Heterogeneous Informational Shocks a new Framework for Bias Testing of Consensus Forecasts

Luciano Vereda

Fluminense Federal University
Department of Economics

e-mail: *lvereda@gmail.com*
ORCID: 0000-0001-7437-8872

Helder Ferreira de Mendonça
Fluminense Federal University
Department of Economics and
National Council for Scientific and Technological Development (CNPq)
Address: Rua Royolze Carvalho de Mendonça, 88 – Alto da Boa Vista -
Miguel Pereira – Rio de Janeiro – Brazil
CEP: 26900-000
e-mail: helderfm@hotmail.com
ORCID: 0000-0003-1465-1755

George Morcerf

Fluminense Federal University
Department of Economics
e-mail: *george.morcerf@yahoo.com*

ABSTRACT

Our study advances the modeling of forecast revisions by accounting for the nuanced impact of informational shocks across different time horizons. Specifically, we introduce modifications to the error structure of regression models used to detect biases in macroeconomic forecasts. Drawing on consensus forecasts of inflation and output growth from the central banks of Brazil, Chile, and Mexico, our approach offers a nuanced understanding of bias estimation uncertainty, leading to a more robust rejection of the null hypothesis of no biases. By elucidating the differential effects of informational shocks on forecast accuracy across time periods, our findings not only contribute to the refinement of forecasting methodologies but also have implications for policymakers and economic analysts striving for more accurate and reliable predictions in dynamic economic environments.

JEL codes: C52, C53, D84, E37, E58

Keywords: forecast evaluation; unbiasedness; inflation forecasts; output growth forecasts.

1. Introduction

The pivotal role of expectations in shaping economic outcomes has long been recognized, prompting extensive investigation by academics, central bankers, and economists. This scholarly pursuit encompasses a multifaceted approach, ranging from the measurement and evaluation of expectations' performance to unraveling the intricacies of their formation. While the bulk of research has traditionally focused on developed economies, a growing body of literature also delves into the dynamics of expectations in emerging economies. Notably, a significant portion of this literature scrutinizes the accuracy, unbiasedness, and efficiency of both individual and consensus forecasts. While output growth commands the lion's share of attention, scholarly interest extends to other vital economic variables such as inflation, exchange rates, and fiscal indicators. For instance, Mitchell and Pearce (2007) and Ince and Molodtsova (2017) have contributed to understanding the behavior of exchange rates and inflation forecasts, respectively. Similarly, studies by Jalles et al. (2015) and de Deus and de Mendonça (2017) have shed light on the forecasting challenges associated with fiscal variables. Through these diverse lenses, researchers strive to deepen our understanding of the intricate interplay between expectations and economic outcomes across diverse economic landscapes.

Many studies use fixed-event forecasts to analyze these issues. These are defined as a sequence of projections for the value of some macroeconomic variable at the end of a given year, being these projections calculated period by period. Fixed-event forecasts are often used to study biases in macroeconomic forecasts. Davies and Lahiri (1995) (henceforth DL) proposed an influential method where they are used to form a panel of forecast errors.[1] Under their framework, each individual i is subject to an error when he or she predicts, h months before the end of year t, the value of the target variable at the end of the same year. This error is written as the sum of an individual bias, idiosyncratic errors and an error component that represents the cumulative effect of all the shocks that occurred from h months prior to the end of year t until the end of year t. This framework allows computing the error covariance matrix, which they use in regressions *a la* Mincer and Zarnowitz (1969) to assess

[1] Examples that also use or extend the methodology proposed by Davies and Lahiri (1995) are Boero, Smith and Wallis (2008), Clements, Joutz and Stekler (2007), Ager, Kappler and Osterloh (2009) and Dovern and Weisser (2011).

the null of no biases in inflation and output growth forecasts. Later, Ager, Kappler and Osterloh (2009) (henceforth AKO) modified DL's methodology to deal with consensus forecasts.

However, the error generating process proposed by DL has an important drawback: it implies that the shock occurring h months prior to the end of year t affects the consensus forecasts made for the value of the target variable at the end of years t and $t+1$ in the same way. This implication neither corresponds to observed data nor is justified in economic terms. Take, for example, the case of inflation forecasts. If new information points to a temporary inflationary shock, then inflation forecasts for the current year would change, but those for the next year would not. If incoming news point to a persistent shock, then agents would update their inflation forecasts for the next year – but only moderately. These two examples show that the hypothesis of an equally strong impact is very unlikely.

Our paper makes a significant contribution by revising the conventional error structure proposed by DL and AKO. In our approach, we introduce a dynamic framework where each month is characterized by the arrival of two distinct types of informational shocks: one influencing forecasts for the current year's end and another for the following year's end. While these shocks exhibit correlation, they convey different information and vary in magnitude.[2] By integrating these nuanced adjustments, we construct an error covariance matrix that better captures the complex interplay of informational shocks over time. Consequently, our bias tests yield more reliable results, offering a clearer understanding of forecast accuracy and enhancing the robustness of economic analysis.

We apply our framework to a database comprising 24 fixed-event forecasts of inflation and output growth for each year. They come from the surveys carried out by the central banks of Brazil, Chile, and Mexico.[3] They are "consensual" in nature, are collected monthly and refer to the figures prevailing at the end of the current and the following years. Observations belong to the period between the beginning of the 2000's until December 2020, which

[2] The structure we propose considers some of the issues raised by Dovern and Weisser (2011). They point out that, under DL, the variance of forecast errors decays linearly when the forecast horizon goes to 1. They recommend the use of more general functional forms, capable of better matching the data.

[3] We choose Brazil, Chile, and Mexico for some reasons. First, their central banks give free access to the bulk of their surveys, in particular macroeconomic forecasts informed monthly (or even daily, in the Brazilian case) by market experts. The monthly availability of such data is not common by international standards since most official institutions release information on a quarterly basis. Second, the consensus forecasts collected by these surveys have a strong impact on economic discussions and attract wide media coverage.

includes important events such as the subprime crisis of 2008, the European public debt crisis of 2010, the huge drop of commodity prices in international markets (which took place after 2010) and the pandemic crisis. For each country and variable, we construct a panel of 24 fixed-event forecasts in its transversal dimension and up to 20 years in its temporal dimension.

Econometric exercises show that our framework yields a less conservative picture of the uncertainty surrounding bias estimates. This increased precision facilitates the rejection of the null of no biases in consensus forecasts. Indeed, under the "traditional" framework and considering a common bias, the null is rejected in Chile (in the case of output growth) and in Brazil – this time in both cases. However, under our framework and considering output growth forecasts, the null is rejected in Brazil, Chile, and Mexico. In the case of inflation and under our framework only in Brazil the null of no bias is rejected. Results are similar if biases vary with the forecast horizon. Their absolute values tend to increase with the forecast horizon, and Brazil stands out as the country where biases are higher and easier to detect.

The remainder of the paper is structured as follows: Section 2 delves into the error decomposition proposed by DL and AKO, highlighting the shortcomings and presenting our novel enhancements. Section 3 details the methodologies employed to estimate both common biases and vectors of horizon-specific biases. Section 4 provides a comprehensive overview of the dataset utilized in our analysis, shedding light on its composition and relevance. Section 5 constitutes the core of our empirical investigation, where we meticulously estimate biases and thoroughly discuss the obtained results. Finally, Section 6 offers concluding remarks.

2. Modelling forecasts errors

Let $F_{i,t,h}$ be the forecast for the target variable for year t, made by individual i, h months prior to the end of year t; and let A_t be the actual value for year t. The forecast error $e_{i,t,h}$ is defined as the difference between A_t and $F_{i,t,h}$, i.e., $e_{i,t,h} = A_t - F_{i,t,h}$. Davies and Lahiri (1995) decompose this error as:

$$e_{i,t,h} = \alpha_i + \lambda_{t.h} + \varepsilon_{i,t,h} \quad (1)$$

The term α_i represents the bias for individual i, while $\varepsilon_{i,t,h}$ represents idiosyncratic errors due to "other factors" (e.g., private information, measurement error, etc.) that are specific to a given individual at a given point in time. The term $\lambda_{t,h}$ represents the cumulative effect of all the shocks occurring from h months prior to the end of year t to the end of year t. It is given by:

$$\lambda_{t,h} = \sum_{j=1}^{h} u_{t,j} \quad (2)$$

Equation (2) shows that $\lambda_{t,h}$ is the sum of the monthly shocks $u_{t,j}$, which DL interpret as a sequence of news shocks. Each shock in (2) reflects the arrival of new information, which induces agents to update their forecasts.

Ager et al. (2009) change the decomposition proposed by Davies and Lahiri to deal with consensus forecasts.[4] They define $F_{t,h}$ as the consensus forecast for the target variable for year t made h months before the end of that year and $e_{t,h} = A_t - F_{t,h}$ as the error made by it. According to them, this error is given by:

$$e_{t,h} = \alpha_h + \lambda_{t,h} \quad (3)$$

The forecast error $e_{t,h}$ is the sum of the bias α_h, which may depend on the forecast horizon h, and the term $\lambda_{t,h}$ given by (2). The idiosyncratic errors $\varepsilon_{i,t,h}$ in (1) would cancel each other out when calculating the average forecast. Both DL and AKO assume that the shocks $u_{t,j}$ are independent and identically distributed, that their unconditional means are zero and that their unconditional variances are σ_u^2 for all h and t.

Under these assumptions it is easy to show that the mean value of $e_{t,h}$ is zero in the absence of biases, a result that abides to rational expectations. One can also show that forecast revisions are efficient in the sense of Nordhaus (1987), that is, they are i.i.d. as the shocks themselves. It seems that both DL and AKO characterization of errors aims to comply with some minimum requirements of rationality. Indeed, it is not based on any specific time series

[4] The focus on consensus forecasts is often criticized (see, for example, Keane and Runkle (1990)). However, this focus is still warranted because consensus forecasts are good proxies of agents' general opinion. Furthermore, Gallo et al. (2002) argue that consensus forecasts work as attractors of individual expectations. Finally, Lahiri and Sheng (2010) and Glas and Hartmann (2016) argue that individual-specific biases are small and can be neglected.

model for the monthly evolution of the target variable. It is also not based on the observed behavior of errors and revisions.

Nevertheless, the frameworks proposed by DL and AGO have some important drawbacks. First, they allow the shock occurring h months prior to the end of year t (which can be interpreted as the arrival of new information in that month) to exert the same impact on $F_{t,h}$ and $F_{t+1,h+12}$. To see this under the AKO framework, define the revision of the consensus forecast occurred between h and $h-1$ as:

$$r_{t;h-1,h} = F_{t,h-1} - F_{t,h} \quad (4)$$

Equation (3) implies that $F_{t,h} = A_t - \alpha_h - \lambda_{t,h}$ and $F_{t,h-1} = A_t - \alpha_{h-1} - \lambda_{t,h-1}$. Moreover, definition (2) implies that $\lambda_{t,h} = \sum_{j=1}^{h} u_{t,j}$ and $\lambda_{t,h-1} = \sum_{j=1}^{h-1} u_{t,j}$. Substituting into (4) yields:

$$r_{t;h-1,h} = F_{t,h-1} - F_{t,h} = A_t - \alpha_{h-1} - \sum_{j=1}^{h-1} u_{t,j} - A_t + \alpha_h + \sum_{j=1}^{h} u_{t,j} = \alpha_h - \alpha_{h-1} + u_{t,h} \quad (5)$$

Following the same path, one can show that:

$$r_{t+1;h+11,h+12} = \alpha_{h+12} - \alpha_{h+11} + u_{t+1,h+12} \quad (6)$$

Results (5) and (6) imply that revisions $r_{t;h-1,h}$ and $r_{t+1;h+11,h+12}$ have the same variance (which is σ_u^2) and are perfectly correlated, given that both revisions are affected by the same shock ($u_{t,h} = u_{t+1,h+12}$). Putting into words, consensus forecasts for the current and the next years are equally revised.

This outcome is not reasonable, though. Indeed, the information disclosed in each month does not necessarily affects agents' perceptions about the behavior of the target variable in the current and the next years in the same way. Take, for example, the case of inflation. If new information points to a temporary inflationary shock, then inflation forecasts for the current year would change, but expectations for the next year would not. If incoming news suggest that the inflationary shock is persistent, then agents would update their inflation forecasts for the next year – but changes would be attenuated. These two examples show that the hypothesis of an equally strong impact is very unlikely.

A second issue is the ability of their framework to reproduce the observed behavior of agents' forecasts. In Section 4 we use monthly data on inflation and output growth forecasts collected by the central banks of Brazil, Chile, and Mexico to provide evidence that forecast revisions $r_{t;h-1,h}$ and $r_{t+1;h+11,h+12}$ neither have the same variance nor are perfectly correlated. This point will be further discussed in Section 4 and supplemental Appendix 4.

Our proposal to overcome these problems is changing the way $\lambda_{t,h}$ is formed. As DL and AKO, our framework preserves the minimum requirements of rationality mentioned earlier and is not based on a time series model for the dynamics of the target variable. Our contribution is increasing the adherence to observed data by imposing that different news arriving each month do not exert the same impact on short- and long-term forecasts.

More specifically, let $u_{t,h}^{s}$ and $u_{t+1,h+12}^{l}$ be the two informational shocks (or news) that arrive h months prior to the end of year t. These shocks affect the values of the target variable at the end of years t and $t+1$, respectively.[5,6] We assume that $Var\left(u_{t,h}^{s}\right)=\sigma_s^2$, $Var\left(u_{t+1,h+12}^{l}\right)=\sigma_l^2$ and that the correlation between $u_{t,h}^{s}$ and $u_{t+1,h+12}^{l}$ is φ. We maintain the hypothesis that shocks $u_{t,h}^{s}$ and $u_{t+1,h+12}^{l}$ are serially uncorrelated.

The structure above is based on some ideas. First, a real economy is hit by various shocks of different magnitudes and durations.[7] Second, economic agents gather information from different sources to unveil these characteristics. A part of this information (the news shock $u_{t,h}^{s}$) signals the occurrence of temporary shocks, thus affecting the forecasts made for the current year but not for the next. The other part (which is represented by the news shock $u_{t+1,h+12}^{l}$) signals the occurrence of persistent disturbances, leading agents to revise the forecasts made for the next year.

The aforementioned structure generates a forecast updating process in which, from $t-24$ until $t-13$ (that is, as long as $h>12$), the forecast $F_{t,h}$ responds to the type "*l*"

[5] The superscripts "*s*" and "*l*" denote short-term and long-term forecasts. Short-term forecasts target the current year, while long-term forecasts target the next. If $h=1$, then the informational shock $u_{t,h}^{s}$ would occur in December of year t and would refer to the value of the target variable at the end of year t. The shock $u_{t+1,h+12}^{l}$ would occur in the same month, but it would refer to year $t+1$.

[6] The sub index j varies between 12 and 1 because we deal with data observed on a monthly frequency.

[7] Appendix 4 shows the consequences of allowing inflation and output growth to respond to several structural shocks. Ultimately, under this more realistic framework, the result of forecast revisions being the same regardless of the target (the current year or the next) is no longer valid.

informational shocks that arrive during year $t-1$. From $t-12$ until $t-1$ (that is, when $h \leq 12$) $F_{t,h}$ is affected by the type "*s*" informational shocks that arrive during year t. The error made by long-term forecasts reflects the sum of the news shocks occurring throughout years $t-1$ and t, while the error made by short-term forecasts is driven by the news shocks that arrive during year t.

Under this framework, the term $\lambda_{t,h}$ does not obey (2); instead, it is given by:

• If $h \leq 12$, then $\lambda_{t,h} = \sum_{j=1}^{h} u_{t,j}^{s}$;

• If $h > 12$, then $\lambda_{t,h} = \sum_{j=13}^{h} u_{t,j}^{l} + \sum_{j=1}^{12} u_{t,j}^{s}$. (7)

The perfect correlation between $r_{t;h-1,h}$ and $r_{t+1;h+11,h+12}$ disappears because, given that $h \leq 12$, $Corr\left(r_{t;h-1,h}, r_{t+1;h+11,h+12}\right) = Corr\left(u_{t,h}^{s}, u_{t+1,h+12}^{l}\right) = \varphi$.[8] Moreover, it is easy to see that $Var\left(r_{t;h-1,h}\right) = \sigma_s^2$ and $Var\left(r_{t+1;h+11,h+12}\right) = \sigma_l^2$, meaning that forecast revisions for years t and $t+1$ are not equally "strong". Indeed, we expect σ_s^2 to be higher than σ_l^2.

Regarding the error structure, under the framework proposed by AKO (in which $\lambda_{t,h}$ follows (2)), the covariance between two typical forecast errors is given by:

• If $\tilde{t} = t$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = \min\left(h, \tilde{h}\right)\sigma_u^2$;

• If $\tilde{t} = t+1$ and $\tilde{h} \geq 13$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = \min\left(h, \tilde{h}-12\right)\sigma_u^2$; (8)

• $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = 0$ otherwise.

where $\tilde{t} = t+1$ denotes the first year after t and $E\left(u_{t,h}^2\right) = \sigma_u^2$ over all t and h. In supplemental Appendix 1 we show that, under our framework (in which $\lambda_{t,h}$ follows (7)), the covariance between two typical forecast errors is given by:

• If $\tilde{t} = t$, $h \leq 12$ and $\tilde{h} \leq 12$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = \min\left(h, \tilde{h}\right)\sigma_s^2$;

• If $\tilde{t} = t$, $h \leq 12$ and $\tilde{h} \geq 13$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = h\sigma_s^2$;

[8] We prove this result in supplemental Appendix 1.

• If $\tilde{t} = t$, $h \geq 13$ and $\tilde{h} \leq 12$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = \tilde{h}\sigma_s^2$; (9)

• If $\tilde{t} = t$, $h \geq 13$ and $\tilde{h} \geq 13$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = 12\sigma_s^2 + \min\left(h-12, \tilde{h}-12\right)\sigma_l^2$;

• If $\tilde{t} = t+1$ and $\tilde{h} \geq 13$, then $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = \min\left(h, \tilde{h}-12\right)\varphi\sigma_s\sigma_l$;

• $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right) = 0$ otherwise.

The structures described above are very parsimonious. Under AKO the only unknown is σ_u^2, being the correlation coefficient φ and the variances σ_l^2 and σ_s^2 the unknowns of our framework. Apparently both DL and AKO try to avoid the dangers of estimating models with many parameters - and we follow the same approach.

Both structures are used to characterize the uncertainty surrounding the estimation of a constant bias α and of a vector of biases that depend on the forecast horizon ($\boldsymbol{\alpha}_h$). These subjects are discussed in the next section.

3. Bias estimation

We split this section into three subsections. The first describes the estimation of a common bias under the AKO framework, the second is dedicated to the same procedure based on our framework and the third to the estimation of horizon-specific biases.

3.1. The AKO model

First, we estimate the common bias α through the pooling procedure proposed by DL and AKO. It starts by forming a panel in which the cross-section units are the data observed for each forecast horizon and the time dimension contains the target years. The sample comprises T target years and forecast horizons h that range from 1 to H. Under this framework one can show that the regression used to estimate α can be written as:[9]

$$\mathbf{e} = \mathbf{i}_{TH}\alpha + \boldsymbol{\lambda} \quad (10)$$

In this regression $\mathbf{e}$ is a TH dimensional column vector of forecast errors arranged

[9] More details about (10) to (18) can be seen in supplemental Appendix 2.

in stacked form, with all the observations of a given target year grouped together. The scalar α is the bias, $\mathbf{i}_{TH}$ is a column vector with dimension TH whose elements are all equal to 1 and $\boldsymbol{\lambda}$ is a column vector with the same dimension. The elements of $\boldsymbol{\lambda}$ reflect the sum of the informational shocks that occurred between $t-h$ and t. They are formed following (2) under the AKO framework.

The ordinary least squares estimator of α is given by:

$$\hat{\alpha} = \left(\mathbf{i}_{TH}^{tr}\mathbf{i}_{TH}\right)^{-1}\mathbf{i}_{TH}^{tr}\mathbf{e} \tag{11}$$

which is equivalent to:

$$\hat{\alpha} = \frac{1}{TH}\sum_{t=1}^{T}\sum_{h=1}^{H} e_{t,h} \tag{12}$$

The consistent variance estimator of $\hat{\alpha}$ is given by:

$$Var\left(\hat{\alpha}\right) = \left(\mathbf{i}_{TH}^{tr}\mathbf{i}_{TH}\right)^{-1}\mathbf{i}_{TH}^{tr}\boldsymbol{\Sigma}\mathbf{i}_{TH}\left(\mathbf{i}_{TH}^{tr}\mathbf{i}_{TH}\right)^{-1} \tag{13}$$

where $\boldsymbol{\Sigma} = E\left(\boldsymbol{\lambda\lambda}^{tr}\right)$. It's easy to prove that (13) can be written as:

$$Var\left(\hat{\alpha}\right) = \frac{1}{\left(TH\right)^{2}}\sum_{l=1}^{TH}\sum_{c=1}^{TH}\sigma_{l,c} \tag{14}$$

where $\sigma_{l,c}$ denotes the element at row l and column c of $\boldsymbol{\Sigma}$. Therefore, $Var\left(\hat{\alpha}\right)$ is the mean value of the elements of $\boldsymbol{\Sigma}$.

The precise form of $\boldsymbol{\Sigma}$ depends on how errors $e_{t,h}$ are generated. Under the error structure proposed by AKO (see (8)), $Var\left(\hat{\alpha}\right)$ is given by:

$$Var\left(\hat{\alpha}\right) = \frac{\sigma_u^2}{\left(TH\right)^{2}}\left(T\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c} + 2\left(T-1\right)\sum_{l=1}^{H}\sum_{c=1}^{H}b_{l,c}\right) \tag{15}$$

where $a_{l,c}$ and $b_{l,c}$ represent the elements at row l and column c of the $H \times H$ matrices $\mathbf{A}$ and $\mathbf{B}$, respectively. Both are defined as:

$$\mathbf{A} = \begin{bmatrix} 24 & 23 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 23 & 23 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \end{bmatrix} \qquad (16)$$

$$\mathbf{B}^{tr} = \begin{bmatrix} 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \qquad (17)$$

Matrix $\mathbf{\Sigma}$ equals $\sigma_u^2\mathbf{\Psi}$. $\mathbf{\Psi}$ is formed from $\mathbf{A}$ and $\mathbf{B}$ in the following way:

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{B}^{tr} & \mathbf{A} & \mathbf{B} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}^{tr} & \mathbf{A} & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}^{tr} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}^{tr} & \mathbf{A} \end{bmatrix} \qquad (18)$$

3.2. The new model of forecast errors

The new forecast error model considers shocks of two types, $u_{t,j}^s$ and $u_{t+1,j+12}^l$. Under this framework the elements of $\boldsymbol{\lambda}$ (see (10)) are formed following (7) and (9). The consistent variance estimator of $\hat{\alpha}$ turns out to be:[10]

[10] More details about (19) to (25) are in supplemental Appendix 2.

$$Var\left(\hat{\alpha}\right)=\frac{1}{\left(TH\right)^2}\left(T\sigma_l^2\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c}^{l}+T\sigma_s^2\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c}^{s}+2\left(T-1\right)\varphi\sigma_s\sigma_l\sum_{l=1}^{H}\sum_{c=1}^{H}b_{l,c}^{ls}\right) \tag{19}$$

where $a_{l,c}^{l}$, $a_{l,c}^{s}$ and $b_{l,c}^{ls}$ denote the elements at row l and column c of matrices $\mathbf{A}_s$, $\mathbf{A}_l$ and $\mathbf{B}_{ls}$, respectively. These matrices are defined as:

$$\mathbf{A}_s=\begin{bmatrix}12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1\\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1\\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1\\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1\\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1\end{bmatrix} \tag{20}$$

$$\mathbf{A}_l=\begin{bmatrix}12 & 11 & \dots & 1 & 0 & 0 & \dots & 0 & 0\\ 11 & 11 & \dots & 1 & 0 & 0 & \dots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 0\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\end{bmatrix} \tag{21}$$

$$\mathbf{B}_{ls}^{tr}=\begin{bmatrix}12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1\\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1\\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1\\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0\end{bmatrix} \tag{22}$$

Matrices $\mathbf{A}_s$, $\mathbf{A}_l$ and $\mathbf{B}_{ls}$ are used to form $\mathbf{\Sigma}$. Indeed, we define two $H\times H$ submatrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ as:

$$\tilde{\mathbf{A}} = \sigma_s^2 \mathbf{A}_s + \sigma_l^2 \mathbf{A}_l \tag{23}$$

$$\tilde{\mathbf{B}} = \mathbf{B}_{ls} \varphi \sigma_s \sigma_l \tag{24}$$

Using definitions (23) and (24) one can show that $\boldsymbol{\Sigma}$ is given by:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} & \tilde{\mathbf{B}} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{B}}^{\mathbf{tr}} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} \end{bmatrix} \tag{25}$$

3.3. Horizon-specific biases

Regarding the case in which biases may depend on h, we define the column vector $\boldsymbol{\alpha}_H$ with dimension H as $\boldsymbol{\alpha}_H = \left[\alpha_H \quad \alpha_{H-1} \quad \dots \quad \alpha_1\right]^{tr}$. The elements of $\boldsymbol{\alpha}_H$ are the biases α_h that are typical of each forecast horizon. We also define $\mathbf{i}_T$ as a column vector with dimension T whose elements are all equal to 1, $\mathbf{I}_H$ as the $H \times H$ identity matrix and $\mathbf{M}$ as the Kronecker product between $\mathbf{i}_T$ and $\mathbf{I}_H$. Under this framework one can show that the regression used to estimate $\boldsymbol{\alpha}_H$ can be written as:

$$\mathbf{e} = \mathbf{M}\boldsymbol{\alpha}_H + \boldsymbol{\lambda} \tag{26}$$

The ordinary least squares estimator of $\boldsymbol{\alpha}_H$ is given by:

$$\hat{\boldsymbol{\alpha}}_H = \left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1} \mathbf{M}^{tr}\mathbf{e} \tag{27}$$

which is equivalent to:

$$\hat{\boldsymbol{\alpha}}_H^{tr} = \left[\sum_{t=1}^{T} e_{t,H} \quad \sum_{t=1}^{T} e_{t,H-1} \quad \dots \quad \sum_{t=1}^{T} e_{t,1}\right]_{1\times H} \tag{28}$$

It follows that $\hat{\boldsymbol{\alpha}}_H$ is a column vector whose element α_h equals the mean value of all h-month-ahead forecast errors.

The covariance matrix $Var\left(\hat{\boldsymbol{\alpha}}_H\right)$ is given by:

$$Var\left(\hat{\boldsymbol{\alpha}}_H\right) = \left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1} \mathbf{M}^{tr}\boldsymbol{\Sigma}\mathbf{M}\left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1} \tag{29}$$

or the equivalent expression below:

$$Var\left(\hat{\boldsymbol{\alpha}}_H\right)=\frac{1}{T^2}\sum_{c=1}^{T}\sum_{l=1}^{T}\boldsymbol{\sigma}_{l,c} \qquad (30)$$

Therefore, $Var\left(\hat{\boldsymbol{\alpha}}_H\right)$ is the $H \times H$ matrix that equals the average of the T^2 submatrices with dimensions $H \times H$ that comprise $\boldsymbol{\Sigma}$. Under the AKO framework $\boldsymbol{\Sigma}=\sigma_u^2\boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is given by (18). Under our framework matrix $\boldsymbol{\Sigma}$ is given by (25). The *t*-statistics regarding the null $H_0:\alpha_h=0$ is obtained by using the (h,h) element of $Var\left(\hat{\boldsymbol{\alpha}}_H\right)$.

We close Section 3 by mentioning that matrix $\boldsymbol{\Sigma}$ is not observable, therefore it must be replaced by an estimate $\hat{\boldsymbol{\Sigma}}$. Under AKO only the variance σ_u^2 is needed, but under our framework we need to estimate the correlation coefficient φ and the variances σ_l^2 and σ_s^2 .[11]

4. Inflation and Output Growth Forecasts in Brazil, Chile, and Mexico

This section presents the data to which we apply the methodologies described in the previous sections. The database comprises inflation and output growth forecasts observed in Brazil, Chile, and Mexico from the beginning of the 2000's until 2020. They are collected monthly and refer to the so-called "market consensus" (i.e., to the mean value of individual forecasts in the case of Brazil and Mexico; and to the median in the case of Chile).[12] Forecasts are made for the values of each variable at the end of the current and the next years. Each year is targeted by 24 consecutive forecasts, whose forecast horizons range from 24 months (in the case of a forecast made in January of year $t-1$ for the value of the variable at the end of year t) to only 1 month (if the forecast is made in December of year t). The measures of inflation and output growth that forecasters try to predict are year-over-year percentage changes. In the case of inflation, the referred measure is expressed in the end of the period, not annual average data. Actual inflation rates come from the April 2021 World Economic Outlook database. Regarding output growth, we follow Dovern and Weisser (2011) and

[11] We discuss how these parameters are estimated in supplemental Appendix 3.

[12] It is worth mentioning that, in the case of Chile, Vereda et al. (2021) do not find any substantial differences between these medians and the consensus forecasts collected from the Focus Economics survey (https://www.focus-economics.com/), which are measured through mean values.

extract the data for a given year from the World Economic Outlook published in April or May of the following year.[13] Forecasts are extracted from the central bank's websites of Brazil, Chile, and Mexico.[14,15]

The sample analyzed for each country depends on the availability of data, as reported below:

- Brazil: data belong to the period between January 2001 and December 2020. The Central Bank of Brazil (CBB) publishes daily series of inflation and output growth forecasts. We collect figures released on the first business day of each month.
- Chile: actuals and forecasts come from the period between January 2005 and December 2020. The Central Bank of Chile (CBC) publishes inflation and output growth forecasts collected during the first ten days of each month.
- Mexico: data come from the period between January 2002 and December 2020.[16] The Bank of Mexico (BM) publishes forecasts of inflation and output growth at the beginning of each month.

Figures 1a, 1c and 1e (1b, 1d and 1f) show the paths of fixed-event forecasts of output growth (inflation) in Brazil, Chile, and Mexico. Actual values are depicted by black squares labeled with the years. The forecast paths are represented by black, gray, and light gray solid lines. The *x*-axis indicates the date and the *y*-axis the growth rates, which are measured in percent per year. One can see that the paths of fixed-event forecasts tend to actual values and that, in most cases, the sequences of 24 forecasts move steadily up or down towards actual outcomes.

In the Chilean case, both market expectations and observed inflation rates fluctuate

[13] According to Dovern and Weisser (2011), "*... for the evaluation of macroeconomic forecasts, it has become standard in the literature to use data from the initial releases rather than revised ex-post data*" (page 455). Therefore, our procedure is an attempt to follow this principle. We have also performed the same estimations using data collected from the World Economic Outlook of October, which is the second release of the year. Our results did not change significantly.

[14] These forecasts are reported by experts from financial institutions, consulting firms, real sector firms and academic institutions. Currently the Brazilian, Mexican, and Chilean surveys follow approximately 130, 30 and 60 regular respondents, respectively.

[15] There are missing values in consensus forecasts of inflation and output growth in all three countries. However, the proportion of missing values is small – indeed, it is less than 1% in Mexico and less than 8% in Chile. Missing values were filled by linear interpolations based on the nearest neighboring observations.

[16] Regarding the Mexican case, we use data observed after the formal implementation of the inflation-targeting regime, which occurred in 2001 (De Pooter et al. (2014)). Since then, the inflation target was fixed at 3% per year.

around 3% per year, which is the long-term target pursued by the CBC. Figure 1f shows that, in the Mexican case, expectations and actuals rover around 4% per year, which is the upper limit of the tolerance band established by Mexican monetary authorities. Consensus forecasts of inflation in Brazil are quite unstable, as it becomes evident in Figure 1b.

Regarding output growth, Figures 1a and 1c show that 2011 marked a significant decrease in consensus forecasts in Brazil and Chile. The fall in commodity prices that took place after 2010 is one of the causes, given that Brazil and Chile depend on the revenues of selling primary products in international markets. Figure 1e shows that Mexico was somewhat spared, maybe due to the existing economic ties with the USA.

Tables 5, 6 and 7 report results regarding forecast errors. Each line refers to a given forecast horizon, which ranges from 24 to 1. Tables 5, 6 and 7 refer to Brazil, Chile, and Mexico, respectively. For now, we are interested in columns 2, 3 and 4 and 7, 8 and 9 of each table. The first group concerns output growth forecasts, while the second refers to inflation forecasts. Columns 2 and 7 report the mean value of errors – in the absence of biases, these figures should be close to zero. Columns 3 and 8 inform the root mean square error (RMSE), while columns 4 and 9 show the mean absolute error (MAE).

Results confirm that the magnitudes of forecast errors shrink as the forecast horizon goes to 1. This outcome is not surprising, since agents improve their forecasts with the passage of time and the arrival of new information. Regarding output growth forecasts, mean errors are usually negative and attain absolute values above 1.5% in all three countries when h goes to 24. Regarding inflation expectations, mean errors are usually positive. They are higher in Brazil, where mean errors increase substantially as h goes to 24 – even going beyond 1%. In Chile and Mexico mean errors are slightly positive for longer forecast horizons, but they go to zero when h goes to 1.

Taken together, these results suggest that agents are initially optimistic about future output growth. Indeed, actual growth rates observed two years later are 1% lower, on average. Inflation expectations are also initially optimistic, since inflation rates observed two years later tend to be higher – in the Brazilian case, by almost 1.5%. Finally, we find evidence that biases may depend on the forecast horizon, since the absolute values of mean errors increase with h. In the next section we reassess these results by applying the methodologies discussed in the previous sections. $r_{t;h-1,h} = F_{t,h-1} - F_{t,h}$

We close this section by analyzing the results displayed in Tables 1, 2 and 3. They show some basic statistics regarding inflation and output growth forecast revisions in Brazil, Chile, and Mexico. The notation used is such that $r_{t;j-1,j}$ ($r_{t+1;j+11,j+12}$) ($j=1,\ldots,12$) denote the revisions pertaining to the target years t and $t+1$ that occur between j ($j+12$) and $j-1$ ($j+11$) periods before the end of year t ($t+1$). In other words, both revisions occur in the same month but focus on different years. The months are identified in the rows of Tables 1, 2 and 3. Variances and correlation coefficients are informed in the columns.

Regarding inflation forecasts, the correlation coefficients between $r_{t;j-1,j}$ and $r_{t+1;j+11,j+12}$ are always less than 1 for all countries. The correlation coefficients observed in Brazil, Chile and Mexico varies between 0.93 and 0.80, 0.92 and 0.77, and 0.97 and 0.78, respectively. Results for output growth are such that correlation coefficients are between 0.72 and 0.18, 0.94 and 0.28, and 0.72 and 0.18, respectively. In sum, inflation forecast revisions pertaining to the next year are quite sensitive to news released during the current year, but output growth forecast revisions are relatively unaffected. Furthermore, the variances of inflation (output growth) forecast revisions regarding the next year are smaller than current year ones. All in all, these results provide some support to the notion that forecast revisions $r_{t;j-1,j}$ and $r_{t+1;j+11,j+12}$ (as well as the informational shocks that affect them) neither have the same variance nor are perfectly correlated.

5. Results of the bias tests

In this section we estimate the constant bias α and the elements of the vector $\boldsymbol{\alpha}_H$, which comprises the biases α_h that depend on the forecast horizon.

We perform the procedures described in Section 3 separately for each country and variable. In the constant case we use the OLS estimator of α, which is given by (11) or (12) . Its robust standard error is given by (15) under the "traditional" error structure and by (19) under the structure that we propose. Regarding the case in which biases may depend on the forecast horizon, the OLS estimator of $\boldsymbol{\alpha}_H$ is given by (27) or (28), being $Var\left(\hat{\boldsymbol{\alpha}}_H\right)$ given by (29) or (30). Expression (29) yields (A.14) under the "traditional" error structure and (A.15) under ours (see supplemental Appendix 3). As we have seen before, our structure is not only more convincing in economic terms, but also displays a better fit to the data – in particular, it conforms well with the fact that informational shocks arriving in each month do not exert the same impact on short- and long-term forecasts.

Table 4 shows results of the common bias case. Columns 2, 3 and 4 (5, 6 and 7) refer to output growth (inflation). Columns 2 and 5 show the estimated biases. Columns identified as "***t-stat-old***" show the t statistics obtained under the "traditional" error structure, while those labeled "***t-stat-new***" show the same statistics obtained under our structure. Results of the case in which biases depend on the forecast horizon are displayed in Tables 5 (Brazil), 6 (Chile) and 7 (Mexico). Columns 5 and 6 (10 and 11) refer to output growth (inflation) forecasts. Columns 5 and 10 report the t statistics obtained under the "traditional" error structure, while columns 6 and 11 report the same statistics calculated under ours.

We begin with the big picture and then move on to a more detailed analysis. First, the error structure we propose usually yields t-statistics of greater magnitude, thus providing a better picture of the accuracy. Second, this greater accuracy increases the probability of finding statistically significant biases, thus leading to more rejections of the null of no biases. Finally, our results show that professional forecasters in Brazil tend to underestimate inflation, being this phenomenon more acute in the "long term" – that is, when forecasts are made for the next year. Regarding output growth, under the "traditional" error structure we find a statistically significant tendency of overestimating future economic growth in Brazil, Chile, and Mexico for various forecast horizons. Under our structure, though, this result is clearer.

We can now perform a more detailed analysis of the constant bias case. Referring to inflation forecasts in Brazil, we reject the null $H_0 : \alpha = 0$ regardless the error structure, but we do not reject the null in Chile and Mexico under both structures. We find that the common bias that affects inflation forecasts in Brazil equals $\hat{\alpha} = 0.92$, meaning that market analysts underestimate actual inflation by almost 1 percentage point. This value is statistically significant at the 10% confidence level under the "traditional" error structure, and at the 5% confidence level under ours.

In the case of output growth forecasts, we reject the null in Brazil and Chile regardless the error structure. We do not reject the null in Mexico under the "traditional" error structure, but we reject the null under our structure. We estimate a common bias of -1.13 in Brazil, which is statistically significant at the 5% confidence level under both structures. We also estimate common biases of -0.96 in Chile and Mexico. Both are statistically significant at the 5% confidence level under our error structure. These figures suggest that professional forecasters tend to overestimate actual growth by almost 1 percentage point in Brazil, Chile, and Mexico.

Now we turn to the case of horizon-specific biases. Regarding output growth and considering the "traditional" error structure, we do not reject the null at the 10% confidence level in Brazil from $h=7$ until $h=1$, in Chile from $h=24$ until $h=22$ and from $h=9$ until $h=1$, and in Mexico from $h=10$ until $h=1$. Under our error structure the null is rejected in Brazil from $h=24$ until $h=8$, in Chile from $h=24$ until $h=8$ and in Mexico $h=24$ until $h=10$. The *t*-statistics are always higher, meaning that rejections of the null of no bias at forecast horizon h often happen at lower confidence levels. Biases may depend on h, being an increasing function of it.

Regarding inflation forecasts and considering the "traditional" error structure, we do not reject the null $H_0 : \alpha_h = 0$ at the 10% confidence level in Chile and Mexico for all forecast horizons. In Brazil the null is rejected from $h=20$ until $h=1$. Under our error structure, we do not reject the null in Chile and Mexico for all forecast horizons, but in Brazil we reject it from $h=24$ until $h = 14$ and between $h=5$ and $h=2$. We confirm that biases may depend on h; indeed, their magnitudes increase with the forecast horizon.

6. Conclusion

Our paper belongs to the literature that studies the unbiasedness and efficiency of fixed-event forecasts. Its main contribution is changing the error characterization put forward by Davies and Lahiri (1995), which was later improved by Ager et al. (2009) to deal with consensus forecasts. The two frameworks share the same feature, namely, that the informational shock occurring h months prior to the end of year t affects the consensus forecasts made for the value of the target variable at the end of years t and $t+1$ by exactly the same amount. We have shown that this feature is not only unreasonable in economic terms, but it also fails to match real data. Therefore, we propose a new framework in which two types of informational shocks arrive each month: the first refers to the value of the target variable at the end of the current year, while the second concerns its value at the end of the next year. We assume that both shocks are correlated, but they neither have the same information nor are equally "powerful". We believe the error covariance matrix arising from these assumptions is more realistic and better grounded theoretically. Consequently, the results of the bias tests become more reliable.

Our framework and the "traditional" one proposed by DL and AKO are used to analyze consensus forecasts of inflation and output growth collected by the central banks of Brazil, Chile, and Mexico from the beginning of the 2000's until 2020. Observations are collected monthly and refer to the figures prevailing in the end of the current and the following years. Results suggest that our error structure delivers a less pessimistic picture of the uncertainty surrounding parameter estimates. It also leads to more (and stronger) rejections of the null of no biases in macroeconomic forecasts, especially when assessing projections made many months before the end of the target year. It seems that our characterization of forecast errors is not only more realistic, but it also has some tangible implications.

We leave to future research the task of revisiting our general conclusions using data from other countries and projections for other economic and financial variables. One could reassess the issue of comparing biases in developed and emerging economies using our procedure. Another contribution would be further improving the characterization of errors by considering, for example, that the impact of news may depend on the month in which they are released. Indeed, the news released in December of the current year may have little effect on the forecasts made for this year, but they may exert a sizeable impact on those made for the next. We deem important, though, that new characterizations are as parsimonious as those analyzed in this paper.

References

Ager, Philipp, Marcus Kappler, and Steffen Osterloh. (2009) "The accuracy and efficiency of the Consensus Forecasts: A further application and extension of the pooled approach." *International Journal of Forecasting*, 25 (1), 167-181.

Baghestani, Hamid, and Cassia Marchon. (2015) "On the accuracy of private forecasts of inflation and growth in Brazil." *Journal of Economics and Finance*, 39 (2), 370-381.

Bakhshi, Hasan, George Kapetanios, and Yates Tony. (2005) "Rational expectations and fixed-event forecasts: An application to UK inflation." *Empirical Economics*, 30 (3), 539-

553.
Batchelor, Roy. (2007), "Bias in macroeconomic forecasts." *International Journal of Forecasting*, 23 (2), 189-203.
Boero, Gianna, Jeremy Smith, and Kenneth F. Wallis. (2008) "Evaluating a three-dimensional panel of point forecasts: The Bank of England Survey of External Forecasters." *International Journal of Forecasting*, 24 (3), 354-367.
Capistrán, Carlos, and Gabriel López-Moctezuma. (2014) "Forecast revisions of Mexican inflation and GDP growth." *International Journal of Forecasting*, 30 (2), 177-191.
Carvalho, Fabia A., and Mauricio S. Bugarin. (2006) "Inflation Expectations in Latin America." *Economía*, 6 (2), 101-145.
Chen, Qiwei, Mauro Costantini, and Bruno Deschamps. (2016) "How accurate are the professional forecasts in Asia? Evidence from ten countries." *International Journal of Forecasting*, 32 (1), 154-167.
Clements, Michael P., Fred Joutz, and Herman O. Stekler. (2007) "An Evaluation of the Forecasts of the Federal Reserve: a Pooled Approach." *Journal of Applied Econometrics*, 22, 121-136.
Davies, Anthony, and Kajal Lahiri. (1995) "A new framework for analyzing survey forecasts using three-dimensional panel data." *Journal of Econometrics*, 68, 205-227.
de Deus, Joseph D.B.V., and Helder F. de Mendonça (2017). "Fiscal forecasting performance in an emerging economy: An empirical assessment of Brazil." *Economic Systems*, 41 (3), 408-419.
De Pooter, Michiel, Patrice Robitaille, Ian Walker, and Michael Zdinak. (2014) "Are Long-Term Inflation Expectations Well Anchored in Brazil, Chile, and Mexico?" *International Journal of Central Banking*, 10 (2), 337-400.
Dovern, Jonas, Ulrich Fritsche, Prakash Loungani, and Natalia Tamirisa. (2015) "Information rigidities: Comparing average and individual forecasts for a large international panel." *International Journal of Forecasting*, 31 (1), 144-154.
Dovern, Jonas, and Johannes Weisser. (2011) "Accuracy, Unbiasedness and Efficiency of Professional Macroeconomic Forecasts: An Empirical Comparison for the G7." *International Journal of Forecasting*, 27 (2), 452-65.
Glas, A., and Matthias Hartmann. (2016). "Inflation uncertainty, disagreement and monetary policy: evidence from the ECB Survey of Professional Forecasters." *Journal of Empirical Finance*, 39(B), 215-228.
Ince, Onur, and Tanya Molodtsova. (2017) "Rationality and forecasting accuracy of exchange rate expectations: Evidence from survey-based forecasts." *Journal of International Financial Markets, Institutions & Money*, 47, 131-151.
Isiklar, Gultekin, Kajal Lahiri, and Prakash Loungani. (2006) "How quickly do forecasters incorporate news: Evidence from cross-country surveys." *Journal of Applied Econometrics*, 21 (6), 703-725.
Isiklar, Gultekin, and Kajal Lahiri. (2007) "How far ahead can we forecast? Evidence from cross-country surveys." *International Journal of Forecasting*, 23, 167-187.
Jalles, João T. (2017a) "Inflation Forecasts' Performance in Latin America." *Review of Development Finance*, 7, 157-168.
Jalles, João T. (2017b) "On the rationality and efficiency of inflation forecasts: Evidence from advanced and emerging market economies." *Research in International Business and Finance*, 40, 175-189.
Jalles, João T, Iskander Karibzhanov, and Prakash Loungani. (2015) "Cross-country

evidence on the quality of private sector fiscal forecasts." *Journal of Macroeconomics*, 45, 186-201.

Keane, Michael P., and David E. Runkle. (1990) "Testing the rationality of price forecasts: New evidence from panel data." *American Economic Review*, 80 (4), 714-735.

Lahiri, K., and Xuguang Sheng. (2010). "Measuring forecast uncertainty by disagreement: the missing link". *Journal of Applied Econometrics*, 25, 514-538.

Loungani, Prakash. (2001) "How accurate are private sector forecasts? Cross-country evidence from consensus forecasts of output growth." *International Journal of Forecasting*, 17 (3), 419-432.

Loungani, Prakash, Herman Stekler, and Natalia Tamirisa. (2013) "Information rigidity in growth forecasts: Some cross-country evidence." *International Journal of Forecasting*, 29 (4), 605-621.

Mincer, Jacob, and Victor Zarnowitz. (1969) "The evaluation of economic forecasts." In J. Mincer (ed.), *Economic Forecasts and Expectations* (New York: Columbia University Press).

Mitchell, Karlyn, and Douglas K. Pearce. (2007) "Professional forecasts of interest rates and exchange rates: Evidence from the Wall Street Journal's panel of economists." *Journal of Macroeconomics*, 29 (4), 840-854.

Nordhaus, William D. (1987) "Forecasting efficiency: concepts and applications." *The Review of Economics and Statistics*, 69 (4), 667–674.

Öller, Lars-Erik, and Bharat Barot. (2000) "The Accuracy of European Growth and Inflation Forecasts." *International Journal of Forecasting*, 16, 293-215.

Vereda, Luciano, João Savignon, and Tarciso G. da Silva. (2021) "A new method to assess the degree of information rigidity using fixed-event forecasts". *International Journal of Forecasting*, 37(4), 1576-1589.

Table 1

Variances and correlation coefficients of inflation and output growth forecast revisions in Brazil

	Output Growth			Inflation		
	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$
January	3.02	0.94	0.91	2.67	0.82	0.90
February	3.38	1.05	0.92	3.08	0.95	0.92
March	3.80	1.09	0.92	3.91	1.02	0.93
April	4.86	1.17	0.89	4.15	1.02	0.91
May	6.87	1.17	0.73	4.48	1.01	0.89
June	8.68	1.09	0.62	4.44	0.93	0.87
July	8.85	1.17	0.58	4.24	0.86	0.84
August	8.36	1.31	0.60	3.74	0.75	0.80
September	8.27	1.48	0.60	3.65	0.72	0.80
October	8.71	1.82	0.58	3.95	0.81	0.84
November	8.74	2.17	0.57	4.10	1.45	0.80
December	8.75	2.47	0.56	4.85	2.28	0.84

Notes**:** This table shows the variances and correlation coefficients of inflation and output growth forecast revisions in Brazil. Observations come from the period between the early 2000's and 2020. The lines identify the months in which forecast revisions occur. Columns identified by $Var\left(r_{t;j-1,j}\right)$ ($Var\left(r_{t+1;j+11,j+12}\right)$) show the variances of forecast revisions regarding the current year (next year). Lines identified by $Corr\left(r_{t;j-1,j}, r_{t+1;j+11,j+12}\right)$ inform the correlation coefficients between both types of revisions.

Table 2

Variances and correlation coefficients of inflation and output growth forecast revisions in Chile

	Output Growth			Inflation		
	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$
January	2.37	1.05	0.94	0.22	0.01	0.81
February	2.59	1.23	0.93	0.23	0.01	0.83
March	3.01	1.31	0.92	0.44	0.04	0.81
April	5.47	1.14	0.84	0.51	0.04	0.77
May	6.05	1.17	0.77	0.67	0.03	0.84
June	8.15	1.07	0.63	1.13	0.04	0.82
July	10.13	1.25	0.48	2.21	0.16	0.92
August	10.20	1.33	0.50	2.96	0.16	0.89
September	9.72	1.27	0.46	3.71	0.25	0.89
October	9.47	1.37	0.38	3.98	0.21	0.89
November	9.47	1.62	0.33	4.87	0.20	0.89
December	10.02	1.99	0.28	5.85	0.11	0.88

Notes**:** This table shows the variances and correlations of inflation and output growth forecast revisions in Chile. The comments made for Table 1 also apply here.

Table 3

Variances and correlation coefficients of inflation and output growth forecast revisions in Mexico

	Output Growth			Inflation		
	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$	$Var(r_{t;j-1,j})$	$Var(r_{t+1;j+11,j+12})$	$Corr(r_{t;j-1,j}, r_{t+1;j+11,j+12})$
January	1.63	0.70	0.72	2.97	1.74	0.97
February	2.02	0.75	0.71	2.63	1.67	0.96
March	4.86	0.76	0.75	2.18	1.43	0.95
April	7.98	0.67	0.60	2.13	1.46	0.93
May	10.24	0.62	0.55	1.97	1.12	0.93
June	11.53	0.60	0.48	1.97	1.09	0.91
July	13.35	0.59	0.40	1.80	1.08	0.90
August	13.82	0.53	0.33	1.80	0.99	0.86
September	13.81	0.54	0.30	1.85	0.94	0.86
October	13.55	0.81	0.26	1.88	0.86	0.85
November	13.37	1.04	0.23	2.14	0.88	0.81
December	13.41	1.21	0.18	2.23	0.84	0.78

Notes**:** This table shows the variances and correlations of inflation and output growth forecast revisions in Mexico. The comments made for Table 1 also apply here.

Table 4

Inflation and Output Growth Forecasts ($H_0 : \alpha = 0$)

	Output Growth			Inflation		
	bias	t-stats old	t-stats new	bias	t-stats old	t-stats new
Brazil	-1.125	-2.22	-2.54	0.916	1.91	1.99
Chile	-0.960	-1.70	-2.08	0.203	0.42	0.46
Mexico	-0.956	-1.58	-2.13	0.223	1.00	1.18

Notes: This table shows the results of testing for the presence of a common bias α . The columns denominated "bias" report the estimated bias $\hat{\alpha}$, which we measure in % per year. The columns identified as "*t-stats old*" report the t statistics calculated when the error structure follows the "traditional" framework of Davies and Lahiri (1995) and Ager et al. (2009). The columns identified as "*t-stats new*" show the t statistics derived from our structure. Cells having a light gray (dark gray, black) background indicate rejection of the null $H_0 : \alpha = 0$ at the 10% (5%, 1%) significance level.

Table 5

Brazil - inflation and output growth forecasts ($H_0 : \alpha_h = 0$)

	Output Growth					Inflation				
h	Mean Error	RMSE	MAE	t-stats old	t-stats new	Mean Error	RMSE	MAE	t-stats old	t-stats new
24	-1.95	3.40	2.88	-2.15	-2.54	1.36	2.86	1.90	1.55	1.97
23	-1.93	3.38	2.89	-2.21	-2.59	1.32	2.89	1.92	1.55	1.95
22	-1.88	3.35	2.87	-2.23	-2.61	1.28	2.88	1.90	1.56	1.93
21	-1.87	3.35	2.88	-2.29	-2.67	1.27	2.87	1.86	1.60	1.95
20	-1.86	3.30	2.83	-2.38	-2.76	1.27	2.83	1.83	1.67	1.99
19	-1.80	3.20	2.72	-2.40	-2.76	1.26	2.79	1.80	1.73	2.02
18	-1.70	3.08	2.60	-2.37	-2.71	1.26	2.75	1.80	1.81	2.05
17	-1.61	2.96	2.48	-2.37	-2.69	1.25	2.68	1.77	1.89	2.09
16	-1.54	2.85	2.39	-2.39	-2.68	1.21	2.57	1.71	1.95	2.08
15	-1.51	2.74	2.29	-2.51	-2.78	1.13	2.43	1.61	1.94	1.99
14	-1.39	2.60	2.14	-2.49	-2.71	0.98	2.26	1.47	1.82	1.76
13	-1.32	2.52	2.05	-2.59	-2.75	0.88	2.28	1.52	1.77	1.62
12	-1.24	2.43	1.96	-2.72	-2.80	0.86	2.32	1.54	1.94	1.64
11	-1.21	2.37	1.90	-2.77	-2.86	0.79	2.31	1.51	1.87	1.57
10	-1.10	2.26	1.80	-2.64	-2.72	0.75	2.29	1.50	1.86	1.57
9	-0.82	1.86	1.53	-2.08	-2.14	0.72	2.18	1.40	1.88	1.58
8	-0.66	1.59	1.25	-1.77	-1.82	0.69	2.12	1.36	1.91	1.61
7	-0.41	1.47	1.17	-1.18	-1.22	0.62	2.05	1.26	1.84	1.55
6	-0.30	1.32	1.00	-0.94	-0.97	0.59	1.90	1.13	1.88	1.59
5	-0.25	1.09	0.85	-0.85	-0.88	0.61	1.73	0.99	2.13	1.79
4	-0.17	0.83	0.66	-0.66	-0.68	0.61	1.60	0.91	2.37	2.00
3	-0.18	0.67	0.54	-0.77	-0.79	0.60	1.48	0.80	2.69	2.27
2	-0.16	0.63	0.52	-0.87	-0.90	0.45	1.11	0.67	2.46	2.08
1	-0.11	0.51	0.38	-0.86	-0.89	0.23	0.53	0.35	1.83	1.54

Notes: This table shows mean errors, RMSEs, MAEs and t statistics regarding the null $H_0 : \alpha_h = 0$ as a function of the forecast horizon h in the Brazilian case. The columns denominated "Mean Error" report the estimated bias $\hat{\alpha}_h$, which is the mean value of h-month-ahead forecast errors. Biases are measured in % per year. The columns denominated "RMSE" and "MAE" report two common measures of forecast performance, the root mean squared error and the mean absolute error. The columns identified as "*t-stats old*" report the t statistics calculated when the error structure is the "traditional" one (Davies and Lahiri (1995), Ager et al. (2009)). The columns identified as "*t-stats new*" report the t statistics that come from our error structure. Cells having a light gray (dark gray, black) background indicate rejection of the null $H_0 : \alpha_h = 0$ at the 10% (5%, 1%) significance level.

Table 6

Chile - inflation and output growth forecasts ($H_0 : \alpha_h = 0$)

	Output Growth					Inflation				
h	Mean Error	RMSE	MAE	t-stats old	t-stats new	Mean Error	RMSE	MAE	t-stats old	t-stats new
24	-1.53	3.26	2.28	-1.50	-1.69	0.34	2.17	1.46	0.38	0.51
23	-1.56	3.24	2.25	-1.58	-1.78	0.35	2.16	1.46	0.41	0.53
22	-1.51	3.17	2.21	-1.57	-1.78	0.31	2.22	1.50	0.38	0.49
21	-1.52	3.18	2.21	-1.65	-1.87	0.31	2.22	1.50	0.39	0.50
20	-1.54	3.16	2.20	-1.73	-1.97	0.33	2.21	1.49	0.42	0.53
19	-1.51	3.13	2.15	-1.78	-2.03	0.30	2.22	1.47	0.40	0.49
18	-1.42	3.05	2.08	-1.75	-2.01	0.27	2.34	1.55	0.38	0.46
17	-1.37	3.03	2.06	-1.78	-2.05	0.26	2.31	1.51	0.39	0.45
16	-1.38	2.99	2.00	-1.90	-2.21	0.23	2.38	1.55	0.36	0.40
15	-1.35	2.85	1.88	-1.98	-2.32	0.24	2.34	1.54	0.41	0.44
14	-1.19	2.57	1.71	-1.89	-2.24	0.26	2.33	1.54	0.47	0.48
13	-1.09	2.34	1.59	-1.88	-2.27	0.31	2.19	1.48	0.61	0.59
12	-1.04	2.23	1.51	-2.00	-2.48	0.30	2.09	1.43	0.66	0.59
11	-1.01	2.16	1.40	-2.04	-2.53	0.31	2.04	1.40	0.73	0.65
10	-0.92	2.06	1.27	-1.94	-2.41	0.18	1.94	1.36	0.45	0.40
9	-0.69	1.34	1.01	-1.55	-1.92	0.21	1.84	1.28	0.53	0.48
8	-0.63	1.19	0.92	-1.49	-1.85	0.23	1.69	1.18	0.62	0.55
7	-0.42	0.83	0.70	-1.05	-1.31	0.25	1.47	1.05	0.73	0.65
6	-0.31	0.73	0.55	-0.85	-1.05	0.15	1.19	0.88	0.46	0.41
5	-0.29	0.66	0.47	-0.85	-1.06	0.07	1.03	0.80	0.24	0.22
4	-0.31	0.60	0.41	-1.04	-1.29	-0.04	0.86	0.69	-0.14	-0.12
3	-0.24	0.50	0.34	-0.93	-1.16	-0.07	0.79	0.66	-0.32	-0.29
2	-0.18	0.37	0.26	-0.85	-1.06	-0.10	0.83	0.67	-0.56	-0.50
1	-0.06	0.27	0.18	-0.37	-0.46	-0.13	0.92	0.70	-1.03	-0.92

Notes: This table shows the same information of Table 5, but regarding the Chilean case. The comments made for Table 5 also apply here.

Table 7

Mexico - inflation and output growth forecasts ($H_0 : \alpha_h = 0$)

	Output Growth					Inflation				
h	Mean Error	RMSE	MAE	t-stats old	t-stats new	Mean Error	RMSE	MAE	t-stats old	t-stats new
24	-1.87	3.76	2.43	-1.73	-2.11	0.33	1.22	0.79	0.82	1.07
23	-1.86	3.76	2.42	-1.78	-2.17	0.35	1.21	0.79	0.89	1.16
22	-1.81	3.73	2.42	-1.79	-2.19	0.35	1.20	0.76	0.91	1.17
21	-1.77	3.69	2.41	-1.82	-2.23	0.35	1.20	0.75	0.94	1.20
20	-1.79	3.67	2.39	-1.91	-2.35	0.37	1.20	0.74	1.04	1.31
19	-1.75	3.64	2.36	-1.95	-2.40	0.37	1.20	0.74	1.10	1.36
18	-1.73	3.56	2.31	-2.02	-2.50	0.39	1.20	0.76	1.20	1.46
17	-1.64	3.47	2.23	-2.02	-2.50	0.37	1.20	0.78	1.20	1.43
16	-1.52	3.37	2.16	-1.99	-2.47	0.32	1.18	0.75	1.11	1.30
15	-1.30	3.09	1.97	-1.81	-2.27	0.29	1.17	0.78	1.08	1.22
14	-1.18	2.96	1.88	-1.77	-2.23	0.22	1.11	0.78	0.86	0.94
13	-1.16	2.90	1.84	-1.89	-2.41	0.20	1.10	0.80	0.87	0.91
12	-1.07	2.73	1.70	-1.95	-2.52	0.18	0.93	0.69	0.87	0.86
11	-1.03	2.59	1.58	-1.96	-2.53	0.16	0.91	0.66	0.82	0.80
10	-0.68	1.59	1.18	-1.37	-1.77	0.13	0.89	0.68	0.67	0.66
9	-0.48	1.14	0.92	-1.02	-1.32	0.18	0.84	0.65	0.98	0.96
8	-0.31	0.81	0.64	-0.69	-0.90	0.20	0.80	0.62	1.20	1.18
7	-0.23	0.77	0.61	-0.56	-0.72	0.20	0.71	0.57	1.25	1.23
6	-0.15	0.86	0.66	-0.38	-0.49	0.16	0.65	0.56	1.09	1.07
5	0.03	0.63	0.48	0.10	0.12	0.13	0.53	0.48	0.94	0.92
4	0.07	0.58	0.44	0.23	0.30	0.08	0.48	0.41	0.67	0.66
3	0.10	0.45	0.35	0.38	0.49	0.04	0.42	0.32	0.39	0.38
2	0.09	0.31	0.24	0.42	0.55	-0.01	0.27	0.22	-0.09	-0.08
1	0.08	0.29	0.23	0.53	0.69	-0.02	0.19	0.15	-0.28	-0.27

Notes: This table shows the same information of Table 5, but regarding the Mexican case. The comments made for Table 5 also apply here.

Figure 1a

Brazil - output growth

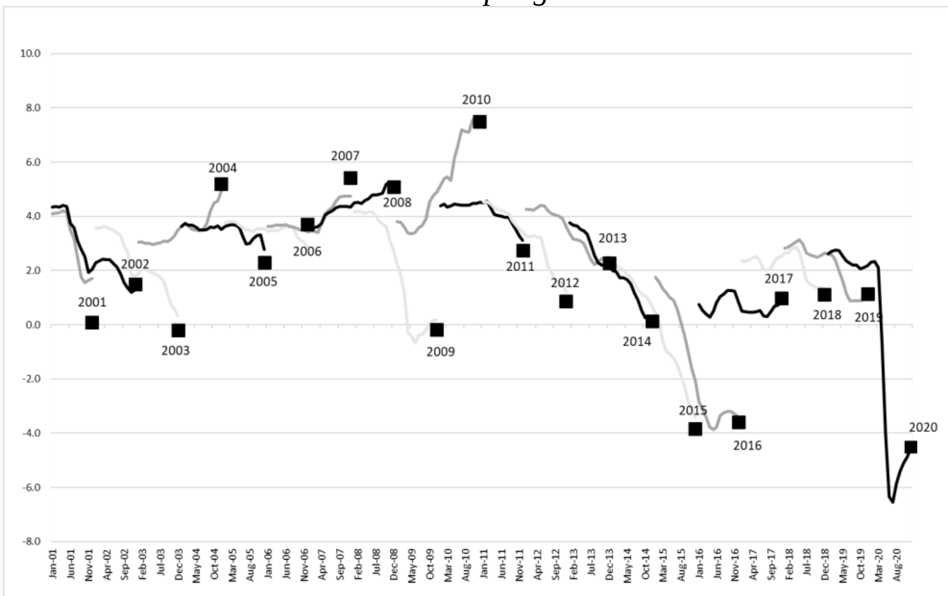
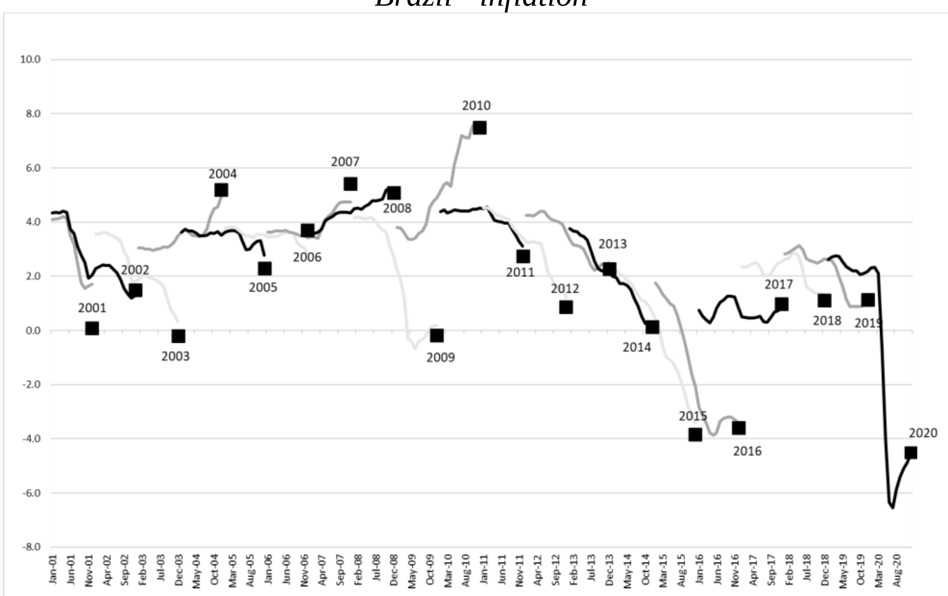


Figure 1b

Brazil - inflation



Notes to Figures 1a and 1b: These figures show the paths of consensus forecasts of output growth (1a) and inflation (1b) in Brazil from 2001 until 2020, comparing them with realized values at the end of each year. Actual values are depicted by black squares labeled with the years. The forecast paths are represented by black, gray, and light gray solid lines. The *x*-axis indicates the date and the *y*-axis the growth rates, which are measured in percent per year.

Figure 1c

Chile - output growth

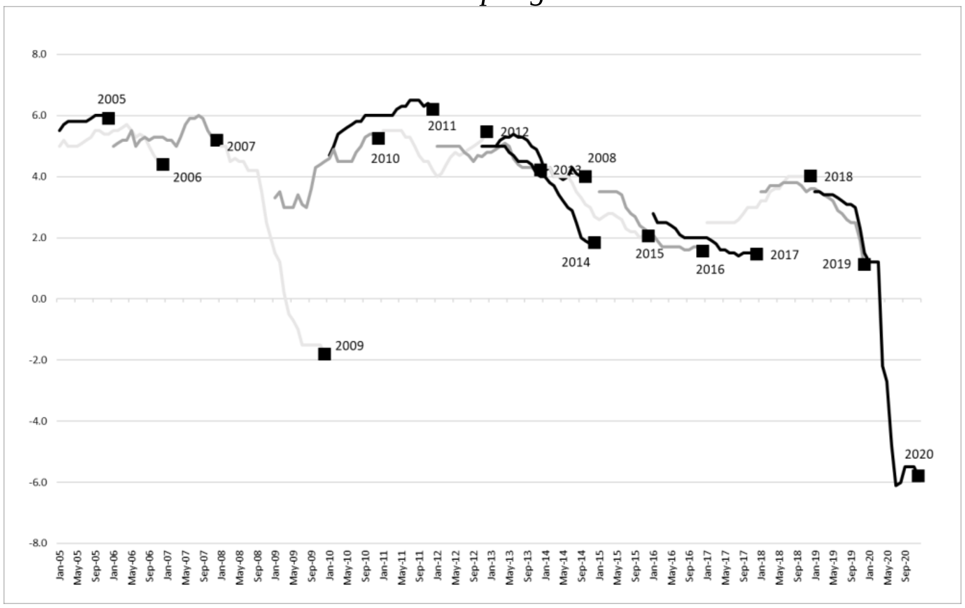
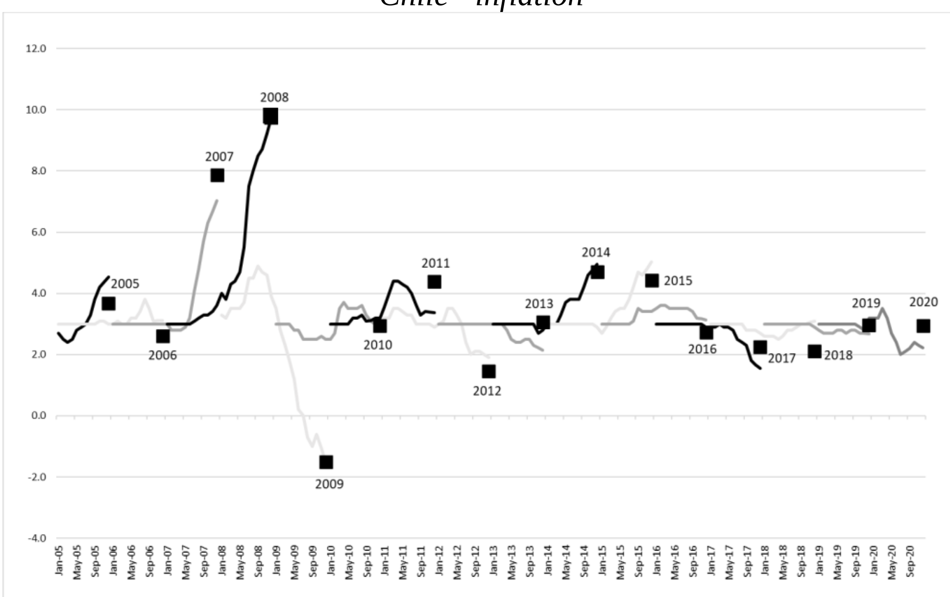


Figure 1d

Chile - inflation



Notes: Figures 1c and 1d show the paths of consensus forecasts of output growth and inflation in Chile from 2005 until 2020, comparing them with realized values at the end of each year. The comments made for Figures 1a and 1b also apply.

Figure 1e

Mexico - output growth

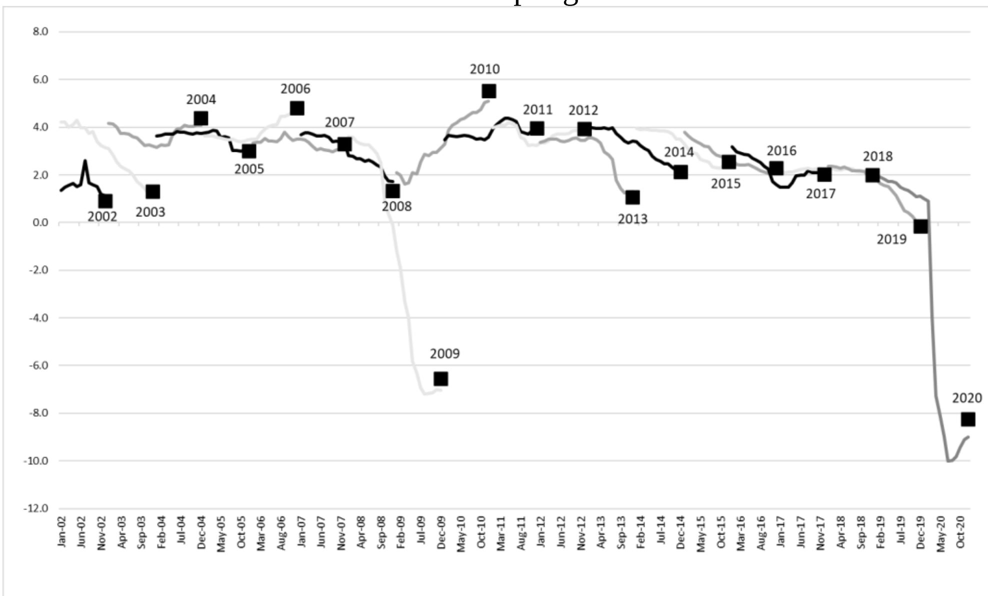
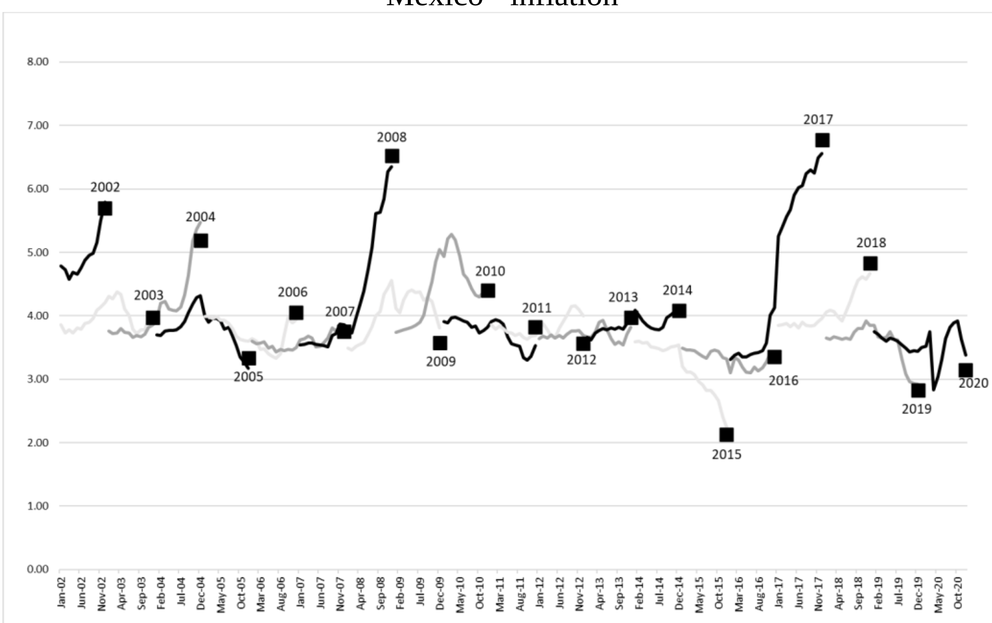


Figure 1f

Mexico - inflation



Notes: Figures 1e and 1f show the paths of consensus forecasts of output growth and inflation in Mexico from 2002 until 2020, comparing them with realized values at the end of each year. The comments made for Figures 1a and 1b also apply.

Supplemental Appendix

APPENDIX 1

In this appendix we show that if (i) there are two types of shocks $u_{t,j}^{s}$ and $u_{t+1,j+12}^{l}$; (ii) each one has its own variance, i.e., $Var\left(u_{t,j}^{s}\right)=\sigma_s^2$ and $Var\left(u_{t+1,j+12}^{l}\right)=\sigma_l^2$; (iii) if shocks $u_{t,j}^{s}$ and $u_{t+1,j+12}^{l}$ are not perfectly correlated, that is, $Corr\left(u_{t,j}^{s},u_{t+1,j+12}^{l}\right)=\varphi<1$; and (iv) the term $\lambda_{t,h}$ obeys (7), then $Corr\left(r_{t;h-1,h},r_{t+1;h+11,h+12}\right)=\varphi$ and the covariance between two typical forecast errors is given by (9).

Under this framework and supposing $h\leq 12$, one can use expressions (3) and (7) in the main paper to show that:

- $F_{t,h}=A_t-\alpha_h-\lambda_{t,h}$ and $\lambda_{t,h}$ equals $\sum_{j=1}^{h}u_{t,j}^{s}$;
- $F_{t,h-1}=A_t-\alpha_{h-1}-\lambda_{t,h-1}$ and $\lambda_{t,h-1}$ equals $\sum_{j=1}^{h-1}u_{t,j}^{s}$;
- $F_{t+1,h+12}=A_{t+1}-\alpha_{h+12}-\lambda_{t+1,h+12}$ and $\lambda_{t+1,h+12}$ equals $\sum_{j=13}^{h+12}u_{t+1,j}^{l}+\sum_{j=1}^{12}u_{t+1,j}^{s}$;
- $F_{t+1,h+11}=A_{t+1}-\alpha_{h+11}-\lambda_{t+1,h+11}$ and $\lambda_{t+1,h+11}$ equals $\sum_{j=13}^{h+11}u_{t+1,j}^{l}+\sum_{j=1}^{12}u_{t+1,j}^{s}$).

We use the results above to calculate revisions $r_{t;h-1,h}$ and $r_{t+1;h+11,h+12}$ as following:

$$r_{t;h-1,h}=F_{t,h-1}-F_{t,h}=A_t-\alpha_{h-1}-\lambda_{t,h-1}-A_t+\alpha_h+\lambda_{t,h}=\dots$$
$$\dots=\alpha_h-\alpha_{h-1}+\sum_{j=1}^{h}u_{t,j}^{s}-\sum_{j=1}^{h-1}u_{t,j}^{s}=\alpha_h-\alpha_{h-1}+u_{t,h}^{s}$$

$$r_{t+1;h+11,h+12}=F_{t+1,h+11}-F_{t+1,h+12}=A_{t+1}-\alpha_{h+11}-\lambda_{t+1,h+11}-A_{t+1}+\alpha_{h+12}+\lambda_{t+1,h+12}=\dots$$
$$\dots=\alpha_{h+12}-\alpha_{h+11}+\sum_{j=13}^{h+12}u_{t+1,j}^{l}+\sum_{j=1}^{12}u_{t+1,j}^{s}-\sum_{j=13}^{h+11}u_{t+1,j}^{l}-\sum_{j=1}^{12}u_{t+1,j}^{s}=\alpha_{h+12}-\alpha_{h+11}+u_{t+1,h+12}^{l}$$

This implies that:

$$Corr\left(r_{t;h-1,h}, r_{t+1;h+11,h+12}\right) = Corr\left(\alpha_h - \alpha_{h-1} + u^s_{t,h}, \alpha_{h+12} - \alpha_{h+11} + u^l_{t+1,h+12}\right) = \dots$$

$$\dots = Corr\left(u^s_{t,h}, u^l_{t+1,h+12}\right) = \varphi .$$

Regarding the covariance between two typical forecast errors, first suppose that the target year is t (that is, $\tilde{t} = t$), $h \le 12$ and $\tilde{h} \le 12$. In this case we calculate $Cov\left(e_{t,h}, e_{t,\tilde{h}}\right)$ as following:

$$Cov\left(e_{t,h}, e_{t,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t,\tilde{h}}\right) = Cov\left(\sum_{j=1}^{h} u^s_{t,j}, \sum_{j=1}^{\tilde{h}} u^s_{t,j}\right) = \min\left(h, \tilde{h}\right)\sigma^2_s$$

The last equality comes from the fact that both sums contain either the sequence of i.i.d. shocks $u^s_{t,h}, u^s_{t,h-1}, \dots, u^s_{t,1}$ (if $h \le \tilde{h}$), or the sequence of i.i.d. shocks $u^s_{t,\tilde{h}}, u^s_{t,\tilde{h}-1}, \dots, u^s_{t,1}$ (if $h > \tilde{h}$).

Now suppose that $h \le 12$ and $\tilde{h} \ge 13$. In this case $Cov\left(e_{t,h}, e_{t,\tilde{h}}\right)$ is calculated as following:

$$Cov\left(e_{t,h}, e_{t,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t,\tilde{h}}\right) = Cov\left(\sum_{j=1}^{h} u^s_{t,j}, \sum_{j=13}^{\tilde{h}} u^l_{t,j} + \sum_{j=1}^{12} u^s_{t,j}\right) = h\sigma^2_s$$

The last equality comes from the fact that the two arguments of the covariance operator contain the same sequence of i.i.d. shocks $u^s_{t,h}, u^s_{t,h-1}, \dots, u^s_{t,1}$.

Now suppose that $h \ge 13$ and $\tilde{h} \le 12$. In this case $Cov\left(e_{t,h}, e_{t,\tilde{h}}\right)$ comes from:

$$Cov\left(e_{t,h}, e_{t,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t,\tilde{h}}\right) = Cov\left(\sum_{j=13}^{h} u^l_{t,j} + \sum_{j=1}^{12} u^s_{t,j}, \sum_{j=1}^{\tilde{h}} u^s_{t,j}\right) = \tilde{h}\sigma^2_s$$

The last equality comes from the fact that the two arguments of the covariance operator contain the same sequence of i.i.d. shocks $u^s_{t,\tilde{h}}, u^s_{t,\tilde{h}-1}, \dots, u^s_{t,1}$.

If h and $\tilde{h}$ are either greater than or equal to 13, then $Cov\left(e_{t,h}, e_{t,\tilde{h}}\right)$ is evaluated as follows:

$$Cov\left(e_{t,h}, e_{t,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t,\tilde{h}}\right) = Cov\left(\sum_{j=13}^{h} u^l_{t,j} + \sum_{j=1}^{12} u^s_{t,j}, \sum_{j=13}^{\tilde{h}} u^l_{t,j} + \sum_{j=1}^{12} u^s_{t,j}\right) = \dots$$
$$\dots = 12\sigma_s^2 + \min\left(h-12, \tilde{h}-12\right)\sigma_l^2$$

The last equality comes from the fact that the two arguments of the covariance operator either have the sequence of i.i.d. shocks $u^l_{t,h}, u^l_{t,h-1}, \dots, u^l_{t,13}, u^s_{t,12}, u^s_{t,11}, \dots, u^s_{t,1}$ (if $h \le \tilde{h}$), or the sequence of i.i.d. shocks $u^l_{t,\tilde{h}}, u^l_{t,\tilde{h}-1}, \dots, u^l_{t,13}, u^s_{t,12}, u^s_{t,11}, \dots, u^s_{t,1}$ (if $h > \tilde{h}$).

In the case of two adjacent target years (t and $\tilde{t} = t+1$), $h < 12$ and $\tilde{h} > 12$, the covariance between two typical forecast errors would be:

$$Cov\left(e_{t,h}, e_{t+1,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t+1,\tilde{h}}\right) = Cov\left(\sum_{j=1}^{h} u^s_{t,j}, \sum_{j=13}^{\tilde{h}} u^l_{t+1,j} + \sum_{j=1}^{12} u^s_{t+1,j}\right) = \dots$$
$$\dots = Cov\left(\sum_{j=1}^{h} u^s_{t,j}, \sum_{j=13}^{\tilde{h}} u^l_{t+1,j}\right) = \min\left(h, \tilde{h}-12\right)\varphi\sigma_s\sigma_l$$

This result comes from the facts that $\sum_{j=1}^{h} u^s_{t,j}$ contains the sequence of i.i.d. shocks $u^s_{t,h}, u^s_{t,h-1}, \dots, u^s_{t,1}$, and $\sum_{j=13}^{\tilde{h}} u^l_{t+1,j}$ contains the sequence of i.i.d. shocks $u^l_{t+1,\tilde{h}}, u^l_{t+1,\tilde{h}-1}, \dots, u^l_{t+1,13}$. It also results from the fact that, if $h \le \tilde{h}-12$, then the pairs $\left(u^s_{t,h}, u^l_{t+1,12+h}\right), \left(u^s_{t,h-1}, u^l_{t,11+h}\right), \dots, \left(u^s_{t,1}, u^l_{t,13}\right)$ are common to both summations and are correlated. Similarly, if $h > \tilde{h}-12$, then the pairs $\left(u^s_{t,\tilde{h}-12}, u^l_{t+1,\tilde{h}}\right), \left(u^s_{t,\tilde{h}-13}, u^l_{t+1,\tilde{h}-1}\right), \dots, \left(u^s_{t,1}, u^l_{t+1,13}\right)$ are common to both summations and are correlated.

The same result would apply in the case of two adjacent target years (t and $t+1$), $h>12$ and $\tilde{h}>12$. The proof follows below:

$$Cov\left(e_{t,h}, e_{t+1,\tilde{h}}\right) = Cov\left(\lambda_{t,h}, \lambda_{t+1,\tilde{h}}\right) = Cov\left(\sum_{j=13}^{h} u_{t,j}^{l} + \sum_{j=1}^{12} u_{t,j}^{s}, \sum_{j=13}^{\tilde{h}} u_{t+1,j}^{l} + \sum_{j=1}^{12} u_{t+1,j}^{s}\right) = \ldots$$

$$\ldots = Cov\left(\sum_{j=1}^{12} u_{t,j}^{s}, \sum_{j=13}^{\tilde{h}} u_{t+1,j}^{l}\right) = \min\left(h, \tilde{h}-12\right)\varphi\sigma_s\sigma_l$$

The last equality comes from the fact that the pairs $\left(u_{t,\tilde{h}-12}^{s}, u_{t+1,\tilde{h}}^{l}\right), \left(u_{t,\tilde{h}-13}^{s}, u_{t+1,\tilde{h}-1}^{l}\right), \ldots, \left(u_{t,1}^{s}, u_{t+1,13}^{l}\right)$ (which are all inside $\sum_{j=1}^{12} u_{t,j}^{s}$ and $\sum_{j=13}^{\tilde{h}} u_{t,j}^{l}$) are correlated. It also reflects the fact that there are $\tilde{h}-12$ such pairs and that $\tilde{h}-12 = \min\left(h, \left(\tilde{h}-12\right)\right)$ if $h>12$ and $\tilde{h}>12$.

In all other cases, the summations inside the covariance operator would not contain shocks occurring at the same time and $Cov\left(e_{t,h}, e_{\tilde{t},\tilde{h}}\right)$ would be zero.

APPENDIX 2

It is straightforward to show that results (11) and (12) in the main paper are equivalent. First notice that $\mathbf{i}_{TH}^{tr}\mathbf{i}_{TH}$ equals TH, that its inverse equals $1/TH$ and that $\mathbf{i}_{TH}^{tr}\mathbf{e} = \sum_{t=1}^{T}\sum_{h=1}^{H} e_{t,h}$, where we define vector $\mathbf{e}$ as $\mathbf{e}^{tr} \equiv \begin{bmatrix} \mathbf{e}_1^{tr} & \mathbf{e}_2^{tr} & \dots & \mathbf{e}_T^{tr} \end{bmatrix}$. Its "representative" component $\mathbf{e}_t^{tr}$ is defined as $\mathbf{e}_t^{tr} \equiv \begin{bmatrix} e_{t,H} & e_{t,H-1} & \dots & e_{t,2} & e_{t,1} \end{bmatrix}_{1\times H}$.

The proof that expressions (13) and (14) in the main paper are equivalent goes as follows: first we recognize that the product $\mathbf{i}_{TH}^{tr}\mathbf{\Sigma}$ yields the row vector $\begin{bmatrix} \sum_{l=1}^{TH}\sigma_{l,1} & \sum_{l=1}^{TH}\sigma_{l,2} & \dots & \sum_{l=1}^{TH}\sigma_{l,TH} \end{bmatrix}_{1\times TH}$, then that the product between this vector and $\mathbf{i}_{TH}$ yields $\sum_{c=1}^{TH}\sum_{l=1}^{TH}\sigma_{l,c}$, then that $\left(\mathbf{i}_{TH}^{tr}\mathbf{i}_{TH}\right)^{-1} = \frac{1}{TH}$ and finally that $\frac{1}{TH}$ appears twice in (13).

When biases depend on h, the error vector $\mathbf{e}$ follows (17) and the OLS estimator of $\boldsymbol{\alpha}_H$ comes from:

$$\hat{\boldsymbol{\alpha}}_H = \left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1}\mathbf{M}^{tr}\mathbf{e}$$

where the matrix $\mathbf{M}$ is the Kronecker product between $\mathbf{i}_T$ and $\mathbf{I}_H$, $\mathbf{i}_T$ is a column vector with dimension T whose elements are all equal to 1 and $\mathbf{I}_H$ is an $H \times H$ identity matrix. Consequently, $\mathbf{M}$ can be written as:

$$\mathbf{M} = \begin{bmatrix} I_{H\times H} \\ I_{H\times H} \\ \vdots \\ I_{H\times H} \end{bmatrix}_{TH\times H}$$

The transpose of $\mathbf{M}$ is $\mathbf{M}^{tr} = \begin{bmatrix} I_{H\times H} & I_{H\times H} & \dots & I_{H\times H} \end{bmatrix}_{H\times TH}$, since the transpose of the identity matrix is equal to itself. Therefore, the product $\mathbf{M}^{tr}\mathbf{e}$ yields a column vector with dimension H whose element at row h corresponds to $\sum_{t=1}^{T} e_{t,h}$.

The next step is recognizing that the product $\mathbf{M}^{tr}\mathbf{M}$ is a matrix with dimensions $T \times T$ and that the elements located on its main diagonal are all equal to T. As a result, the inverse of $\mathbf{M}^{tr}\mathbf{M}$ is the scalar $1/T$ multiplied by the identity matrix $\mathbf{I}_H$. Finally, one can show that:

$$\hat{\boldsymbol{\alpha}}_H = \left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1}\mathbf{M}^{tr}\mathbf{e} = \frac{1}{T} I_{H\times H}\mathbf{M}^{tr}\mathbf{e} = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^{T} e_{t,H} \\ \frac{1}{T}\sum_{t=1}^{T} e_{t,H-1} \\ \vdots \\ \frac{1}{T}\sum_{t=1}^{T} e_{t,1} \end{bmatrix}_{H\times 1}$$

Expression (19) in the main paper results from calculating the transpose of the vector $\hat{\boldsymbol{\alpha}}_H$ derived above.

To show that expression (20) is equivalent to (21), first partition matrix $\boldsymbol{\Sigma}$ into T^2 submatrices $\boldsymbol{\sigma}_{l,c}$, each one with dimensions $H \times H$:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\sigma}_{1,1} & \boldsymbol{\sigma}_{1,2} & \cdots & \boldsymbol{\sigma}_{1,T} \\ \boldsymbol{\sigma}_{2,1} & \boldsymbol{\sigma}_{2,2} & \cdots & \boldsymbol{\sigma}_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\sigma}_{T,1} & \boldsymbol{\sigma}_{T,2} & \cdots & \boldsymbol{\sigma}_{T,T} \end{bmatrix}$$

Next, calculate the product $\mathbf{M}^{tr}\boldsymbol{\Sigma}\mathbf{M}$ as:

$$\mathbf{M}^{tr}\boldsymbol{\Sigma}\mathbf{M} = \begin{bmatrix} I_{H\times H} & I_{H\times H} & \cdots & I_{H\times H} \end{bmatrix} \begin{bmatrix} \boldsymbol{\sigma}_{1,1} & \boldsymbol{\sigma}_{1,2} & \cdots & \boldsymbol{\sigma}_{1,T} \\ \boldsymbol{\sigma}_{2,1} & \boldsymbol{\sigma}_{2,2} & \cdots & \boldsymbol{\sigma}_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\sigma}_{T,1} & \boldsymbol{\sigma}_{T,2} & \cdots & \boldsymbol{\sigma}_{T,T} \end{bmatrix} \begin{bmatrix} I_{H\times H} \\ I_{H\times H} \\ \vdots \\ I_{H\times H} \end{bmatrix} = \ldots$$

$$\ldots = \begin{bmatrix} \sum_{l=1}^{T}\boldsymbol{\sigma}_{l,1} & \sum_{l=1}^{T}\boldsymbol{\sigma}_{l,2} & \cdots & \sum_{l=1}^{T}\boldsymbol{\sigma}_{l,T} \end{bmatrix} \begin{bmatrix} I_{H\times H} \\ I_{H\times H} \\ \vdots \\ I_{H\times H} \end{bmatrix} = \sum_{c=1}^{T}\sum_{l=1}^{T}\boldsymbol{\sigma}_{l,c}$$

Finally, recall that $\left(\mathbf{M}^{tr}\mathbf{M}\right)^{-1} = \frac{1}{T}\mathbf{I}_H$ and that this term appears twice in (20).

Ager, Kappler and Osterloh point out that the precise form of $\mathbf{\Sigma} = E\left(\boldsymbol{\lambda\lambda}^{tr}\right)$ depends on the forecast structure and forecast horizon of the variable of interest. They show that, if the error terms in (3) meet conditions (8), then $\mathbf{\Sigma}$ can be written as follows:

$$\mathbf{\Sigma} = \sigma_u^2 \mathbf{\Psi} \tag{A.1}$$

where $\mathbf{\Psi}$ is the $TH \times TH$ matrix defined on page 172 of their paper. Matrix $\mathbf{\Psi}$ is formed by T submatrices $\mathbf{A}$, $T-1$ submatrices $\mathbf{B}$ and $T-1$ submatrices $\mathbf{B}^{tr}$, all of them with dimensions $H \times H$ and defined on the same page. The rest of $\mathbf{\Psi}$ is formed by the $H \times H$ submatrix $\mathbf{0}$, whose elements are all equal to zero. We repeat these definitions below for convenience:

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{B}^{tr} & \mathbf{A} & \mathbf{B} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}^{tr} & \mathbf{A} & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}^{tr} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}^{tr} & \mathbf{A} \end{bmatrix} \tag{A.2}$$

$$\mathbf{A} = \begin{bmatrix} 24 & 23 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 23 & 23 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \end{bmatrix} \tag{A.3}$$

$$\mathbf{B}^{tr} = \begin{bmatrix} 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{A.4}$$

Therefore, under the error structure assumed by Ager, Kappler and Osterloh, the mean value of the elements of $\mathbf{\Sigma}$ is σ_u^2 multiplied by the mean value of the elements of $\mathbf{\Psi}$, which conforms to:

$$\frac{1}{(TH)^2}\sum_{l=1}^{TH}\sum_{c=1}^{TH}\psi_{l,c} = \frac{1}{(TH)^2}\left(T\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c} + 2(T-1)\sum_{l=1}^{H}\sum_{c=1}^{H}b_{l,c}\right) \tag{A.5}$$

where $\psi_{l,c}$, $a_{l,c}$ and $b_{l,c}$ denote the elements at row l and column c of matrices $\mathbf{\Psi}$, $\mathbf{A}$ and $\mathbf{B}$, respectively.[17]

When the error structure follows our framework (see (7) and (9)), then $\mathbf{\Sigma}$ conforms to:

$$\mathbf{\Sigma} = \begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} & \tilde{\mathbf{B}} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} & \ddots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{B}}^{\mathbf{tr}} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \tilde{\mathbf{B}}^{\mathbf{tr}} & \tilde{\mathbf{A}} \end{bmatrix} \tag{A.6}$$

The $H \times H$ submatrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ are defined below:

$$\tilde{\mathbf{A}} = \sigma_s^2\mathbf{A}_s + \sigma_l^2\mathbf{A}_l \tag{A.7}$$

[17] We use the fact that the sum of the elements of $\mathbf{B}$ equals the sum of the elements of its transpose. The same property is used to derive result (A.12).

$$\tilde{\mathbf{B}} = \mathbf{B}_{ls}\varphi\sigma_s\sigma_l \qquad \text{(A.8)}$$

… where $\mathbf{A}_s$, $\mathbf{A}_l$ and $\mathbf{B}_{ls}$ are given by:

$$\mathbf{A}_s = \begin{bmatrix} 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \end{bmatrix} \qquad \text{(A.9)}$$

$$\mathbf{A}_l = \begin{bmatrix} 12 & 11 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ 11 & 11 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \qquad \text{(A.10)}$$

$$\mathbf{B}_{ls}^{tr} = \begin{bmatrix} 12 & 12 & \dots & 12 & 11 & 10 & \dots & 2 & 1 \\ 11 & 11 & \dots & 11 & 11 & 10 & \dots & 2 & 1 \\ 10 & 10 & \dots & 10 & 10 & 10 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & \dots & 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \qquad \text{(A.11)}$$

We call attention to the fact that the matrix $\mathbf{A}$ defined in (A.3) equals the sum of $\mathbf{A}_l$ and $\mathbf{A}_s$ defined above. Furthermore, the matrix $\mathbf{B}^{tr}$ defined in (A.4) matches matrix $\mathbf{B}_{ls}^{tr}$ defined in (A.11). Both facts will be useful shortly.

Under our framework, the mean value of the elements of $\mathbf{\Sigma}$ comes from:

$$\frac{1}{(TH)^2}\sum_{l=1}^{TH}\sum_{c=1}^{TH}\sigma_{l,c} = \frac{1}{(TH)^2}\begin{pmatrix} T\sigma_l^2\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c}^{l} + T\sigma_s^2\sum_{l=1}^{H}\sum_{c=1}^{H}a_{l,c}^{s} + \ldots \\ \ldots + 2(T-1)\varphi\sigma_s\sigma_l\sum_{l=1}^{H}\sum_{c=1}^{H}b_{l,c}^{ls} \end{pmatrix} \tag{A.12}$$

where $\sigma_{l,c}$, $a_{l,c}^{l}$, $a_{l,c}^{s}$ and $b_{l,c}^{ls}$ denote the elements at row l and column c of matrices $\mathbf{\Sigma}$, $\mathbf{A}_s$, $\mathbf{A}_l$ and $\mathbf{B}_{ls}$, respectively.

Regarding the case in which biases may depend on h, we have seen that $Var(\hat{\boldsymbol{\alpha}}_H) = \frac{1}{T^2}\sum_{c=1}^{T}\sum_{l=1}^{T}\boldsymbol{\sigma}_{l,c}$ (see (22)). Under the error structure assumed by Ager, Kappler and Osterloh, one can show that:

$$Var(\hat{\boldsymbol{\alpha}}_H) = \frac{\sigma_u^2}{T^2}\left[T\mathbf{A} + 2(T-1)\mathbf{B}\right] \tag{A.14}$$

where matrices $\mathbf{A}$ and $\mathbf{B}$ are defined in (A.3) and (A.4). The result above follows from (i) for $l = 1, 2, \ldots, T$, matrices $\boldsymbol{\sigma}_{l,l}$ are all equal to $\sigma_u^2\mathbf{A}$; (ii) for $l = 1, 2, \ldots, T-1$, matrices $\boldsymbol{\sigma}_{l+1,l}$ ($\boldsymbol{\sigma}_{l,l+1}$) are all equal to $\sigma_u^2\mathbf{B}^{tr}$ ($\sigma_u^2\mathbf{B}$); and (iii) $\boldsymbol{\sigma}_{l,c} = \mathbf{0}$ elsewhere.

Under our framework, matrices $\mathbf{A}$ and $\mathbf{B}$ are replaced by $\tilde{\mathbf{A}} = \sigma_s^2\mathbf{A}_s + \sigma_l^2\mathbf{A}_l$ and $\tilde{\mathbf{B}} = \mathbf{B}_{ls}\varphi\sigma_s\sigma_l$, being $\mathbf{A}_s$, $\mathbf{A}_l$ and $\mathbf{B}_{ls}$ given by (A.9), (A.10) and (A.11). Consequently, $Var(\hat{\boldsymbol{\alpha}}_H)$ comes from:

$$Var(\hat{\boldsymbol{\alpha}}_H) = \frac{1}{T^2}\left[T\left(\sigma_s^2\mathbf{A}_s + \sigma_l^2\mathbf{A}_l\right) + 2(T-1)\left(\varphi\sigma_s\sigma_l\mathbf{B}_{ls}\right)\right] \tag{A.15}$$

Result (A.15) follows from (i) for $l = 1, 2, \ldots, T$, matrices $\boldsymbol{\sigma}_{l,l}$ are all equal to $\sigma_u^2\tilde{\mathbf{A}}$; (ii) for $l = 1, 2, \ldots, T-1$, matrices $\boldsymbol{\sigma}_{l+1,l}$ ($\boldsymbol{\sigma}_{l,l+1}$) are all equal to $\sigma_u^2 \tilde{\mathbf{B}}^{tr}$ ($\sigma_u^2 \tilde{\mathbf{B}}$); and (iii) $\boldsymbol{\sigma}_{l,c} = \mathbf{0}$ elsewhere.

Finally, expressions (A.14) and (A.15) yield the same result only if $\sigma_l^2 = \sigma_s^2 = \sigma_u^2$ and $\varphi = 1$. Indeed, under these assumptions we get $\tilde{\mathbf{A}} = \sigma_u^2 \left(\mathbf{A}_s + \mathbf{A}_l \right) = \sigma_u^2 \mathbf{A}$ and $\tilde{\mathbf{B}} = \mathbf{B}_{ls} \varphi \sigma_s \sigma_l = \varphi \sigma_s \sigma_l \mathbf{B} = \sigma_u^2 \mathbf{B}$, thus (A.15) converges to (A.14). Incidentally, notice that $Var\left(\hat{\boldsymbol{\alpha}}_H \right) = \frac{\sigma_u^2}{T} \mathbf{A}$ when shocks $u_{t,j}^s$ and $u_{t+1,j+12}^l$ have the same variance but are uncorrelated.

APPENDIX 3

In this appendix we discuss the methods used to estimate σ_u^2, σ_l^2, σ_s^2 and φ. We adopt the procedure followed by Davies and Lahiri (1995), Clements et al. (2007) and Ager et al. (2009) to estimate σ_u^2. If the bias does not depend on h, first subtract the estimated bias from the residuals to yield $\hat{\boldsymbol{\lambda}} = \mathbf{e} - \mathbf{i}_{TH} \hat{\alpha}$, then estimate the auxiliary OLS regression $\hat{\boldsymbol{\lambda}} \odot \hat{\boldsymbol{\lambda}} = \phi \boldsymbol{\tau} + \boldsymbol{w}$.[18] The column vector $\boldsymbol{\tau}$ is defined as the Kronecker product between $\mathbf{i}_T$ and $\boldsymbol{\tau}_H$, where $\mathbf{i}_T$ is a column vector with dimension T whose elements are all equal to 1 and $\boldsymbol{\tau}_H$ is another column vector with H elements obeying $\boldsymbol{\tau}_H^{tr} = \left[H \quad H-1 \quad \ldots \quad 2 \quad 1 \right]_{1 \times H}$. The estimated value of σ_u^2 corresponds to the $\hat{\phi}$ estimated from this auxiliary regression. If there is dependence with the forecast horizon, then estimate $\hat{\boldsymbol{\lambda}} = \mathbf{e} - \mathbf{M} \hat{\boldsymbol{\alpha}}_H$ and proceed as before.

When the error structure follows (9) and the bias does not depend on the forecast horizon, an estimator of σ_s^2 can be obtained by first calculating $\hat{\boldsymbol{\lambda}} = \mathbf{e} - \mathbf{i}_{TH} \hat{\alpha}$ and then

---

[18] The operator $\odot$ refers to the element by element multiplication (Hadamard product).

forming the column vector $\hat{\mathbf{v}}_s$, which has $12T$ elements and complies with $\hat{\mathbf{v}}_s^{tr} = \begin{bmatrix} \hat{\mathbf{v}}_{s,1}^{tr} & \hat{\mathbf{v}}_{s,2}^{tr} & \dots & \hat{\mathbf{v}}_{s,T-1}^{tr} & \hat{\mathbf{v}}_{s,T}^{tr} \end{bmatrix}$.[19] Each row vector $\hat{\mathbf{v}}_{s,t}^{tr}$ comprising $\hat{\mathbf{v}}_s^{tr}$ has 12 elements and obeys $\hat{\mathbf{v}}_{s,t}^{tr} = \begin{bmatrix} \hat{\lambda}_{t,12} & \hat{\lambda}_{t,11} & \dots & \hat{\lambda}_{t,2} & \hat{\lambda}_{t,1} \end{bmatrix}$. Next, estimate the auxiliary OLS regression $\hat{\boldsymbol{v}}_s \odot \hat{\boldsymbol{v}}_s = \phi\boldsymbol{\tau}_s + \boldsymbol{w}_s$, where the column vector $\boldsymbol{\tau}_s$ is defined as the Kronecker product between $\mathbf{i}_T$ and $\boldsymbol{\tau}_{12}$, and vector $\boldsymbol{\tau}_{12}$ has 12 elements and conforms to $\boldsymbol{\tau}_{12}^{tr} = \begin{bmatrix} 12 & 11 & \dots & 2 & 1 \end{bmatrix}_{1\times 12}$. The estimated value of σ_s^2 corresponds to the $\hat{\phi}$ estimated from this auxiliary regression. If the bias depends on h, then estimate $\hat{\boldsymbol{\lambda}} = \mathbf{e} - \mathbf{M}\hat{\boldsymbol{\alpha}}_H$ and proceed as before.

The procedure used to estimate σ_l^2 goes as follows: form the column vector $\hat{\mathbf{v}}_l$ with $12T$ elements and complying with $\hat{\mathbf{v}}_l^{tr} = \begin{bmatrix} \hat{\mathbf{v}}_{l,1}^{tr} & \hat{\mathbf{v}}_{l,2}^{tr} & \dots & \hat{\mathbf{v}}_{l,T-1}^{tr} & \hat{\mathbf{v}}_{l,T}^{tr} \end{bmatrix}$. Each row vector $\hat{\mathbf{v}}_{l,t}^{tr}$ inside $\hat{\mathbf{v}}_l^{tr}$ has 12 elements and conforms to $\hat{\mathbf{v}}_{l,t}^{tr} = \begin{bmatrix} \hat{\lambda}_{t,24} & \hat{\lambda}_{t,23} & \dots & \hat{\lambda}_{t,14} & \hat{\lambda}_{t,13} \end{bmatrix}_{1\times 12}$. Next, estimate the auxiliary OLS regression $\hat{\boldsymbol{v}}_l \odot \hat{\boldsymbol{v}}_l = 12\hat{\sigma}_s^2 + \phi\boldsymbol{\tau}_l + \boldsymbol{w}_l$, where the column vector $\boldsymbol{\tau}_l$ is defined as the Kronecker product between $\mathbf{i}_T$ and $\boldsymbol{\tau}_{12}$. The estimated value of σ_l^2 corresponds to the $\hat{\phi}$ estimated from this last regression. If the bias depends on h, then estimate $\hat{\boldsymbol{\lambda}} = \mathbf{e} - \mathbf{M}\hat{\boldsymbol{\alpha}}_H$ and proceed as before.

Lastly, the procedure used to estimate φ goes as follows: the first step is calculating estimates of the shocks $u_{t,j}^s$ and $u_{t+1,j+12}^l$ by using:

$$\hat{u}_{t,j}^s = F_{t,j-1} - F_{t,j} - \left(\hat{\alpha}_j - \hat{\alpha}_{j-1}\right) \tag{A.16}$$

$$\hat{u}_{t+1,j+12}^l = F_{t+1,j+11} - F_{t+1,j+12} - \left(\hat{\alpha}_{j+12} - \hat{\alpha}_{j+11}\right) \tag{A.17}$$

[19] We have already taken into consideration that $H = 24$.

where j varies from 12 to 1 and $t = 1, 2, \ldots, T$.[20] Then estimate φ by calculating:

$$\hat{\varphi} = \frac{1}{\sqrt{\hat{\sigma}_l^2 \hat{\sigma}_s^2}} \frac{1}{12(T-1)} \sum_{t=1}^{T-1} \sum_{j=1}^{12} \hat{u}_{t,j}^s \times \hat{u}_{t+1,j+12}^l \qquad \text{(A.18)}$$

[20] If $j = 1$, then $\hat{u}_{t,1}^s = X_t - F_{t,1} - \hat{\alpha}_1$.

APPENDIX 4

In this appendix we try to strengthen the arguments in favor of our framework. First, take inflation as an example and assume that $x_t = \ln(1+\pi_t)$, where π_t is realized inflation (monthly frequency). Suppose that x_t follows an AR(1), that is, $x_t = \varphi x_{t-1} + \varepsilon_t$, being φ between 0 and 1 and ε_t a white noise ($\varepsilon_t \sim N(0,\sigma^2)$). Under these circumstances we know that the expectation of x_{t+j} made at *t* is $E_t x_{t+j} = \varphi^j x_t$.

Assume that *t* corresponds to January of year *a*. Hence, the rational forecast of the accumulated inflation over this year is:

$$x_t + E_t x_{t+1} + E_t x_{t+2} + \ldots + E_t x_{t+11} = \left(1+\varphi+\varphi^2+\ldots+\varphi^{11}\right)x_t \quad \text{(A.19)}$$

The rational forecast for the accumulated inflation over the next year (*a* + 1) is:

$$E_t x_{t+12} + E_t x_{t+13} + \ldots + E_t x_{t+23} = \left(\varphi^{12}+\varphi^{13}+\ldots+\varphi^{23}\right)x_t \quad \text{(A.20)}$$

Consider a shock that occur in February of year *a*, which is equivalent to $t+1$. This shock will affect the expectations for years a and $a+1$ in the following way:

Forecast for the current year:

$$\begin{aligned}
&x_t + x_{t+1} + E_{t+1}x_{t+2} + E_{t+1}x_{t+3} + \ldots + E_{t+1}x_{t+11} = \ldots \\
&\ldots = x_t + x_{t+1} + \varphi x_{t+1} + \varphi^2 x_{t+1} + \ldots + \varphi^{10} x_{t+1} = \ldots \\
&\ldots = x_t + \left(1+\varphi+\varphi^2+\ldots+\varphi^{10}\right)x_{t+1} = \ldots \\
&\ldots = x_t + \left(1+\varphi+\varphi^2+\ldots+\varphi^{10}\right)\left(\varphi x_t + \varepsilon_{t+1}\right) = \ldots \\
&\ldots = x_t + \varphi x_t + \varphi^2 x_t + \varphi^3 x_t + \ldots + \varphi^{11} x_t + \varepsilon_{t+1} + \varphi\varepsilon_{t+1} + \varphi^2\varepsilon_{t+1} + \ldots + \varphi^{10}\varepsilon_{t+1} = \ldots \\
&\ldots = \left(1+\varphi+\varphi^2+\ldots+\varphi^{11}\right)x_t + \left(1+\varphi+\varphi^2+\ldots+\varphi^{10}\right)\varepsilon_{t+1} \quad \text{(A.21)}
\end{aligned}$$

Forecast for the next year:

$$E_{t+1}x_{t+12}+E_{t+1}x_{t+13}+\ldots+E_{t+1}x_{t+23}=\ldots$$
$$\ldots=\varphi^{11}x_{t+1}+\varphi^{12}x_{t+1}+\ldots+\varphi^{22}x_{t+1}=\ldots$$
$$\ldots=\left(\varphi^{11}+\varphi^{12}+\ldots+\varphi^{22}\right)x_{t+1}=\ldots$$
$$\ldots=\left(\varphi^{11}+\varphi^{12}+\ldots+\varphi^{22}\right)\left(\varphi x_t+\varepsilon_{t+1}\right)=\ldots$$

$$\ldots=\left(\varphi^{12}+\varphi^{13}+\ldots+\varphi^{23}\right)x_t+\left(\varphi^{11}+\varphi^{12}+\ldots+\varphi^{22}\right)\varepsilon_{t+1} \tag{A.22}$$

The changes in the consensus forecasts made for the current and next years are:

Revision in the forecast for the current year: (A.21)-(A.19)

$$\left(1+\varphi+\varphi^2+\ldots+\varphi^{11}\right)x_t+\left(1+\varphi+\varphi^2+\ldots+\varphi^{10}\right)\varepsilon_{t+1}-\left(1+\varphi+\varphi^2+\ldots+\varphi^{11}\right)x_t=\ldots$$

$$\ldots=\left(1+\varphi+\varphi^2+\ldots+\varphi^{10}\right)\varepsilon_{t+1}=F\left(\varphi\right)\varepsilon_{t+1} \tag{A.23}$$

Revision in the forecast for the next year: (A.22)-(A.20)

$$\left(\varphi^{12}+\varphi^{13}+\ldots+\varphi^{23}\right)x_t+\left(\varphi^{11}+\varphi^{12}+\ldots+\varphi^{22}\right)\varepsilon_{t+1}-\left(\varphi^{12}+\varphi^{13}+\ldots+\varphi^{23}\right)x_t=\ldots$$

$$\ldots=\left(\varphi^{11}+\varphi^{12}+\ldots+\varphi^{22}\right)\varepsilon_{t+1}=G\left(\varphi\right)\varepsilon_{t+1} \tag{A.24}$$

Both $F\left(\varphi\right)$ and $G\left(\varphi\right)$ depend on φ. Because results (A.23) and (A.24) rely on constants multiplied by the same shock (ε_{t+1}), they are perfectly correlated. Hence, if the model AR(1) could represent well the inflation process, then the correlation between forecast revisions made for the current and the next years would equal 1.

But this assumption is not very realistic. In practice, inflation is affected at the same time by several structural shocks of different natures (productivity, preferences, taxes) and persistence (see, for example, Woodford (2003)). This situation would be better represented by a model in which $x_t=x_{1,t}+x_{2,t}$, where $x_{1,t}$ and $x_{2,t}$ are AR(1) processes with different autoregressive coefficients (φ_1,φ_2; $0<\varphi_{1(2)}<1$) and error variances ($\varepsilon_{1,t}\sim N(0,\sigma_1^2)$, $\varepsilon_{2,t}\sim N(0,\sigma_2^2)$). Under these circumstances we know that the expectation of x_{t+j} made at *t* is given by $E_t x_{t+j} = E_t x_{1,t+j} + E_t x_{2,t+j} = \varphi_1^j x_{1,t} + \varphi_2^j x_{2,t}$. The rational forecasts of the accumulated inflation over years a and $a+1$ are:

$$\left(1+\varphi_1+\varphi_1^2+\ldots+\varphi_1^{11}\right)x_{1,t}+\left(1+\varphi_2+\varphi_2^2+\ldots+\varphi_2^{11}\right)x_{2,t} \tag{A.25}$$

$$\left(\varphi_1^{12}+\varphi_1^{13}+\ldots+\varphi_1^{23}\right)x_{1,t}+\left(\varphi_2^{12}+\varphi_2^{13}+\ldots+\varphi_2^{23}\right)x_{2,t} \tag{A.26}$$

A shock occurring in February of year *a* would affect the expectations for years a and $a+1$ in the following way:

Forecast for the current year:

$$\begin{aligned}&\left(1+\varphi_1+\varphi_1^2+\ldots+\varphi_1^{11}\right)x_{1,t}+\left(1+\varphi_1+\varphi_1^2+\ldots+\varphi_1^{10}\right)\varepsilon_{1,t+1}+\ldots\\&\ldots+\left(1+\varphi_2+\varphi_2^2+\ldots+\varphi_2^{11}\right)x_{2,t}+\left(1+\varphi_2+\varphi_2^2+\ldots+\varphi_2^{10}\right)\varepsilon_{2,t+1}\end{aligned} \tag{A.27}$$

Forecast for the next year:

$$\begin{aligned}&\left(\varphi_1^{12}+\varphi_1^{13}+\ldots+\varphi_1^{23}\right)x_{1,t}+\left(\varphi_1^{11}+\varphi_1^{12}+\ldots+\varphi_1^{22}\right)\varepsilon_{1,t+1}+\ldots\\&\ldots+\left(\varphi_2^{12}+\varphi_2^{13}+\ldots+\varphi_2^{23}\right)x_{2,t}+\left(\varphi_2^{11}+\varphi_2^{12}+\ldots+\varphi_2^{22}\right)\varepsilon_{2,t+1}\end{aligned} \tag{A.28}$$

The changes in the forecasts made for the current and next years are:

Revision in the forecast for the current year: (A.27)-(A.25)

$$\left(1+\varphi_1+\varphi_1^2+\ldots+\varphi_1^{10}\right)\varepsilon_{1,t+1}+\left(1+\varphi_2+\varphi_2^2+\ldots+\varphi_2^{10}\right)\varepsilon_{2,t+1}=\ldots$$

$$\ldots=F_1\left(\varphi_1\right)\varepsilon_{1,t+1}+F_2\left(\varphi_2\right)\varepsilon_{2,t+1} \tag{A.29}$$

Revision in the forecast for the next year: (A.28)-(A.26)

$$\left(\varphi_1^{11}+\varphi_1^{12}+\ldots+\varphi_1^{22}\right)\varepsilon_{1,t+1}+\left(\varphi_2^{11}+\varphi_2^{12}+\ldots+\varphi_2^{22}\right)\varepsilon_{2,t+1}=\ldots$$

$$\ldots=G_1\left(\varphi_1\right)\varepsilon_{1,t+1}+G_2\left(\varphi_2\right)\varepsilon_{2,t+1} \tag{A.30}$$

The covariance coefficient between (A.29) and (A.30) is:

$$
\begin{aligned}
&E\left(\left(F_1(\varphi_1)\varepsilon_{1,t+1}+F_2(\varphi_2)\varepsilon_{2,t+1}\right)\left(G_1(\varphi_1)\varepsilon_{1,t+1}+G_2(\varphi_2)\varepsilon_{2,t+1}\right)\right)=\dots\\
&\dots=E\begin{pmatrix} F_1(\varphi_1)G_1(\varphi_1)\varepsilon_{1,t+1}^2+F_2(\varphi_2)G_1(\varphi_1)\varepsilon_{2,t+1}\varepsilon_{1,t+1}+\dots \\ \dots+F_1(\varphi_1)G_2(\varphi_2)\varepsilon_{1,t+1}\varepsilon_{2,t+1}+F_2(\varphi_2)G_2(\varphi_2)\varepsilon_{2,t+1}^2\end{pmatrix}=\dots\\
&\dots=F_1(\varphi_1)G_1(\varphi_1)E\left(\varepsilon_{1,t+1}^2\right)+F_2(\varphi_2)G_1(\varphi_1)E\left(\varepsilon_{2,t+1}\varepsilon_{1,t+1}\right)+\dots\\
&\dots+F_1(\varphi_1)G_2(\varphi_2)E\left(\varepsilon_{1,t+1}\varepsilon_{2,t+1}\right)+F_2(\varphi_2)G_2(\varphi_2)E\left(\varepsilon_{2,t+1}^2\right)=\dots\\
&\dots=F_1(\varphi_1)G_1(\varphi_1)\sigma_1^2+F_2(\varphi_2)G_2(\varphi_2)\sigma_2^2
\end{aligned} \tag{A.31}
$$

Result (A.31) comes from the hypothesis that structural shocks are independent (that is, $\varepsilon_{1,t}\perp\varepsilon_{2,t}$), therefore $E\left(\varepsilon_{1,t+1}\varepsilon_{2,t+1}\right)=E\left(\varepsilon_{1,t+1}\right)E\left(\varepsilon_{2,t+1}\right)=0\times 0=0$.

The variance of (A.29) is:

$$
\begin{aligned}
&E\left(\left(F_1(\varphi_1)\varepsilon_{1,t+1}+F_2(\varphi_2)\varepsilon_{2,t+1}\right)^2\right)=\dots\\
&\dots=E\left(\left(F_1(\varphi_1)\right)^2\varepsilon_{1,t+1}^2\right)+2E\left(F_1(\varphi_1)F_2(\varphi_2)\varepsilon_{1,t+1}\varepsilon_{2,t+1}\right)+E\left(\left(F_2(\varphi_2)\right)^2\varepsilon_{2,t+1}^2\right)=\dots\\
&\dots=\left(F_1(\varphi_1)\right)^2E\left(\varepsilon_{1,t+1}^2\right)+2F_1(\varphi_1)F_2(\varphi_2)E\left(\varepsilon_{1,t+1}\varepsilon_{2,t+1}\right)+\left(F_2(\varphi_2)\right)E^2\left(\varepsilon_{2,t+1}^2\right)=\dots\\
&\cdots=\left(F_1(\varphi_1)\right)^2\sigma_1^2+\left(F_2(\varphi_2)\right)^2\sigma_2^2
\end{aligned} \tag{A.32}
$$

Following the same lines one can show that:

$$
E\left(\left(G_1(\varphi_1)\varepsilon_{1,t+1}+G_2(\varphi_2)\varepsilon_{2,t+1}\right)^2\right)=\left(G_1(\varphi_1)\right)^2\sigma_1^2+\left(G_2(\varphi_2)\right)^2\sigma_2^2 \tag{A.33}
$$

Combining (A.31), (A.32) and (A.33) yields the correlation between (A.29) and (A.30):

$$
\begin{aligned}
&Corr(\dots)=\frac{F_1(\varphi_1)G_1(\varphi_1)\sigma_1^2+F_2(\varphi_2)G_2(\varphi_2)\sigma_2^2}{\sqrt{\left(F_1(\varphi_1)\right)^2\sigma_1^2+\left(F_2(\varphi_2)\right)^2\sigma_2^2}\times\sqrt{\left(G_1(\varphi_1)\right)^2\sigma_1^2+\left(G_2(\varphi_2)\right)^2\sigma_2^2}}=\dots\\
&\dots=\frac{F_1(\varphi_1)G_1(\varphi_1)\sigma_1^2+F_2(\varphi_2)G_2(\varphi_2)\sigma_2^2}{\sqrt{\left(\left(F_1(\varphi_1)\right)^2\sigma_1^2+\left(F_2(\varphi_2)\right)^2\sigma_2^2\right)\left(\left(G_1(\varphi_1)\right)^2\sigma_1^2+\left(G_2(\varphi_2)\right)^2\sigma_2^2\right)}}=\dots\\
&\dots=\frac{F_1(\varphi_1)G_1(\varphi_1)\sigma_1^2+F_2(\varphi_2)G_2(\varphi_2)\sigma_2^2}{\sqrt{\begin{matrix}\left(F_1(\varphi_1)G_1(\varphi_1)\sigma_1^2\right)^2+\left(F_1(\varphi_1)G_2(\varphi_2)\sigma_1\sigma_2\right)^2+\dots\\ \dots+\left(F_2(\varphi_2)\left(G_1(\varphi_1)\right)\sigma_1\sigma_2\right)^2+\left(F_2(\varphi_2)G_2(\varphi_2)\sigma_2^2\right)^2\end{matrix}}}
\end{aligned} \tag{A.34}
$$

In general result (A.34) is always below 1. This means that, under a more realistic framework, the correlation between forecast revisions made for the current and the next years is not perfect.

We reinforce result (A.34) empirically by running the following regression:

$$r_{t+1;\, j+11,\, j+12} = \beta r_{t;\, j-1,\, j} + \chi_t \qquad \text{(A.35)}$$

where $r_{t;\, j-1,\, j}$ and $r_{t+1;\, j+11,\, j+12}$ denote the revisions of the consensus forecasts pertaining to the target years t and $t+1$ that occur between j and $j-1$ periods before the end of year t or, equivalently, between $j+12$ and $j+11$ periods before the end of year $t+1$ (that is, both revisions occur during the same month but focus on different years). The table below shows that the β coefficients and the coefficients of determination R^2 are smaller than 1 regardless the country and the macroeconomic variable. This means that news shocks are not only attenuated from one year to the next, but also that they impact differently the forecasts made for the current and the next years.

Table A.1: Results of estimating regression (A.35) by variable and country

	Output Growth		Inflation	
	R^2	β	R^2	β
Brazil	0.018	0.067 *	0.503	0.552 ***
Chile	0.005	-0.060	0.163	0.151 ***
Mexico	0.005	-0.036	0.199	0.167 ***

Notes: the superscripts '***', '**' and '*' indicate rejection of the null $H_0: \beta = 0$ at the 1% (5%, 10%) significance level.