ESTUDO DO EFEITO AMORTECEDOR DE CAIXAS D'ÁGUA DEVIDO A SISMOS

Study of the damper effect of water tanks due to earthquakes

Lívio Pires de C. Melo (1); William Verissimo Nakamura (2); Tereza Denyse P. de Araújo (3)

(1) Engenheiro Civil, Universidade Federal do Ceará, Fortaleza - CE, Brasil.
(2) Engenheiro Civil, Universidade Federal do Ceará, Fortaleza - CE, Brasil.
(3) Dra. Profa., Universidade Federal do Ceará, Fortaleza - CE, Brasil.
Email para Correspondência: denyse@ufc.br; (2) Apresentador

Resumo: Edifícios altos vêm sendo construídos em todo mundo devido ao aumento da densidade populacional nas grandes cidades, unido ao grande desenvolvimento tecnológico. Esses edifícios, contudo, são suscetíveis aos efeitos gerados por cargas horizontais provenientes de terremotos ou ventos. No Brasil, as caixas d'água usadas para o abastecimento estão localizadas frequentemente no topo das edificações onde estão situados os amortecedores de líquido sintonizado (ALS). O objetivo deste trabalho é avaliar o comportamento destas caixas d'água como um potencial ALS, considerando-se modelos teóricos encontrados na literatura para estudos de reservatórios e ALS. Esses modelos são o de Housner, para reservatórios elevados, e o de Yu, para ALS. Uma estrutura de quatro pavimentos em concreto armado com caixa d'água é analisada devido a ação sísmica. Três tipos de fatores de amortecimento são considerados nesses modelos: nulo, constante e não linear. As análises são realizadas pelo método dos elementos finitos, utilizando o software SAP2000 v14. Os resultados indicam que a caixa d'água pode funcionar como um amortecedor, diminuindo os deslocamentos da estrutura. No entanto, a redução desses deslocamentos não foi o suficiente para chegar a níveis aceitáveis. Também foi possível constatar que maiores volumes de água na caixa d'água geram maiores fatores de amortecimento.

Palavras chaves: ALS; modelo de Housner; modelo de Yu; análise numérica.

Abstract: Tall buildings have been built around the world due to the increase in population density in large cities, united to the significant technological development. These buildings, however, are more susceptible to the effects generated by horizontal loads caused by earthquakes or winds. In Brazil, the supply water tanks are often located at the top of the buildings where are placed the Tuned Liquid Dampers (TLD). The purpose of this work is to evaluate the behavior of these water tanks as a likely TLD, considering theoretical models found in the literature for the studies reservoirs and TLD. These models are Housner's model for elevated tanks, and Yu's model for TLD. A four-storied structure of reinforced concrete with water tank is analyzed due to seismic action. The analysis is carried out by the finite element method, using the SAP2000 v14 software. These analyses involve three types of damping ratios: null, constant, and nonlinear. The results indicate that the water tank could function as a damper, decreasing the structure displacements. However, the reduction of displacements was not enough to reach acceptable levels. It was also possible to see that larger volumes of water inside the water tank generate greater damping ratios.

Keywords: TLD; Housner's model; Yu's model; numerical analysis.

1   INTRODUCTION

Vibrations caused by horizontal dynamic loads in the building structures can compromise not only the building stability and safety but also the feeling of well-being of its occupants. An alternative to eliminating these effects is increasing the structure stiffness by adding mass or changing the cross-sections of the structural elements. However, these solutions can make the enterprise unfeasible, because it requires more material or more space on the ground for the structure construction what is very expensive.

On the other hand, the introduction of dampers in the structure is an alternative that allows the design of a more flexible structure, but it could also increase the cost of the building. In this way, using existing water tanks in the building as a damping device brings the benefit of designing less robust structures that meet the comfort and safety limits without the additional charge of introducing dampers. From this point of view, it becomes plausible to check whether these reservoirs can function as Tuned Liquid Dampers (TLD).

A TLD is a tank of liquid (in general is water) that reduces the dynamic response through the sloshing energy of the water when the system is excited by a dynamic load. There are many mathematical models in literature to represent the liquid movement into the tank. The most well-known are the Housner's and Yu's models. Housner (1963) developed the first model established a linear equation set to characterize the dynamic behavior of reservoirs when subjected to seismic action. Yu et al. (1999) developed the second model to represent the TLD behavior based on the Tuned Mass Damper (TMD) model. These authors introduce nonlinear parameters of stiffness coefficient (spring) and damping (damper) to model the sloshing phenomenon and the fluid-structure interaction.

In Brazil, it is common to design supply water tanks for domestic use on the top of buildings. These reservoirs placed on the roofing slab supported by columns have a rigid structure, and its material is in reinforced concrete. The water level varies throughout a day but never is empty. It is necessary to have inside the container at least the firefighting water reserve (NBR 5626:1998). In this way, these reservoirs meet some requirements to function as a TLD, which are: rigid walls, full of water and located at the structure top. Some authors (Kareem et al., 1999; Livaoğlu et al., 2011) suggested using the existing water tanks in buildings by merely configuring internal partitions to increase the energy dissipation and damping. Accioly et al. (2017) proposed associating the two models above mentioned to evaluate the potential damping of water tanks. Its results indicated that the water level height in the reservoir altered the dynamic behavior of the structure when subjected to seismic action. Besides, the higher the water level in the container, the displacement of the structure will be smaller.

The purpose of this work is to evaluate the behavior of the water tanks as a potential TLD, considering the theoretical models of Housner (1963) and Yu et al. (1999). In this sense, it realizes the analysis of a four-storied building with a water tank due to a seismic load. The numerical dynamic analyses are executed using the finite element general purpose program SAP2000 v14 (CSI, 2009). The studies consider two different water height and three damping ratios.

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2 FLUID MODELS FOR RESERVOIRS

Reservoirs with fluids began to be used as dampers in the shipbuilding industry and date back to the 1950s. In this case, the goal was to stabilize marine vessels and prevent its rocking and rolling motions (Nanda, 2010). Its use in civil engineering was studied later, and today there are several buildings spread around the world that use these dampers to absorb the vibrations caused by dynamic actions.

It is noteworthy that the water behavior inside the tanks is nonlinear and involving the fluid-structure interaction. However, it is a hard task elaborating on mathematical models that characterize this behavior. Besides, the models presented here are valid only to rectangular reservoirs, because the equations are specific for each tank shape.

2.1 Housner's model

According to Housner (1963), single-mass structures represent the empty reservoirs or filled with water without a free surface. On the other hand, containers with a water-free surface are described by dividing the water mass into two different portions (Figure 1): one that moves together with the tank structure named the impulsive mass (m_0), and another that moves along with the tank wall called the convective mass (m_1). This last one mass behaves as if it were a mass attached to the reservoir walls by a spring that applies variable forces on the tank walls.
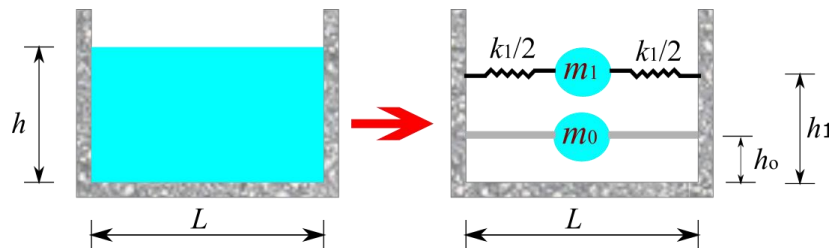

Figure 1. Housner's model for water inside the tank

The equations that characterize both masses and the stiffness coefficient of spring were elaborated by Housner (1963) and reviewed by Epstein (1976 cited by Livaoğlu et al., 2011). For a rectangular tank which ratio between water height (h) and length (L) is less and equal to 1.5 ($h/L \leq 1.5$), the equations are:

$$\frac{m_0}{M} = 0.577 \frac{h}{L} \tanh\left(1.732 \frac{L}{h}\right) \tag{1}$$

$$\frac{m_1}{M} = 0.527 \frac{L}{h} \tanh\left(1.581 \frac{h}{L}\right) \tag{2}$$

$$k_1 = 1.581 \frac{m_1 g}{L} \tanh\left(1.581 \frac{h}{L}\right) \tag{3}$$

In which k_1 is the stiffness coefficient of the convective mass-spring, M is the total mass of water inside the reservoir, and g is the gravitational acceleration. The height of the impulsive mass (h_0) and the convective mass (h_1), and the sloshing frequency (f) are as follows:

$$h_0 = \frac{3}{8}h \tag{4}$$

$$h_1 = \left(1 - \frac{\cosh\left(1.581\frac{h}{L}\right) - 1}{1.581\frac{h}{L}\sinh\left(1.581\frac{h}{L}\right)}\right)h \tag{5}$$

$$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k_1}{m_1}} \tag{6}$$

The elevated shallow reservoir (Figure 2a) in this case is modeled mathematically as a 2-degree of freedom system (Figure 2c) where the tank structure mass (m_s) is added to impulsive mass (m_0). k_s is the stiffness coefficient of the structure.
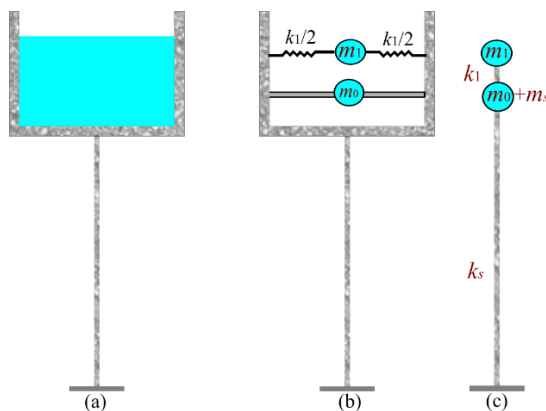

Figure 2. (a) Elevated water tank, (b) Housner's model for water, (c) 2-DOF system

2.2   Yu's model

Yu was one of the pioneers in the studies of TLD addressing the nonlinearities of dynamic water behavior. His work consisted in representing a TLD as a TMD with nonlinear stiffness and damping coefficients, that is, these parameters change with each oscillating cycle of the structure. This representation was possible using an equivalence between the energies dissipated by the two devices (Figure 3).

Yu et al. (1999) realized experiments that allowed concluding that the best parameter to represent the nonlinear behavior of the fluid motion is the dimensionless amplitude for an oscillation cycle. This parameter is the ratio between the displacement amplitude (A) and the tank length (L). Then, the following equations define the stiffness and damping coefficients of the liquid, for the ratio h/L ranging from 0.04 to 0.5 (Malekghasemi et al., 2015):
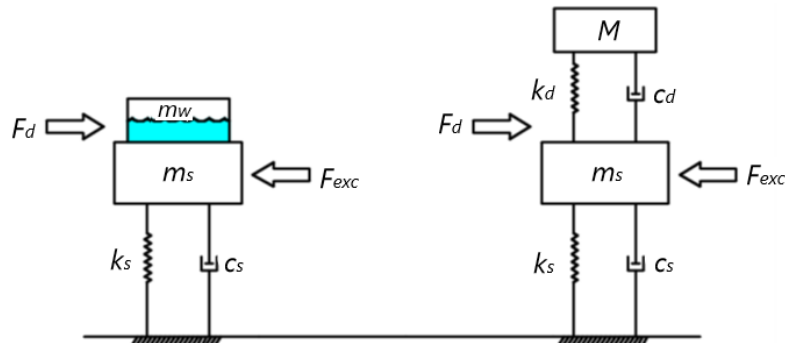
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Figure 3. Representation of a TMD

Font: (Yu et al., 1999)

$$k_d = \begin{cases} 1.075\left(\dfrac{A}{L}\right)^{0.007} M\left(2\pi f\right)^2, \text{ if } \left(\dfrac{A}{L}\right) \le 0.03 \\ 2.52\left(\dfrac{A}{L}\right)^{0.25} M\left(2\pi f\right)^2, \text{ if } \left(\dfrac{A}{L}\right) > 0.03 \end{cases} \tag{7}$$

$$c_d = 1.04\sqrt{Mk_d}\left(\frac{A}{L}\right)^{0.35} \tag{8}$$

Which k_d is the nonlinear damper stiffness coefficient, c_d is the nonlinear damping coefficient, M is the total water mass equal to m_w in Figure 3, and f is the fundamental frequency of the fluid that depends on the surface waves analysis. The fundamental frequency for rectangular tanks is calculated by:

$$f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{\pi g}{L}\tanh\left(\frac{\pi h}{L}\right)} \tag{9}$$

The TLD is most efficient when the fundamental frequency of the fluid is tuned to the fundamental frequency of the structure, causing both water movement and main structure to be in resonance (Malekghasemi et al., 2015). In this way, the energy dissipation increases considerably. Nevertheless, in the water tanks, it is not possible to tune the frequencies because the water amount inside the tank varies throughout a day.

3   CASE STUDY: GEOMETRICAL AND NUMERICAL MODELS

The case study is an office concrete building of four-stories (Figure 4a) with a rectangular shape. The ceiling height is 2.8 m, and the free span between columns is 4.0 m (Figure 4b). All columns have (35x20) cm² cross-sections, while the beams have (15x40) cm². The total area of the building is 1053.36 m² and for each floor is 263.34 m². This building was designed by

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Miranda (2010) to evaluate its seismic vulnerability, and Accioly et al. (2017) analyzed it analytically.
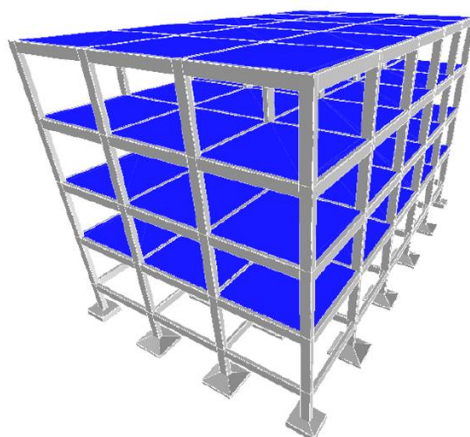
(a)
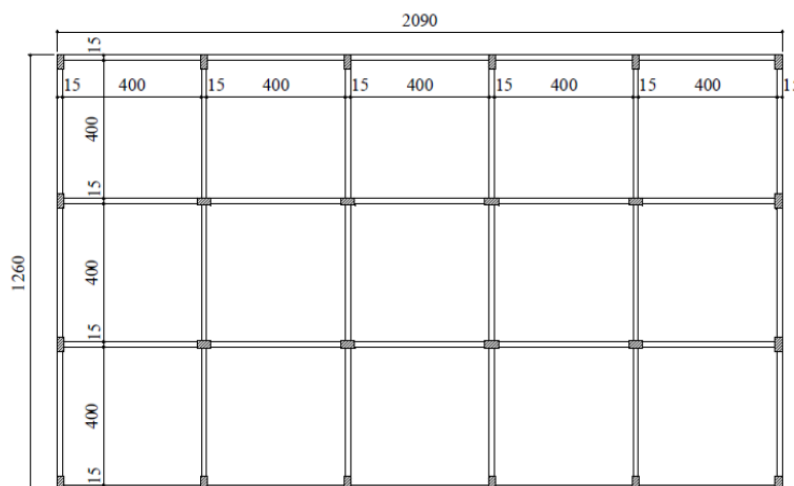
(b)

Figure 4. Case study (a) Four-storey building, and (b) Structural plan for each floor

Font: (Miranda, 2010)

The water tank has dimensions of 2.0 x 4.0 x 2.5 (m³), which corresponds to a water height of 2.34 m. This water tank is positioned above the roofing slab, being supported by four columns with (20x20) cm² cross-section, each one with high of 1.5 m. Table 1 shows the mass and the equivalent stiffness of each floor and tank structure.

Bar finite elements discretize the beams and columns of the structure, according to the building architecture (Figure 4a). The slab mass of each floor is discrete and concentrated in the junction of columns and beams. The slab of each level considers the rigid diaphragm condition. The columns of the first floor are clamped on the base to simulate a rigid soil-structure interaction (Figure 4b).

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Table 1. Mass and equivalent stiffness

Floor	Mass (kg)	Equivalent Stiffness (N/m)
1	749620.00	$1.2915 \cdot 10^8$
2	562806.00	$1.2915 \cdot 10^8$
3	375992.00	$1.2915 \cdot 10^8$
4	189178.00	$1.2915 \cdot 10^8$
Empty tank	9174.31	$4.7407 \cdot 10^7$

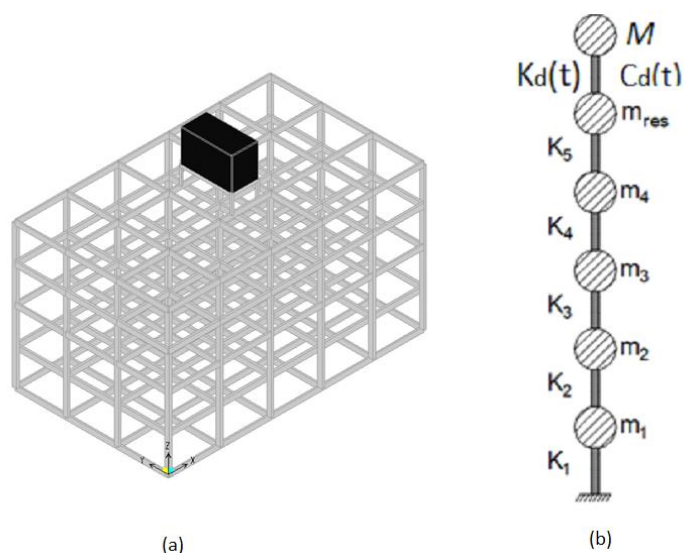(a)   (b)

Figure 5. (a) Finite element model of the building; (b) 6-DOF system

The water mass inside the tank is a punctual mass located as stated by each model. Spring and damper elements are in parallel and attached to the reservoir walls and the concentrated mass (Figure 6). The values of the stiffness and damping coefficients vary according to the models and analysis.
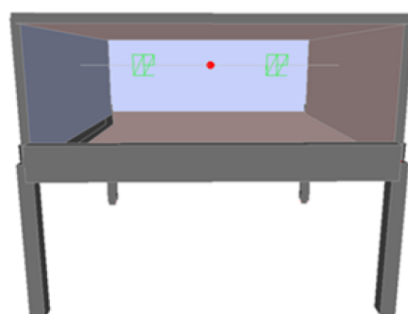

Figure 6. Finite element model of the water mass attached to spring and damper element

4   RESULTS AND DISCUSSION

Firstly, modal analysis is carried out to know the natural frequencies of the building in different situations: without water tank, with an empty water tank, water tank with a firefighting water reserve, and a full water tank. Then a forced vibration analysis of the building is realized applying a resonant earthquake load. The structure is analyzed without a water tank and with an empty container. Besides, two water levels are analyzed: a firefighting water reserve tank and a full tank. Table 2 shows the masses, the stiffness coefficients, and water height for each water model.

Table 2. Parameters for each water model

Water models	Parameters	Firefighting water reserve tank ($h = 1.24$ m)	Full tank ($h = 2.34$ m)
Housner's model	m_1 (kg)	6350.65	8024.94
	h_1 (m)	0.665	1.419
	m_0 (kg)	3522.29	11393.28
	h_0 (m)	0.465	0.877
	k_1 (N/m)	37091.74	59227.55
Yu's model	M (kg)	9920.00	18720.00
	h_M (m)	0.62	1.17

4.1   Modal analysis

In this analysis, it should say that the water masses and the stiffness coefficients used for two conditions of water inside the tank are the linear parameters of the Housner's model (Table 2). The natural frequencies and its correspondent shape mode for each situation of the water tank is shown in Table 3. It is verified that the presence of the empty container in the building reduces its frequencies by about 0.5%. Also, the building with water inside the reservoir has a new shape mode with low frequencies that rises with the increase of the water inside the tank. These frequencies agree to slosh frequency of the water given by Eq. (6) that are 0.3846 Hz and 0.4324 Hz, for each condition of water height respectively. Besides, it can say that these frequencies become smaller with the water reduction in the reservoir, approximately 1%.

4.2   Forced vibration analysis

The seismic action is displacement-type loading that describes the soil motion at the building base in *x*-axes direction (Figure 5a). Mathematically, this motion can be represented by a time-dependent sine function defined by displacement amplitude and excitation frequency (rad/s). For all analysis, its values are 0.01 m and 1.0297 Hz, respectively. This last value is

equal to the fundamental frequency of the structure without a tank (Table 3), characterizing the resonance condition.

Table 3. Natural frequencies of the finite element model

Shape mode	Frequency (Hz)			
	No tank	Empty tank	Firefighting water reserve tank (h = 1.24 m)	Full tank (h = 2.34 m)
Bending in X-axis	-	-	0.3839	0.4313
Bending in X-axis	1.0297	1.0248	1.0235	1.0196
Bending in Y-axis	1.2170	1.2110	1.2043	1.1974
Bending in Y-axis	1.3712	1.3705	1.3703	1.3697
Torsion in Z-axis	2.6250	2.6098	2.6033	2.5861
Bending in X-axis	3.1024	3.0821	3.0450	2.9787
Bending in Y-axis	3.4956	3.4934	3.4922	3.4886
Torsion in Z-axis	3.9060	3.8833	3.8711	3.8303

Displacement history in time is the response analyzed for the fourth floor of the building. The structure without a tank is in resonance as expected (Figure 7a). For the structure with an empty tank, the beat phenomenon occurs (Figure 7b). In this case, the behavior of structure exhibits maximum displacements in the time constant intervals. This behavior is characteristic of the system in which excitation frequency is close to the fundamental frequency of the system without damping. The presence of the reservoir changed the fundamental frequency of the structure ($f = 1.0248$ Hz) while the applied load remained constant, justifying this phenomenon.
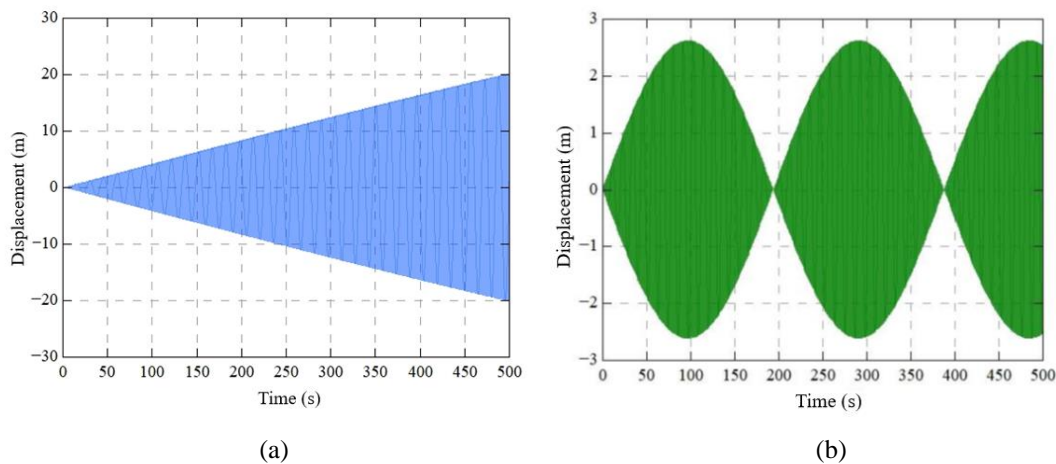

(a)   (b)

Figure 7. Displacement vs. time of the building (a) without water tank, (b) with an empty tank

4.3   Water model evaluations

4.3.1   Yu's model

SAP2000 (CSI, 2009) software only has two manners to enter the damping ratio: a constant value or exponential damping. About the stiffness coefficient of springs, its value is just constant or uses link elements with different properties. However, the use of nonlinear equations (Eq. 7 and Eq. 8) for Yu's model is not possible. So, the application of this model is analytically analyzed. In this case, a mass-spring system represents the building with the water reservoir as a 6-DOF system (Figure 5b), where the total fluid mass contributes to the damping of the structure. The movement equation is nonlinear due to the nonlinear coefficients of damping and stiffness. The Newmark's method solves this equation which algorithm is described by Accioly et al. (2017).

Figure 8 shows the displacement history for the last floor of the building of this model. In this case, the presence of the water reduces the displacements severely. The displacement results are lesser to full tank, but the firefighting water reserve also diminished it in less intensity. In both analyses, the beat phenomenon does not happen.
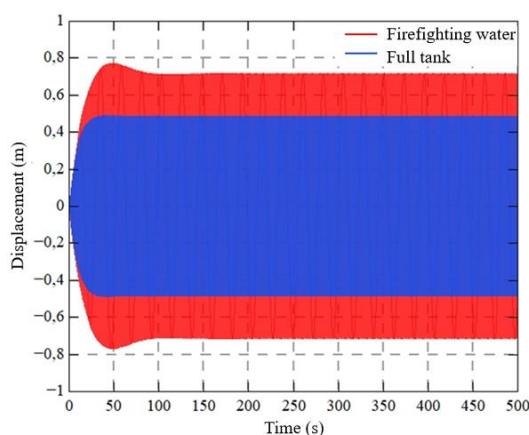

Figure 8. Displacement vs. time of the building – Analytical solution of the Yu's model (f_{exc} = 1.0297 Hz)

Curves of damping ratio (Figure 9a) and stiffness coefficient (Figure 9b) along the time allow evaluating better this model. Both parameters converge to a constant value. However, the higher ratios are to firefighting water reserve for damping ratio and a full tank for stiffness coefficient (Table 4).

Table 4. Damping ratio and the stiffness coefficient

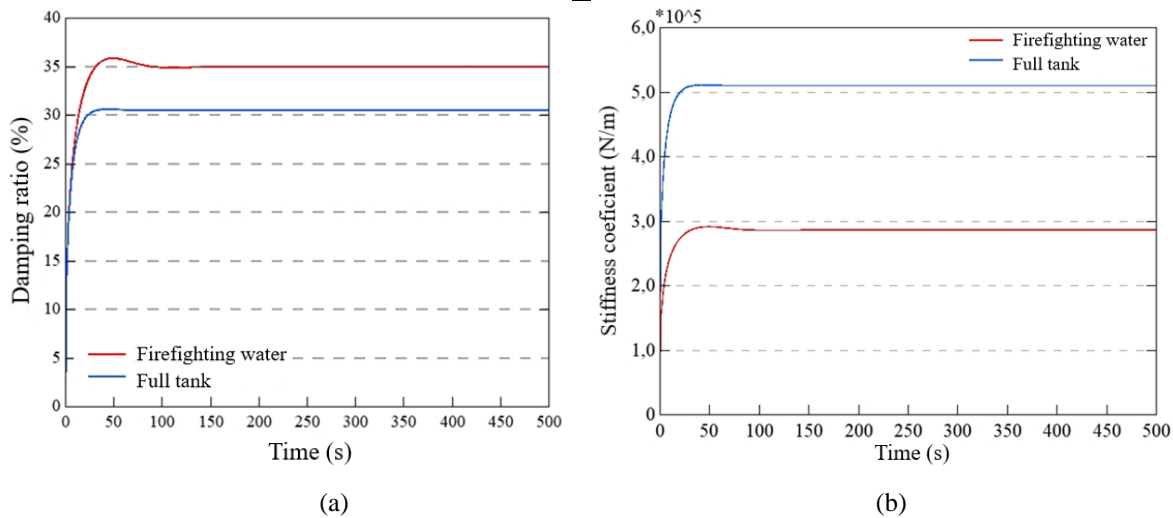| Parameters	Firefighting water reserve tank (h = 1.24 m)	Full tank (h = 2.34 m)
Damping ratio (%)	34.9	30.5
Stiffness coefficient (N/m)	$2.86 \cdot 10^5$	$5.10 \cdot 10^5$

Figure 9. An analytic solution for Yu's model of the building. (a) Damping ratio vs. time, (b) Stiffness coefficient vs. time of the building

The nonlinear damping force $\left(F_d\right)$ depends on the relative velocity among two points. For Yu's model, this relative speed is between the velocity of water mass ($\dot{x}_d$) and the velocity of reservoir mass ($\dot{x}_r$). For the expression available in SAP2000 (CSI, 2009), this relative speed is given by the velocities of nodes at the bar ends ($\dot{x}_1$ and $\dot{x}_2$). These forces are written as follows:

$$F_d\left(t\right) = c_d(t)(\dot{x}_d - \dot{x}_r) \tag{10}$$

$$F_d\left(t\right) = \pm C\left|\dot{x}_1 - \dot{x}_2\right|^E \tag{11}$$

The signal of the damping force (Eq. 11) depends on the relative speed between the bar end nodes. This force function represents an exponential curve where the user defines C and E parameters, where C is the constant damping (Ns/m), and E is the dimensionless exponent. So, it wishes that the damper element of the finite element model produce the same force of the Yu's model (Eq. 10). Besides, the relative velocity is the same for two equations. By trial and error, it determines the C and E parameters that are shown in Table 5.

Table 5. Damping coefficient and dimensionless exponent for Yu's model

Parameters	Firefighting water reserve tank (h = 1.24 m)	Full tank (h = 2.34 m)
C (Ns/m)	25400	30990
E	1.2	1.4

Yu's model is numerically analyzed using the constant values specified in Table 4, and the parameters indicated in Table 5. In this last case, the stiffness coefficient is the same used in

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the first case. The results (Figure 10a and Figure 10b, respectively) show the same behavior found in the analytical solution (Figure 8).
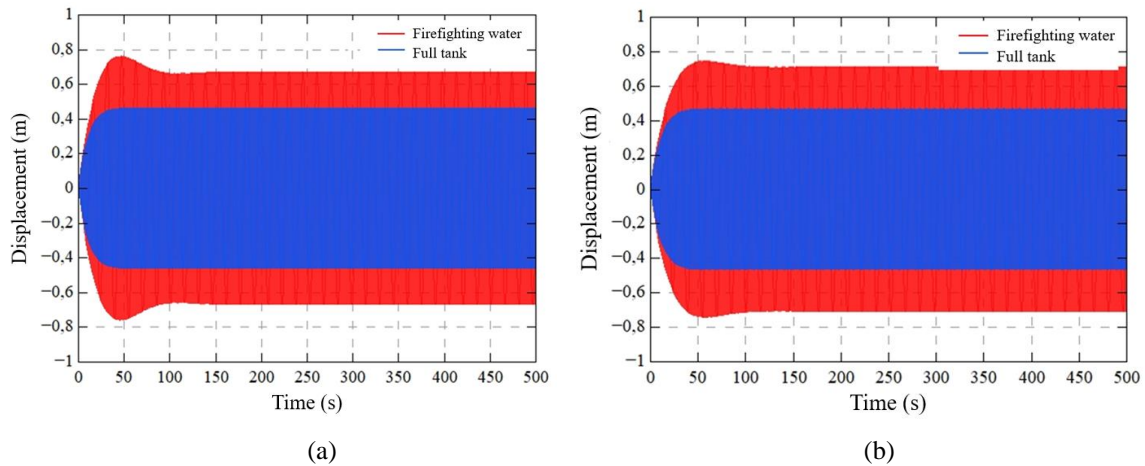

Figure 10. Displacement vs. time of the building of the Yu's model (a) Constant damping, (b) Nonlinear exponential damping

4.3.2 Housner's model

The original model of Housner does not consider the water with the damping effect. Some authors (Hashemi & Barji, 2016) have used this model by adding a damping ratio to represent the viscous behavior of the water. This work evaluates this model with three different damping ratio types: null, constant, and nonlinear exponential. Thus, the impulsive mass is added to the mass of the reservoir structure, while a spring of constant stiffness coefficient (Eq. 3) links the convective mass to the tank walls.

Figure 11 shows the displacement history for the last floor of the building with a damping ratio equal null value. In this case, the water presence reduces the displacements even without considering its damper effect. The structure displacements are smaller whether there is more water in the reservoir. Also, the occurrence of beating is observed again, which may be justified by the proximity between both frequencies, of the excitation and the structure ($f = 1.0235$ Hz for firefighting water reserve tank; $f = 1.0196$ Hz for full water tank).
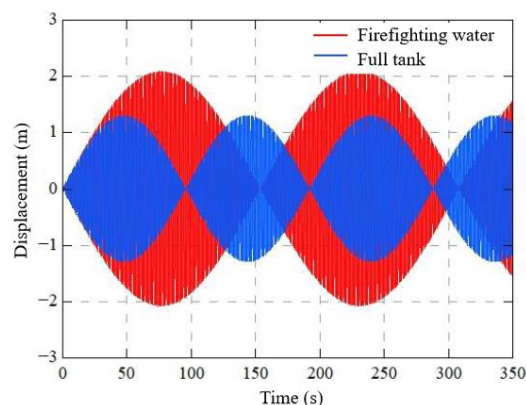

Figure 11. Housner's model without damping ($f_{exc} = 1.0297$ Hz)

The water model proposed by Accioly et al. (2017) is also analyzed here, i.e., the damping coefficient is nonlinear (Eq. 8). So, it is necessary to solve this case analytically using the same procedure previously described for Yu's model. In this case, the reservoir mass increases by the addition of the impulsive mass, and the convective mass substitutes the total mass for the 6-DOF system (Figure 5b).

Figure 12 shows the displacement history for the last floor of the building for this analysis. The presence of the nonlinear damping causes a stabilizing effect on the structure that exhibits smaller displacements when the tank is full of water. The firefighting water reserve tank also reduces the displacements but more slowly. Besides, the beat phenomenon no more happens.
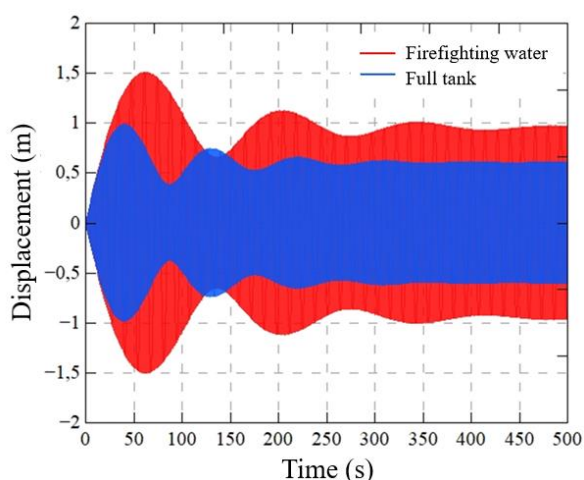

Figure 12. Displacement vs. time – Analytical solution for Housner's model with Yu's damping

Figure 13 shows the behavior of the damping ratio obtained by the analytical solution. This ratio converges to a constant value for both tank types. These values are 38.9% for firefighting water reserve tank, and 33.1% for a full tank. It is applied the same procedure described for the Yu's model to obtain the parameters of Eq. (11). Thus, two numerical analyses are realized. The first one uses the constant damping (Figure 13), and the second one uses the parameters shown in Table 6. The results show (Figure 14a and Figure 14b, respectively) the same behavior found in the analytical solution (Figure 12).
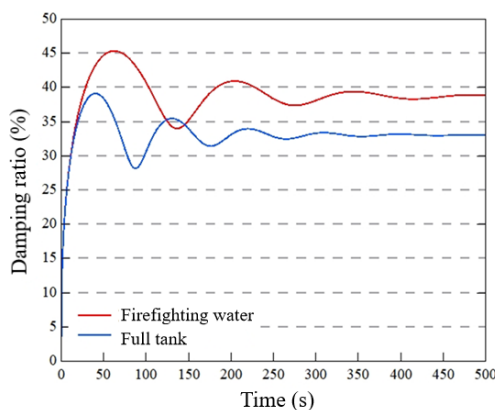

Figure 13 – Damping ratio vs. time - Analytical solution for Housner's model with Yu's damping

Table 6. Damping coefficient and dimensionless exponent for Housner's model

Parameters	Firefighting water reserve tank (h = 1.24 m)	Full tank (h = 2.34 m)
C (Ns/m)	7000	11000
E	1.3	1.2

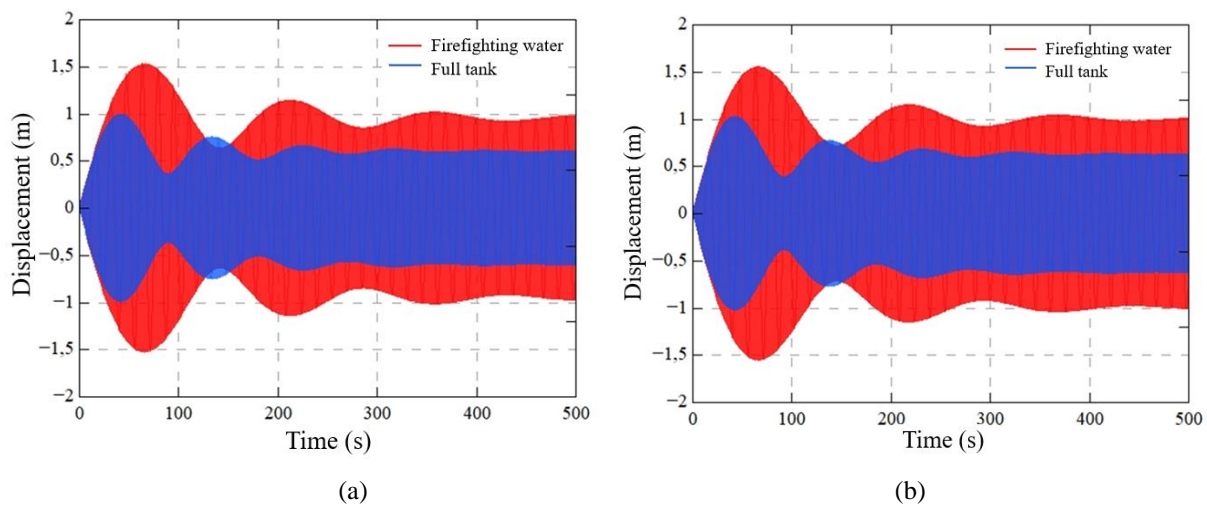(a)   (b)

Figure 14 - Displacement vs. time of the building for Housner's model (a) Constant damping, (b) Nonlinear exponential damping

5   CONCLUSIONS

The primary focus of this work is to evaluate the behavior of the water tanks as a potential TLD, considering theoretical models of Housner (1963) and Yu et al. (1999). By the results found, it can say that a higher water amount inside the tank provokes smaller displacements in the structure eliminating the resonance condition and the beat phenomenon, independent of the theoretical model used. However, the Yu's model reduces more rapidly the displacements comparing with the Housner's model. Maybe because this model considers all water mass as a damper, and the fluid-structure interaction is nonlinear. The consideration of constant or nonlinear damping ratio does not change the final result because the curves founded are similar. The same affirmative can say for stiffness coefficient. So, the water tank acts as a damper, but the reservoir as designed in this work was not enough to reduce the displacements of the structure to acceptable levels.

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