Do Inflation-Linked Bonds Predict Future Inflation? A Reassessment Using Novel Methodologies and Instruments

Gustavo Silva Araujo*

José Ignacio Ándres Bergallo †

Abstract

This study revisits the question "Do inflation-linked bonds contain information about future inflation?" posed by Vicente and Guillen (2013), by addressing two critical issues related to the breakeven inflation rate (BEIR) that were not considered by the authors: the inflation lag embedded in inflation-indexed securities and the seasonality inherent in inflation indices. The analysis evaluates three methods for calculating the BEIR for the purpose of forecasting inflation: the one used by Vicente and Guillen (2013), which we refer to as Naïve BEIR; an alternative measure derived from government bonds but incorporating corrections for inflation lag and seasonality (Bond Market BEIR – BM-BEIR),; and a BEIR based on the Futures Market (Futures Market BEIR– FM-BEIR), which also includes these adjustments. The results show that the BM-BEIR significantly outperforms the Naïve BEIR for short-term horizons (3 and 6 months), closely tracking realized inflation. Moreover, both the BM-BEIR and the FM-BEIR match the predictive accuracy of survey expectations, while offering the advantage of daily updates - highlighting their utility for short-term inflation forecasting and decision-making.

Keywords: Break-Even Inflation Rate (BEIR), Inflation Forecasting, Inflation-linked Bonds, DI x IPCA Spread Futures (DAP), Inflation Seasonality, Inflation Lag, FOCUS survey.

* Central Bank of Brazil and EPGE/FGV. E-mail: gustavo.araujo@bcb.gov.br

† Master in Economics and Finance from EPGE/FGV. E-mail: ignacio.bergallo@gmail.com

1. INTRODUCTION

Inflation expectations play a critical role in shaping economic decisions, influencing monetary policy, financial market dynamics, and investment strategies. Policymakers should pay close attention to market-based expectations of short-term inflation, especially when these diverge significantly from survey-based projections (Anttonen and Laine, 2024). The Break-Even Inflation Rate (BEIR), defined as the spread between nominal and real interest rates, is widely interpreted as the primary market-based indicator of expected inflation (Vicente and Guillen, 2013). Indeed, inflation-linked bonds are considered one of the most reliable sources of future inflation expectations (Bernanke, 2004).

However, the BEIR reflects not only inflation expectations but also a risk premium. Further complicating BEIR calculations are the seasonal behavior of inflation and the lag between inflation-linked bonds and their associated price indices, which create distortions, particularly for short-term estimates (Val and Araujo, 2019).

This study revisits the question posed by Vicente and Guillen (2013) "Do inflation-linked bonds contain information about future inflation?" Using their model-free approach as a foundation, this work introduces two key refinements to address the identified limitations: Adjustments for lagged inflation embedded in Brazilian inflation-indexed securities and the incorporation of seasonal adjustments in BEIR calculations. These enhancements build on the methodologies proposed by Araujo and Vicente (2017) and Val and Araujo (2019) to improve the BEIR, which was previously calculated in a more naïve manner, as in Vicente and Guillen (2013). In this study, we assess whether these improvements enhance the BEIR's predictive accuracy and explanatory power.

The analysis employs three methodologies to calculate the BEIR: (1) the one used by Vicente and Guillen (2013), which we refer to as the Naïve BEIR, which uses government bonds without adjustments for lagged inflation or seasonality; (2) the Bond Market BEIR (BM-BEIR), which also uses government bonds but incorporates corrections for lagged inflation and seasonality; and (3) the BEIR derived from the Futures Market (FM-BEIR), which is also adjusted for lagged inflation and seasonality.[1] We evaluate the performance of these methodologies in forecasting inflation in the Brazilian market across horizons ranging from 3

[1] In this study we do not combine government bonds with futures markets to obtain the BEIR due to differences in liquidity and operational discrepancies. According to Fleckenstein et al. (2014), even in the U.S. market, instruments with the same issuer, indexation, cash flows, and maturity may exhibit pricing differences.

to 30 months. Additionally, we compare their predictive accuracy with inflation expectations published in the Central Bank of Brazil's FOCUS Report.[2]

The analysis relies on OLS regressions to evaluate the BEIR's explanatory power for future inflation, using datasets from two distinct periods to perform two experiments. The first set ranges from 2005 to 2011, to replicate Vicente and Guillen (2013), and the second spans from 2016 to 2023, to assess more recent market conditions for the futures market.

The results show that the BM-BEIR significantly outperforms the Naïve BEIR in the first experiment, particularly over short-term horizons of 3 and 6 months, where it closely tracks realized inflation.[3] In the second experiment, the FM-BEIR demonstrates slightly better explanatory power for short-term horizons compared to the BM-BEIR. Both methods perform comparably to survey-based inflation expectations but underscore the challenges of predicting inflation over longer horizons, when the so-called expectation hypothesis of the BEIR often fails. The expectation hypothesis states that the risk premium of the BEIR is constant over time but possibly maturity-dependent.

These findings reinforce the value of the BEIR as a dynamic tool for inflation forecasting, particularly for short-term decision-making since its daily updates offer a key advantage over survey-based expectations like the FOCUS survey.[4] Moreover, this study contributes to the existing literature on the topic by providing critical refinements to the BEIR calculation and offering practical insights for policymakers, financial institutions, and investors navigating inflation-driven economic landscapes.

The remainder of the paper is structured in the following segments. Section 2 outlines the theoretical framework for analyzing the explanatory power of the Break-Even Inflation Rate (BEIR), discussing the nominal and real interest rate securities and the model-free approach applied. Section 3 describes the methodology, addressing the challenges of BEIR calculation and explaining how adjustments for inflation lag and seasonality were made. Two methods are detailed: one using government bonds (BM-BEIR), as developed in Val and Araujo (2019), and the other leveraging the IPCA Coupon Futures Market (FM-BEIR), as proposed by

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[2] The FOCUS Report is a weekly publication by the Central Bank of Brazil that summarizes the market's expectations for key macroeconomic indicators. It serves as a reference for economic expectations in Brazil and is widely followed by policymakers, economists, and financial markets.

[3] The FM-BEIR is not used in the first experiment due to the lack of liquidity in the futures market during that period.

[4] Söderlind (2011) also emphasizes that implied price expectations do not require incentives for frequent price updates.

Araujo and Vicente (2017). Section 4 presents the results of the empirical analysis, comparing the alternative measures both among themselves and against survey-based inflation expectations. Finally, Section 5 presents our concluding remarks.

2. THEORETICAL FRAMEWORK

This section provides a comprehensive foundation for understanding and analyzing the explanatory power of the Break-Even Inflation Rate (BEIR). It begins with a discussion of the government bonds and derivatives that negotiate nominal and real interest rates in Brazil, including the conceptual framework for the calculation of the BEIR. Then, we introduce a model-free approach, as proposed by Vicente and Guillen (2013), to evaluate the BEIR's relationship with future inflation.

2.1. Nominal and Real Interest Rate Securities and Derivatives in Brazil

The implied inflation traded in the financial market is calculated as the difference between the nominal interest rate and the real interest rate (Fisher equation) of risk-free assets. This measure is called Break-Even Inflation Rate (BEIR). Nominal and real rates are traded in fixed-income instruments and derivatives.

In Brazil, nominal and real interest rates can be derived from fixed-rate financial instruments - such as National Treasury Bills (LTN), Series F National Treasury Bonds (NTN-F), and One-Day Interbank Deposit Futures (DI1) - and from inflation-indexed instruments, including Series B National Treasury Notes (NTN-B) and DI x IPCA Spread Futures (DAP). LTNs, NTN-Fs and NTN-Bs are bonds issued by The National Treasury of Brazil and are considered risk-free assets in the Brazilian market. DI1 and DAP are futures contracts traded on the Brazilian stock exchange (B3).

LTN and NTN-F are fixed-rate bonds with a face value of R$1,000. LTNs are zero-coupon bonds, while NTN-Fs pay semiannual interest at a rate of 10% per year. Series B National Treasury Notes (NTN-B) are inflation-indexed securities which also pay semi-annual interest. They offer an interest rate agreed upon at the time of purchase plus the IPCA variation.[5] The

[5] IPCA is the Broad Consumer Price Index in Brazil. It is the inflation index used in the inflation target system of Brazil. For more information about IPCA, see the website of Brazil’s statistical bureau (IBGE), http://www.ibge.gov.br/english/.

semi-annual coupon rate is 6% per year. The updated nominal value (or Valor Nominal Atualizado – VNA, in Portuguese) of the NTN-B was set at R$ 1,000.00 on July 15th, 2000. Since then, this value has been monthly updated by the IPCA variation. It is important to note that the VNA value of the NTN-B is known once a month when the IPCA index is disclosed. The interest rates of all these instruments are based on a 252-business day year comprising 12 months with 21 business days – the so-called 252 exponential convention.[6]

DI1 is a Brazilian interest rate futures contract that reflects the expected accumulated overnight interbank deposit rate (DI rate) plus a term premium for a specific date in the future, trading the same underlying asset as LTNs and NTN-Fs. DAP, on the other hand, is a Brazilian futures contract that trades the difference (spread) between the floating interest rate (DI rate) and the inflation rate (IPCA) over a specified period, negotiating the same underlying asset as NTN-Bs. This rate is equivalent to an IPCA-indexed bond. Both DI1 and DAP reflect zero-coupon (spot) rates, as they are traded in the futures market.

Since the IPCA variation is calculated monthly rather than daily and it is disclosed with a time lag, the interest rate traded in the inflation linked instruments are not exactly the real interest rate.[7] Therefore, the difference between the nominal interest rate and the interest rate traded in the inflation linked instruments are not the BEIR. For calculating the BEIR, we must use the true interest rate.

Vicente and Guillen (2013) use a model-free approach to analyze the relationship between future inflation and the BEIR calculated from financial securities, but they do not consider the inflation lag in their analysis (and the inflation seasonality). In other words, they did not use the true interest rate. This work implements the same model-free approach used by Vicente and Guillen (2013), treating the inflation lag and seasonality using the methods developed by Araujo and Vicente (2017) and Val and Araujo (2019).

2.2. Theoretical Framework for BEIR Estimation

This section describes the theoretical framework used to calculate the BEIR, based on the work of Vicente and Guillen (2013).

[6] Holidays can be downloaded from the ANBIMA website in https://www.anbima.com.br/feriados/feriados.asp.
[7] In the U.S., the lags for inflation-indexed government bonds are 3 months; in Brazil, 15 calendar days; and in the U.K., both 8 months and 3 months, depending on the issuance.

The BEIR (π^e) is defined by the Fisher equation as the difference between $R_{t,T}$ and $r_{t,T}$, the nominal and real yields at time t with a maturity of T:

$$\pi_{t,T}^e = \frac{(1 + R_{t,T})}{(1 + r_{t,T})} - 1$$

The accumulated inflation rate between t and $t + T$ is given by:

$$\pi_{t,T} = \left[\prod_{j=t}^{t+T-1} (1 + \pi_{j,1}) \right]^{\frac{T}{12}} - 1$$

where $\pi_{t,1}$ denotes the annual rate of change between two observations of a price index (from t to $t + 1$).

When the BEIR is calculated from financial securities and future contracts it can be decomposed as the sum of the expected inflation rate plus a risk premium and a convexity term (see Grishchenko and Huang, 2010):

$$\pi_{t,T}^e = E\left[\pi_{t,T}\right] + IRP_t(T) - Convexity_t(T) \quad (1)$$

where $E\left[\pi_{t,T}\right]$ denotes the inflation expectation conditional on information available at time t; $IRP_t(T)$ is the risk premium required by investors as protection against real losses; and $Convexity_t(T)$ is a mathematical correction that arises from the transformation of prices into rates through a convex function of interest rates known as Jensen's correction coefficient. The Fisher equation does not consider the last two terms in equation (1).

As demonstrated in Vicente and Graminho (2015), the effects of convexity are negligible in the Brazilian market. Therefore, this term can be ignored, and equation (1) can be rewritten as:

$$\pi_{t,T}^e = E\left[\pi_{t,T}\right] + IRP_t(T) \quad (2)$$

The expectation hypothesis states that the term premium, $IRP_t(T)$, probably depends on maturity but is constant over time. Under this hypothesis and the assumption of rational expectation, the econometric specification of equation (2) is given by:

$$\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T} \quad (3)$$

where $\pi_{t,T}$ and $\pi_{t,T}^{e}$ are the realized inflation and the BEIR, respectively, for period t and horizon T. Using equation (3), it is possible to test if the BEIR contains some information about future inflation. If c_1 is nonzero, the answer to this question is positive. Moreover, it can be verified if the BEIR is an unbiased forecast of realized inflation if it finds that $c_1 = 1$ and $c_2 = 0$.

Vicente and Guillen (2013) used this model to study whether the risk premium and the Jensen's correction are negligible. More generally, they asked: Does the BEIR efficiently predict future inflation? However, they did not consider the indexation lag and seasonality of real rates in their BEIR calculation (they used the Naïve BEIR approach which did not consider the true real interest rate). Nonetheless, Araujo and Vicente (2017) show that these concerns are relevant, especially for the short-term BEIR.

Vicente and Guillen (2013) calculated the BEIR using the interest rate from Brazilian Government securities, which are publicly available through ANBIMA.[8] The term structure of nominal rates is extracted from NTN-F and LTN bonds using the Svensson interpolation model (see Svensson, 1994). Similarly, the term structure of the pseudo real rates (IPCA Coupon) is constructed using the Svensson model, but the curve is fitted using NTN-B bonds.[9]

The BEIR extracted from the Brazilian term structure of nominal and IPCA Coupon rates, used by Vicente and Guillen (2013), is given by the formula:

$$\pi_{t,T}^{e} = \frac{\left(1 + R_{t,T}\right)}{\left(1 + C\_IPCA_{t,T}\right)} - 1 \quad (4)$$

where $R_{t,T}$, is the nominal interest rate and $C\_IPCA_{t,T}$ is the IPCA Coupon rate traded on NTN-Bs.

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[8] ANBIMA is an association of Brazilian financial service providers. For more information about ANBIMA, see the website http://investors.anbima.com.br/Pages/Home.aspx.

[9] The Svensson model is a parametric extension of the Nelson-Siegel model, commonly used to fit the yield curve. As an empirical specification, it does not rely on or enforce no-arbitrage conditions. As a result, the fitted curve does not necessarily pass through observed market data points.

As in Vicente and Guillen (2013), this paper uses the model in equation (3) to study whether the risk premium and the Jensen's correction are negligible. The study offers a contribution by considering the effect of the inflation lag embedded in inflation-indexed securities and the seasonality of inflation in the calculation of the BEIR.

The next sections will address the practical problems in calculating the BEIR and introduce two methods: one using only government bonds as proposed by Val and Araujo (2019), and another using futures contracts based on the work of Araujo and Vicente (2017).

3. METHODOLOGY

This section addresses the challenges involved in calculating the BEIR, focusing on the issues of indexation lag embedded in inflation-indexed securities and seasonality of inflation. It outlines the methodology referred to as Bond Market BEIR (BM-BEIR), which uses government bonds as developed in Val and Araujo (2019), and the approach using the Futures Market (FM-BEIR) from Araujo and Vicente (2017).

3.1. Challenges in Calculating the BEIR

There are two problems in calculating the BEIR: the inflation lag embedded in inflation-indexed securities and the seasonality of inflation.

The inflation lag occurs because inflation is disclosed monthly and with a delay. Consequently, inflation cannot index a financial instrument up to the maturity day, as the inflation data is not known in the days approaching maturity.

The price of any zero-coupon fixed-income security can be calculated by:

$$P_{t,T} = \frac{VNA_T}{(1 + R_{t,T})} \quad (5)$$

Where t is the current day, T is the maturity day, $P_{t,T}$ represents the price at time t of a security that matures on day T, VNA_T is the nominal value on T (this value is constant for fixed-rate securities) and $R_{t,T}$ is the nominal rate from t to T.

To exemplify the lag problem, a zero-coupon NTN-B is used. The traded rate on this bond is known as IPCA Coupon. On a day when the VNA is known (the 15th day of each month or the next business day), the VNA_T will be VNA_t adjusted by the lagged inflation of 15 calendar days:

$$VNA_T = VNA_t\left(1 + \pi_{t-15,T-15}\right) \quad (6)$$

Where $\pi_{t-15,T-15}$ is the inflation 15 calendar days before the security is priced up to 15 days before its maturity.

The nominal rate, $R_{t,T}$, in equation (5) can be decomposed into real rate, $r_{t,T}$, plus inflation, $\pi_{t,T}$:

$$P_{t,T} = \frac{VNA_t(1 + \pi_{t-15,T-15})}{(1 + r_{t,T})(1 + \pi_{t,T})} \quad (7)$$

From equation (7), the inflation in the numerator and denominator do not correspond to the same period. Decomposing the inflation rates in the previous equation:

$$P_{t,T} = \frac{VNA_t(1 + \pi_{t-15,t})(1 + \pi_{t,T-15})}{(1 + r_{t,T})(1 + \pi_{t,T-15})(1 + \pi_{T-15,T})}$$

$$P_{t,T} = \frac{VNA_t\left(1 + \pi_{t-15,t}\right)}{\left(1 + r_{t,T}\right)\left(1 + \pi_{T-15,T}\right)} \quad (8)$$

The IPCA Coupon is implicitly defined by the following equation:

$$P_{t,T} = \frac{VNA_t}{(1 + C\_IPCA_{t,T})} \quad (9)$$

From (8) and (9)

$$(1 + C_{IPCA_{t,T}}) = \left(1 + r_{t,T}\right)\frac{\left(1 + \pi_{T-15,T}\right)}{\left(1 + \pi_{t-15,t}\right)} \quad (10)$$

If we assume that $\left(1 + \pi_{T-15,T}\right) \approx \left(1 + \pi_{t-15,t}\right)$ then the IPCA Coupon equals the real rate: $C\_IPCA_{t,T} = r_{t,T}$.[10]

For long-term bonds, this approximation does not present issues, even if the two inflation rates differ significantly, since the real interest rate remains valid for a much longer period

[10] The Naïve BEIR refers to the Break-Even Inflation Rate calculated under the assumption that $C\_IPCA_{t,T} = r_{t,T}$.

than 15 calendar days. However, for short-term bonds, this difference may become significant as demonstrated in Val and Araujo (2019).

Equation (10) indicates that the rate traded for these securities is not the true real rate. This reasoning also applies to coupon bonds. Thus, the indexation lag problem of Brazilian government securities arises because NTN-Bs do not reflect inflation from the moment of purchase to maturity. Instead, they account for the inflation accrued between 15 days before purchase and 15 days before maturity.

This indexation lag is a common feature in many countries with inflation-indexed government bonds. The United States and the United Kingdom, which have the largest stocks and highest liquidity of such securities, trade Treasury Inflation-Protected Securities (TIPS) and Index-linked Gilts (Linkers), respectively. Linkers issued before 2005 have an 8-month lag for incorporating the Retail Price Index (RPI) into their face value, while those issued after 2005 have a 3-month lag, similar to the TIPS.

The second issue concerns the seasonality of inflation. Price indexes such as the IPCA, used in calculating NTN-B prices, display pronounced monthly seasonal patterns. As a result, the monthly real interest rates derived from these securities also exhibit significant seasonality. Therefore, when calculating the BEIR on a monthly basis, it is essential to account for this seasonal effect.

3.2. BEIR using government bonds (Bond Market BEIR or BM-BEIR)

This section outlines the methodology developed by Val and Araujo (2019) for calculating the BEIR using only government bonds. Their approach accounts for the indexation lag in inflation-linked securities and the seasonality of inflation.

NTN-Bs trade the inflation rate from fifteen days before the last known nominal value to fifteen days before their maturity (see Araujo and Vicente, 2017). Therefore, they do not trade the real interest rate.

Defining VNA_{t^*} as the last known nominal value and t^* as the day when this value was released, and using Zero Coupon NTN-Bs, equation (5) becomes:

$$P_{t,T} = \frac{VNA_{t^*}(1 + \pi_{t^*-15,T-15})}{(1 + R_{t,T})}, \text{or}$$

$$\pi_{t^*-15,T-15} = \frac{P_{t,T}(1+R_{t,T})}{VNA_{t^*}} - 1 \quad (11)$$

where $\pi_{t^*-15,T-15}$ is the implied inflation embedded in the bond within a 15-calendar day lag between time t^* and maturity day T. Thus, the BEIR traded via a zero-coupon bond is the implied inflation rate extracted from equation (11). t^* is defined as the 15th day of a given month or the next business day. Therefore, due to the 15-calendar-day lag, equation (11) implies that the BEIR refers not for the trading date itself, but for a month whose inflation figure is still unknown.

In practice, coupon NTN-Bs are preferred for BEIR calculation due to the lack of liquidity in zero-coupon NTN-Bs. There are two possible situations with coupon NTN-Bs: when they have no further intermediary payments, and when they do.

When there are no further intermediary payments, equation (11) can be used with a modification in VNA_{t^*} since these bonds pay a coupon at maturity. Given that the semiannual coupon rate corresponds to 6% annually on the bond's face value (calculated under the 252-day exponential convention), equation (11) becomes:

$$\pi_{t^*-15,T-15} = \frac{P_{t,T}(1+R_{t,T})}{VNA_{t^*}(1+2{,}956301\%)} - 1$$

When there are intermediate payments, Val and Araujo (2019)'s method consists of:

i) Calculating the nominal and IPCA Coupon spot rates to the maturity date of each NTN-B using the Svensson model; and

ii) Finding the price of a synthetic zero-coupon bond at the valuation date:

$$P_{t,T} = \frac{VNA_t}{(1+C\_IPCA_{t,T})}, \quad (12)$$

where $P_{t,T}$ is the synthetic zero-coupon NTN-B price on day t with maturity date T, estimated using the nominal updated value at date t (VNA_t), and the IPCA Coupon from t to T obtained in step i.[11]

iii) Using equation (12) in equation (11) to find the spot rate of the BEIR with a lag of 15 days from each NTN-B maturity:

$$\pi_{t^*-15,T-15} = \frac{VNA_t(1 + R_{t,T})}{VNA_{t^*}(1 + C_{IPCA_{t,T}})} - 1 \quad (13)$$

3.3. BEIR using the Futures Market (Futures Market BEIR or FM-BEIR)

This section discusses the method to calculate the BEIR using only the futures market as developed by Araujo and Vicente (2017) and used in Val and Araujo (2019). Specially, in the Brazilian futures market, we use the One-day Interbank Deposit Futures contracts (DI1 contracts) and DI x IPCA Spread Futures (DAP contracts).

The DI1 contracts negotiate the nominal interest rate, and the DAP contracts negotiate the IPCA Coupon rate (the spread between the compounded DI rate and the IPCA variation - with a 15-day delay). Equation (11) must be resolved to find the BEIR with 15-days lag between t^* and T:

$$\pi_{t^*-15,T-15} = \frac{PU_{t,T}(1 + R_{t,T})}{VNA_{t^*}} - 1$$

where t^* is the date of the last known VNA, T is the date at maturity, $\pi_{t^*-15,T-15}$ is the BEIR with a 15-day lag from t^* to T, $PU_{t,T}$ is the current unit price of the DAP contract maturing at T and $R_{t,T}$ is the nominal interest rate traded on the one-day Interbank Deposit Futures contracts (DI-1 contract).

If NTN-Bs were being used, the VNA_{t^*} would be the last known value of VNA. Instead, for DAP contracts:

$$PU_{t,T} = \frac{100{,}000}{(1 + C\_IPCA_{t,T})}$$

[11] VNA_t is published daily by ANBIMA on business days. The inflation used for its indexation is projected by the institution.

Comparing this formula with equation (9), VNA_t of the DAP is 100,000. Therefore, the VNA_{t^*} of the DAP is equal to 100,000 discounted by the inflation with a lag of 15-days from the current day:

$$VNA_{t^*} = \frac{VNA_t}{(1 + \pi_{t^*-15,t-15})}$$

where the inflation in the denominator ($\pi_{t^*-15,t-15}$) is not yet known although having already occurred. However, since the daily settlement of the DAP contract consider the projection of IPCA inflation published by ANBIMA, the IPCA Coupon used in the calculation of its PU takes this expectation into account from the VNAs released by ANBIMA:

$$\pi_{t^*-15,t-15} = \frac{VNA_t^{ANBIMA}}{VNA_{t^*}^{ANBIMA}} - 1$$

From this, inflation can be calculated the BEIR using equation (11).

3.4. Incorporating Seasonality in the BEIR

The methods for BEIR calculation in sections 3.2 and 3.3 generate spot BEIR estimates, which are subsequently converted into forward rates for specific maturities. These maturities correspond to those of NTN-Bs and DAP contracts used in the analysis, ensuring alignment with the underlying financial instruments. The forward BEIRs span multiple months. Araujo and Vicente (2017) incorporate monthly inflation seasonality to break down forward BEIR by month. To include information that may affect short-term inflation and agents' expectations, forecasts from the BCB's Focus Report are used. Specifically, the median of the Top 5 Short-Term forecasts for the IPCA is chosen, based on the most accurate recent predictions. An example of the incorporation of Focus seasonality into forward BEIRs can be found in Section 3.6 of Val and Araujo (2019).

In cases where the Focus report shows a negative value, distributing the forward rate according to Araujo and Vicente (2017) results in a negative rate lower than expected. To address this issue, the average difference between the accumulated Focus rate for the forward rate period and the forward rate itself is calculated and then added to each Focus estimation.

4. RESULTS

The primary objective of this paper is to revisit and address the question posed by Vicente and Guillen (2013): "Do inflation-linked bonds contain information about future inflation?".

The study involves a series of Ordinary Least Squares (OLS) regressions between realized inflation (dependent variable) and the BEIR (independent variable) – equation 3 – over horizons of 3, 6, 12, 18, 24, and 30 months.[12] The inflation index used is the IPCA, which benchmarks inflation-linked bonds and the corresponding futures market. The significance of the parameter and the R^2 serve as a measure of the predictive ability and explanatory power of the BEIR.

Initially, the Naïve BEIR and the BM-BEIR are used in the regressions from 2005 to 2011 - the same period analyzed by Vicente and Guillen (2013) - to assess whether relying on the Naïve BEIR impacted the results.[13] Subsequently, the BM-BEIR (using the same coefficients of the Svensson model employed in Val and Araujo, 2017) and the FM-BEIR are used in the regressions for longer time series, and the outcomes of these methods are also compared.

Finally, the results of future inflation estimation using the BM-BEIR and BEIR DAP are compared with those derived from the survey expectations (FOCUS Top 5 Short-Term survey) for 3-, 6-, and 12-month horizons, emphasizing the BEIR's effectiveness as a tool for inflation forecasting.

4.1. Naïve BEIR Versus BM-BEIR

Following Vicente and Guillen (2013), this section analyzes a sample consisting of a monthly series of Brazilian real and nominal yields from April 2005 to April 2011. The starting point of April 2005 was chosen by Vicente and Guillen (2013) to avoid liquidity issues in the NTN-B market observed between 2001 and 2005.

[12] The choice to use the OLS method for these regressions stems from the robustness of this method, as demonstrated in the work of Vicente and Guillen (2013). They extensively tested their model using various estimation techniques, including Two-Stage Least Squares (TSLS) and the Generalized Method of Moments (GMM), to control for potential endogeneity. Their findings revealed that the results were remarkably consistent across all methods, indicating that OLS provides reliable and robust estimates. The estimated coefficients in their regressions remain stable regardless of the estimation method.
[13] As we previously observed, it is not possible to use the FM-BEIR during this period due to the lack of liquidity in the futures market, specifically the illiquidity of DAP contracts.

The BEIR is calculated using two methods: the Naïve Method and the BM-BEIR. Both methods employ the Svensson interpolation with the coefficients released by ANBIMA.[14] The BEIR used in the analysis - as in all other experiments in this study - corresponds to the value calculated on the last business day of each month.

4.1.1. Descriptive Statistics

Tables 1 and 2 present descriptive statistics for the Naïve BEIR and the BM-BEIR, respectively, across different horizons from April 2005 to April 2011. The last column in both tables also displays the realized inflation (IPCA). Both tables indicate that the averages of IPCA and BEIR for all horizons are around 5% p.a. The Naïve BEIR, the BM-BEIR, and IPCA exhibit positive skewness in this period, indicating a long right tail. Additionally, the kurtosis values in both tables are below 3, suggesting that these distributions are platykurtic (flatter than a normal distribution).

Table 1 - Descriptive Statistics: Naïve BEIR and IPCA per year (April 2005 to April 2011)

	Naïve BEIR-3	Naïve BEIR-6	Naïve BEIR-12	Naïve BEIR-18	Naïve BEIR-24	Naïve BEIR-30	IPCA
Mean	5,29%	4,97%	4,78%	4,79%	4,86%	4,95%	5,11%
Median	4,84%	4,70%	4,66%	4,66%	4,72%	4,87%	5,03%
Maximum	9,90%	7,75%	6,99%	7,43%	7,53%	7,52%	10,95%
Minimum	1,66%	2,33%	3,20%	3,24%	3,28%	3,35%	-2,49%
Std. Dev.	1,64%	1,07%	0,85%	0,82%	0,80%	0,78%	2,96%
Skewness	0,59	0,55	0,54	0,64	0,63	0,57	0,07
Kurtosis	0,15	0,17	- 0,04	0,44	0,73	0,63	- 0,32
Jarque-Bera	**4,01**	**3,54**	**3,49**	**5,10**	**5,77**	**4,62**	**0,48**
Jarque-Bera p-value	**13,5%**	**17,0%**	**17,5%**	**7,8%**	**5,6%**	**9,9%**	**78,5%**
Observations	73	73	73	73	73	73	73
Correlation with IPCA	36%	46%	54%	52%	48%	43%	100%

This table presents descriptive statistics for the Break-Even Inflation Rate (BEIR) across different forecast horizons ($\pi^e_{t,T}$, with T=3,6,12,18,24 e 30 months) and the variation in the consumer price index (IPCA). The number following each variable name in the column headers represents the forecast horizon in months (e.g., "Naïve BEIR-3" corresponds to a 3-month horizon or $\pi^e_{t,3}$). Naïve BEIR refers to the Break-Even Inflation Rate calculated using the IPCA Coupon rate instead of the real rate, employing a simple method without adjustments for lagged inflation or seasonality. Skewness measures the asymmetry of the distribution; a symmetric distribution has skewness equal to zero. Positive skewness indicates a long right tail, while negative skewness suggests a long-left tail. Kurtosis indicates the "tailedness" of a distribution. A normal distribution has a kurtosis of 3. Distributions with kurtosis greater than 3 are peaked (leptokurtic), while those with kurtosis less than 3 are flatter (platykurtic) compared to the normal. The Jarque-Bera test assesses normality, with p-values greater than 5% indicating that the null hypothesis of normality cannot be rejected.

[14] We use the Svensson Model coefficients published by ANBIMA to ensure comparability between the BM-BEIR and the results of Vicente and Guillen (2013), who also relied on ANBIMA's coefficients. These coefficients are available on the ANBIMA website.

The Jarque-Bera test evaluates whether a dataset follows a normal distribution by examining skewness and kurtosis. In Table 1, all p-values are greater than 5%, indicating that the null hypothesis of normality cannot be rejected for the BEIR and IPCA across horizons. However, for the horizons of 18, 24, and 30 months, the p-values are close to the 5% threshold, suggesting that for these periods, the test only marginally supports the assumption of normality.

Table 2 - Descriptive Statistics: BM-BEIR and IPCA per Year (April 2005 to April 2011)

	BM-BEIR_3	BM-BEIR_6	BM-BEIR_12	BM-BEIR_18	BM-BEIR_24	BM-BEIR_30	IPCA
Mean	5,34%	5,26%	5,22%	5,30%	5,34%	5,45%	5,11%
Median	5,05%	5,06%	4,96%	5,14%	5,36%	5,43%	5,03%
Maximum	9,81%	8,18%	8,27%	8,48%	8,59%	8,53%	10,95%
Minimum	2,49%	3,00%	3,65%	3,45%	3,53%	3,61%	-2,49%
Std. Dev.	1,75%	1,23%	1,06%	0,99%	0,94%	0,88%	2,96%
Skewness	0,47	0,51	0,86	0,69	0,70	0,63	0,07
Kurtosis	- 0,55	- 0,22	0,22	0,56	1,08	1,22	- 0,32
Jarque-Bera	**3,61**	**3,34**	8,61	6,18	8,38	7,97	**0,48**
Jarque-Bera p-value	**16,4%**	**18,8%**	1,4%	4,5%	1,5%	1,9%	**78,5%**
Observations	73	73	73	73	73	73	73
Correlation with IPCA	79%	67%	62%	54%	51%	44%	100%

This table presents descriptive statistics for the Break-Even Inflation Rate (BEIR) across different forecast horizons ($\pi^e_{t,T}$, T=3,6,12,18,24 e 30 months) and the variation in the consumer price index (IPCA). The number following each variable name in the column headers represents the forecast horizon in months (e.g., " BM-BEIR_3" corresponds to a 3-month horizon or $\pi^e_{t,3}$). BM-BEIR_ refers to the Break-Even Inflation Rate calculated using only government bonds while adjusting for lagged inflation and seasonality, for forecast horizons of 3, 6, 12, 18, 24, and 30 months, respectively. Skewness measures the asymmetry of the distribution; a symmetric distribution has skewness equal to zero. Positive skewness indicates a long right tail, while negative skewness suggests a long-left tail. Kurtosis indicates the "tailedness" of a distribution. A normal distribution has a kurtosis of 3. Distributions with kurtosis greater than 3 are peaked (leptokurtic), while those with kurtosis less than 3 are flatter (platykurtic) compared to the normal. The Jarque-Bera test assesses normality, with p-values greater than 5% indicating that the null hypothesis of normality cannot be rejected.

In Table 2, for the horizons of 3 and 6 months, as well as for IPCA, the p-values are greater than 5%, indicating that the null hypothesis of normality cannot be rejected. However, for the horizons of 12, 18, 24, and 30 months, the p-values are below the 5% threshold, which indicates that the assumption of normality is rejected for these periods.

Moreover, Table 2 shows higher correlation values between the BM-BEIR and IPCA for shorter horizons (3, 6, and 12 months) compared to the correlations observed between Naïve BEIR and IPCA in Table 1. This highlights a stronger relationship between the BM-BEIR and realized inflation for shorter time horizons.

In both tables, the correlation between BEIRs and IPCA is higher for short horizons. This means that the BEIR provides a better explanation of realized inflation over short horizons than over longer ones, a finding confirmed by Vicente and Guillen (2013) and further demonstrated in this paper.

Figure 1 and Figure 2 depict the time evolution for the Naïve BEIR and the BM-BEIR, respectively, and for the IPCA from April 2005 to April 2011. Both figures show the BEIR values across horizons of 3, 6, 12, 18, 24, and 30 months. Note that Both BEIRs and IPCA exhibit no trend in the period.

In Figure 1, which uses the Naïve BEIR method, there is noticeable divergence from the IPCA, particularly in periods of economic volatility. The BEIR values tend to underperform in tracking realized inflation, especially for longer horizons. This observation aligns with the lower correlation values observed in Table 1, which indicate a weaker relationship between the Naïve BEIR and the IPCA.

In contrast, Figure 2, which uses the BM-BEIR, accounting for the inflation lag and seasonality, demonstrates a closer alignment with IPCA, especially for shorter horizons. The BM-BEIR time series follow the trends in realized inflation more accurately, with less pronounced deviations. This is consistent with the higher correlation values observed in Table 2, confirming that the BM-BEIR provides a more reliable estimation of future inflation compared to the Naïve approach.

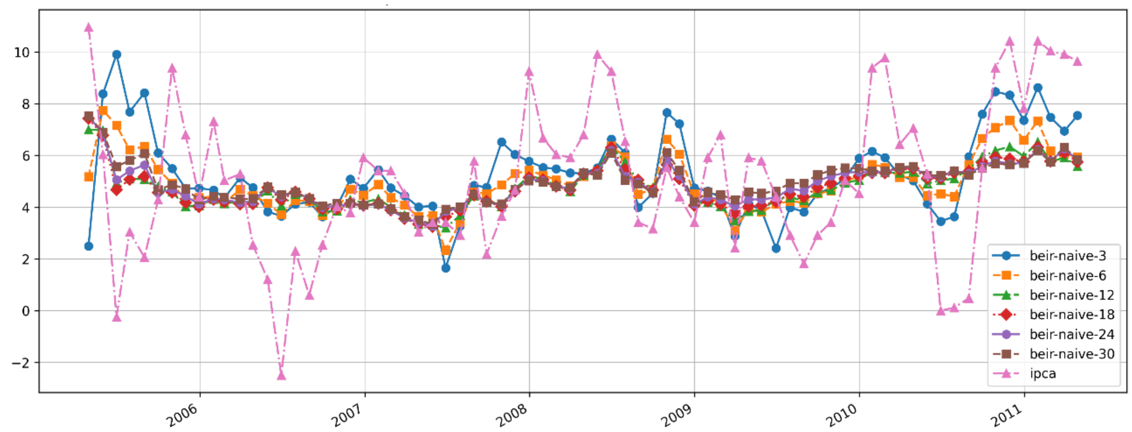

Figure 1 – Naïve BEIR and IPCA (per year) from April 2005 to April 2011

This figure shows time series data for the 3-, 6-, 12-, 18-, 24-, and 30-month Naïve BEIR alongside the rate of change in the IPCA from April 2005 to April 2011. The Naïve BEIR is calculated as the difference between nominal yields and IPCA Coupon yields from government bonds. The IPCA represents the main Brazilian consumer price index.

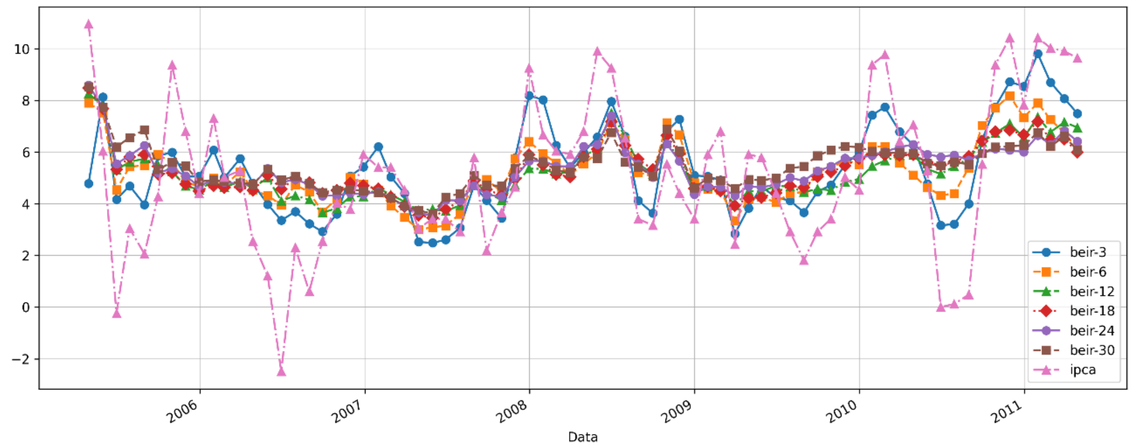

Figure 2 - BM-BEIR and IPCA (per year) from April 2005 to April 2011

This figure shows the time series data for the 3-, 6-, 12-, 18-, 24-, and 30-month BEIR calculated using the BM-BEIR, which accounts for the inflation lag and seasonality, alongside the rate of change in the IPCA, from April 2005 to April 2011. The IPCA represents Brazil's primary consumer price index.

Overall, these results highlight the superiority of the BM-BEIR in capturing inflation dynamics compared to the Naïve BEIR used by Vicente and Guillen (2013), underscoring the importance of adjusting for seasonality and indexation lag.

4.1.2. Empirical Results

Tables 3 and 4 present the regression results for the Naïve BEIR and BM-BEIR, respectively, based on equation (3): $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$, where $\pi_{t,T}^e$ is the BEIR and $\pi_{t,T}$ is the IPCA. The coefficients in bold are statistically significant.

Table 3 - Naïve BEIR: OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ *. Period: April 2005 to April 2011.*

Horizons	3	6	12	18	24	30
c_1	0,71	**0,96**	0,38	-0,04	**-0,27**	-0,24
Std. Err.	0,41	0,34	0,24	0,24	0,11	0,13
c_2	0,01	0,00	**0,03**	**0,05**	**0,06**	**0,06**
Std. Err.	0,02	0,02	0,01	0,01	0,01	0,01
R^2	24,00%	31,80%	7,30%	0,10%	9,50%	14,80%
$c_1 = 1, c_2 = 0$	**0,30**	**0,04**	3,47	10,94	73,34	91,48

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ *(Equation (3) in this paper) for the horizons of 3, 6, 12, 18, 24 and 30 months. Here,* $\pi_{t,T}^e$ *denotes the Naïve BEIR at time* t *and horizon* τ *and* $\pi_{t,T}$ *denotes the IPCA accumulated between* t *and* $t+\tau$*. The bottom row shows the F-statistic of the joint hypothesis* $c_1 = 1$ *and* $c_2 = 0$*. The Std. Err. below* c_1 *and* c_2 *denote the standard errors of* c_1 *and* c_2*. Boldface values mean significance at a 95% confidence level.*

Table 4 - BM-BEIR: OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$*. Period: April 2005 to April 2011.*

Horizons	3	6	12	18	24	30
c_1	**0,97**	**0,91**	0,23	-0,08	**-0,24**	-0,21
Std. Err.	0,18	0,24	0,17	0,17	0,10	0,12
c_2	-0,00	0,00	**0,04**	**0,05**	**0,06**	**0,06**
Std. Err.	0,01	0,01	0,01	0,01	0,01	0,01
R^2	51,00%	37,00%	4,00%	0,60%	11,80%	15,10%
$c_1 = 1, c_2 = 0$	**1,06**	**0,25**	10,82	19,54	77,76	82,10

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ *(Equation (3) in this paper) for the horizons of 3, 6, 12, 18, 24 and 30 months. Here,* $\pi_{t,T}^e$ *denotes the BM-BEIR at time* t *and horizon* τ *and* $\pi_{t,T}$ *denotes the IPCA accumulated between* t *and* $t+\tau$*. The bottom row shows the F-statistic of the joint hypothesis* $c_1 = 1$ *and* $c_2 = 0$*. The Std. Err. below* c_1 *and* c_2 *denote the standard errors of* c_1 *and* c_2*. Boldface values mean significance at a 95% confidence level.*

The results from Tables 3 and Table 4 are similar to those obtained by Vicente and Guillen (2013). In Table 4, the slope coefficient c_1 is significant for the horizons of 3, 6, and 24 months. This suggests that the BEIR contains information about future inflation for both short- and long-term horizons.

For the 3- and 6-month horizons, the F-statistic of the joint hypothesis $c_1 = 1$ and $c_2 = 0$ indicate that the BEIR is an unbiased estimator of future inflation. In other words, the expectation hypothesis holds for short-term horizons. However, for the 24-month horizon, the relationship between the BEIR and future inflation turns negative. As noted by Vicente and Guillen (2013), this result implies that the expectation hypothesis fails for longer horizons, pointing to the need for a more generalized specification.

For horizons of 12, 18, and 30 months, the slope coefficient c_1 is not significant, and the R^2 values are very low. Additionally, the F-statistic rejects the joint hypothesis $c_1 = 1$ and $c_2 = 0$, indicating that the BEIR does not contain information about inflation for these horizons. The value of c_2, when c_1 is statistically different from 1, can be interpreted as a risk premium. Furthermore, c_2 ranges from 400 to 600 basis points for medium and long-term horizons, suggesting that investors require higher returns to hold inflation-linked bonds with maturities longer than one year.

Comparing the results in Tables 3 and 4, we observe several improvements when using the BM-BEIR instead of the Naïve BEIR. The standard errors of the coefficients are significantly smaller, particularly for shorter horizons. Additionally, the R^2 values consistently indicate higher predictive accuracy and explanatory power for the BM-BEIR compared to the Naïve BEIR. Similar conclusions can be drawn when comparing Tables 5 and 6, which present the results obtained by Vicente and Guillen (2013).

These findings confirm that the BM-BEIR has a stronger relationship with future inflation, making it a more reliable measure for predicting the IPCA than the Naïve approach.

Table 5 – Vicente and Guillen (2013) OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$

Horizons	3	6	12	18	24	30
c1	**0,91**	**0,98**	0,26	-0,14	**-0,30**	**-0,26**
Std. Err.	0,16	0,19	0,17	0,16	0,11	0,08
c2	0,00	0,00	**0,04**	**0,05**	**0,06**	**0,06**
Std. Err.	0,01	0,01	0,01	0,01	0,01	0,00
R2	31,49%	28,18%	3,81%	1,35%	12,62%	17,64%
c1 = 1, c2= 0	**0,54**	**0,89**	0,00	0,00	0,00	0,00

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ *(Equation 1 in Vicente and Guillen, 2013) for the horizons of 3, 6, 12, 18, 24 and 30 months. Here,* $\pi_{t,T}^e$ *denotes the Naïve BEIR at time* t *and horizon* τ *and* $\pi_{t,T}$ *denotes the IPCA accumulated between* t *and* $t+\tau$*. The bottom row shows the F-statistic of the joint hypothesis* $c_1 = 1$ *and* $c_2 = 0$*. The Std. Err. below* c_1 *and* c_2 *denote the standard errors of* c_1 *and* c_2*. Boldface values mean significance at a 95% confidence level.*

Table 6 – Vicente and Guillen (2013) TSLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$*.*

Horizons	3	6	12	18	24	30
c1	**1,07**	**1,06**	0,20	-0,02	-0,04	-0,15
Std. Err.	0,27	0,32	0,37	0,28	0,22	0,22
c2	0,00	0,00	**0,04**	**0,05**	**0,07**	**0,06**
Std. Err.	0,01	0,01	0,02	0,01	0,01	0,01
R2	41,06%	36,05%	4,32%	-0,28%	1,53%	-0,68%
c1 = 1, c2= 0	**0,89**	**0,82**	0,04	0,00	0,00	0,00

This table presents the TSLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ *(Equation 1 in Vicente and Guillen, 2013) for the horizons of 3, 6, 12, 18, 24 and 30 months. Here,* $\pi_{t,T}^e$ *denotes the Naïve BEIR at time* t *and horizon* τ *and* $\pi_{t,T}$ *denotes the IPCA accumulated between* t *and* $t+\tau$*. The bottom row shows the F-statistic of the joint hypothesis* $c_1 = 1$ *and* $c_2 = 0$*. The Std. Err. below* c_1 *and* c_2 *denote the standard errors of* c_1 *and* c_2*. Boldface values mean significance at a 95% confidence level.*

4.2. BM-BEIR Versus FM-BEIR

This section analyzes the relationship between realized inflation (IPCA) and the BEIR calculated using two approaches: one based on government bonds (BM-BEIR, presented in Section 3.2), and the other based on the futures market (FM-BEIR, presented in Section 3.3), over the period from January 2016 to May 2023. Both methods employ the Svensson interpolation with the same coefficients used by Val and Araujo (2019). The starting point of January 2016 was chosen to avoid liquidity issues in the DI x IPCA Spread Futures contracts (DAP contracts) observed prior to 2016.

4.2.1. Descriptive Statistics

Tables 7 and 8 provide descriptive statistics for the BM-BEIR and FM-BEIR, respectively, over various time horizons, as well as for IPCA variation. The averages of BEIR and IPCA for all horizons are approximately 5% over the period from January 2016 to May 2023. The BM-BEIR (Table 7), the FM-BEIR (Table 8), and the realized inflation (IPCA) exhibit negative skewness for the horizons of 3 and 6 months, suggesting a long-left tail. Conversely, positive skewness is observed for horizons of 12, 18, 24, and 30 months, as well as for the IPCA, indicating a long right tail. Additionally, the kurtosis values in both tables are below 3, implying that the distributions are platykurtic, meaning they are flatter than a normal distribution.

Table 7 - Descriptive Statistics: BM-BEIR and IPCA per year (January 2016 to May 2023)

	BM-BEIR_3	BM-BEIR_6	BM-BEIR_12	BM-BEIR_18	BM-BEIR_24	BM-BEIR_30	IPCA
Mean	4,64%	4,54%	4,66%	4,69%	4,79%	4,83%	5,52%
Median	4,85%	4,43%	4,45%	4,48%	4,56%	4,62%	4,78%
Maximum	11,86%	9,28%	8,94%	8,75%	8,71%	9,04%	21,27%
Minimum	-6,22%	-1,29%	1,10%	1,62%	1,96%	2,21%	-7,86%
Std. Dev.	2,85%	1,84%	1,36%	1,17%	1,20%	1,11%	5,35%
Skewness	-0,52	-0,25	0,30	0,38	0,61	0,84	0,39
Kurtosis	1,79	1,18	0,88	1,21	0,96	2,13	0,37
Jarque-Bera	13,78	**5,02**	**3,50**	6,42	8,04	24,42	**2,46**
Jarque-Bera p-value	0,10%	**8,13%**	**17,38%**	4,04%	1,79%	0,00%	**29,23%**
Observations	90	90	90	90	90	90	90
Correlation with IPCA	82%	70%	60%	52%	52%	49%	100%

This table presents descriptive statistics for the Break-Even Inflation Rate (BEIR) across different forecast horizons ($\pi^e_{t,T}$, T =3, 6, 12, 18, 24 and 30 months) and the variation in the consumer price index (IPCA). The number following each variable name in the column headers represents the forecast horizon in months (e.g.," BM-BEIR_3" corresponds to a 3-month horizon or $\pi^e_{t,3}$).BM-BEIR_ refers to the Break-Even Inflation Rate calculated using only government bonds while adjusting for lagged inflation and seasonality, for forecast horizons of 3, 6, 12, 18, 24, and 30 months, respectively. Skewness measures the asymmetry of the distribution; a symmetric distribution has skewness equal to zero. Positive skewness indicates a long right tail, while negative skewness suggests a long-left tail. Kurtosis indicates the "tailedness" of a distribution. A normal distribution has a kurtosis of 3. Distributions with kurtosis greater than 3 are peaked (leptokurtic), while those with kurtosis less than 3 are flatter (platykurtic) compared to the normal. The Jarque-Bera test assesses normality, with p-values greater than 5% indicating that the null hypothesis of normality cannot be rejected.

In Table 7, the Jarque-Bera p-values suggest that for horizons of 6 and 12 months, as well as for the IPCA, the null hypothesis of normality cannot be rejected, indicating the data aligns with a normal distribution for these horizons. However, for the horizons of 3, 18, 24, and 30 months, the p-values fall below the 5% threshold, rejecting the assumption of normality.

In Table 8, the Jarque-Bera p-values are greater than 5% for the horizons of 3, 6, 12, and 18 months, indicating the null hypothesis of normality cannot be rejected for these periods. This suggests that the FM-BEIR shows better adherence to a normal distribution over shorter and medium horizons compared to the BM-BEIR.

Table 8 - Descriptive Statistics: FM-BEIR and IPCA per year (January 2016 to May 2023)

	beir_dap_3	beir_dap_6	beir_dap_12	beir_dap_18	beir_dap_24	beir_dap_30	IPCA
Mean	4,57%	4,53%	4,61%	4,61%	4,66%	4,73%	5,52%
Median	4,56%	4,44%	4,35%	4,33%	4,32%	4,40%	4,78%
Maximum	10,80%	9,38%	9,08%	8,96%	8,74%	9,01%	21,27%
Minimum	-3,99%	-1,71%	0,74%	1,21%	1,79%	2,00%	-7,86%
Std. Dev.	2,76%	1,97%	1,45%	1,30%	1,23%	1,18%	5,35%
Skewness	-0,26	-0,11	0,31	0,36	0,65	0,76	0,39
Kurtosis	0,63	0,9	0,55	0,97	0,92	1,51	0,37
Jarque-Bera	**2,00**	**2,47**	**2,18**	**4,65**	8,47	15,31	**2,46**
Jarque-Bera p-value	**37,0%**	**29,0%**	**34,0%**	**10,0%**	1,0%	0,0%	**29,0%**
Observations	90	90	90	90	90	90	90
Correlation with IPCA	86%	74%	64%	58%	53%	52%	100%

This table presents descriptive statistics for the Break-Even Inflation Rate (BEIR) across different forecast horizons ($\pi_{t,T}^{e}$, T =3, 6, 12, 18, 24 and 30 months) and the rate of change in the consumer price index (IPCA). The number following each variable name in the column headers represents the forecast horizon in months (e.g.," BM-BEIR_3" corresponds to a 3-month horizon or $\pi_{t,3}^{e}$).beir_dap_ refers to the Break-Even Inflation Rate calculated using DAP while adjusting for lagged inflation and seasonality (FM-BEIR), for forecast horizons of 3, 6, 12, 18, 24, and 30 months, respectively. Skewness measures the asymmetry of the distribution; a symmetric distribution has skewness equal to zero. Positive skewness indicates a long right tail, while negative skewness suggests a long-left tail. Kurtosis indicates the "tailedness" of a distribution. A normal distribution has a kurtosis of 3. Distributions with kurtosis greater than 3 are peaked (leptokurtic), while those with kurtosis less than 3 are flatter (platykurtic) compared to the normal. The Jarque-Bera test assesses normality, with p-values greater than 5% indicating that the null hypothesis of normality cannot be rejected.

When comparing correlations with the IPCA, Table 8 shows stronger correlations for shorter horizons (3, 6, and 12 months) than those in Table 7. Both methods indicate a high correlation between the BEIR and the IPCA, but the FM-BEIR outperforms the BM-BEIR in shorter-term predictability.

Figure 3 and Figure 4 illustrate the time evolution of the BEIR and the IPCA from January 2016 to May 2023. Both figures confirm the descriptive statistical results: the BEIR performs better in capturing inflation expectations for shorter horizons. Neither approach shows a clear trend in the time series, reflecting a lack of systematic bias in either method.

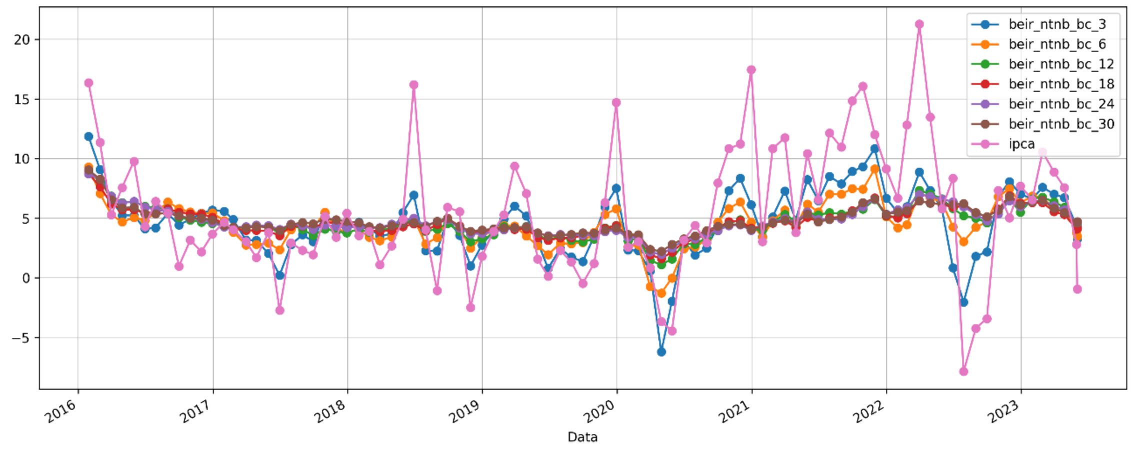

Figure 3 - BM-BEIR and IPCA (Annual Rates) from January 2016 to May 2023.

This figure shows the time series data for the 3-, 6-, 12-, 18-, 24-, and 30-month BEIR calculated using the method using only government bonds, which accounts for the inflation lag and seasonality, alongside the variation in the IPCA, from January 2016 to May 2023. The IPCA represents Brazil's primary consumer price index.

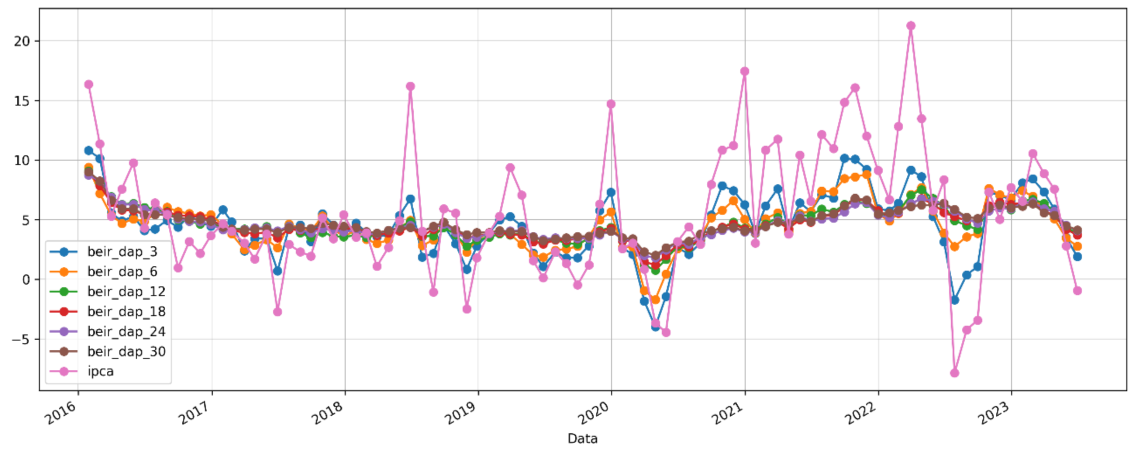

Figure 4 - BEIR calculated from Futures Contracts (BF-BEIR) and IPCA from January 2016 to May 2023.

This figure shows the time series data for the 3-, 6-, 12-, 18-, 24-, and 30-month BEIR calculated from futures contracts, which accounts for the inflation lag and seasonality, alongside the variation in the IPCA, from January 2016 to May 2023. The IPCA represents Brazil's primary consumer price index.

4.2.2. Empirical Results

Tables 9 and 10 display the regression results for the BM-BEIR and FM-BEIR, respectively, following equation (3): $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$, where $\pi_{t,T}^e$ is the BEIR and $\pi_{t,T}$ is the IPCA. Statistically significant coefficients are highlighted in bold. For both BEIRs, the slope coefficient c_1 is significant for the 3- and 6-month horizons, which indicates that the BEIR provides meaningful information about the future inflation over short horizons.

For longer horizons (18, 24 and 30 months) c_1 is also significant in Table 9, while in Table 10 it is significant for 24- and 30-months horizons. This suggests that the BEIR calculated using both methods is somewhat effective at capturing inflation expectations for longer horizons, though its explanatory power deceases as the horizons increases. However, the relationship between the BEIR and future inflation turns negative suggesting again that the expectation hypothesis fails for longer horizons and requires a more generalized specification.

Table 8 - BM-BEIR: OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$. Period: January 2016 to May 2023.

Horizons	3	6	12	18	24	30
c_1	**0,83**	**0,82**	0,25	**-0,22**	**-0,74**	**-0,77**
Std. Err.	0,10	0,16	0,24	0,02	0,33	0,36
c_2	0,00	0,01	**0,04**	**0,06**	**0,08**	**0,08**
Std. Err.	0,01	0,01	0,02	0,02	0,02	0,02
R^2	58,90%	27,80%	0,50%	1,10%	15,20%	20,70%
$c_1 = 1$, $c_2 = 0$	**1,80**	**0,89**	4,92	7,22	16,23	12,89

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ (Equation (3) in this paper) for the horizons of 3,6,12,18,24 and 30 months. Here, $\pi_{t,T}^e$ denotes the BM-BEIR at time t and horizon τ and $\pi_{t,T}$ denotes the IPCA accumulated between t and $t + \tau$. The bottom row shows the F-statistic of the joint hypothesis $c_1 = 1$ and $c_2 = 0$. The $Std. Err.$ below c_1 and c_2 denote the standard errors of c_1 and c_2. Boldface values mean significance at a 95% confidence level.

Table 9 - BM-BEIR: OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$. Period: January 2016 to May 2023.

Horizons	3	6	12	18	24	30
c_1	**0,88**	**0,85**	0,30	-0,14	**-0,66**	**-0,74**
Std. Err.	0,09	0,15	0,24	0,33	0,33	0,34
c_2	-0,00	0,01	**0,03**	**0,06**	**0,08**	**0,08**
Std. Err.	0,00	0,01	0,01	0,02	0,02	0,02
R^2	62,10%	33,70%	3,00%	0,50%	13,10%	20,70%
$c_1 = 1$, $c_2 = 0$	**1,06**	**0,74**	4,22	5,96	15,48	13,69

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ (Equation (3) in this paper) for the horizons of 3,6,12,18,24 and 30 months. Here, $\pi_{t,T}^e$ denotes the BEIR calculated from DAP at time t and horizon τ and $\pi_{t,T}$ denotes the IPCA accumulated between t and $t + \tau$. The bottom row shows the F-statistic of the joint hypothesis $c_1 = 1$ and $c_2 = 0$. The $Std. Err.$ below c_1 and c_2 denote the standard errors of c_1 and c_2. Boldface values mean significance at a 95% confidence level.

For medium-term horizons, 12- months in Table 9 and 12- and 18-months for Table 10, the slope coefficient c_1 is not significant, and the R^2 values are very low. Additionally, the F − statistic rejects the joint hypothesis $c_1 = 1$ and $c_2 = 0$, indicating that BEIR does not contain information about inflation for these horizons.

Coefficient c_1 is consistently below one in both tables, which reflects the presence of a positive risk premium. This aligns with economic theory, as investors demand additional compensation for holding inflation-linked securities. The result implies that the BEIR overestimates the realized inflation (IPCA).

The R^2 values for the 3- and 6-month horizons are much higher in both tables, particularly in Table 10 (62.10% and 33.70%, respectively). This indicates that the BEIR has a stronger explanatory power for realized inflation over short horizons compared to medium or long horizons.

The F-statistic values in both tables for the 3- and 6-month horizons suggest that the null hypothesis of c_1= 1 and c_2= 0 is not rejected. This supports the "expectation hypothesis" for these short horizons, implying that the BEIR can be considered an unbiased predictor of future inflation in the short term.

For longer horizons (18, 24, and 30 months), the F-statistic values reject the null hypothesis. This indicates that the expectation hypothesis does not hold over longer periods, and a more general model may be required to capture the dynamics of BEIR and future inflation for these horizons.

The results in Table 10 (BEIR DAP) generally show slightly higher R^2 values for the shorter horizons (3 and 6 months) compared to Table 9 (BM-BEIR). This suggests that the FM-BEIR may better explain short-term inflation expectations. For longer horizons, the R^2 values are relatively similar between the two methods, indicating that neither method has a distinct advantage in capturing long-term inflation expectations.

The value of c_2, when c_1 is statistically different from 1, can be interpreted as a risk premium. Furthermore, c_2 ranges from 300 to 800 basis points for medium and long-term horizons, confirming that investors require higher returns to hold inflation-linked bonds with maturities longer than one year. For longer horizons (12, 18, 24, and 30 months), c_2 grows, reflecting a risk premium over time. This aligns with the observed rejection of the expectation hypothesis in the F-tests for these horizons. For short horizons (3 and 6 months), c_2 is close to zero, which supports the expectation hypothesis for these periods. This is consistent with the interpretation that the BEIR at short horizons is a more reliable predictor of realized inflation.

4.3. BM-BEIR and FM-BEIR Versus Survey Expectations

FOCUS Top 5 Short-Term Survey is a component of Brazil's FOCUS report, which is published weekly by the Central Bank of Brazil. The survey aggregates the inflation expectations of the five most accurate forecasters based on their historical performance, including financial institutions and economists. In this section, we compare BM-BEIR and FM-BEIR with these survey expectations from January 2016 to May 2023.

Table 11 presents the results of estimating future inflation using the FOCUS Top 5 Short-Term forecast inflation for 3-, 6-, and 12-month horizons (equation (3), using survey expectations in place of the BEIR). The comparison is limited to these horizons because the FOCUS Top 5 Short-Term survey provides forecasts for a maximum of 17 months ahead in the beginning of our sample, which restricts its applicability for longer horizons.

When comparing the results from Table 11 with those in Tables 9 and 10, it is evident that BM-BEIR and FM-BEIR perform on par with the FOCUS Top 5 expectations in predicting short-term inflation. For the 3- and 6-month horizons, the slope coefficient c_1 is significant and close to 1 across all methods, indicating that the BM-BEIR and the FM-BEIR contains predictive power comparable to the survey-based expectations.

Table 10 – Survey Expectations (Focus): OLS results of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$. Period: from January 2016 to May 2023

Horizons	3	6	12
c_1	**0,92**	**0,92**	0,14
Std. Err.	0,08	0,16	0,31
c_2	-0,00	0,00	**0,04**
Std. Err.	0,00	0,01	0,02
R^2	64,20%	29,90%	0,40%
$c_1 = 1$, $c_2 = 0$	**2,46**	**0,14**	4,03

This table presents the OLS estimate of $\pi_{t,T} = c_1^T \pi_{t,T}^e + c_2^T + \varepsilon_{t+T}$ (Equation (3) in this paper) for the horizons of 3,6 and 12 months. Here, $\pi_{t,T}^e$ denotes the inflation expectation from FOCUS Survey at time t and horizon τ and $\pi_{t,T}$ denotes the IPCA accumulated between t and $t+\tau$. The bottom row shows the F-statistic of the joint hypothesis $c_1 = 1$ and $c_2 = 0$. The Std.Err. below c_1 and c_2 denote the standard errors of c_1 and c_2. Boldface values mean significance at a 95% confidence level.

5. FINAL REMARKS

This study revisits the question posed by Vicente and Guillen (2013): 'Do inflation-linked bonds contain information about future inflation?'. Building on their model-free approach, it refines the BEIR used by the authors by incorporating two key adjustments: accounting for the inflation lag embedded in inflation-linked securities and addressing the seasonal patterns of inflation. These enhancements, inspired by the methodologies of Araujo and Vicente (2017) and Val and Araujo (2019), significantly improve the accuracy of BEIR estimation.

Vicente and Guillen (2013) demonstrated that the BEIR provides valuable insights into future inflation, particularly for short-term horizons (3 and 6 months). However, their analysis relied on a Naïve BEIR approach that fails to incorporate the necessary adjustments. By directly addressing these limitations, this study evaluates whether these refinements yield a more robust measure of inflation expectations.

The findings indicate that incorporating lagged inflation and seasonality enhances the BEIR's ability to explain future inflation, particularly over short-term horizons. However, for longer horizons, the expectation hypothesis of the BEIR breaks down, aligning with previous research. The results suggest that investors require higher compensation for holding inflation-linked securities with longer maturities.

Furthermore, the analysis highlights the differences between the BEIR calculated using government bonds (BM-BEIR) and the BEIR derived from futures contracts (FM-BEIR). While both methods show strong correlations with future inflation for short term horizons, FM-BEIR has a slightly greater explanatory power. For longer term horizons, neither method exhibits a clear advantage, emphasizing the challenges of predicting inflation over extended periods. Additionally, when compared to inflation survey expectations, both methods perform equally well, while offering the key advantage of daily updates. This makes BEIR a valuable tool for entities requiring real-time inflation forecasts for short-term decision-making.

In conclusion, this study validates the findings of Vicente and Guillen (2013) but the refinements to the BEIR methodology (the adjustments for lagged inflation and seasonality) not only enhance the BEIR's predictive accuracy but also contribute to a more comprehensive understanding of the relationship between inflation-linked instruments and future inflation. These findings underscore the importance of incorporating these adjustments in future studies and in practical applications of the BEIR for inflation forecasting.

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