

TECHNOLOGY, ARTIFICIAL INTELLIGENCE, AND DIGITAL TRANSFORMATION
IN ADMINISTRATION

**OPTIMIZATION SOFTWARE FOR SHIFT SCHEDULING OF HEALTH
PROFESSIONALS USING QUANTITATIVE EVALUATION IN STROKE
REHABILITATION.**

ABSTRACT

This paper presents the development of software designed to optimize shift scheduling for healthcare professionals in hospitals and rehabilitation centers, with a focus on stroke rehabilitation. The proposed optimization is crucial to addressing the shortage of specialized professionals and the increasing demand for this type of care. Additionally, staffing costs represent a substantial share of operational costs in the healthcare sector. In this way, this study proposes integrating Operations Research techniques with quantitative patient assessments, using technologies such as assistive robotics and serious games. The developed software optimizes shift schedules to meet the specific needs of patients and minimize operational costs, incorporating functional assessment data such as Fugl-Meyer and Motor Activity Log scores, as well as quantitative scores collected by robotic equipment. The methodology also accounts for Brazilian labor laws, including mandatory breaks during shifts, overtime hours, and night shift allowances. The results demonstrate that optimization can improve resource allocation, ensuring personalized care for each patient while minimizing operational costs. Thus, implementing this software in healthcare facilities represents a significant opportunity to modernize resource management and enhance clinical outcomes, providing a robust tool for strategic decision-making in hospital administration.

Keywords: Healthcare Resource Management. Operations Research. Software Development. Quantitative Evaluation. Robotic Stroke Rehabilitation.

RESUMO

Este artigo apresenta o desenvolvimento de um software para otimizar a alocação de profissionais de saúde em hospitais e centros de reabilitação, com foco na reabilitação do Acidente Vascular Cerebral. A otimização proposta é fundamental para enfrentar a escassez de profissionais especializados e a crescente demanda por esse tipo de atendimento. Além disso, os custos com pessoal representam uma parcela substancial dos gastos operacionais no setor da saúde. Assim, este trabalho propõe a integração de técnicas de Pesquisa Operacional com avaliação quantitativa dos pacientes, utilizando tecnologias como robótica assistida e jogos sérios. O software projetado otimiza as escalas de trabalho para atender às necessidades específicas dos pacientes e minimizar os custos operacionais, integrando dados de avaliação funcional dos pacientes como as avaliações Fugl-Meyer e Motor Activity Log, bem como pontuações quantitativas coletadas por equipamentos robóticos. A metodologia também considera as leis trabalhistas brasileiras, incluindo o intervalo obrigatório durante os turnos, horas extras e adicional noturno. Os resultados demonstram que a otimização pode melhorar a alocação de recursos, garantindo um atendimento personalizado às necessidades de cada paciente e minimizando os custos operacionais. Assim, a implementação deste software em clínicas e ambientes hospitalares representa uma oportunidade significativa para modernizar a gestão de recursos e melhorar os resultados clínicos, oferecendo uma ferramenta robusta para a tomada de decisões estratégicas na administração hospitalar.

Palavras-chave: Gestão de Recursos da Saúde. Desenvolvimentos de Software. Pesquisa Operacional. Avaliação Quantitativa. Reabilitação Robótica do AVC.

1. INTRODUCTION

In recent years, the difficulty in finding specialized health professionals and the increasing market demand have led to a focus on more efficient ways to plan and schedule the workforce in hospitals and rehabilitation centers. This task is critical because the workforce represents, on average, 50% of the total costs in healthcare facilities (ERHARD et al, 2018). Research investment in health subjects is of global interest due to the increasing average age and life span, leading to a rising research field in resource optimization with Operations Research (RAIS; VIANA, 2011).

Manual elaboration of the work schedule is not uncommon. In many Brazilian institutions, these schedules are made by hand and without any mathematical basis. Using a proper mathematical/computational model can lead to better resource and workforce management, thus reducing costs (NOGUEIRA; NEVES, 2019).

Stroke affects over 100 million people globally, remaining a leading cause of disability [WSO, 2023]. It is crucial that post-stroke patients undergo high-intensity training with repetitive tasks, but conventional therapies are often monotonous and repetitive, leading to motivation issues (STIENEN et al, 2007). Additionally, a one-on-one manual interaction with a physical therapist is required and fulfill this intensive task may become inviable due to demand and budget restrictions. In this way, the use of new technologies, such as robot assisted therapy, is well-suited to improving therapy effectiveness (POLI et al, 2013; GONÇALVES; ALVES; CARBONE, 2021).

An approach based on serious games strategy has emerged in rehabilitation recently. This approach provides a detachment from efforts by providing a more immersive, engaging, and interactive therapy (PROENÇA; QUERESMA; VIEIRA, 2018). The combination of robots and serious games provides a safe and reliable alternative to enhance the effectiveness of the therapists and offer a means to objectively measure and quantify patient improvements, aligning with an evidence-based approach (ALVES; GONÇALVES; CARBONE, 2022; MOULAEI et al., 2023).

By quantifying and measuring these improvements, it is possible to correlate professional demands with scales such as the Fugl-Meyer (FM) assessment and the Motor Activity Log (MAL). Software to quantitatively measure patient outcomes was developed by ALVES, GONÇALVES and CARBONE (2021). This software (BiEval) was developed to be used with a robotic device/serious game and tracks measures such as duration, motion ranges, speeds, and forces over time for computing a quantitative assessment of the rehabilitation progress (score).

A positive correlation between the FM and MAL scales was identified by SILVA et al. (2017), suggesting these scales are interchangeable and provide an accurate representation of the clinical condition of post-stroke patients. Furthermore, these assessment tools are clinically reproducible, making them valuable for developing functional physiotherapy diagnoses and patient progress tracking.

This paper introduces a feature for efficiently optimizing healthcare professionals' schedules by integrating patient assessment with the scheduling model and including considerations for night shift allowances and overtime hours. This enhancement aligns work schedules with patient specific requirements.

2. LITERATURE REVIEW

This section explores the key themes to provide a comprehensive understanding of the present study.

2.1. Operational Efficiency and Customer Satisfaction/Quality of Service

Operational efficiency in hospitals and healthcare involves ensuring optimal management of materials, financial resources, and human resources. Good operational efficiency can also influence professionals' retention and loyalty. The low offer of good health professionals on the job market highlights the importance of ways to improve these professionals' satisfaction (THIELEN, 2018; MCKONE-SWEET; HAMILTON; WILLIS, 2005).

Customer satisfaction evaluation is a key part of healthcare facilities. High quality services increase patient loyalty and quality perception. Satisfied patients return for future treatment and recommends the institution to others. Hospital management is also benefited: knowing the dimensions that affect patients' satisfaction allows managers to manage resources in a more efficient way. Quality of service is crucial, it impacts directly the efficacy of treatments, patient loyalty, and the image of the healthcare institution (CARVALHO; OLIVEIRA; CAGNANI, 2023).

2.2. Operations Research

Operations Research (OR) is the study, development, and application of advanced analytical methods to support decision-making across many fields. OR uses robust mathematical modeling techniques to find the best operating conditions for a represented system. Recent technological progress has expanded the scope of OR, leading to a shift in terminology towards the term Business Analytics (GOLDBARG, 2005; SOBRAPO, 2024). The main phases of OR are defined as:

- 1) *Scenario and Problem Definition*: the primary output of this phase is a clear statement of the problem that will help guiding the subsequent phases. (MBA INSTITUTE, 2024). This phase involves analyzing, understanding, and defining the problem and its objectives, decision variables, and associated constraints. An incorrect problem definition can lead to wasted resources, time, and investment (RAGSDALE, 2007).
- 2) *Model Construction*: this phase involves translating the problem definition into a mathematical formulation (TAHA, 2008). The mathematical model consists of a system of equations and mathematical expressions that captures the essence of the business analytics problem (HILLIER; LIEBERMAN, 2013, p. 31). If the mathematical relationships are too complex or indefinite, simulation methods may be more suitable (ANDRADE, 2015, p. 11). The mathematical model or simulation should be sufficiently simple to solve using the available techniques while addressing all essential aspects of the problem. Several modeling techniques can be employed to address a specific problem (MBA INSTITUTE, 2024).
- 3) *Solution Generation and Analysis*: this phase relies on optimization algorithms, software, and techniques to find an optimal solution (TAHA, 2008; MBA INSTITUTE, 2024). Mathematical models are solved with the best algorithm in terms of speed, processing, and accuracy. For some models, the optimal solution concept may not be defined, resulting in an approximate measure of the objective (ANDRADE, 2015, p. 11).

- 4) *Model Validation and Verification*: the proposed model is tested to ensure it accurately predicts the expected behavior (TAHA, 2007). Its predictions are compared with real-world data to verify their accuracy (MBA INSTITUTE, 2024). A model is considered valid if it improves decision quality by providing acceptable predictions (ANDRADE, 2015, p.12).
- 5) *Implementation and Monitoring*: implementing solutions from OR involves applying conclusions and recommendations derived from problem analysis and modeling. It is good practice to have an implementation team to monitor and control the process, supported by a plan that addresses any challenges that arise. New solution values may require reformulations and corrections (ANDRADE, 2015, p. 12; MBA INSTITUTE, 2024).

2.3. Linear Programming

According to the MBA INSTITUTE (2024), Linear Programming (LP) is a mathematical technique used to determine the optimal solution for a linear function, while satisfying its constraints. ANDRADE (2015) adds that by saying that LP is a solution technique to address various problems, including resource allocation which involves distributing limited resources among competing activities.

Several authors explain the mathematical model formulation of a Linear Programming Problem (LPP): decision variables represent quantifiable decisions aimed at maximizing or minimizing a function Z (objective function) (HILLIER; LIEBERMAN, 2013; FAVERO; BELFIORE, 2013; MBA INSTITUTE, 2024). The optimal solution is the set of decision variables values that yields the best results for the objective function within the given constraints.

The set of decision variables can be represented as:

$$x_1, x_2, x_3, \dots, x_n$$

Decision variables are key parameters for solving a problem, requiring a clear definition of each variable's meaning in the context of the problem (RAGSDALE, 2007).

The constraints are mathematically represented as:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n & (\leq, =, \geq) b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n & (\leq, =, \geq) b_2 \\ & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n & (\leq, =, \geq) b_m \\ x_1, x_2, \dots, x_n & \geq 0 \text{ (non-negativity constraint)} \end{aligned}$$

In the equation, the independent term b_i corresponds to the available resources associated with the i -th constraint, a_{ij} represents the constant associated with the i -th constraint and j -th variable (FAVERO; BELFIORE, 2013).

The objective function represents the primary goal of decision-making, which can be either minimization or maximization, and is mathematically expressed as:

$$\text{MAX or MIN } Z = f(x_1, x_2, x_3, \dots, x_n) = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n$$

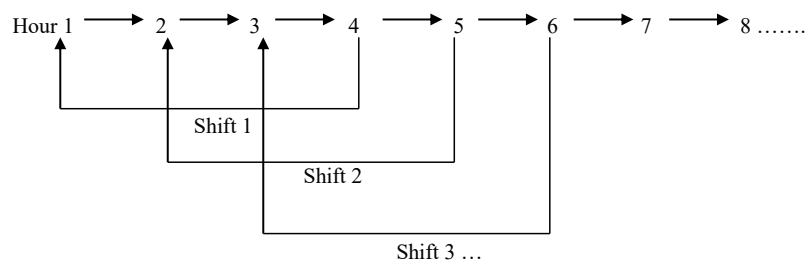
Where c_j denotes the coefficients of the j -th variable in the objective function Z .

2.4. Resource Allocation: Shift Scheduling Problem

The Shift Scheduling problem aims at determining the number of staff on each shift to minimize the total cost or minimize total excess hours. A particular research case involving this problem was to determine shift schedules for many telephone operators working full-time hours, who received half hour or one hour lunch break during several divided periods between their shifts. This problem is also very common in restaurants, hospitals, factories, drugstore, etc. (HENDERSON; BERRY 1976).

HOJATI and PATIL (2007) presented a solution method to manually optimize shift scheduling models with fixed-length intervals, including a lunch break at the midpoint of each shift. Most methods for solving shift scheduling use (integer) LPP, which is the case. If the minimum number of staff required during an hour is integer and shifts use consecutive hours, the problem can be formulated as a capacitated network flow problem, Figure 1.

Figure 1 – Capacitated network flow representation for shift scheduling problem with 4-hour duration.



Source: HOJATI and PATIL (2007).

Network optimization models are, in fact, special cases of LPP. The present study incorporated night shift allowance and overtime hours into the model.

2.5. Consolidation of Brazilian Labor Laws

The Consolidation of Brazilian Labor Laws (CLT, 1943) established a wide array of provisions, ranging from employees' contracts, maximum working hours, night shift allowances, and minimal wages, among others. A break interval (one hour minimum) is mandatory for rest or lunch in any shift exceeding six hours (Art. 71). The daily shift work may be extended by up to two extra hours, subject to an additional payment at least 50% higher than the regular hour (Art. 59). Finally, for night shifts between 22:00 and 5:00 h, an additional payment 20% higher than the regular hour is stipulated (Art. 73).

The CLT sought to ensure comprehensive protection for workers while fostering a stable and predictable environment for employers (ROSA; NOGUEIRA, 2017). For healthcare professionals, the disrespect of these laws can lead to serious risks in performance, for example, exposure to long working hours or long night shifts can result in reduced cognitive performance and a higher propensity for erroneous decisions. These laws are considered in the scope of this study.

3. METHODOLOGY

The proposed methodology in this study focuses on optimizing shift scheduling in hospitals and/or stroke rehabilitation centers using Operations Research tools to efficiently allocate healthcare professionals. The approach combines the use of

robotics, serious games, and quantitative assessment to determine the staffing requirements based on patient disability severity. This assessment (score) is automatically provided by robotic equipment and integrated software and is used to determine the proportion of professionals (multiplier) required for each patient during rehabilitation sessions.

Optimized scheduling within each work shift also enhances client satisfaction, as the number of professionals required is accurately calculated to address each patient's specific needs and ensure more personalized and effective treatment.

3.1. Scenario and Problem Definition

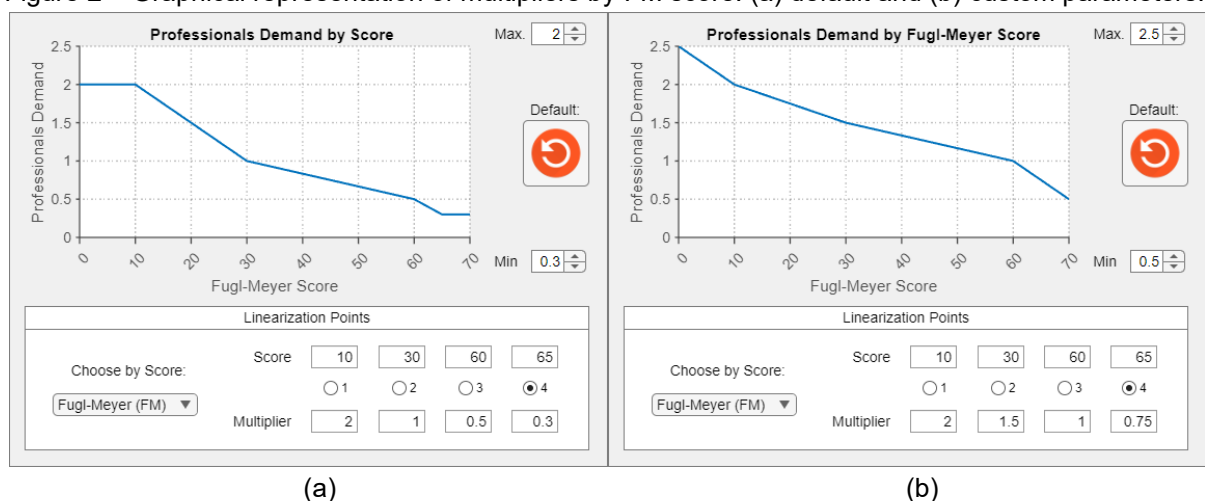
The optimization aims to efficiently allocate the required professionals per work shift to minimize the staffing costs associated with patient care, thus generating greater operational efficiency and service quality.

The patients' condition, quantified by their scores, is used by the software to calculate the multipliers for each patient in a linearly proportional manner, as shown in Figure 2(a). The multiplier determines the optimal number of professionals for efficient and effective rehabilitation sessions.

A maximum multiplier of 2 professionals per patient is used for an FM score of 10 or less (severe disability). Conversely, for patients with a score of 65 or higher (mild disability), the multiplier is 0.3 professionals per patient, allowing one professional to supervise three patients simultaneously. For moderate impairments, multipliers are derived through linear interpolation within their respective range.

It is important to note that these default values can be customized in the software interface to accommodate regional and application-specific needs. Figure 2(b) provides an example where the maximum/minimum multipliers are set to 2.5/0.5 professionals per patient. The intermediate points were also readjusted.

Figure 2 – Graphical representation of multipliers by FM score: (a) default and (b) custom parameters.



Source: Author

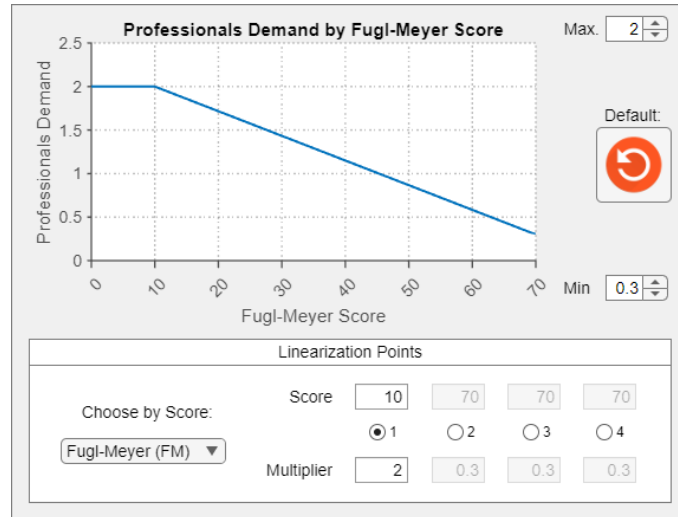
The software also allows customization of the number and position of linearization/interpolation points (for precision adjustment) and score type (FM, MAL, or Bimanual), as shown in Figure 3. Figure 4 illustrates the effect of using a single linearization/interpolation point on multiplier calculation.

In addition to adjusting the professional multiplier, session duration is typically scaled based on the patient's impairment level. Figure 5 illustrates the customizable session lengths, which can be modified within each score range in the software.

Figure 3 – Selection of score type and number/position of linearization points.

Source: Author

Figure 4 – Effect single-point linearization on multiplier calculation (graphical representation).



Source: Author

Figure 5 – Software interface displaying a table of session duration intervals by patient score.

Score	Duration	Multiplier
0-10	2:00	2.00
10-20	1:40	2.00
10-30	1:20	1.50
30-40	1:10	1.00
40-50	1:00	0.83
50-60	0:40	0.67
60-65	0:40	0.50
65-70	0:30	0.30

Source: Author

The work shift schedules were initially divided into segments, with each type of employee starting at a specific hour (1, 2, 3, ..., 24) and working 8 hours daily, including a 1-hour break in the mid-shift, at a fixed cost per employee (C_f). Additionally, each employee can work up to 2 extra hours per day (overtime), as per Brazilian labor laws, at a 50% additional cost, which represents a variable cost (C_v). Finally, night shifts (10:00 PM to 5:00 AM) incur an extra 20% cost (C_n) for the hours worked during this period, while night overtime hours attract a cumulative cost of 80% ($C_v \times C_n$).

Given that sessions are in 10-min intervals, each hour is divided into six 10-min packages. This division ensures a more accurate alignment between professionals' schedules and patient demand, minimizing rough allocation and idle periods. Thus, each employee type F_i starts their shift at a designated time i ($i = 1, 2, 3, \dots, 144$), takes a 6-package break (1 hour), starting at $i + 24$ and ending at $i + 30$, and concludes their shift at $i + 54$. Table 1 succinctly presents this allocation pattern.

Table 1 – Employee allocation pattern for each employee type F_i .

F_i	Scheduling				Night shift
	Start	Interval		End	
		Start	End		
F_1	0:00	4:00	5:00	9:00	5:00 (00:00-5:00)
F_2	0:10	4:10	5:10	9:10	4:50 (00:10-5:00)
F_3	0:20	4:20	5:20	9:20	4:40 (00:20-5:00)
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
F_{49}	8:00	12:00	13:00	17:00	-
F_{50}	8:10	12:10	13:10	17:10	-
F_{51}	8:20	12:20	13:20	17:20	-
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
F_{142}	23:30	3:30	4:30	8:30	5:30 (23:30-5:00)
F_{143}	23:40	3:40	4:40	8:40	5:20 (23:40-5:00)
F_{144}	23:50	3:50	4:50	8:50	5:10 (23:50-5:00)

Source: Author

Accurate and efficient shift scheduling is highly complex without optimization and computational algorithms. Therefore, an optimization model was implemented within the software interface to generate an optimized work schedule based on patient and professional demand. This model is presented in the following subsection.

3.2. Model Construction and Objectives Identification

The optimization model was built into the software using the information, rules, and patterns outlined in the problem definition. Operations Research techniques were employed to derive the optimal solution, which is the quantity (x_i) of each employee type (F_i) scheduled and, if applicable, the amount of overtime hours (y_{ik}) performed by each F_i employee to fully meet patient demand in each interval (D_j) and minimize total operational cost (C_{total}). The decision variables, objective function, and constraints of the OR model are detailed below.

3.2.1. Decision Variables

The problem involves two types of decision variables:

Decision variables $\begin{cases} x_i: \text{number of professional scheduled for shift } i \\ y_{ik}: \text{number of professionals } x_i \text{ working } k \text{ overtime hours.} \end{cases}$

with $i = 1, 2, 3, \dots, 144$ (shift) and $k = 1, 2, 3, \dots, 12$ (overtime hours).

The decision variables x_i represent the number of professionals scheduled for each shift i , while y_{ik} denote the number of k overtime hours performed by a professional assigned to shift i . Since each employee can work up to two extra hours per day and each hour is divided into six 10-minute workload packages, a professional can work up to $k = 12$ extra workload packages.

An excerpt of the allocation pattern with a 1-hour interval (simplified) is illustrated in Table 2. This excerpt shows the coefficient matrix $A_{ij} (a_{ij} \times x_i) + A'_{ijk} (a_{ijk} \times y_{ik})$ demonstrating how shift scheduling cyclically allocates each decision variable x_i to meet the demand D_j . The representation of the original matrix, even

when simplified, is impractical as the 10-minute intervals result in a matrix A_{ij} ($A_{144 \times 144}$) containing over 20,000 elements.

Table 2 – Excerpt of the allocation pattern (simplified for visualization) for decision variables and demands in the matrix A_{ij}/A'_{ijk} .

F'_i	Shift i (Start Time)																							
	1 0:00	2 1:00	3 2:00	4 3:00	5 4:00	6 5:00	7 6:00	8 7:00	9 8:00	10 9:00	11 10:00	12 11:00	13 12:00	14 13:00	...	21 20:00	22 21:00	23 22:00	24 23:00					
F'_1	x_1	x_1	x_1	x_1	0	x_1	x_1	x_1	x_1	y_{11}	y_{12}	0	0	0	...	0	0	0	0					
F'_2	0	x_2	x_2	x_2	x_2	0	x_2	x_2	x_2	x_2	y_{21}	y_{22}	0	0	...	0	0	0	0					
F'_3	0	0	x_3	x_3	x_3	x_3	0	x_3	x_3	x_3	x_3	y_{31}	y_{32}	0	...	0	0	0	0					
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots					
F'_{22}	x_{22}	0	x_{22}	x_{22}	x_{22}	x_{22}	$y_{22,1}$	$y_{22,2}$	0	0	0	0	0	0	...	0	x_{22}	x_{22}	x_{22}					
F'_{23}	x_{23}	x_{23}	0	x_{23}	x_{23}	x_{23}	x_{23}	$y_{23,1}$	$y_{23,2}$	0	0	0	0	0	...	0	0	x_{23}	x_{23}					
F'_{24}	x_{24}	x_{24}	x_{24}	0	x_{24}	x_{24}	x_{24}	x_{24}	$y_{24,1}$	$y_{24,2}$	0	0	0	0	...	0	0	0	x_{24}					
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}	...	D_{21}	D_{22}	D_{23}	D_{24}					

Source: Author

3.2.2. Objective Function

The Objective Function was formulated based on Henderson and Berry (1976). It aims to minimize the sum of two components: the cost (c_i) of each employee assigned to shift i multiplied by the number of professionals scheduled for that shift, and the cost of the k -th overtime hours (c_{ki}) for each shift i multiplied by the number of overtime hours performed (y_{ik}). Thus:

$$\text{Costs} \begin{cases} c_i: \text{cost of an employee assigned to shift } i \\ c_{ki}: \text{cost of } k \text{ - th overtime hour in shift } i \end{cases}$$

- $i = 1, 2, 3, \dots, 144$ (shift);
- $k = 1, 2, 3, \dots, 12$ (overtime hour).

Objective Function (minimization):

$$\text{Minimize } C_{total} = \sum_{i=1}^{144} c_i \times x_i + \sum_{k=1}^{12} \sum_{i=1}^{144} c_{ki} \times y_{ik}$$

Due to the sum's extensions, the terms will not be expanded.

3.2.3. Constraints

Two types of constraints are identified in this study. First, Type I constraints ensure that the professionals demand (D_j) in each period j is met. Therefore, the sum of operational (and overtime) hours within each interval j must be greater than or equal to the respective demand D_j . Thus:

a_{ij} : shift scheduling coefficient matrix; **1** if hour j is in shift i ; **0** otherwise

a'_{ijk} : overtime coefficient matrix; **1** if k - th overtime hour in j is in shift i ; **0** otherwise

- $i = 1, 2, 3, \dots, 144$ (shift);
- $j = 1, 2, 3, \dots, 144$ (operating hours);
- $k = 1, 2, 3, \dots, 12$ (overtime hour).

Since a_{ij} is a square matrix, we can use $a_{ij} = a_{ji}$ in this context. Therefore, the constraints are given by:

Constraints Type I:

$$\text{Subject to: } \sum_{i=1}^{144} a_{ij} \times x_i + \sum_{k=1}^{12} \sum_{i=1}^{144} a_{ijk} \times y_{ik} \geq D_j \quad \text{for } j = 1, 2, 3, \dots, 144$$

Due to the sum's extensions, the terms will not be expanded.

The Type II constraints stem from the use of overtime hours and the necessity to maintain continuous shiftwork. These constraints prevent the optimization process from finding solutions where employees x_i not scheduled for the shift perform overtime hours y_{ik} to meet demand within a specific interval. Thus, Table 3 demonstrates the required association to ensure that only employees scheduled for a regular shift can perform overtime hours.

Table 3 – Truth table showing the relationship between x_i e y_{ik} to satisfy Type II constraint .

x_i	y_{ik}	Allowed
1	0	Yes
1	1	Yes
0	0	Yes
0	1	No
$x_i \geq y_{ik}$		Restriction

Source: Author

Similarly, to ensure the continuity between overtime hours, each subsequent overtime interval can only be scheduled if the employee has completed the previous one, thus avoiding gaps in the work schedule. In this way:

$$y_{i,m} \geq y_{i,m+1} \quad \text{for } m = 1, 2, \dots, k - 1; i = 1, 2, 3, \dots, 144$$

Type II Constraints:

$$\text{Subject to: } x_i \geq y_{i,1} \geq y_{i,2} \geq \dots \geq y_{i,12} \quad \text{for } i = 1, 2, 3, \dots, 144$$

3.2.4. Domain of the Variables

The decision variables x_i and y_{ik} must be non-negative, as they represent the number of professionals and overtime hours, respectively. The variables x_i must be integers, as they represent the count of professionals. Although y_{ik} variables do not need to be integers (e.g., a professional working 6 minutes of overtime), but as a session interval of 10-minute was used, these variables were also adopted as integers for consistency. Thus, variables domain and non-negativity constraints are given by:

$$x_i, y_{ik} \in \mathbb{Z}^* \text{ (integers } \geq 0) \forall i, k$$

3.3. Linear Programming Problem Solution

Initially, the Solver tool (Microsoft Excel) was selected for solving the linear programming problem. However, this tool is limited to 100 variables and 200 constraints making it impractical, given our model has 1,872 variables and 1,872 constraints. Consequently, the model was ported for the numerical computing software MATLAB, restructured into a matrix format, and the LPP was solved using the *linprog* function for integers $\{[x, C] = \text{linprog}(f, A, b)\}$, where:

$$\text{Min } f'x \quad \text{subject to: } Ax \leq b$$

C

- x : vector containing both the decision variables x_i and y_{ik} ;
- C : value of the objective function evaluation in the optimal solution;
- A : aggregated matrix with coefficients a_{ij} and a'_{ijk} ;
- f' : objective function coefficients for minimization;
- b : right-hand side constants of the constraint equations (demand D_j).

The results obtained from solving the linear programming problem and model validation will be discussed in the next section.

4. RESULTS

To solve and validate the optimization model, a test data set was initially created. This data set can be partially viewed in the “Patients Data” tab of the software, as shown in Figure 6. The last column displays the scheduled sessions for each patient, with session durations adjusted based on their score. Using this data, the daily or weekly (average) patient schedules can be observed, as illustrated in Figure 7.

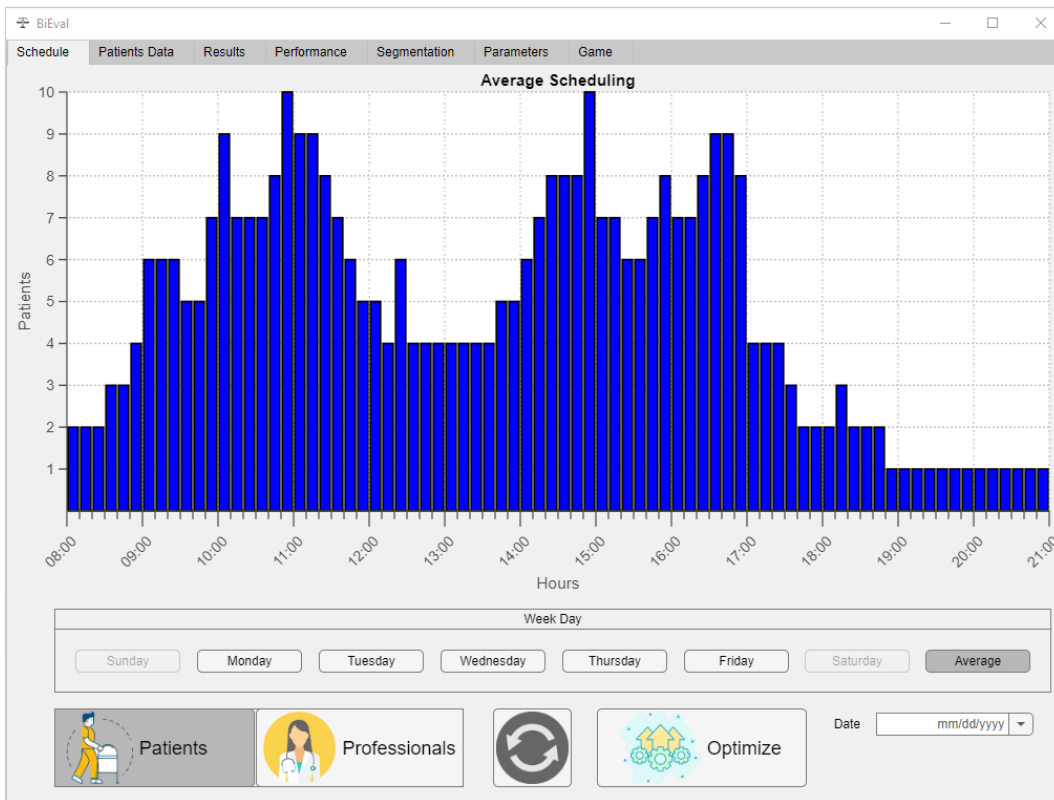
Figure 6 – Software “Patients Data” tab with the test data set used for model solution and validation.

BiEval										
Schedule	Patients Data				Results	Performance	Segmentation	Parameters	Game	
Code	Name	Gender	Age	Weight	Health Status	Score	FM	MAL	Session Schedule	
P2783	Alice	F	45	62	Chronic	90	20.0	2.00	Segunda-Feira,08:00-09:20; ;Quarta-Feira,08:00-09:20; ;Sexta-Feira,08:30-09:50;	
P3019	Bob	M	56	78	Acute	120	26.5	2.65	Segunda-Feira,09:00-10:20; ;Quarta-Feira,09:00-10:20; ;Sexta-Feira,08:30-09:50;	
P3564	Carol	F	62	70	Subacute	67	15.0	1.50	Segunda-Feira,10:00-11:40; ;Quarta-Feira,10:00-11:40; ;Sexta-Feira,09:30-11:10;	
P4876	Dave	M	50	85	Chronic	163	29.5	2.95	Segunda-Feira,11:00-12:20; ;Quarta-Feira,11:00-12:20; ;Sexta-Feira,10:30-11:50;	
P5390	Eve	F	70	80	Acute	198	31.5	3.15	Terça-Feira,08:00-09:10; ;Quinta-Feira,08:00-09:10; ;Sexta-Feira,11:20-12:30;	
P1048	Frank	M	33	55	Subacute	21	5.0	0.50	Terça-Feira,09:00-11:00; ;Quinta-Feira,09:00-11:00; ;Sexta-Feira,13:30-15:30;	
P5741	Grace	F	58	68	Chronic	44	10.0	1.00	Terça-Feira,10:00-11:40; ;Quinta-Feira,10:00-11:40;	
P4293	Henry	M	40	74	Acute	83	18.5	1.85	Terça-Feira,13:30-15:10; ;Quinta-Feira,11:00-12:40;	
P5172	Ivy	F	65	69	Subacute	42	9.5	0.95	Segunda-Feira,13:30-15:30; Terça-Feira,14:40-16:40; ;Quarta-Feira,13:30-15:30;	
P5869	Jack	M	73	90	Chronic	134	27.5	2.75	Segunda-Feira,15:10-16:30; Terça-Feira,16:20-17:40; ;Quarta-Feira,15:10-16:30;	
P4391	Kelly	F	49	60	Acute	97	21.5	2.15	Segunda-Feira,16:00-17:20; Terça-Feira,17:10-18:30; ;Quarta-Feira,16:00-17:20;	
P4802	Liam	M	55	82	Subacute	92	20.5	2.00	Segunda-Feira,17:00-18:20; ;Quarta-Feira,17:00-18:20;	
P4054	Mia	F	61	75	Chronic	85	19.0	1.75	Segunda-Feira,08:00-09:40; ;Quarta-Feira,08:00-09:40; ;Sexta-Feira,08:30-10:10;	
P4723	Noah	M	68	85	Acute	110	24.0	2.50	Segunda-Feira,09:00-10:20; Terça-Feira,09:50-11:10; ;Sexta-Feira,11:10-12:30;	
P4588	Olivia	F	34	65	Subacute	128	27.5	2.80	Segunda-Feira,11:40-13:00; ;Quarta-Feira,11:40-13:00; ;Sexta-Feira,12:20-13:40;	

Source: Author

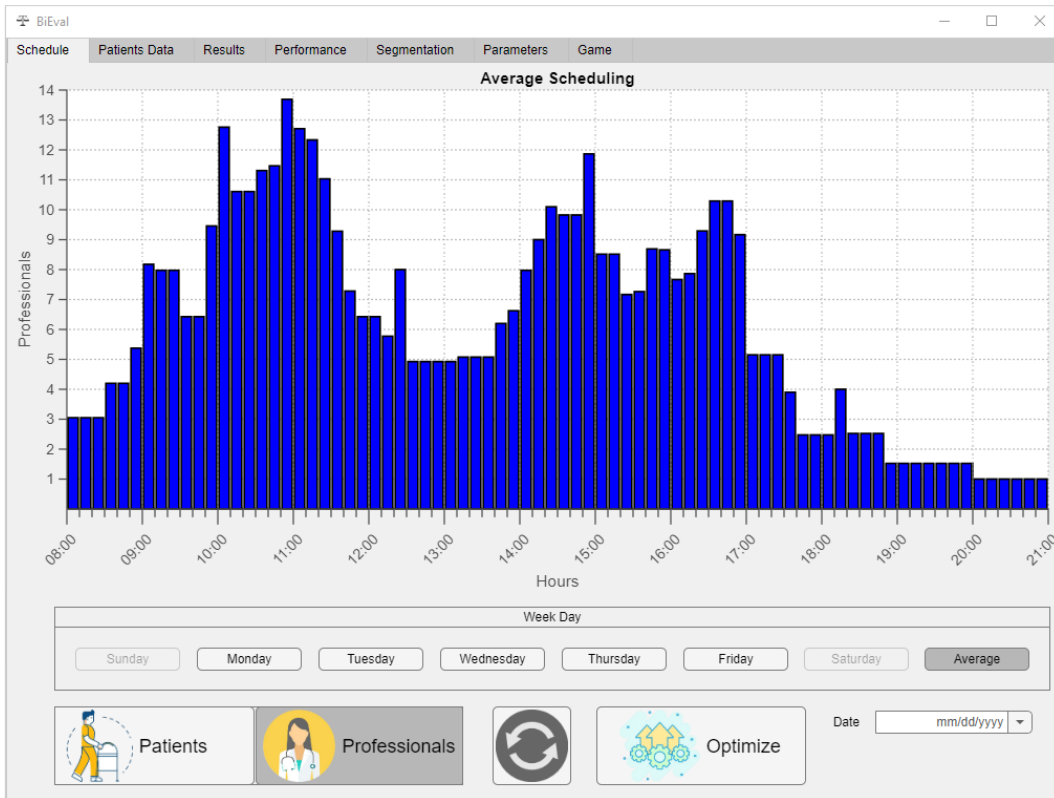
Multipliers are calculated from one of the three selected scores in the software, scaling the number of professionals' according to each patient requirements or specifics. Default linearization parameters, Figure 1(a), were employed. Figure 8 shows the generation of professional demand (D_j) in each interval.

Figure 7 – Optimization software showing the average weekly schedule for patients.



Source: Author

Figure 8 – Optimization software showing the average weekly demand for professionals in each interval.



Source: Author

The cost values (per hour) adopted to solve the model and find the optimal solution are shown in Table 4.

Table 4 – Cost values adopted to find the optimal solution.

Shift Class	Regular (C_f)	Night Shift (C_n)	Overtime (C_v)	Night Shift Overtime ($C_v \times C_n$)
Cost (per hour)	\$ 30.00	\$ 36.00 (+20 %)	\$ 45.00 (+50 %)	\$ 54.00 (+80 %)

Source: Author

4.1. Model Solution Results

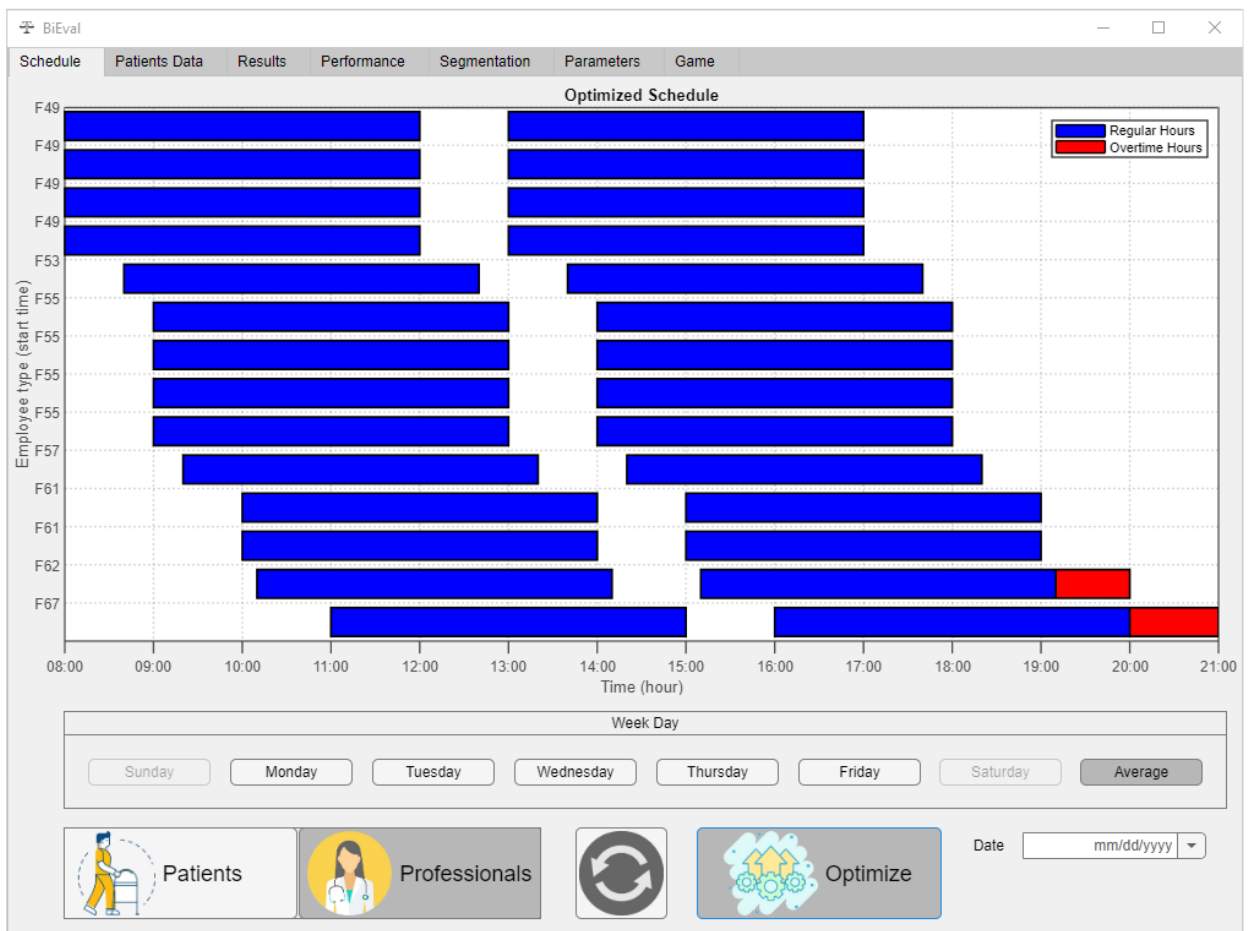
The results obtained after solving the model with the test data set is shown in Table 5. Due to the quantity (1872), variables with null values were omitted. The optimized professionals schedule found a minimal total cost $C_{total} = \$3,450.00/day$ and is shown in Figure 9.

Table 5 – Optimized professionals schedule found with the optimizations results.

Quantity	Professional shift	Working time	Overtime Hour
4	F49	08:00-17:00h	-
1	F53	08:40-17:40h	-
4	F55	09:00-18:00h	-
1	F57	09:20-18:20h	-
2	F61	10:00-19:00h	-
1	F62	10:10-19:10h	50 min
1	F67	11:00-20:00h	60 min (1h)

Source: Author

Figure 9 – Optimization software showing the optimized schedule with Operations Research.

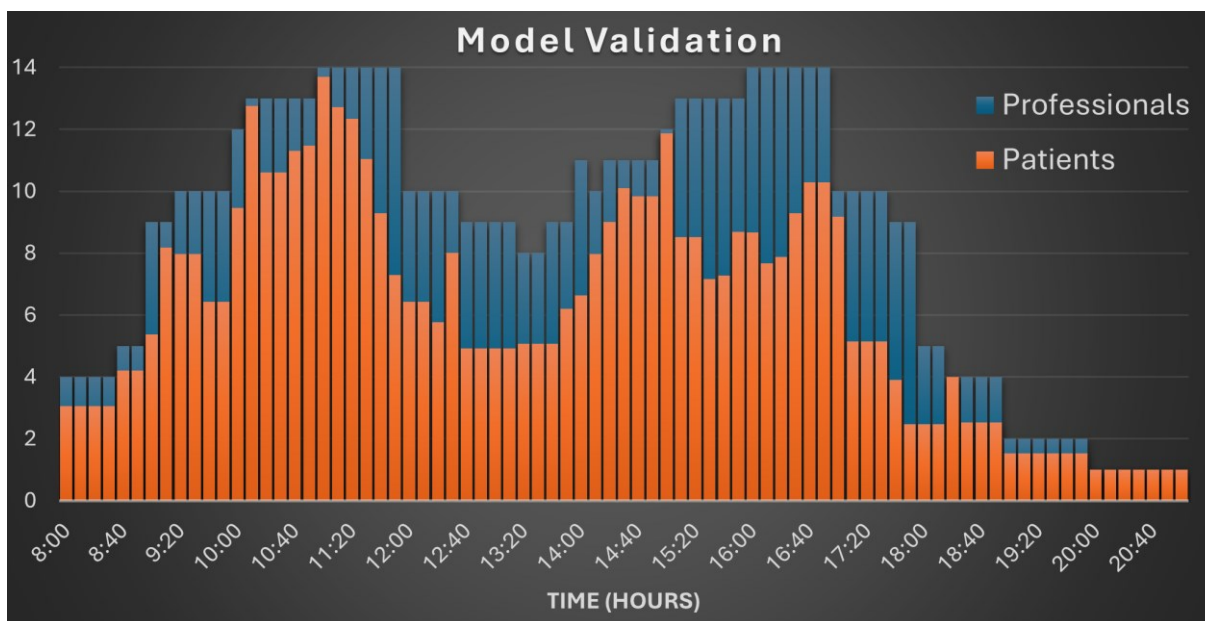


Source: Author

4.2. Model Validation

To ensure that the solution meets the expected result, it was verified if the values found for the decision variables satisfy all constraints while minimizing the objective function. The continuity of the work schedule can be observed graphically in Figure 9. Demand fulfillment can be confirmed in the validation shown in Figure 10, where the number of professionals scheduled in each interval is always equal to or greater than the respective demand.

Figure 10 – Graph for model validation confirming demand fulfillment.



Source: Author

5. CONCLUSIONS AND FUTURE RESEARCH

Accurate and efficient shift scheduling for healthcare professionals is highly complex without computational support. Given the substantial impact of staffing costs, optimizing this process is critical for enhancing operational efficiency in hospitals and rehabilitation centers.

This paper introduces the development of automated scheduling software for healthcare professionals, leveraging patient evaluation data and an optimization model grounded in Operations Research. By integrating patient assessment with the scheduling model, the software not only enhances patient outcomes but also optimizes the allocation of professional resources, including considerations for night shift allowances and overtime hours.

The software calculates the required number of professionals per shift using multipliers derived from patient conditions, based on three interchangeable options: software score, FM, and MAL. Using this weighted demand, OR techniques are employed to solve the optimization model embedded in the software and to generate an optimal shift schedule aligned with patients' specific needs. This approach allows for improving operational efficiency by minimizing costs and improving patient satisfaction through effective demand fulfillment. The solution met all imposed constraints, ensuring continuity in the work schedule and demand fulfillment, with a minimized total cost of \$3,450.00 per day.

Implementing this solution in a hospital or rehabilitation center offers a significant opportunity for future research, enabling practical expansion and validation of the results presented in this paper. Additionally, future studies could explore additional variables, such as professional levels, experience, and skills, among others, to further enhance the model's effectiveness.

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