# Analysis of Factors Affecting Punching Shear Resistance in Reinforced Concrete Flat Slabs

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**Abstract**
Reinforced concrete flat slabs have been widely adopted in civil construction owing to the several advantages offered by this structural system. However, one of the main concerns with this system is the high concentration of forces, particularly shear forces, at the slab-column connection, which can result in a phenomenon known as punching shear. This article presents verifications based on the normative formulations of ABNT NBR 6118 (2023), EUROCODE 2 (2004), ACI 318 (2019), and *fib* Model Code (2020), addressing the main parameters that influence punching shear resistance, such as concrete compressive strength, column geometry, slab effective depth, and tensile bending reinforcement ratio. The results show that, in structures already equipped with shear reinforcement, increasing the slab's effective depth is the most effective factor in improving punching shear resistance. Among the standards analyzed, ACI 318 (2019) and *fib* MODEL CODE (2020) proved to be the most conservative.

## 1 INTRODUCTION

The flat slab is an alternative to the conventional reinforced concrete system, eliminating the use of beams by resting directly on the columns. Its advantages include formwork savings, faster construction, greater architectural flexibility, and increased ceiling height. However, highlighted that, despite its apparent simplicity, the structural behavior of flat slabs is complex, particularly in support regions, where it is susceptible to punching shear effects [1]. That punching shear occurs owing to shear forces generated by concentrated loads near the columns, resulting in to brittle failures and, in severe cases, progressive collapse of lower floors [2].

## 2 LITERATURE REVIEW

### 2.1 Concrete Compressive Strength

Punching shear resistance in concrete structures is directly related to compressive strength, with studies indicating that tensile strength also influences structural behavior. A formulation for punching shear resistance based on the square root of the concrete's compressive strength limits the compressive strength to 69 MPa [3]. Other standards use the cube root of the compressive strength to estimate punching shear resistance, considering this approach safer for concrete with strengths ranging from 20 to 90 MPa [4]-[5].

### 2.2 Column Dimensions and Geometry

The stress in a concrete structure varies according to the dimensions and shape of the column, affecting the slab-to-column connection. Circular columns distribute stresses more uniformly, whereas rectangular and square columns concentrate stresses at the corners. Observed that punching shear resistance can be up to 15% higher in circular columns [6]. Column geometry, particularly the use of column capitals to increase the top section, can and in better distributing stresses and reduce punching shear effects.

## 2.3 Slab Effective Depth

Increasing the effective depth of the slab increases the compressed concrete area and, consequently, the compressive strength. However, there is the size effect. This effect refers to the phenomenon in which the nominal shear strength does not vary linearly with the slab thickness [7]-[8]. For example, a slab that is twice the size of another with similar properties will not have twice the strength of the smaller one due to the size effect. Consequently, the larger slab will exhibit a lower average resistant stress.

## 2.4 Use of Punching Shear Reinforcement

The best solution for increasing punching shear resistance is the use of shear reinforcement, as modifying the column geometry or increasing the slab thickness can impact the architectural design and raise costs. Concrete compressive strength is also not very effective in this case. It is worth noting that the main difficulty in using shear reinforcement is the proper placement of the bars near the column, ensuring good anchorage, even in thin sections [9]. Achieving the ideal reinforcement layout is challenging, as the stress region is already occupied by flexural reinforcement. Typically, a radial or cross-shaped distribution is adopted.

# 3 REGULATORY REQUIREMENTS FOR PUNCHING SHEAR RESISTANCE

This paper presents four verifications according to existing standards, using characteristic values, since the analysis was based on this type of approach [4]-[5]-[10]-[11]. In the various calculation formulas, the resisting load is verified using a critical perimeter around the column, and when the design load exceeds the design resisting load, the ultimate limit state is reached. Similarly, in experimental scenarios, when the characteristic design load exceeds the characteristic resisting load, punching shear failure occurs. The possible failure modes in reinforced concrete flat slabs with and without shear reinforcement are evaluated. Thus, the maximum applied load can be calculated and compared with the resisting load according to the prescriptions of the adopted standard [12].

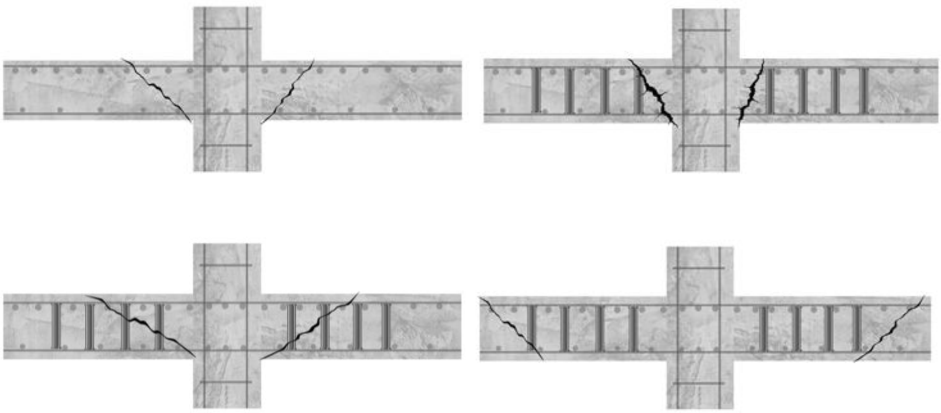


Figure 1 Without shear reinforcement;

Figure 2 At the maximum crushing capacity of the compressed strut at the column perimeter;

Figure 3 Within the shear reinforcement region;

Figure 4 Outside the shear reinforcement region.

## 4 ABNT NBR 6118 (2023)

Punching shear resistance in flat slabs of reinforced concrete is verified under the following conditions: without shear reinforcement, considering diagonal tension; within the shear reinforcement region; and outside the shear reinforcement region. These verifications are carried out to ensure the safety of the structure against punching failures under different reinforcement conditions [4].

The punching shear resistance without shear reinforcement is verified by:

$$V_{\mathrm{Rk,c}} = 0{,}18 \cdot \xi \cdot \left((100 \cdot \rho \cdot f_{ck})^{\frac{1}{3}}\right) \cdot U_1 \cdot d \quad (1)$$

The diagonal compression resistance at the column perimeter with punching reinforcement verified from:

$$V_{\mathrm{Rk,máx}} = 0{,}27 \cdot (1 - \frac{f_{ck}}{250}) \cdot f_{ck} \cdot U_0 \cdot \mathrm{d} \quad (2)$$

For load capacity within the shear reinforcement region:

$$V_{Rk,cs} = [0{,}75 \cdot 0{,}18 \cdot \xi \cdot ((100 \cdot \rho \cdot f_{ck})^{1/3}) + (1{,}5 \cdot \frac{d}{s_r} \cdot \frac{A_{sw} \cdot F_{ywk}}{U_1 . d})] \cdot U_1 \cdot \mathrm{d} \quad (3)$$

For regions outside the shear reinforcement:

$$V_{Rk,out} = 0{,}18 \cdot \xi \cdot ((100 \cdot \rho \cdot f_{ck})^{1/3}) \cdot U_{out} \cdot \mathrm{d} \quad (4)$$

## 5 EUROCODE 2 (2004)

The punching shear resistance without shear reinforcement is obtained using Equation 5, with a critical perimeter ($u_1$) located at a distance of 2d from the column face. The punching shear resistance when shear reinforcement is provided is determined based on the control perimeter $u_0$, located at the column perimeter, and $u_1$, defined at a distance of 2d from the column, and should be adopted in a way that minimizes its length. The control perimeter outside the shear reinforcement region (uout) is positioned at a distance of 1.5d [5].

$$V_{\mathrm{Rk,c}} = 0{,}18 \cdot \xi \cdot \left((100 \cdot \rho \cdot f_{ck})^{\frac{1}{3}}\right) \cdot U_1 \cdot d \quad (5)$$

The diagonal compression resistance at the column perimeter is verified according to the following equation:

$$V_{\mathrm{Rk,máx}} = 0{,}24 \cdot (1 - \frac{f_{ck}}{250}) \cdot f_{ck} \cdot U_0 \cdot \mathrm{d} \quad (6)$$

The load capacity within the region with shear reinforcement can be obtained using the following equation:

$$V_{Rk,cs} = [0{,}75 \cdot 0{,}18 \cdot \xi \cdot ((100 \cdot \rho \cdot f_{ck})^{1/3}) + (1{,}5 \cdot \frac{d}{s_r} \cdot \frac{A_{sw} \cdot F_{ywk}}{U_1 . d})] \cdot U_1 \cdot \mathrm{d} \quad (7)$$

The verification of the load capacity in the region outside the shear reinforcement is verified by the equation:

$$V_{Rk,out} = 0{,}18 \cdot \xi \cdot ((100 \cdot \rho \cdot f_{ck})^{1/3}) \cdot U_{out} \cdot \mathrm{d} \quad (8)$$

## 6 ACI 318 (2019)

The determination of punching shear resistance in flat slabs of reinforced concrete without shear reinforcement must be verified using the maximum diagonal compression stress at the critical perimeter ($B_0$), located at a distance of d/2 from the column face, using Equation 9 [10].

$$V_{Rk,c} = \min \quad 0{,}035 \cdot \sqrt{f_c} \cdot B_0 \cdot \mathrm{d} \tag{9}$$

The diagonal compression resistance at a distance of d/2 from the column with punching shear reinforcement is verified by the equation:

$$V_{Rk,máx} = \lambda \cdot \sqrt{f_c} \cdot B_0 \cdot \mathrm{d} \tag{10}$$

The ACI 318 (2019) standard proposes the verification of load capacity within the shear reinforcement region, which should be considered according to the equation:

$$V_{Rk,cs} = p_v \cdot B_0 \cdot F_{yd} \tag{11}$$

The load capacity in the region outside the shear reinforcement is verified by Equation 12, and its critical perimeter is determined :

$$V_{Rk,out} = \mathrm{k} \cdot V_{Rk,máx} \tag{12}$$

## 7 FIB MODEL CODE (2020)

Employs a model based on the critical shear crack theory by Aurelio Muttoni, which relates punching shear resistance to the opening of the critical crack, depending on the slab rotation under load [11]. The larger the crack opening, the lower the shear resistance owing to reduced contact between the concrete surfaces. The model also considers the contribution of coarse aggregate interlock to shear resistance [13].

When there is no punching shear reinforcement, $k_{sys}$ equals 1.0, and in this case, $V_{Rkmax}$ is equal to the diagonal tensile resistance:

$$V_{Rk,máx} = \min \begin{pmatrix} \{ K_{\text{sys}} \,.\, K_{\omega,\text{d},1} \,.\, \frac{\sqrt{f_{ck}}}{\Upsilon_c} \,.\, B_{0,1} \,.\, d_v \\ \frac{\sqrt{f_{ck}}}{\Upsilon_c} \,.\, B_{0,1} \,.\, d_v \end{pmatrix} \tag{13}$$

The maximum punching shear resistance is limited by the integrity of the concrete in the compressed strut:

$$V_{Rk,c} = K_{\omega,\text{d},1} \,.\, \frac{\sqrt{f_{ck}}}{\Upsilon_c} \,.\, B_{0,1} \,.\, d_v \tag{14}$$

Diagonal tension resistance in the presence of punching reinforcement is verified using:

$$V_{Rk,cs} = K_{\omega,\text{d},1} \,.\, \frac{\sqrt{f_{ck}}}{\Upsilon_c} \,.\, B_{0,1} \,.\, d_v + \left(\sum A_{sw}\right) K_{\text{e,l}} \,.\, \sigma_{\text{swk}} \,.\, \text{sen}\alpha \tag{15}$$

Diagonal tensile strength in the region outside the shear reinforcements is verified using:

$$V_{Rk,out} = K_{\omega,\text{d},1} \,.\, \frac{\sqrt{f_{ck}}}{\Upsilon_c} \,.\, B_{0,out} \,.\, d_{v,out} \tag{16}$$

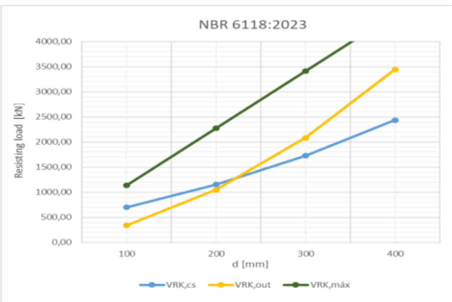
Where:

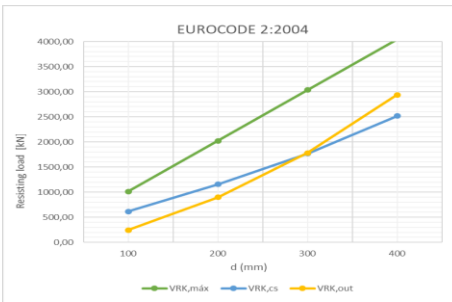
$V_{Rk,c}$ = Characteristic resisting load related to diagonal tension;
$V_{Rk,\,max}$ = Characteristic resisting load related to the diagonal compression of concrete;
$V_{Rk,cs}$ = Characteristic resisting load related to diagonal tension in the presence of punching shear reinforcement;
$V_{Rk,out}$ = Characteristic resisting load related to the diagonal tension of concrete outside the reinforcement region;
$\xi$ = Size effect, defined as $\xi = 1 + \sqrt{(200/d)}$; (NBR6118 (2023))
$\xi$ = Size effect, defined as $\xi = 1 + \sqrt{(200/d)} \leq 2{,}0$; (EUROCODE 2 (2004))
$B_0$ = Critical perimeter within the punching reinforcement region;
$B_{out}$ = Critical perimeter outside the punching shear reinforcement region;
$\sigma_{swk}$ = Characteristic stress activated in the punching shear reinforcement
$k_{e,1}$ = Eccentricity coefficient for critical perimeter
$k_{w,d,1}$ = Coefficient related to slab design rotation calculated based on the perimeter $b_1$
$B_{0,1}$ = Critical perimeter within the punching reinforcement region;
$\alpha_s$ = Constant that characterizes the position of the slab-column connection in the floor;
$\rho$ = Bending reinforcement ratio;
$p_v$= is a measure of the shear reinforcement density per unit area or critical perimeter.
$f_{ck}$ = Characteristic compressive strength of concrete;
d = Effective slab depth;
$d_v$ = Effective slab depth;
$U_0$ = Critical perimeter within the punching reinforcement region;
$U_1$ = Critical perimeter at the column face;
$U_{out}$ = Critical perimeter outside the punching shear reinforcement region;
$S_r$ = Distance between shear reinforcement layers;$A_{sw}$ = Area of shear reinforcement steel;
$F_{ywk}$ = Characteristic yield strength of punching shear reinforcement.

## 8 RESULTS AND DISCUSSIONS

The punching shear resistance results were calculated based on the formulations of the design codes [4]-[5]-[10]-[11]. The configuration included eight lines and five layers, S0 = 70 mm, Sr = 70 mm, d = 159 mm, c = 350 mm, fck = 35 MPa, 8 mm studs, bending reinforcement with Ø16 mm bars spaced at 125 mm, 25 mm concrete cover, a square column, cruciform reinforcement arrangement, and an internal column to examine the slab-column connection behavior. These were the standard characteristics; however, modifications were made when varying the resistance parameters.

### 8.1 Influence of Effective Depth



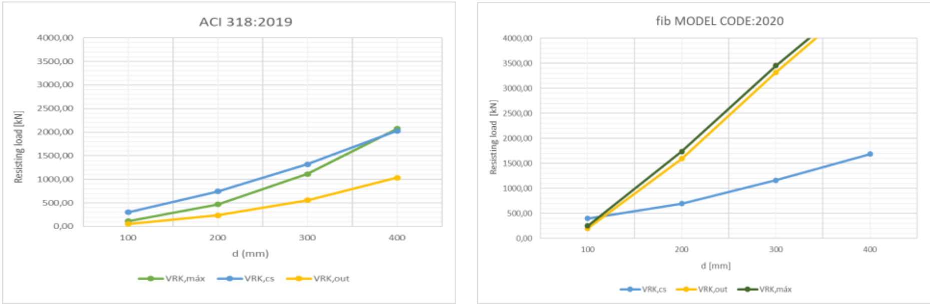



Figure 5 to 8 - Influence of the effective depth, d;

The analysis of the ABNT NBR 6118 (2023), EUROCODE 2 (2004), ACI 318 (2019), and fib Model Code (2020) standards revealed significant differences in punching shear resistance. The ABNT NBR 6118 (2023) and EUROCODE 2 (2004) standards showed similar trends, with a significant increase in resistance as the effective depth increases, highlighting the influence of concrete diagonal compression. The ACI 318 (2019) adopted more conservative criteria, resulting in lower resistance values, while the fib Model Code (2020) presented the highest values, demonstrating the strong influence of punching shear reinforcement. It is concluded that punching shear reinforcement is essential for increasing resistance, with differences among the standards in how they consider this reinforcement.

## 8.2 Influence of Column Geometry

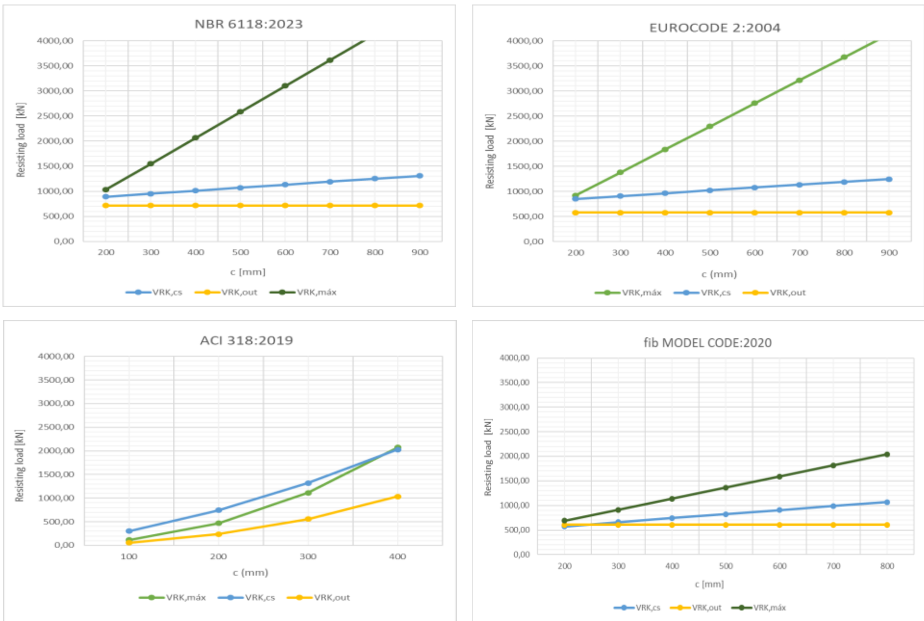



Figure 9 to 12 Influence of the column geometry, c;

The comparative analysis of the standards ABNT NBR 6118 (2023), EUROCODE 2 (2004), ACI 318 (2019), and fib Model Code (2020) regarding the variation in column dimensions revealed significant differences in punching shear resistance. The ABNT NBR 6118 (2023) and EUROCODE 2 (2004) standards exhibited similar trends, with an increase in resistance as the column dimension grows, highlighting the influence of concrete diagonal compression. ACI 318 (2019) adopted more conservative criteria, resulting in lower resistance values, while the fib Model Code (2020) presented the highest values, emphasizing the positive impact of punching shear reinforcement on the structural load-bearing capacity.

## 8.3 Influence of Fck

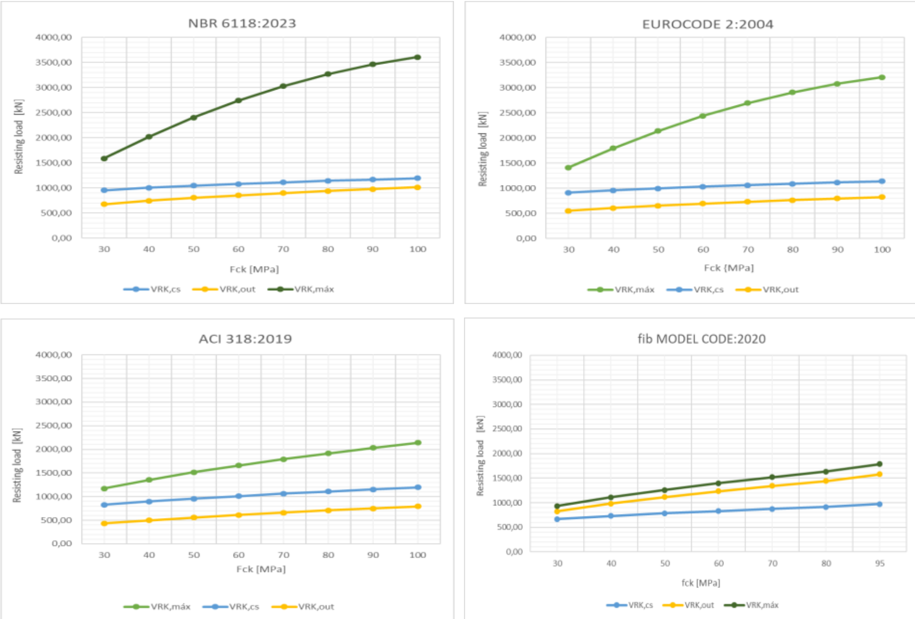



Figure 13 to 16 Influence of Fck

This graph analyzes the punching shear resistance in concrete slabs according to the standards NBR 6118 (2023), Eurocode 2 (2004), ACI 318 (2019), and fib Model Code (2020). The characteristic resisting loads were compared: $V_{Rk,c}$ (diagonal tension of concrete), $V_{Rk,max}$ (diagonal compression of concrete), $V_{Rk,cs}$ (diagonal tension with punching shear reinforcement), and $V_{Rk,out}$ (diagonal tension outside the reinforcement) varying the fck parameter.

The results show that NBR 6118 (2023) and Eurocode 2 (2004) provide the highest $V_{Rk,max}$ values, while ACI 318 (2019) and fib Model Code (2020) adopt more conservative criteria. Thus, the choice of the standard directly impacts the safety and efficiency of the design.

# 9 CONCLUSIONS

During the research on the factors influencing punching shear resistance in flat slabs, we found that the ABNT NBR 6118 (2023) and EUROCODE 2 (2004) standards have similar mathematical formulations. However, the European standard sets limits for the flexural reinforcement ratio ($\leq 0.02$) and the size effect ($\leq 2.0$).

The analysis of variations in effective depth, column dimensions, concrete strength ($F_{ck}$), flexural reinforcement ratio, and punching shear reinforcement indicated a general increase in punching shear resistance. In particular, changes in effective depth at the slab-column connection resulted in strength gains across all four analyzed standards, with emphasis on ABNT NBR 6118 (2023).

Based on these results, we concluded that the most effective strategy to enhance punching shear resistance is the use of specific reinforcements, with the cross arrangement being an efficient and viable alternative. Additionally, the variation in effective depth had the greatest impact on the results. Finally, the graph analysis showed that the ACI 318 (2019) and *fib* MODEL CODE (2020) standards provided the most conservative results.

As a suggestion for future studies, it is recommended to analyze punching shear resistance in slabs with corner, edge, and re-entrant columns, as well as to expand the study by evaluating new sets of experimental and numerical data.

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