# MÉTODO DOS ELEMENTOS DE CONTORNO APLICADO À IDENTIFICAÇÃO DE PARÂMETROS DE INTERFACE

## Boundary Element Method applied to joint parameter identification

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**Resumo:** As juntas são essenciais para o comportamento global de sistemas estruturais, pois são responsáveis por comunicar as tensões entre os seus diversos componentes. Em serviço, essas junções estão sujeitas a efeitos não-lineares como fissuras, danos, perdas de aderência entre outros. A modelagem precisa desses fenômenos ainda é uma tarefa desafiadora nos dias atuais. Para contornar esse problema, as junções são frequentemente consideradas como perfeitas, ou seja, com plena capacidade de transmitir esforços. Tal suposição pode ser conservadora em muitos casos, e uma abordagem mais realista seria considerar essas juntas como entidades elásticas caracterizadas por um conjunto de parâmetros. A tarefa complexa é determinar qual o conjunto de parâmetros mais adequado, mesmo sob condições de carga estática. O presente estudo propõe um procedimento inovador de identificação numérica baseado no Método dos Elementos de Contorno (MEC). O conjunto de parâmetros é identificado através de uma forma inversa por meio da teoria do Erro em Relação Constitutiva (ERC). A formulação é aplicada a problemas de elasticidade plana, por meio de ensaios numéricos comparando com respostas de referências ora analíticas, ora numéricas, utilizando carregamentos exclusivamente estáticos. A investigação numérica particular mostra características interessantes, sobretudo o comportamento quadrático nas vizinhanças do ponto de otimalidade, mantendo a estabilidade numérica mesmo sob fortes discrepâncias nos parâmetros esperados. Os parâmetros da interface são corretamente identificados em todos os casos numéricos testados.

***Palavras chaves: MEC; ERC; juntas; identificação de parâmetros.***

**Abstract:**

Joints are essential for the global behavior of the structural system because they are responsible for communicate stresses among their numerous components. In service, these joints are subjected to nonlinear affects such as cracks, damages, adherence losses among others. Modeling accurately these phenomena is still a challenging task in the present days. To circumvent this problem, the joints are often admitted perfect, which is complete capacity of transmitting efforts. Such assumption is conservative in many cases, and a more realistic approach is considering the joints as elastic entities characterized by a set of parameters. The complex task is to determine which set of parameters is most appropriate, even under static loading conditions. The present study proposes an innovative numerical identification procedure based on the Boundary Element Method (BEM). The set of parameters is identified through inverse formulation according to the Constitutive Relation Error (CRE) theory. The formulation is applied to plane elasticity problems, where the numerical tests are performed comparing to reference responses, either analytical or numerical. The particular numerical investigation shows interesting features, especially the quadratic behavior near the point of convergence, maintaining the numerical stability even under strong discrepancies in the expected parameters. The interface parameters are correctly identified in all tested cases.

***Keywords: BEM; CRE; Joints; Parameter identification.***

# 1 INTRODUCTION

The increasing complexities of structural designs make the task of accurate predicting more and more challenging, even under static load conditions. The understanding of how the interconnection between elements affect the global response is an important and active research field (Mehrpouya, Graham, & Park, 2013). In this study, a new numerical technique based on the Constitutive Error Theory coupled with the Boundary Element Method is presented for identifying interface parameters.

The engineering community has long been concerned with quality of model predictions. The idea that has been receiving attention is improving the numerical predictions by computational model updating based on experimental measures. In this case, the model has finite dimension and it is assumed to be well known. The governing parameters are assumed to be doubtful and susceptible to be identifiable according to some criterion. The Model Updating has been developed since the middle of World War II in the airplane industry (Natke, 1988). The developments are mainly driven by domain methods such as Finite Element Method (FEM) (Ben Azzouna, Feissel, & Villon, 2015).

The first appearance of the direct BEM used for Model Updating is related to beam vibration using measured Frequency Response Function (FFR), instead of modal input data (Dos Santos, Campos, & Neto, 2000). This study is reedited after. The benchmark utilized is the Friswell and Mottershead's beam where the estimated parameters are the flexural rigidity, the translational stiffness and rotational stiffness of a flexible joint (Friswell & Mottershead, 1995). It is interesting to remark that the dynamical behavior of frame structures can be accurately modeled by BEM (de Mesquita Neto, Barretto, & Pavanello, 2000). No more progress in this specific field has been communicated since then.

Other fields of engineering have investigated the capability of the BEM identification. The advantages of BEM in representing the accurate solutions in the domain has led to the development of numerical techniques for modeling site effects (Álvarez-Rubio, Benito, Sánchez-Sesma, & Alarcón, 2005). The idea is using two-dimensional analysis for identifying relevant parameters for characterize the seismic response of the site depending on the estimated subsoil structure. It is a practical application of "best estimates" for practical engineering challenges.

Besides reliability, the existence and uniqueness of the model parameters is a constant concern among the engineers, in particular for joints. In general it is possible to relate the identifiability of the problem to its parameter sensitivity (Beck & Arnold, 1977). This is a strong motivation for adopting gradient approaches taking advantages of continuous dependencies among the design parameters, what is a reasonable assumption for practical purposes.

It is frequent the observation of random responses coming out from experimental tests. This is due to the inherit variabilities concerning the material properties and load conditions, besides the lack of knowledge. Recent investigations have applied the BEM with stochastic optimization for better identifying the parameters of cohesive laws (Cordeiro, Leonel, & Beaurepaire, 2017).

Concerning identification methods, the Constitutive Relation Error (CRE) approach is an important numerical tool. At its birth, it was proposed for quantifying the divergencies between numerical approximations and the exact solutions (Ladeveze & Leguillon, 1983). Posteriorly, it evolved for quantification of the stiffness and the mass matrices of linear dynamical structural systems basing on modal tests (Ladeveze, Nedjar, & Reynier, 1994). The CRE problems are generally formulated as inverse forms, which also needs to include all available information. In this sense the augmented version (Modified Constitutive Relation Error - MCRE), an additional term acts as regularization parameter. It broadens the formulation in order to include the evidence measures, instead of making additional assumptions (Guchhait & Banerjee, 2016). The CRE technique has permitted the proposition of improved predictive algorithms for pressure levels decreasing the needs for prototyping which represent expressive saving costs (Decouvreur, Bouillard, Deraemaeker, & Ladevèze, 2004).

It is recognized that structural interconnections play a significant role in the behavior of complex structures because they govern the energy flow and concentrate dissipative phenomena (Arruda & Santos, 1992). In medium-frequency range of vibration, the traditional domain methods (such as FEM) become very expensive due to the pollution errors. These numerical errors can be circumvented at the expense of refining the mesh as the frequency increases. For practical structures this option is often not possible. In addition, a major problem is the joint parameter identification including damping effects. In such a case, integral methods reveals to be an efficient alternative, such as the Variational Theory of Complex Rays which is a Trefftz approach (O. Dorival, Rouch, & Allix, 2008; Olivier Dorival, Rouch, & Allix, 2006) because the test functions utilized satisfy the governing equation interior to the domain as well as the constitutive relations. The boundary conditions are satisfied in a variational sense. In the Trefftz Element Method (TEM) the unknowns are the amplitudes of the basis functions and the boundary does not need to be discretized.

These studies let clear the availability of using integral formulations for parameter identification purposes. In particular, the present study presents a specific BEM procedure that takes advantage of the fact that all information is available only at the exterior boundary. In addition, the proposed approach is the seed for many brunches of investigation such as structural health monitoring.

## 2 FORWARD FORMULATION

The media of interest for the present study occupies a domain $\Omega \subset \mathbb{R}^2$ with boundary denoted by $\partial\Omega$. The strain tensor is locally defined as the symmetric part of the gradient displacement because only small perturbations are admissible. In this case, the initial configuration remains unchanged along time. The scenario is presented in Figure 1. In any instant of time, $t$, the set of external actions can be outlined in the synthetic way:

- A surface displacement field $\underline{u_i}$ on a portion $\Gamma_u$,
- A surface force density field $\underline{t_i}$ applied on $\Gamma_t$,
- A volume force density $b_i$ on the domain $\Omega$.

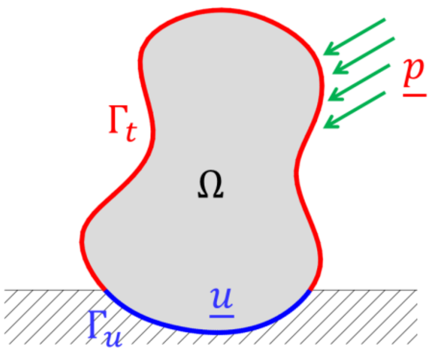


**Figure 1 Solid configuration**

The classical elasticity problem on its differential form can then be formulated as following:

Problem 1.

Find the pair of displacement fields, $u_i$, and stress field, $\sigma_{ik}$, such that:

The displacement field must be kinematically admissible;

$$U^{ad} = \left\{\forall t \in [0,T], u_i|_{\Gamma_u} = \underline{u_i}\,, \forall y \in \partial\Omega : u_i|_{t_0} = u_{i_0}\,, \dot{u}_i|_{t_0} = \dot{u}_{i_0}\right\} \tag{1}$$

The stress tensor field must be statically admissible;

$$S^{ad} = \left\{\forall t \in [0,T], \forall y \in \Omega : \sigma_{ik,k} + b_i = 0, \forall y \in \partial\omega : \sigma_{ik} n_k = \underline{t_i}\right\} \tag{2}$$

The constitutive relation must hold along the whole domain.

$$\sigma_{jk}|_t = A\left[\varepsilon_{jk}(\dot{u}_i|_\tau), 0 \le \tau \le t\right], \forall y \in \partial\Omega, \forall t \in [0,T] \tag{3}$$

The strong form of equation (2) is difficult to solve for general boundary conditions even for linear materials. However, the Betti's theorem is a classic approach for joining the constitutive law to the admissible sets $U^{ad}$ and $S^{ad}$. This is the essence of the classical BEM computational approach (Aliabadi & Wen, 2010). The mechanical response of a linear elastic solid can be formulated as:

Problem 2.

Find the pair of boundary fields $(u_i|_\Gamma, t_i|_\Gamma) \in V^{ad}$such that:

The boundary field must respect the boundary conditions;

$$V^{ad} = \left\{\forall t \in [0,T], u_i|_{\Gamma_u} = \underline{u_i}\,, \forall y \in \partial\Omega : u_i|_{t_0} = u_{i_0}\,, \dot{u}_i|_{t_0} = \dot{u}_{i_0}, \sigma_{ik} n_k = \underline{t_i}\right\} \tag{4}$$

The displacement field, $u_j$,must respect the boundary integral equation;

$$u_i(y) = \int_\Gamma \left[u_{ij}^*(x,y)t_j(x) - p_{ij}^*(x,y)u_j(x)\right]d\Gamma + \int_\Omega u_{ij}^*(x,y)b_j(x)d\Omega \qquad (5)$$

In this integral form of the elasticity form, $u_{ij}^*$ and $p_{ij}^*$ are known as fundamental solutions. For 2D domains these quantities are expressed as follows:

$$u_{ij}^*(q,y) = \frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)ln\left(\frac{1}{r}\right)\delta_{ij} + r_{,i}r_{,j}\right] \qquad (6)$$

$$p_{ij}^*(q,y) = -\frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right] + (1-2\nu)(n_i r_{,j} - n_j r_{,i})\right. \qquad (7)$$

## 3 INCLUDING JOINTS

Consider the elastic solid under consideration has a joint $\Gamma_j$ (Figure 2) dividing it in two separated parts.

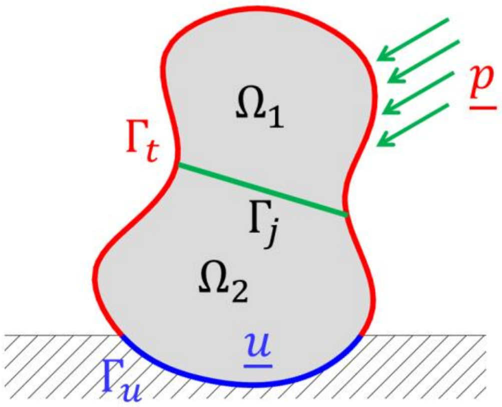


**Figure 2 Solid with a joint**

To maintain the equilibrium, the joint needs to respect the continuity conditions, which now include new constraints to the precedent problem. The new version of the problem is then:

Problem 3.

Find the pair of boundary fields $(u_i|_\Gamma, t_i|_\Gamma) \in V^{ad}$such that:

The boundary field must respect the boundary conditions;

$$V^{ad} = \left\{\forall t \in [0,T], u_i|_{\Gamma_u} = \underline{u_i}\,, \forall y \in \partial\Omega : u_i|_{t_0} = u_{i_0}\,, \dot{u}_i|_{t_0} = \dot{u}_{i_0}, \sigma_{ik}n_k = \underline{t_i}\right\} \qquad (8)$$

The displacement field, $u_j$,must respect the boundary integral equation;

$$u_i(y) = \int_\Gamma [u_{ij}^*(x,y)t_j(x) - p_{ij}^*(x,y)u_j(x)]d\Gamma + \int_\Omega u_{ij}^*(x,y)b_j(x)d\Omega \tag{9}$$

The equilibrium at the interface must be respected

$$u_i|_{\partial\Omega_1} - u_i|_{\partial\Omega_2} = 0 \tag{10}$$

$$p_i|_{\partial\Omega_1} + p_i|_{\partial\Omega_2} = 0 \tag{11}$$

This is the new version of the forward problem to be solved. Note that the equilibrium at interface is imposed on a strong form. This is an advantage of BEM compared to other numerical approaches that try to impose these conditions on a variational sense.

## 4 CONSTITUTIVE RELATION ERROR FORM

The key aspect of the present formulation is modifying the way of transmitting displacements through the interface (equation 10). The imposition of equilibrium will be assured via the solution of an inverse problem. It is well known that inverse problems belong to the class of ill-posed problems in a Hadamard sense. Violation of Hadamard's conditions often can be fixed by reformulating the problem in conjunction to adding more available information (Hansen, 2010).

The inverse problem is written following the formalism of the CRE theory (Ladevèze & Pelle, 2003). This numerical approach was initially proposed for estimating the error concerning the numerical approximations. Lately, the procedure was extended to identification of parameters.

The core aspect of the CRE formulation is dividing the governing equations in two distinct categories. Category 1: it is named *Reliable equations* and include all equations that must strictly be respected in order to assure the fundamental Mechanics basis. Category 2: it is named *Unreliable equations*, which encompasses all expressions that has any doubtful aspect. For example, in *Problem 1*, among the set of governing equations, equation (3) is the most doubtful. The reason is because it depends on parameters that are often not fully known. However, this choice is particular to each problem the designer has on hands. To guarantee the convergence properties, in the present study the following choice is made:

=Reliable fields=

Let $U^{ad}$ be the displacement functional space preserving the minimum requirements of regularity such that:

$$U^{ad} = \left\{\forall y \in \partial\Omega, \forall t \in [0,T]: u_i|_{\Gamma_u} = \underline{u_i}, \dot{u}_i|_{t_0} = \dot{u}_{i0}\right\}$$

Let $S^{ad}$ be the stress functional space preserving the quasi-static admissible conditions:

$$S^{ad} = \left\{ \begin{array}{c} \forall t \in [0,T], \forall y \in \partial\Omega : \\ v_i(y) = \int_{\Gamma} \left[u_{ij}^*(x,y)s_j(x) - p_{ij}^*(x,y)v_j(x)\right]d\Gamma + \int_{\Omega} u_{ij}^*(x,y)b_j(x)d\Omega, \\ \forall y \in \partial\Omega : \sigma_{ik}n_k|_{\Gamma} = \underline{t_i} \end{array} \right\}$$

=Unreliable fields=

The force among the structural faces $\Gamma_a$ and $\Gamma_b$ (each side of the interface $\Gamma_j$) is calculated by the following expression:

$$p_i|_{\Gamma_a} = -p_j|_{\Gamma_b} = R_{ij}(u_j|_{\Gamma_a} - u_j|_{\Gamma_b})$$

The matrix operator $R_{ij}$ is a Hookean and it is dependent on a set of independent parameters $\{p\}$. The interior constitutive relation is considered to be reliable and it is already included in the admissible space $S^{ad}$.

Note that a kinematical and statically admissible fields are completely independent between each other. For this reason, it is necessary to have some measure for quantifying the quality of a given vector pair $(u,\sigma) \in U^{ad} \times S^{ad}$. This measure is the CRE norm having the following properties:

$$\begin{gathered} J: U^{ad} \times S^{ad} \mapsto \mathbb{R} \\ J \geq 0 \\ J(u,\sigma) = 0 \Leftrightarrow (u,\sigma)\ is\ the\ exact\ solution\ of\ the\ problem \end{gathered} \tag{12}$$

The BEM problems are defined in terms of boundary fields. In this case the CRE norm will be defined such as the interface behaves independently from the remaining solid. For practical purposes, one can specify other material models than Hookean without changing the core formulation. Let the functional norm be defined as:

$$\begin{aligned} J(u,t) = \frac{1}{2}\int_{\Gamma_j} & \left(t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b})\right) \\ & \cdot [R_{ik}]^{-1}\left(t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b})\right)d\Gamma \end{aligned} \tag{13}$$

Since this norm assumes zero only when the set of admissible displacement field and traction field are the actual solution of the problem, one can define:

Problem 4.

Find the pair of boundary fields $(u_i|_{\Gamma}, t_i|_{\Gamma}) \in U^{ad} \times S^{ad}$such that:

$$(u_i|_{\Gamma}, t_i|_{\Gamma}) = \operatorname{argmin} J(u,t) \tag{14}$$

Suppose that there are some real measures over some part of the boundary field. Let $\breve{u}$ represent these measures. It is possible to modify the performance measure in order to find a compromise between BEM model and the experimental tests. The modified function is defined as follows:

$$J^*(u,t) = \frac{1}{2}\int_{\Gamma_j} (t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b}))R_{ik}^{-1}(t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b}))d\Gamma_{ij} + \frac{r}{1-r}\frac{1}{N_K}\|u_k - \breve{u}_k\| \tag{15}$$

In equation (15) $r$ is real parameter, $r \in [0,1)$, that controls the degree of assurance conferred to the measures. $N$ is a parameter that assures the compatibility between both terms for preventing numerical instabilities.

Note that the admissible solution in *Problem 4* is dependent on the set of parameters defining the interface. Let $P$ be the set of admissible parameters, thus, the searched parameters are found through a min-min approach:

Problem 5.

Find the set of parameters {p} such that:

$$\{p\} = \underset{p \in P}{\operatorname{argmin}} \left[ \min_{(u_i|_\Gamma, t_i|_\Gamma) \in U^{ad} \times S^{ad}} J^*(u,t) \right] \tag{16}$$

## 5 APPLICATION

Consider the case illustrated in Figure 3. The structure consists of two plates under tensile static loading. It is assumed plane stress conditions. The material parameters are $E_1 = 100\ N/mm^2$, $E_2 = 100\ N/mm^2$, $\upsilon_1 = 0$, $\upsilon_2 = 0$. The geometric characteristics are $l_1 = 100\ mm$, $l_2 = 100\ mm$, $h = 50\ mm$. The idea is identifying the joint parameter represented by a spring along the axis direction. The left edge is constrained along horizontal and vertical directions. The surface load is constant $P = 10\ N/mm^2$.

The displacement boundary field is analytically given by:

$$u(x) = \begin{cases} \dfrac{Px}{E_1} & ;\ 0 < x < l_1 \\ \dfrac{P(l_1 + x)}{E_1} + \dfrac{P}{k} & ;0 < x < l_2 \end{cases} \tag{17}$$

Note that the displacement at the charged edge is inversely proportional to the joint parameter $k$, for a fixed charge and location. In this case, the perfect joint is achieved doing the values of $k$ tend to infinity. The expression (17) is used for generating the reference displacement measures along the boundary. The adopted joint parameter is $k = 2$.

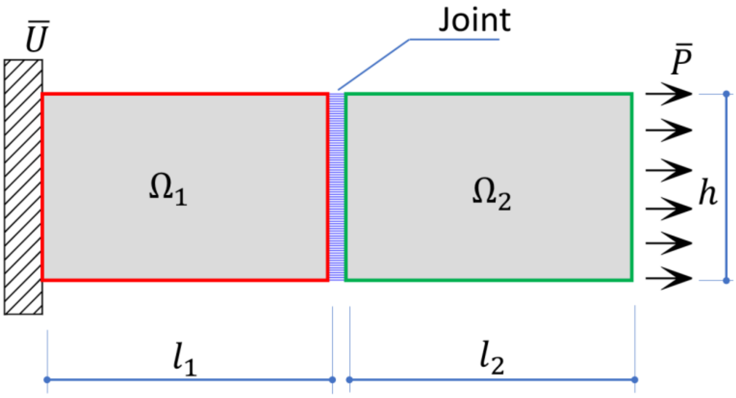


**Figure 3 Two-plate structure with a center joint**

The boundary mesh has 8 quadratic elements because the boundary fields behave linearly in this case. See Figure 4 for more details. For cases where no priori information is available, it is recommended the mesh converge test for finding the appropriate number of degrees of freedom to be utilized.

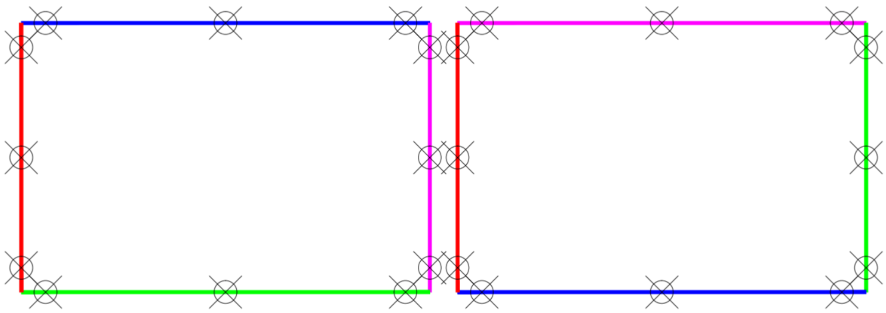

**Figure 4 Mesh adopted**

This simple example permits to draw some interesting observations. The linear system derived from the equilibrium equations is non-symmetric. In Figure 5, it is showed the matrix plot of the present example. Each pixel indicates a specific position in the matrix. If the pixel is white, it means that the position contains a zero value. Colored pixels indicate values different from zero according to an arbitrary scale. One can see that the number of degrees of freedom is increased. The classical direct BEM system would have roughly a matrix (48,48) to treat. In the proposed approach, this system increases to (180,180), which represent an important decrease in the computational efficiency specially for three dimensional cases. However, one can see that the majority of these positions do not need to be created because they are occupied by zero. It is possible to use the storage techniques similar to those utilized for FEM approaches for gaining better performance. It is also possible to observe the placement of matrices $H_i$, $i = 1{,}2$, in color blue, and the corresponding matrices $G_i$, $i = 1{,}2$ in color yellow. The joint matrix is distributed according to the pertinent degrees of freedom in the top left region. In Figure 6, one depicts the zoom into the corresponding joint region of the global matrix from Figure 5.

The evolution of the values according to the search interval is presented in Figure 7. Each point of this curve corresponds to a solution of the posed *Problem 4*. It is possible to see that the point of minimum of this curve corresponds exactly to the reference joint parameter value stipulated, $k = 2$. At this point the functional value tends to zero as stated in the properties of the CRE theory. The quadratic behavior of this functional is evident and it helps to keep stability along the optimization process. It is now clear the chosen min-min approach according to *Problem 5*.

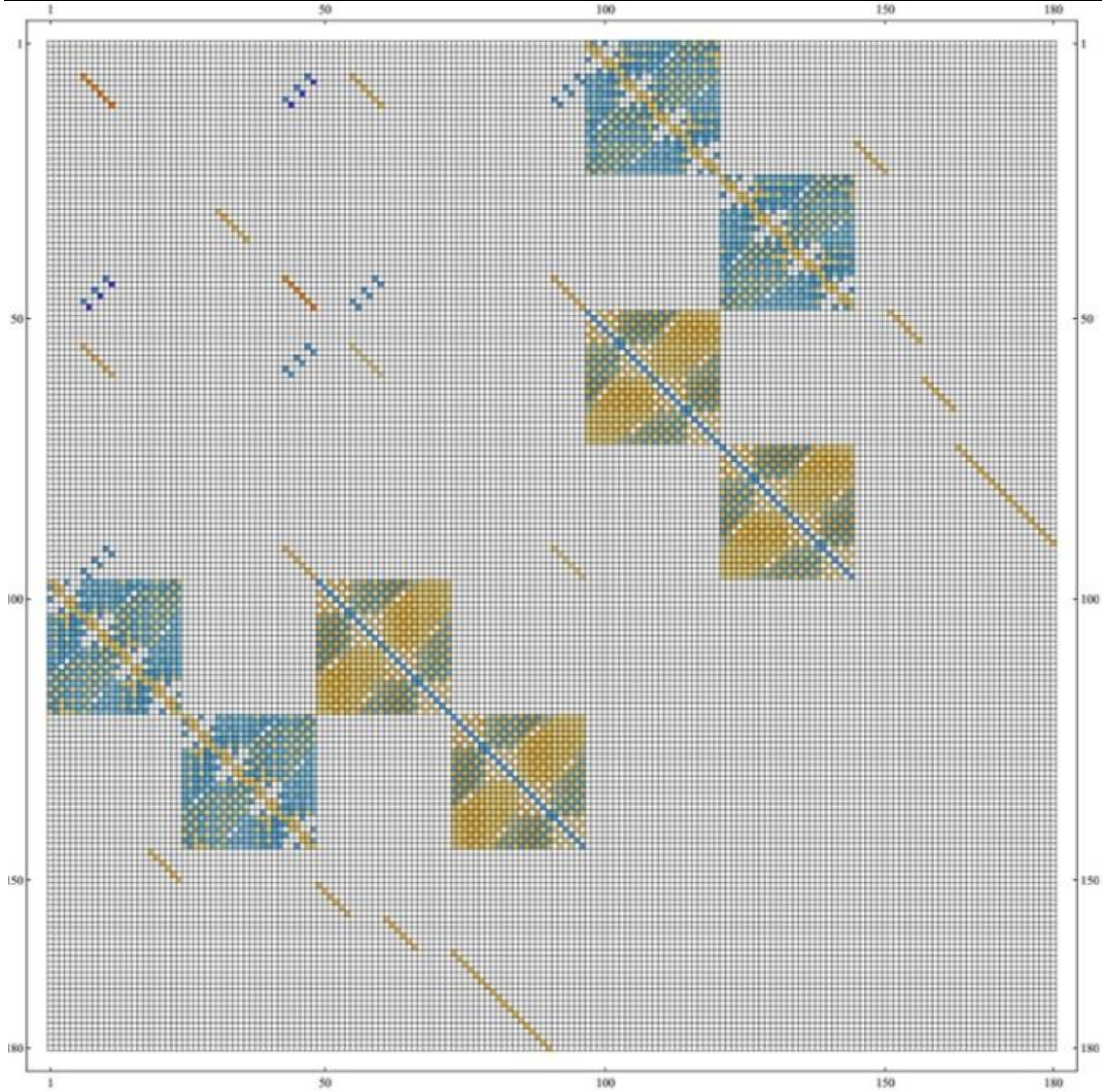


**Figure 5 Matrix plot of linear system. White positions mean zero value. Colored positions mean values different of zero**

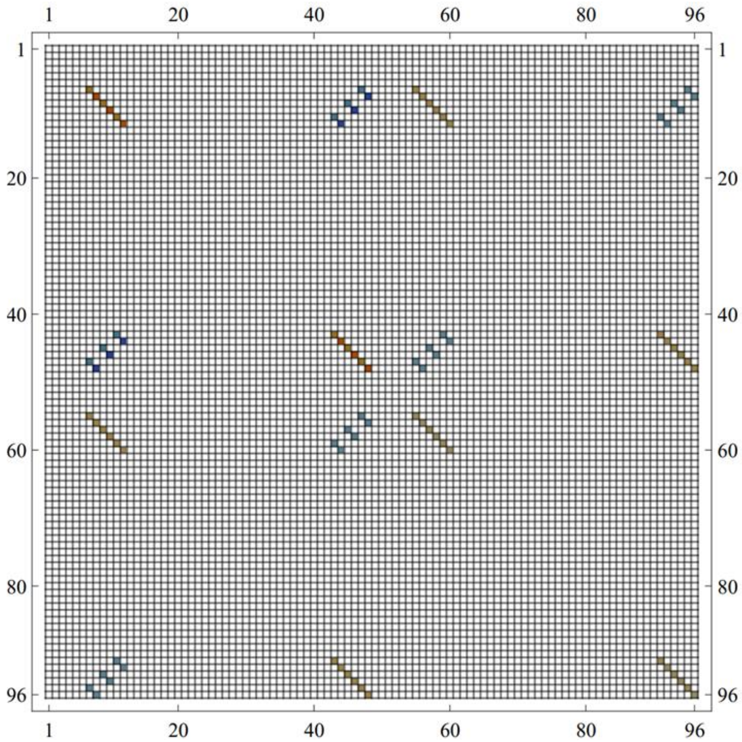


**Figure 6 Matrix plot of the joint matrix**

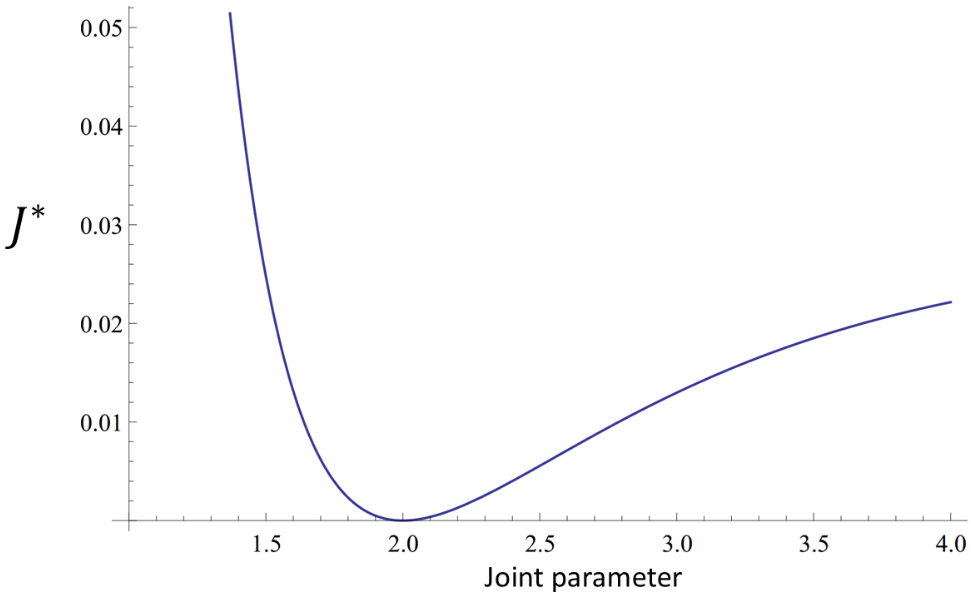


**Figure 7 Convergence evolution along the parameter search**

## 6 CONCLUDING REMARKS

The present study proposed a BEM formulation for identification of joint parameters of structures under static loading conditions. The joints are admitted to behave linearly, depending on a limited set of intrinsic parameters. The proposed approach permits to determine the value of these parameters based on the structural static response and the inverse analysis. The example explored in this study showed the accuracy in the prediction of the referred parameters. The stability properties are also an important remark since the inverse problems are known for being of difficult solution, especially considering general geometries. In addition, it demonstrates numerically the viability of this numerical procedure for parameter identification.

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