{-1}(PD_{it}^c) = -0.2$, then it will default in both scenarios at the end of the simulation. Figure 2 gives a synthetic illustration of this default dynamics.

Figure 2: Events of default from a normal distribution.



Once the EL_t^s is constructed for each s, the LD_t is obtained by ordering its values. The expected loss of LD_t corresponds to the average EL_t, and $VaR_{LD_t}^{\alpha}$ is calculated as the quantile α of the distribution. In contrast, the unexpected loss is derived as the difference between the $VaR^{\alpha}_{LD_t}$ and the expected loss. Because we are dealing with extreme events, the vector LD_t usually has a positive skew distribution, or a right-skewed distribution, and leptokurtosis, i.e., fat tails with excess of kurtosis (Gupton et al., 2007; Bellini, 2017). The representation of a hypothetical loss distribution of the banking system is shown in Figure 3.

Figure 3: Banking system loss distribution.



3 Data

We used quarterly data from December 2000 to September 2022 for all 26 publicly traded Brazilian banks, resulting in a unbalanced panel data with 1.586 observations[26]. All balance sheet data used in this paper are publicly provided by the Central Bank of Brazil (BCB, 2022a), while the daily stock price and market capitalization data were

[26]Although we have available information from 2000:I-2000:III in the database, we used these first three quarters to calculate assets volatility considering that bank capital information are available from December 2000. Brazilian banks (with national headquarters) that are traded on foreign markets were also considered. Also, it is important to mention that, although we have a data set for 244 Brazilian financial institutions, which would result in 9.653 observations, we work only with a restricted subset that has market data available.

obtained from Bloomberg for the same period. The data set used in this study considers the financial conglomerates and independent institutions until December 2014 and the prudential conglomerates and independent institutions before March 2015[27] with the business model category of $b1$, $b2$, $b4$ and $n1$[28], considering a minimum of six valid observations in the studied period. The final data set represents 79.89% of total assets, 80.46% of total credit, 89.71% of total deposits and 13.19% of total member institutions in September 2022. For the interest rate, we used public data provided by B3, the Brazilian financial market infrastructure company (B3, 2022).

In order to estimate the probability of default on a one-year horizon for each FI using the Merton (1974)'s structural model, we used the adjusted total assets[29] for A, total liabilities to calculate DB, annualized interbank interest rate DI for r and the annualized standard deviation of the logarithmic returns of adjusted total assets, that is, $\log(A_t/A_{t-1})$, for asset volatility σ_A. Table 3 presents the aggregate descriptive statistics of these variables.

[27] Note that until December 2013 the Central Bank of Brazil only registered the institution type of financial conglomerates and independent institution. Before March 2014, the prudential conglomerate and independent institution perspective was included, but the capital information from bank's DLO (Statement of Operating Limits) was only published in the prudential conglomerate and independent institution perspective before March 2015. The difference between the two filter is that the latter include, besides the institutions belonging to the financial conglomerate: (i) consortium administrators, (ii) payment institutions, (iii) companies that perform acquisition of credit operations, including real estate or credit rights, (iv) other legal entities domiciled in the country that have as an exclusive objective an equity interest in the aforementioned entities and (v) investment funds in which the entities that compose a prudential conglomerate take or retain substantial risks and benefits (BCB, 2022a).

[28] We only used these four business model category to account for institutions that issue covered deposits by the deposit insurance of Brazil, where ($b1$) includes commercial bank, universal bank with commercial portfolio or savings bank; ($b2$), universal bank without commercial portfolio or investment bank or foreign exchange banks and ($n1$), non banking credit company. The member institutions are: (i) multiple banks; (ii) commercial banks; (iii) investment banks; (iv) development banks; (v) Caixa Econômica Federal (Brazilian federal savings bank); (vi) savings banks; (vii) finance and investment companies; (viii) building societies; (ix) mortgage companies savings and (x) loan associations (FGC, 2022; BCB, 2022a). For more information on FGC covered deposits and the guaranteed financial instruments, see BCB (2021b).

[29] The adjusted total assets comprise total assets after netting and reclassification of the balance sheet items. Netting is performed on the following balance sheet items: repurchase agreements, interbank relations and relations within branches, foreign exchange portfolio and debtors due to litigation. Reclassifications are performed within foreign exchange and leasing portfolios.

Table 3: Descriptive statistics.

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
ATA[a]	211.72	418.76	0.04	5.71	137.19	2,184.86
TL[a]	194.04	387.44	0.00	5.05	123.11	2,018.16
Loans[a]	89.23	185.98	0.00	2.44	39.73	978.56
DI[b]	11.19	5.11	1.90	6.93	14.13	26.23
AV	0.47	0.93	0.00	0.11	0.42	14.86
PD[b]	16.22	23.34	0.00	0.16	25.94	100.00
SRISK[a]	12.98	36.53	0.00	0.00	2.40	187.02
SES[a]	9.69	32.37	0.00	0.00	0.64	161.89
Beta	0.87	0.68	−0.60	0.45	1.10	5.79
MES	0.05	0.17	0.00	0.02	0.04	1.59
LRMES	0.30	0.15	−0.30	0.18	0.39	0.92
CoVaR	0.04	0.04	0.01	0.03	0.04	0.28
ΔCoVaR	0.01	0.03	0.00	0.01	0.02	0.24
II[a]	34.27	84.11	0.00	0.40	17.74	634.07
ID[a]	1.54	4.83	0.00	0.02	0.69	41.48
TD[a]	72.68	147.03	0.00	1.71	53.11	854.76
CET1[a]	10.26	26.70	0.00	0.00	2.37	142.78
TRC[a]	20.44	38.70	0.00	0.60	14.29	180.30
RWA[a]	123.00	234.02	0.00	3.62	88.56	1,225.17
DoA[b]	36.82	20.47	0.00	21.86	52.35	93.64
CoA[b]	42.04	21.06	0.00	29.30	54.53	97.87
LR[b]	4.15	8.31	0.00	0.00	7.11	100.00
CAR[b]	20.89	25.04	0.00	14.25	18.73	542.27
NSFR	1.61	19.92	0.15	0.88	1.12	793.98

Notes: The sample period runs from 2000:IV-2021:IV for 25 publicly traded Brazilian banks. *ATA* = adjusted total assets; *TL* = total liabilities; *Loans* = loan operations by risk level; *DI* = interest rate; *AV* = assets volatility; *PD* = probability of default; *SRISK* = systemic risk metric; *SES* = systemic expected shortfall; *Beta* = market beta; *MES* = marginal expected shortfall; *LRMES* = long-run marginal expected shortfall; *CoVaR* = conditional value-at-risk; Δ*CoVaR* = delta conditional value-at-risk; *II* = interbank investments; *ID* = interbank deposits; *TD* = total deposits; *CET1* = common equity tier I; *TRC* = total regulatory capital; *RWA* = risk-weighted assets; *DoA* = total deposits over total assets; *CoA* = total credit over total assets; *LR* = leverage ratio; *CAR* = capital adequacy ratio (tier I and II) and *NSFR* = net stable funding ratio.
[a] In BRL billion.
[b] In percentage.

4 Results and Discussion

This section summarizes and discusses the empirical results obtained in this paper. First, we present some statistics on the banks' risk measures, the contagion process, and the estimation of the loss distribution of the banking system considering a reduced sample (RS) of only the listed banks. Second, we compare the results of the reduced sample with a full sample (FS) with some adaptations. Then, we propose and discuss a new capital adequacy ratio based on this framework considering the RS.

4.1 Banks' Probability of Default and Contagion Process in the Reduced Sample

Table 4 presents summary statistics for the risk measures estimated to build the loss distribution of the Brazilian banking system in September 2022 with a reduced sample of the 24 listed banks. The first risk measure is the probability of default given by the Merton (1974)'s structural model (structural probability of default) used to compute the idiosyncratic PD of each bank. The second measure is the banks' PD that represents the default rates in the Monte Carlo simulation. As expected and reflecting the construction of our model, the mean of the structural PD and the Banks' PD without contagion are practically the same and equal to 19.3%. In addition, note that the contagion process described in Section 2.2.2 increases the average probability of default of banks by 42%.

Table 4: Summary statistics of structural PD and banks' PD with and without contagion in the reduced sample.

		Structural PD of Merton model (%)	Banks' probability of default (%)	
			Panel A: without contagion process	Panel B: with contagion process
Mean		19,29	19,28	27,39
Std. Deviation		27,41	5,91	6,25
Minimum		0,00	4,17	12,50
Maximum		99,93	45,83	54,17
Percentiles:	25%	0,04	16,67	20,83
	50%	7,02	20,83	25,00
	75%	24,30	25,00	33,33

Notes: This table shows statistics of PD estimates for 24 Brazilian banks in September 2022. Structural PD represents the estimates of the Merton (1974)'s model used as an input for the idiosyncratic PD of each bank. Banks' PD are the default rates of the sample banks in the Monte Carlo simulation. In details, Panel A shows summary statistics of Banks' PD with only idiosyncratic default (without contagion process). Panel B presents the same statistics when all the contagion process described in Section 2.2.2 are considered. As expected, note that the mean of the structural PD and the Banks' PD without contagion are practically the same.

The clusters of the 24 Brazilian banks for September 2022 are presented in Table 5. We specify four clusters based on the statistics described in Section 2.2.2.1 and taking into account the relative consistency of their size, operation, regulation, and market similarity. These results were used in the first contagion process (panic) when the idiosyncratic default of a bank within a cluster causes a 36% shock to the other banks' PD within the cluster. Note that banks classified in the S1 prudential segment were clustered in the same group.

Table 5: Cluster segmentation of the Brazilian banking system.

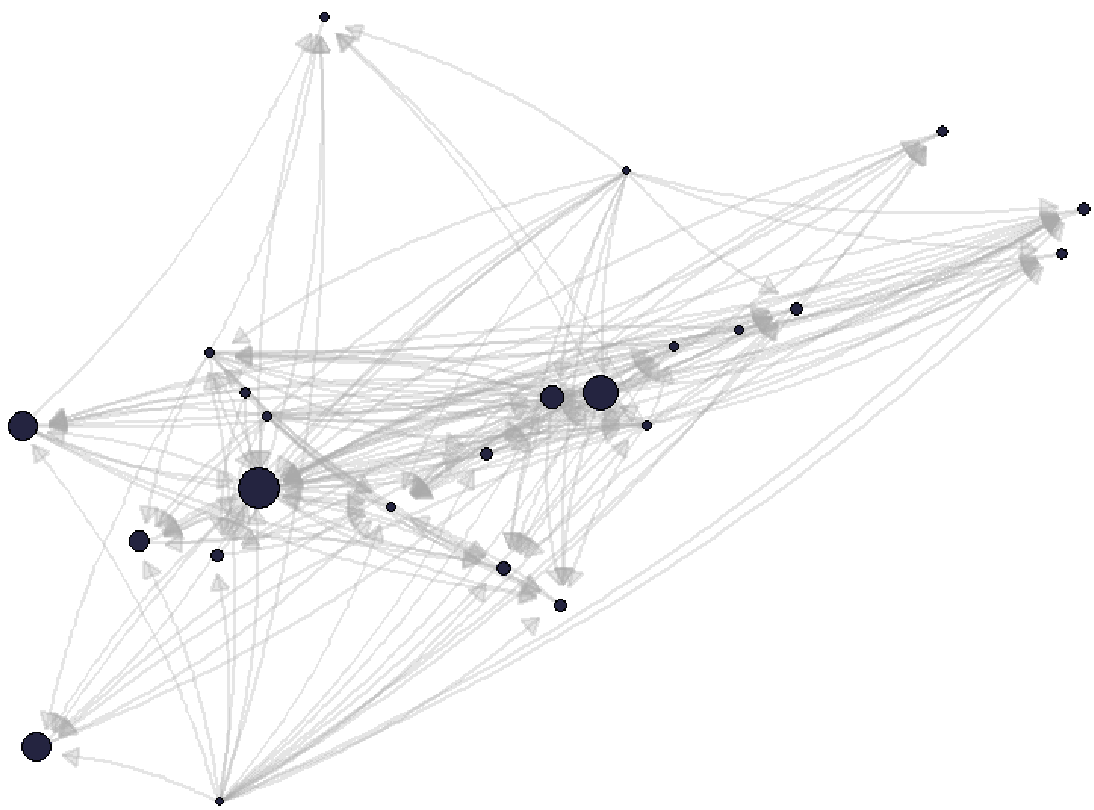
Cluster	Bank	Ticker	DoA[b]	CoA[b]	RWA[a]	SRISK[a]	ΔCoVaR
1	Itaú	ITUB4 BZ	38,7	40,3	1225,17	94,44	0,0148
	Bradesco	BBDC4 BZ	35,9	39,2	988,41	67,18	0,0144
	Santander	SANB11 BZ	39,7	47,0	637,46	48,44	0,0114
	BB	BBAS3 BZ	34,7	40,5	1039,39	185,10	0,0136
	CEF	CXSE3 BZ	34,9	62,7	704,62	168,25	0,0039
2	BTG	BPAC11 BZ	26,6	26,3	300,75	0,00	0,0117
	XP	XP US	17,0	15,5	37,38	0,00	0,0077
	B3	B3SA3 BZ	3,8	0,0	0,17	0,00	0,0174
	BR Partners	BRBI11 BZ	17,4	3,5	2,68	0,00	0,0032
	Nordeste	BNBR3 BZ	16,2	22,1	78,78	0,46	0,0026
3	Porto Seguro	PSSA3 BZ	1,5	96,8	14,76	0,00	0,0093
	Alfa	BRIV4 BZ	24,4	58,2	19,37	1,67	0,0025
	ABC	ABCB4 BZ	19,1	49,5	41,35	1,44	0,0114
	Amazônia	BAZA3 BZ	14,4	52,2	36,72	2,03	0,0044
4	Nubank	NU US	86,7	33,7	24,22	0,00	0,0131
	Inter	BIDI11 BZ	48,8	48,0	24,04	1,18	0,0027
	BMG	BMGB4 BZ	53,1	52,4	22,99	3,06	0,0076
	Modal	MODL11 BZ	45,7	19,6	5,51	0,00	0,0023
	Pine	PINE4 BZ	47,9	30,8	7,23	1,26	0,0055
	Banrisul	BRSR6 BZ	57,1	41,9	51,56	7,64	0,0099
	Banestes	BEES4 BZ	54,3	19,3	12,78	2,45	0,0027
	Mercantil	BMEB4 BZ	71,7	71,3	8,55	0,25	0,0004
	Mercado Crédito	MELI34 BZ	84,4	22,5	0,89	0,00	0,0018
	Est. Sergipe	BGIP4 BZ	77,4	46,6	4,97	0,56	0,0029

Notes: This table shows the four estimated clusters for 24 Brazilian banks in September 2022. *DoA* = total deposits over total assets; *CoA* = total credit over total assets; *RWA* = risk-weighted assets; *SRISK* = systemic risk metric; $\Delta CoVaR$ = delta conditional value-at-risk.
[a] In BRL billion.
[b] In percentage.

The estimated interbank network for the Brazilian economy in September 2022 is shown in Figure 4. The main statistics of our estimated network, such as density, assortativity, r, and average degree, are 0.31, -0.54 and 14.4, respectively. In terms of comparison, the density, assortativity and average degree reported by BCB and other works using the true

Brazilian interbank network vary between [0.03,0.07], [-0.31,-0.54] and [4.6,7.8] (or [21,26] for large banks), respectively, during March 2010 and September 2015[30] (Castro Miranda et al., 2014; BCB, 2016; Souza et al., 2016; Silva et al., 2016; Anand et al., 2018; Alexandre et al., 2022). Note that even considering 13.19% of FIs, the adapted minimum density method was still able to simulated the expected properties of the Brazilian interbank network.

Figure 4: Estimated interbank network for the Brazilian financial system.



Notes: The size of each financial institution represented in the interbank network reflects the sum of its interbank investments, interbank deposits and CET1 (Common Equity Tier I) in September 2022. This is the same weight used by the DebtRank algorithm when calculating the process of bank failure due to contagion.

Because $r < 0$, the interbank market network is disassortative, indicating that the Brazilian financial system has highly connected FI's that are frequently connected to others with very few connections. This result follows the conclusion of Silva et al. (2016)

[30]The last release of the assortativity and degree metric of the Brazilian interbank network made by the BCB was in April 2016 with data until September 2015. All other data comes from works that use the true Brazilian interbank network. For more information, see BCB (2016).

and Alexandre et al. (2022), indicating the existence of money centers in which a few large banks have several connections with the market. This network topology makes the onset of a default in these money centers directly affect a large portion of the system. Thus, vulnerable neighbors to these money centers may, in turn, default, leading to a contagion process throughout the network. Evidence of negative assortativity in financial networks has also been reported in other empirical studies (Bottazzi et al., 2020).

Taking into account all the steps of our modeling process in the RS, the results of the loss distribution described in Section 2.2.3 are shown in Tables 6 and 7. Table 6 shows how many of the 100.000 simulated scenarios have at least one idiosyncratic default (default scenarios) and how many have at least one contagion default (systemic risk scenarios). This analysis is separated into systemically important financial institution (SIFI) and non-systemically important financial institution (N-SIFI). This result shows the importance of the contagion process for our model, as it is present in 96% of the simulated scenarios with all banks.

Table 6: Summary statistics of the loss distribution of the banking system in the reduced sample.

	All banks	SIFI	N-SIFI
N. of default scenarios	100.000	70.515	99.999
N. of systemic risk scenarios	96.135	72.909	85.682

Notes: This table shows statistics of the loss distribution for 24 Brazilian banks in September 2022. SIFI stands for systemically important financial institution and N-SIFI for non-systemically important financial institution. The number of default scenarios represents the number of scenarios in which at least one bank idiosyncratically default. On the other hand, the number of systemic risk scenarios represents the number of scenarios in which at least one bank default due to the contagion process described in Section 2.2.2.

In detail, Table 7 shows summary statistics of the loss distribution of the Brazilian banking system in the RS. Panel A shows that, whiteout contagion, the maximum cost of capital shortfall is close to BRL 293 billion and the one-year probability of the Brazilian DIA default is close to 1.8% when considering only this operation. For the scenario where the DIA makes the reimbursement of the covered deposits, the maximum cost is close to BRL 1.1 trillion and it's associate PD is close to 53.6%. Lastly, when considering the cost of all eligible deposits, the maximum cost is close to BRL 2 trillion and the PD is close to

72.5%.

On the other hand, Panel B shows that, with contagion, the maximum cost of capital shortfall is close to BRL 414 billion and the probability of the DIA default is close to 44%. Note that even the contagion process increases banks' PD by 42% in average, as reported in Table 4, it increases 25 times the DIA's PD. Finally, when considering the reimbursement of the covered deposits, the maximum cost is close to BRL 2.8 trillion and the DI default in all simulated scenarios.

Table 7: Summary statistics of the loss distribution of the banking system with and without contagion in the reduced sample.

	Cost of capital shortfall			Cost of eligible deposits			Cost of covered deposits		
	All banks	SIFI	N-SIFI	All banks	SIFI	N-SIFI	All banks	SIFI	N-SIFI
Panel A: without contagion process									
Mean	25,63	24,73	0,90	326,26	252,43	73,83	170,76	132,12	38,64
Percentiles:									
0%	0,00	0,00	0,00	4,95	4,95	0,00	2,59	2,59	0,00
25%	0,00	0,00	0,00	97,38	30,08	67,29	50,97	15,75	35,22
50%	2,47	2,47	0,00	204,28	127,21	77,07	106,92	66,58	40,34
75%	51,51	50,25	1,26	510,63	419,84	90,79	106,92	59,40	47,52
90%	53,27	50,25	3,02	613,72	513,11	100,61	321,22	268,56	52,66
95%	55,24	51,80	3,44	626,34	515,61	110,73	327,83	269,87	57,96
99%	154,42	149,51	4,92	1.333,40	1.211,09	122,31	697,90	633,89	64,02
99,9%	158,15	151,27	6,89	1.468,69	1.331,34	137,35	768,71	696,82	71,89
100,0%	293,09	284,44	8,65	2.049,06	1.876,77	172,30	1.072,48	982,30	90,18
N. of LOLR scenarios	1.777	1.479	0	72.510	70.515	6.961	53.640	42.939	0
Panel B: with contagion process									
Mean	108,07	106,83	1,24	727,26	634,79	92,47	380,65	332,25	48,40
Percentiles:									
0%	0,00	0,00	0,00	180,79	113,50	67,29	94,63	59,40	35,22
25%	0,42	0,00	0,42	198,48	119,66	78,82	103,89	62,63	41,25
50%	2,88	2,47	0,42	221,93	134,45	87,48	116,16	70,37	45,79
75%	238,95	237,27	1,68	1.347,64	1.246,45	101,19	705,35	652,39	52,96
90%	240,71	237,27	3,44	1.367,59	1.257,74	109,85	715,80	658,30	57,49
95%	242,68	239,11	3,57	1.384,06	1.262,88	121,18	724,42	660,99	63,43
99%	341,86	336,53	5,33	2.185,54	2.055,39	130,15	1.143,91	1.075,79	68,12
99,9%	345,02	337,72	7,30	2.219,65	2.074,72	144,93	1.161,77	1.085,91	75,86
100,0%	414,34	405,28	9,06	2.796,99	2.624,64	172,34	1.463,94	1.373,74	90,20
N. of LOLR scenarios	43.952	43.952	0	100.000	100.000	18.408	72.180	43.952	0

Notes: This table shows statistics of the loss distribution for 24 Brazilian banks in September 2022. SIFI stands for systemically important financial institution and N-SIFI for non-systemically important financial institution. Cost of capital shortfall uses the SRISK as the EAD in the process described in Section 2.2.3, while cost of eligible deposits uses the total deposits and cost of covered deposits uses a share θ over the total deposits. Because the exact covered deposits are not publicly available for each bank, but only the total covered and eligible deposits of the system, we used this share θ over the total deposits as a proxy. Panel A shows summary statistics of the banking system LD with only idiosyncratic default (without contagion process). Panel B presents the same statistics when all the contagion process described in Section 2.2.2 are considered. The number of LOLR scenarios represents the number of scenarios in which the consumption of resources due to default exceeded the equity of the Brazilian deposit insurance in all simulations. All values are in BRL billion except for the number of LOLR scenarios.

4.2 Banks' Probability of Default and Contagion Process in the Full Sample

This section presents the exercise of comparing the results between the reduced sample, discussed in Section 4.1, and the full sample. While the reduced sample considers all 24 listed banks in September 2022, a necessary condition for calculating the main systemic risk measures, but with the downside of representing only 13.19% of the total member institutions, this full sample considers all 182 banks in its analysis. Since we cannot calculate measures such as SRISK and others to discuss the capital shortfall in time of crisis, we focus on comparing the results of LD in terms of eligible and covered deposits.

Table 8 presents summary statistics for the risk measures estimated to build the loss distribution of the Brazilian banking system in September 2022 with a full sample of all member institutions. Compared to Table 4, we can observe that the level of average default is well captured in the RS but the contagion process is underestimated; while the contagion process in the FS intensifies the average default by 104%, in the RS only intensifies by 42%.

Table 8: Summary statistics of structural PD and banks' PD with and without contagion in the full sample.

		Structural PD of Merton model (%)	Banks' probability of default (%)	
			Panel A: without contagion process	Panel B: with contagion process
Mean		22,62	22,63	46,23
Std. Deviation		24,60	2,53	1,84
Minimum		0,00	11,67	38,33
Maximum		99,93	35,00	54,44
Percentiles:	25%	0,17	21,11	45,00
	50%	13,65	22,78	46,11
	75%	43,51	24,44	47,22

Notes: This table shows statistics of PD estimates for 182 Brazilian banks in September 2022. Structural PD represents the estimates of the Merton (1974)'s model used as an input for the idiosyncratic PD of each bank. Banks' PD are the default rates of the sample banks in the Monte Carlo simulation. In details, Panel A shows summary statistics of Banks' PD with only idiosyncratic default (without contagion process). Panel B presents the same statistics when all the contagion process described in Section 2.2.2 are considered. As expected, note that the mean of the structural PD and the Banks' PD without contagion are practically the same.

Regarding the estimation of the interbank network, the full sample database produces an even more adherent result with the moments of the true Brazilian interbank network. While in the RS the statistics for the density, assortativity and average degree are 0.31, -0.54 and 14.4, respectively, the statistics in the FS are 0.02, -0.55 and 4.8, respectively. Again, in the FS we are able to better capture the level of density and average degree when we compare to the true Brazilian interbank network.

Lastly, considering that the RS represents 13.19% of the total member institutions but 89.71% of the total deposits, we analyze whether the loss distribution constructed using the RS maintains the proportions of total deposits with respect to the FS. Comparing the results of the LD for eligible and covered deposits in the FS in Table 9 with the results of the RS in Table 7, we can verify that the RS was able to capture the proportional level of the LD in the FS, especially in the tails (above the 99th quantile) of the distribution, being close to 90%. On average, it was able to capture 78% of the simulated distress, but it was unable to correctly capture the beginning and middle of the distribution.

Table 9: Summary statistics of the loss distribution of the banking system with and without contagion in the full sample.

	Cost of elegible deposits			Cost of covered deposits		
	All banks	SIFI	N-SIFI	All banks	SIFI	N-SIFI
Panel A: without contagion process						
Mean	420,09	251,86	168,23	219,87	131,83	88,05
Percentiles:						
0%	30,68	4,28	26,40	16,06	2,24	13,82
25%	202,70	59,70	142,99	106,09	31,25	74,84
50%	306,48	138,95	167,53	160,41	72,73	87,68
75%	617,22	423,40	193,81	323,05	221,61	101,44
90%	707,30	486,14	221,16	370,20	254,45	115,75
95%	753,35	514,61	238,74	394,30	269,35	124,96
99%	1.428,60	1.156,21	272,39	747,73	605,16	142,57
99,9%	1.583,37	1.268,04	315,34	828,74	663,69	165,05
100,0%	2.270,49	1.889,96	380,54	1.188,38	989,20	199,17
N. of LOLR scenarios	97.867	70.543	92.904	75.421	42.728	20.113
Panel B: with contagion process						
Mean	1.949,51	1.604,16	345,35	1.020,37	839,62	180,75
Percentiles:						
0%	1.051,57	741,73	309,84	550,39	388,22	162,17
25%	1.111,10	786,09	325,02	581,55	411,44	170,11
50%	1.753,96	1.411,92	342,04	918,02	739,00	179,02
75%	2.363,07	2.006,54	356,53	1.236,83	1.050,22	186,61
90%	2.994,73	2.614,65	380,08	1.567,44	1.368,51	198,94
95%	3.011,91	2.616,86	395,05	1.576,44	1.369,67	206,77
99%	3.050,51	2.637,04	413,46	1.596,63	1.380,23	216,41
99,9%	3.610,31	3.180,95	429,37	1.889,64	1.664,91	224,73
100,0%	3.689,39	3.233,81	455,58	1.931,03	1.692,58	238,45
N. of LOLR scenarios	100.000	100.000	100.000	100.000	100.000	100.000

Notes: This table shows statistics of the loss distribution for 182 Brazilian banks in September 2022. SIFI stands for systemically important financial institution and N-SIFI for non-systemically important financial institution. Cost of eligible deposits uses the total deposits and cost of covered deposits uses a share θ over the total deposits. Because the exact covered deposits are not publicly available for each bank, but only the total covered and eligible deposits of the system, we used this share θ over the total deposits as a proxy. Panel A shows summary statistics of the banking system LD with only idiosyncratic default (without contagion process). Panel B presents the same statistics when all the contagion process described in Section 2.2.2 are considered. The number of LOLR scenarios represents the number of scenarios in which the consumption of resources due to default exceeded the equity of the Brazilian deposit insurance in all simulations. All values are in BRL billion except for the number of LOLR scenarios.

4.3 Optimal Capital Requirement

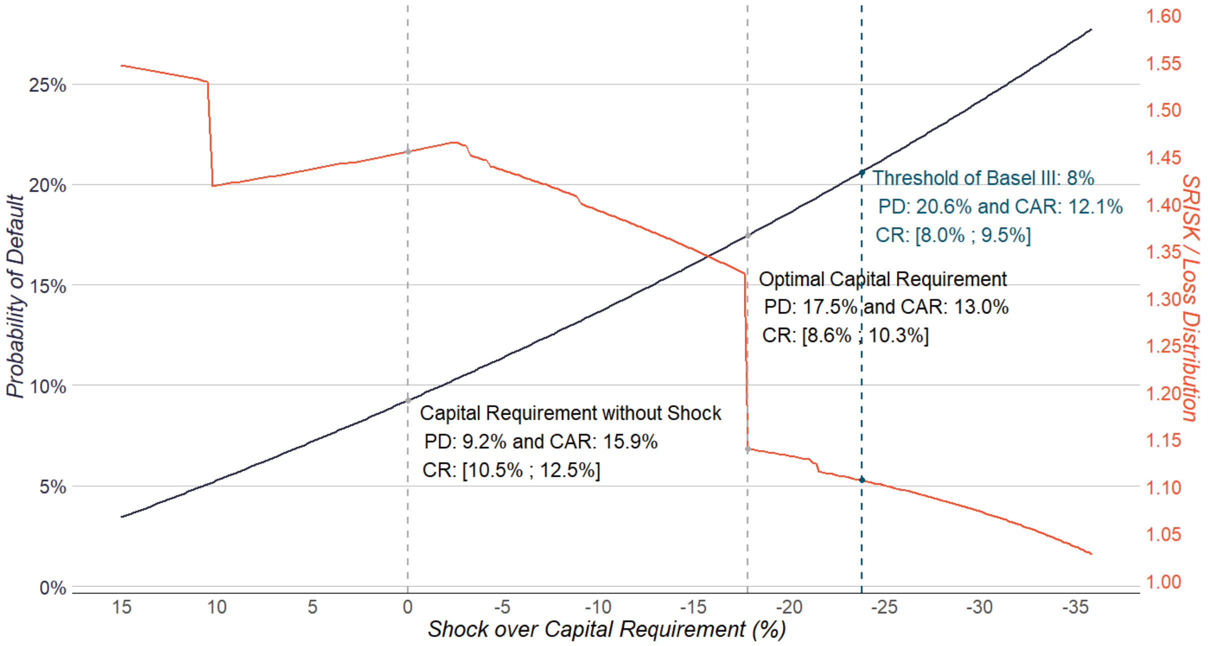
In order to simulate the optimal capital requirement (CR) for the Brazilian financial system, we used a similar theoretical framework in which portfolio risk is calculated in banking organizations. We also adopted a heterogeneous CR regime in which we have different CRs for each bank depending on the prudential segment as in Alexandre et al. (2022). Banks can also hold different levels of capital adequacy ratio (CAR) based on their own strategy and balance structure. Through this analysis, we used granular balance sheet information to estimate the PD of each FI as presented in Section 2.2.1, the amount of capital expected in times of crisis (SRISK) presented in Section 2.1.2, and the systemic risk contagion process presented in Section 2.2, which models the banking segmentation cluster to capture bank runs due to panic and market similarity, the interbank network to estimate the financial contagion in the banking system, and the net stable funding ratio to capture long-run liquidity shortages.

Thus, we estimated the minimum CR that maintains stable the relationship between SRISK and the economy's loss distribution (LD) and that takes into account the international framework of Basel III. The fundamentals of this approach over the LD consider that (i) a lower CR decreases the CAR hold by banks and the amount of SRISK necessary in the economy, which also decreases the total cost of bailout by the central bank and the LD, and (ii) increased the bank's probability of default, which increases, on the other hand, the frequency and costs of extrajudicial settlements, and the LD. Therefore, because there are effects of costs and benefits acting in opposite directions through PD and SRISK, respectively, the maintenance of the relation between SRISK and LD through several shocks characterizes an optimal trade-off scenario. To this purpose, we calculated the PD of the whole system as a weight average of all banks' PD in relation to its total deposits.

Because the CAR of each bank is the result of total regulatory capital over the risk-weighted assets (RWA), we calculated a two-way fixed effects model to estimate the impact that the probability of default has over these two variables to reconstruct the individual and aggregate capital adequacy ratio at every new shock on the bank's PD. We also controlled this effect by considering loan operations by risk level, since it is expected that banks may try to meet a higher capital requirement by either reducing assets, which decreases loan supply, or increasing the loan interest rate, which leads to a reduction in loan demand (Thakor, 2014; Alexandre et al., 2022). It is important to mention that we also control this reduction in loan demand on its reflection on total deposits. Thus, we are able to construct a link between our interest variables and also check the individual consistency of the simulated CAR with the minimum required by Basel III. The results of our equation estimate that, on average, 1% increase in the bank's PD reduces total regulatory capital by 0.10%, increases RWA by 0.12%, and increases loan operations by 0.35%, which also increases the total deposits by 0.13%[31].

Through Figure 5 it is possible to observe a more accelerated drop in SRISK in relation to the LD of the economy as we decrease the CR and CAR of banks. Thus, the moment when a next marginal drop maintains this relationship relatively stable for the next simulations and it is in accordance with Basel III framework is characterized as the optimal CR and CAR. Additional decreases from this point in CR would not bring long-term benefits in reducing the LD and would continue to increase the burden of a greater PD for the financial system. Note that this optimization problem is subject to binding regulatory constraints such that an indefinite reduction in CR would be incompatible with international accords.

Figure 5: Impact of the Capital Requirement shock on SRISK/LD and Probability of Default.



Notes: CAR stands for capital adequacy ratio and *CR* for minimal capital requirement, which varies depending on the bank prudential segment. The blue dashed line represents the threshold in which the capital requirement of at least one bank would be bellow the minimum required of 8% by Basel III.

Our results show that the optimal capital adequacy ratio for the Brazilian financial system in September 2022 is close to 13%. Also, the optimal interval for the minimum

[31]The complete results are shown in Table 10 and Table 11 in Appendix A and B. It is important to mention that, although we are simulating the optimal capital requirement for 26 banks that has available market data, we estimated the average effect considering all 244 Brazilian financial institutions, which results in 9.653 observations between December 2000 and September 2022.

capital requirement depending on the prudential segment varies between 8.6% and 10.3%. Considering that the CAR calculated in our model is 15.9% and the reported by the BCB (2022b) is 16.15%[32], this reduction of 18% in our model implies an optimal CAR in the entire financial system of 13.2%. Because a lower CAR is expected to lead to a financial system more fragile and susceptible to systemic risk, the probability of default of the entire system calculated by our model increases from 9.2% to 17.5%, a variation of 89%.

Taking into account that the percentage of extrajudicial settlements or interventions made by the BCB on the Brazilian banking market is 8.2%, this positive variation of 89% in the PD implies a probability of default of the entire financial system of 15.5%. Considering that the PD of the US financial system is 11.9% and the lowest historical level of CAR in Brazil since 2000 was 12.9% in May 2001, the level of 15.5% for the Brazilian PD as a consequence of a CAR of 13.2% can be argued as reasonable in view of the benefits of a financial system with lower cost for the central bank in a financial crisis scenario, greater loan supply and lower credit cost[33].

In terms of impact of this new CAR, because the simulated capital requirement is lower, the total amount of capital needed in an extreme financial crisis event would also be lower. For our data set, which covers 79.89% of total assets but only 13.19% of total member institutions, we estimate a total cost of bailout in the base scenario (without shock) between 3.9 and 5.7 times the size of the Brazilian deposit insurance agency. Considering the scenario with a 18% shock in the CAR, the total cost of bailout would be between 3.4 and 3.9[34]. Therefore, one of the estimated benefits of this new CAR and CR would be a reduction between 13% and 32% in the cost of bailout. In addition, another positive estimated impact would be an increase in loan operations by 31.1%.

It is important to note that the optimal CAR of the system does not necessarily mean

[32] This indicator measures the capital adequacy of financial institutions in the Brazilian financial system and is based on the definitions used in the Basel Capital Accord. The scope of the data coverage commercial banks, universal banks, investment banks, savings banks or any financial conglomerate comprising any of these entities. For more information, see BCB (2022b) series 21424 (I001 - Regulatory Capital to Risk-Weighted Assets).

[33] It is worth to mention that our modeling process share similar premises presented by Alexandre et al. (2022), but our results do not generate unrealistic minimum CAR for some banks (while the authors mention results close to 1%, our minimum is close to 9.1%). Furthermore, we also consider several balance sheet variables of each bank (used to construct our probability of default, systemic risk metrics, clusters, interbank network, and NSFR) in our model instead of calibrating or assuming baseless values for these parameters.

[34] The lower bound of the range is composed by the maximum value of the loss distribution and the upper bound is composed by the sum of SRISK in the specific scenario. Note that the reduction in the gap between the lower and upper bound after the shock is a consequence of the increase in PD in the simulation.

the optimal minimum CR established by the regulator. Banks can hold levels of regulatory capital ratio higher than the minimum demanded by the Central Bank for strategic reasons or for expectations about its own portfolio or future macroeconomic conditions. Thus, considering that in our model we allow a heterogeneous CR regime, while the official minimum regulatory capital ratio in September 2022 varies between 10.5% and 12.5% depending on the bank prudential segments, the minimum regulatory capital ratio with a 18% shock would result in a range between 8.6% and 10.3%, which is compatible with the 8% minimum established by Basel III (BCBS, 2011).

5 Final Remarks

This paper estimated different measures to understand how much systemic risk each bank brings to the Brazilian market and proposed a bank run model that accounts for idiosyncratic banks' probability of default and a systemic risk process in which additional defaults occur through different channels of contagion.

Our approach follows a similar theoretical framework in which portfolio risk is calculated in banking organizations and in which banking losses are estimated in deposit insurance schemes. Through this analysis, we used granular balance sheet information to estimate the PD of each FI and the systemic risk contagion process captured through the channels of (i) panic due to deposit withdrawals and market similarity, (ii) interbank network, and (iii) funding illiquidity.

The application of our model to a sample of 24 Brazilian banks for September 2022 was able to estimate the loss distribution, the PD of the deposit insurance agency and simulate the optimal capital adequacy ratio of the Brazilian banking system. We find that the DI would be able to bailout the system without contagion in 98.6% of the simulated scenarios and 56% when considering the contagion process. However, if the banks were liquidated and the DI needed to reimburse the covered deposits in the worst-case scenario, the PD of the DI without contagion would be 72.5% and, with contagion, the DI would default in all simulated scenarios.

Regarding the optimal capital adequacy ratio, our results show a value close to 13.2% with a minimum capital requirement interval depending on the prudential segment between 8.6% and 10.3%, 18% lower than the practiced in the Brazilian financial market but compatible with the 8% minimum established by Basel III. This reduction would increase loan operations by 31.1%, but would also increase the PD of the banking system to the level of 15.5%.

Finally, this paper also shows the need to consider different channels of contagion as a key element when designing the overall financial safety net.

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A Effects of Total Regulatory Capital, RWA and Loans on FI's PD

Table 10: Effects of Total Regulatory Capital, RWA and Loans on FI's PD.

	Probability of Default
Total Regulatory Capital	-9.6151^{**}
	(3.9319)
RWA	8.5554^{***}
	(3.2882)
Loans	2.8652
	(2.1761)
Observations	8,780
R^2	0.0149
Adjusted R^2	−0.0222
F Statistic	42.7658^{***}

Notes: This table presents the two-way fixed effects estimates of the FI's total regulatory capital, risk-weighted assets (RWA) and loan operations by risk level on their PD and Z-Score. The equations uses quarterly data from December 2000 to September 2022 for 244 Brazilian financial institutions, resulting in public 9.653 observations provided by the Central Bank of Brazil (BCB, 2022a). All variables are in the natural log. Robust standard errors double-clustered are in parentheses. ***, **, and * denote statistical significance at 1%, 5%, and 10%, respectively.

B Effect of Loans on FI's Total Deposits

Table 11: Effect of Loans on FI's Total Deposits.

	Total Deposits
Loans	0.3727***
	(0.0665)
Observations	7,665
R^2	0.0962
Adjusted R^2	0.0602
F Statistic	784.5530***

Notes: This table presents the two-way fixed effects estimates of the FI's loan operations by risk level on their total deposits. The equations uses quarterly data from December 2000 to September 2022 for 244 Brazilian financial institutions, resulting in public 9.653 observations provided by the Central Bank of Brazil (BCB, 2022a). All variables are in the natural log. Robust standard errors double-clustered are in parentheses. ***, **, and * denote statistical significance at 1%, 5%, and 10%, respectively.

C Codes

Listing 1: Adapted Minimum Density method in R language.

```
min_dens_improved = function(rowsums, colsums, c = 1, lambda = 1, k =
    100, alpha = 1/sum(rowsums), delta = 1/sum(rowsums), theta = 1,
    remove.prob = 0.01, max.it = 1e5, abs.tol = 1e-3, match_moments =
    TRUE, pr_max = TRUE, clear_memory = TRUE, clear_memory_it = 500,
    target_density = 0.0296, target_assortativity = -0.3872, target_
    degree = 6.7696, target_pr_max = 25, target_tol = 0.1, sufficient_
    moments = TRUE, sufficient_alocation = 85, verbose = TRUE){

  emp_results = data.table()
  a = rowsums
  l = colsums

  if (lambda > 1 | lambda < 0) stop("lambda must be between 0 and 1")

  # number of vertices
  n = length(a)

  # initial matrix
  X =  matrix(0, n, n)

  # matrix of indices
  mindex = matrix(1:length(X), n, n)

  # Position vector for sampling
  mu = 1:length(X)

  # remove diagonal (it will be zero)
  mu = mu[mu != diag(mindex)]

  # sampled positions vector
  v = numeric(0)

  # Asset and liabilities deficit
  ad = a - rowSums(X)
  ld = l - colSums(X)
```

```r

# 'prior' probabilities
probs = Q(ad, ld, n)

if (verbose) cat("Starting Minimum Density estimation.\n\n")

for (t in 1:max.it) {

  if (t > k) lambda = 1

  if ((runif(1) < remove.prob && t > 1 && length(v) > 0) || sum(probs[
 mu]) == 0) {

    # sample position to be removed
    ij = sample(v, 1)

    # check the position row and column indices
    index = which(mindex == ij, arr.ind = T)
    i = index[1]
    j = index[2]

    # sum the value back
    ad[i] = ad[i] + X[ij]
    ld[j] = ld[j] + X[ij]

    # Remove the position value from X
    X[ij] = 0

    # include position back to the ones to be sampled
    mu = c(mu, ij)

    # remove it from the sampled
    v = v[v != ij]
  } else {
    # position of the sample to be filled
    ci = sample.int(length(mu), 1, prob = probs[mu])

    # takes the position value
    ij = mu[ci]
```

```

    # takes the position row and column indices
    index = which(mindex == ij, arr.ind = T)
    i = index[1]
    j = index[2]

    # adds value to X
    Xnew = X
    Xnew[ij] = lambda*min(ad[i], ld[j])

    # computes new deficits
    ## assets
    adnew = ad
    adnew[i] = adnew[i] - Xnew[ij]

    ## liabilities
    ldnew = ld
    ldnew[j] = ldnew[j] - Xnew[ij]

    # checks the new value function against the old value function
    dif  = V(Xnew, adnew, ldnew, c = c, alpha = alpha, delta = delta)
 - V(X, ad, ld, c = c, alpha = alpha, delta = delta)

    comp1 = dif > 0
    comp2  = exp(theta*dif) > runif(1)

    if (comp1 || comp2) {
      # updates X, ad and ld
      X = Xnew
      ad = adnew
      ld = ldnew
      # includes the position in the sampled vector
      v = c(v, ij)
      # removes it from the vector to be sampled
      mu = mu[mu != ij]
    }
  }
  # updates probabilities
  probs = Q(ad, ld, n)
```

```
  if (pr_max) {
    # limiting the total regulatory capital (TRC)
    limit_pr_emp = data.table()
    for (i in 1:nrow(X)) {
      aux5 = X[i,] # limiting the asset exposure
      aux7 = sum(aux5>0) # number of connections with the market
      aux8 = (aux5/PR_T1[i])*100 # the exposure compared to the FI's TRC
      aux9 = a[i] # checking if there is a connection
      aux10 = sum(aux5)/aux9*100
      if (aux9==0) { aux7 = NA }
      aux11 = ifelse(aux8>target_pr_max, (PR_T1[i]*target_pr_max/100), aux5)
      aux12 = (aux11/PR_T1[i])*100
      aux13 = sum(aux11)/aux9*100
      if (aux9==0) { aux13 = NA }
      limit_pr_aux = data.table(conexoes = aux7,pr_max = max(aux8), completude = aux10,pr_max_new = max(aux12),completude_new = aux13)
      limit_pr_emp = rbind(limit_pr_emp,limit_pr_aux)
      X[i,] = aux11
    }

    # Additional information about the matrix
    X = round(X,0)
    matrix_temp = graph_from_adjacency_matrix(X, weighted = T)
    matrix_den =  edge_density(matrix_temp) # network density
    matrix_assort = assortativity_degree(matrix_temp) # assortativity
    matrix_degree = mean(igraph::degree(matrix_temp)) # average degree

    if (verbose) cat("- Iteration number: ", t,  " -- total alocated: ", round(100*(sum(a - ad)/sum(a)),6)," %"," | Dens: ",round(matrix_den,4)," | Assort: ",round(matrix_assort,4)," | Degree: ",round(matrix_degree,4)," | Conex Min e Max: ",min(limit_pr_emp$conexoes,na.rm = TRUE)," e ",max(limit_pr_emp$conexoes,na.rm = TRUE)," | PR Max: ",percent_format(max(limit_pr_emp$pr_max_new,na.rm = TRUE),0)," - ",nrow(limit_pr_emp[pr_max_new>target_pr_max])," | Complet. Min e Max: ",percent_format(min(limit_pr_emp$completude_new,na.rm = TRUE),0)," e ",percent_format(max(limit_pr_emp$completude_new,na.rm = TRUE),0),"
```

```
\n", sep = "")

    if (match_moments) {
      if (matrix_den < (target_density*(1+target_tol)) & matrix_den > (target_density*(1-target_tol)) & matrix_assort < (target_assortativity*(1-target_tol)) & matrix_assort > (target_assortativity*(1-target_tol)) & matrix_degree > (target_degree*(1-target_tol)) & matrix_degree < (target_degree*(1+target_tol)) & max(limit_pr_emp$pr_max,na.rm = TRUE) <= target_pr_max ) break
    }
    if (sufficient_moments) {
      if (matrix_assort < (target_assortativity*(1-target_tol)) & matrix_assort > (target_assortativity*(1-target_tol)) & (round(100*(sum(a - ad)/sum(a)),6)) > sufficient_alocation & matrix_degree > (target_degree*(1-target_tol)) & matrix_degree < (target_degree*(1+target_tol))  ) break
    }
  } else {
    if (verbose) cat("- Iteration number: ", t," -- total alocated: ", round(100*(sum(a - ad)/sum(a)),6)," %"," | Dens: ",round(matrix_den,4)," | Assort: ",round(matrix_assort,4)," | Degree: ",round(matrix_degree,4)," \n", sep = "")
  }

  emp_results_aux = data.table(Iteration = t,Total_Alocated=round(100*(sum(a - ad)/sum(a)),6),Densid=matrix_den,Assort=matrix_assort,Degree=matrix_degree,Conex_Min=min(limit_pr_emp$conexoes,na.rm = TRUE),Conex_Max = max(limit_pr_emp$conexoes,na.rm = TRUE),PR_Min = min(limit_pr_emp$pr_max_new,na.rm = TRUE),PR_Max = max(limit_pr_emp$pr_max_new,na.rm = TRUE),PR_Target = target_pr_max,PR_Exess = nrow(limit_pr_emp[pr_max_new>target_pr_max]),Completude_Min=min(limit_pr_emp$completude_new,na.rm = TRUE),Completude_Max=max(limit_pr_emp$completude_new,na.rm = TRUE), Delta_Dens = abs(matrix_den-target_density),Delta_Assort = abs(matrix_assort-target_assortativity),Delta_Degree = abs(matrix_degree-target_degree),Tol = target_tol)
  emp_results = rbind(emp_results,emp_results_aux)
  if (clear_memory & (t %% clear_memory_it)==0) {
    print(paste0("Cleaning the memory at every ",clear_memory_it," iteration"))
```

```
149       Sys.sleep(1) ; gc() ; Sys.sleep(1)
150     }
151     if (match_moments) {
152       if (matrix_den < (target_density*(1+target_tol)) & matrix_den > (target_density*(1-target_tol)) & matrix_assort < (target_assortativity*(1-target_tol)) & matrix_assort > (target_assortativity*(1-target_tol)) & matrix_degree > (target_degree*(1-target_tol)) & matrix_degree < (target_degree*(1+target_tol)) ) break
153     }
154     if (sum(abs(ad) - 0) < abs.tol) break
155   }
156   if (verbose) {
157     if (t >= max.it) cat("\nMaximum number of iterations reached! Change the max.it parameter or other settings.\n")
158     cat("\nMinimum Density estimation finished.","\n * Total Number of Iterations: ", t, "\n * Total Alocated: ", round(100*(sum(a - ad)/sum(a)), 6), " % \n", sep = "")
159   }
160   rownames(X) = colnames(X) = names(rowsums)
161   return(list(Matrix = X,Final_Results = emp_results))
162 }
163
164 V = function(z, ad, ld, c = 1, alpha = 1, delta = 1)
165   -c*sum(z > 0) - sum((alpha^2)*ad) - sum((delta^2)*ld)
166
167 Q = function(ad, ld, n){
168   Q = rep.int(ad,  n)/rep(ld, each = n)
169   index = (Q < 1 | is.na(Q)) # Q < 1/Q
170   Q[index]  = (1/Q)[index]
171   Q[is.na(Q) | is.infinite(Q)] = 0
172   return(Q) }
```

Notes: The original R Statistical Software (R Core Team, 2022) code is publicly available in the `NetworkRiskMeasures` package published by Cinelli and Silva (2022). The authors implemented the Minimum Density method based on Anand et al. (2015). All significant changes to the original code begin at line 107.