Crowding in or crowding out? A time-frequency analysis of the investment

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ABSTRACT

I use conditional continuous wavelet tools to address the relationship between private investment and public investment, both by the federal government and state and local government in U.S. I use quarterly data for total investment to GDP, and disaggregated investment in structures, equipment, and intellectual property products. I use net saving to GDP, debt to GDP, growth, interest rate and CPI as instruments, to isolate the crowding in and crowding out effects. I find unprecedented results varying over time and across a range of frequencies during recessionary and non-recessionary periods over more than seventy years. I show under which fiscal and monetary scenarios crowding in/out effects occur. The main findings suggest that the divergence on the previous findings – whether crowding in or crowding out – is partly due to aggregation of investments (by type), aggregation of the public sector, lack of flexibility to allow different short-term, medium-term and long-term reactions, lack of flexibility to allow time-varying reactions, and the impossibility to modelling reactions from the public sector in response to private sector investment (reverse causality).

1. Introduction

The role of public investment measured by its impact on macroeconomic variables has been addressed by a large literature on growth. Arrow and Kurz (1970) are one of the first to discuss this issue, and Barro (1990) is a breaking point in this literature. He assumes that tax-financed public services and goods work as an input to private production and enter into household utility function. There is a recent literature analyzing whether an increase in public investment can affect other variables, as inflation, interest rates, public debt, welfare, and even income inequality [e.g. Furceri and Li (2017), Chaterjee et al. (2018), Bom (2019), and IMF (2020)].

There are three kernel issues in this discussion on economic effects of the public investment: the efficiency, the capital marginal productivity, and the crowding in or crowding out response.

According to Berg et al. (2015), the marginal contribution of an additional dollar of investment spending to output can be broken down into the amount of capital actually installed (efficiency) and the marginal productivity of that capital. According to IMF (2014) if the efficiency of the public investment process is low, then project selection and execution are poor, and only a fraction of the amount invested is converted into productive capital stock. Low investment efficiency implies that less than a dollar of capital is installed. However, a country with persistent low efficiency has been installing less capital over time and then has a lower public capital stock. Assuming decreasing returns on production factor, this implies a higher marginal productivity of public capital. In summary, both effects could go in opposite directions in terms of the effect of additional investment spending on output.

The third issue depends on efficiency and productivity, and it addresses whether private investment reacts positively or negatively to a public investment increase, which can also affect economic activity.

Regarding this relationship between private and public investment, on the one hand, the Global Integrated Monetary and Fiscal Model (GIMF) developed by International Monetary Fund (IMF) – described in Kumhof et al. (2010) – predicts an immediate crowding in response. On the other hand, based on the model developed by Traum and Yang (2015), one should expect short-run crowding-out effects. Both models predict crowding-in effects in the long-term as more productive public capital stock can encourage private investment. Empirically, crowding out effects of investment are supported by some papers, as Mountford and Uhlig (2009), and Voss (2000), while Argimon et al. (1997) and Blackley (2014) find crowding in effect, for instance.

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All these studies are part of a complex literature, and one of the primitive reasons of this lack of theoretical or empirical consensus is that classical, neo classical, Keynesian, and new Keynesian models, etc., all of them assume different hypothesis on the role, the sign or the magnitude of wealth effect, income effect, elasticities of substitution and of output (with respect to the capital stock).

Moreover, there is also a divergence on the size of fiscal multipliers due to increasing in public investment and consumption, which is usually associated with the positive or negative effects of private investment. Observing recent contributions applied to developed economies, Boehm (2020) uses real-time forecasts of government consumption and investment to purify changes in purchases of their predicted components for a panel of OECD countries. He finds that the government investment multiplier is near zero and the government consumption multiplier close to 0.8. Regarding the specific role played by U.S. federal procurement, Hebous and Zimmermann (2021) propose measuring government demand shocks at the firm level, by linking federal procurement contracts with firms′ financial information. It enables them to filter out potential anticipation effects and focus on unexpected changes to a firm′s future cash flows. Their resulting dataset is novel and includes more than 90,000 observations from 1999:4 to 2018:3. They find that 1 dollar of federal purchases increases capital investment of financially constrained firms by 10 to 13 cents, but has no effect on investment of unconstrained firms, corroborating the result expected by the financial accelerator model.

In short, there are theoretical frameworks predicting both directions, there is an empirical literature with developed or emerging countries with divergent findings, as well as there are acceptable and intuitive explanations justifying both responses by the private sector. The empirical divergences remain, even analyzing American economy [e.g. Erenburg and Wohar (1995), Voss (2000), Mountford and Uhlig (2009), Traum and Yang (2015), and Blackley (2014)]. However, there seems to be common issues in this divergent discussion: the papers do not use to isolate the investment decisions of federal, state, and local governments, and they do not allow reverse causality, i.e., public investment reacting to private investment. There are few studies modeling such effects for investment by type and allowing for state-dependent results.

In this context, this paper adds to the debate on macroeconomic based relationship between private and public investment, by proposing an innovative empirical exercise applied to U.S., aiming to fil well in this relevant literature from a methodological and empirical point of view.

First, I decompose public sector. I assume that private investment cycles can be affected by or affect public investment cycles, both by the federal government and state and local government in U.S. Second, I study this relationship using investments by type: structures, equipment, and intellectual property products. Third, the wavelet-based approach allows me to identify and measure lead-lag relationship in both directions varying over time and across different ranges of frequency.

Fourth, by using the partial wavelet tools developed by follow Aguiar-Conraria et al. (2014) and Aguiar-Conraria et al. (2018), I can isolate the effects, by filtering out the effects of a set of instruments: GDP growth, relevant monetary variables, as interest rate and inflation (CPI), and fiscal variables that usually drive public investments (net saving to GDP, and debt to GDP). In other words, I find when, whether and in which macroeconomic scenarios there is crowding in or crowding out. Concerning the fiscal drivers of public investment decisions, see Traum and Yang (2015) for the effects of debt level, and Ardanaz et al. (2021) for the role played the fiscal rule design. See Arin, Devereux, and Mazur (2023) for firm-level investment response to unanticipated narrative shocks to average personal and corporate tax rates. See Döttling and Ratnovski (2023) for the role played by monetary policy in investment of firms.

Regarding the results, this paper offers an unprecedented range of evidence and insights on distinct types of public and private investment in U.S., during recessionary or non-recessionary periods controlled by contractionary or expansionary fiscal and monetary policies, over the period from 1952:1 to 2023:2.

To summarize, there is a significant difference in results considering the short-term, medium term and long-term; the effects are dynamic over time; the relationship between private investment is different depending on whether the public player is the federal government, or the governments state and local; investments by type present very specific crowding in and crowding out behavior; and private investment is able to affect public investment in the phasic and antiphasic directions.

It is relevant highlighting that I do not use a specific theoretical framework already developed. Instead, the thesis of this paper follows the purpose of Hodrick and Prescott (1997), by assuming that studying the co-movements of aggregate economic variables using an efficient, and replicable technique can provide insights into the features of the economy that an equilibrium theory should incorporate. From a normative point of view, the relationship proposed here between the investment by households, firms, federal, state and local government filtered by a relevant and parsimonious set of macroeconomic variables can be useful for modelling and better understanding private and public investment decisions. The main findings reported here can be useful to add to the discussion on private investment, based on corporate variables or related to the household cycle of life model [e.g. Gala et al. (2020), and Rannenberg (2021), respectively], and can help to address the drivers of public investment [e.g. Matos (2024)]

This paper proceeds as follows. In Section 2 I show the related literature. In Section 3 I briefly describe the methodology. In Section 4 I analyze the data and discuss the results. Section 5 offers the concluding remarks.

2. Related Literature

The macroeconomic effects of the public investments depend on the efficiency, the marginal productivity of capital, and finally, on whether private investment flows into the industries. Actually, this complex and relevant (crowding in or out) effect has a long history in economic theory. During the seventies, due to a sequence of recessions, the dangers of public spending crowding out private spending, as well as the benefits and need of crowding in reaction, both have been emphasized in the literature [e.g. Friedman (1972), Blinder and Solow (1973), and Tobin and Buiter (1976)]. Although the empirical economic literature has reported cases where public investments can crowd out and crowd in private investment, there are well known specific circumstances when public investments is be able to mobilize private investments, while the displacement of private economic activity by public economic activity is expected under other circumstances.

One of the main characteristics in this discussion is the divergence over the choice of the theoretical model, and consequently, over the value of the fiscal multiplier of government investment and consumption. In addition to such divergences due to "economic schools", Buiter (1977) argues that: "often neither the impact effect nor the long-run, steady-state effect correspond to the 'run' one is most interested in for policy purposes. For policy, the real (i.e. calendar) time effects of policy changes over a period of, say, a few years tend to be most pertinent. In principle this represents no great problems."

According to him, the method of solving the dynamic system and comparing trajectories under different assumptions about initial conditions, other parameter values or the behavior of policy control variables permits one to find the degree of crowding out and crowding in for any time interval. In practice, more than half a century ago, the complexity of this relationship between public and private investment was already recognized, but analytical solutions were difficult to obtain. Empirically, in the seventies it could be complicated to study this phenomenon with results varying in frequency and time, and enabling reverse causality, but it is no longer a problem nowadays. Fortunately, for my purpose in this study, it is not necessary to use a specific model that impose or assume that there must be crowding in or crowding out, in the short or long-term, and the conditional continuous wavelet-based tools enables me to trace transitional changes across time and frequencies, and also accounts for the possible leadership by private investment cycles.

In order to characterize the relevance and novelty of this paper, it is important to mention the main empirical contributions on crowding in and crowding out effects.

Regarding applications on the relationship between government spending and private investment based in panel estimations, Argimon et al. (1997) find the existence of a significant crowding-in effect of private investment by public investment, through the positive impact of infrastructure on private investment productivity, for a set with 14 OECD countries. They argue that deficit reductions engineered through cuts in public investment could severely impinge on private capital accumulation and growth prospects. A decade later, Ahmed and Miller (2007) revisit this issue by using fixed- and random-effect methods. Their analysis explores the effects of tax- and debt-financed expenditure for the full sample, and for subsamples of developed and developing countries. They find that tax-financed government expenditure crowds out more investment than debt-financed expenditure.

More recently, Dreger and Reimers (2016) propose exploring the long run relationship between public and private investment in the euro area, by applying a stock-flow approach to control for the different orders of integration. They find that the lack of public investment may have restricted private investment and thus GDP growth in the euro area. Bom (2017) aims to study the dynamic effects of public investment on private capital accumulation in a general equilibrium macroeconomic model of a small open economy with factor- biased public capital. He calibrates the values of key aggregate ratios in order to match the corresponding euro area averages over the period 1995–2015, and he finds that whether private investment is crowded in or out during transition critically depends on parameters that are empirically hard to measure, such as the labor supply elasticity and the elasticity of substitution between private inputs.

Considering the exercise proposed here, it is even more relevant to observe the related empirical literature applied to the United States. In Erenburg and Wohar (1995), the influence of the provision of public infrastructure on private investment activity is examined by including public sector investment spending and public capital stock along with the variables specified in the major theoretical private investment models. They use the multivariate Granger-Causality testing procedure considering annual data over the period 1954-1989. They find that the effect of government investment spending on private investment exhibits a significant overall effect, but the direction of the impact on private investment is ambiguous: there is an inverse statistical effect of the first three lags, however, the fourth lag is statistically significant and positive. Given the lag between spending on the public project and project completion, firms who benefit from public capital may postpone equipment investment plans until the public capital is available for use. In fact, it is precisely the accumulation of public sector investment spending that results in the addition to the public capital stock.

Voss (2000) examines private and public investment during the last four decades to determine what support there is, if any, for the crowding in hypothesis, by using data for the U.S. and Canada. Based on neo-classical theories of investment, simple VAR models of private and public investment behavior are used to examine the short- and long-run interactions between these variables. For both countries they find no evidence of crowding in due to complementarities between public and private investment; in fact, innovations to public investment tend to crowd out private investment. Mountford and Uhlig (2009) develop a novel methodology for distinguishing the effects of fiscal policy shocks, using the information in the macroeconomic time series of the vector autoregression together with minimal assumptions to identify fiscal policy shocks. Considering post war data on the U.S. economy, they find that a deficit spending scenario stimulates the economy for the first 4 quarters but only weakly compared to that for a deficit financed tax cut. They also report that both types of spending scenario had the effect of crowding out investment.

Another study aligned with this fiscal issue impacting investment in U.S. is Traum and Yang (2015). They use Bayesian inference methods to measure the extent of crowding out in a dynamic stochastic general equilibrium model with quarterly data from 1983:1 to 2008:1. They find that higher government debt can crowd in investment if the debt is generated by a reduction in capital tax rates or by an increase in productive government investment, because both raise the net return to capital. Over a longer horizon, distortionary financing plays a key role in the negative investment response following a debt expansion.

An even more connected contribution to this study is Blackley (2014). He aims to study the effect on private investment based on an autoregressive model considering cointegration from 1956:1 – 2010:2, and he finds that there is no crowding out effect in the long run in America, but there is crowding in in the short run.

3. Methodology

3.1. Wavelet overview

In the early 1800's, Joseph Fourier discovered a powerful tool to model time series on frequency domain, assuming that any periodic function can be expressed by an infinite sum of trigonometric functions. The Fourier transform's utility lies in its ability to analyze a signal in the time domain for its frequency content. The transform works by first translating a function in the time domain into a function in the frequency domain. An inverse transform does what it is expected, transform data from the frequency domain into the time domain.

Besides the appealing of this approach to evaluate financial time series on frequency domain, the function does not allow decompose the time series into different time scales, a limitation of its applicability. Moreover, these techniques are only appropriate for time series with stable statistical properties, i.e. stationary time series. However, typical economic time series are noisy, complex and strongly non-stationary. In other words, while several questions about time series economic data are connected to the understanding of the behavior of key variables at different frequencies over time, this type of information is difficult to uncover using pure time-domain or pure frequency-domain methods.

To overcome the problems of analyzing non-stationary data, Gabor (1946) introduced the short time Fourier transform. The basic idea is to break a time series into smaller sub-samples and apply the Fourier transform to each sub-sample. However, as Raihan et al. (2005) pointed out, this approach is inefficient because the frequency resolution is the same across all different frequencies. As an alternative to overcome all these problems, wavelet analysis has been proposed, and its tools were generalized to accommodate the analysis of time–frequency dependencies between two time series. In this context, some researchers in this field feel that, by using wavelets, one is adopting a whole new mindset or perspective in processing data.

According to some surveys on this issue,[1] wavelet analysis performs the estimation of the spectral characteristics of any time series as a function of time, revealing how the different periodic components of the time series change over time. Aiming to compare, while the Fourier transform breaks down a time series into constituent sinusoids of different frequencies and infinite duration in time, the wavelet transform expands the time series into shifted and scaled versions of a function (mother wavelet) that has limited spectral band and limited duration in time. The major advantage of using wavelet expansion is analyzing a time series with the length of wavelets varying endogenously, since it stretches into a long wavelet function (wide window) to measure the low frequency movements, and it compresses into a short function (narrow window) to measure the high frequency movements.

More specifically, first, it is necessary to use wavelet algorithms to process data by decomposing each time series as a wave-shaped function of the position and scale.[2] Next, most of empirical applications of such technique have followed Theiler et al. (1992) to construct the surrogate series of and randomize the phase of the original data. With this transformation, the surrogate series keep the mean, standard deviation, autocorrelation and partial auto-correlation functions and the power spectrum of the original data. In the method of surrogate data, this distribution is estimated by direct Monte Carlo simulation. An ensemble of surrogate data sets are generated which share given properties of the observed time series (such as mean, variance, and Fourier spectrum).

Finally, I utilize the multiple coherency and three partial cross-wavelet tools: the partial wavelet coherency, the partial wavelet phase difference, and the partial wavelet gain. Since theoretical distributions for wavelet coherency have not been derived yet, to assess the statistical significance of the estimated wavelet coherency, one has to rely on Monte Carlo simulation methods. With these instruments I am able to unravel some economic time–frequency relations that have remained hidden so far.

3.2. Wavelet analysis

The wavelet transform decomposes a time series in terms of small wavelets. They grow and decay in a limited time period, and they result from a mother wavelet that can be expressed as function of the time position and the scales, which is related with the frequency. Based on the approximation using superposition of such functions one can measure the variance distribution of a given variable and address the instrumentalized co-movements between a set of variables in the time-frequency domain.

Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau,s) = \int_{-\infty}^{+\infty} x(t)\,\psi^*_{\tau,s}(t)dt \tag{1}$$

[1] See Graps (1995), Torrence and Compo (1998), Crowley (2007), Aguiar-Conraria et al. (2008), In and Kim (2013), and Gallegati and Semmler (2014).

[2] The continuous wavelet-based filter can be compared to the filter proposed by Baxter and King (1995), since it approximates a pass band filter, letting pass frequencies between high and low frequency. Second, to capture the high and low frequencies of the signal, we need a basic function or a "mother wavelet" that is stretched and shifted.

where τ determines the position, s is the scaling factor, * denotes the complex conjugate, and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the decomposition of the time series both in space and scale. To capture the high and low frequencies of the signal, we need a basic function or a "mother wavelet" that is stretched and shifted (In and Kim, 2013):

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{2}$$

The factor $1/\sqrt{s}$ is added aiming the preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequency movements. The CWT is defined by:

$$W_x(\tau,s) = \int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)dt \tag{3}$$

The basis function $\psi_{\tau,s}$ must obey some criteria, such as: admissibility, similarity, invertibility, regularity, and vanishing moments. There are many options of wavelet mother functions to select, which includes Daubechies, Haar, Mexican Hat, Morlet and Meyer wavelets. Aguiar-Conraria and Soares (2011) highlight the importance of that choice and suggest picked up an analytic wavelet to study the synchronism between oscillatory signals because its corresponding transform contains information on both amplitude and phase, providing an estimate of the instantaneous amplitude and instantaneous phase of the signal in the neighboring of each time/scale location (τ, s). On subset of analytic wavelet, the Morlet wavelet mother seems to be the most popular alternative because some properties. This basic function is given by:

$$\psi_{\omega_0}(t) = \pi^{-1/4}e^{i\omega_o t}e^{-t^2/2} \tag{4}$$

where the non-dimensional frequency ω_0 is set $\omega_0 = 6$ to satisfy the admissibility condition (Torrence and Compo, 1998). As the wavelet transform decomposes the original signal in a time-scale domain, which put us the necessity to convert scale into frequency. In this sense, the Morlet wavelet is an ideal alternative because it provides us a unique relation between frequency and scale (the peak frequency, the energy frequency and the central instantaneous frequency are all equal) which makes it easier the conversion from scales to frequencies. The choice of $\omega_0 = 6$ give us a conversion ratio equal $f = \frac{6}{2\pi s} \approx \frac{1}{s}$, that direct correspondence between scale and frequency is ideal to simplify an effective interpretation of the results.

Finally, because the CWT is applied on finite-length time series, border distortions will occur due the fact that values of the transform at the beginning and the end of the sample are imprecisely computed, which involve artificial padding on the extremes of the sample (the most common is set zero to extend the time series). As larger scales decrease the amplitude near the edges as more zeroes enter the analysis (Torrence and Compo, 1998), the region that suffers from these edge effects is function of s. The Cone of Influence (COI) is the region of the wavelet spectrum in which edge effects become important by a factor of e^{-2}. In the case of the Morlet wavelet this is given by $\sqrt{2s}$.

Based on the approximation using superposition of such functions, I follow Aguiar-Conraria et al. (2014) and Aguiar-Conraria et al. (2018), by using high-order wavelet tools. As for the notation, given p time-series, $x_1, \dots, x_p$ with $p > 2$, this literature usually denotes by W_i the wavelet spectrum corresponding to the ith time-series and by W_{ij} the cross-wavelet spectrum of two series, x_i and x_j, such that $1 \le i \le p, 1 \le j \le p$. It is common denoting by S_{ij} the smoothed version of W_{ij}, and using φ to denote the $p \times p$ matrix of all the smoothed cross-wavelet spectra S_{ij}. Still according to Aguiar-Conraria et al. (2018) notation, the squared multiple wavelet coherency between x_1 and all other series $x_2, \dots, x_p$ is denoted by $R^2_{1(2\dots p)}$ given by

$$R^2_{1(2\dots p)} = 1 - \frac{\varphi^d}{S_{11}\varphi^d_{11}} \tag{5}$$

where for a given matrix φ, φ_{ij} denotes the sub-matrix obtained by deleting its ith row and jth column and φ^d_{ij} denotes the co-factor of the element in position (i, j) of matrix φ.

The partial wavelet coherency of x_1 and $x_j, 2 \leq j \leq p$ allowing for all the other series is denoted by $R_{1j.q_j}$ and given by

$$R_{1j.q_j} = \frac{|\varphi_{j1}^d|}{\sqrt{\varphi_{11}^d}\sqrt{\varphi_{jj}^d}} \tag{6}$$

The partial phase difference of x_1 and $x_j, 2 \leq j \leq p$ given for all other series, denoted by $\phi_{1j.q_j}$, is given by the angle of the complex partial wavelet coherency, $\varrho_{1j.q_j}$, given by $\varrho_{1j.q_j} = -\frac{\varphi_{j1}^d}{\sqrt{\varphi_{11}^d}\sqrt{\varphi_{jj}^d}}$.

Concerning the trigonometric interpretations of the partial phase difference applied to the relationship between investment by private sector and federal government, for example, a null phase-difference indicates that the time-series move together (phasic co-movement) at such frequency, without leadership. If $\phi_{1j.q_j} \epsilon \left(0, \frac{\pi}{2}\right)$ the series move in phase, with the time-series of private investment cycle leading the federal government investment cycle, while if $\phi_{1j.q_j} \epsilon \left(-\frac{\pi}{2}, 0\right)$ then federal government investment cycle is leading in the same direction. A phase-difference of $\phi_{1j.q_j} = \pm\pi$ indicates an anti-phase relation without leadership. If $\phi_{1j.q_j} \epsilon \left(\frac{\pi}{2}, \pi\right)$, then federal government investment cycle is leading private investment in the opposite direction, and if $\phi_{1j.q_j} \epsilon \left(-\pi, -\frac{\pi}{2}\right)$, then private investment cycle is leading this anti-phasic co-movement. The same procedure applies when studying private investments versus state and local government, instead of federal government.

Finally, following Aguiar-Conraria et al. (2018) it is usual to define the partial wavelet gain as follows:

$$G_{1j.q_j} = \frac{|\varphi_{j1}^d|}{\varphi_{11}^d} \tag{7}$$

I use this concept of partial wavelet gain proposed by Aguiar-Conraria et al. (2018) as a regression coefficient in the multiple regression given by

$$PRIV_t = \alpha + \beta FED_t + \gamma STALOC_t + \boldsymbol{\theta Z_t} + \varepsilon_t \tag{8}$$

where $PRIV_t$ is private investment to GDP, FED_t means investment to GDP by federal government, and $STALOC_t$ denotes investment to GDP by state and local government. The set of macroeconomic variables as instruments is given by $\boldsymbol{Z_t}$, and ε_t means the residual. Concerning the coefficients, $\boldsymbol{\theta}$ is the vector associated to macro variables, while β and γ are the intensity of crowding in (positive) and crowding out (negative) effects in response to variations in investments by federal government and state and local government, respectively. The main difference and advantage is the possibility of tracing transitional changes across time and frequencies.

3.3. Wavelet-based applications

Summarizing the extensive literature, I mention here only some studies published in different journals in the last decade, addressing macrofinance issues in the United States.

In Lo Cascio (2015), the author revisits the fiscal reaction over the period 1795-2012, using wavelet power spectrum, wavelet coherency, and phase-difference. Aguiar-Conraria et al. (2018) estimate the Taylor rule over the period 1965:4 to 2017:2 using wavelet, and they also propose a novel wavelet-based tool: the partial wavelet gain. Verona (2020) is closely aligned both methodologically and in the sense of modeling the behavior of private investment. The biggest difference is that he studies the relationship between private investment and Tobin's Q and cash flow. The study uses quarterly aggregate US data from 1952:1 to 2017:4.

Matos et al. (2021a) assess the conditional relationship in the time-frequency domain between the return on S&P 500 and confirmed cases and deaths by COVID-19 in Hubei, China, countries with record deaths and the world, for the period from January 29 to June 30, 2020. In Matos et al. (2021b), the authors address instrumentalized co-movements across time and frequencies between macro-finance variables and household decisions in terms of consumer loans, home mortgage and its respective delinquency rates, based on partial wavelet tools covering the period from 1991:2 to 2018:1.

Giri (2022) investigates if core inflation is a good approximation of headline inflation that considers the entire bundle, using the wavelet methodology covering a time span from January 1968 to June 2021. Aguiar-Conraria et al. (2023) the U.S. New Keynesian Phillips Curve in the time-frequency domain with continuous wavelet tools using quarterly time-series for 1978:1–2022:2 of inflation, the unemployment gap, expectations of inflation and energy inflation.

Aiming to position this paper in this wavelet-based applied macrofinance literature, I must highlight that my purpose is to capture the conditional comovements, to identify the lead-lag relationship and to measure its magnitude based on the estimation of regression coefficients in a time-frequency domain, considering investment by private sector, by federal government, and by state and local government. The macroeconomic data used here is most of the times noisy, strongly nonstationary, with well documented nonlinear and dynamic relations. The nature of wavelet analysis makes it suitable to use with this kind of data.

4. Empirical Exercise

4.1. Data

Wavelet analysis is quite data demanding, and using data with the highest frequency and the longest time span as possible is a bonus. I use quarterly data for the American economy over the period 1952:1–2023:2 (286 observations). As for the main variable, I use investment as a percentage of GDP (annual rate) by federal government, private sector,[3] and state and local government. I use total investment, as well as investment by type: structures, equipment, and intellectual property products. Regarding the macro instruments, I use: i) Net federal government saving to GDP (FGDEF); ii) Federal government debt securities and loans to GDP (FGSDODNS); iii) Variation of real GDP (GDP); iv) 3-Month treasury bill rate (TB3MS), and v) CPI (CPIAUCSL). The data source if the Federal Reserve Economic Data (FRED).[4]

In Table 1, I report the main summary statistics for the series of investment.

First, I find that private sector, households and firms, invest on average more than the federal government and state and local government, considering total investment and investment broken down by types. Except for investment in structures, the federal government invests more than state and local government. This average pattern is observed in recessionary and non-recessionary periods.

Observing the "market share" extremes, it is noteworthy that the public sector's greatest role occurs in investments in intellectual property products, with investment by federal government nearly half the amount invested by the private sector. On the other hand, the difference between private and federal government investment is greater considering structures, a type of investment in which the private sector invests from 19 to 24 times more, depending on whether there is or not a recession. Finally, the superiority of the private sector in relation to state and local government is more pronounced in investments in intellectual property products and equipment, with values twenty (or more) times greater.

Considering total investment, or by types, for both the private and public sector, it is clear that during the period of recession there is less volatility of the investment to GDP ratios. Except for investment in structures, average private investment to GDP is higher during non-recession periods. On the other hand, federal government uses to invest more during recession periods considering all types of investment, while this is true for state and local government only for investment in structures and consequently total investment.

It is also interesting analyzing the correlations between public and private investments. Considering total investment there is a negative correlation of private investment and investment by federal government, as well as by state and local government, a prior indication of possible short-term crowding out response. The absolute order of magnitude of these correlations is higher during recessions. However, it is curious, because when I analyze the correlations by type of investment, I find new results.

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[3] Private fixed investment (PFI) measures spending by private businesses, nonprofit institutions, and households on fixed assets in the U.S. economy. PFI assets consist of structures, equipment, and intellectual property products that are used in the production of goods and services. PFI encompasses the creation of new productive assets, the improvement of existing assets, and the replacement of worn out or obsolete assets, and is an integral part of the U.S. NIPA.

[4] The respective FRED series codes of the instruments are reported in parenthesis and the codes of investment series are reported in the notes of Table 1.

Table 1. Summary statistics for investment series over the period 1952:1 to 2023:2.

Statistics	Total Investment			Investment in Equipment			Invest. in Intell. Proper. Produc			Investment in Strucutres		
	Private	Fed Gov	Sta/Loc Gov	Private	Fed Gov	Sta/Loc Gov	Private	Fed Gov	Sta/Loc Gov	Private	Fed Gov	Sta/Loc Gov
Statistics during NBER recession periods (39 quarterly observations) from 1952:1 to 2023:2												
Mean	16.51%	2.78%	2.44%	6.18%	1.15%	0.27%	2.38%	1.21%	0.11%	7.95%	0.42%	2.06%
St. Dev.	5.70%	1.02%	0.84%	2.15%	0.48%	0.10%	0.93%	0.43%	0.04%	2.75%	0.18%	0.72%
Minimum	14.32%	1.54%	1.99%	4.60%	0.47%	0.16%	0.96%	0.67%	0.02%	5.83%	0.13%	1.61%
Maximum	18.85%	4.86%	2.82%	7.65%	3.11%	0.40%	5.57%	1.61%	0.25%	9.13%	1.08%	2.57%
Var. Coef.	0.35	0.37	0.35	0.35	0.41	0.35	0.39	0.35	0.42	0.35	0.43	0.35
Correl | Private		-0.63	-0.55		-0.40	0.38		-0.33	0.98		0.52	0.10
Statistics during NBER non-recession periods (247 quarterly observations) from 1952:1 to 2023:2												
Mean	16.96%	2.51%	2.28%	6.33%	0.97%	0.28%	2.78%	1.21%	0.13%	7.86%	0.33%	1.88%
St. Dev.	5.98%	1.22%	0.84%	2.28%	0.63%	0.11%	1.52%	0.50%	0.08%	2.90%	0.25%	0.73%
Minimum	13.81%	1.45%	1.78%	4.64%	0.39%	0.16%	0.75%	0.61%	0.02%	4.70%	0.09%	1.35%
Maximum	20.00%	4.96%	3.06%	8.17%	3.16%	0.40%	5.49%	1.85%	0.24%	10.08%	1.24%	2.75%
Var. Coef.	0.35	0.49	0.37	0.36	0.65	0.41	0.55	0.42	0.60	0.37	0.76	0.39
Correl | Private	-	-0.51	-0.43	-	-0.32	0.55	-	-0.56	0.97	-	0.45	0.34

Data source: FRED. Series codes: A787RC1Q027SBEA, A801RC1Q027SBEA, Y058RC1Q027SBEA, Y059RC1Q027SBEA, FPI, GDP, B013RC1Q027SBEA, Y033RC1Q027SBEA, Y001RC1Q027SBEA, B009RC1Q027SBEA, A012RC1Q027SBEA, A842RC1Q027SBEA, Y070RC1Q027SBEA, SLINV, Y071RC1Q027SBEA.

The correlation between investment by the private sector and federal government remains negative, except for investment in structures. The correlation between private investment and state and local government becomes positive for all three types of investments. This heterogeneity of results shows how important it is to study these relationships by disaggregating the public sector and investment by type, the purpose of this paper.

Fig. 1 displays the time series of investment by player and by type.

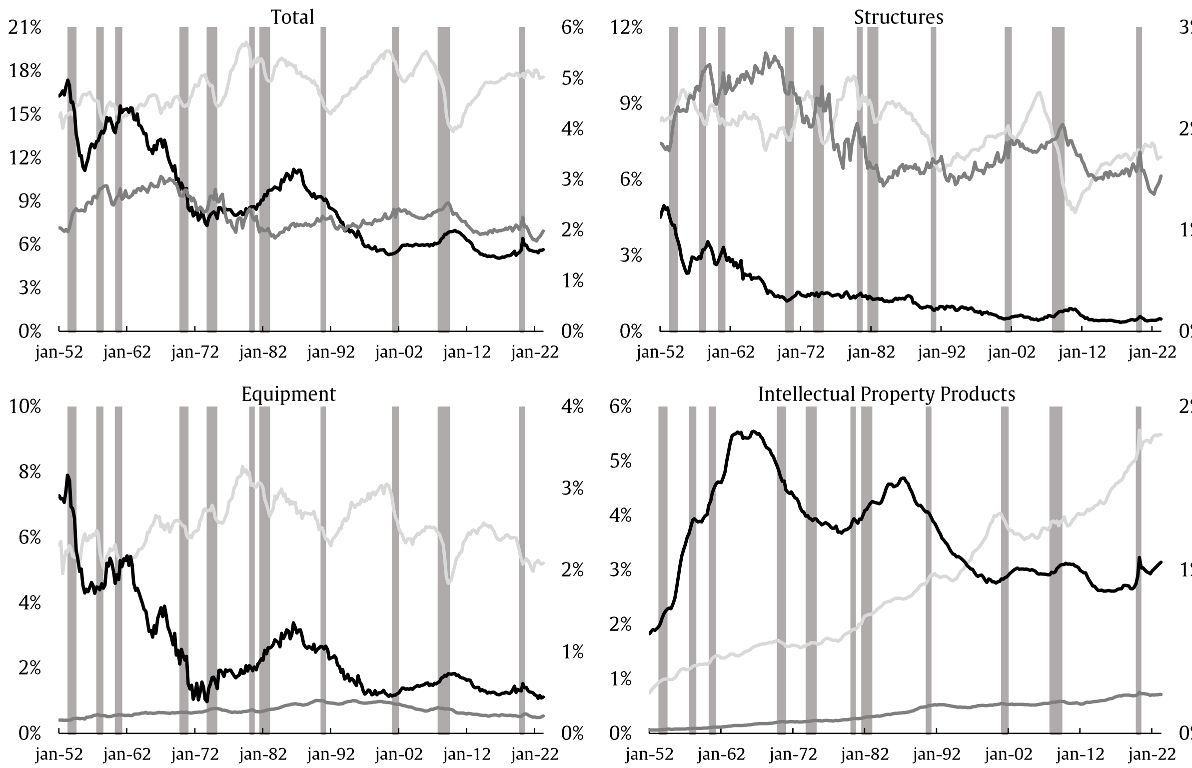

Fig. 1. Investment to GDP (annual rate) by Federal Government (dark gray on the right axis), State and Local Government (medium gray on the right axis) and Private Sector (light gray on the left axis). Quarterly series from 1952:1 to 2023:2. Data source: FRED. Series codes: A787RC1Q027SBEA, A801RC1Q027SBEA, Y058RC1Q027SBEA, Y059RC1Q027SBEA, FPI, GDP, B013RC1Q027SBEA, Y033RC1Q027SBEA, Y001RC1Q027SBEA, B009RC1Q027SBEA, A012RC1Q027SBEA, A842RC1Q027SBEA, Y070RC1Q027SBEA, SLINV, Y071RC1Q027SBEA. Notes: Shaded areas correspond to NBER recession periods.

In all figures, only the private investment series is associated with the left axis. It is important to highlight that the order of magnitude of public investments is always smaller than private investment, except for the investment in intellectual property products from 1957:2 to 1969:2, when the federal government used to invest more than the private sector. From then on, this type of investment by firms and households increased almost linearly, except for the effects of the crisis in the early 2000s, while federal government has varied and, in the end, stagnated at the level of 1% of GDP. This type of investment by state and local government shows a smooth growth trend.

Regarding investments in structures, while the federal government presents a low average level and a smooth downward trend from the 1960s onwards, the private sector and state and local government hold a relationship oscillating between moments with opposite movements and periods with series in the same direction. This variation between phasic and antiphasic unconditional co-movements between private and state and local government also appears to be present in investments in equipment, while federal government shows a growth trend until the end of the nineties, with a smooth and declining trend thereafter.

Corroborating the evidence already identified in Table 1, drops of private investment as a result of recessions are visible in all figures. Fig. 2 displays the scatter plot considering total investment and by type, private and public players, during recession and non-recession periods.

All figures on the left side relate to private and federal government investment, while the figures on the right side relate to state and local government investment. The blue points (247 observations) refer to the non-

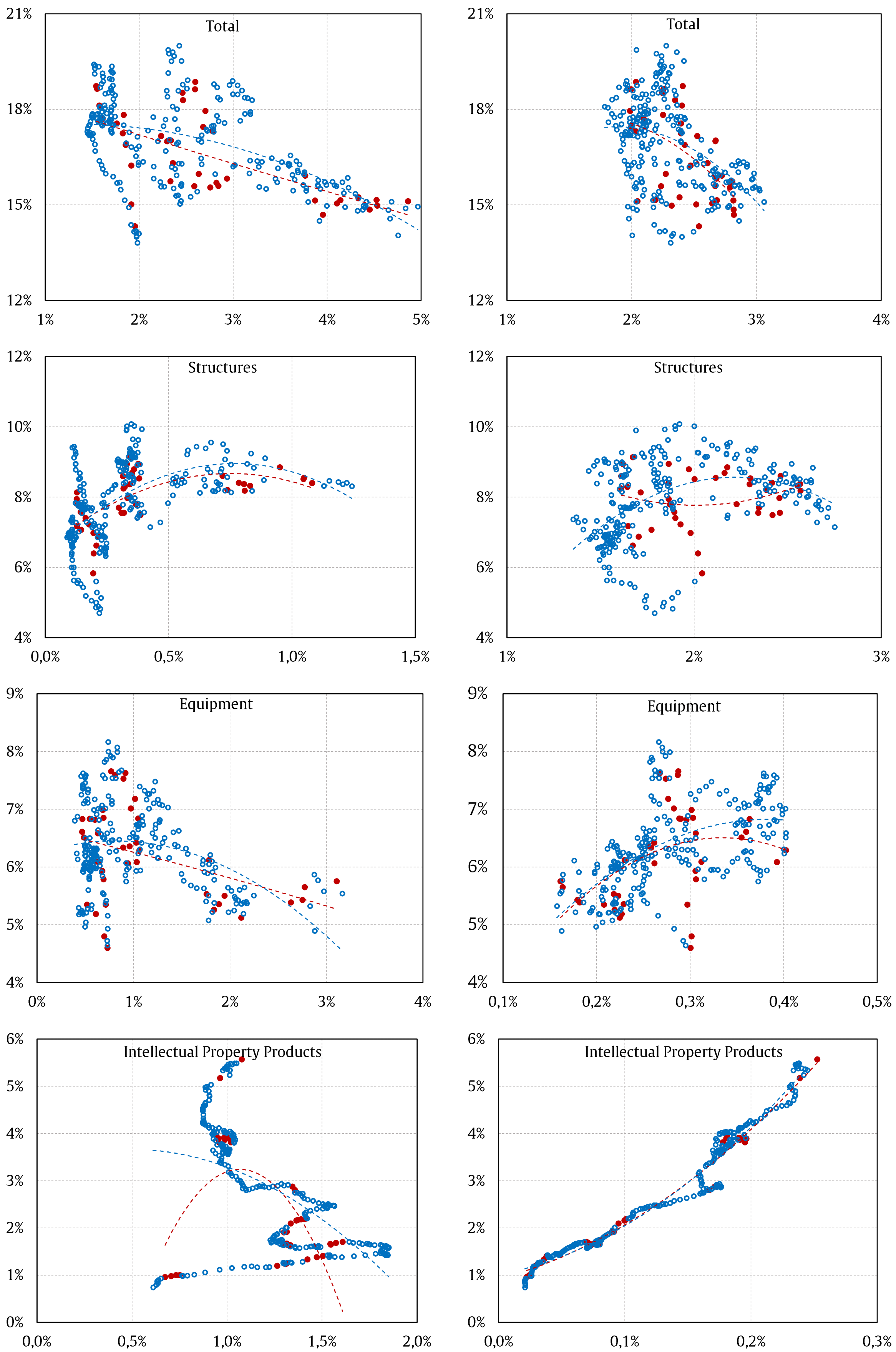

Fig. 2. Investment to GDP by Private sector (vertical axis) versus Federal Government (horizontal axis) in the left column and versus State and Local Government (horizontal axis) in the right column. Quarterly series from 1952:1 to 2023:2. Data source: FRED. Notes: 39 observations during recessions (red) and 247 observations otherwise (blue).

recession period and the red points (39 observations) to the recession period. The importance of analyzing crowding in and crowding out effects breaking down the public sector is clear. As an example, the positive linear relationship between investments in intellectual property products by the private sector and by state and local government is strong, with a correlation of 0.97 (Table 1), while the relationship with the federal government is apparently much more dynamic and complex. The behavior in periods of recession and non-recession can also be different, and it is important to consider GDP growth as an instrument in this analysis. Dispersion clouds are very heterogeneous depending on the type of investment, reinforcing the importance of disaggregated analysis. Finally, the lines able to fit the points suggest that there has been a change in the slope and curvature over the seven decades, intuitive prior evidence that needs to be incorporated into the analysis, through a technique that allows time-varying results.

4.2. Preliminary Results: Explanatory Power

The first step is measuring the joint explanatory power of the set of public investment, and economics variables used here to explain the responses of total private investment to GDP, and private investment by type. I use the squared multiple wavelet coherency given by $R^2_{1(2....8)}$, reported in equation (1), assuming that x_1 is the private investment, and all other series $x_2, \ldots, x_8$ are investment by federal government, investment by state and local government, net saving to GDP, debt to GDP, growth, CPI, and interest rate. This feature resembles and is comparable to the explanatory power of temporal regressions, measured by R^2, for instance.

I show the results on this multivariate coherency in Fig. 3.

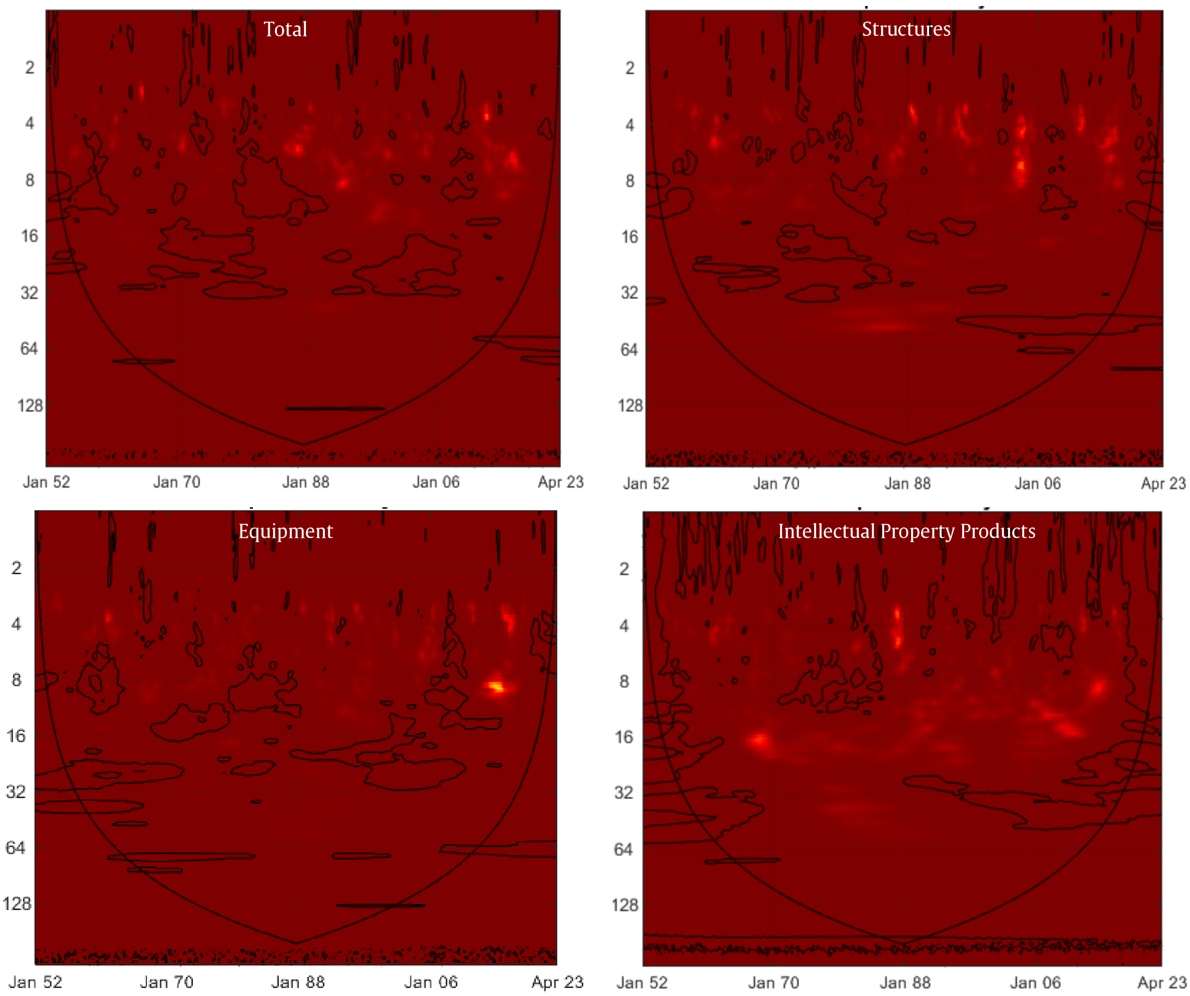

Fig. 3. Multiple Coherency. Notes: Frequency (quarters) in the vertical axis

One must realize that $R^2_{1(2....8)}$ ranges from low (blue) to high (red) values, as well as the respective cone of influence (5% significance level) with a black line. The color and the significance (5% given by the black line) in the heat map measure the degree of adjustment in the time-frequency domain. There are four heat maps, because I use total investment cycle, and the cycles of three types of investment. In all heat maps, there are no blue areas, and there are only occasional yellow areas, with red predominating. In other words, I know that the set of variables was not defined based on a specific theoretical model. Despite this, the potential for joint explanation in the location-scale dimension suggests that theoretical models should incorporate these fiscal, monetary and public investment variables to study private investment cycles. Analyzing Fig. 3 in detail, a common aspect of all heat maps is the presence of several significant areas, mainly in the frequency ranging between 4 and 32 quarters, i.e., from 1 to 8 years. In the heat map of investments in intellectual property products, there is a concentration of significant areas at a very high frequency, between 0 and 1 year.

To summarize, the presence of successive areas with significance statistics in the multiple coherencies color map denotes a good overall fit in the model and the next step is analyzing partial coherency, partial phase-difference with two standard deviations and partial gain or coefficient regression.

4.3. General Results: Partial Wavelet-based Evidence

I report in Fig. 4 (top) the conditional wavelet-based results between private total investment to GDP and total investment to GDP by federal government, controlled by total investment to GDP by state and local government, net saving to GDP, debt to GDP, CPI, interest rates, and growth. In Fig. 4 (bottom), I show the co-movement between private total investment to GDP and total investment to GDP by state and local government, controlled by total investment to GDP by federal government, net saving to GDP, debt to GDP, CPI, interest rates, and growth. In an analogous way, in Fig. 5, Fig. 6, and Fig. 7, I show the same results for investment to GDP in equipment, intellectual property products and structures, respectively.

The way to understand each result is similar. First, I look at the partial wavelet coherency heat map (left side). For each red and significant area, I identify the respective period and frequency. For a frequency: i) lower than 8 quarters (2 years), the next step is looking at the first partial phase difference (top graphic), ii) between 8 and 16 quarters (from 2 to 4 years) I look at the second graph (intermediate graphic), and iii) between 16 and 32 quarters (from 4 to 8 years) I look at the last graph (bottom graphic). This same procedure goes for the graphs of partial gain reported (right side), where I report the intensity of the regression coefficient.

Once the results based on Figures 4, 5, 6 and 7 have been analyzed, in subsection 4.4, I analyze the results on at what time and with what frequency there is crowding in or crowding out. In subsection 4.5, I show under which fiscal and monetary scenarios crowding in/out effects occur. In subsection 4.6, I analyze the conditions under which private investment cycles are capable of anticipating public investment cycles. [5]

4.4. Main Results: Crowding In and Crowding Out Effects

Using the heat maps and phase difference diagrams, I identify the periods in which there are crowding in/out effects. It is usual in this literature to assume ranges that delimit high, medium and low frequencies, and I assume that between 0 and 2 years, the co-movements are of high frequency, between 2 years and 4 years, medium frequency and between 4 years and 8 years, low frequency. The intensities of the crowding in (positive values) and crowding out (negative values) effects varying over time and in the range of frequencies are measured based on the estimation of the coefficients β and γ, depending on whether the government is the federal government, or the state and local government. I report the results for total investment, and investment in equipment, intellectual property products and structures, in Tables 2, 3, 4 and 5, respectively. Observing these tables, it is possible to highlight a heterogeneity of results over time and depending on the frequency. An interesting and didactic way of representing the results obtained in Tables 2, 3, 4 and 5 is through Figures 8, 9, 10 and 11, which follow the same sequence of types of investment in the Tables mentioned.

[5] The mother wavelet function coefficients work as a smoothing filter (like a moving average) and work to bring data′s detail information. For this reason, it is usual in this literature on empirical wavelet analysis using the term cycles.

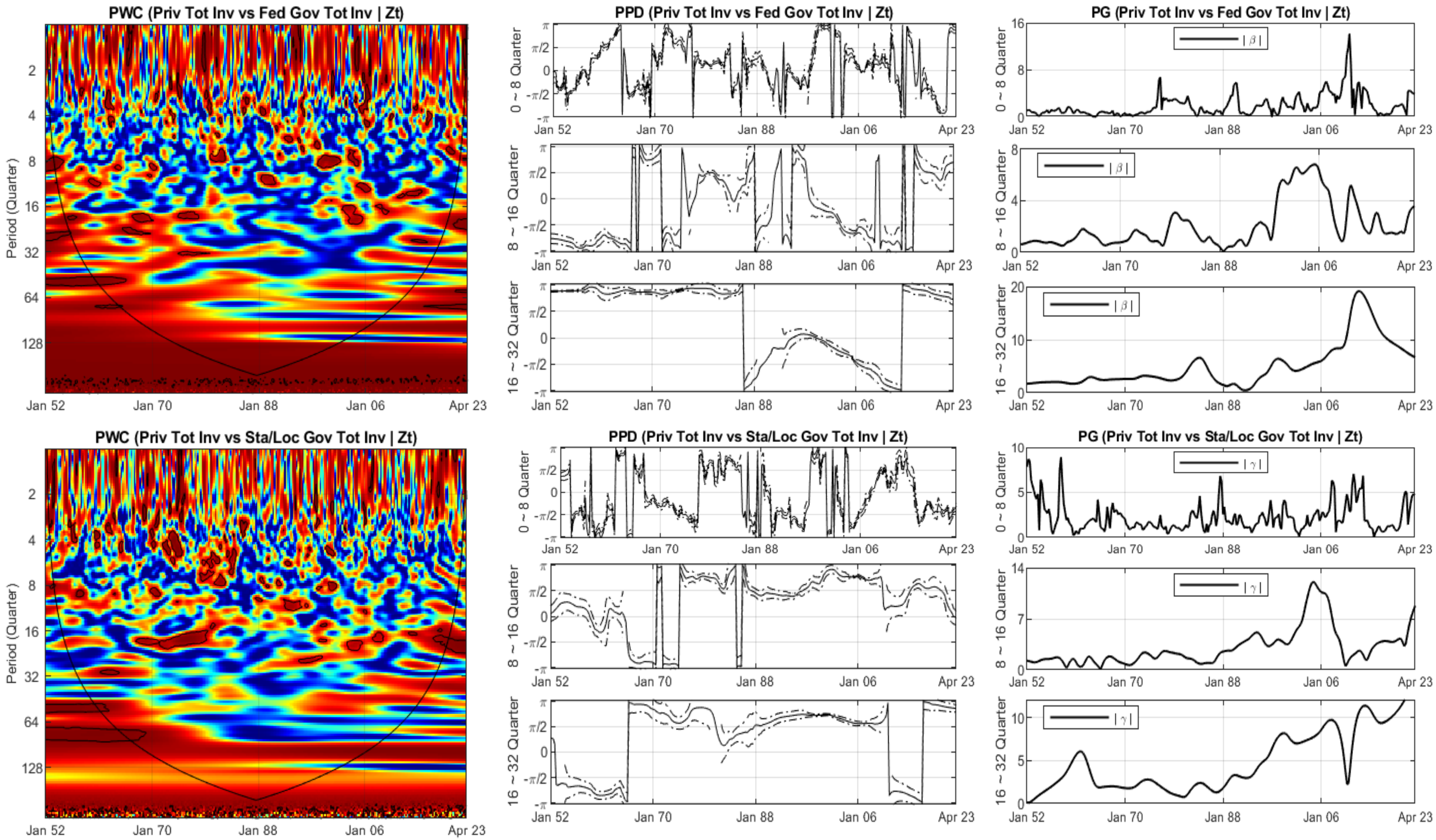

Fig. 4. Conditional Relationship between Total Investment by Private Sector, Federal Government, and State and Local Government: Partial Wavelet coherency (left), Partial Phase-difference (center) and Partial Gain (right).

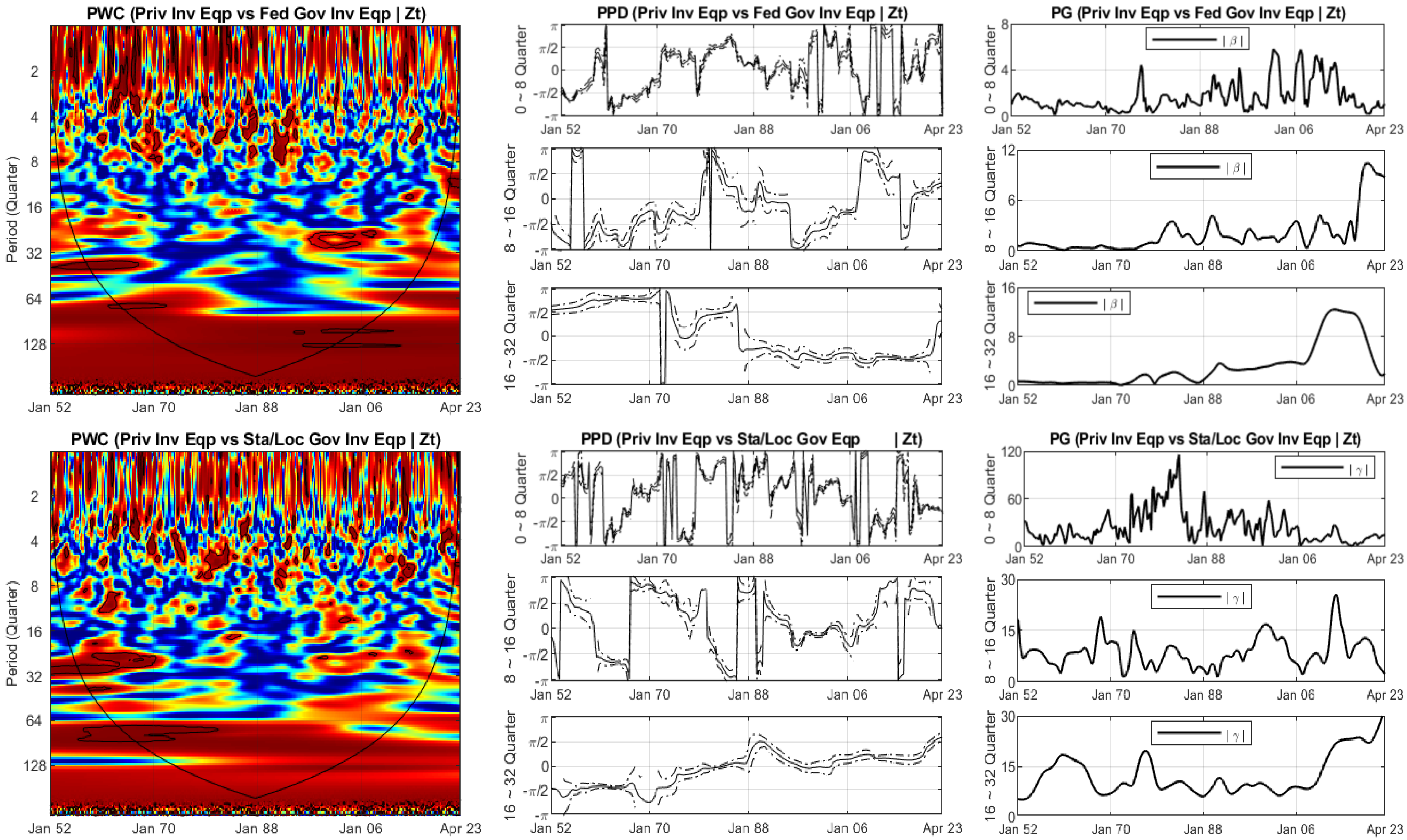

Fig. 5. Conditional Relationship between Investment in Equipment by Private Sector, Federal Government, and State and Local Government: Partial Wavelet coherency (left), Partial Phase-difference (center) and Partial Gain (right).

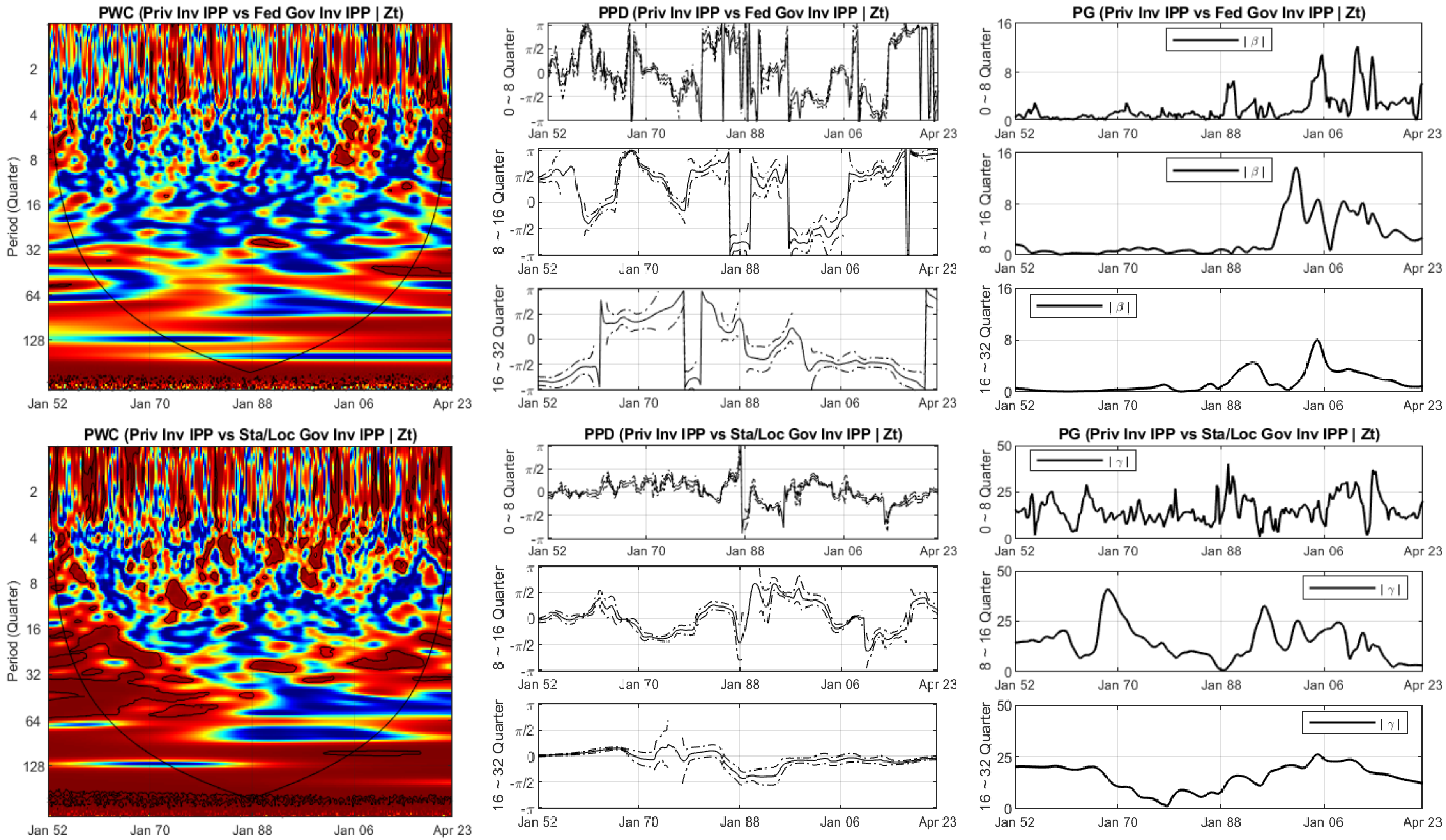

Fig. 6. Conditional Relationship between Investment in Intellectual Property Products by Private Sector, Federal Government, and State and Local Government: Partial Wavelet coherency (left), Partial Phase-difference (center) and Partial Gain (right).

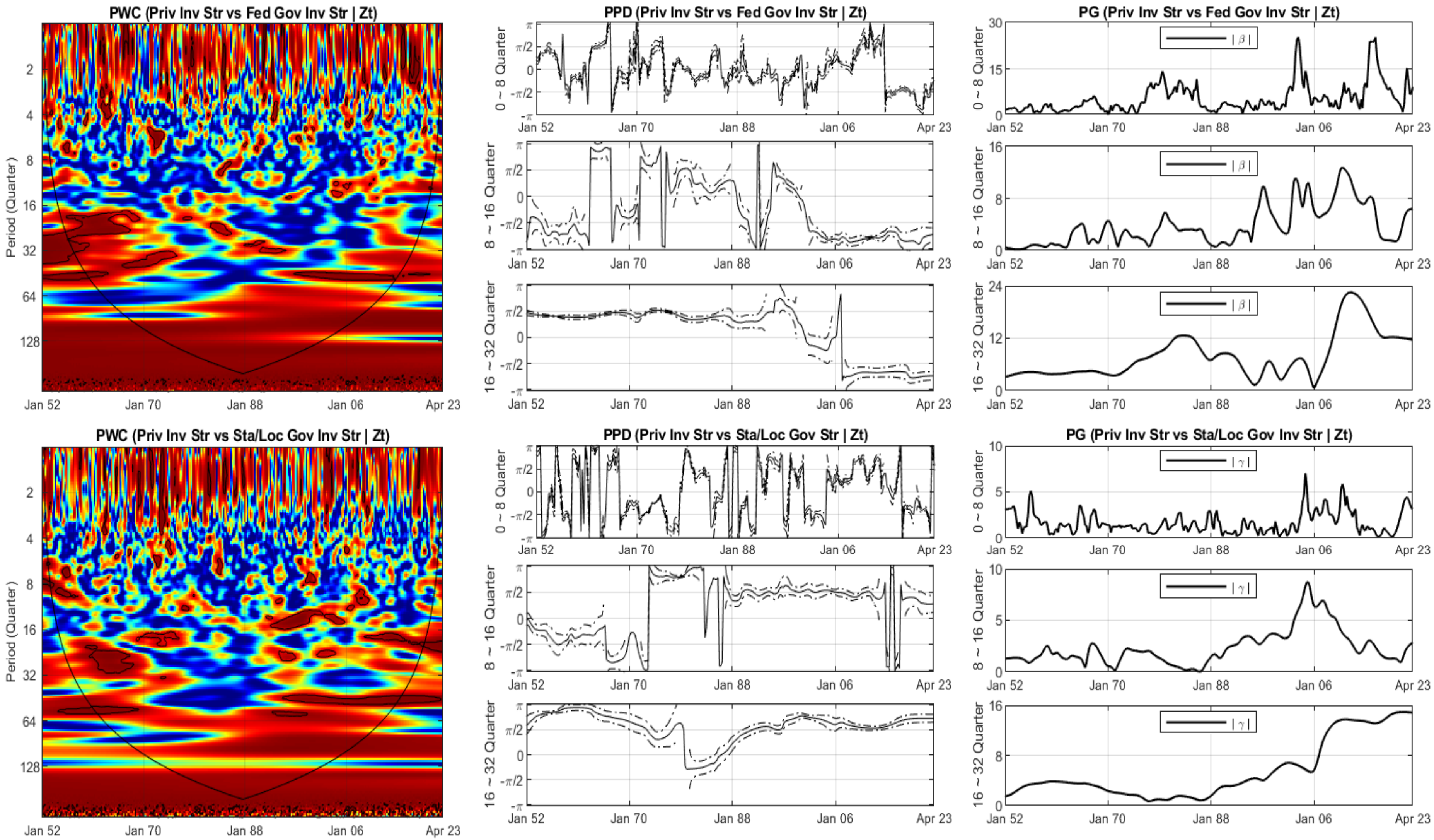

Fig. 7. Conditional Relationship between Investment in Structures by Private Sector, Federal Government, and State and Local Government: Partial Wavelet coherency (left), Partial Phase-difference (center) and Partial Gain (right).

In such figures, I summarize when there is crowding in (blue) and crowding out (red) in the high, medium and low frequency regions over time, making it possible to associate them with periods of recession and non-recession.

A first piece of evidence, comparing all the graphs, is that there are significant differences between the behavior of private investment in response to public investment, depending on the type of investment.

It is important to start with the total investment to GDP (Fig. 8). Regarding crowding in effects, there is some short-term, mid-term, and long-term evidence in response to federal government investment, both at the beginning of the sample and in the 90s and 2000s. Phasic responses in reaction to state government investment and local are all short-term, short-lived and distributed over the seven decades analyzed.

However, most of the effects evidenced in Fig. 8 suggest crowding out reactions.

Despite occurring in all decades, most of such negative responses of private investment occurred in the 60s and 70s. In the nineties and in the 2000s, there was again a greater incidence of crowding out, however, in response to investments only of state and local governments, mainly in the medium and long-term. In the last decade, there is occasional evidence of low duration and short-term. According to Table 2, looking only at the reactions of total private investment in reaction to federal government investment, the crowding in effects are more intense than the crowding out effects. Reactions to state and local government investments are, on average, more intense than reactions to federal government investments.

Table 2. Crowding in and crowding out conditional effects of total investment in the time-frequency domain

Private sector versus federal government							Private sector versus state and local government						
Period		Range of frequency (years)		Effect	Range of elasticity (β)		Period		Range of frequency (years)		Effect	Range of elasticity (γ)	
Begin	End	From	To		From	To	Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)							*Main Results - Long term (from 4 to 8 years)*						
Jul-55	Jun-57	4.6	5.0	Crowding out	2.00	2.04	Jan-70	Jun-73	4.0	5.3	Crowding out	1.80	2.66
Oct-64	Dec-69	5.6	6.2	Crowding out	2.60	2.68	Apr-76	Sep-79	4.0	5.3	Crowding out	0.96	2.11
Jul-66	Jun-70	4.5	5.1	Crowding out	2.42	2.60	Oct-94	Dec-98	5.9	6.6	Crowding out	4.52	8.13
Apr-74	Dec-76	4.3	4.9	Crowding out	2.66	3.16	Apr-98	Sep-99	4.5	4.7	Crowding out	7.90	8.13
Apr-74	Dec-76	8.0	8.5	Crowding out	2.66	3.16	Jan-03	Dec-05	4.6	5.3	Crowding out	7.28	8.37
Jan-02	Sep-05	4.0	5.3	Crowding in	4.25	5.81	Apr-07	Mar-08	4.1	4.7	Crowding out	9.60	9.73
Main Results - Mid term (from 2 to 4 years)							*Main Results - Mid term (from 2 to 4 years)*						
Jan-69	Jun-70	2.4	2.6	Crowding out	0.74	0.79	Oct-79	Dec-80	3.5	3.9	Crowding out	1.87	2.19
Jul-69	Dec-70	2.1	2.3	Crowding out	0.75	0.85	Jan-91	Mar-92	2.0	2.1	Crowding out	3.48	3.93
Apr-99	Dec-01	2.0	2.2	Crowding in	5.29	6.59	Apr-92	Sep-95	2.5	2.9	Crowding out	3.86	5.06
Apr-99	Sep-99	3.7	3.7	Crowding in	5.29	5.40	Oct-95	Sep-96	2.6	2.8	Crowding out	3.24	3.25
Jan-00	Sep-00	3.7	3.7	Crowding in	6.25	6.62	Oct-97	Dec-98	2.1	2.2	Crowding out	3.94	4.17
Apr-02	Dec-02	3.9	4.0	Crowding in	6.13	6.45	Jul-98	Sep-99	3.6	3.8	Crowding out	3.53	4.06
Main Results - Short term (from 0 to 2 years)							Jan-00	Jun-01	2.0	2.2	Crowding out	3.56	5.18
Apr-53	Sep-54	1.8	2.0	Crowding in	0.21	0.85	Oct-05	Jun-06	2.9	3.1	Crowding out	10.58	10.84
Jul-57	Sep-58	0.7	0.7	Crowding in	0.73	1.60	*Main Results - Short term (from 0 to 2 years)*						
Oct-61	Sep-63	0.7	0.9	Crowding out	0.49	0.86	Oct-55	Mar-56	0.7	1.0	Crowding in	2.66	3.17
Jul-62	Sep-63	1.0	1.2	Crowding out	0.49	0.86	Jan-62	Jun-63	0.8	0.9	Crowding out	0.96	1.52
Apr-66	Dec-66	0.3	0.5	Crowding in	0.84	0.88	Jul-68	Mar-69	0.6	0.8	Crowding in	2.73	3.03
Apr-70	Mar-71	0.7	0.8	Crowding out	0.46	0.49	Jul-69	Jun-71	0.6	1.1	Crowding in	0.92	2.32
Oct-72	Jun-73	0.6	0.7	Crowding out	0.78	0.79	Jul-71	Dec-72	0.9	1.5	Crowding in	1.12	1.68
Jan-73	Mar-74	0.9	1.1	Crowding out	0.57	0.86	Jul-78	Sep-79	1.2	2.0	Crowding out	1.03	1.36
Jan-74	Dec-74	0.5	0.8	Crowding out	0.88	1.06	Oct-82	Jun-84	1.1	1.7	Crowding out	1.38	4.61
Jul-94	Jun-95	0.3	0.4	Crowding in	1.94	1.97	Jan-91	Dec-91	1.8	2.0	Crowding in	1.35	2.52
Jul-94	Jun-95	1.4	1.6	Crowding in	1.94	1.97	Jul-97	Jun-98	0.4	0.8	Crowding out	2.04	2.62
Oct-94	Dec-95	0.8	1.0	Crowding in	2.00	2.98	Oct-06	Jun-07	0.3	0.5	Crowding in	2.24	2.72
Jan-95	Dec-95	1.6	1.7	Crowding in	1.92	2.98	Jan-18	Dec-18	0.4	0.5	Crowding in	0.97	1.22
Jul-14	Mar-15	0.8	0.9	Crowding out	2.23	4.38							
Apr-22	Dec-22	0.3	0.5	Crowding out	4.17	4.32							

Table 3. Crowding in and crowding out effects of investment in equipment in the time-frequency domain

Private sector versus federal government

Period		Range of frequency (years)		Effect	Range of elasticity (β)	
Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)						
Apr-67	Mar-68	4.5	4.7	Crowding out	0.44	0.47
Jul-03	Mar-06	5.9	7.6	Crowding in	3.69	3.80
Jan-06	Jun-07	5.7	6.4	Crowding in	3.49	3.70
Main Results - Mid term (from 2 to 4 years)						
Jul-70	Mar-71	2.0	2.1	Crowding in	0.22	0.31
Apr-76	Dec-76	2.7	2.9	Crowding in	0.69	0.84
Main Results - Short term (from 0 to 2 years)						
Jul-56	Dec-56	0.3	0.6	Crowding in	1.18	1.22
Apr-57	Sep-57	0.3	0.4	Crowding in	0.84	0.92
Apr-67	Dec-67	0.3	0.4	Crowding in	0.79	0.83
Jul-67	Mar-68	0.9	2.0	Crowding in	0.80	0.90
Jul-68	Dec-68	0.8	0.8	Crowding in	1.19	1.31
Jul-68	Jun-71	0.9	2.0	Crowding in	0.26	1.31
Jan-69	Sep-69	0.6	0.7	Crowding in	0.66	0.88
Jul-82	Sep-83	0.3	0.4	Crowding out	1.30	1.49
Jan-84	Sep-84	0.3	0.4	Crowding out	0.69	1.07
Oct-90	Dec-91	0.8	2.0	Crowding out	2.33	3.61
Jan-01	Mar-02	0.6	0.9	Crowding out	1.59	5.82
Apr-13	Dec-13	0.4	0.7	Crowding out	3.97	4.70
Jan-16	Dec-16	1.1	1.5	Crowding in	1.25	1.87

Private sector versus state and local government

Period		Range of frequency (years)		Effect	Range of elasticity (γ)	
Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)						
Oct-59	Jun-64	5.6	7.8	Crowding in	15.76	18.47
Oct-59	Jun-60	4.9	5.0	Crowding in	18.28	18.47
Main Results - Mid term (from 2 to 4 years)						
Jul-86	Mar-87	2.1	2.1	Crowding out	3.58	4.39
Jul-00	Dec-00	2.6	2.7	Crowding in	16.39	16.64
Main Results - Short term (from 0 to 2 years)						
Jul-58	Jun-59	0.3	0.5	Crowding out	22.17	27.74
Jan-59	Sep-59	0.7	0.9	Crowding out	17.03	26.53
Apr-70	Dec-70	0.7	1.4	Crowding out	9.20	24.06
Jan-73	Sep-73	0.9	1.4	Crowding out	23.62	66.39
Apr-77	Dec-78	1.0	1.8	Crowding out	20.60	62.67
Jul-80	Dec-82	1.0	1.8	Crowding out	73.55	115.74
Oct-94	Sep-95	0.7	2.0	Crowding out	15.64	47.21
Apr-04	Mar-05	0.6	0.9	Crowding in	13.72	23.14
Apr-08	Sep-08	0.4	0.5	Crowding out	5.96	6.39
Jul-10	Jun-11	1.2	1.9	Crowding in	6.10	11.80
Jan-11	Jun-11	0.5	0.6	Crowding in	8.91	17.62
Jul-13	Jun-14	1.2	1.9	Crowding in	11.40	25.45
Jul-13	Mar-14	0.3	0.9	Crowding in	16.45	25.45
Oct-14	Sep-15	1.2	1.9	Crowding out	4.52	6.14
Oct-14	Sep-15	0.8	1.0	Crowding out	4.52	6.14
Jul-19	Dec-21	0.8	1.2	Crowding in	10.87	15.65
Oct-19	Sep-21	0.3	0.4	Crowding in	10.87	15.65
Jul-22	Dec-22	0.3	0.3	Crowding in	9.97	15.65

According to Fig. 9, there is little evidence of a response from private investment in equipment in the medium term. In the long-term the highlight is the crowding in effects, in response to investment by state and local governments in the first decades, and in response to the federal government in the 2000s. In the short-term, there is a variety of responses by firms and households. There is plenty of evidence of crowding in and crowding out of short duration, both in response to investment from the federal government, state and local governments, in periods of recession and non-recession. The intensities of the effects reported in Table 3 show that the reactions of private investment (crowding and crowding out) are stronger in response to investment by state and local governments.

In Fig. 10, there is a predominance of crowding in effects, and mainly in response to investment by state and local governments. There are short-term crowding-in effects in all decades analyzed, with the exception of the 60s and 70s, when there was medium-term crowding-in. The positive long-term responses of private investment following investment by state and local government occurred from the 1990s onwards and were very notable from the subprime crisis in 2008 until the pandemic in 2020. Observing the eleven recessions during this period, in 10 of them there was evidence of a crowding in of investment in intellectual property products. The unique exception is the first crisis (1953 - 1954). Still on the reactions of private investment in intellectual property products, there are occasionally a few crowding out responses in reaction to federal government investment, mainly concentrated in the last decade. These are short-term and medium-term effects. Among these antiphasic responses of private investment, it is appropriate to highlight the occurrences before, during and after the period of the recent crisis due to the pandemic. In relation to the intensities of reactions to private investment, the crowding in effects in response to investment by state and local governments are the strongest (Table 4).

Table 4. Crowding in and crowding out effects of investment in intellectual property products in the time-frequency domain

Private sector versus federal government						
Period		Range of frequency (years)		Effect	Range of elasticity (β)	
Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)						
Jul-93	Sep-94	6.8	7.8	Crowding in	4.12	4.54
Main Results - Mid term (from 2 to 4 years)						
Apr-62	Jun-63	3.1	3.3	Crowding in	0.28	0.32
Jan-12	Sep-12	3.2	3.3	Crowding out	6.31	7.86
Apr-15	Jun-16	2.9	3.2	Crowding out	3.01	5.12
Jan-19	Dec-20	2.0	2.1	Crowding out	2.62	3.79
Main Results - Short term (from 0 to 2 years)						
Jan-54	Sep-54	1.3	1.6	Crowding in	1.14	1.94
Jan-55	Jun-55	1.3	1.6	Crowding in	1.79	1.93
Apr-73	Mar-74	0.8	0.9	Crowding in	0.68	1.48
Oct-77	Mar-78	0.9	1.0	Crowding in	1.48	2.24
Jan-81	Dec-81	1.9	2.0	Crowding out	0.51	1.29
Jul-99	Mar-00	1.1	1.3	Crowding in	1.07	1.17
Jul-99	Mar-00	0.4	0.9	Crowding in	1.07	1.17
Apr-03	Sep-03	1.6	2.0	Crowding in	2.28	2.77
Apr-09	Dec-09	1.2	1.6	Crowding in	3.08	3.66
Apr-09	Mar-10	1.7	1.9	Crowding in	2.66	3.66
Jul-09	Mar-10	0.3	0.3	Crowding in	2.66	3.66
Oct-09	Mar-10	0.6	0.6	Crowding in	3.15	3.66
Oct-15	Jun-16	1.0	1.1	Crowding out	2.81	3.18
Jan-16	Dec-16	0.3	0.6	Crowding out	2.50	3.18
Jul-17	Jun-19	0.5	0.9	Crowding out	2.46	3.03
Jan-18	Jun-18	0.4	0.5	Crowding out	3.02	3.03
Jul-18	Mar-19	0.3	0.5	Crowding out	2.61	2.87
Apr-20	Jun-21	0.5	0.9	Crowding out	2.77	3.86
Apr-20	Mar-21	1.5	2.0	Crowding out	2.77	3.86
Apr-20	Jun-21	0.9	1.6	Crowding out	2.77	3.86

Private sector versus state and local government						
Period		Range of frequency (years)		Effect	Range of elasticity (γ)	
Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)						
Apr-89	Dec-90	6.4	7.8	Crowding in	13.33	15.95
Jan-95	Mar-96	6.4	7.8	Crowding in	11.19	15.40
Jan-00	Sep-01	5.3	6.2	Crowding in	20.18	20.82
Jul-04	Mar-06	5.6	7.4	Crowding in	24.04	26.16
Jul-09	Jun-18	5.6	7.4	Crowding in	15.43	23.80
Jul-17	Jun-20	4.3	5.0	Crowding in	13.87	16.07
Main Results - Mid term (from 2 to 4 years)						
Oct-67	Jun-69	2.0	2.3	Crowding in	37.28	40.93
Jul-71	Jun-72	2.5	2.6	Crowding in	21.17	25.83
Jul-72	Dec-74	3.0	3.7	Crowding in	16.11	20.02
Oct-72	Mar-79	2.0	3.0	Crowding in	9.89	19.17
Jan-79	Mar-80	2.4	2.5	Crowding in	7.15	9.89

Private sector versus state and local government						
Period		Range of frequency (years)		Effect	Range of elasticity (γ)	
Begin	End	From	To		From	To
Main Results - Short term (from 0 to 2 years)						
Jan-56	Sep-56	1.5	1.9	Crowding in	9.72	18.42
Oct-57	Mar-59	1.5	1.9	Crowding in	18.53	25.06
Jan-58	Jun-58	0.3	0.8	Crowding in	23.96	25.06
Jan-80	Jun-80	1.0	1.6	Crowding in	17.60	26.96
Jan-81	Sep-81	1.0	1.6	Crowding in	11.87	14.35
Jan-81	Mar-82	1.7	1.9	Crowding in	11.87	14.53
Apr-81	Dec-82	0.3	0.9	Crowding in	13.32	14.53
Jan-82	Sep-83	0.9	1.6	Crowding in	9.42	16.20
Oct-88	Mar-89	1.9	2.0	Crowding in	21.97	27.24
Jan-90	Dec-90	1.8	1.9	Crowding in	16.55	33.75
Apr-90	Dec-93	0.6	1.5	Crowding in	10.91	31.41
Oct-93	Jun-94	0.8	1.2	Crowding in	9.16	20.48
Apr-04	Jun-05	0.8	0.9	Crowding in	14.89	20.75
Jul-04	Dec-04	0.4	0.5	Crowding in	16.50	16.57
Jul-04	Jun-05	1.1	1.3	Crowding in	14.89	20.75
Jul-07	Jun-08	0.8	0.9	Crowding in	21.03	27.98
Jul-07	Jun-08	0.6	0.8	Crowding in	21.03	27.98
Apr-09	Jun-10	0.3	0.7	Crowding in	22.09	30.56
Oct-09	Jun-11	1.1	1.3	Crowding in	22.09	30.56
Oct-10	Mar-11	0.3	0.6	Crowding in	22.53	24.49
Jul-11	Mar-12	0.3	0.6	Crowding in	27.93	30.65
Apr-14	Jun-15	1.4	1.7	Crowding in	27.44	36.92
Oct-14	Mar-16	0.7	1.1	Crowding in	21.15	36.31
Apr-15	Dec-15	0.5	0.6	Crowding in	23.58	29.43
Oct-15	Mar-16	0.6	0.7	Crowding in	21.15	23.58
Jan-16	Mar-17	0.4	0.5	Crowding in	14.66	21.15
Jul-17	Jun-20	0.7	1.1	Crowding in	10.40	16.90
Jul-18	Dec-18	0.3	0.4	Crowding in	12.42	13.68
Jul-19	Mar-20	0.3	0.4	Crowding in	11.25	11.90
Jan-20	Jun-21	0.5	0.8	Crowding in	10.40	12.21
Jul-20	Sep-21	1.0	1.3	Crowding in	11.42	13.73
Oct-20	Sep-21	0.3	0.4	Crowding in	12.16	13.73

Table 5. Crowding in and crowding out effects of investment in structures in the time-frequency domain

Private sector versus federal government

Period		Range of frequency (years)		Effect	Range of elasticity (β)	
Begin	End	From	To		From	To
Main Results - Short term (from 0 to 2 years)						
Apr-56	Sep-56	1.5	1.8	Crowding in	3.49	3.64
Jul-58	Dec-58	0.5	0.7	Crowding in	1.86	3.09
Apr-59	Mar-60	0.9	1.2	Crowding in	2.42	3.86
Apr-66	Sep-66	0.3	0.4	Crowding in	4.43	4.69
Jul-67	Jun-68	0.3	0.4	Crowding in	3.58	6.33
Jan-80	Sep-80	1.1	1.2	Crowding in	5.12	10.32
Oct-80	Mar-81	0.4	0.5	Crowding in	6.07	6.89
Oct-80	Dec-81	0.8	1.0	Crowding in	6.07	9.32
Oct-89	Jun-91	1.5	1.7	Crowding in	2.86	4.41
Oct-90	Dec-91	0.3	0.8	Crowding in	3.15	3.70
Apr-94	Dec-94	0.3	0.4	Crowding in	3.50	4.86
Jan-97	May-98	1.2	1.6	Crowding in	3.21	4.78
Jan-02	Jun-02	0.8	0.9	Crowding in	7.70	9.15
Apr-10	Dec-10	0.9	1.1	Crowding out	2.03	5.34
Apr-17	Sep-18	0.3	0.7	Crowding in	10.24	15.96

Private sector versus state and local government

Period		Range of frequency (years)		Effect	Range of elasticity (γ)	
Begin	End	From	To		From	To
Main Results - Long term (from 4 to 8 years)						
Apr-63	Jun-68	5.1	7.8	Crowding out	3.04	3.67
Jul-68	Mar-70	4.1	5.1	Crowding out	2.34	2.96
Apr-74	Mar-76	4.1	5.1	Crowding out	1.28	1.73
Oct-00	Jun-04	5.4	6.1	Crowding out	5.92	6.83
Apr-09	Mar-10	4.2	5.3	Crowding out	12.89	13.43
Apr-14	Mar-20	4.2	5.3	Crowding out	13.14	14.74
Main Results - Mid term (from 2 to 4 years)						
Jul-53	Mar-54	2.0	2.1	Crowding in	1.37	1.38
Apr-54	Mar-56	2.1	2.5	Crowding in	0.91	1.37
Oct-76	Mar-78	2.8	3.0	Crowding out	1.71	2.06
Oct-78	Mar-79	2.4	2.4	Crowding out	1.35	1.42
Main Results - Short term (from 0 to 2 years)						
Jul-58	Dec-58	1.6	1.7	Crowding out	1.47	1.74
Jul-58	Mar-59	0.3	0.4	Crowding out	1.47	1.74
Jul-62	Dec-62	0.6	1.0	Crowding out	1.21	1.36
Apr-65	Sep-65	1.2	1.4	Crowding out	2.81	3.48
Jan-69	Sep-69	0.4	0.6	Crowding in	1.50	1.68
Jul-71	Dec-71	0.3	0.5	Crowding in	1.24	1.25
Oct-71	Mar-73	1.3	1.6	Crowding in	1.16	1.45
Oct-72	Jun-74	0.3	0.9	Crowding in	1.06	1.47
Oct-73	Mar-75	1.0	1.3	Crowding in	0.93	1.47
Jul-84	Mar-85	0.3	0.5	Crowding in	1.33	1.88
Jan-86	Jun-86	0.4	0.5	Crowding out	0.53	0.69
Jul-00	Dec-00	1.6	2.0	Crowding in	0.61	0.88
Jan-09	Jun-09	0.5	0.6	Crowding out	2.44	2.57
Jul-09	Jun-10	0.6	0.8	Crowding out	1.46	2.08
Oct-18	Mar-19	0.3	0.5	Crowding in	0.53	0.71
Oct-18	Mar-19	0.6	0.7	Crowding in	0.53	0.71

Analyzing Fig. 11, it is possible to identify that in the medium and long-term, there is a predominance of the crowding out effect of investments in structures, in reaction to investment by state and local governments. These negative reactions of private investment were mainly concentrated during the recessions of the 60s and 70s, and in the 3 most recent crises that occurred in the last two decades. In the short-term, despite little evidence of crowding out, most private investment responses were in the same direction, complementing investments from all spheres of the public sector, in all 7 decades studied. There are short-term crowding-in effects during recessions, but most of the private sector's phasic responses to investment in structures have been in non-recession periods. This short-term finding is intuitive, because the most well-known mechanisms for the crowding-in effect are based on public investment in specific areas. The construction or improvement of physical infrastructure increases the productivity of the private sector.

Examples of physical infrastructure investment that exhibit this property are transportation infrastructure such as roads and ports, health and sanitation infrastructure such as sewage systems, and communications infrastructure such as broadband networks. According to Table 5, the intensities of long-term reactions are greater than those of medium or short-term.

More specifically regarding the short-term effects during the recent pandemic crisis, which was very short and had the greatest impact on growth, there is evidence about crowding in effects considering investments in equipment and intellectual property products in response to investment by state governments and local and crowding out in response to investments in intellectual property products by the federal government.

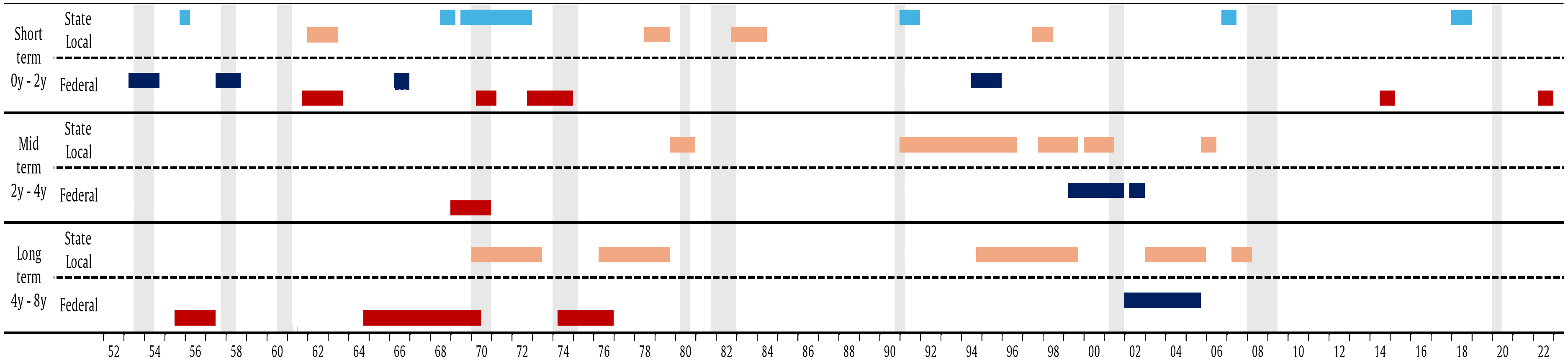

Fig. 8. Crowding in (blue color) and crowding out (red color) effects on total investment.

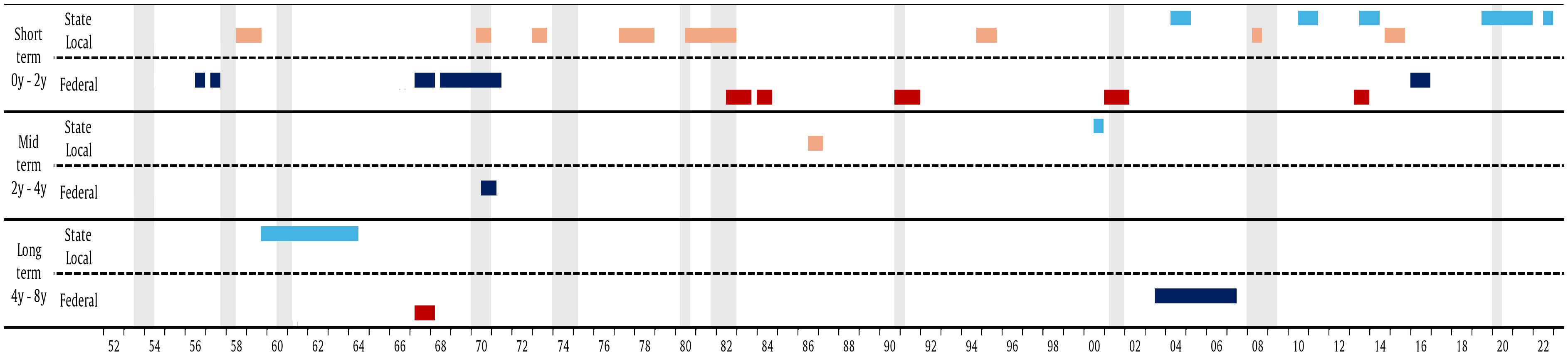

Fig. 9. Crowding in (blue color) and crowding out (red color) effects on investment in equipment.

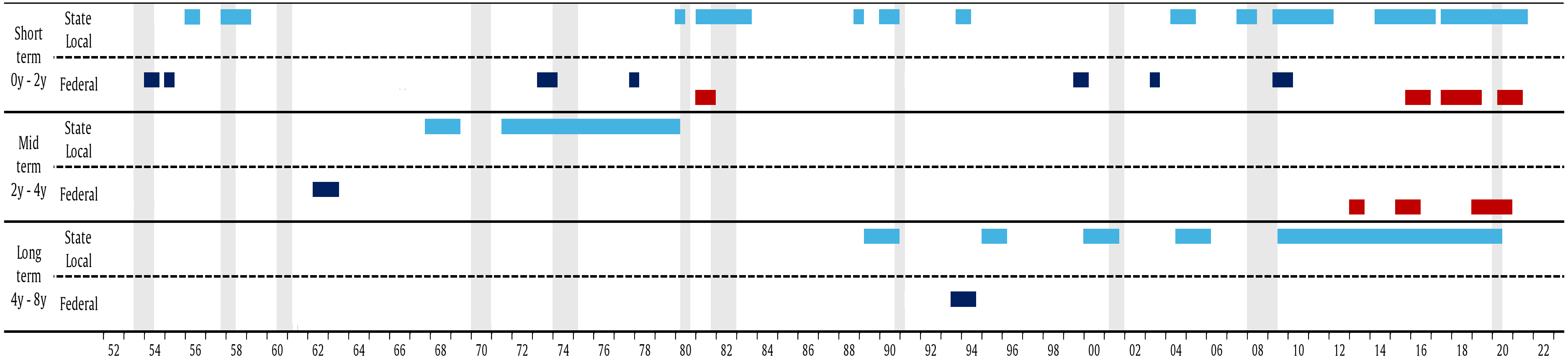

Fig. 10. Crowding in (blue color) and crowding out (red color) effects on investment in intellectual property products.

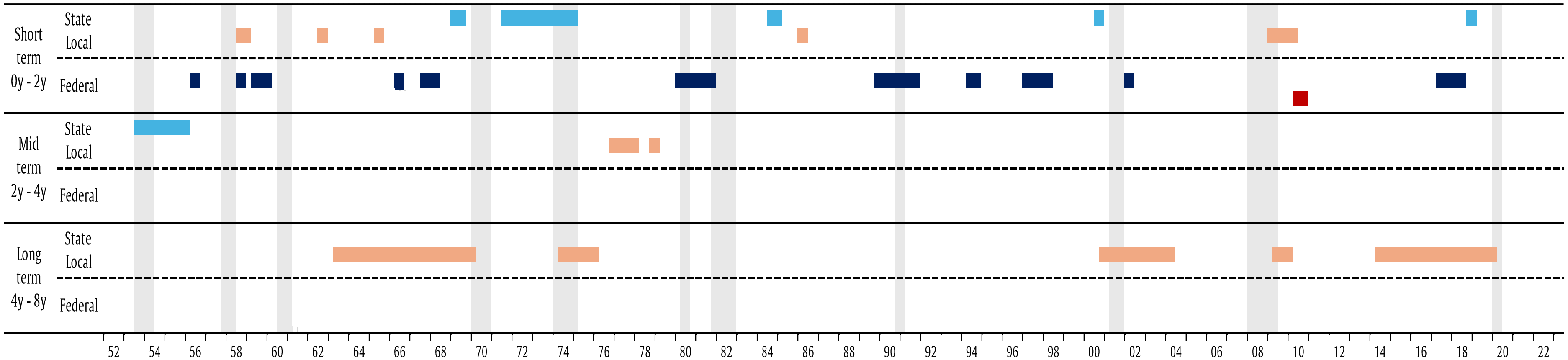

Fig. 11. Crowding in (blue color) and crowding out (red color) effects on investment in structures.

In relation to the eleven recessions occurred in these more than seventy years, there was crowing in reaction in all of them. Except for the first three recessions, which occurred in the 50s and 60s, there was crowding out effect in all other crises. During the so-called mild recession in the beginning of the 1970s (FED tightened its monetary policy, and the Nixon Administration moved to cut government spending), there was the highest incidence of crowding out effects, six in total, with emphasis on long-term reactions. During the recession in the early 1990s (restrictive monetary policy), there was the highest incidence of crowding in effects, four in total, with emphasis on the short-term responses of private investments.

4.5. Complementary Results: Crowding In and Out Effects Controlled by Fiscal and Monetary Scenarios

I have discussed about instrumentalized results varying over time and across a range of frequencies during recessionary and non-recessionary periods over more than seventy years. It is also relevant and highly informative correlating the crowding in/out effects with some macroeconomic background variables used here as instruments. In this context, I report the results showing under which fiscal and monetary scenarios crowding in/out effects occur. See the details in the Appendix Tables (A.1, A.2, A.3 and A.4).

The variables that summarize the fiscal context used as instruments are net saving to GDP and debt to GDP. The variables that describe the monetary context are inflation (CPI) and short-term interest rates. For each (fiscal or monetary instrument) instrument, its respective 286 temporal observations are identified as belonging to one of the four quartiles. In order to make the explanation more didactic, the net saving series was replaced by the deficit, i.e., the opposite of net saving. Therefore, when a debt or deficit observation is in the first quartile, there is a more adequate or controlled fiscal scenario, characterizing a more contractionary policy. As you move from the first quartile to the fourth quartile, fiscal policy deteriorates, and it can be seen as an expansionist fiscal policy. The analysis is similar for the interest rate and inflation series. The main difference is that now a contractionary (expansionist) monetary policy is characterized by high (low) interest or inflation values, i.e., in the third or fourth (second or first) quartile.

Throughout the quarters in which any effect (crowding in/out) is found, the modal quartile of each of the variables (CPI, interest rates, deficit or debt) is identified. When there is more than one modal quartile in that period, the result is not reported.

Regarding total investments (Table A.1) most of the long-term and medium-term crowding out effects in response to the federal government are in periods with low debt level and high interest rates and very high inflation. In the short term, there is a concentration of negative responses from the private sector in the 1970s, when the fiscal sector had low debt, but very high interest rates and inflation. In the 90s, there was a new concentration, but now with crowding in effects, with the expansionary fiscal, high interest rates and low inflation. Analyzing the effects in relation to state and local governments, there is a pattern in most crowding out effects in the medium and long term, with high debt and high interest rates. At high frequency, crowding in effects occur in times of fiscal contraction, especially with very low debt, while the few crowding out effects occur in periods of high debt.

The main findings on private investment decision in equipment (Table A.2) in response to federal government investment shows 18 moments with significant effects, twelve of which are crowding in. During these effects, debt and deficit are very low, with a contractionary monetary scenario in most cases. As for the effects of crowding out, they are concentrated in the most recent period, and in common, the worrying fiscal deficit. Considering state and local governments, there is no clear pattern in the limited medium- and long-term evidence. Already with high frequency, in the 1970s, evidence occurs in periods with low debt and compromised monetary pillars, while from 2010 onwards, there is a predominance of crowding in effects in periods with expansionary fiscal and monetary pillars characterized by low interest rates.

Observing the evidence from Intellectual Property Products in relation to the federal government (Table A.3), there are five significant effects in the medium and long term, mostly characterized by expansionary fiscal policy, with emphasis on high debt, and a favorable monetary context, mainly influenced by very low interest rates. With rare exceptions, most of the 8 short-term crowding out effects are characterized by a fiscal scenario with worrying debt and a monetary framework with low interest rates. Regarding high-frequency crowding-in effects, only from the 2000s onwards, I find a scenario with a pattern, with expansionary fiscal (very low debt), very low interest rates and very high inflation. The private reactions to state and local government investments in IPP are all positive. The patterns here become clearer over time. In the 1950s, the monetary policy was favorable, low interest and inflation, while in the 1980s, the fiscal policy is given by very low debt, but the monetary policy seems contractionary. In the 1990s, the fiscal system returned to a more expansionist behavior, and in the 2000s we had a combination of a contractionary fiscal policy, with very low interest rates. From 2010 onwards, there was evidence of crowding in, with an expansionary fiscal driven by high debt and an expansionary monetary one, with low interest rates.

There is only one crowding out effect, when I look at investment in federal government structures (Table A.4), and 14 evidence of crowding in. Until the mid-1960s, positive responses from private investment occurred at times with low levels of deficit. From then on, the favorable fiscal with low debt combined with contractionary monetary characterized the scenario until the end of the 80s. From then on, with occasional exceptions, the fiscal and monetary remained expansionary, with greater intensity from 2010 onwards. I observe the low and medium frequency effects associated with investment by state and local governments, there is a mix of scenarios, with no defined pattern. In relation to short-term effects, a concentration of crowding in effects in the 70s and 80s, with fiscal control due to low debt and monetary contraction, mainly due to high interest rates. The evidence (both crowding and out) after the subprime recession is characterized by a worryingly expansionary fiscal and monetary policy with low interest rates and inflation.

4.6. Final Results: Public Investment Response

The main purpose of this study is to model the reactions of private investment cycles in response to public investment cycles, controlling for the macroeconomic context. However, when I use a framework based on partial wavelet tools, the main advantage is the possibility of tracing transitional changes across time and frequencies. Another advantage is being able to deal with reverse causality, by modelling the possibility that public investment is able to react to private investment over time and with different frequency ranges. In this case, the analysis based on partial wavelet coherence and partial phase difference are useful and informative. The elasticities β and γ given by partial gain are not useful, as they specifically measure the reaction of private investment, in response to public investment. The results on the occurrences of phasic or antiphasic co-movements (without leadership), and co-movements led by private investment are reported in the Appendix Tables (A.5, A.6, A.7 and A.8).

The federal government's total and equipment investment cycles are anticipated in both directions (phasic and anti-phasic) by the respective private investment cycles, while federal government's investment in intellectual property products and structures are more commonly led in the same direction. Except for the total investment, investment cycles by type in the private sector are capable of anticipating in most cases the phasic co-movements of state and local government investment cycles. This phasic influence on state and local government reaction is still stronger considering investment in intellectual property products and structures.

5. Conclusion

The discussion on the complex relationship between the investment decisions by private players (firms and households), and public players (federal, state and municipal governments) is strategic and relevant for better understanding the effects of investments on employment, income inequality, and economic activity.

Summarizing the empirical literature on crowding in and crowding out effects applied to U.S., there is a lack of consensus [e.g. Erenburg and Wohar (1995), Voss (2000), Mountford and Uhlig (2009), Blackley (2014) and Traum and Yang (2015)]. In this context, the main evidence reported here allows me to argue that the divergence on the directions of such relationship – whether crowding in or crowding out effects – is partly due to: i) aggregation of investments (by type), ii) aggregation of the public sector, iii) lack of flexibility to allow different short-term, medium-term and long-term reactions, iv) lack of flexibility to allow time-varying reactions, and v) theoretical or empirical impossibility to modelling reactions from the public sector in response to private sector investment. The choice of some class of theoretical models can also play a significant role in this polarized discussion because some of them impose the expected effect and because whether private investment is crowded in or out during transition critically depends on parameters that are empirically hard to measure, according to Bom (2017).

This study deals with all these issues by using investment by type (equipment, intellectual property products, and structures), considering federal government and state and municipal government separately, allowing for effects changing across time and frequencies, and accounting for reverse causality. This flexibility plays a crucial role in this discussion. I also allow for (fiscal, monetary and growth) state-dependent results. According to the main results reported here, the restrictions usually assumed in previous literature are binding.

Now, I try to summarize the most relevant findings reported here.

Looking at the total amount of crowding in/out effects, I find 51 crowding in and 34 crowding out reactions in relation to the federal government. In relation to state and local governments, I highlight 72 opportunities in which there is crowding in, and 44 with crowding out. In short, there is a greater intensity of private sector investment decisions in response to the public sector, considering state and local governments.

Observing these occurrences in fiscal or monetary scenarios, the pattern that draws the most attention is that among the 51 occurrences of crowding in (in relation to the federal government), 30 (59%) occurred in periods in which the deficit was in the first or second quartile, i.e., a more balanced or contractionary fiscal scenario. Among the 34 crowding out events, also in response to the federal government, 22 (65%) occurred in periods with deficits in the third and fourth quartiles, i.e., in a scenario where the flow of revenues and expenses may be worrying. In the other occurrences, there are no percentages so high as to suggest an explicit or more robust pattern.

Possibly, the best way to identify more patterns is to disaggregate by type of investment and frequency.

Considering low or medium frequency, for example, the evidence reported here is contrary to the result expected by models that predict crowding in the long term. Considering, for example, total investment by the aggregate public sector, there are 12 events identified, of which 11 are crowding out in the long term. Even in the medium term, among the 14 events identified, 10 are crowding out. However, looking at long term investment in intellectual property products, there is only evidence of crowding in, and also a strong predominance of crowding in when considering investment in equipment. The pattern changes when I consider investment in structures, where long-term crowding out effects predominate, in response to investment by state and local governments.

Among the 85 occurrences (crowding in/out) in relation to the federal government, 63 (74%) are short-term, and among the 116 occurrences in relation to state and local governments, 75 (65%) are also short-term. These decisions with a reaction within 2 years deserve special emphasis in the fiscal and monetary context.

There is a well-defined fiscal/monetary standard, based on the evidence that this scenario is present during (between 61% and 89% of) short-term crowding in/out of investment in intellectual property products (both from the federal government and the state and local governments) as well as investment in equipment by state and local governments. This pattern is characterized by a level of debt and deficit in the highest quartiles, and interest rates in the lowest quartiles. On the other hand, there seems to be a monetary pattern that suggests that in periods of higher interest rates, there is a greater incidence of effects (crowding in/out) in response to the federal government, whether in equipment or structure.

Just as a specific example regarding the role played by interest rates, observing the incidence of effects (crowding in/out) of investment in intellectual property products in Fig. 10 and comparing with the interest time series, in the 1980s (decade with the highest average level of interest in that over 70 years) there is a reduction in the incidence of effects, while there is a greater intensity of occurrence following the subprime recession, when interest rates reach historically low levels for a long period.

To summarize this discussion of fiscal and monetary context driving investment decisions by public or private sector, it should be noted that the continuity or sustainability of any government's capital expenditures must be associated with a balanced fiscal framework, considering the possible sources of resources for a developed economy like the American one: domestic and external borrowing, and tax minus current spending. Concerning the relationship between debt, interest rates, and growth, according to Reinhart et al. (2003), there is a nonlinear effect of debt on growth related to a response of market interest rates as economies reach debt tolerance limits. According to them, as countries reach debt tolerance limits, sharply rising interest rates, in turn, force painful fiscal adjustment in the form of tax hikes and spending cuts, or outright default. This pass-through is even more complex, since it may depend on the term structure of the interest rate, the maturity of such debt, and its solvency, i.e., its relationship with to the flow of surplus/deficits.

Therefore, an unbalanced fiscal can indirectly influence the rise of inflation and interest rates, and the monetary scenario itself characterized by high interest rates directly influences the consumption and investment decisions of firms, and especially households. It is well known that a more developed capital market may also be important for large companies to be able to accommodate negative shocks in the supply of credit. In other words, with a stronger capital market, the pass through of the increase in basic interest rates may be such that the impact on corporate financing is smaller, and thus, less undesirable impacts on GDP and inflation. With the exception of some smaller companies, firms can still access external credit, and depend less on the effects of interest rates in their investment decisions.

Empirically, if quarterly data is available, and considering the differences in fiscal and mainly monetary influences on decisions of households and firms (even depending on the size of the firm), it is very informative to disaggregate firms and households. Some other next steps could be replicating this study for other economies and using a broader set of variables accounting for political cycles [6]. It could be useful analyzing state and local governments separately, as well as using stock of capital as instrument. Regarding the role of this last issue, Aschauer (1989) finds that nonmilitary public capital stock is much more important in determining productivity than is either the flow of nonmilitary or military spending, and a 'core' infrastructure – streets, highways, airports, sewers, water systems – has most explanatory power for productivity.

Theoretically, from a normative point of view, the main findings, and methodological innovations proposed here can be seen as useful for modelling investment decisions.

In practice, this study is useful to identify when public investments can provide additional certainty for the private sector and help overcome market failures like financial frictions that may impede the development of early industries. This crowding in response is more likely to occur in certain contexts, including when the economy is not at full capacity and in the presence of spillover benefits that limit returns for the private sector. The American government itself recognizes that crowding in may involve shifts in investment across sectors. Depending on the state of the economy, public spending could theoretically increase private investment in one sector but reduce it in another. The idea is motivating an applied research agenda considering the specificities of each type of investment. These three types of investment are completely different, with regard to planning, choice of projects, financial sources of financing, duration of purchase or execution, cost, efficiency, and financial or productivity marginal impacts. Moreover, this research should account for each area where the resource was spent, using advanced quantitative techniques that allow freedom of results, which accommodate the complexity of this relationship between the private sector and governments. A very interesting and recent contribution is Azhgaliyeva et al. (2021). They provide a first quantitative estimate of the effect of government renewable energy policies on private investments across different sources of financing for 13 global economies over the period 2008 to 2018, with a focus on a sub-panel of Asian economies.

Data Availability

The data underlying this article will be shared on reasonable request to the corresponding author.

Conflict of interest

I have no conflicts of interest to declare. I agree with the contents of the manuscript and there is no financial interest to report. I certify that the submission is an original work and is not under review at any other publication.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this study, I have not used any AI technology.

[6] See Belo et al. (2013) for the influence of political cycles.

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Appendix

Table A.1. Total investment: Crowding in/out effects over fiscal and monetary scenarios [a]

Private sector versus federal government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Jul-55	Jun-57	Crowding out	1st	3rd	2nd	3rd
Oct-64	Dec-69	Crowding out	1st	1st	3rd	-
Jul-66	Jun-70	Crowding out	1st	1st	3rd	4th
Apr-74	Dec-76	Crowding out	4th	1st	3rd	4th
Apr-74	Dec-76	Crowding out	4th	1st	3rd	4th
Jan-02	Sep-05	Crowding in	2nd	2nd	1st	2nd
Main Results - Mid term (from 2 to 4 years)						
Jan-69	Jun-70	Crowding out	1st	1st	4th	4th
Jul-69	Dec-70	Crowding out	3rd	1st	4th	4th
Apr-99	Dec-01	Crowding in	1st	2nd	3rd	2nd
Apr-99	Sep-99	Crowding in	1st	3rd	3rd	2nd
Jan-00	Sep-00	Crowding in	1st	2nd	4th	3rd
Apr-02	Dec-02	Crowding in	2nd	2nd	1st	2nd
Main Results - Short term (from 0 to 2 years)						
Apr-53	Sep-54	Crowding in	2nd	4th	1st	1st
Jul-57	Sep-58	Crowding in	2nd	3rd	2nd	2nd
Oct-61	Sep-63	Crowding out	1st	2nd	2nd	1st
Jul-62	Sep-63	Crowding out	1st	2nd	2nd	1st
Apr-66	Dec-66	Crowding in	1st	1st	3rd	3rd
Apr-70	Mar-71	Crowding out	3rd	1st	4th	-
Oct-72	Jun-73	Crowding out	2nd	1st	4th	4th
Jan-73	Mar-74	Crowding out	2nd	1st	4th	4th
Jan-74	Dec-74	Crowding out	2nd	1st	4th	4th
Jul-94	Jun-95	Crowding in	3rd	3rd	3rd	2nd
Jul-94	Jun-95	Crowding in	3rd	3rd	3rd	2nd
Oct-94	Dec-95	Crowding in	3rd	3rd	3rd	2nd
Jan-95	Dec-95	Crowding in	3rd	3rd	3rd	2nd
Jul-14	Mar-15	Crowding out	3rd	4th	1st	1st
Apr-22	Dec-22	Crowding out	3rd	4th	-	4th

Private sector versus state and local government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Jan-70	Jun-73	Crowding out	3rd	1st	-	3rd
Apr-76	Sep-79	Crowding out	3rd	1st	4th	4th
Oct-94	Dec-98	Crowding out	1st	3rd	3rd	1st
Apr-98	Sep-99	Crowding out	1st	3rd	3rd	2nd
Jan-03	Dec-05	Crowding out	-	2nd	1st	3rd
Apr-07	Mar-08	Crowding out	2nd	2nd	-	4th
Main Results - Mid term (from 2 to 4 years)						
Oct-79	Dec-80	Crowding out	3rd	1st	4th	4th
Jan-91	Mar-92	Crowding out	4th	3rd	3rd	2nd
Apr-92	Sep-95	Crowding out	3rd	3rd	2nd	2nd
Oct-95	Sep-96	Crowding out	2nd	3rd	3rd	-
Oct-97	Dec-98	Crowding out	1st	3rd	3rd	2nd
Jul-98	Sep-99	Crowding out	1st	3rd	3rd	2nd
Jan-00	Jun-01	Crowding out	1st	2nd	4th	3rd
Oct-05	Jun-06	Crowding out	2nd	2nd	3rd	3rd
Main Results - Short term (from 0 to 2 years)						
Oct-55	Mar-56	Crowding in	1st	3rd	2nd	1st
Jan-62	Jun-63	Crowding out	1st	2nd	2nd	1st
Jul-68	Mar-69	Crowding in	1st	1st	3rd	4th
Jul-69	Jun-71	Crowding in	3rd	1st	4th	4th
Jul-71	Dec-72	Crowding in	-	1st	3rd	3rd
Jul-78	Sep-79	Crowding out	2nd	1st	4th	4th
Oct-82	Jun-84	Crowding out	4th	1st	4th	3rd
Jan-91	Dec-91	Crowding in	-	3rd	3rd	-
Jul-97	Jun-98	Crowding out	1st	3rd	3rd	-
Oct-06	Jun-07	Crowding in	2nd	2nd	3rd	-
Jan-18	Dec-18	Crowding in	4th	4th	2nd	1st

[a] Throughout the quarters in which an effect (crowding in or out) has occurred, the modal quartile of each variable (CPI, interest rates, deficit or debt) is identified. When there is more than one modal quartile in that period, the result will not be reported.

Table A.2. Investment in Equipment: Crowding in/out effects over fiscal and monetary scenarios [a]

Private sector versus federal government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Apr-67	Mar-68	**Crowding out**	2nd	1st	3rd	3rd
Jul-03	Mar-06	**Crowding in**	2nd	2nd	2nd	2nd
Jan-06	Jun-07	**Crowding in**	2nd	2nd	3rd	3rd
Main Results - Mid term (from 2 to 4 years)						
Jul-70	Mar-71	**Crowding in**	3rd	1st	-	3rd
Apr-76	Dec-76	**Crowding in**	3rd	1st	3rd	4th
Main Results - Short term (from 0 to 2 years)						
Jul-56	Dec-56	**Crowding in**	1st	3rd	2nd	3rd
Apr-57	Sep-57	**Crowding in**	1st	3rd	2nd	3rd
Apr-67	Dec-67	**Crowding in**	2nd	1st	3rd	-
Jul-67	Mar-68	**Crowding in**	2nd	1st	3rd	3rd
Jul-68	Dec-68	**Crowding in**	1st	1st	3rd	4th
Jul-68	Jun-71	**Crowding in**	1st	1st	4th	4th
Jan-69	Sep-69	**Crowding in**	1st	1st	4th	4th
Jul-82	Sep-83	**Crowding out**	4th	1st	4th	1st
Jan-84	Sep-84	**Crowding out**	4th	1st	4th	3rd
Oct-90	Dec-91	**Crowding out**	3rd	3rd	3rd	-
Jan-01	Mar-02	**Crowding out**	1st	2nd	2nd	1st
Apr-13	Dec-13	**Crowding out**	3rd	4th	1st	1st
Jan-16	Dec-16	**Crowding in**	3rd	4th	1st	2nd

Private sector versus state and local government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Oct-59	Jun-64	**Crowding in**	1st	2nd	2nd	1st
Oct-59	Jun-60	**Crowding in**	1st	3rd	2nd	2nd
Main Results - Mid term (from 2 to 4 years)						
Jul-86	Mar-87	**Crowding out**	3rd	2nd	3rd	2nd
Jul-00	Dec-00	**Crowding in**	1st	2nd	4th	-
Main Results - Short term (from 0 to 2 years)						
Jul-58	Jun-59	**Crowding out**	-	3rd	2nd	1st
Jan-59	Sep-59	**Crowding out**	1st	3rd	2nd	1st
Apr-70	Dec-70	**Crowding out**	3rd	1st	4th	4th
Jan-73	Sep-73	**Crowding out**	2nd	1st	4th	4th
Apr-77	Dec-78	**Crowding out**	3rd	1st	4th	4th
Jul-80	Dec-82	**Crowding out**	3rd	1st	4th	4th
Oct-94	Sep-95	**Crowding out**	3rd	3rd	3rd	2nd
Apr-04	Mar-05	**Crowding in**	-	2nd	-	-
Apr-08	Sep-08	**Crowding out**	-	-	1st	4th
Jul-10	Jun-11	**Crowding in**	4th	4th	1st	3rd
Jan-11	Jun-11	**Crowding in**	4th	4th	1st	-
Jul-13	Jun-14	**Crowding in**	3rd	4th	1st	2nd
Jul-13	Mar-14	**Crowding in**	3rd	4th	1st	2nd
Oct-14	Sep-15	**Crowding out**	3rd	4th	1st	1st
Oct-14	Sep-15	**Crowding out**	3rd	4th	1st	1st
Jul-19	Dec-21	**Crowding in**	4th	4th	1st	4th
Oct-19	Sep-21	**Crowding in**	4th	4th	1st	4th
Jul-22	Dec-22	**Crowding in**	-	4th	-	-

[a] Throughout the quarters in which an effect (crowding in or out) has occurred, the modal quartile of each variable (CPI, interest rates, deficit or debt) is identified. When there is more than one modal quartile in that period, the result will not be reported.

Table A.3. Investment in Intellectual Property Products: Crowding in/out effects over fiscal and monetary scenarios [a]

Private sector versus federal government						
Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Jul-93	Sep-94	Crowding in	3rd	3rd	2nd	2nd
Main Results - Mid term (from 2 to 4 years)						
Apr-62	Jun-63	Crowding in	1st	2nd	2nd	1st
Jan-12	Sep-12	Crowding out	4th	4th	1st	2nd
Apr-15	Jun-16	Crowding out	3rd	4th	1st	1st
Jan-19	Dec-20	Crowding out	4th	4th	1st	-
Main Results - Short term (from 0 to 2 years)						
Jan-54	Sep-54	Crowding in	2nd	4th	1st	1st
Jan-55	Jun-55	Crowding in	1st	4th	1st	1st
Apr-73	Mar-74	Crowding in	2nd	1st	4th	4th
Oct-77	Mar-78	Crowding in	3rd	1st	4th	4th
Jan-81	Dec-81	Crowding out	2nd	1st	4th	4th
Jul-99	Mar-00	Crowding in	1st	3rd	3rd	2nd
Jul-99	Mar-00	Crowding in	1st	3rd	3rd	2nd
Apr-03	Sep-03	Crowding in	3rd	2nd	1st	1st
Apr-09	Dec-09	Crowding in	4th	4th	1st	3rd
Apr-09	Mar-10	Crowding in	4th	4th	1st	3rd
Jul-09	Mar-10	Crowding in	4th	4th	1st	3rd
Oct-09	Mar-10	Crowding in	4th	4th	1st	3rd
Oct-15	Jun-16	Crowding out	3rd	4th	1st	1st
Jan-16	Dec-16	Crowding out	3rd	4th	1st	2nd
Jul-17	Jun-19	Crowding out	4th	4th	2nd	1st
Jan-18	Jun-18	Crowding out	4th	4th	2nd	2nd
Jul-18	Mar-19	Crowding out	4th	4th	2nd	1st
Apr-20	Jun-21	Crowding out	4th	4th	1st	4th
Apr-20	Mar-21	Crowding out	4th	4th	1st	-
Apr-20	Jun-21	Crowding out	4th	4th	1st	4th

Private sector versus state and local government						
Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Apr-89	Dec-90	Crowding in	3rd	3rd	4th	4th
Jan-95	Mar-96	Crowding in	3rd	3rd	3rd	2nd
Jan-00	Sep-01	Crowding in	1st	2nd	4th	3rd
Jul-04	Mar-06	Crowding in	2nd	2nd	2nd	-
Jul-09	Jun-18	Crowding in	3rd	4th	1st	-
Jul-17	Jun-20	Crowding in	4th	4th	-	1st
Main Results - Mid term (from 2 to 4 years)						
Oct-67	Jun-69	Crowding in	-	1st	3rd	4th
Jul-71	Jun-72	Crowding in	-	1st	-	1st
Jul-72	Dec-74	Crowding in	2nd	1st	4th	4th
Oct-72	Mar-79	Crowding in	2nd	1st	4th	4th
Jan-79	Mar-80	Crowding in	2nd	1st	4th	4th

Private sector versus state and local government						
Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Short term (from 0 to 2 years)						
Jan-56	Sep-56	Crowding in	1st	3rd	2nd	-
Oct-57	Mar-59	Crowding in	2nd	3rd	2nd	1st
Jan-58	Jun-58	Crowding in	-	3rd	-	-
Jan-80	Jun-80	Crowding in	-	1st	4th	4th
Jan-81	Sep-81	Crowding in	2nd	1st	4th	4th
Jan-81	Mar-82	Crowding in	2nd	1st	4th	4th
Apr-81	Dec-82	Crowding in	3rd	1st	4th	4th
Jan-82	Sep-83	Crowding in	4th	1st	4th	4th
Oct-88	Mar-89	Crowding in	2nd	2nd	4th	-
Jan-90	Dec-90	Crowding in	3rd	3rd	4th	4th
Apr-90	Dec-93	Crowding in	4th	3rd	2nd	-
Oct-93	Jun-94	Crowding in	3rd	3rd	2nd	2nd
Apr-04	Jun-05	Crowding in	2nd	2nd	2nd	2nd
Jul-04	Dec-04	Crowding in	-	2nd	-	-
Jul-04	Jun-05	Crowding in	2nd	2nd	2nd	2nd
Jul-07	Jun-08	Crowding in	2nd	2nd	2nd	4th
Jul-07	Jun-08	Crowding in	2nd	2nd	2nd	4th
Apr-09	Jun-10	Crowding in	4th	4th	1st	-
Oct-09	Jun-11	Crowding in	4th	4th	1st	-
Oct-10	Mar-11	Crowding in	4th	4th	1st	3rd
Jul-11	Mar-12	Crowding in	4th	4th	1st	2nd
Apr-14	Jun-15	Crowding in	3rd	4th	1st	1st
Oct-14	Mar-16	Crowding in	3rd	4th	1st	1st
Apr-15	Dec-15	Crowding in	2nd	4th	1st	1st
Oct-15	Mar-16	Crowding in	-	4th	1st	1st
Jan-16	Mar-17	Crowding in	3rd	4th	1st	2nd
Jul-17	Jun-20	Crowding in	4th	4th	-	1st
Jul-18	Dec-18	Crowding in	-	4th	2nd	1st
Jul-19	Mar-20	Crowding in	4th	4th	1st	1st
Jan-20	Jun-21	Crowding in	4th	4th	1st	-
Jul-20	Sep-21	Crowding in	4th	4th	1st	4th
Oct-20	Sep-21	Crowding in	4th	4th	1st	4th

[a] Throughout the quarters in which an effect (crowding in or out) has occurred, the modal quartile of each variable (CPI, interest rates, deficit or debt) is identified. When there is more than one modal quartile in that period, the result will not be reported.

Table A.4. Investment in Structures: Crowding in/out effects over fiscal and monetary scenarios [a]

Private sector versus federal government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Short term (from 0 to 2 years)						
Apr-56	Sep-56	Crowding in	1st	3rd	2nd	-
Jul-58	Dec-58	Crowding in	2nd	3rd	-	1st
Apr-59	Mar-60	Crowding in	1st	3rd	2nd	-
Apr-66	Sep-66	Crowding in	1st	-	3rd	3rd
Jul-67	Jun-68	Crowding in	2nd	1st	3rd	3rd
Jan-80	Sep-80	Crowding in	3rd	1st	4th	4th
Oct-80	Mar-81	Crowding in	-	1st	4th	4th
Oct-80	Dec-81	Crowding in	2nd	1st	4th	4th
Oct-89	Jun-91	Crowding in	3rd	3rd	4th	4th
Oct-90	Dec-91	Crowding in	3rd	3rd	3rd	-
Apr-94	Dec-94	Crowding in	3rd	3rd	3rd	2nd
Jan-97	Jun-98	Crowding in	1st	3rd	3rd	2nd
Jan-02	Jun-02	Crowding in	2nd	2nd	-	-
Apr-10	Dec-10	Crowding out	4th	4th	1st	1st
Apr-17	Sep-18	Crowding in	3rd	4th	1st	-

Private sector versus state and local government

Period		Effect	Fiscal context		Monetary context	
Begin	End		Deficit	Debt	Int. rate	CPI
Main Results - Long term (from 4 to 8 years)						
Apr-63	Jun-68	Crowding out	1st	2nd	3rd	-
Jul-68	Mar-70	Crowding out	1st	1st	4th	4th
Apr-74	Mar-76	Crowding out	4th	1st	4th	4th
Oct-00	Jun-04	Crowding out	-	2nd	1st	-
Apr-09	Mar-10	Crowding out	4th	4th	1st	3rd
Apr-14	Mar-20	Crowding out	3rd	4th	1st	1st
Main Results - Mid term (from 2 to 4 years)						
Jul-53	Mar-54	Crowding in	2nd	4th	1st	1st
Apr-54	Mar-56	Crowding in	1st	4th	1st	1st
Oct-76	Mar-78	Crowding out	3rd	1st	3rd	4th
Oct-78	Mar-79	Crowding out	2nd	1st	4th	4th
Main Results - Short term (from 0 to 2 years)						
Jul-58	Dec-58	Crowding out	2nd	3rd	-	1st
Jul-58	Mar-59	Crowding out	2nd	3rd	2nd	1st
Jul-62	Dec-62	Crowding out	1st	2nd	2nd	1st
Apr-65	Sep-65	Crowding out	-	2nd	-	-
Jan-69	Sep-69	Crowding in	1st	1st	4th	4th
Jul-71	Dec-71	Crowding in	4th	1st	3rd	-
Oct-71	Mar-73	Crowding in	3rd	1st	3rd	3rd
Oct-72	Jun-74	Crowding in	2nd	1st	4th	4th
Oct-73	Mar-75	Crowding in	2nd	1st	4th	4th
Jul-84	Mar-85	Crowding in	4th	2nd	4th	3rd
Jan-86	Jun-86	Crowding out	4th	2nd	4th	-
Jul-00	Dec-00	Crowding in	1st	2nd	4th	-
Jan-09	Jun-09	Crowding out	4th	-	1st	-
Jul-09	Jun-10	Crowding out	4th	4th	1st	-
Oct-18	Mar-19	Crowding in	4th	4th	2nd	1st
Oct-18	Mar-19	Crowding in	4th	4th	2nd	1st

[a] Throughout the quarters in which an effect (crowding in or out) has occurred, the modal quartile of each variable (CPI, interest rates, deficit or debt) is identified. When there is more than one modal quartile in that period, the result will not be reported.

Table A.5. Response of Public Total Investment

Period		Range of frequency (years)		Effect on the investment by federal government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jan-59	Dec-59	2.6	2.9	**Antiphasic**
Oct-76	Mar-78	4.3	4.9	**Antiphasic**
Oct-76	Sep-79	8.0	8.0	**Antiphasic**
Apr-98	Dec-00	1.8	2.0	**Antiphasic**
Apr-00	Dec-00	0.5	0.8	**Antiphasic**
Apr-11	Dec-12	3.3	3.6	**Antiphasic**
Apr-14	Sep-16	6.6	7.0	**Antiphasic**
Oct-83	Jun-84	2.2	2.4	**Phasic**
Oct-97	Jun-99	2.0	2.2	**Phasic**
Jan-99	Jun-99	3.7	3.7	**Phasic**
Jul-06	Mar-07	0.4	0.6	**Phasic**
Comovements with private investment leading public investment				
Jul-53	Mar-55	2.0	2.3	**Private inv. antiphasic lead.**
Oct-57	Mar-58	2.2	2.3	**Private inv. antiphasic lead.**
Oct-59	Jun-61	2.6	2.9	**Private inv. antiphasic lead.**
Jul-63	Dec-64	2.1	2.3	**Private inv. antiphasic lead.**
Jul-03	Jun-04	3.9	4.0	**Private inv. antiphasic lead.**
Jul-03	Dec-04	2.0	2.2	**Private inv. antiphasic lead.**
Dec-05	Jun-06	2.8	3.2	**Private inv. antiphasic lead.**
Jan-08	Dec-08	3.9	4.0	**Private inv. antiphasic lead.**
Jul-09	Mar-10	4.0	4.6	**Private inv. antiphasic lead.**
Jul-19	Jun-21	1.6	1.8	**Private inv. antiphasic lead.**
Apr-78	Mar-79	0.3	0.5	**Private inv. phasic lead.**
Oct-78	Sep-79	0.7	0.8	**Private inv. phasic lead.**
Apr-79	Jun-80	0.8	1.0	**Private inv. phasic lead.**
Jul-79	Jun-82	1.6	2.0	**Private inv. phasic lead.**
Jul-80	Jun-81	0.4	0.5	**Private inv. phasic lead.**
Jul-80	Sep-81	1.3	1.4	**Private inv. phasic lead.**
Apr-81	Jun-82	2.0	2.1	**Private inv. phasic lead.**
Apr-81	Dec-81	1.0	1.1	**Private inv. phasic lead.**
Jan-88	Jun-89	0.9	1.0	**Private inv. phasic lead.**
Apr-97	Mar-98	1.5	1.7	**Private inv. phasic lead.**
Oct-04	Mar-05	0.3	0.5	**Private inv. phasic lead.**
Oct-04	Sep-06	0.6	1.0	**Private inv. phasic lead.**
Apr-05	Mar-06	0.5	0.6	**Private inv. phasic lead.**
Jul-05	Sep-06	1.1	1.2	**Private inv. phasic lead.**
Apr-09	Dec-09	0.5	0.6	**Private inv. phasic lead.**
Oct-09	Dec-10	1.0	1.1	**Private inv. phasic lead.**

Period		Range of frequency (years)		Effect on the investment by state and local government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Apr-64	Mar-65	1.0	1.2	**Antiphasic**
Apr-67	Mar-70	4.0	5.3	**Antiphasic**
Jul-69	Sep-71	2.1	2.4	**Antiphasic**
Jan-74	Sep-74	2.6	2.6	**Antiphasic**
Jul-17	Jun-20	4.3	5.1	**Antiphasic**
Apr-18	Jun-19	3.9	4.0	**Antiphasic**
Apr-63	Dec-63	2.2	2.3	**Phasic**
Comovements with private investment leading public investment				
Apr-61	Sep-64	7.2	7.8	**Private inv. antiphasic lead.**
Jan-75	Sep-75	0.3	0.8	**Private inv. antiphasic lead.**
Jan-00	Dec-00	1.1	1.3	**Private inv. antiphasic lead.**
Jan-00	Dec-00	1.9	2.0	**Private inv. antiphasic lead.**
Apr-00	Dec-00	0.6	0.7	**Private inv. antiphasic lead.**
Apr-13	Mar-15	4.1	4.3	**Private inv. antiphasic lead.**
Apr-57	Sep-58	2.2	2.4	**Private inv. phasic lead.**

Table A.6. Response of Public Investment in Equipment

Period		Range of frequency (years)		Effect on the investment by federal government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jan-89	Sep-89	0.3	0.4	**Phasic**
Oct-90	Dec-94	0.8	2.0	**Phasic**
Oct-91	Sep-92	2.0	2.1	**Phasic**
Oct-91	Jun-93	0.8	2.0	**Phasic**
Comovements with private investment leading public investment				
Oct-53	Sep-54	1.9	2.0	**Private inv. antiphasic lead.**
Oct-60	Dec-61	1.3	1.5	**Private inv. antiphasic lead.**
Oct-61	Mar-63	0.3	0.6	**Private inv. antiphasic lead.**
Jul-62	Jun-65	0.4	0.9	**Private inv. antiphasic lead.**
Jan-66	Dec-66	0.3	0.6	**Private inv. antiphasic lead.**
Jan-66	Dec-66	0.7	1.0	**Private inv. antiphasic lead.**
Apr-66	Dec-66	0.9	2.0	**Private inv. antiphasic lead.**
Apr-00	Dec-00	0.6	0.9	**Private inv. antiphasic lead.**
Oct-58	Mar-59	0.3	0.4	**Private inv. phasic lead.**
Jul-70	Mar-71	0.7	0.8	**Private inv. phasic lead.**
Jan-73	Sep-74	0.3	0.8	**Private inv. phasic lead.**
Apr-76	Jun-77	1.4	1.6	**Private inv. phasic lead.**
Jan-78	Dec-78	1.1	1.4	**Private inv. phasic lead.**
Jul-78	Sep-79	0.7	1.0	**Private inv. phasic lead.**
Oct-78	Jun-79	0.3	0.4	**Private inv. phasic lead.**
Apr-85	Mar-88	0.7	1.9	**Private inv. phasic lead.**
Jul-86	Mar-87	0.3	0.4	**Private inv. phasic lead.**
Apr-93	Jun-94	0.8	2.0	**Private inv. phasic lead.**
Jul-19	Dec-19	0.8	1.1	**Private inv. phasic lead.**
Jan-21	Sep-21	2.6	2.8	**Private inv. phasic lead.**

Period		Range of frequency (years)		Effect on the investment by state and local government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Oct-54	Mar-55	1.1	1.3	**Antiphasic**
Jul-80	Mar-84	1.0	1.8	**Antiphasic**
Jul-80	Mar-81	0.6	0.8	**Antiphasic**
Jul-81	Dec-81	0.4	0.5	**Antiphasic**
Jul-81	Dec-81	0.3	0.3	**Antiphasic**
Apr-84	Dec-84	2.1	2.2	**Antiphasic**
Oct-64	Jun-65	0.6	0.7	**Phasic**
Oct-82	Jun-85	4.2	4.6	**Phasic**
Jul-98	Jun-00	5.9	6.2	**Phasic**
Apr-02	Mar-05	3.3	3.6	**Phasic**
Apr-05	Mar-06	0.6	0.9	**Phasic**
Comovements with private investment leading public investment				
Oct-60	Jun-61	1.3	1.5	**Private inv. antiphasic lead.**
Apr-61	Mar-64	2.2	3.0	**Private inv. antiphasic lead.**
Jul-62	Mar-63	0.3	0.4	**Private inv. antiphasic lead.**
Jul-62	Jun-63	0.7	1.0	**Private inv. antiphasic lead.**
Oct-73	Jun-75	0.9	1.4	**Private inv. antiphasic lead.**
Jul-11	Dec-12	1.2	1.9	**Private inv. antiphasic lead.**
Jul-12	Dec-12	0.3	0.9	**Private inv. antiphasic lead.**
Jan-17	Jun-17	0.7	0.8	**Private inv. antiphasic lead.**
Jan-66	Sep-67	0.6	1.0	**Private inv. phasic lead.**
Apr-67	Mar-68	0.3	0.4	**Private inv. phasic lead.**
Jul-68	Sep-69	0.6	0.8	**Private inv. phasic lead.**
Oct-68	Mar-70	0.7	1.4	**Private inv. phasic lead.**
Oct-78	Sep-80	1.0	1.8	**Private inv. phasic lead.**
Jan-79	May-80	0.5	0.6	**Private inv. phasic lead.**
Apr-86	Sep-86	0.3	0.3	**Private inv. phasic lead.**
Apr-86	Dec-86	1.3	1.4	**Private inv. phasic lead.**
Apr-87	Mar-88	1.1	1.2	**Private inv. phasic lead.**
Jul-91	Dec-94	0.7	2.0	**Private inv. phasic lead.**
Oct-91	Sep-92	2.0	2.1	**Private inv. phasic lead.**
Oct-99	Dec-01	0.8	1.5	**Private inv. phasic lead.**
Oct-02	Sep-03	0.3	0.5	**Private inv. phasic lead.**
Jan-04	Jun-04	0.6	0.9	**Private inv. phasic lead.**
Apr-07	Jan-09	5.6	5.9	**Private inv. phasic lead.**
Jul-13	Mar-15	5.4	5.7	**Private inv. phasic lead.**

Table A.7. Response of Public Investment in Intellectual Property Products

Period		Range of frequency (years)		Effect on the investment by federal government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jan-84	Dec-84	2.1	2.2	**Antiphasic**
Oct-20	Dec-21	2.2	2.6	**Antiphasic**
Apr-62	Mar-63	1.4	1.6	**Phasic**
Jan-69	Dec-69	1.0	1.3	**Phasic**
Oct-03	Mar-05	1.6	2.0	**Phasic**
Oct-03	Mar-05	0.8	0.9	**Phasic**
Jan-04	Mar-05	0.9	1.7	**Phasic**
Comovements with private investment leading public investment				
Apr-00	Sep-00	1.1	1.3	**Private inv. antiphasic lead.**
Apr-00	Sep-00	0.4	0.9	**Private inv. antiphasic lead.**
Oct-01	Sep-02	0.5	0.7	**Private inv. antiphasic lead.**
Jul-02	Dec-02	1.6	2.0	**Private inv. antiphasic lead.**
Jul-11	Jun-12	0.3	0.6	**Private inv. antiphasic lead.**
Jan-56	Sep-56	0.8	1.3	**Private inv. phasic lead.**
Oct-68	Mar-69	1.0	1.3	**Private inv. phasic lead.**
Oct-69	Jul-70	1.0	1.3	**Private inv. phasic lead.**
May-70	Mar-71	0.8	0.9	**Private inv. phasic lead.**
Oct-68	Mar-69	0.3	0.5	**Private inv. phasic lead.**
Jan-70	Jun-70	0.4	0.6	**Private inv. phasic lead.**
Apr-05	Dec-05	1.6	2.0	**Private inv. phasic lead.**
Apr-05	Jun-06	0.9	1.7	**Private inv. phasic lead.**

Period		Range of frequency (years)		Effect on the investment by state and local government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jan-53	Dec-53	0.3	0.7	**Phasic**
Jul-53	Jun-54	2.1	2.2	**Phasic**
Oct-54	Sep-56	0.9	1.6	**Phasic**
Jan-55	Sep-56	4.1	4.6	**Phasic**
Sep-56	Sep-57	1.5	1.9	**Phasic**
Oct-56	Mar-58	3.5	3.7	**Phasic**
Jul-58	Dec-60	0.3	0.8	**Phasic**
Jul-61	Mar-62	0.4	0.8	**Phasic**
Apr-68	Mar-72	4.0	7.2	**Phasic**
Oct-68	Jun-69	1.1	1.6	**Phasic**
Oct-68	Mar-69	0.3	0.4	**Phasic**
Apr-74	Jun-75	7.4	7.6	**Phasic**
Jul-82	Jun-85	4.0	4.1	**Phasic**
Jul-01	Sep-02	5.3	6.2	**Phasic**
Jul-05	Mar-06	0.9	1.0	**Phasic**
Apr-06	Jun-09	5.6	7.4	**Phasic**
Comovements with private investment leading public investment				
Apr-54	Sep-54	0.9	1.6	**Private inv. phasic lead.**
Jul-54	Dec-54	2.1	2.2	**Private inv. phasic lead.**
Oct-55	Mar-68	4.0	7.2	**Private inv. phasic lead.**
Jul-56	Dec-58	5.7	6.8	**Private inv. phasic lead.**
Jan-58	Dec-61	7.4	8.0	**Private inv. phasic lead.**
Apr-59	Sep-60	2.8	3.0	**Private inv. phasic lead.**
Jul-59	Jun-60	2.0	2.2	**Private inv. phasic lead.**
Jul-60	Dec-62	3.8	4.0	**Private inv. phasic lead.**
Jul-62	Dec-62	0.3	0.3	**Private inv. phasic lead.**
Oct-62	Sep-63	3.2	3.4	**Private inv. phasic lead.**
Jul-63	Sep-64	3.8	4.0	**Private inv. phasic lead.**

Period		Range of frequency (years)		Effect on the investment by state and local government
Begin	End	From	To	
Comovements with private investment leading public investment				
Oct-64	Jul-95	7.6	8.0	**Private inv. phasic lead.**
Oct-64	Sep-65	0.6	0.7	**Private inv. phasic lead.**
Jan-65	Jun-65	0.4	0.5	**Private inv. phasic lead.**
Jul-65	Mar-68	0.3	0.5	**Private inv. phasic lead.**
Oct-65	Jun-66	1.0	1.1	**Private inv. phasic lead.**
Jan-66	Jun-67	1.5	1.7	**Private inv. phasic lead.**
Jul-69	Mar-71	1.1	1.6	**Private inv. phasic lead.**
Apr-73	Oct-07	1.9	2.0	**Private inv. phasic lead.**
Jan-75	Jun-77	0.4	0.9	**Private inv. phasic lead.**
Apr-78	Sep-79	1.0	1.6	**Private inv. phasic lead.**
Oct-78	Dec-79	1.7	1.9	**Private inv. phasic lead.**
Jul-80	Dec-80	1.0	1.6	**Private inv. phasic lead.**
Jul-80	Dec-80	1.7	1.9	**Private inv. phasic lead.**
Jan-82	Dec-84	0.9	1.6	**Private inv. phasic lead.**
Jul-82	Mar-84	2.9	3.2	**Private inv. phasic lead.**
Jul-82	Dec-85	3.8	4.0	**Private inv. phasic lead.**
Apr-96	Mar-97	1.0	1.2	**Private inv. phasic lead.**
Jul-96	Mar-97	1.6	2.0	**Private inv. phasic lead.**
Apr-98	Jun-99	1.6	2.0	**Private inv. phasic lead.**
Apr-98	Mar-01	0.9	1.6	**Private inv. phasic lead.**
Oct-98	Jun-99	0.4	0.6	**Private inv. phasic lead.**
Jul-01	Jun-03	0.3	0.9	**Private inv. phasic lead.**
Jan-02	Sep-02	2.0	2.1	**Private inv. phasic lead.**
Jul-03	Mar-04	0.8	0.9	**Private inv. phasic lead.**
Jul-06	Mar-07	1.0	1.1	**Private inv. phasic lead.**
Oct-21	Sep-22	1.0	1.3	**Private inv. phasic lead.**
Oct-21	Sep-22	0.8	0.9	**Private inv. phasic lead.**

Table A.8. Response of Public Investment in Structures

Period		Range of frequency (years)		Effect on the investment by federal government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jul-64	Mar-65	2.1	2.2	Antiphasic
Oct-20	Jun-21	3.0	3.1	Antiphasic
Oct-66	Jun-67	0.3	0.4	Phasic
Apr-78	Mar-79	0.3	0.5	Phasic
Apr-83	Sep-85	2.1	2.6	Phasic
Jul-87	Sep-88	1.5	1.7	Phasic
Jul-92	Jun-93	0.4	0.6	Phasic
Oct-92	Mar-94	0.8	1.1	Phasic
Jul-98	Jun-00	1.2	1.6	Phasic
Jan-08	Jun-08	0.7	0.8	Phasic
Comovements with private investment leading public investment				
Jul-03	Mar-07	2.8	3.6	Private inv. antiphasic lead.
Jan-10	Sep-10	2.8	3.0	Private inv. antiphasic lead.
Oct-10	Mar-11	2.6	2.7	Private inv. antiphasic lead.
Apr-11	Dec-11	3.6	3.7	Private inv. antiphasic lead.
Apr-12	Mar-13	2.0	2.2	Private inv. antiphasic lead.
Oct-15	Mar-16	0.3	0.5	Private inv. antiphasic lead.
Jul-19	Jun-20	1.2	1.3	Private inv. antiphasic lead.
Oct-19	Jun-20	0.7	0.8	Private inv. antiphasic lead.
Apr-53	Dec-53	0.9	1.0	Private inv. phasic lead.
Oct-54	Sep-55	1.1	1.3	Private inv. phasic lead.
Jul-55	Mar-56	1.5	1.8	Private inv. phasic lead.
Jan-56	Jun-66	4.3	8.0	Private inv. phasic lead.
Jan-71	Dec-71	1.2	1.6	Private inv. phasic lead.
Jul-72	Mar-73	1.2	1.6	Private inv. phasic lead.
Apr-77	Dec-77	0.7	0.8	Private inv. phasic lead.
Oct-77	Dec-78	4.9	5.0	Private inv. phasic lead.
Oct-81	Sep-85	5.3	6.4	Private inv. phasic lead.
Jan-88	Sep-91	7.0	7.6	Private inv. phasic lead.
Jul-90	Mar-93	7.6	8.0	Private inv. phasic lead.
Oct-05	Jun-06	0.8	0.9	Private inv. phasic lead.
Jan-06	Jun-07	1.1	1.2	Private inv. phasic lead.
Oct-07	Mar-08	0.3	0.5	Private inv. phasic lead.
Oct-08	Sep-09	0.5	0.7	Private inv. phasic lead.

Period		Range of frequency (years)		Effect on the investment by state and local government
Begin	End	From	To	
Phasic and antiphasic comovements without leadership				
Jul-59	Jun-63	5.1	7.8	Antiphasic
Oct-78	Jun-79	0.6	0.7	Antiphasic
Comovements with private investment leading public investment				
Apr-53	Mar-54	1.9	2.0	Private inv. antiphasic lead.
Jan-56	Sep-56	1.5	1.8	Private inv. phasic lead.
Apr-81	Sep-81	0.3	0.5	Private inv. phasic lead.
Oct-98	Mar-99	1.6	2.0	Private inv. phasic lead.
Jan-06	Sep-06	1.1	1.3	Private inv. phasic lead.
Jan-08	Jun-08	0.3	0.4	Private inv. phasic lead.
Oct-08	Mar-10	2.2	2.8	Private inv. phasic lead.