

Performance of Funds of Funds (FOFs) in Brazil

Abstract

FOFs, or fund of funds, are a widely developed investment instrument globally, including in Brazil. The goal of this study is to evaluate the performance of equity FOFs in Brazil by assessing it through three methodologies and three different views of the sample. The first, composed of panel regressions, aims to identify the determinants of returns and value creation of FOFs. The second identifies the probability of FOFs outperforming mirror and master funds. Using logit regressions, the study seeks to identify the determinants of these probabilities. Lastly, FOF indices were constructed and their performance, return determinants, and market timing ability were analyzed in comparison with BOVA11 ETF, the most liquid in the Brazilian market. This paper contributes by clearly defining FOFs because of observation of fund portfolios. The final sample comprises 3350 stock funds, of which 307 are equity FOFs, 1242 Master funds and 1801 Mirror funds. These funds are managed by a total of 404 asset managers, of which 89 are FOF managers. The analysis period is between 12/31/2013 and 12/31/2023, 10 years of data. The results show that overall, on average, FOFs have a lower return and performance measured by the Sharpe ratio than the others. Fama-French and Carhart factors, together with exchange rate exposure, and the management fee charged being the main determinants of the return. The probability of FOF having a higher return than other equity funds is less than 50%. However, it is possible to construct index or FOF strategies that outperform passive investing such as a general market index ETF, but do not have market timing skill.

Keywords: mutual funds, hedge funds, fund of funds, FOF, stock funds, performance, Sharpe ratio, market timing

Introduction

The investment fund industry in Brazil was born in the late 1950s and has experienced great growth since the 1990s. The total AUM at the end of 2023 was R\$8.3 trillion, equivalent to US\$1.5 trillion, which places it as the 11th largest in the world according to data from IAFA (2023). If we eliminate Luxembourg and

Ireland from the ranking, which operate as investment hubs, the Brazilian industry is 9th in the world in terms of AUM. It is an industry highly regulated by the CVM – Real Estate Securities Commission – and self-regulated by ANBIMA – Brazilian Association of Financial and Capital Market Entities – and therefore has not presented major or systemic problems for investors, and the Brazilian market in recent decades. In December 2023, according to ANBIMA's ranking of funds, there were 970 assets in Brazil with more than 50,000 funds and 30 million shareholders. In addition to regulation, both CVM and ANBIMA contribute to the classification, survey and organization of data relating to investment funds, which are used in this work.

A fund of funds (FOF) investment strategy, classified as hedge funds in some markets, that comprises investing in a portfolio of shares of other funds, rather than directly in individual securities like stocks or bonds. These underlying funds can be diverse, encompassing various asset classes, investment styles, and geographic regions.

FOFs provide instant access to a diversified portfolio, reducing risk by spreading investment across multiple funds and underlying assets. This minimizes the impact of any single fund underperforming. FOFs are managed by experienced professionals who conduct extensive research and select underlying funds based on their specific investment objectives. This eliminates the need for individual investors to research and choose funds themselves, and often benefit from economies of scale, which can lead to lower transaction costs compared to investing in individual funds directly. These funds can provide access to specialized asset classes or investment strategies that may be otherwise inaccessible to individual investors, such as private equity or hedge funds, and by investing in FOF, investors avoid the risk of selecting underperforming underlying funds. The FOF manager handles the selection and monitoring of individual fund managers. However, it's important to consider potential drawbacks of FOFs, such as higher fees and potential for double taxation.

Asset managers can also benefit from FOFs, because they attract a wider pool of investors, leading to larger AUM (assets under management) and potentially higher revenue for asset managers. FOFs charge their own management fees in addition to the fees charged by the underlying funds, creating an additional revenue stream for asset managers, and they can leverage economies of scale to reduce operational costs associated with managing individual investments. Managing a successful FOF can enhance the brand recognition and reputation of the asset manager.

Funds of funds in Brazil have existed for decades, and in the Brazilian market there are FOFs of equity, multimarket, fixed income, and real estate funds. This work aims to answer some questions that concern

investment in equity FOFs in the Brazilian market: What are the determinants of FOFs ' performance? What are the determinants of the probability of do FOFs outperform other types of equity funds? Is there any investment strategy in FOFs that results in superior performance compared to investing in a generic stock index in Brazil?

Previous studies

Compared to individual funds, research on FOFs is less extensive. Few works were identified in Brazil, and even in Latin America. There are studies abroad on the subject, but with opposite conclusions. Existing papers often explore specific aspects like:

- Performance: Studies investigate whether FOFs outperform their benchmark indices or offer diversification benefits. Findings are mixed, with some showing limited alpha generation and others highlighting potential diversification value.
- Costs: FOFs typically have higher fees than individual funds, leading to research on their cost-effectiveness. While fees might be justified for specialized expertise or access to unique strategies, the net benefit remains a critical question.
- Selection Process: Examining how FOFs select underlying funds reveals crucial insights into potential biases and performance drivers. Some studies analyze the impact of manager skill, investment style preferences, and risk management practices on overall FOF returns.

Ang, Zhao and Rhodes-Kropf (2008) indicate that 25% of hedge funds in the United States are funds of funds. And that is the appropriate benchmark to evaluate your performance. This benchmark is developed in the work from concepts of certainty equivalents and preference arguments. This way they estimate the distribution of the benchmark for funds of funds. The rationale behind this estimate concerns the differences found among investors in terms of volume invested. Small investors will mitigate their risks through funds of funds, while large investors will allocate their resources directly to hedge funds. The period of analysis is between 1992 and 2003 with a total of 3695 funds, 748 of which are funds of funds. The alphas of fund of funds are lower than those of hedge funds, but using the benchmark proposed by the authors, and then fund of funds add value to investors' portfolios, especially for less sophisticated investors.

Ramadorai et al (2008) analyze 1603 funds of funds in the United States between 1995 and 2004. The authors justify the choice of FOFs due to the low quality of information available for other hedge funds in the United States. They conclude that on average these funds presented Jensen alphas only in the period between October 1998 and March 2000, less than a year and a half. But they indicate that there is a subset of funds that showed positive alpha throughout the period, and that, unlike the others, they were very successful in raising funds. This led to greater difficulty in generating positive alphas. Another interesting result of the study refers to the importance of positive alpha for the survival of the fund. Among those that generated positive alphas, only 7% were liquidated, compared to 22% of those that did not generate positive alphas.

Elton, Gruber and Souza (2017) make an interesting comparison between funds of funds that invest in in-house funds and those that invest outside the home. The authors indicate that managers have non-public information about their house funds, but still choose funds that perform worse than randomly selected funds. Negative point for FOFs.

Gowri and Deo (2018) evaluate the performance of FOFs in India by comparing the performance of these funds with the risk-free rate (91-day Treasury bills) and a benchmark (BSC 100). Analyzing data from 2007 to 2014, the authors conclude that all FOFs perform better in relation to the risk-free rate and most of them also perform better in relation to the benchmark. Performance was measured using Sharpe and Treynor indices as well as Jensen's alpha. The authors indicate that this result must be due to the diversification strategy of FOFs.

In Brazil, studies are also found with opposite conclusions regarding the benefits of FOFs for investors.

Assali (2008) analyzes 32 Brazilian funds of funds in the period from 1998 to 2007. The author focused his analysis on multimarket funds (as hedge funds are known in Brazil), only one of which was a multimarket fund without variable income. Almost all the funds in the sample do not exceed the risk-free rate (DI rate) during the analysis period, indicating that FOFs do not add value to investors. However, the author concludes that the risk-return relationship, when viewed from the optimal perspective of an efficient frontier, improved when these funds are included in the analysis.

Cotrim (2012) analyzes 1421 multi-manager funds in Brazil from 2005 to 2011. Using Jensen's alpha, he concludes that only 3.03% of the funds studied manage to add value for the shareholder. Even with the elimination of costs, the percentage of funds that produced positive alpha is only 6.39%. The author also

concludes that managers do not have market timing and are unable to select the best funds. However, these managers get the strategic allocation right, that is, the choice of strategy for their fund portfolio.

Fonseca (2012) analyzes 18 multi-manager funds in the period between 2005 and 2010, indicating that in all years, except 2008, most of these funds generated positive alpha, surpassing the benchmark used, the Ibovespa, and generating value for investors.

Data sample

The Brazilian investment fund industry may be at an advanced stage of evolution and complexity, but as there are no clear definitions and characterizations of certain types of investment funds that make up or are confused with FOFs, that is why in this work the following were developed and adopted fund categories:

- “Cash” Fund: is a highly liquid fixed income investment fund used to meet redemptions and investments made by fund shareholders. Many investment funds, including FOFs, use “Cash” funds to handle redemptions and applications requested by investors.
- Master Fund: is an investment fund that invests directly in individual assets, which are not investment funds, and in some other investment funds, but cannot invest only in investment funds. Master funds receive resources from FOFs or mirror funds or directly from investors, in accordance with the distribution strategy of the resource management company.
- Mirror Fund: is an investment fund that invests only in another fund, disregarding “Cash” funds. Mirror funds are used by asset managers to distribute their funds across different investor channels and audiences, charging fees or a minimum initial investment amount differentiated by audience or channel.
- FOF Mirror Fund: it is a mirror fund that invests only in a certain FOF, disregarding “Cash” funds. In this work, this type of fund is considered as a mirror, but not as FOF. The FOF mirror is used to distribute the FOF across different investor segments or distribution channels.
- FOF: is a fund that invests only in other funds, it must invest in at least 1 fund from a manager other than the FOF manager, and in at least 2 funds in total, excluding “cash” funds. Invested funds can be national, international, Exchange Traded National Funds (ETFs), international ETFs, Participation Funds, or multimarket funds. The exception is FOFs that invest in funds from the FOF manager itself, but there is at least 1 mirror fund from another manager other than the FOF manager.

A FOF cannot have individual assets such as shares or BDRs, but it can have fixed income securities issued by the Brazilian National Treasury or private securities such as certificates of deposit, which have a similar function to “Cash” funds, due to their liquidity and meeting redemptions and applications. It may have derivatives for hedging purposes for investing abroad.

The previous definitions are essential to distinguish equity funds and classify them into 3 basic categories: mirror, FOF and master. Within these 3 categories, other forms of fund classification can be applied, such as, for example, by active or passive management, by investment strategy, etc.

To construct the sample, equity funds were selected from the ANBIMA database, available in the Economatica system on 12/31/2023. Funds that were recently canceled were considered, but the study may be subject to some level of survival bias, as mainly older funds did not have data on the investment portfolio, which did not allow the fund to be categorized. All data on shares, characteristics, and investment fund portfolios were obtained from the Economatica system (<http://www.economatica.com>), as well as ETF shares, IBOVESPA stock index, and DI rate.

Funds classified as exclusive in the ANBIMA database were excluded from the sample, as they are owned by an individual investor who can influence the fund's resource management; funds managed by foundations and social security institutes, and social security funds, as they have restrictions on investments in shares; privatization funds (e.g. Petrobras or Vale privatization mutual fund) and mono share funds as they only invest in one asset. Funds that only invest in funds from the same manager and that do not have mirror funds from other managers were reclassified as master, instead of being considered as FOF. There are funds in the sample with only 1 shareholder, which are not classified as exclusive by the resource manager, as they are a master fund that receives financial resources from a mirror fund, for example.

The Economatica system has the investment portfolios of equity funds at the end of each month, to identify the mirror, master and FOF funds, the portfolios were investigated, and the investment funds were classified. The Economatica database that will be used in this paper, after applying the exclusion criteria, contains 3350 equity funds, of which 307 equity FOFs, 1242 Master funds and 1801 mirror funds, 61 of which are mirror FOFs. These 3350 funds are managed by a total of 404 asset managers, but 315 of them do not have FOFs on their investor product shelf, or in other words, there are 89 FOF managers in the sample.

The analysis period for equity funds is between 12/31/2013 and 12/31/2023, ten years therefore. Daily, monthly, and annual data were used depending on the analysis or method applied. The rates of return on fund shares and indices or references were calculated logarithmically to perform the aggregation in months or years.

IBOVESPA was chosen as a benchmark for the Brazilian stock market, which is the most widespread stock index on B3 (Brazil's main stock exchange) and a reference for investors in this market. The one year risk-free rate adopted in this paper is computed from the 360-day DI Swap, that will be called DI rate, and factors used comes from data available in the NEFIN data base, that is a Brazilian Center for Research in Financial Economics of the university of São Paulo, (https://nefin.com.br/data/risk_factors.html). For some analyses, BOVA11 was used, which is the ETF referenced on IBOVESPA with the highest liquidity on B3 and available since the end of 2008, and it is a passive investment on IBOVESPA, with high liquidity in the Brazilian market and low cost (management fee) of 0.10% per year in December 2023 for investors. The choice of these benchmarks, which are a reference in the stock market in Brazil, is justified, given that the study sample is composed exclusively of equity funds.

Table 1, below, presents the descriptive statistics of the sample in aggregate and separated by type of investment fund (mirror, master or FOF). Contrary to what is normally expected, the average management fee of FOFs was lower than that of other types of funds (0.49% per year), approximately 43% of the average management fee charged by mirror funds, which was 1.13% per year, and 60% of the average management fee for master funds. The management fee disclosed by investment funds in Brazil is the maximum that can be charged, including the management fees of other invested funds, that is, apparently FOFs invest in other funds with lower fees than those charged by funds in general, and managers exchange lower management fees for greater fundraising and allocation of resources. It can also be noted that fewer FOFs charge a performance fee, approximately 22% of the sample, compared to mirrors (43%) and masters (33%). Therefore, on average the FOFs charge lower fees than expected, the transparency and regulation of the Brazilian fund industry should have this effect.

Table 1: Descriptive statistics for the sample of equity funds.

Statistic	AUM Dec. 29th 2023 (R\$ mi)	Number of Shareholders on Dec. 31st 2023	Management fee (% per year)	Dummy fund charges performance fee = 1	Minimum initial investment (R\$ x1000)	Dummy fund allows leverage = 1	Dummy fund follows ESG = 1	Redemption conversion period (days)	Redemption payment period (days)	Years since fund inception	Maximum overseas investment (%)
MIRROR FUNDS											
Average	130.5	2634.0	1.13	0.43	670.8	0.50	0.01	31.5	33.9	5.9	38.4
Trimmed average	89.5	351.2	1.08	0.42	50.3	0.50	0.00	26.8	29.2	5.3	37.8
Standard deviation	558.7	33020.3	0.95	0.49	23588.7	0.50	0.08	46.1	45.9	6.7	32.4
Median	36.5	28.0	1.05	0.00	5.0	1.00	0.00	30.0	32.0	3.7	40.0
Minimum	0.0	1.0	0.00	0.00	0.0	0.00	0.00	0.0	0.0	0.3	0.0
Maximum	20346.7	818315.0	10.00	1.00	1000000.0	1.00	1.00	540.0	542.0	56.5	100.0
Observations	1778	1781	1751	1801	1801	1801	1801	1800	1800	1800	1800
MASTER FUNDS											
Average	248.9	511.8	0.81	0.33	186.8	0.48	0.03	11.2	13.5	7.2	33.7
Trimmed average	151.1	123.4	0.77	0.32	42.4	0.48	0.00	7.7	9.9	6.8	32.9
Standard deviation	858.2	3501.5	0.83	0.47	2016.3	0.50	0.16	30.8	31.1	6.6	35.2
Median	47.2	3.0	0.50	0.00	0.0	0.00	0.00	1.0	3.0	4.4	20.0
Minimum	0.9	1.0	0.00	0.00	0.0	0.00	0.00	0.0	0.0	0.3	0.0
Maximum	13452.1	61656.0	7.00	1.00	50000.0	1.00	1.00	365.0	367.0	52.2	100.0
Observations	1223	1225	1217	1239	1242	1242	1242	1238	1238	1238	1238
FUND-OF-FUNDS (FOFS)											
Average	152.8	176.4	0.49	0.22	244.1	0.82	0.00	23.6	26.0	7.1	56.5
Trimmed average	115.0	57.8	0.48	0.21	167.0	0.83	0.00	22.6	25.0	6.8	56.9
Standard deviation	338.4	960.6	0.40	0.42	1187.6	0.39	0.00	23.0	22.8	5.6	35.4
Median	49.1	4.0	0.41	0.00	0.1	1.00	0.00	30.0	32.0	4.6	40.0
Minimum	1.8	1.0	0.00	0.00	0.0	0.00	0.00	0.0	0.0	0.4	0.0
Maximum	3666.7	12926.0	2.25	1.00	20000.0	1.00	0.00	90.0	92.0	27.0	100.0
Observations	304	305	301	307	307	307	307	307	307	307	307
ALL FUNDS											
Average	176.3	1622.5	0.95	0.37	452.2	0.52	0.01	23.3	25.6	6.5	38.3
Trimmed average	111.1	229.6	0.91	0.36	58.1	0.52	0.00	19.1	21.5	6.0	37.7
Standard deviation	673.9	24337.6	0.90	0.48	17344.5	0.50	0.12	40.4	40.4	6.6	34.3
Median	40.9	8.0	0.80	0.00	1.0	1.00	0.00	9.0	11.0	3.9	40.0
Minimum	0.0	1.0	0.00	0.00	0.0	0.00	0.00	0.0	0.0	0.3	0.0
Maximum	20346.7	818315.0	10.00	1.00	1000000.0	1.00	1.00	540.0	542.0	56.5	100.0
Observations	3305	3311	3269	3347	3350	3350	3350	3345	3345	3345	3345

* Trimmed average calculated excluding the 5% lowest and the 5% highest values. AUM = Assets Under Management.

As can be seen, the funds do not yet have major commitments to ESG issues, which generally account for approximately 1% of the sample on average. On average 82% of FOFs allow leverage compared to approximately 50% for mirror and master funds, and FOFs can invest a maximum of an average of 56% of their portfolio abroad compared to somewhere between 33% and 38% for mirrors and masters. FOFs seek to differentiate themselves from other investment fund strategies, which is why they seek to have greater flexibility in their regulations to invest both in terms of leverage and investing in the foreign market or local market assets but with foreign risk. Mirror funds are generally newer than master funds and FOFs, as they are created after the period that the funds in which they invest need to create a track record of performance.

In Table 2, below, 56.7% of FOFs (174 funds) were created and classified as free equity funds according to ANBIMA's classification, precisely to have greater flexibility in portfolio allocation. Another 37.1% of FOFs (114 funds) are classified as equity funds for investment abroad, which allows for as great flexibility

in investment policy as those categorized as free equity. As Brazilian regulation of investment funds requires managers to indicate the investment policy in the fund regulations, approximately 81% of funds are concentrated in free classifications. and investment abroad precisely to have greater freedom and flexibility in allocating the amounts invested.

Table 2: Funds segregated by ANBIMA classification.

ANBIMA classification for STOCK funds	Number of funds					
	Espelho			Master	FOF	Total
	Not FOF	of FOF	Total			
Dividends	28		28	34		62
Index	44		44	42		86
Index active	84	7	91	133	18	242
Investment abroad	317	12	329	239	114	682
Free	1131	42	1173	686	174	2033
Sector	12		12	23		35
Small Caps	36		36	21		57
Sustainability/Governance	8		8	2		10
Value/Growth	80		80	62	1	143
Number of funds	1740	61	1801	1242	307	3350

The redemption and conversion deadlines for FOF shares were between those of the master funds (shorter deadlines) and the mirror funds. This occurs due to the distribution structure of the products because if the shareholder invests in a FOF mirror fund, he will have to wait for the FOF make redemptions from the master funds, and the master funds will have to redeem and sell assets to meet the demand of the FOFs, and consequently of the shareholders.

The average number of shareholders of FOFs is approximately 176 investors, which represents only 6.7% of the average number of shareholders of mirror funds, but the average amount invested per shareholder (AUM divided by the number of shareholders) of FOFs was R\$866,442 which was 16.5 times greater than the average amount invested per shareholder of mirror funds, that is, there are still few investors for FOFs but they invest more of their resources in these types of funds, as they already guarantee good diversification, in the sample on average a FOF allocates resources in 9.52 investment funds, already disregarding “Cash” funds. Table 3 below presents the distribution of the number of FOFs in relation to the number of funds that receive some allocation of financial resources from the FOFs. The data that on average a FOF performs allocation in approximately 10 funds will be used later to construct the FOF indices.

Table 3: Distribution of number of FOFs in relation to the number of funds that receive some allocation from FOFs, excluding “Cash” funds.

Number of funds invested by each FOF	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Number of FOFs	0	2	23	7	16	26	20	21	32	34	31	26	25	22	5	9	1	5	2	0	307
Proportion of FOFs over the total (%)	0.0	0.7	7.5	2.3	5.2	8.5	6.5	6.8	10.4	11.1	10.1	8.5	8.1	7.2	1.6	2.9	0.3	1.6	0.7	0.0	100.0
Accumulated proportion of FOFs over the total (%)	0.0	0.7	8.1	10.4	15.6	24.1	30.6	37.5	47.9	59.0	69.1	77.5	85.7	92.8	94.5	97.4	97.7	99.3	100.0	100.0	100.0

Methodology and results

To answer the questions proposed in the introduction of this paper, the methodology was divided into 3 parts. In the first, panel analysis is used with the total sample and subsamples to identify the determinants of equity fund performance and the value creation of FOFs for investors. The second part estimates the probability of FOFs having a higher rate of return than master and mirror funds and applies logistic (or logit) regressions to identify the determinants of the estimated probabilities. Finally, the third part presents the construction of FOF indices and analyzes their performance, determinants of profitability, and market timing ability in comparison with the ETF BOVA11.

Table 4, below, presents the descriptive statistics of the annualized rates of return of the investment funds in the sample in an aggregated manner and separately by type of fund (mirror, master or FOF), for each of the years of the 10-year sample period which ended at the end of December 2023. The annualized rates of return, using the number of business days in each year, were obtained from the sum of the daily logarithmic rates of return of investment funds, using the Equation (1):

Equation (1)
$$\text{Annualized return } i = e^{\left(\sum_{t=1}^{N_i} \ln\left(\frac{\text{Share}_t}{\text{Share}_{t-1}}\right)\right)} - 1$$

Where Annualized return i = the arithmetic annualized rate of return of any investment fund in year i .

i = a given year within the 10-year period from 2014 to 2023, inclusive.

N_i = is the number of working days in year i .

t = a given business day.

Share t = is the share of the investment fund on business day t , coming from the Economatica system.

The standard deviation, a measure used to estimate risk, is calculated in this paper with daily fund return data and annualized by multiplying the result by the square root of 252 or the effective number of business days in the time window under evaluation. The annualized standard deviation can be converted from logarithmic to arithmetic with the formula used to annualize returns, by replacing the sum of the log returns with the annualized standard deviation.

Daily logarithmic rates of return on investment funds were used for this work to facilitate the aggregation of data into monthly and annual periods, as well as to work with moving windows of profitability that are employed by the study methodology, however the profitability results were converted from logarithmic form for arithmetic, more natural for general people.

The Sharpe ratio, known as reward-to-variability ratio, was calculated following the traditional form of Sharpe (1966), based on the average return rates and annualized standard deviations of the funds in each year using the Equation (2):

Equation (2)
$$\text{Sharpe ratio} = \frac{\text{Average annualized return } i - \text{RiskFree } i}{\text{StdDev } i}$$

Where Average annualized return i = is the average of the annualized arithmetic return rates of investment funds in year i .

RiskFree i = is the risk-free rate in year i , in this case it is the DI rate in year i .

StdDev i = is the average annualized standard deviation of investment funds in year i .

Table 4 below presents the descriptive statistics of the return rates of investment funds segregated into mirror, FOF, and master, for each of the 10 years of the sample period. The Sharpe ratios of the aggregate

funds for the various years are also presented. It can be seen in Table 4 that there are two periods in which the performance of equity funds in total and by category of funds had a negative Sharpe ratio, that is, it was not possible to surpass the risk-free interest rate. These two periods were 2014-2015 when there was an economic crisis and recession in Brazil that culminated in the impeachment of President Dilma Rousseff, and 2021-2022 which was the period of recession and reflections of the global crisis resulting from the COVID-19 pandemic.

The data shows that generally the risk, measured by the standard deviation of returns, is lower in FOFs than in other categories of funds, which is a result of the natural diversification of this investment strategy, after all FOFs diversify financial resources in investment funds that already have some degree of diversification. However, it can be seen that in most years of the analysis period the average return rate of FOFs was lower than the average of each of the other types of funds, this may have been caused by the fees charged by FOFs and their invested funds, or problems in the selection and diversification of funds in which FOFs invest.

The Sharpe ratios of FOFs were generally lower than those of mirror and master funds, mainly due to the lower average profitability of FOFs, as the risk of FOFs was generally lower than that of other funds, that is, the lower risk was unable to compensate for the lower profitability of FOFs to provide a higher Sharpe ratio.

Table 4: Descriptive statistics of the return rates of funds and benchmarks.

Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Business days in the year	248	246	249	246	245	248	249	247	250	248
MIRROR FUNDS										
Average Annualized Returns (% p.a.)	-0.93	-4.54	27.29	27.21	15.12	38.79	9.23	-8.72	-7.50	20.69
Trimmed Average Annualized Returns (% p.a.)	-0.50	-5.04	26.92	26.31	14.84	37.68	8.57	-9.46	-7.23	20.65
Mean Standard Deviation of Annualized Returns (% p.a.)	19.10	18.06	21.30	17.73	20.72	16.70	51.29	22.96	26.82	19.12
Trimmed Mean Standard Deviation of Annualized Returns (% p.a.)	18.85	17.62	20.84	17.45	20.44	16.51	51.09	22.77	26.37	18.86
Median Annualized Returns (% p.a.)	0.16	-7.31	27.75	24.99	14.50	35.66	3.85	-12.53	-7.32	20.35
Minimum Annualized Returns (% p.a.)	-72.34	-55.02	-20.86	-13.87	-19.38	-8.60	-45.38	-54.44	-65.85	-39.24
Maximum Annualized Returns (% p.a.)	26.17	65.49	129.20	189.03	105.17	278.73	120.97	217.79	34.23	85.98
Sharpe ratio	-0.615	-0.985	0.624	0.975	0.420	1.966	0.126	-0.573	-0.742	0.400
Sharpe ratio using trimmed means	-0.600	-1.037	0.620	0.939	0.412	1.921	0.114	-0.609	-0.744	0.404
Number of funds	317	353	381	417	477	575	804	1150	1468	1674
MASTER FUNDS										
Average Annualized Returns (% p.a.)	-3.65	-6.74	30.27	30.99	15.88	42.95	6.21	-8.48	-5.35	23.05
Trimmed Average Annualized Returns (% p.a.)	-3.34	-7.44	29.32	29.45	15.68	41.64	5.95	-8.73	-5.09	21.97
Mean Standard Deviation of Annualized Returns (% p.a.)	21.36	20.98	24.87	19.80	22.66	19.06	56.24	25.75	29.52	23.31
Trimmed Mean Standard Deviation of Annualized Returns (% p.a.)	20.79	19.94	23.69	19.14	22.00	18.45	54.77	24.76	27.90	21.58
Median Annualized Returns (% p.a.)	-2.84	-9.27	29.63	27.63	15.42	39.06	3.75	-11.69	-3.35	20.90
Minimum Annualized Returns (% p.a.)	-66.98	-71.75	-28.69	-16.52	-62.53	-34.70	-71.95	-86.84	-97.10	-98.66
Maximum Annualized Returns (% p.a.)	29.64	81.36	143.07	277.27	81.64	190.92	97.07	68.10	54.41	530.89
Sharpe ratio	-0.677	-0.952	0.654	1.064	0.417	1.940	0.061	-0.501	-0.601	0.429
Sharpe ratio using trimmed means	-0.681	-1.037	0.647	1.020	0.421	1.934	0.058	-0.531	-0.627	0.414
Number of funds	341	395	423	449	505	591	684	838	1045	1159
FUND-OF-FUNDS (FOFS)										
Average Annualized Returns (% p.a.)	1.28	-0.59	22.76	24.49	13.06	36.03	7.05	-12.88	-5.39	20.90
Trimmed Average Annualized Returns (% p.a.)	1.41	-0.62	23.06	24.42	13.29	36.07	6.87	-13.43	-5.28	21.04
Mean Standard Deviation of Annualized Returns (% p.a.)	15.81	13.87	16.42	14.37	17.24	14.56	47.47	20.27	22.27	16.02
Trimmed Mean Standard Deviation of Annualized Returns (% p.a.)	15.65	13.49	16.16	14.43	17.33	14.57	47.92	20.41	22.34	16.11
Median Annualized Returns (% p.a.)	1.12	-1.00	22.91	24.69	14.04	36.36	6.69	-15.34	-5.45	21.71
Minimum Annualized Returns (% p.a.)	-20.52	-31.42	-30.03	0.20	-11.68	11.87	-17.57	-28.52	-34.32	-5.51
Maximum Annualized Returns (% p.a.)	15.86	30.72	46.13	55.55	23.86	65.67	38.19	31.24	25.56	56.12
Sharpe ratio	-0.603	-0.997	0.534	1.013	0.385	2.066	0.090	-0.853	-0.798	0.491
Sharpe ratio using trimmed means	-0.600	-1.027	0.561	1.005	0.397	2.066	0.086	-0.875	-0.791	0.496
Number of funds	85	97	104	109	119	146	176	218	271	299
ALL FUNDS										
Average Annualized Returns (% p.a.)	-1.92	-5.12	28.16	28.65	15.24	40.36	7.76	-9.04	-6.49	21.58
Trimmed Average Annualized Returns (% p.a.)	-1.51	-5.74	27.45	27.47	15.00	39.15	7.36	-9.59	-6.22	21.11
Mean Standard Deviation of Annualized Returns (% p.a.)	19.76	18.95	22.40	18.31	21.23	17.53	52.92	23.75	27.39	20.38
Trimmed Mean Standard Deviation of Annualized Returns (% p.a.)	19.37	18.17	21.53	17.83	20.77	17.08	52.02	23.13	26.39	19.35
Median Annualized Returns (% p.a.)	-0.91	-7.68	27.68	26.06	14.85	37.08	4.10	-12.46	-5.47	20.73
Minimum Annualized Returns (% p.a.)	-72.34	-71.75	-30.03	-16.52	-62.53	-34.70	-71.95	-86.84	-97.10	-98.66
Maximum Annualized Returns (% p.a.)	29.64	81.36	143.07	277.27	105.17	278.73	120.97	217.79	54.41	530.89
Sharpe ratio	-0.645	-0.969	0.632	1.023	0.415	1.963	0.094	-0.567	-0.689	0.419
Sharpe ratio using trimmed means	-0.636	-1.044	0.625	0.984	0.413	1.943	0.088	-0.606	-0.705	0.417
Number of funds	743	845	908	975	1101	1312	1664	2206	2784	3132
BENCHMARKS										
Risk-free rate (DI rate, % p.a.)	10.81	13.24	14.00	9.93	6.42	5.96	2.76	4.42	12.39	13.04
BOVA11 (IBOVESPA ETF) annual returns (% p.a.)	-3.38	-13.21	38.24	26.75	14.60	31.38	2.79	-11.88	5.11	22.81
IBOVESPA index annual returns (% p.a.)	-2.91	-13.31	38.94	26.86	15.03	31.58	2.92	-11.93	4.69	22.28

* Trimmed averages and mean values calculated excluding the 5% lowest and the 5% highest values.

Firstly, an analysis will be carried out of the determinants of the return rates of the equity funds in the sample, especially FOFs. To this end, panel regressions of the forms pooling, fixed effects, and random effects, each of which is for 3 different sample views. The first sample view is made up of monthly observations, monthly return rates and end-of-month data, for all available funds and all 10 years. For this sample, the panel regression will be unbalanced, as not all data is available for everyone. funds on all dates as can be seen in Table 4 of descriptive statistics. The second sample view is composed of annual

observations, annual return rates and end-of-year data, for all available funds and all 10 years. For this sample, the panel regression will also be unbalanced as in the first sample view, as it does not have all data available for all funds on all dates. The third sample view is made up of annual observations, annual rates of return and end-of-year data, for only funds that have all data from 2018 to 2023 (6 years in total), for this sample the panel regression will also be balanced, as there will be all data available for all funds within this period from 2018 to 2023.

The regressions will use as a basis the factors of Fama and French (1993), the momentum factor of Carhart (1997), the illiquidity from Acharya and Pedersen (2005), and a factor was added relating to the exchange rate variation of the Brazilian Real in relation to the United States Dollar, as part of the equity investment funds in Brazil invest in assets abroad, generally denominated in dollars from the USA. Table 4 even shows that FOFs can invest on average up to 56.5% of the portfolio in assets abroad, and in the sample of 307 FOFs, 92.8% of them can invest abroad.

The values of the Fama and French factors, as well as Carhart and illiquidity were obtained from the NEFIN website, while the quotations for the exchange rate of the Brazilian Real in relation to the US Dollar were obtained from the Economatica system, commercial dollar asset code PTAX 800.

In addition to the factors already mentioned, we will add variables specific to equity investment funds, coming from the Economatica system to the regression models. These specific variables may vary over time, for example, in the 10 years of the study, 16.54% of funds reduced the management fee, and 9.85% increased the fee charged to shareholders. The specific factors and variables that will be used in this work are described below:

- Risk- free rate (Rf): is the 360-day DI Swap, that it is called DI rate in this paper. Source: NEFIN.
- Market Factor (Mkt-Rf): is the difference between the value-weighted daily return of the market portfolio (IBOVESPA) and the daily risk-free rate. Source: Economatica, NEFIN.
- Small Minus Big (SMB): is the return of a portfolio long on stocks with low market capitalization (“Small”) and short on stocks with high market capitalization (“Big”). Every January of year t , NEFIN ascending sort the eligible stocks according to their December of year $t-1$ market capitalization and separate them into 3 quantiles. Then, NEFIN compute the equal-weighted returns of the first portfolio (“Small”) and the third portfolio (“Big”). The SMB Factor is the return of the “ Small ” portfolio minus the return of the “Big” portfolio. Source: NEFIN.

- High Minus Low (HML): is the return of a portfolio long on stocks with high book- to - market ratio (“High”) and short on stocks with low book- to - market ratio (“Low”). Every January of to year t, NEFIN ascending sort the eligible stocks into 3 quantiles (portfolios) according to the book- to - market ratio of the firms in June of year t-1. Then, NEFIN compute the equal-weighted returns of the first portfolio (“Low”) and the third portfolio (“High”). The HML Factor is the return of the “High” portfolio minus the return of the “ Low ” portfolio. Source: NEFIN.
- Momentum (MOM): is the return of a portfolio long on stocks with high past returns (“Winners”) and short on firms with low past returns (“Losers”). Every month t, NEFIN ascending sort the eligible stocks into 3 quantiles (portfolios) according to their cumulative returns between month t-12 and t-2. Then NEFIN compute the equal-weighted returns of the first portfolio (“ Losers ”) and the third portfolio (“Winners”). The MOM Factor is the return of the “ Winners ” portfolio minus the return of the “ Losers ” portfolio. Source: NEFIN.
- Illiquid Minus Liquid (ILLIQ): is the return of a portfolio long on stocks with high illiquidity (“Illiquid”) and short on stocks with low illiquidity (“Liquid”). Every month t, NEFIN ascending sort the eligible stocks into 3 quantiles (portfolios) according to their previous twelve-month illiquidity moving average, stock illiquidity is computed as in Acharya and Pedersen 2005. Then NEFIN compute the equal-weighted returns of the first portfolio (“Liquid”) and the third portfolio (“Illiquid”). The IML Factor is the return of the “ Illiquid ” portfolio minus the return of the “Liquid” portfolio. Source: NEFIN.
- Exchange Rate Variation Factor (BRLUSD- Rf): is the difference in exchange rate variation, calculated as log- return, of the Brazilian Real in relation To the United States Dollar and the daily risk-free rate. Source: Economatica.
- dummy variable that takes on a value equal to 1 if the fund is FOF, according to the definition adopted in this paper, and zero otherwise. Source: Economatica.
- Mgmt Fee: maximum management fee charged by the investment fund according to the ANBIMA database. Source: Economatica.
- LN(age): natural logarithm of the fund's age measured in years from the date the investment fund began operating, coming from the ANBIMA database. Source: Economatica.
- LN(AUM): natural logarithm of the fund's net equity measured in R\$, coming from the ANBIMA database. Source: Economatica.
- Leverage: dummy variable that takes on a value equal to 1 if the fund allows financial leverage and zero otherwise, according to the ANBIMA database. Source: Economatica.
- Perf Fee: dummy variable that takes on a value equal to 1 if the fund charges a performance fee and zero otherwise, according to the ANBIMA database. Source: Economatica.

- ESG: dummy variable that takes on a value equal to 1 if the fund has any commitment to ESG (Environmental, Social, and Governance) practices and zero otherwise, according to the ANBIMA database. Source: Economatica.

In general terms, the panel regression equation with fund-specific factors and variables can be written as follows:

Equation (3)

$$Ret_{i,t} - Rf_t = constant + \left(\sum_{j=1}^N a_j \cdot Factor_{j,t} \right) + b_i \cdot FOF_i + \left(\sum_{k=1}^M c_{k,i} \cdot Specific_{k,i,t} \right) + \varepsilon_t$$

Where $Ret_{i,t} - Rf_t$ = log- return of fund i in period t minus the risk-free interest rate on date t;

$Factor_{j,t}$ is factor j, one of Mkt-Rf, SMB, HML, MOM, ILLIQ, and BRLUSD- Rf, on date t;

FOF_j is the dummy equal to 1 when the fund is FOF, or zero otherwise;

$Specific_{k,i,t}$ is the specific variable k of fund i, one among Mgmt Fee, LN(age), LN(AUM),

Leverage, Perf Fee, ESG, on date t;

N = is the number of factors used in the regression specification;

M = is the number of fund-specific variables used in the regression specification;

a, b, c = are the coefficients resulting from the regression execution;

The following Table 5 shows the correlations between the different factors that are the independent variables of the model, excluding dummies. Apart from the correlation between the SMB and ILLIQ factors, the other variables do not show a high correlation with each other, so multicollinearity should not be a problem.

Table 5: Pearson correlation matrix between variables independent of the regression model.

Correlations	Mkt-Rf	SMB	HML	MOM	ILLIQ	BRLUSD-Rf	Mgmt Fee	LN(age)
Mkt-Rf	1.000	0.452	0.272	-0.533	0.380	-0.332	-0.030	-0.052
SMB	0.452	1.000	0.411	-0.461	0.776	-0.518	0.024	0.030
HML	0.272	0.411	1.000	-0.368	-0.028	-0.702	0.057	0.080
MOM	-0.533	-0.461	-0.368	1.000	-0.134	0.514	0.112	0.070
ILLIQ	0.380	0.776	-0.028	-0.134	1.000	-0.045	0.021	-0.020
BRLUSD-Rf	-0.332	-0.518	-0.702	0.514	-0.045	1.000	-0.058	-0.100
Mgmt Fee	-0.030	0.024	0.057	0.112	0.021	-0.058	1.000	0.431
LN(age)	-0.052	0.030	0.080	0.070	-0.020	-0.100	0.431	1.000

All regressions and econometric models in this paper were run in the R system, with functions `lm ()`, `plm ()` or `pglm ()` depending on the situation and model.

The first regressions in Table 6, are unbalanced panel regressions, were conducted with monthly observations of the sample of all funds (FOFs, masters and mirrors) from 2014 to 2023 inclusive, using the Fama- French and Carhart factors and variation of the Real against the Dollar of the US minus the risk-free interest rate. Inclusion of the FOF dummy to differentiate the performance of FOFs from masters and mirrors, and with other registration characteristics of the funds such as management fee, fund age, leverage, performance rate and finally indication whether it is a fund in any way oriented to ESG themes.

The dependent variable is the difference between the stock fund's monthly rate of return and the risk-free interest rate. All regressions include period (month) fixed effects. T- statistics in parentheses. Standard errors clustered at the investment fund-level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 6: Unbalanced panel regressions with monthly data.

The dependent variable is the difference between the stock fund's monthly rate of return and the risk-free interest rate.									
Panel regression model	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Mkt-Rf	0.9600*** (0.00470)	1.8341*** (0.50780)	0.9612*** (0.02620)	0.9600*** (0.00470)	1.8329*** (0.50720)	0.9612*** (0.02620)	0.9637*** (0.00480)	1.8374*** (0.50170)	0.9634*** (0.02620)
SMB	0.0141*** (0.00530)	1.2742** (0.63950)	-0.0482 (0.03280)	0.0140*** (0.00530)	1.2747** (0.63930)	-0.0481 (0.03280)	0.0136*** (0.00520)	1.2746** (0.64780)	-0.0485 (0.03220)
HML	-0.0603*** (0.00300)	1.2340*** (0.37460)	-0.0163 (0.02040)	-0.0602*** (0.00300)	1.2341*** (0.37430)	-0.0163 (0.02040)	-0.0603*** (0.00300)	1.2394*** (0.39290)	-0.0168 (0.02010)
MOM	-0.0827*** (0.00380)	0.9070*** (0.34750)	-0.1151*** (0.02270)	-0.0829*** (0.00380)	0.9072*** (0.34730)	-0.1152*** (0.02270)	-0.0812*** (0.00400)	0.8982*** (0.34830)	-0.1163*** (0.02260)
BRLUSD-Rf	0.1230*** (0.00550)	0.7005 (0.56850)	0.1272*** (0.03360)	0.1233*** (0.00550)	0.6994 (0.56770)	0.1273*** (0.03360)	0.1280*** (0.00550)	0.6958 (0.55100)	0.1320*** (0.03280)
FOF				-0.00003*** (0.00001)	-0.00004*** (0.00001)	-0.00004*** (0.00001)	-0.0001*** (0.00001)	-0.0001*** (0.00001)	-0.0001*** (0.00001)
Mgmt Fee							-0.00005*** (0.00000)	-0.00004*** (0.00000)	-0.00004*** (0.00000)
LN(age)							0.00003*** (0.00001)	0.00001 (0.00004)	0.00002 (0.00001)
Leverage							0.00003*** (0.00001)	0.000004 (0.00001)	0.000004 (0.00001)
Perf Fee							0.0001*** (0.00001)	0.00003*** (0.00001)	0.00003*** (0.00001)
ESG							0.0001*** (0.00002)	0.00003 (0.00002)	0.00004** (0.00002)
Constant	-0.00003*** (0.00001)		-0.00004 (0.00003)	-0.00003*** (0.00001)		-0.00004 (0.00003)	-0.0001*** (0.00001)		-0.00002 (0.00004)
Observations	193,298	193,298	193,298	193,298	193,298	193,298	193,298	193,298	193,298
R2	0.2967	0.0002	0.0187	0.2968	0.0004	0.0188	0.2986	0.0015	0.0205
Adjusted R2	0.2967	-0.0111	0.0186	0.2967	-0.0109	0.0188	0.2986	-0.0099	0.0204
F Statistic	16,309.1***	9.42***	3,682.7***	13,594.6***	12.24***	3,709.8***	7,482.3***	25.3***	4,046.2***

The first three regressions (1), (2) and (3) tested the Fama-French (1993), Carhart (1997) factors and the exchange rate variation for the entire sample of 3350 equity funds. The Hausman test was applied, and the fixed effects model (2) proved to be more consistent, at a significant level of 5%. The results of this model were in line with the literature of Fama-French and Carhart, indicating the dependence of the returns of equity funds on the excess return of the market index, SMB, HML, MOM, all coefficients with a positive sign. SMB positive implies that smaller companies outperform larger companies in the sample. A positive coefficient for HML meanings that value stocks outperform growth stocks. A positive coefficient for MOM suggests that stocks with past momentum (winners) continue to outperform those with negative momentum

(losers) in the panel. However, the exchange rate factor did not prove to be significant, despite the increase in the internationalization of investments that can be observed in stock fund portfolios.

The factor referring to illiquidity (ILLIQ) was eliminated since the VIF test showed a multicollinearity signal, as ILLIQ had a coefficient greater than 5 in the test.

We then insert a dummy variable to indicate when the fund is an FOF. An unbalanced panel with monthly observations of all funds in the sample was conducted and models (4), (5) and (6) were tested. The Hausman test was applied, and the fixed effects model (5) proved to be more consistent, at a significant level of 5%. The results of this model were aligned with the Fama- French and Carhart literature, as was the case with model (2), and the FOF dummy (takes the value 1 when the fund is FOF, and zero otherwise) proved to be significant. and with a negative sign, that is, FOFs have, on average, lower returns than other types of equity funds.

Independent variables related to specific characteristics of investment funds were then inserted, in addition to the FOF dummy variable. The unbalanced panel with monthly observations of all funds in the sample was conducted and models (7), (8) and (9) were tested. The Hausman test was applied, and the fixed effects model (8) proved to be more consistent, at a significant level of 5%. The results of this model were aligned with the Fama- French and Carhart literature, as was the case with models (2) and (5), and the FOF dummy was significant and had a negative sign, as was the case with model (5), that is, FOFs destroy value compared to other types of equity funds, mirror and master.

Model (8) identified as significant the variables management fee with a negative sign, and performance fee with a positive sign, indicating how it is expected that the higher the management fee, the higher the cost for the shareholder and the lower the profitability of the investment fund. On the other hand, the positive performance fee shows that funds that charge this fee must have managers who are more focused and aligned with the interests of shareholders, as the higher the fund's rate of return, the greater the manager's gain, that is, apparently the incentive system works.

The Breusch-Pagan test indicated heteroscedasticity, and the Breusch-Godfrey/Wooldridge test indicated serial correlation, so the coefficients were corrected by White and Arellano. The VIF test indicated that there is no multicollinearity among the independent variables, after removing ILLIQ. The Breusch-Pagan LM and Pesaran CD tests indicated cross-dependency. All tests performed were significant considering the significant level of 5 %.

A robust test of the previous results was conducted, now with panel regressions with annual data, which can be found in Table 7. An unbalanced panel was conducted with annual observations ending in December of each year of the sample of all funds (FOFs, masters and mirrors) using the Fama- French and Carhart factors and variation of the Real against the US Dollar minus the interest rate risk free. Furthermore, a FOF dummy was included to differentiate the performance of FOFs from masters and mirrors.

The dependent variable is the difference between the stock fund's annual rate of return and the risk-free interest rate. All regressions include period (year) fixed effects. T- statistics in parentheses. Standard errors clustered at the investment fund-level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Unbalanced panel regressions with annual data.

The dependent variable is the difference between the stock fund's monthly rate of return and the risk-free interest rate.									
Panel regression model	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects
	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Mkt-Rf	1.1250*** (0.02290)	2.0250*** 0.000	1.1515*** (0.03280)	1.1247*** (0.02290)	4.1559*** (1.17300)	1.1509*** (0.03270)	1.0920*** (0.02150)	3.3949*** (1.27750)	1.1151*** (0.03090)
SMB	0.0482*** (0.01340)	-0.0576*** 0.000	0.0427** (0.02030)	0.0483*** (0.01340)	-0.0196 (0.02090)	0.0429** (0.02030)	0.0488*** (0.01320)	0.0296 (0.02560)	0.0444** (0.01980)
HML	-0.0609*** (0.01490)	0.0971*** 0.000	-0.0529** (0.02250)	-0.0610*** (0.01490)	0.6978** (0.33070)	-0.0532** (0.02250)	-0.0758*** (0.01450)	0.4988 (0.36010)	-0.0700*** (0.02160)
MOM	-0.0349** (0.01510)	0.6314*** 0.000	-0.0268 (0.02140)	-0.0354** (0.01510)	2.5347** (1.04770)	-0.0278 (0.02140)	-0.0787*** (0.01520)	1.9361* (1.13850)	-0.0728*** (0.02140)
BRLUSD-Rf	0.2507*** (0.01660)		0.2659*** (0.02640)	0.2512*** (0.01660)		0.2663*** (0.02650)	0.2456*** (0.01640)		0.2592*** (0.02580)
FOF				-0.0001*** (0.00001)	0.0004* (0.00020)	-0.0001*** (0.00002)	-0.00005*** (0.00002)	0.0003 (0.00020)	-0.0001*** (0.00002)
Mgmt Fee							-0.000004 (0.00001)	-0.00001 (0.00001)	-0.00001 (0.00001)
LN(age)							-0.0001*** (0.00001)	-0.0001*** (0.00002)	-0.0001*** (0.00001)
Leverage							0.00002 (0.00002)	0.000005 (0.00002)	0.00002 (0.00002)
Perf Fee							-0.00001 (0.00002)	-0.00002 (0.00002)	-0.00002 (0.00002)
ESG							0.0001 (0.00010)	0.0001 (0.00010)	0.0001 (0.00010)
Constant	-0.00004* (0.00002)		-0.00005 (0.00003)	-0.00003 (0.00002)		-0.00004 (0.00003)	0.0001*** (0.00003)		0.0002*** (0.00004)
Observations	17,330	17,330	17,330	17,330	17,330	17,330	17,330	17,330	17,330
R2	0.3244	0.0004	0.1903	0.3245	0.0004	0.1905	0.3377	0.0137	0.2081
Adjusted R2	0.3242	-0.1888	0.19	0.3245	-0.1889	0.1902	0.3373	-0.1734	0.2076
F Statistic	1,663.5***	1.44	4,021.6***	1,387.2***	1.25	4,027.1***	802.7***	20.3***	4,501.3***

The Hausman test was applied and models (12), (15) and (18) of random effects were more consistent than the others, at a significant level of 5%. The results of this model were in line with the Fama- French literature, but Carhart's MOM factor was only significant in the specification (18). A positive SMB implies that smaller companies outperform larger companies in the sample. In these models, it was found to be a negative coefficient for HML, it means that growth stocks outperform value stocks. In the specification (18) the coefficient for MOM was negative, that suggests negative momentum (losers) continue to outperform those with past momentum (winners) in the panel. In the 3 specifications, the exchange rate factor was significant with a positive sign, capturing the increase in the internationalization of equity fund portfolios.

Dummy variable proved to be significant and with a negative sign, that is, FOFs have, on average, lower returns than other types of equity funds.

Model (18) identified only the specific variable LN(age) as significant and with a negative sign, that is, the older the investment fund, the lower its profitability, a similar result is found in Chen and Zhou (2004), but The relationship between fund age and performance is a controversial issue in the literature.

The Breusch-Pagan test indicated heteroscedasticity, and the Breusch-Godfrey/ Wooldridge test indicated serial correlation, so the coefficients were corrected by White and Arellano. The VIF test indicated that there is no multicollinearity among the independent variables, after removing ILLIQ. The Breusch-Pagan LM and Pesaran CD tests indicated cross-dependency. Finally, the Hausman test indicated that the random effects model is superior to the fixed effects model, at the 5% significant level. All tests performed were significant considering the significant level of 5 %.

The third sample view was used to conduct balanced panel regressions with annual data, which can be found in Table 8. Observations are annual ending in December of each year of the sample of all funds (FOFs, masters and mirrors) that have all data during the period 2018 to 2023 using the factors used in the previous specifications.

The dependent variable is the difference between the stock fund's annual rate of return and the risk-free interest rate. All regressions include period (year) fixed effects. T- statistics in parentheses. Standard errors clustered at the investment fund-level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Balanced panel regressions with annual data.

The dependent variable is the difference between the stock fund's monthly rate of return and the risk-free interest rate.									
Panel regression model	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects	Pooling	Fixed effects	Random effects
	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
Mkt-Rf	1.6960*** (0.11570)	1.6960*** (0.12000)	1.6960*** (0.11170)	1.6960*** (0.11570)	1.6960*** (0.12000)	1.6960*** (0.11170)	1.6775*** (0.11690)	1.7592*** (0.13050)	1.6797*** (0.11380)
SMB	0.0846*** (0.01460)	0.0846*** (0.01480)	0.0846*** (0.01410)	0.0846*** (0.01460)	0.0846*** (0.01480)	0.0846*** (0.01410)	0.0861*** (0.01460)	0.0798*** (0.01540)	0.0859*** (0.01420)
HML	0.3421*** (0.05020)	0.3421*** (0.05180)	0.3421*** (0.04850)	0.3421*** (0.05020)	0.3421*** (0.05180)	0.3421*** (0.04850)	0.3338*** (0.05080)	0.3708*** (0.05760)	0.3348*** (0.04940)
MOM	0.4837*** (0.09390)	0.4837*** (0.09580)	0.4837*** (0.09070)	0.4837*** (0.09390)	0.4837*** (0.09580)	0.4837*** (0.09070)	0.4702*** (0.09460)	0.5294*** (0.10270)	0.4718*** (0.09200)
BRLUSD-Rf	-0.008 (0.02620)	-0.008 (0.02910)	-0.008 (0.02530)	-0.008 (0.02620)	-0.008 (0.02910)	-0.008 (0.02530)	-0.0076 (0.02610)	-0.0103 (0.02900)	-0.0077 (0.02530)
FOF				-0.00005*** (0.00001)		-0.00005*** (0.00001)	-0.0001*** (0.00002)		-0.0001*** (0.00002)
Mgmt Fee							-0.00004*** (0.00001)	-0.0001* (0.00003)	-0.00004*** (0.00001)
LN(age)							-0.00001 (0.00001)	0.00003 (0.00004)	-0.00001 (0.00001)
Leverage							0.00001 (0.00002)		0.00001 (0.00002)
Perf Fee							0.00002* (0.00001)		0.00003 (0.00002)
ESG							-0.00002 (0.00010)		-0.00002 (0.00010)
Constant	-0.0011*** (0.00020)		-0.0011*** (0.00020)	-0.0011*** (0.00020)		-0.0011*** (0.00020)	-0.0010*** (0.00020)		-0.0010*** (0.00020)
Observations	6,504	6,504	6,504	6,504	6,504	6,504	6,504	6,504	6,504
R2	0.563	0.6238	0.5802	0.5634	0.6238	0.5803	0.5663	0.6241	0.5816
Adjusted R2	0.5627	0.5483	0.5799	0.563	0.5483	0.58	0.5656	0.5484	0.5809
F Statistic	1,674.6***	1,796.1***	8,980.6***	1,397.2***	1,796.1***	8,984.5***	770.6***	1,283.9***	9,023.3***

The Hausman test was applied and the random effects models (21), (24) and (27) proved to be more consistent, at a significant level of 5%. The results of these three regressions were similar in significant and sign to those of (2), (5) and (8) which were the most appropriate when the database was daily returns and an unbalanced panel. The results of this model were also aligned with the literature by Fama- French and Carhart, indicating the dependence of the returns of equity funds on the excess return of the market index, SMB, HML, MOM, all coefficients with a positive sign. This implies that smaller companies outperform larger companies in the sample. A positive coefficient for HML meanings that value stocks outperform growth stocks. A positive coefficient for MOM suggests that stocks with past momentum (winners) continue to outperform those with negative momentum (losers) in the panel. However, the exchange rate factor did

not prove to be significant, despite the increase in the internationalization of investments that can be observed in stock fund portfolios.

The FOF dummy proved to be significant and with a negative sign, that is, FOFs have, on average, lower returns than other types of equity funds, and destroy value for shareholders. Model (27) identified the management fee variable with a negative sign as significant, that is, the lower the cost to the shareholder will be the fund's profitability. But unlike model (8), the performance fee was not significant.

It is interesting to note that even changing the aggregation of fund returns from monthly to annual, and the panel from unbalanced with 10 years to balanced with 6 years (2018 to 2023), the results were similar in terms of sign and significance.

The Breusch-Pagan test indicated heteroscedasticity, and the Breusch-Godfrey/ Wooldridge test indicated serial correlation, so the coefficients were corrected by White and Arellano. The VIF test indicated that there is no multicollinearity among the independent variables, after removing ILLIQ. The Breusch-Pagan LM and Pesaran CD tests indicated cross-dependency. All tests performed were significant considering the significant level of 5 %.

The next step was to empirically estimate the probability of FOFs having greater profitability than mirror and master funds, within a certain time horizon. To do this, first the probability of each FOF having a rate of return higher than the basis of comparison funds is estimated, then the average of the estimated probabilities is calculated for each FOF, or for a given set of FOFs.

The probability of each FOF having a rate of return greater than the basis of comparison funds is calculated as follows: (a) a period of time is fixed to calculate the rate of return of the FOF, in this work the window of 252 working days; (b) the return rate of the FOF and each of the funds that will be compared with the FOF is calculated; (c) the number of times the FOF return rate was higher than the return rates of the compared funds is counted; (d) the estimated probability is the result of dividing the number of times the FOF return rate was greater than the return rates of the compared funds, and the number of funds compared.

Equation (4)
$$Prob_i = \frac{\sum_{j=1}^N I(Ret_i > Comp_j)}{N}$$

Where $Prob_i$ is the probability of each FOF having a higher rate of return than the comparison fund base;

Ret_i is the FOFi return rate in a given time window;

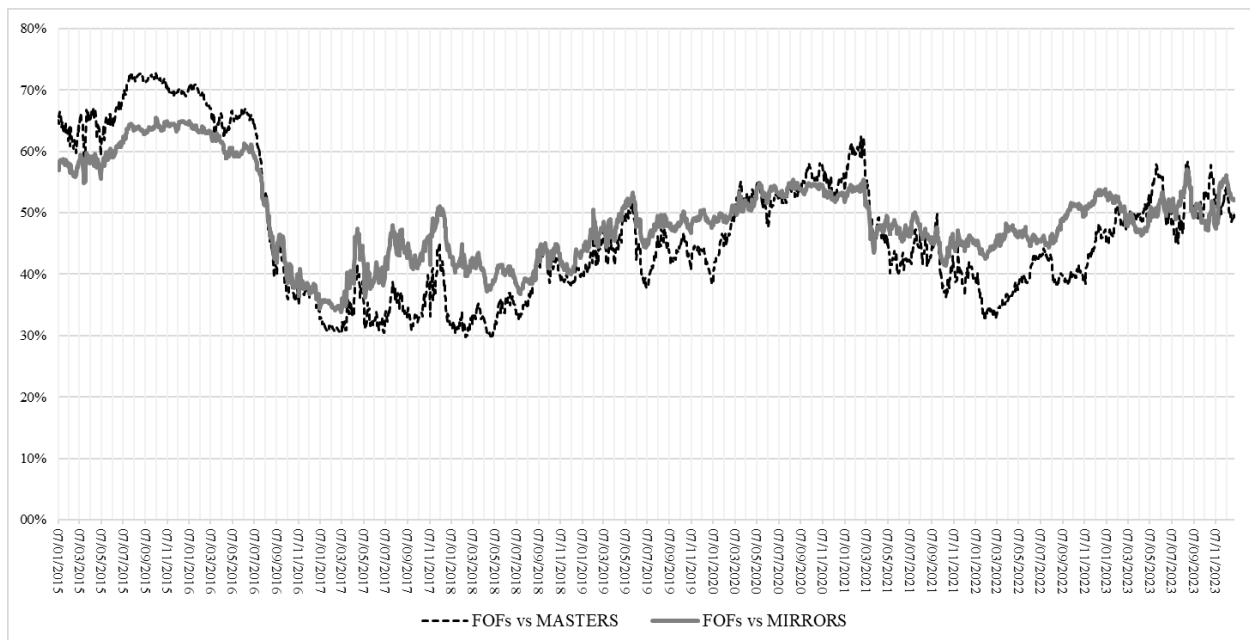
$Comp_j$ is the rate of return of stock fund j in a given time window;

N is the number of equity funds that will be compared with FOFi ;

$I(k)$ is the indicator __ function, denoted by $I(k)$, takes the value 1 if the condition k is true and 0 if k is false.

We calculated the probabilities of FOF having a higher rate of return, in windows of 252 business days, than the other funds, separating them into masters and mirror funds, within the period from 2014 to 2023, and the results can be seen in Chart 1 below.

Chart 1. Probability of FOF having higher returns compared to funds mirror and master.



It is interesting because in the period between January 2015 and July 2016, FOFs presented a high probability of having higher returns than other funds, both when compared to mirrors and masters. This period coincides with the economic crisis and the biggest recession in Brazil's history, with a 6.9% drop in GDP in the 2015-2016 biennium. However, after the economic crisis, the probability of FOFs having a higher return fell to something close to 30%, and over time it remained around a 50% chance, that is, FOFs on average do not have a high probability of having a return superior to other types of equity funds.

The probability of the FOFs having a higher return segregated by asset manager, more specifically, the average probability of their FOFs having a higher return than the master and mirror funds that the manager itself created and has to distribute to its investors was calculated for each manager.

The results, which can be seen in Table 9 for the managers that had at least 4 FOFs, show that the managers' FOFs in general have a probability of less than or close to 50% of having a higher return than other funds from the same asset manager. Furthermore, FOFs had a lower return rate on average than the BOVA11 ETF, which confirms the results previously presented that FOFs had a lower return than mirrors and masters.

Table 9: Main asset managers probability of own FOFs having higher returns than other funds.

Period from 2014 to 2023, with windows of 252 business days, asset managers that have with 4 or more FOFs	Number of FOFs	Comparison of FOFs with the manager's own MIRROR funds					Comparison of FOFs with the manager's own MASTER funds				
		Average Estimated Probability (%)	Standard deviation of the estimated Probability (%)	Difference between the annualized rate of return of FOF and BOVA11 (% p.a.)	Standard deviation difference between rate of return of FOF and BOVA11 (% p.a.)	Number of MIRROR funds	Average Estimated Probability (%)	Standard deviation of the estimated Probability (%)	Difference between the annualized rate of return of FOF and BOVA11 (% p.a.)	Standard deviation difference between rate of return of FOF and BOVA11 (% p.a.)	Number of MASTER funds
Asset manager											
BB	4	52.96	17.72	-0.21	0.17	74	44.14	17.61	-1.72	0.26	43
BRADESCO	12	54.24	15.92	0.38	0.19	162	45.72	15.15	-1.28	0.28	61
BTGPACTUAL	32	51.85	17.98	0.13	0.27	80	48.99	14.60	-1.24	0.52	79
CARPA	13	50.42	10.13	2.06	1.32	5	-	-	-	-	-
CSHG	8	42.78	24.65	-1.82	0.28	31	46.54	34.30	-0.97	0.24	10
ITAU	57	53.57	21.09	1.13	0.27	213	46.33	18.70	-0.34	0.37	37
JULIUS BAER	18	45.34	20.83	-1.55	0.27	19	51.34	20.10	0.44	0.42	11
PERFIN	6	61.59	8.97	2.13	0.70	6	45.48	12.04	-2.71	0.58	9
SANTANDER	5	66.25	13.71	3.68	0.25	34	58.58	10.75	3.89	0.29	38
TURIM 21	5	47.10	26.14	0.67	0.30	5	54.80	13.09	-0.09	0.56	3
UBS BRASIL	8	43.72	18.25	-1.90	0.41	4	-	-	-	-	-
UV	5	-	-	-	-	-	45.59	15.50	0.09	0.67	4
V&B	4	15.32	16.87	-3.78	0.18	3	-	-	-	-	-
VINCI PARTNERS	10	43.73	11.68	-1.59	0.61	4	50.11	15.91	0.34	0.24	22
WE CAPITAL	6	45.99	29.53	-2.17	0.36	6	-	-	-	-	-
XP	10	43.46	7.96	-0.93	0.50	102	32.37	7.40	-4.32	0.36	38
ALL funds	307	49.25	19.68	-0.22	0.15	1801	47.25	18.56	-0.52	0.26	1242

It can be seen in Table 9 that some asset management firms specialize in allocating resources in investment funds and do not have their own master funds, as is the case with CARPA, UBS Brasil, V&B, and WE Capital. From the results, it is possible to verify that this specialization does not imply FOFs with a greater probability of success.

The sample of asset managers comprises Brazilian and international companies, such as Julius Baer, Credit Suisse (CSHG), UBS, among others that are in the general sample but do not appear in Table 9. The estimated probability for international managers was on average less than 50%.

We sought to identify the determinants of the probabilities of FOFs having a higher rate of return than other funds, using panel logistic regressions (logit) with random effects due to the characteristics of the data and non-linearity of the model. The regressions will employ the specific variables of FOFs, and only these types of funds, with data already collected from the Economatica system. The Fama, French and Carhart factors will not be used, as we want to identify characteristics specific to the funds to explain the probabilities.

The dependent variable is 1 if the probability of a FOF having a higher rate of return than other types of funds is greater than 50%, within the time horizon of 252 business days, or zero otherwise. The samples used to identify the determinants were:

- Unbalanced panel with MONTHLY observations of the sample that contains only FOFs.
- Unbalanced panel with ANNUAL observations, ending in December of each year, from the sample of FOFs only.
- Balanced panel with ANNUAL observations, ending in December of each year, from 2018 to 2023, from the sample of FOFs only.

The Risk variable was added to the model, which represents the risk or volatility of the fund's daily returns and was estimated by the standard deviation of the returns.

Table 10 presents the results of the 3 logit panel regressions performed for each of the FOFs samples.

Table 10: Logit panel regressions performed.

The dependent variable is 1 if the probability of a FOF having a higher rate of return than other types of funds is greater than 50%, within the time horizon of 252 business days, or zero otherwise.

Sample	Unbalanced panel with MONTHLY observations of the sample that contains only FOF	Unbalanced panel with ANNUAL observations, ending in December of each year, sample of FOFs only	Balanced panel with ANNUAL observations, ending in December of each year, from 2018 to 2023, sample of FOFs only
	Logistic panel regression model	Random effects	Random effects
	(1)	(2)	(3)
Intercept	0.3507 (0.3142)	-5.9480*** (1.462)	-2.7500* (1.188)
Risk	3.3100 (4.878)	63.0200** (19.22)	87.5700*** (14.78)
Ret-Rf	7674.0*** (132.4)	10630.0*** (693.9)	1263.0*** (140.5)
Mgmt Fee	-0.5480*** (0.05399)	-0.3295 (0.2532)	-0.6497** (0.2031)
LN(age)	0.8910*** (0.04189)	-0.1910 (0.1282)	0.6220*** (0.1693)
LN(AUM)	0.0701*** (0.01661)	0.0788 (0.0755)	0.0058 (0.0584)
Leverage	0.2648*** (0.0605)	0.2497 (0.2648)	-0.0350 (0.241)
Perf Fee	0.0052 (0.0565)	-0.1481 (0.2497)	0.0530 (0.2188)
sigma	3.8640*** (0.0676)	8.6700*** (0.5776)	0.0000 (0.1607)
Observations	33,156	2,763	708
Log-Likelihood	-7,570.58	-741.56	-417.72

The logit regressions show that the return and risk assumed by the FOF contribute positively to having a greater probability of surpassing the profitability of other types of funds, which is expected and natural in the stock market. The Management Fee was significant in 2 of the 3 models, and with a negative sign, confirming that the higher the cost of the fund, the lower its probability of having a higher return than the others.

The variable LN(age) was also significant in 2 of the 3 models, and with a positive sign, that is, the longer the fund's lifetime (and consequently the manager's experience) the greater the fund's profitability. This result conflicts with another that was previously found in this work, but in the literature there are works that converge.

Although it is clear in this work that FOFs on average have a lower return than other types of investment funds, as seen in previous regressions, at this stage FOF indices were constructed to identify simple strategies for constructing FOFs that obtain a return greater than a passive and cheap alternative, which was chosen as the BOVA11 ETF. Table 11 shows the ETFs that fully reference the IBOVESPA and are traded on the B3 stock exchange, located in São Paulo, Brazil.

Table 11: ETFs that reference IBOVESPA traded in B3 stock exchange.

ETF name	Ticker	Number of trades in 10 years ending on 12/31/2023	Accumulated financial volume in 10 years ended 12/31/2023 (R\$ mi)	Securities issued (in thousands)	Start date
Ishares Bova Ci	BOVA11	117.03	1165913.29	8,815,059.5	03/12/2008
It Now Ibov	BOVV11	31.24	261420.90	8,414,375.0	29/07/2016
Trend Ibovx	BOVX11	2.14	9263.04	44,821.5	18/05/2023
Etf Brad Bov	BOVB11	0.68	16676.05	8,500,000.0	05/04/2019
BB Etf Ibov	BBOV11	0.52	6686.28	8,800.0	14/08/2020
Safraetfibov	BOVS11	0.22	288.63	100.0	16/04/2021
Pactual Ibov	IBOB11	0.05	1352.49	800.0	08/07/2021
Caixaetfbov	XBOV11	0.04	1654.61	8,414,375.0	06/11/2012

Source: B3 stock exchange (https://www.b3.com.br/pt_br/produtos-e-servicos/negociacao/renda-variavel/etf/renda-variavel/etfs-listados/)

An index was created that comprises all FOFs available at the time under study, but this makes it difficult for investors or asset managers to construct and manage an index or portfolio of FOFs. Then, the number of 10 FOFs was adopted to construct indices, which was determined based on data observed in the sample of 307 FOFs, with the value 10 being on average the most common amount of funds to which a FOF allocates its resources, as can be seen in Table 3.

In this way, four indices were created with two different weighting systems, equal weighting and market value (FOF net equity). Equal-weighted indices achieved profitability 5% to 18% lower than value-weighted indices, which is why the results of these indices were omitted. Below are the indexes created and presented in this paper:

1. FOFs index VW: composed of all FOFs in the sample weighted by the value of their respective net assets, and with monthly rebalancing;
2. FOFs index 10VW: composed of the 10 largest FOFs in the sample by AUM, weighted by the value of their respective net assets, and with monthly rebalancing;

3. FOFs index 10CapVW: composed of the 10 largest FOFs in the sample by total funding at the end of the current month, weighted by the value of their respective net assets, and with monthly rebalancing;
4. FOFs index 10CapVW3m: composed of the 10 largest FOFs in the sample by accumulated funding in the 3 months prior to the current month, weighted by the value of their respective net assets, and with monthly rebalancing.

FOF indices. IBOVESPA, the B3 stock exchange index, continues to be the Brazilian market reference in the paper.

The FOFs index 10CapVW3m will be the main target of analysis, as its construction allows replication in the market by using data from previous periods to select the funds that will make up the index in the current month. Furthermore, this index not only achieved the highest annualized rate of return over the full period from 2014 to 2023 compared to the other indices, BOVA11 and DI rate, but also achieved the highest Sharpe ratio among index alternatives such as can be seen in Table 12. Also, in Table 12, the diversification effect is noticeable with the four FOF indices presenting risks (standard deviation) much lower than BOVA11. And it is important to remember that returns are after costs, so management fees are already included in them.

Table 12: Average return, risk and Sharpe ratio of indexes and benchmarks.

Index or benchmark	Average annualized return rate during 10 year period (% p.a.)	Annualized standard deviation of the returns during 10 year period (% p.a.)	Sharpe ratio [(AR) - Rf] / (SD)
	(AR)	(SD)	
BOVA11	10.19	28.99	0.0274
FOFs index VW	10.58	17.70	0.0665
FOFs index 10VW	10.95	14.81	0.1048
FOFs index 10CapVW	10.62	22.51	0.0543
FOFs index 10CapVW3m	11.74	22.26	0.1051
Rf (DI rate)	9.40	0.25	

Chart 2 shows the performance of BOVA11 and the indices created considering 100 as a base on 12/31/2013, the beginning of the 10-year series of data ending on 12/28/2023, where the superiority of the indices over the ETF can be seen BOVA11.

Chart 2: Indexes and BOVA11 during all sample period.

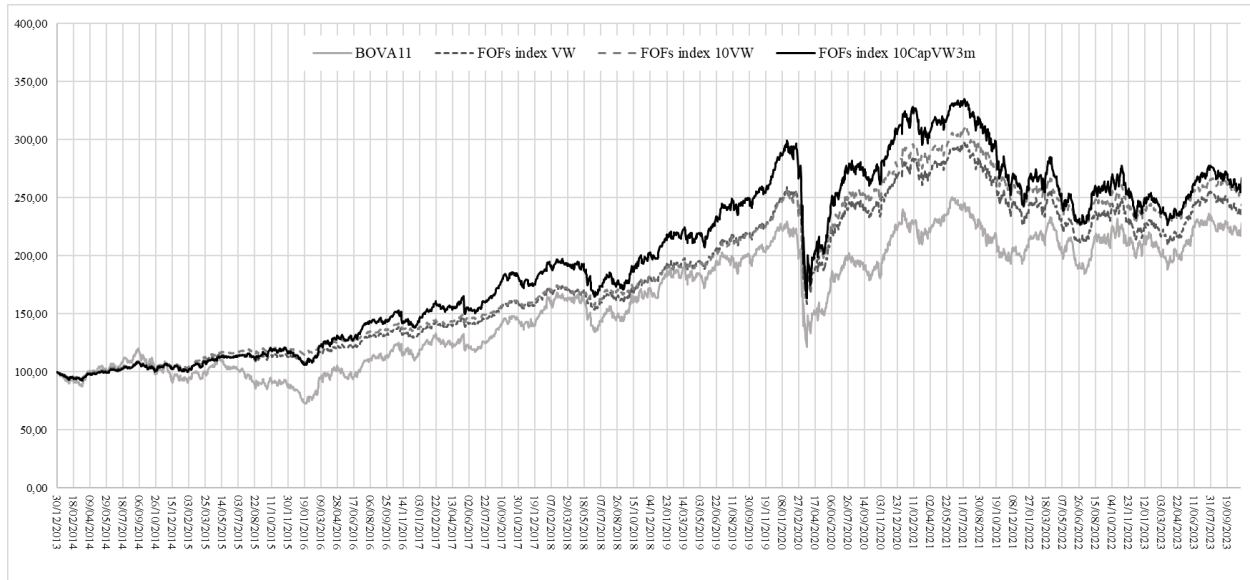


Table 13 presents the correlations between the Fama- French and Carhart factors, which will be the independent variables to study the determinants of the returns of the FOF indices, and the dependent variables, represented by the FOF indices created.

Table 13: Pearson correlation matrix among regression model variables.

Correlations	Mkt	Rf	SMB	HML	MOM	ILLIQ	BOVA11	VW	10VW	10CapVW	10CapVW3m	BRLUSD
Mkt	1.000	-0.007	-0.132	0.359	-0.101	-0.296	0.981	0.926	0.898	0.938	0.929	-0.373
Rf	-0.007	1.000	-0.002	0.012	-0.017	-0.023	-0.004	-0.010	-0.014	-0.005	-0.004	-0.032
SMB	-0.132	-0.002	1.000	-0.181	-0.129	0.733	-0.160	0.047	0.039	0.037	0.046	-0.091
HML	0.359	0.012	-0.181	1.000	-0.040	-0.321	0.375	0.174	0.145	0.207	0.196	-0.140
MOM	-0.101	-0.017	-0.129	-0.040	1.000	0.039	-0.105	-0.073	-0.056	-0.074	-0.076	0.087
ILLIQ	-0.296	-0.023	0.733	-0.321	0.039	1.000	-0.328	-0.087	-0.080	-0.099	-0.086	-0.008
BOVA11	0.981	-0.004	-0.160	0.375	-0.105	-0.328	1.000	0.903	0.875	0.916	0.906	-0.355
VW	0.926	-0.010	0.047	0.174	-0.073	-0.087	0.903	1.000	0.991	0.987	0.986	-0.309
10VW	0.898	-0.014	0.039	0.145	-0.056	-0.080	0.875	0.991	1.000	0.966	0.967	-0.252
10CapVW	0.938	-0.005	0.037	0.207	-0.074	-0.099	0.916	0.987	0.966	1.000	0.995	-0.347
10CapVW3m	0.929	-0.004	0.046	0.196	-0.076	-0.086	0.906	0.986	0.967	0.995	1.000	-0.339
BRLUSD	-0.373	-0.032	-0.091	-0.140	0.087	-0.008	-0.355	-0.309	-0.252	-0.347	-0.339	1.000

In general terms, the multivariate linear regression equation with the factors can be written as follows:

$$\text{Equation (5)} \quad \text{Ind}_{i,t} - Rf_t = \text{constant} + \left(\sum_{j=1}^N a_j \cdot \text{Factor}_j, t \right) + \varepsilon_t$$

Where $\text{Ind}_{i,t} - Rf_t =$ log- return of index i on day t minus the risk-free interest rate on day t, where index i can be one of the following: BOVA11, FOFs index VW, FOFs index 10VW, FOFs index 10CapVW, FOFs index 10CapVW3m;

$\text{Factor}_j, t =$ is factor j, one of Mkt-Rf, SMB, HML, MOM, ILLIQ, and BRLUSD- Rf, on day t;

$N =$ is the number of factors used in the regression specification;

$a_j =$ is the coefficient of factor j resulting from the execution of linear regression.

The regressions used daily observations from 01/02/2014 to 12/28/2023, and the Newey-West correction was applied to the coefficient statistics. The results of the FOFs index 10CapVW3m and BOVA11 regressions (this serves as a reference for analysis) are in Table 14.

Table 14: FOF index 10CapVW3m and BOVA11 linear regression results.

Daily observations from 01/02/2014 to 12/28/2023, 10 years of data	Daily rate of return of the FOF index "10CapVW3m" minus daily risk-free interest rate (DI rate)			BOVA11 daily return rate (IBOVESPA ETF) minus daily risk-free interest rate (DI rate)		
	(1)	(2)	(3)	(4)	(5)	(6)
Mkt-Rf	0.8584*** (0.0508)	0.8742*** (0.0456)	0.8860*** (0.0451)	1.0513*** (0.0055)	1.0466*** (0.0063)	1.0498*** (0.0078)
SMB	0.1834*** (0.0143)	0.0786*** (0.0280)	0.0826*** (0.0278)	-0.0445*** (0.0088)	-0.0135 (0.0118)	-0.0124 (0.0115)
HML	-0.1273*** (0.0199)	-0.1103*** (0.0175)	-0.1090*** (0.0174)	0.0269*** (0.0073)	0.0218** (0.0086)	0.0222** (0.0087)
MOM	0.0351* (0.0186)	0.0229 (0.0156)	0.0217 (0.0150)	-0.0114** (0.0051)	-0.0078* (0.0044)	-0.0081* (0.0044)
ILLIQ		0.1624*** (0.0391)	0.1654*** (0.0384)		-0.0481** (0.0199)	-0.0473** (0.0206)
BRLUSD-Rf			0.0469*** (0.0122)			0.0127 (0.0104)
Constant	0.00001 (0.00010)	-0.00001 (0.00010)	-0.00001 (0.00010)	-0.0001*** (0.00004)	-0.0001*** (0.00004)	-0.0001*** (0.00004)
Observations	2,476	2,476	2,476	2,476	2,476	2,476
R2	0.9085	0.9151	0.9161	0.9643	0.9647	0.9647
Adjusted R2	0.9084	0.9149	0.9159	0.9643	0.9646	0.9647
Res.Std.Err.	0.0038	0.0037	0.0037	0.003	0.003	0.003
F Statistic	6,135.9***	5,322.0***	4,494.4***	16,705.2***	13,498.8***	11,260.4***

The VIF test was performed and no variable had a value greater than or equal to 5, considering the level of significant.

The factors Mkt-Rf, SMB, HML and ILLIQ were significant in all specifications for the 10CapVW3m index. Mkt -Rf presented the highest coefficient, with a positive sign, which was expected as funds of funds contain many assets and naturally approach the market portfolio. The SMB coefficient is positive, it implies that smaller companies outperform larger companies in the sample. A negative coefficient for HML meanings that growth stocks outperform value stocks. A positive ILLIQ implies that illiquid stocks outperform liquid ones. The exchange rate factor proved to be positive and significant. These results show that the superior performance of the FOFs index 10CapVW3m over BOVA11 comes from a greater investment allocation in small and growth stocks, and exposure to investments abroad. The MOM factor was not significant in all specifications, which was expected because the index's FOFs are incorporated after 3 months of positive funding, which should reduce the momentum effect.

The BOVA11 ETF, due to its concentration in large and liquid shares, due to the construction of its reference index, IBOVESPA, did not have a significant SMB factor in most model specifications, but had a positive and significant positive HML, as well as ILLIQ and Negative and significant MOM in all specifications, which are expected signs due to consistency with the IBOVESPA methodology. The exchange rate factor was not significant, which is also consistent with IBOVESPA.

The results of the regressions of the FOFs index VW, FOFs index 10VW, FOFs index 10CapVW were omitted as they are like those obtained and presented from the FOFs index 10CapVW3m.

Finally, to evaluate if there is skill of market timing in the FOFs index 10CapVW3m, it employed the Treynor and Mazuy (1966) regression, which augments the traditional CAPM with a quadratic term, adding the squared market excess return. If the index has market forecasting ability, it will overweight their market exposure when there are positive excess market returns and underweight in down markets, leading to a convex relationship between their portfolio returns and the market return. The equation presents the regression model:

$$\text{Equation (6)} \quad \text{Ind}_{i,t} - Rf_t = \text{constant} + \beta \cdot (\text{Mkt}_t - Rf_t) + \gamma \cdot (\text{Mkt}_t - Rf_t)^2 + \varepsilon_t$$

Where $\text{Ind}_{i,t} - Rf_t$ = log- return of index i on day t minus the risk-free interest rate on day t, where index i can be one of the following: BOVA11, FOFs index VW, FOFs index 10VW, FOFs index 10CapVW, FOFs index 10CapVW3m.

The key measure of market timing ability is the gamma coefficient. A positive gamma (γ) indicates positive timing ability. The fund manager, or index, has historically outperformed the benchmark by making good market timing decisions. The negative gamma indicates negative timing ability. The fund manager, or index, has historically underperformed the benchmark due to poor market timing decisions. Zero gamma indicates no timing ability.

Table 15 presents the results for the FOFs index 10CapVW3m and BOVA11. The results of the regressions of the FOFs index VW, FOFs index 10VW, FOFs index 10CapVW were omitted as they are like those obtained and presented from the FOFs index 10CapVW3m.

Table 15: FOFs index 10CapVW3m and BOVA11 market timing results.

Daily observations from 01/02/2014 to 12/28/2023, 10 years of data	10CapVW3m minus Rf	BOVA11 minus Rf
	(1)	(2)
Mkt-Rf	0.7799*** (0.0490)	1.0629*** (0.0090)
(Mkt-Rf) ²	-1.5659*** (0.2130)	-0.2346* (0.1208)
Constant	0.0003*** (0.00010)	-0.0001** (0.00004)
Observations	2476	2,476
R2	0.8735	0.963
Adjusted R2	0.8734	0.9629
Res. Std. Error	0.0045	0.0031
F Statistic	3.452***	12.996***

The results for both the FOFs index 10CapVW3m and BOVA11 were similar, both with negative and significant gamma, which indicates that neither of them has market timing skills. It is interesting to note that the gamma coefficient is much more negative for the 10CapVW3m index compared to BOVA11, which can be explained in part by the 10CapVW3m methodology that only includes funds in the portfolio after verifying how much positive financial resources were raised in the last 3 months. It is worth noting that the 10CapVW3m constant, which would be equivalent to the alpha measure of value creation, is positive and significant, which makes sense given the reconstructed historical performance of this index in comparison with IBOVESPA.

Conclusions

This study proposed an analysis of equity FOFs in Brazil through three methodologies. However, before the application of these methodologies, the analysis of descriptive statistics from the sample yielded interesting conclusions. The first pertains to management fees. While most investors believe that FOFs charge higher fees compared to other funds, our analysis showed the opposite. Another conclusion is that these funds do not demonstrate significant commitment to ESG-related aspects. A final point to highlight concerns the type of investor in FOFs: the number of unit holders per fund is small, but the volume invested by each is significantly higher than in other funds.

Returning to the applied methodologies, the first, composed of panel regressions, seeks to identify the determinants of returns and value creation of FOFs. A preliminary analysis of the data regarding volatility revealed that FOFs exhibit lower volatility than other equity funds, indicating the beneficial effect of diversification. However, as FOFs' profitability is lower than that of other funds, their average Sharpe ratio is also lower. The regressions indicated that fund returns are dependent on the three Fama-French factors as well as the Carhart momentum factor. It is noteworthy that the exchange rate variation factor (BRL/USD) showed no significant despite the observed increase in Brazilian funds' exposure to foreign assets in recent years. When a dummy variable representing FOFs was included in the regression, the result showed a negative and significant coefficient, reflecting the lower returns of these funds compared to the total sample. Another interesting result concerns the management fee, which, as expected, exhibited a negative coefficient. Conversely, the performance fee showed a positive coefficient, indicating it to be an effective instrument for aligning the interests of managers and investors.

The second methodology aimed to analyze the probability of FOFs outperforming master and mirror funds. The initial result indicates that, on average, FOFs do not have a higher probability of achieving greater returns than other funds. However, during the period between January 2015 and July 2016, the probability of FOFs achieving better returns was significant, coinciding with a period of deep economic crisis in Brazil. This is a topic that warrants further exploration in future studies. Analyzing managers with at least 4 FOFs, it was concluded that the probability of these funds outperforming other funds is 50% or less. It was also found that FOFs, on average, exhibit lower returns than BOVA11 ETF, the most liquid in the Brazilian market, which tracks the same portfolio as the Ibovespa. The determinants of this probability were identified through logit regressions and include volatility, excess return relative to the benchmark, management fee (negative coefficient), and fund age.

In the third methodology, indices were constructed with the aim of seeking simple portfolio construction alternatives that could potentially be profitable. Indeed, alternatives more profitable than the BOVA11 ETF were obtained, proving that profitable strategies can be devised through selection and investment in FOFs. The determinants of index returns were the market excess return relative to the risk-free rate, the Fama-French factors SMB and HML, as well as the liquidity factor (ILLIQ) referring to asset liquidity. One possible conclusion regarding this factor is that less liquid assets outperform liquid ones. Finally, the possibility of market timing capability by fund managers was analyzed using the methodology proposed by Treynor and Mazuy. The conclusion, consistent with various studies conducted, is that such ability does not exist.

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