Energy Shocks and Business Cycle Dynamics *

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June 25, 2024

Abstract

Transition to renewable energy might affect sensitivity to different types of energy supply and demand shocks economy wide. This paper develops a DSGE model that features renewable energy production, stochastic growth, and external habit formation to tackle this issue. The model is estimated by Bayesian techniques for Brazil, a large country highly dependent on renewable sources with an energy matrix that may soon reflect other countries' matrices. We assess historical decompositions of energy supply and demand shocks, address measurement errors due to regulated energy prices, account for the sharp increase in volatility during the pandemic period, compute structural impulse response functions, and calculate price-elasticities of energy demand. Energy supply shocks are the major driving force of energy prices. Output growth variations are mostly explained by non-energy shocks. Nevertheless, energy shocks account for 4.6% of its fluctuations, decomposed in 2% to energy-price (supply) shocks and 1.3% to each residential and industrial consumption (demand) shocks. Price-elasticities for residential energy usage are -0.150%, -0.364%, and -0.459% after one, five, and ten years, respectively. Accordingly, price increases would have a limited impact to refrain energy consumption in times of climate change and adverse shocks in renewable sources.

Keywords: Renewable energy; Historical decomposition; Supply shocks; Demand shocks; Energy prices.

JEL Codes: E32; Q41; Q43; C13.

*We would like to thank comments and suggestions by Eurilton Araujo. J. A. Divino thanks CNPq for financial support through grant number 302632-2019-0. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. The authors declare that they have no conflict of interest that could have appeared to influence the work reported in this paper. All remaining errors are the authors' sole responsibility. *The views expressed in this paper are those of the authors and should not be interpreted as representing the positions of the Banco Central do Brasil or its board members.*

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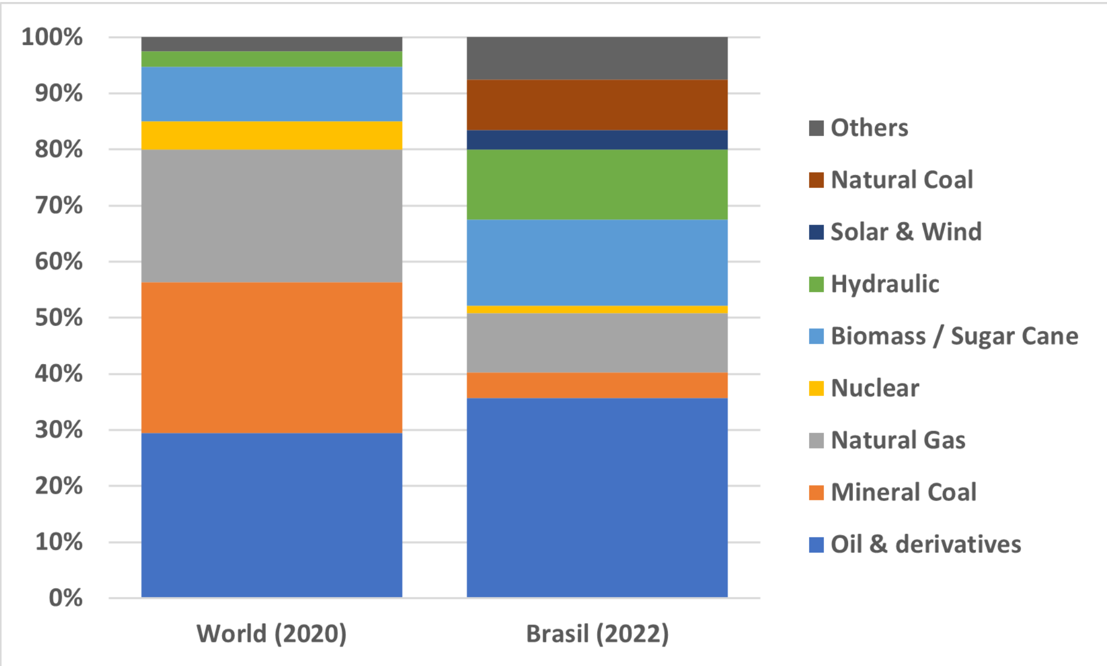
1 Introduction

There is a consensual view in the literature about the importance of energy shocks to the business cycle, especially in the current scenario of climate change and widespread search for renewable and non-pollutant sources of energy. Understanding the nature of energy supply and demand shocks and their effects on energy prices, economic activity, and inflation is paramount for policy makers.

However, much of the literature has focused on the role of fossil fuels, such as oil, natural gas, and mineral coal, for business cycle dynamics. This is justified not only because fossil energy accounts for about 80% of the world's energy matrix (see Figure 1) but also because most countries are not self-sufficient in this energy production and must rely on imports. These economies are more susceptible to exogenous supply shocks and energy price variations.

Figure 1: Energy matrix: World and Brazil

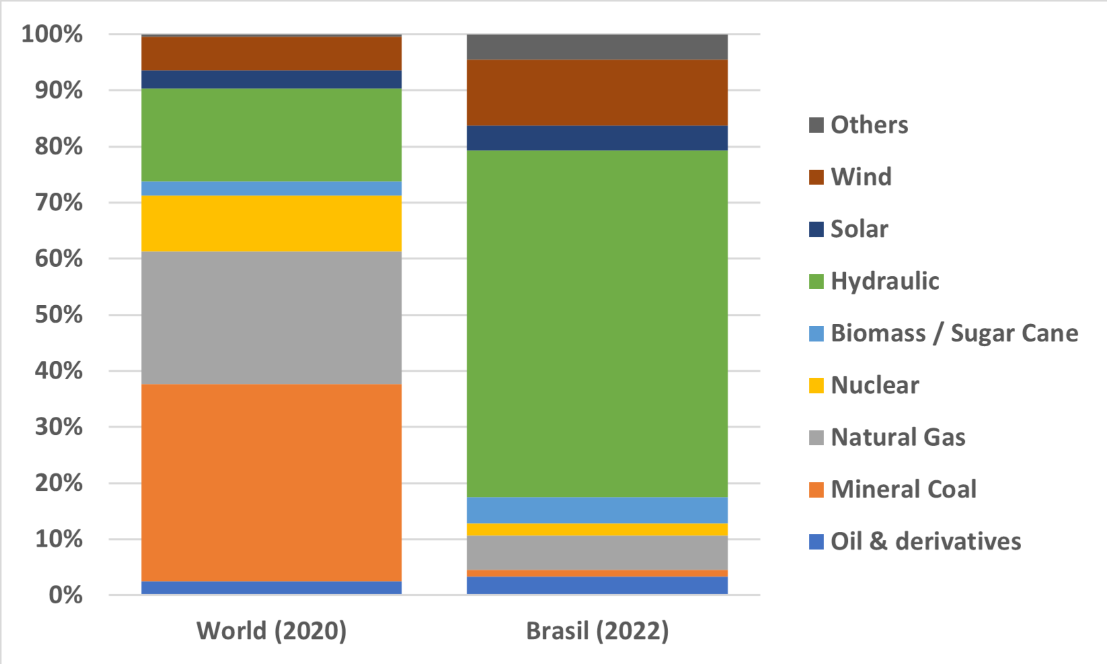


Source: Empresa de Pesquisa Energética, BEN, 2023; original world's data from IEA, 2022. (https://www.epe.gov.br/pt/abcdenergia/matriz-energetica-e-eletrica. Accessed 27 Nov. 2023.)

Nevertheless, there is a growing concern about global warming caused by excessive consumption of fossil energy, which has led to an unprecedented search for alternative renewable and clean energy sources. First, the world is increasing utilization of electrical energy. A stark example is the substitution of combustion engines for electric ones in cars and other vehicles. As a result, the concern is gradually shifting to the electrical energy matrix composition, as reported in Figure 2. Second, the world is rapidly migrating from fossil fuels to renewable energy sources, such as hydraulic, solar and wind generation. Looking at Figure 2, one might even say the world's electrical energy matrix is moving towards becoming more like Brazil's, a leading country worldwide in the transition to renewable energy production.

The objective of this paper is to develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model that includes production and consumption of renewable energy, stochastic

Figure 2: Electrical energy matrix: World and Brazil



Source: Empresa de Pesquisa Energética, BEN, 2023; original world's data from IEA, 2022. (https://www.epe.gov.br/pt/abcdenergia/matriz-energetica-e-eletrica. Accessed 27 Nov. 2023.)

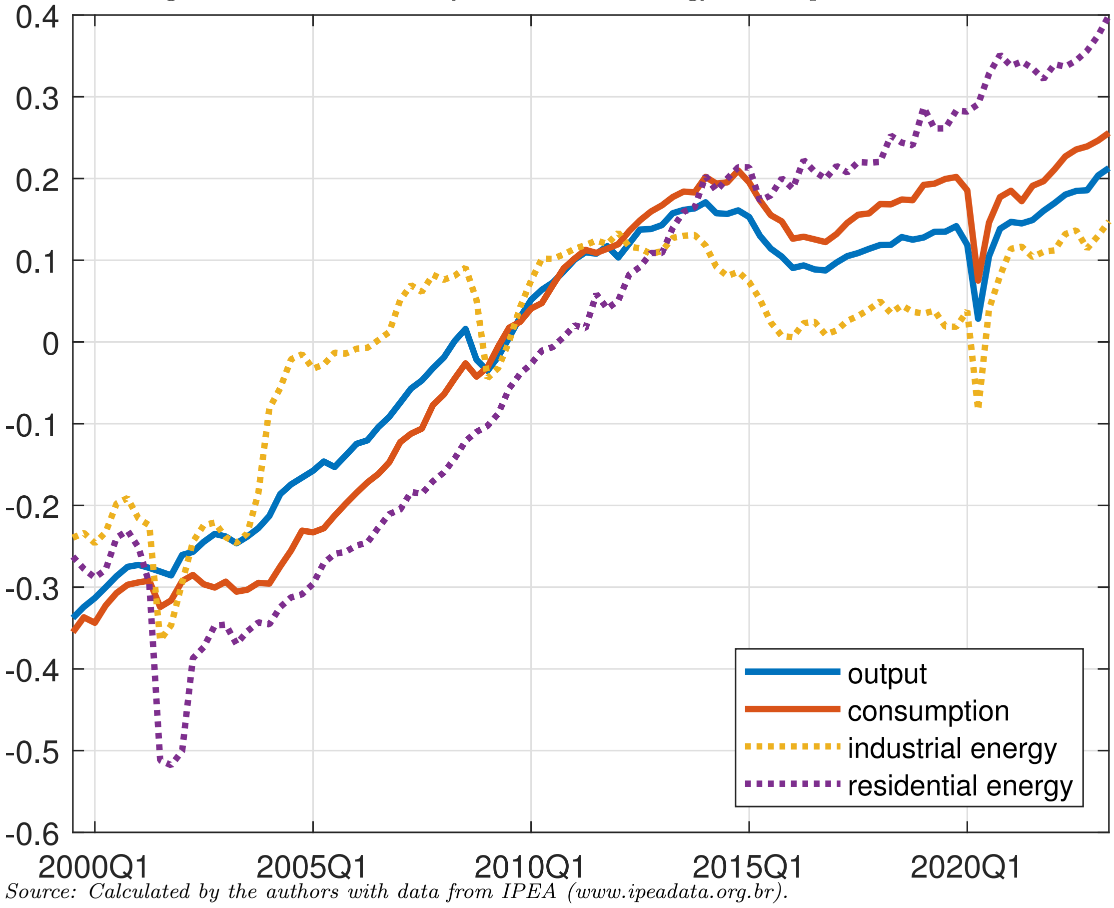
economic growth, and external habit formation to investigate how the transition to renewable energy may affect sensitivity to different types of energy supply and demand shocks in the economy. The model is applied to the data for Brazil as a case study that may currently reflect energy matrices of many countries following their transition periods to renewable energy production as an attempt to mitigate the under-way climate change.

Migrating to renewable energy sources, however, does not mean exogenous energy shocks will lose their importance. They might just shift from oil shocks, which are typically caused by exogenous factors, such as geopolitical conflicts and OPEC decisions (and have been extensively studied), to different exogenous causes, such as climate change. Transition to renewable sources might affect sensitivity to different types of energy supply and demand shocks. In the case of a country that is self-sufficient in clean electricity production, such as Brazil, the main driving forces of supply shocks is climate change. The greater or lesser incidence of rain, solar irradiation, and wind speed would act as supply shocks to the electrical energy production.

We estimate the model structural parameters by Bayesian techniques, assess the historical decomposition of supply and demand shocks on renewable sources, compute structural impulse response functions, and estimate price-elasticity of energy consumption. Our major goal is, from a data centric point of view, to address the relative importance of each type of shock to explain variations in production and prices of renewable energy and the business cycle dynamics. We chose Brazil for application because over 80% of its electricity production comes from renewable sources such as hydroelectric, solar, and wind. This composition might soon reflect energy matrices of many countries that are currently facing a transition period from fossil fuels to clean sources.

The Bayesian estimation uses aggregate data for output, consumption, and investment as well as for usage and prices of residential and industrial energy for Brazil. Figure 3 reports some of these time series in demeaned logarithmic values. Even though industrial energy consumption is a bit more volatile, the growth rates share a common trend and are steadily increasing over time, suggesting the inclusion of a stochastic growth component in the theoretical model. We follow Schmitt-Grohé and Uribe (2012) and consider an economy-wide permanent productivity shock to account for the stochastic balanced growth path.

Figure 3: Economic activity and electrical energy consumption in Brazil



Source: Calculated by the authors with data from IPEA (www.ipeadata.org.br).
Note: All series are in demeaned logs.

To allow for deviations from the balanced growth path, we include a temporary productivity shock in the energy-producing sector and a total factor productivity (TFP) shock in the goods-producing sector. A practical example of the energy supply shock is climate risk. When there is a severe drought, hydroelectric plants may have to operate with reduced capacity by using fewer turbines. Or there may be a need to resort to less efficient and more polluting energy generation methods, such as thermoelectric plants.

The model includes a typical government spending shock and separate disturbances for residential and industrial energy consumptions as demand shifters. The installation of more energy efficient appliances and light bulbs, for instance, would work as a negative demand shock. But we can also think of climate shocks. In a hot summer, consumers not only buy, but also use more air-conditioners and consume more energy, as happened in the year of 2023 with one of the hottest summers ever for many countries.

In addition to carefully accounting for the shock structure, we also have to handle a few other issues that might affect the estimates. First, we need to address the fact that electrical energy prices are regulated by the government and, therefore, might not be fully subject to market clearing conditions. We deal with this by allowing for only a fraction of the variation of observed energy prices be explained by the market conditions, while the rest is treated as measurement errors. This includes everything that affects energy prices but is not explicitly modeled, such as short-run price rigidities in energy contracts, government taxes, subsidies, and price regulation.

Second, we address the huge impact the COVID-19 pandemic had on the time series variation. Even though affecting only a few observations of the full sample, if not properly treated, it would bias our estimates (Lenza and Primiceri, 2022). We follow Cardani et al. (2022) and apply a heteroskedastic filter, while performing a deterministic scaling up of the shocks during the pandemic period. More details on this procedure will be provided later on in Section 3.2.

Using the estimated parameters and shock structure, we compute historical shock decompositions to understand how alternative shocks explain key features of the data. We also evaluate the theoretical impulse response functions to structural shocks and compare the results with the literature. Finally, we calculate price-elasticity of residential energy demand for distinct time horizons to assess consumption sensitivity to price changes.

Results suggest energy supply shocks are the major driving force of variations in energy prices. Energy shocks accounts for 4.6% of output growth fluctuations, decomposed in 2% for energy price variations (supply shocks) and 1.3% for each residential and industrial consumption variations (demand shocks). The estimated price-elasticity of residential energy demand is -0.150, -0.364, and -0.459% after one, five, and ten years, respectively.

Our work is related to the wide literature on the effects of energy and oil shocks to the business cycle, where early contributions include Hamilton (1983) and Rotemberg and Woodford (1996). Typical empirical studies at the time used reduced form regressions or vector autoregressions (VAR) and assumed that oil prices were determined in the international market and were exogenous to any individual country. They usually found large effects of oil shocks on output and economic activity.

In the 2000s, however, the effects of oil price shocks on output became smaller than in the 1970s, as noted by Blanchard and Galí (2010).[1] Among possible explanations, Segal (2011) claims the effect of oil price shocks is indirect, working through monetary policy: "oil prices raise inflation, then monetary authorities raise interest rates, slowing down economic activity".

This assessment calls for the application of general equilibrium models to investigate the transmission channels of energy shocks to the economy. Dhawan and Jeske (2008) divided output into durable and non-durable goods, where consumption of durables requires imported energy that is exogenous. They found an even smaller fraction of output fluctuations explained by energy shocks. Harrison et al. (2011) separated energy into oil and gas (utilities) but allowed for domestic production (and consumption) of both types. In their setup, permanent energy price shocks had important implications for monetary policy but less for output.

Despite the vast literature on oil shocks, only a small strand has addressed the effects of renewable energy supply and demand shocks on energy prices and the business cycle. Huynh (2016) extended the model of Dhawan and Jeske (2008) to include domestic production of electrical energy and usage in consumption of durables and in other production sectors of the economy. Impulse response functions suggested significant impacts of energy shocks on output fluctuations. We question these results, however, because substantial profits from the energy-producing sector were not properly accounted for and distributed back to households in his model.

[1]See Kilian (2008) for an up-to-date review of the literature.

We contribute to the literature by developing a DSGE model that includes stochastic growth, external habit formation, and government consumption. We account for regulated energy prices and apply a heteroskedastic filter to model shock volatilities during the pandemic period. We estimate the model by Bayesian techniques for Brazil, a leading country in renewable energy usage with an energy matrix composition that may soon reflect other countries' matrices. By mapping and identifying the effects of energy shocks, we demonstrate how the transition to renewable energy might influence sensitivity to different types of energy supply and demand shocks. To the best of our knowledge, Millard (2011) is the study most similar to ours. His shock structure, however, is based on Harrison et al. (2011), who only focused on oil and natural gas. In addition, the model was estimated for the UK economy, which has an energy matrix composition very different from Brazil.

The paper is organized as follows. Section 2 fully describes the model economy, including production sectors, households, government, and the equilibrium. Section 3 reports the data, treatment of added volatility due to the pandemic period, and measurement errors in regulated energy prices. Section 4 reports the empirical strategy and discusses the Bayesian estimates, historical shock decompositions, structural impulse response functions, and price-elasticities of energy consumption. Section 5 is dedicated to the concluding remarks. The Appendix A reports the full set of equations, for both stationary and non-stationary economies.

2 The Model

We follow Huynh (2016) and assume convex costs in energy production, which results in small price-elasticity as usually found in the data. According to the data reported in Figure 3, we disregard the assumption of stationarity and allow for stochastic growth by adding a permanent labor productivity shock similar to Schmitt-Grohé and Uribe (2012). The consumer price index (CPI) is taken as numeraire in line with transformation of energy prices and other nominal variables in real values for estimation purposes. We assume external habit formation to bring consumption volatility closer to the data and include government consumption subject to an exogenous shock as a demand shifter. The government also encompasses other variables such as imports and exports, which are not explicitly accounted for in the closed economy environment. Finally, we explicitly accounted for and evenly distribute profits made by the energy producers when out of the steady-state, while Huynh (2016) did not address this issue and was silent about the effects of undistributed profits on the business cycle dynamics.

The model refers to a multi-sector economy that produces three types of goods: durables, non-durables, and energy (electricity). Production factors capital and labor are rented from households. The main difference here is that energy is necessary to the utilization of capital in the production process.

Households extract utility from leisure and a consumption bundle that is composed of non-durable goods and usage of durables, which requires energy. Households must invest not only in capital stock, but also in durable goods. A clarifying example would be the purchase of an air-conditioner. No utility is gained from the purchase by itself, but from its utilization that consumes electricity.

We employ an end-of-period notation for capital and durable goods. Therefore, K_t represents capital stock at the end of time t, chosen at time t, that will be available for production in $t+1$. In the current period, the capital stock available for production is K_{t-1}. Then, each sector chooses how much of this capital stock will be used as $K_{d,t}$, $K_{n,t}$ and $K_{e,t}$.[2] The same reasoning applies for D_t, the stock of durable goods.

[2] As it will be stated later, market clearing requires that $K_{t-1} = K_{d,t} + K_{n,t} + K_{e,t}$.

2.1 Production Sectors

2.1.1 Durable and Non-durable Production Sectors

The durable and non-durable production sectors are assumed to follow a Cobb-Douglas production function of the form:

$$Y_{j,t} = A_t K_{j,t}^{\gamma_j} (Z_t h_{j,t})^{1-\gamma_j} \tag{1}$$

where $j \in \{d, n\}$ indicates the production sector, $Y_{j,t}$, $K_{j,t}$ and $h_{j,t}$ are, respectively, output, usage of capital and labor hours in each sector, and γ_j is the capital share in production for sector j. A_t is a transitory productivity shock, while Z_t is a permanent productivity shock as in Schmitt-Grohé and Uribe (2012), for instance.

The transitory total factor productivity (TFP) shock, A_t, is assumed to be common across the durable and non-durable sectors, but not for the energy sector. It follows a log-stationary AR(1) process of the form:[3]

$$\log(A_t) = \rho_A \log(A_{t-1}) + \left(\sqrt{1-\rho_A^2}\right) \epsilon_{A,t}, \quad \epsilon_{A,t} \sim \mathcal{N}(0, \sigma_A^2) \tag{2}$$

Stochastic Growth Process

Z_t is an economy-wide, non-stationary, permanent labor productivity shock, which accounts for stochastic growth and its logarithm is assumed to follow a random walk with drift as follows:

$$\log(Z_t) = \log(Z_{t-1}) + \log(z_t^z) \tag{3}$$

It can also be expressed as:

$$\frac{Z_t}{Z_{t-1}} = z_t^z \tag{4}$$

The stochastic growth rate, represented by z_t^z, is log-stationary and assumed to follow an AR(1) process with mean $\overline{z^z}$:[4]

$$\log(z_t^z) = (1-\rho_Z)\log(\overline{z^z}) + \rho_Z \log(z_{t-1}^z) + \left(\sqrt{1-\rho_Z^2}\right) \epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim \mathcal{N}(0, \sigma_Z^2) \tag{5}$$

Which is equivalent to:

$$\log\left(\frac{z_t^z}{\overline{z^z}}\right) = \rho_Z \log\left(\frac{z_{t-1}^z}{\overline{z^z}}\right) + \left(\sqrt{1-\rho_Z^2}\right) \epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim \mathcal{N}(0, \sigma_Z^2) \tag{6}$$

The exogenous labor productivity growth affects many endogenous variables of the model, which fluctuate around this growth path. Therefore, it is important to keep track of which variables are stationary and which are not. Except for shocks (like A_t), across the text capital letters will denote growth variables, while lower case letters will stand for stationary variables. We perform a stationarity-inducing transformation of these endogenous growth variables by dividing them by Z_t. Therefore, for any non-stationary (upper case) variable X_t, we define a stationary (lower case) x_t as:

$$x_t = \frac{X_t}{Z_t} \tag{7}$$

For example, $c_t = \frac{C_t}{Z_t}$ is the stationary consumption after adjusting for the growth level of the economy.

[3]With the specification: $E[\log(A_t)] = 0$ and $Var[\log(A_t)] = Var[\epsilon_{A,t}] = \sigma_A^2$.
[4]With the specification: $E[\log(z_t^z)] = \log(\overline{z^z})$ and $Var[\log(z_t^z)] = Var[\epsilon_{Z,t}] = \sigma_Z^2$.

Energy Used in Production

Even though energy is needed for production in all sectors, including to produce energy itself, it does not enter each sector's production function directly. It is rather modeled as an additional cost that is a function of the amount of capital used in production. This implies in a high degree of complementarity between capital and energy.

As Huynh (2016), we consider a linear function for the amount of energy used for production in each sector:

$$E_{j,t} = f(K_{j,t}) = bA_{b,t}K_{j,t} \tag{8}$$

where $j \in \{d, n, e\}$ indicates each production sector, $E_{j,t}$ and $K_{j,t}$ are, respectively, the amount of energy and capital used by sector j, and b represents the energy intensity of capital. We assume this intensity is the same across all sectors and subject to a common energy demand (for production) shock $A_{b,t}$ that follows an AR(1) process:[5]

$$\log(A_{b,t}) = \rho_b \log(A_{b,t-1}) + \left(\sqrt{1-\rho_b^2}\right)\epsilon_{b,t}, \quad \epsilon_{b,t} \sim \mathcal{N}(0, \sigma_b^2) \tag{9}$$

Durable and Non-durable Producers' Problem

Given this setup, each goods producing sector will solve the following profit maximization problem:

$$\max_{\{h_{j,t}, K_{j,t}\}} \{p_{j,t}Y_{j,t} - W_t h_{j,t} - r_t K_{j,t} - p_{e,t}E_{j,t}\} \tag{10}$$

subject to the production technology (equation 1) and to the energy demand (equation 8). $p_{j,t}$, with $j \in \{d, n\}$, is the price of goods produced by sector j, $p_{e,t}$ is the market price of energy, and W_t and r_t are the wage and the rental rate of capital, respectively. All prices are relative to the numeraire, which will be defined in the Households section.

The solution to these maximization problems, along with all equilibrium conditions, are reported in Appendix A.

2.1.2 Energy Production Sector

The energy production technology includes convex costs and takes the form:

$$Y_{e,t} = A_{e,t}(1-\sigma_{e,t})K_{e,t}^{\gamma_e}(Z_t h_{e,t})^{1-\gamma_e} \tag{11}$$

where, as before, the subscript e indicates the energy sector, $Y_{e,t}$, $K_{e,t}$, and $h_{e,t}$ are energy production, capital, and labor hours used in the energy production, respectively. γ_e is the capital share in production of energy and Z_t is the non-stationary labor productivity shock that affects all production sectors.

The energy sector has a transitory productivity shock, $A_{e,t}$, that is different from the other sectors to allow for separate energy supply shocks. It is also modeled as an AR(1) process as follows:[6]

$$\log(A_{e,t}) = \rho_e \log(A_{e,t-1}) + \left(\sqrt{1-\rho_e^2}\right)\epsilon_{e,t}, \quad \epsilon_{e,t} \sim \mathcal{N}(0, \sigma_{ee}^2) \tag{12}$$

The main difference between the energy and the other production sectors lies in $\sigma_{e,t}$, which represents the fraction of energy that is lost due to inefficiencies in the production process:

$$\sigma_{e,t} = \frac{\omega_{e1}}{1+\omega_{e2}}\left(\frac{K_{e,t}^{\gamma_e}(Z_t h_{e,t})^{1-\gamma_e}}{Z_t}\right)^{1+\omega_{e2}} \tag{13}$$

[5]With the specification: $E[\log(A_{b,t})] = 0$ and $Var[\log(A_{b,t})] = Var[\epsilon_{b,t}] = \sigma_b^2$.
[6]With the specification: $E[\log(A_{e,t})] = 0$ and $Var[\log(A_{e,t})] = Var[\epsilon_{e,t}] = \sigma_{ee}^2$.

This convex cost implies that, when energy demand is higher, supply does not adjust one-to-one immediately. This makes the price of energy more volatile, while the supply itself takes time to adjust to demand changes.

Another implication is that optimal choices will lead energy producers to have profits (or losses) when outside the steady-state. We assume energy producers are owned by households and, therefore, profits are distributed as lump-sum values to them.[7] These profit transfers do not affect the households' optimal decisions, but instead imply in substantial income effects. As a result, the impacts of energy shocks are greatly dampened from those reported by Huynh (2016), who did not account for undistributed profits.

Notice that, because of the labor productivity growth, we have to adjust the term inside parentheses so that the cost $\sigma_{e,t}$ remains stationary. The energy producers' maximization problem is the same as the other producers' problem (equation 10), but subject to their specific production technology (equation 11), including the cost function (equation 13) and energy demand (equation 8).

2.2 Households

The household's consumption bundle is represented by a CES aggregation function of non-durable consumption and durable usage according to:

$$C_t = \left[\alpha^{1-\rho}(u_t D_{t-1})^{\rho} + (1-\alpha)^{1-\rho}(N_t)^{\rho}\right]^{1/\rho} \tag{14}$$

where N_t is the consumption of non-durables, D_t is the stock of durables, u_t is the utilization rate of durable goods, α is the share of durables usage in the consumption bundle, and $\frac{1}{1-\rho}$ is the elasticity of substitution between durable and non-durable goods.

This consumption bundle is part of the instantaneous utility given by:

$$U_t(\frac{C_t}{Z_t}, \frac{\overline{C}_{t-1}}{Z_{t-1}}, h_t) = \varphi \log\left(\frac{C_t}{Z_t} - \phi\frac{\overline{C}_{t-1}}{Z_{t-1}}\right) + (1-\varphi)\log(1-h_t)$$

which is a function of the consumption bundle adjusted by the level of growth of the economy ($\frac{C_t}{Z_t}$), hours worked (h_t), and external habit formation in consumption ($\frac{\overline{C}_{t-1}}{Z_{t-1}}$). The share of past consumption is ϕ and the relative weight of consumption versus leisure in utility is φ.

Energy Used in Consumption

As noticed above, energy does not enter the utility function directly. However, the usage of durables that is a part of the consumption bundle requires energy (electricity). This is an additional cost that enters the households budget constraint.

We follow Huynh (2016) and assume a linear function of durables usage to define the amount of energy required in consumption:

$$E_{c,t} = f(u_t D_{t-1}) = aA_{a,t}u_t D_{t-1} \tag{15}$$

where $E_{c,t}$ is the energy demand for consumption and a defines the energy intensity of durables usage. We assume this intensity is subject to a energy demand shock $A_{a,t}$ that follows an AR(1)

[7]An alternative approach would be that energy firms are owned by the Government (or a fraction by Government and another by households). Because both profits and taxes/transfers will enter the households' budget constraint as lump-sum values, this alternative modelling strategy would not make any difference for our analysis. Profits transferred to the households would be the same as being transferred to the Government, which would in turn reduce taxes necessary to balance public budget.

process:[8]

$$\log(A_{a,t}) = \rho_a \log(A_{a,t-1}) + \left(\sqrt{1-\rho_a^2}\right)\epsilon_{a,t}, \quad \epsilon_{a,t} \sim \mathcal{N}(0,\sigma_a^2) \tag{16}$$

Investment and Depreciation

Households accumulate capital and durable goods. Capital depreciates at a constant rate of δ_k, while the stock of durables depreciation depends on the utilization rate according to:

$$\delta_{d,t} = \frac{a_1}{1+a_2} u_t^{1+a_2} \tag{17}$$

Both investments are subject to adjustment costs that depend on the extent that capital (or durables) are growing above or below the long-run growth rate of the economy. Therefore, the laws of motion for capital and durables are, respectively:

$$I_{k,t} = K_t - (1-\delta_k)K_{t-1} + S_k\left(\frac{K_t}{K_{t-1}}\right)K_t \tag{18}$$

$$I_{d,t} = D_t - (1-\delta_{d,t})D_{t-1} + S_d\left(\frac{D_t}{D_{t-1}}\right)D_t \tag{19}$$

and the adjustment cost functions are given by, respectively:

$$S_k(\frac{K_t}{K_{t-1}}) = \frac{\omega_{k1}}{1+\omega_{k2}}\left(\frac{K_t}{K_{t-1}} - \overline{z^z}\right)^{1+\omega_{k2}} \tag{20}$$

$$S_d(\frac{D_t}{D_{t-1}}) = \frac{\omega_{d1}}{1+\omega_{d2}}\left(\frac{D_t}{D_{t-1}} - \overline{z^z}\right)^{1+\omega_{d2}} \tag{21}$$

Households' Optimization Problem

The representative household's problem is to maximize the expected lifetime utility according to:

$$\max_{\{C_t,h_t,u_t,I_{k,t},I_{d,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\varphi \log\left(\frac{C_t}{Z_t} - \phi\frac{\overline{C}_{t-1}}{Z_{t-1}}\right) + (1-\varphi)\log(1-h_t)\right] \tag{22}$$

subject to the following budget constraint:

$$p_{e,t}E_{c,t} + p_{n,t}N_t + p_{d,t}I_{d,t} + p_{d,t}I_{k,t} = W_t h_t + r_t K_{t-1} + \Pi_{e,t} - T_t \tag{23}$$

where $E_{c,t}$ is energy used in consumption, as in equation 15, N_t is the non-durable goods consumption, $I_{d,t}$ represents investment in durables, $I_{k,t}$ defines investment in capital stock, h_t are hours worked, K_{t-1} is the stock of capital rented for production, $\Pi_{e,t}$ are profits made by the energy sector and distributed to households, and T_t are lump-sum taxes.

$p_{e,t}, p_{n,t}, p_{d,t}, W_t$ and r_t are price of energy, price of non-durables, price of durables, wage and rental rate of capital, respectively. All prices but the wage, which grows with labor productivity, are stationary. We use the consumer price index (CPI) to be defined in the next section as the numeraire. The first-order conditions for the household's optimization problem are reported in Appendix A.

[8] With the specification: $E[\log(A_{a,t})] = 0$ and $Var[\log(A_{a,t})] = Var[\epsilon_{a,t}] = \sigma_a^2$.

Consumer Price Index

Given the consumption bundle from equation 14, cost minimization between their components results the following expression for the aggregate Consumer Price Index (CPI):

$$p_{c,t} = \left[\alpha (p_{d,t} a_1 u_t^{a_2} + p_{e,t} a A_{a,t})^{\frac{\rho}{\rho-1}} + (1-\alpha) p_{n,t}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{24}$$

where the first term in parentheses, $(p_{d,t} a_1 u_t^{a_2} + p_{e,t} a A_{a,t})$, is the marginal price of durables usage and $p_{n,t}$ is the marginal price of non-durable goods. The former is composed of the marginal price to account for durables depreciation plus the marginal price of energy for durables usage.

We consider the consumer price index ($p_{c,t} = 1$) as the numeraire and express all other prices as relative prices. This is in line with the empirical requirement to transform nominal variables into real values for estimation purposes.

2.3 Government

To introduce a demand shifter, the model includes a Government that consumes only non-durable goods according to:

$$G_t = Z_t A_{g,t} \tag{25}$$

where $\overline{g}$ is the stationary level of Government consumption, and $A_{g,t}$ is a demand shock that follows an AR(1) process:[9]

$$\log(A_{g,t}) = (1-\rho_g)\log(\overline{g}) + \rho_g \log(A_{g,t-1}) + \left(\sqrt{1-\rho_g^2}\right)\epsilon_{g,t}, \quad \epsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2) \tag{26}$$

Government's revenues come from lump-sum taxes, T_t, that are levied on households. There is no Government debt, so public budget must balance at every time period:

$$T_t = p_{n,t} G_t \tag{27}$$

2.4 Market Clearing Conditions

An equilibrium in this economy is an allocation that satisfies the agents' optimal conditions and clears all markets.

For the capital market, this implies that all available capital is allocated to the production sectors (recall the end-of-period timing notation):

$$K_{t-1} = K_{d,t} + K_{n,t} + K_{e,t} \tag{28}$$

Labor supply must be equal to aggregate labor demand by all sectors:

$$h_t = h_{d,t} + h_{n,t} + h_{e,t} \tag{29}$$

Durable goods production must be used for either investment in capital or durable goods held by households:

$$Y_{d,t} = I_{d,t} + I_{k,t} \tag{30}$$

Non-durable goods production must be consumed by either households or Government:

$$Y_{n,t} = N_t + G_t \tag{31}$$

[9]With the specification: $E[\log(A_{g,t})] = \log(\overline{g})$ and $Var[\log(A_{g,t})] = Var[\epsilon_{g,t}] = \sigma_g^2$.

The energy produced must be fully consumed by households or one of the production sectors, since there is no possibility of energy accumulation. Denoting energy used in production as:

$$E_{y,t} = E_{d,t} + E_{n,t} + E_{e,t} \tag{32}$$

the market clearing condition for energy becomes:

$$Y_{e,t} = E_{c,t} + E_{y,t} \tag{33}$$

According to the Walras' Law, one of these conditions (or the household's budget constraint) is redundant and can be dropped from the equilibrium definition.

We also define an aggregate (value added) output, which excludes energy used in production, as:

$$Y_t = p_{d,t} Y_{d,t} + p_{n,t} Y_{n,t} + p_{e,t} E_{c,t} \tag{34}$$

3 Model Estimation

We solve the model and estimate the parameters by using Bayesian techniques and the Dynare toolbox (Adjemian et al., 2022) applied to Brazilian data and accounting for added volatility during the pandemic period. We compute historical decompositions to identify the relative importance of each shock hitting the economy to the series dynamics. We also estimate impulse response functions to address the effects and transmission mechanisms of the shocks. In all situations, our major concern is the renewable energy shocks and their effects on the Brazilian business cycle.

3.1 Data

The quarterly time series used in the estimation range from 1999Q3, which is the starting period of the current inflation targeting monetary policy regime in Brazil, to 2023Q2, the most recent information available at the conclusion of the study. All series were obtained from the "ipeadata" repository (www.ipeadata.org.br), a publicly available socio-economic database maintained by IPEA - Instituto de Pesquisa Econômica Aplicada (Applied Economic Research Institute).

From the National Accounts, we gathered data for GDP, household consumption and gross investment. We used a chained index that is already in real terms, quarterly frequency and seasonally adjusted. The only additional treatment was to take logs and apply first difference. From now on, these observed variables are called *dlogy_obs*, *dlogc_obs*, and *dlogi_obs*, respectively.

From the electricity sector, we used monthly data for residential and industrial energy consumption (in GWh) and the average market price for MWh (in BRL). For quantities, we first accumulated the energy consumption within each quarter, seasonally adjusted the quarterly data, took logs and applied first differences. The monthly energy price was deflated by the IGP-M price index, which is calculated by the Fundação Getúlio Vargas and is the reference index for concession contracts in the electricity sector in Brazil. We then computed the average price for each quarter, seasonally adjusted the quarterly data, applied logs and first difference. From now on, these variables are called as *dlogec_obs*, *dlogey_obs*, and *dlogpe_obs*, respectively.

Since we intend to estimate parameters from a stochastic growth process, we did not apply any kind of filtering to extract trend from the data, such as the HP filter due to Hodrick and Prescott (1997).[10]

[10] See Hamilton (2018) for a critique on the widespread use of HP filtering in economic time series.

3.2 Added Volatility during the Pandemic Period

In addition to all the tragic loss of lives, the COVID-19 pandemic produced unprecedented change in volatilities of economic variables. For example, if we take the standard deviation of the Brazilian quarterly output growth up to 2019Q4 as a reference, in 2020Q2 output growth was -8 (minus eight) times this standard deviation! Then in 2020Q3 it completely reversed, going up to 6.8 standard deviations! Then in 2020Q4 receded to 3 standard deviations, and only in 2021 went back to variations below 2 standard deviations.

As Lenza and Primiceri (2022) alerted, even though these are few observations relative to the sample size, the movements are so wild that they can (and indeed do) influence parameter estimates. We follow their suggestion of keeping these data points in the sample, but explicitly model the change in shock volatility to account for these exceptionally large innovations during the pandemic period.

We apply the same approach as Cardani et al. (2022), who keep normally distributed errors, but impose a deterministic heteroskedasticity on the shocks during the abnormal period. Then a heteroskedastic Kalman filter is used to evaluate the likelihood function during the estimation process. This is a feature available in Dynare since version 5 (see Adjemian et al., 2022).

As we have the advantage of hindsight, we know the exact periods and the order of magnitude of the increase in volatility. We use half of the aforementioned standard deviations of output growth as a scaling factor for all shocks in the model.[11] Therefore, in the dates of 2020Q2, 2020Q3, and 2020Q4 all shocks have their standard deviations scaled up by 4.0, 3.4, and 1.5, respectively.

We do not include any specific shocks to explain the changes in supply and demand during the pandemic period, leaving all the variations to be explained by the shocks already enclosed in the model.

The procedure worked very well. Our main evidence is that, in an estimation that only covered the period before the pandemic (up to 2019Q4), we found an estimate for the posterior mean of the standard deviation of the economy-wide permanent productivity shock, σ_Z, of 0.0234. When we included the pandemic period (up to 2023q2), the estimate of this parameter doubled its value, climbing to 0.0467. Then, when we treated the pandemic period with the deterministic heteroskedastic shocks using the scaling factors described above, the estimated mean of the parameter returned to 0.0285. This is still a little above the pre-pandemic value, which was expected because of the added volatility at the end of the sample, but not as high as when no treatment was applied to the pandemic period.

For comparison purposes and as a robustness check, Appendix B presents the complete set of Bayesian estimates for distinct sample periods, with and without application of the heteroskedastic Kalman filter during the pandemic era. Specifically, we report estimates for the pre-pandemic period (1999Q3 to 2019Q4) and the full sample (1999Q3 to 2023Q2). The results demonstrate the relevance of accounting for the increased volatility during the pandemic, as the deterministic heteroskedasticity was able to explain the sharp rise in the standard deviation of the aggregate productivity shock.

3.3 Measurement Errors in Renewable Energy Prices

We allow for measurement errors in the observed prices of electricity as a way of having prices that are not fully determined in equilibrium. As a consequence, the estimation will filter the most likely combination of observed prices into two components. The first is due to the fundamentals of

[11] We consider that a variation of 2 standard deviations is a plausible movement. Therefore, if output growth had a -8 standard deviations movement in 2020Q2, we will use a scaling factor of 4 at that period.

the economy, or the market clearing price resulting from the various shocks affecting demand and supply of energy. The second is a measurement error that is due to other factors not accounted for in the model, such as model miss-specification, short-run price rigidities in concession contracts, government subsidies, taxes, or price regulation, that deviates prices from the market clearing conditions.

We assume that measurement errors follow a MA(1) process of the form:[12]

$$me_{p_e,t} = \frac{\epsilon_{me,t} + \hat{\rho}_{me}\epsilon_{me,t-1}}{\sqrt{1+\hat{\rho}_{me}^2}}, \quad \epsilon_{me,t} \sim \mathcal{N}(0, \sigma_{me}^2) \tag{35}$$

where

$$\hat{\rho}_{me} = 2\rho_{me} - 1 \tag{36}$$

The purpose of $\hat{\rho}_{me}$ is to estimate ρ_{me} with a Beta distribution, which is limited between 0 and 1, and implies in $\hat{\rho}_{me}$ within -1 and 1.

In addition, we wish to limit how much of the variance of observed prices comes from the variance of measurement errors. Thus, we define the observation equation as:

$$dlogpe\_obs = \log(p_{e,t}) - \log(p_{e,t-1}) + 0.0382 * 0.75 * me_{p_e,t} \tag{37}$$

where 0.0382 is the standard deviation of *dlogpe_obs* from the data from 1999Q3 to 2019Q4, excluding the pandemic period, 0.75 is the maximum fraction of the standard deviation that is due to the variance of measurement errors, and σ_{me} is estimated with a Beta distribution, constrained between 0 and 1.

The choice of 75% for the maximum fraction due to measurement errors is not trivial. On the one hand, higher values imply that much of price volatility is explained by measurement errors and then the energy production shock parameters are not identified in the estimation. On the other hand, lower values make the price variation too noisy, even after taking out the measurement errors, leaving the shocks too volatile. We tested many fractions and chose 75% as the best balance between these two effects.

4 Results and Discussion

4.1 Calibration

Some parameters were calibrated at standard values from the literature, some were calibrated to achieve certain steady-state targets, and others were estimated. We calibrated those that were not identified in the estimation.

Table 1 summarizes the calibrated parameters, as well as steady-state targets (variables with an upper bar) and reference values. Brief comments to specific cases are provided below.

- α was calibrated to match the share of durables (9.5%) plus the share of energy (10.3%) in the consumer price index;

- after calibrating a_2 to match an annual steady-state durable goods depreciation rate of 12% ($\overline{\delta_d} = 0.03$), a_1 was calibrated to meet an average utilization rate ($\overline{u}$) of 80%;

- a and b were calibrated to match, respectively, the long run ratio of expenditures in residential and industrial energy to GDP ($p_e e_c/y = 0.96\%$ and $p_e e_y/y = 1.07\%$);

- $\overline{g}$ was calibrated so that the long run ratio of government expenditure to GDP matched the average value from data ($p_n g/y = 19.28\%$);

[12]With the specification: $E[me_{p_e,t}] = 0$ and $Var[me_{p_e,t}] = Var[\epsilon_{me,t}] = \sigma_{me}^2$.

Table 1: Calibrated parameters

Param.	Value	Description	Target/reference
		Stochastic Growth Process	
$\overline{z^z}$	1.0059	Labor productivity growth	mean log GDP growth
		Households	
β	0.99	Utility discount factor	standard value (Huynh, 2016)
ρ	1-(1/0.99)	EoS in consumption bundle	EoS=0.99 (Huynh, 2016)
α	0.1980	CPI weight on durables/energy	data
φ	0.0477	Share of consumption in utility	$\overline{h} = 1/3$
δ_k	0.0150	Capital depreciation	standard value (6% per year)
a_2	0.5366	Durable depreciation function	$\overline{\delta_d} = 0.03$ (12% per year)
a_1	0.0650	Durable depreciation function	$\overline{u} = 80\%$
		Government	
$\overline{g}$	0.1676	Government consumption	s.s. $p_n g/y = 0.1928$ (data)
		Firms	
a	0.0105	Energy intensity for consump.	s.s. $p_e e_c/y = 0.0096$ (data)
b	0.0024	Energy intensity for product.	s.s. $p_e e_y/y = 0.0107$ (data)
γ_n	0.27	Capital share in non-durables	calibrated by the authors
γ_d	0.37	Capital share in durables sect.	calibrated by the authors
γ_e	0.55	Capital share in energy sector	Huynh (2016)
ω_{k2}	1	Power of capital adjust. cost	Huynh (2016)
ω_{d2}	1	Power of durables adjust. cost	Huynh (2016)
ω_{e2}	2	Power of energy cost function	Huynh (2016)
ω_{e1}	9.36	Param. of energy cost function	Huynh (2016)

- capital shares in the production functions were calibrated as $\gamma_e = 0.55$ for the energy sector, as in Huynh (2016), and $\gamma_d = 0.37$ and $\gamma_n = 0.27$ based on previous (unpublished) estimations by the authors;

- for the adjustment costs functions, we used a quadratic form for both capital and durables (therefore $\omega_{k2} = 1$ and $\omega_{d2} = 1$), and a cubic form for energy ($\omega_{e2} = 2$), as in Huynh (2016);

- we tried to estimate the remaining parameters for the adjustment cost functions, but ω_{e1} was not identified. Then, we calibrated it as Huynh (2016) and estimated ω_{k1} and ω_{d1}.

4.2 Bayesian Estimates

Table 2 summarizes the assumed prior distributions for each estimated parameter along with the mean of the posterior distributions and the lower and upper bounds of a 90% Bayesian highest posterior density (HPD) interval. Most of these priors were agnostic on the true mean of the parameters and were basically defined to limit the range of values that the estimates could assume according to the theory. That is, between 0 and 1 for the Beta distribution and greater than zero for the Gamma and Inverse Gamma distributions. For the habit formation parameter, we used a tighter prior in line with other estimations for Brazil, such as Castro et al. (2015) and Fasolo et al. (2024).

For the measurement error, we used a Beta distribution both for ρ_{me} and for σ_{me}. Recall from the error specification that ρ_{me} is the moving average parameter that will be modified by the equations, resulting in a support between -1 and 1. σ_{me} should be interpreted as the fraction of 75% of the standard deviation of energy price variations that will be due to measurement

Table 2: Prior distributions and estimated parameters

Param.	Description	Priors			Posteriors	
		Distrib.	Mean	StdDev	Mean	90% HPD
ϕ	Habit formation	Beta	0.80	0.05	0.8885	0.8662 - 0.9125
ω_{k1}	Capital adjust. cost	Gamma	0.05	0.025	0.1326	0.0454 - 0.2150
ω_{d1}	Durables adjust. cost	Gamma	0.40	0.20	0.3315	0.2385 - 0.4304
ρ_Z	$\log(z_t^z)$ persistence	Beta	0.50	0.25	0.8342	0.8146 - 0.8544
ρ_A	$\log(A_t)$ persistence	Beta	0.50	0.25	0.9604	0.9365 - 0.9859
ρ_e	$\log(A_{e,t})$ persistence	Beta	0.50	0.25	0.7756	0.5731 - 0.9734
ρ_a	$\log(A_{a,t})$ persistence	Beta	0.50	0.25	0.8164	0.7308 - 0.9029
ρ_b	$\log(A_{b,t})$ persistence	Beta	0.50	0.25	0.9705	0.9495 - 0.9923
ρ_g	$\log(A_{g,t})$ persistence	Beta	0.50	0.25	0.9432	0.9274 - 0.9591
ρ_{me}	$me_{p_e,t}$ persistence	Beta	0.50	0.25	0.8699	0.7639 - 0.9928
σ_Z	$\log(z_t^z)$ std deviation	Inv Gam	0.001	inf	0.0285	0.0236 - 0.0335
σ_A	$\log(A_t)$ std deviation	Inv Gam	0.001	inf	0.0135	0.0081 - 0.0189
σ_e	$\log(A_{e,t})$ std deviation	Inv Gam	0.001	inf	0.0443	0.0133 - 0.0733
σ_a	$\log(A_{a,t})$ std deviation	Inv Gam	0.001	inf	0.0597	0.0449 - 0.0734
σ_b	$\log(A_{b,t})$ std deviation	Inv Gam	0.001	inf	0.1295	0.0775 - 0.1833
σ_g	$\log(A_{g,t})$ std deviation	Inv Gam	0.001	inf	0.1251	0.0984 - 0.1515
σ_{me}	$me_{p_e,t}$ std deviation	Beta	0.50	0.25	0.9268	0.8439 - 0.9998

errors.

In general, we were able to successful identify the estimated parameters, as can be seen from the posterior mean of the estimates in Table 2. The 90% Bayesian highest posterior density (HPD) interval indicates all estimates are individually statistically different from zero. This is particularly important for the complete set of supply and demand shock parameters, which are in our greatest interest. The estimated posteriors are used in the historical shock decomposition, structural impulse responses functions, and price-elasticity of energy demand calculations.

4.3 Historical Shock Decompositions

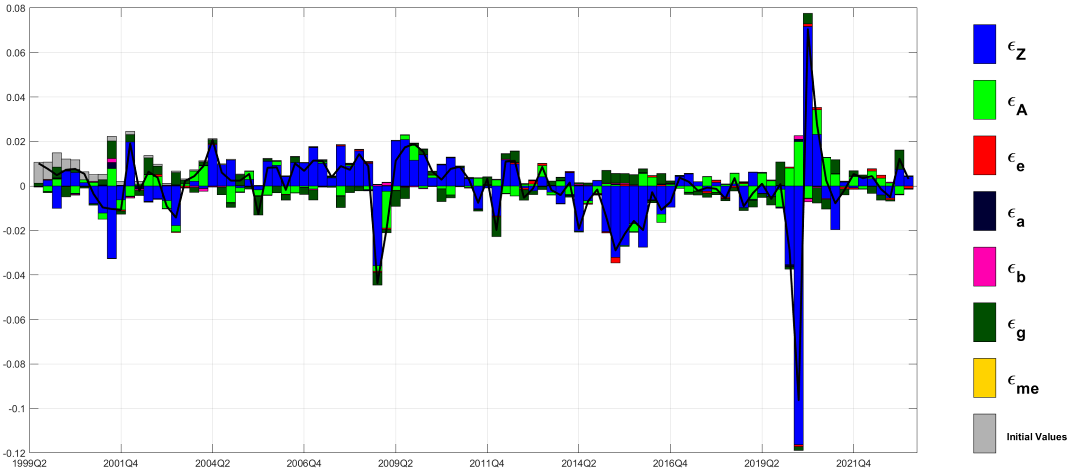
We now analyze the historical shock decompositions of the observed variables to identify which shocks are most relevant to explain the series dynamics. Figure 4 illustrates the historical shock decomposition for observed first difference of log-output (*dlogy_obs*). Values are expressed as log deviations from the steady-state growth rate of $\log(\overline{z^z}) = 0.0059$.

Figure 4 reports 4 deep recessions: the Brazilian energy crisis of 2001, the international financial crisis of 2008-2009, the fiscal crisis that culminated with the impeachment of president Dilma in 2014-2016, and the COVID-19 pandemic crisis. The latter featured a sharp drop in output in 2020Q1 and 2020Q2 followed by a strong recovery in the following quarters.

According to the shock decomposition for the variance of the growth rate of output, the main driving force is the permanent shock to productivity (59%), followed by temporary productivity shock of the goods sector (17%), and shock to government consumption (15%). Exogenous shocks to the energy sector accounted for 4.6% of output growth fluctuations, which can be decomposed in 2% for temporary productivity shock to the energy sector and around 1.3% for each of the two energy intensity (demand) shocks.

Nevertheless, this does not mean energy shocks are unimportant to explain output fluctuations. On the contrary, it is a remarkable effect coming from the renewable energy sector to

Figure 4: Historical Shock Decomposition for *dlogy_obs*



the business cycle. Whenever most of the economic variables move together, including output, consumption, and energy, either above or below the steady-state growth path, it will be filtered as an economy-wide shock to the permanent growth process that affects all these variables and is the most important shock in the decomposition. In this view, shocks to individual sectors (ϵ_A for goods production, ϵ_e for energy production, ϵ_a for residential energy usage, and ϵ_b for industrial energy demand) represent how much these components are "off-path", meaning above or below the rest of the economy.

Figure 5 illustrates the historical shock decomposition for the first difference of log-electricity prices (*dlogpe_obs*). As before, it represents log-deviations of the variable growth rate to its steady-state growth rate. For energy price, however, the steady-state growth rate is zero because price is stationary.

The measurement error shocks filtered out most of the variations in the log-energy prices. These shocks explained 42% of the energy price variations. Energy sector productivity shocks accounted for 37% of the variations, followed by 11% for shocks to permanent productivity, and 8% due to productivity shocks in the other production sectors. All other shocks, individually, accounted for less than 2% of the price variations.

This result allows us to conclude that supply shocks, mostly from productivity of the energy production sector itself but also from the economy-wide labor-augmenting productivity shock, are the major driving forces for energy price movements. To a lesser extent, demand for industrial consumption also had some importance for price movements. However, residential energy consumers are simple price takers, having little influence on price changes.

We perform a similar analysis for renewable energy demand. Figure 6 illustrates the historical shock decomposition for first difference of log-residential electricity consumption (*dlogec_obs*), while Figure 7 focuses on industrial electricity demand (*dlogey_obs*).

Again, both figures report log deviations of the growth rates of these variables to their steady-state growth rates ($\log(\overline{z^z})$). Therefore, they illustrate by how much the variables are growing above (or below) their steady-state values.

According to Figure 6, variations in the demand of energy by households are due mainly to

Figure 5: Historical Shock Decomposition for *dlogpe_obs*

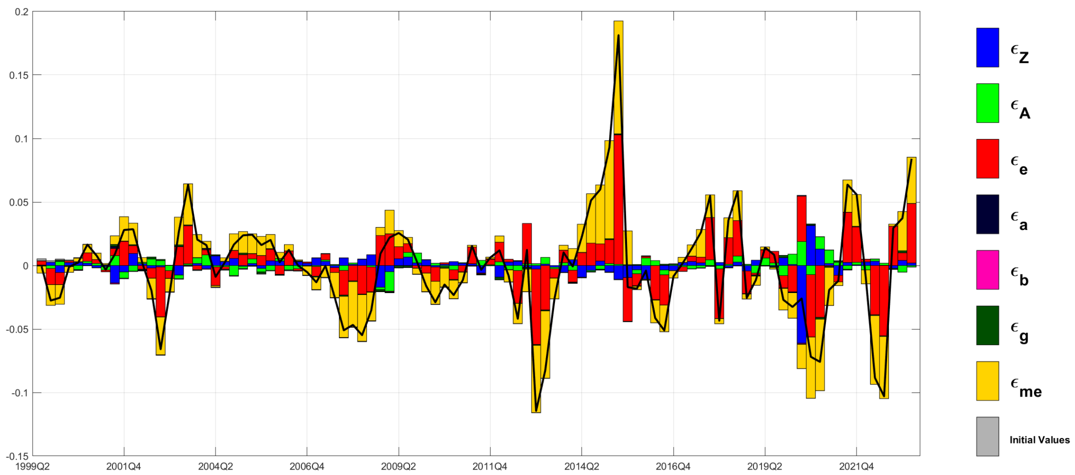
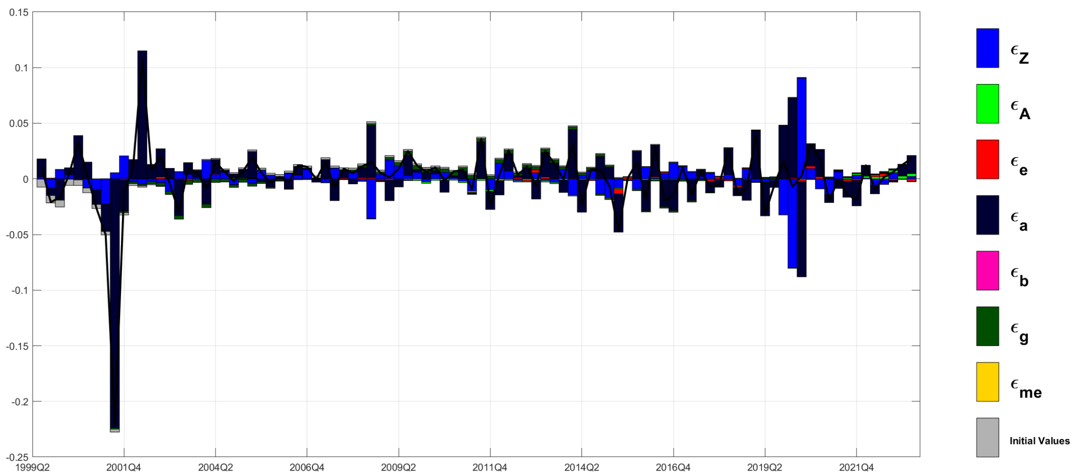
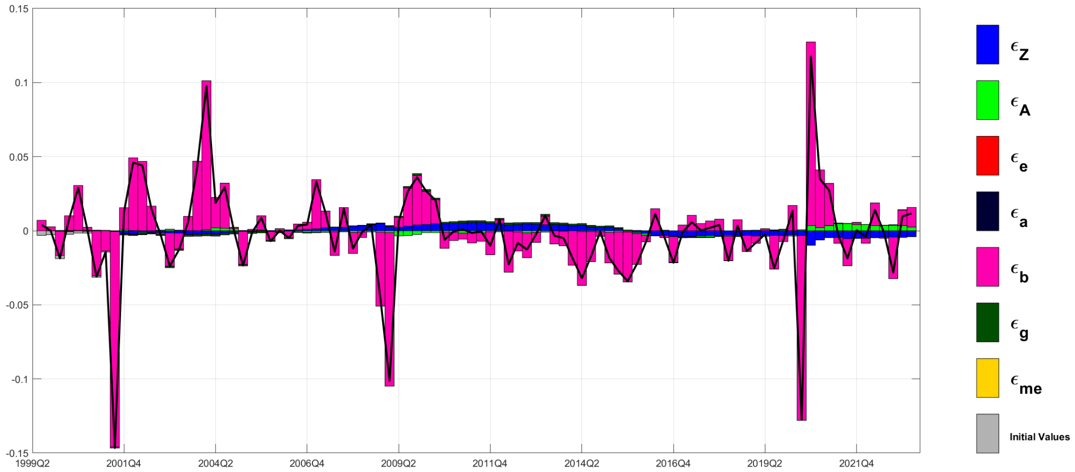


Figure 6: Historical Shock Decomposition for *dlogec_obs*



changes in the energy intensity of durable goods consumption (57%) and shocks to the productivity growth rate of the economy (27%). Shocks to the energy production productivity only account for 4% of changes in residential consumption.

Figure 7: Historical Shock Decomposition for *dlogey_obs*



From Figure 7, changes in the demand of industrial energy are mainly driven by energy intensity of capital consumption (83%) and shocks to the productivity trend of the economy (9%). Shock to energy production productivity is negligible as source of variations in industrial energy consumption.

Our assessment to these shock decompositions is that, because of the linear intensity functions, energy consumption should grow at roughly the same rate as capital (for industrial energy) and durable goods (for residential energy) stocks. Since the energy consumption series in the data are more volatile than capital and durable goods, all extra volatility is attributed to changes in the intensities of energy consumption.

Nevertheless, periods of time that had prominent changes in energy intensities are clearly identified in the shock decompositions. For example, in the Brazilian energy crisis of 2001Q3, the population was called upon to reduce energy consumption by 20% to avoid a collapse of the electrical system due to a severe drought. At that time, 90% of electrical energy generation in the country was hydroelectric. There was a pronounced drop in electrical energy demand, both residential and industrial, which was shortly reversed in the following quarters. This oscillation happened without any major change in energy prices.

On the other hand, during the sharp energy price reduction imposed by the government just before the presidential elections in October 2013 and the subsequent price increase after President Dilma re-election (during the second half of 2014 and the first half of 2015), there were no corresponding fluctuations in energy demand, either residential or industrial. This relative demand stability might be explained by the low price-elasticity of electrical energy consumption in Brazil, as it will be illustrated in the next section.

4.4 Impulse Response Functions

After analyzing historical shock decompositions to verify the relative importance of each shock to the growth rate of the observed variables, we now turn the attention to the the Bayesian impulse response functions (IRFs) obtained from the estimated posterior distributions.

The figures report quarterly impulse response functions to a one standard deviation shock. The black lines show the median IRFs, while the shadow areas represent the 90% posterior density intervals. Variables are shown in log deviations from their respective steady-state values. Therefore, a variation of 0.01 is an approximate increase of 1% from the steady state. We use 1 standard deviation perturbation so that each shock has the actual magnitude that was estimated from the observed data and their effects are comparable. The shock to the energy production productivity is negative, while all the others are positive.

We start with a negative one standard deviation shock to energy productivity, as illustrated in Figure 8. This shock yields a median increase in energy prices of 2.3% at impact. As we already noticed in the shock decompositions, this price increase has very limited effect on quantities. Output drops on impact by only 0.05%, energy used in consumption of durables decreases 0.11%, while the effect is negligible for energy used in production.

This result is in sharp contrast with Huynh (2016). He calibrated the energy productivity shock to produce a 10% raise in energy prices. From our estimated posterior distributions, the magnitude of the shock necessary to reproduce such price variation would be almost 4 standard deviations. Even if we had a $-4\sigma_{ee}$ shock, the drop in output would be less than a quarter of a percentage point, while Huynh (2016) found a decrease of 1%.

We now turn to energy demand shocks represented by increases in residential (Figure 9) and industrial (Figure 10) consumptions of energy.

As already illustrated by the shock decompositions, a positive shock to energy usage in durable goods increases residential energy demand by 3.2%, while only affecting output by -0.04% and having negligible effect on the other variables.

A positive shock to energy usage in production, in turn, raises industrial energy demand by 2.8%, affects output only by -0.03% and has negligible impact on residential energy consumption.

Again, our results are less striking than those of Huynh (2016) by applying 10% shocks to each type of energy consumption, even though the shocks are of similar magnitudes. Recall that the estimated average values are approximately $\sigma_a = 6\%$ and $\sigma_b = 13\%$. The explanation is that, under energy demand shocks, the price of energy sharply increases in Huynh (2016). This leads energy producers to have sizeable profits that are completely disregarded by him. In our model, however, these profits are distributed back to households as lump sum values, which greatly dampens the effects of energy demand shocks.

Figure 11 reports impulse response functions for the temporary productivity shock to the durable and non-durable production sectors. The increase in productivity raises output by 0.41% and grows the demand for energy. Because energy supply takes time to adjust, there is an immediate rise in energy prices of 0.40%. As energy supply adjusts, energy used in production peaks 0.35% after 17 quarters, while energy used in consumption takes 29 quarters to peak a 0.32% increase.

4.5 Price-Elasticity of Residential Energy Consumption

As a final contribution, we use the median of the estimated impulse responses to calculate the implied price-elasticity of residential energy demand. We calibrate a productivity shock to the energy production sector to generate a 1% rise in energy prices. As previously discussed, energy productivity shock is the main driver of energy prices.

Figure 8: IRF to a negative shock in the productivity of the energy sector

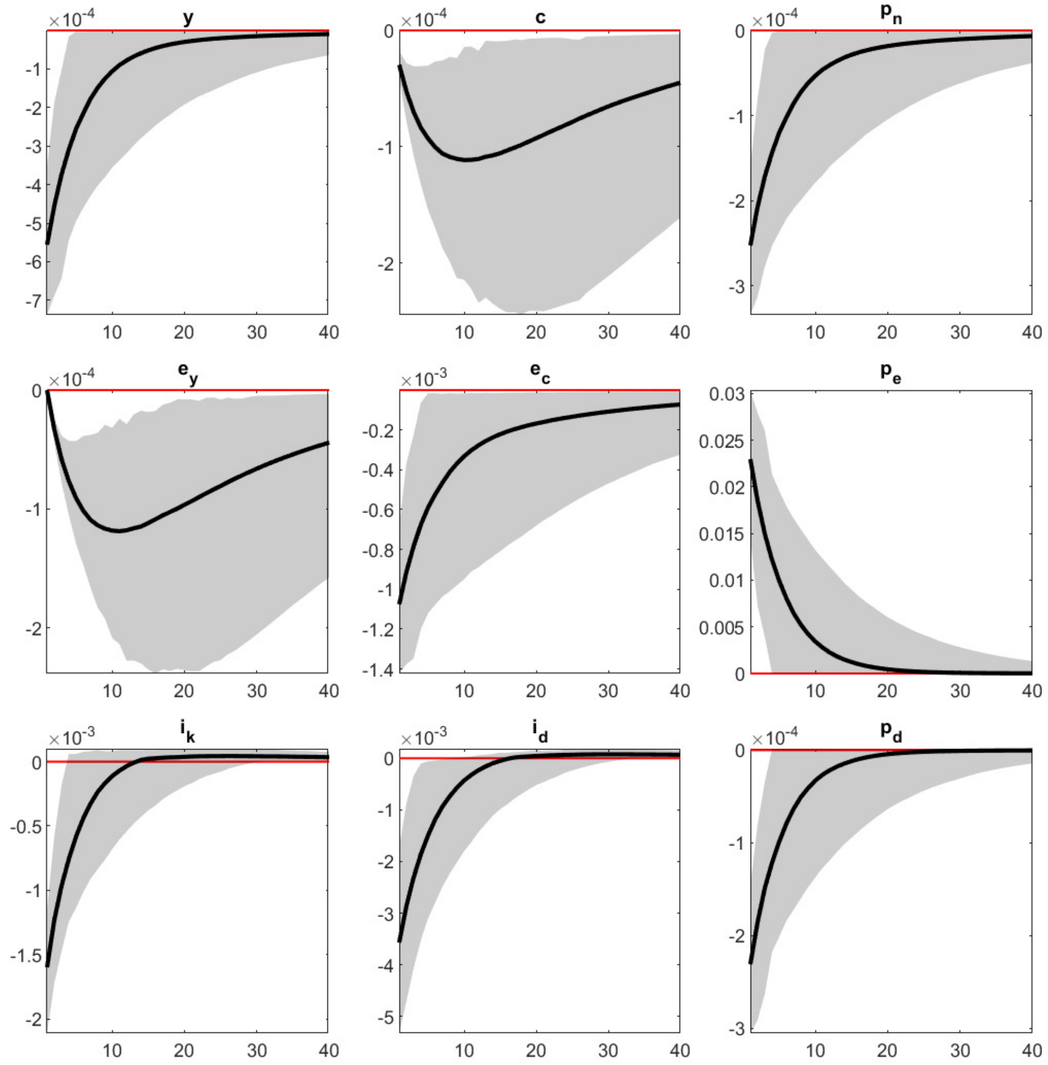


Figure 9: IRF to a positive shock in residential consumption of energy

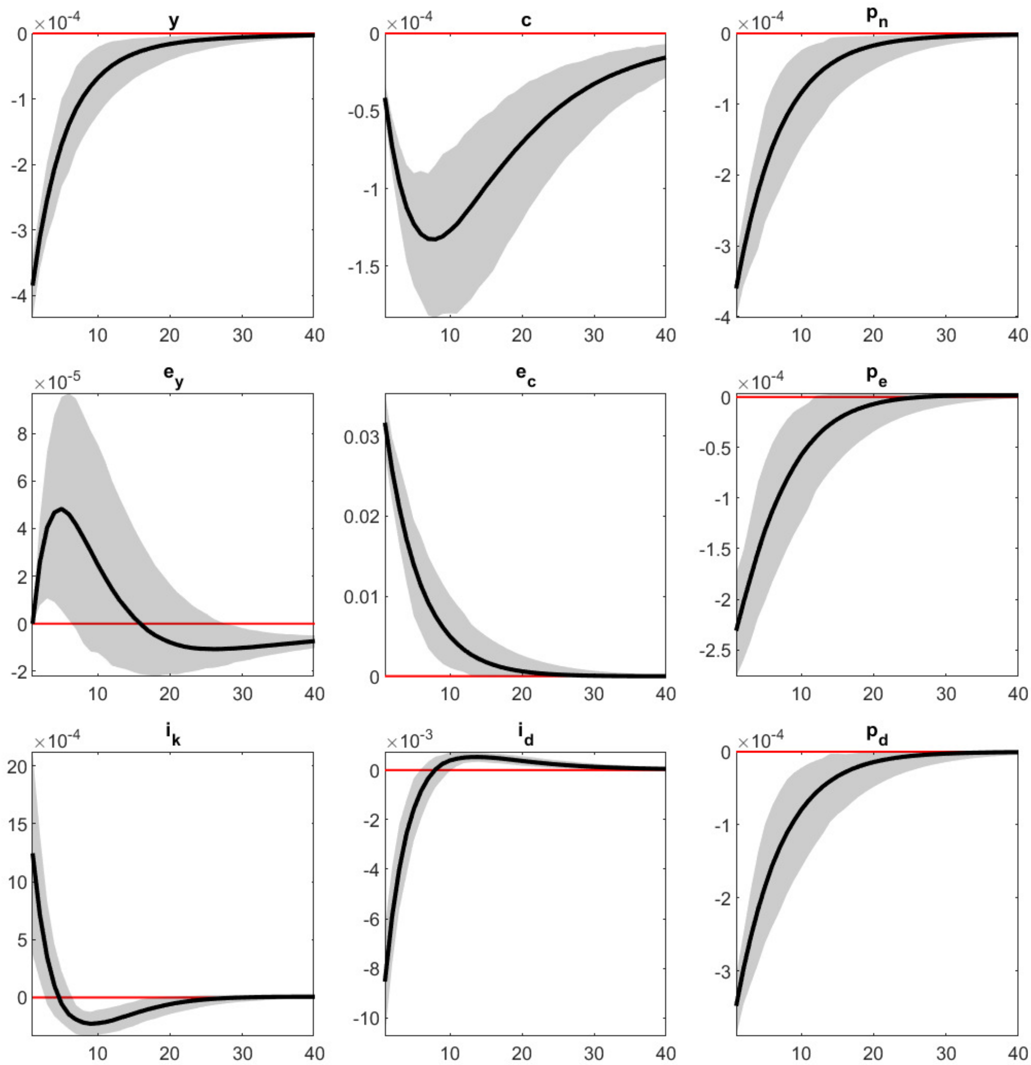


Figure 10: IRF to a positive shock in industrial consumption of energy

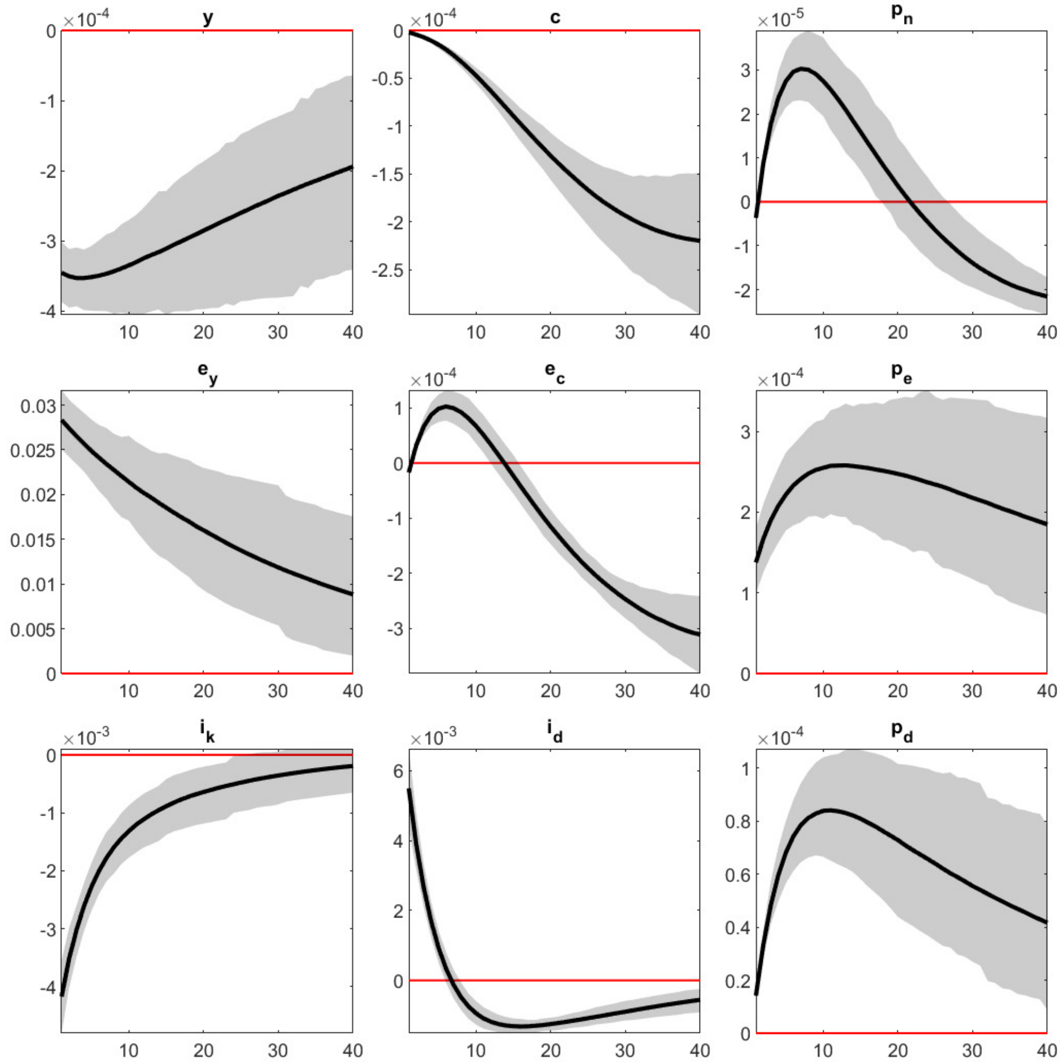
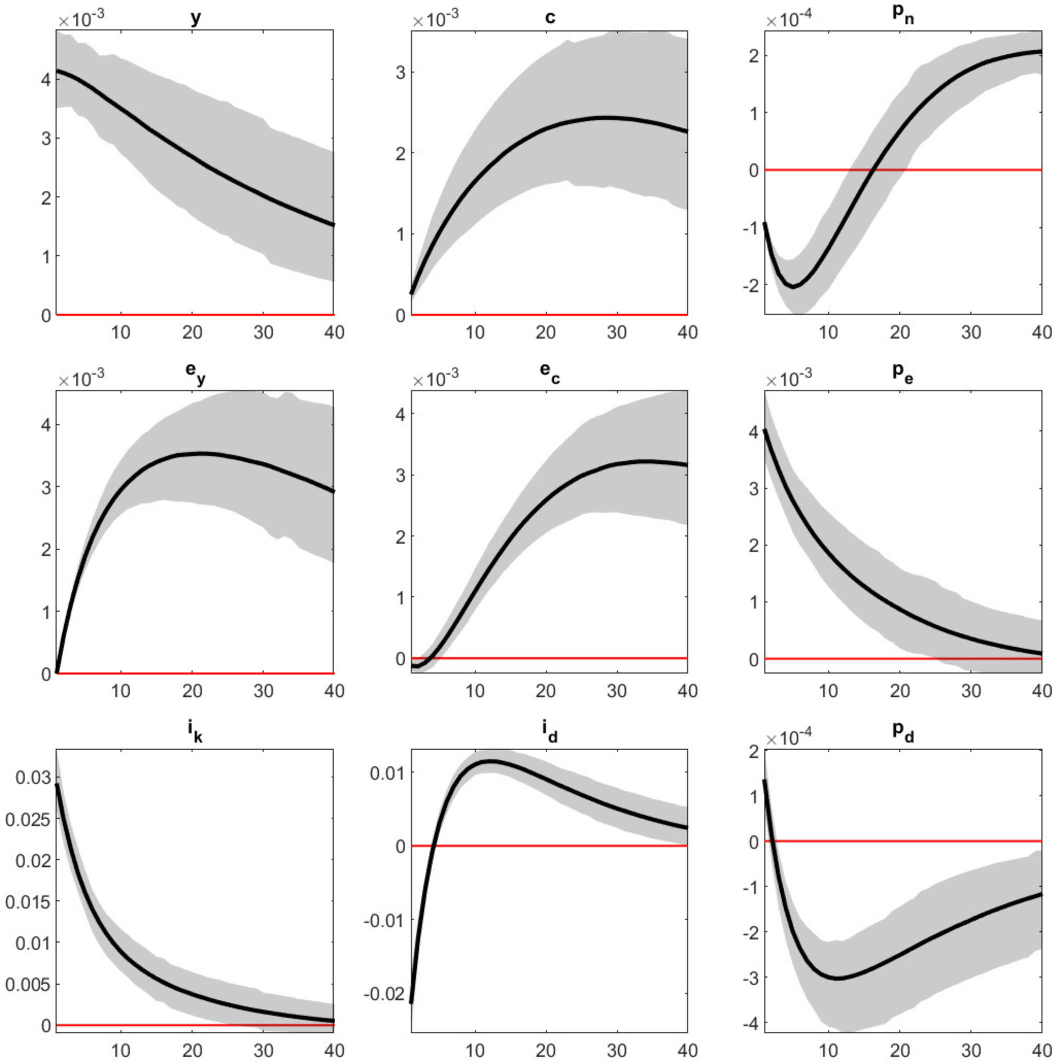


Figure 11: IRF to a positive shock in the goods sector productivity



The result is an immediate reduction of -0.047% in residential energy demand. By accumulating the effects over time, the 1% rise in energy prices yields a drop of -0.150% of energy demand after one year, -0.364% after five years, and -0.459% after ten years.

Comparing our results with other estimates for the Brazilian electrical sector, Andrade and Lobão (1997) used a structural VAR with annual data from 1963 to 1995 and found short-run residential-demand price-elasticities between -0.05 and -0.065%. Schmidt and Lima (2004) also used a structural VAR with annual data but from 1969 to 1999 and estimated a price-elasticity of residential consumption of -0.085%.

Since both studies used yearly data, their results are comparable with our price-elasticity of -0.150% accumulated over one year. This value is more price sensitive than their estimates, suggesting there might be a growing trend in absolute value of the price-elasticity for energy demand over time. It is worth mentioning that Huynh (2016) calibrated his model to generate a price-elasticity of -0.13% at impact, a somewhat high value compared to the estimate of -0.047% for the Brazilian economy.

5 Conclusion

This paper developed a Dynamic Stochastic General Equilibrium (DSGE) model that features production of renewable energy, stochastic growth, and external habit formation to investigate how transition to renewable energy sources may affect sensitivity of the economy to different types of energy supply and demand shocks. The model was estimated by Bayesian techniques for Brazil, a leading country in the transition to renewable sources whose energy matrix may currently reflect energy matrices of many countries in the near future, following their transitions to clean sources. The model includes an economy-wide permanent productivity shock to account for stochastic growth and supply and demand shocks in the goods and energy production sectors. We assessed the relative importance of each shock to explain the business cycle dynamics, accounted for regulated energy price through measurement errors, modelled the increase in volatility during the pandemic period by a heteroskedastic filter, computed structural impulse response functions, and calculated price-elasticities of energy demand.

As expected, output growth variations are mostly explained by non-energy shocks, which include permanent labor-augmenting productivity and temporary productivity of non-energy production sectors. Nevertheless, energy shocks still accounted for 4.6% of variations in the output growth rate, which might be decomposed in 2% for energy supply shocks and 1.3% for each of the two energy demand shocks, residential and industrial. Thus, transition to renewable energy sources may reduce sensitivity to different types of energy supply and demand shocks.

The historical shock decompositions suggested that the energy supply shock, represented by an energy productivity shock, is the major driving force of energy prices. Variations in the residential and industrial energy demands are mainly due to changes in energy utilization rates in the consumption of durable and capital goods, respectively, and in the growth rate of the economy.

According to impulse response functions of comparable magnitudes, the effects of energy demand shocks are greatly dampened once profits from the energy producing sector are properly accounted for and distributed as lump sum values among households. This differs from Huynh (2016), who found large impacts of such shocks on output but mostly driven by undistributed sizeable profits from energy producers.

The demand for residential energy is more price sensitive in the long run, despite the small estimates and low increase over time in absolute terms. The estimated price-elasticities were equal to -0.047% after one quarter and -0.150, -0.364, and -0.459% after one, five, and ten years, respectively. Accordingly, a price increase would have limited impact on the residential demand for electrical energy in Brazil. At times of climate change and adverse shocks in renewable sources, any public policy aiming at reducing energy consumption should not simply rely on increase in prices, but instead be accompanied by awareness campaigns that could be more effective to reduce energy consumption.

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A Appendix: Equilibrium Conditions

A.1 Non-stationary equilibrium conditions

A.1.1 Stochastic Processes

Stochastic growth rate definition:

$$\frac{Z_t}{Z_{t-1}} = z_t^z \tag{A.1}$$

Stochastic process for the growth rate:

$$\log(z_t^z) = (1-\rho_Z)\log(\overline{z^z}) + \rho_Z \log(z_{t-1}^z) + \left(\sqrt{1-\rho_Z^2}\right)\epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim \mathcal{N}(0,\sigma_Z^2) \tag{A.2}$$

Transitory total factor productivity (TFP) shock (durable and non-durable sectors):

$$\log(A_t) = \rho_A \log(A_{t-1}) + \left(\sqrt{1-\rho_A^2}\right)\epsilon_{A,t}, \quad \epsilon_{A,t} \sim \mathcal{N}(0,\sigma_A^2) \tag{A.3}$$

Energy sector transitory TFP shock:

$$\log(A_{e,t}) = \rho_e \log(A_{e,t-1}) + \left(\sqrt{1-\rho_e^2}\right)\epsilon_{e,t}, \quad \epsilon_{e,t} \sim \mathcal{N}(0,\sigma_e e^2) \tag{A.4}$$

Energy intensity in production shock:

$$\log(A_{b,t}) = \rho_b \log(A_{b,t-1}) + \left(\sqrt{1-\rho_b^2}\right)\epsilon_{b,t}, \quad \epsilon_{b,t} \sim \mathcal{N}(0,\sigma_b^2) \tag{A.5}$$

Energy intensity in consumption shock:

$$\log(A_{a,t}) = \rho_a \log(A_{a,t-1}) + \left(\sqrt{1-\rho_a^2}\right)\epsilon_{a,t}, \quad \epsilon_{a,t} \sim \mathcal{N}(0,\sigma_a^2) \tag{A.6}$$

Government demand shock:

$$\log(A_{g,t}) = (1-\rho_g)\log(\overline{g}) + \rho_g \log(A_{g,t-1}) + \left(\sqrt{1-\rho_g^2}\right)\epsilon_{g,t}, \quad \epsilon_{g,t} \sim \mathcal{N}(0,\sigma_g^2) \tag{A.7}$$

A.1.2 Production Sectors

Durable Production Sector

Durable goods production technology:

$$Y_{d,t} = A_t K_{d,t}^{\gamma_d}(Z_t h_{d,t})^{1-\gamma_d} \tag{A.8}$$

Energy demand for production of durables:

$$E_{d,t} = bA_{b,t}K_{d,t} \tag{A.9}$$

Intra-temporal capital demand for production of durables (FOC for $K_{d,t}$):

$$r_t + p_{e,t}bA_{b,t} = p_{d,t}\gamma_d \frac{Y_{d,t}}{K_{d,t}} \tag{A.10}$$

Intra-temporal labor demand for production of durables (FOC for $h_{d,t}$):

$$W_t = p_{d,t}(1-\gamma_d)\frac{Y_{d,t}}{h_{d,t}} \tag{A.11}$$

Non-Durable Production Sector

Non-durable goods production technology:

$$Y_{n,t} = A_t K_{n,t}^{\gamma_n} (Z_t h_{n,t})^{1-\gamma_n} \tag{A.12}$$

Energy demand for production of non-durables:

$$E_{n,t} = bA_{b,t} K_{n,t} \tag{A.13}$$

Intra-temporal capital demand for production of non-durables (FOC for $K_{n,t}$):

$$r_t + p_{e,t} bA_{b,t} = p_{n,t} \gamma_n \frac{Y_{n,t}}{K_{n,t}} \tag{A.14}$$

Intra-temporal labor demand for production of non-durables (FOC for $h_{n,t}$):

$$W_t = p_{n,t} (1-\gamma_n) \frac{Y_{n,t}}{h_{n,t}} \tag{A.15}$$

Energy Production Sector

Energy production technology:

$$Y_{e,t} = A_{e,t} (1-\sigma_{e,t}) K_{e,t}^{\gamma_e} (Z_t h_{e,t})^{1-\gamma_e} \tag{A.16}$$

Energy production costs:

$$\sigma_{e,t} = \frac{\omega_{e1}}{1+\omega_{e2}} \left(\frac{K_{e,t}^{\gamma_e} (Z_t h_{e,t})^{1-\gamma_e}}{Z_t} \right)^{1+\omega_{e2}} \tag{A.17}$$

Energy demand for production of energy:

$$E_{e,t} = bA_{b,t} K_{e,t} \tag{A.18}$$

Intra-temporal capital demand for production of energy (FOC for $K_{e,t}$):

$$r_t + p_{e,t} bA_{b,t} = p_{e,t} \frac{\gamma_e}{K_{e,t}} \left[Y_{e,t} - (1+\omega_{e2}) A_{e,t} \sigma_{e,t} K_{e,t}^{\gamma_e} (Z_t h_{e,t})^{1-\gamma_e} \right] \tag{A.19}$$

Intra-temporal labor demand for production of energy (FOC for $h_{e,t}$):

$$W_t = p_{e,t} \frac{1-\gamma_e}{h_{e,t}} \left[Y_{e,t} - (1+\omega_{e2}) A_{e,t} \sigma_{e,t} K_{e,t}^{\gamma_e} (Z_t h_{e,t})^{1-\gamma_e} \right] \tag{A.20}$$

Optimal choices for capital and labor, due to the energy production costs, will result in profits made by the energy sector of:

$$\Pi_{e,t} = p_{e,t} \left[(1+\omega_{e2}) A_{e,t} \sigma_{e,t} K_{e,t}^{\gamma_e} (Z_t h_{e,t})^{1-\gamma_e} \right] \tag{A.21}$$

A.1.3 Households

Consumption bundle:

$$C_t = \left[\alpha^{1-\rho}(u_t D_{t-1})^{\rho} + (1-\alpha)^{1-\rho}(N_t)^{\rho}\right]^{1/\rho} \tag{A.22}$$

Consumer price index:

$$1 = \left[\alpha(p_{d,t} a_1 u_t^{a_2} + p_{e,t} a A_{a,t})^{\frac{\rho}{\rho-1}} + (1-\alpha)p_{n,t}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{A.23}$$

Energy demand for consumption:

$$E_{c,t} = aA_{a,t}(u_t D_{t-1}) \tag{A.24}$$

Depreciation of durable stock:

$$\delta_{d,t} = \frac{a_1}{1+a_2} u_t^{1+a_2} \tag{A.25}$$

Adjustment costs for capital:

$$S_k(\frac{K_t}{K_{t-1}}) = \frac{\omega_{k1}}{1+\omega_{k2}} \left(\frac{K_t}{K_{t-1}} - \overline{z^z}\right)^{1+\omega_{k2}} \tag{A.26}$$

Derivative of adjustment costs for capital:

$$S_k'(\frac{K_t}{K_{t-1}}) = \omega_{k1} \left(\frac{K_t}{K_{t-1}} - \overline{z^z}\right)^{\omega_{k2}} \tag{A.27}$$

Adjustment costs for durables:

$$S_d(\frac{D_t}{D_{t-1}}) = \frac{\omega_{d1}}{1+\omega_{d2}} \left(\frac{D_t}{D_{t-1}} - \overline{z^z}\right)^{1+\omega_{d2}} \tag{A.28}$$

Derivative of adjustment costs for durables:

$$S_d'(\frac{D_t}{D_{t-1}}) = \omega_{d1} \left(\frac{D_t}{D_{t-1}} - \overline{z^z}\right)^{\omega_{d2}} \tag{A.29}$$

Law of motion for capital:

$$I_{k,t} = K_t - (1-\delta_k)K_{t-1} + S_k\left(\frac{K_t}{K_{t-1}}\right) K_t \tag{A.30}$$

Law of motion for durables:

$$I_{d,t} = D_t - (1-\delta_{d,t})D_{t-1} + S_d\left(\frac{D_t}{D_{t-1}}\right) D_t \tag{A.31}$$

Households budget constraint:

$$p_{e,t}E_{c,t} + p_{n,t}N_t + p_{d,t}I_{d,t} + p_{d,t}I_{k,t} = W_t h_t + r_t K_{t-1} + \Pi_{e,t} - T_t \tag{A.32}$$

Auxiliary variable:

$$\lambda_t = \frac{\varphi}{\frac{C_t}{Z_t} - \phi\frac{\overline{C}_{t-1}}{Z_{t-1}}} \left(\frac{C_t}{N_t}\right)^{1-\rho} (1-\alpha)^{1-\rho} \frac{1}{p_{n,t}} \tag{A.33}$$

Intra-temporal labor supply decision (FOC for h_t):

$$\lambda_t W_t = \frac{1-\varphi}{1-h_t} \tag{A.34}$$

Intra-temporal utilization rate decision (FOC for u_t):

$$\frac{\alpha^{1-\rho}(u_t D_{t-1})^{\rho-1}}{p_{d,t} a_1 u_t^{a_2} + p_{e,t} a A_{a,t}} = \frac{(1-\alpha)^{1-\rho} N_t^{\rho-1}}{p_{n,t}} \tag{A.35}$$

Inter-temporal Euler equation for capital decision (FOC for K_t):

$$\lambda_t p_{d,t}\left[1 + S_k\left(\frac{K_t}{K_{t-1}}\right) + \frac{K_t}{K_{t-1}} S_k'\left(\frac{K_t}{K_{t-1}}\right)\right] =$$
$$\beta E_t\left\{\lambda_{t+1} p_{d,t+1}\left[\frac{r_{t+1}}{p_{d,t+1}} + 1 - \delta_k + \left(\frac{K_{t+1}}{K_t}\right)^2 S_k'\left(\frac{K_{t+1}}{K_t}\right)\right]\right\} \tag{A.36}$$

Inter-temporal Euler equation for durable decision (FOC for D_t):

$$\lambda_t p_{d,t}\left[1 + S_d\left(\frac{D_t}{D_{t-1}}\right) + \frac{D_t}{D_{t-1}} S_d'\left(\frac{D_t}{D_{t-1}}\right)\right] =$$
$$\beta E_t\left\{\frac{\varphi}{\frac{C_t}{Z_t} - \phi\frac{\overline{C}_{t-1}}{Z_{t-1}}} C_{t+1}^{1-\rho}\alpha^{1-\rho}(u_{t+1}D_t)^{\rho-1}u_{t+1}\right\} +$$
$$\beta E_t\left\{\lambda_{t+1} p_{d,t+1}\left[-\frac{p_{e,t+1}}{p_{d,t+1}} a A_{a,t+1} u_{t+1} + 1 - \delta_{d,t+1} + \left(\frac{D_{t+1}}{D_t}\right)^2 S_d'\left(\frac{D_{t+1}}{D_t}\right)\right]\right\} \tag{A.37}$$

A.1.4 Government

Government consumption (of non-durable goods only):

$$G_t = Z_t A_{g,t} \tag{A.38}$$

Government balanced budget constraint:

$$T_t = p_{n,t} G_t \tag{A.39}$$

A.1.5 Market Clearing Conditions

Capital market clearing condition:

$$K_{t-1} = K_{d,t} + K_{n,t} + K_{e,t} \tag{A.40}$$

Labor market clearing condition:

$$h_t = h_{d,t} + h_{n,t} + h_{e,t} \tag{A.41}$$

Durable goods market clearing condition:

$$Y_{d,t} = I_{d,t} + I_{k,t} \tag{A.42}$$

Non-durable goods market clearing condition:

$$Y_{n,t} = N_t + G_t \tag{A.43}$$

Energy market clearing condition:

$$Y_{e,t} = E_{c,t} + E_{d,t} + E_{n,t} + E_{e,t} \tag{A.44}$$

According to Walras' Law, one of the above conditions (or the households budget constraint) is redundant and may be dropped.

Aggregate output definition:

$$Y_t = p_{d,t}Y_{d,t} + p_{n,t}Y_{n,t} + p_{e,t}E_{c,t} \tag{A.45}$$

A.2 Stationary equilibrium conditions

The exogenous labor productivity process will be inherited by several endogenous variables of the model. Therefore, we perform a stationarity-inducing transformation of these endogenous variables by dividing them by Z_t. So, for any non-stationary (uppercase) X_t we define a stationary (lowercase) x_t as:

$$x_t = \frac{X_t}{Z_t} \tag{A.46}$$

In the following subsections we repeat all the equilibrium conditions after this transformation of the non-stationary variables.

A.2.1 Stochastic Processes

Stochastic process for the growth rate:

$$\log(z_t^z) = (1-\rho_Z)\log(\overline{z^z}) + \rho_Z \log(z_{t-1}^z) + \left(\sqrt{1-\rho_Z^2}\right)\epsilon_{Z,t}, \quad \epsilon_{Z,t} \sim \mathcal{N}(0, \sigma_Z^2) \tag{A.47}$$

Transitory total factor productivity (TFP) shock (durable and non-durable sectors):

$$\log(A_t) = \rho_A \log(A_{t-1}) + \left(\sqrt{1-\rho_A^2}\right)\epsilon_{A,t}, \quad \epsilon_{A,t} \sim \mathcal{N}(0, \sigma_A^2) \tag{A.48}$$

Energy sector transitory TFP shock:

$$\log(A_{e,t}) = \rho_e \log(A_{e,t-1}) + \left(\sqrt{1-\rho_e^2}\right)\epsilon_{e,t}, \quad \epsilon_{e,t} \sim \mathcal{N}(0, \sigma_e e^2) \tag{A.49}$$

Energy intensity in production shock:

$$\log(A_{b,t}) = \rho_b \log(A_{b,t-1}) + \left(\sqrt{1-\rho_b^2}\right)\epsilon_{b,t}, \quad \epsilon_{b,t} \sim \mathcal{N}(0, \sigma_b^2) \tag{A.50}$$

Energy intensity in consumption shock:

$$\log(A_{a,t}) = \rho_a \log(A_{a,t-1}) + \left(\sqrt{1-\rho_a^2}\right)\epsilon_{a,t}, \quad \epsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2) \tag{A.51}$$

Government demand shock:

$$\log(A_{g,t}) = (1-\rho_g)\log(\overline{g}) + \rho_g \log(A_{g,t-1}) + \left(\sqrt{1-\rho_g^2}\right)\epsilon_{g,t}, \quad \epsilon_{g,t} \sim \mathcal{N}(0, \sigma_g^2) \tag{A.52}$$

A.2.2 Production Sectors

Durable Production Sector

Durable goods production technology:

$$y_{d,t} = A_t k_{d,t}^{\gamma_d} h_{d,t}^{1-\gamma_d} \tag{A.53}$$

Energy demand for production of durables:

$$e_{d,t} = bA_{b,t}k_{d,t} \tag{A.54}$$

Intra-temporal capital demand for production of durables (FOC for $K_{d,t}$):

$$r_t + p_{e,t}bA_{b,t} = p_{d,t}\gamma_d \frac{y_{d,t}}{k_{d,t}} \tag{A.55}$$

Intra-temporal labor demand for production of durables (FOC for $h_{d,t}$):

$$w_t = p_{d,t}(1-\gamma_d)\frac{y_{d,t}}{h_{d,t}} \tag{A.56}$$

Non-Durable Production Sector

Non-durable goods production technology:

$$y_{n,t} = A_t k_{n,t}^{\gamma_n} h_{n,t}^{1-\gamma_n} \tag{A.57}$$

Energy demand for production of non-durables:

$$e_{n,t} = bA_{b,t} k_{n,t} \tag{A.58}$$

Intra-temporal capital demand for production of non-durables (FOC for $K_{n,t}$):

$$r_t + p_{e,t} bA_{b,t} = p_{n,t} \gamma_n \frac{y_{n,t}}{k_{n,t}} \tag{A.59}$$

Intra-temporal labor demand for production of non-durables (FOC for $h_{n,t}$):

$$w_t = p_{n,t}(1-\gamma_n)\frac{y_{n,t}}{h_{n,t}} \tag{A.60}$$

Energy Production Sector

Energy production technology:

$$y_{e,t} = A_{e,t}(1-\sigma_{e,t}) k_{e,t}^{\gamma_e} h_{e,t}^{1-\gamma_e} \tag{A.61}$$

Energy production costs:

$$\sigma_{e,t} = \frac{\omega_{e1}}{1+\omega_{e2}} \left(k_{e,t}^{\gamma_e} h_{e,t}^{1-\gamma_e} \right)^{1+\omega_{e2}} \tag{A.62}$$

Energy demand for production of energy:

$$e_{e,t} = bA_{b,t} k_{e,t} \tag{A.63}$$

Intra-temporal capital demand for production of energy (FOC for $K_{e,t}$):

$$r_t + p_{e,t} bA_{b,t} = p_{e,t} \frac{\gamma_e}{k_{e,t}} \left[y_{e,t} - (1+\omega_{e2}) A_{e,t} \sigma_{e,t} k_{e,t}^{\gamma_e} h_{e,t}^{1-\gamma_e} \right] \tag{A.64}$$

Intra-temporal labor demand for production of energy (FOC for $h_{e,t}$):

$$w_t = p_{e,t} \frac{1-\gamma_e}{h_{e,t}} \left[y_{e,t} - (1+\omega_{e2}) A_{e,t} \sigma_{e,t} k_{e,t}^{\gamma_e} h_{e,t}^{1-\gamma_e} \right] \tag{A.65}$$

Optimal choices for capital and labor, due to the energy production costs, will result in profits made by the energy sector of:

$$\pi_{e,t} = p_{e,t} \left[(1+\omega_{e2}) A_{e,t} \sigma_{e,t} k_{e,t}^{\gamma_e} h_{e,t}^{1-\gamma_e} \right] \tag{A.66}$$

A.2.3 Households

Consumption bundle:

$$c_t = \left[\alpha^{1-\rho}(u_t \frac{d_{t-1}}{z_t^z})^\rho + (1-\alpha)^{1-\rho}(n_t)^\rho\right]^{1/\rho} \tag{A.67}$$

Consumer price index:

$$1 = \left[\alpha(p_{d,t}a_1 u_t^{a_2} + p_{e,t}aA_{a,t})^{\frac{\rho}{\rho-1}} + (1-\alpha)p_{n,t}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \tag{A.68}$$

Energy demand for consumption:

$$e_{c,t} = aA_{a,t}(u_t \frac{d_{t-1}}{z_t^z}) \tag{A.69}$$

Depreciation of durable stock:

$$\delta_{d,t} = \frac{a_1}{1+a_2} u_t^{1+a_2} \tag{A.70}$$

Adjustment costs for capital:

$$S_k(\frac{k_t}{k_{t-1}} z_t^z) = \frac{\omega_{k1}}{1+\omega_{k2}} \left(\frac{k_t}{k_{t-1}} z_t^z - \overline{z^z}\right)^{1+\omega_{k2}} \tag{A.71}$$

Derivative of adjustment costs for capital:

$$S_k'(\frac{k_t}{k_{t-1}} z_t^z) = \omega_{k1} \left(\frac{k_t}{k_{t-1}} z_t^z - \overline{z^z}\right)^{\omega_{k2}} \tag{A.72}$$

Adjustment costs for durables:

$$S_d(\frac{d_t}{d_{t-1}} z_t^z) = \frac{\omega_{d1}}{1+\omega_{d2}} \left(\frac{d_t}{d_{t-1}} z_t^z - \overline{z^z}\right)^{1+\omega_{d2}} \tag{A.73}$$

Derivative of adjustment costs for durables:

$$S_d'(\frac{d_t}{d_{t-1}} z_t^z) = \omega_{d1} \left(\frac{d_t}{d_{t-1}} z_t^z - \overline{z^z}\right)^{\omega_{d2}} \tag{A.74}$$

Law of motion for capital:

$$i_{k,t} = k_t - (1-\delta_k)\frac{k_{t-1}}{z_t^z} + S_k\left(\frac{k_t}{k_{t-1}} z_t^z\right) k_t \tag{A.75}$$

Law of motion for durables:

$$i_{d,t} = d_t - (1-\delta_{d,t})\frac{d_{t-1}}{z_t^z} + S_d\left(\frac{d_t}{d_{t-1}} z_t^z\right) d_t \tag{A.76}$$

Households budget constraint:

$$p_{e,t}e_{c,t} + p_{n,t}n_t + p_{d,t}i_{d,t} + p_{d,t}i_{k,t} = w_t h_t + r_t \frac{k_{t-1}}{z_t^z} + \pi_{e,t} - t_t \tag{A.77}$$

Auxiliary variable:

$$\lambda_t = \frac{\varphi}{c_t - \phi \bar{c}_{t-1}} \left(\frac{c_t}{n_t} \right)^{1-\rho} (1-\alpha)^{1-\rho} \frac{1}{p_{n,t}} \tag{A.78}$$

Intra-temporal labor supply decision (FOC for h_t):

$$\lambda_t w_t = \frac{1-\varphi}{1-h_t} \tag{A.79}$$

Intra-temporal utilization rate decision (FOC for u_t):

$$\frac{\alpha^{1-\rho}(u_t \frac{d_{t-1}}{z_t^z})^{\rho-1}}{p_{d,t} a_1 u_t^{a_2} + p_{e,t} a A_{a,t}} = \frac{(1-\alpha)^{1-\rho} n_t^{\rho-1}}{p_{n,t}} \tag{A.80}$$

Inter-temporal Euler equation for capital decision (FOC for K_t):

$$\begin{aligned} \lambda_t p_{d,t} \left[1 + S_k \left(\frac{k_t}{k_{t-1}} z_t^z \right) + \frac{k_t}{k_{t-1}} z_t^z S_k' \left(\frac{k_t}{k_{t-1}} z_t^z \right) \right] = \\ \beta E_t \left\{ \frac{\lambda_{t+1} p_{d,t+1}}{z_{t+1}^z} \left[\frac{r_{t+1}}{p_{d,t+1}} + 1 - \delta_k + \left(\frac{k_{t+1}}{k_t} z_{t+1}^z \right)^2 S_k' \left(\frac{k_{t+1}}{k_t} z_{t+1}^z \right) \right] \right\} \end{aligned} \tag{A.81}$$

Inter-temporal Euler equation for durable decision (FOC for D_t):

$$\begin{aligned} \lambda_t p_{d,t} \left[1 + S_d \left(\frac{d_t}{d_{t-1}} z_t^z \right) + \frac{d_t}{d_{t-1}} z_t^z S_d' \left(\frac{d_t}{d_{t-1}} z_t^z \right) \right] = \\ \beta E_t \left\{ \frac{1}{z_{t+1}^z} \frac{\varphi}{c_{t+1} - \phi \bar{c}_t} c_{t+1}^{1-\rho} \alpha^{1-\rho} \left(u_{t+1} \frac{d_t}{z_{t+1}^z} \right)^{\rho-1} u_{t+1} \right\} + \\ \beta E_t \left\{ \frac{\lambda_{t+1} p_{d,t+1}}{z_{t+1}^z} \left[-\frac{p_{e,t+1}}{p_{d,t+1}} a A_{a,t+1} u_{t+1} + 1 - \delta_{d,t+1} + \left(\frac{d_{t+1}}{d_t} z_{t+1}^z \right)^2 S_d' \left(\frac{d_{t+1}}{d_t} z_{t+1}^z \right) \right] \right\} \end{aligned} \tag{A.82}$$

A.2.4 Government

Government consumption (of non-durable goods only):

$$g_t = A_{g,t} \tag{A.83}$$

Government balanced budget constraint:

$$t_t = p_{n,t} g_t \tag{A.84}$$

A.2.5 Market Clearing Conditions

Capital market clearing condition:

$$\frac{k_{t-1}}{z_t^z} = k_{d,t} + k_{n,t} + k_{e,t} \tag{A.85}$$

Labor market clearing condition:

$$h_t = h_{d,t} + h_{n,t} + h_{e,t} \tag{A.86}$$

Durable goods market clearing condition:

$$y_{d,t} = i_{d,t} + i_{k,t} \tag{A.87}$$

Non-durable goods market clearing condition:

$$y_{n,t} = n_t + g_t \tag{A.88}$$

Energy market clearing condition:

$$y_{e,t} = e_{c,t} + e_{d,t} + e_{n,t} + e_{e,t} \tag{A.89}$$

According to Walras' Law, one of the above conditions (or the households budget constraint) is redundant and may be dropped.

Aggregate output definition:

$$y_t = p_{d,t} y_{d,t} + p_{n,t} y_{n,t} + p_{e,t} e_{c,t} \tag{A.90}$$

B Appendix: Complete Estimation Results

Table B.1 provides the prior distributions used for each estimated parameter, together with the mean of the posterior distribution and the lower and upper bounds of a 90% Bayesian highest posterior density (HPD) interval, for all executed estimations. These include an estimation with only the data from the pre-pandemic period (1999Q3 to 2019Q4); an estimation with the full dataset (1999Q3 to 2023Q2), but without controlling for the added volatility of the pandemic period; and finally an estimation for the full period, while also using an heteroskedastic Kalman filter and scaling up the volatilities of all shocks during the pandemic period as described in the main text.

Table B.1: Full estimation results for all datasets and treatments

	Data Sample Period:				1999Q3 - 2019Q4		1999Q3 - 2023Q2		1999Q3 - 2023Q2	
	Kalman Filter Used:				Homoskedastic		Homoskedastic		Heteroskedastic	
		Priors			Posteriors		Posteriors		Posteriors	
Param.	Description	Distrib.	Mean	StdDev	Mean	90% HPD	Mean	90% HPD	Mean	90% HPD
ϕ	Habit formation	Beta	0.80	0.05	0.8890	0.8642 - 0.9126	0.9015	0.8805 - 0.9233	0.8885	0.8662 - 0.9125
ω_{k1}	Capital adjust. cost	Gamma	0.05	0.025	0.0669	0.0157 - 0.1135	0.1040	0.0322 - 0.1730	0.1326	0.0454 - 0.2150
ω_{d1}	Durables adjust. cost	Gamma	0.40	0.20	0.3448	0.2239 - 0.4562	0.3889	0.2791 - 0.5001	0.3315	0.2385 - 0.4304
ρ_Z	$\log(z_t^z)$ persistence	Beta	0.50	0.25	0.8118	0.7857 - 0.8370	0.8239	0.8002 - 0.8473	0.8342	0.8146 - 0.8544
ρ_A	$\log(A_t)$ persistence	Beta	0.50	0.25	0.9134	0.8511 - 0.9883	0.9555	0.9276 - 0.9884	0.9604	0.9365 - 0.9859
ρ_e	$\log(A_{e,t})$ persistence	Beta	0.50	0.25	0.6256	0.2532 - 0.9668	0.7884	0.6147 - 0.9643	0.7756	0.5731 - 0.9734
ρ_a	$\log(A_{a,t})$ persistence	Beta	0.50	0.25	0.7912	0.6993 - 0.8837	0.8156	0.7261 - 0.9092	0.8164	0.7308 - 0.9029
ρ_b	$\log(A_{b,t})$ persistence	Beta	0.50	0.25	0.9722	0.9494 - 0.9973	0.9627	0.9379 - 0.9887	0.9705	0.9495 - 0.9923
ρ_g	$\log(A_{g,t})$ persistence	Beta	0.50	0.25	0.9532	0.9401 - 0.9667	0.9447	0.9284 - 0.9609	0.9432	0.9274 - 0.9591
ρ_{me}	$me_{p_e,t}$ persistence	Beta	0.50	0.25	0.8027	0.6421 - 0.9615	0.8664	0.7574 - 0.9917	0.8699	0.7639 - 0.9928
σ_Z	$\log(z_t^z)$ std deviation	Inv Gam	0.001	inf	0.0234	0.0195 - 0.0271	0.0467	0.0387 - 0.0544	0.0285	0.0236 - 0.0335
σ_A	$\log(A_t)$ std deviation	Inv Gam	0.001	inf	0.0086	0.0045 - 0.0126	0.0138	0.0078 - 0.0201	0.0135	0.0081 - 0.0189
σ_e	$\log(A_{e,t})$ std deviation	Inv Gam	0.001	inf	0.0198	0.0002 - 0.0428	0.0485	0.0193 - 0.0773	0.0443	0.0133 - 0.0733
σ_a	$\log(A_{a,t})$ std deviation	Inv Gam	0.001	inf	0.0574	0.0435 - 0.0701	0.0617	0.0452 - 0.0777	0.0597	0.0449 - 0.0734
σ_b	$\log(A_{b,t})$ std deviation	Inv Gam	0.001	inf	0.1561	0.0770 - 0.2920	0.1335	0.0855 - 0.1834	0.1295	0.0775 - 0.1833
σ_g	$\log(A_{g,t})$ std deviation	Inv Gam	0.001	inf	0.1329	0.1042 - 0.1637	0.1380	0.1069 - 0.1672	0.1251	0.0984 - 0.1515
σ_{me}	$me_{p_e,t}$ std deviation	Beta	0.50	0.25	0.9505	0.8864 - 0.9999	0.9246	0.8394 - 0.9997	0.9268	0.8439 - 0.9998