MACROECONOMICS AND THE STOCK MARKET IN BRAZIL: NEW EVIDENCE FROM DYNAMIC BAYESIAN NETWORKS

Leandro Coghi Bernardelli[1] Carlos Enrique Carrasco-Gutierrez[2] Thiago Christiano Silva[3]

Abstract: This study examines the dynamic interdependencies between year-end and three-year-ahead macroeconomic expectations and the Brazilian stock market, treating financial markets as potential "barometers" of macroeconomic conditions. Using Dynamic Bayesian Networks (DBNs), we identify and characterize the evolving associations between expectations and market performance from 01/02/2007 to 12/06/2023, segmenting the sample into crisis and non-crisis regimes. The main findings indicate that macroeconomic expectations and stock indices exhibit weak or nonexistent directional dependence, both over the full sample and within specific regimes, suggesting that stock market dynamics are primarily driven by firm-level economic–financial fundamentals and sectoral heterogeneity rather than by shifts in expectations. The relatively small size and limited representativeness of the Brazilian stock market compared with those of advanced economies further strengthen this pattern of independence. Additionally, clusters composed exclusively of expectation variables show that, in the short run, economic activity (GDP) and the external sector (Current Account, CA), and in the long run, inflation expectations (CPI) and external accounts (CA) emerge as central nodes influencing the structure of macroeconomic expectations.

Keywords: Dynamic Interdependence, Macroeconomic Expectation Variables, Stock Market, Dynamic Bayesian Networks.
JEL classification: C11; C45; E44; G1.

1. INTRODUCTION

The interaction dynamics between the financial market and the real economy have long attracted the attention of academics, policymakers, and the general public. Since the publication of Schumpeter´s (1949) pioneering work, which advocated the pivotal role of financial markets in fostering productivity and economic growth, a wide range of studies has emerged seeking to understand the true nature of the relationship between financial markets and real economic activity.

[1] Graduate Program in Economics at the Catholic University of Brasília (PPGE/UCB), Brasília and Caixa Econômica Federal, Brasília, DF, Brazil. E-mail address: bernardellileandro@gmail.com.
[2] Graduate Program in Economics at the Catholic University of Brasília (PPGE/UCB) and Graduate Program in Public Policy at the Catholic University of Brasília (MPPP/UCB), Brasília, DF, Brazil. E-mail address: carlos.carrasco.gutierrez@gmail.com.
[3] Graduate Program in Economics at the Catholic University of Brasília (PPGE/UCB) and Brazilian Central Bank, Brasília, DF, Brazil. E-mail address: thiagochris@gmail.com.

The financial market, which encompasses both capital and banking markets, plays a crucial role in mobilizing resources from savers and channeling them to firms seeking to finance expansion projects, thereby converting them into useful and productive capital (Jin & Guo, 2021; Pan & Mishra, 2018). On the other hand, authors such as Seven and Yetkiner (2016) argue that the primary function of the financial system is to enhance the allocative efficiency of capital and to stimulate savings, thus fostering economic growth. However, they also emphasize that there is no consensus in the economic literature regarding the long-term impact of financial markets on economic growth. Pan and Mishra (2018) highlight the ambiguity surrounding the direction of causality between financial market development and economic growth, which still requires careful empirical scrutiny.

In this study, we analyze the degree of interdependence between the stock market and macroeconomic expectation variables in the Brazilian economy. We consider seven sectoral stock indices from the Brazilian stock exchange along with expectation variables related to economic activity, inflation, interest rates, balance of payments, exchange rate, and fiscal conditions. As our methodological approach, we employ Dynamic Bayesian Networks (DBN) over the period from January 2, 2007, to December 6, 2023. This technique was selected due to its ability to capture and analyze information over time, allowing for the classification and identification of relevant patterns (Shiguihara et al., 2021). We examine the temporal evolution of the networks across non-crisis and crisis periods, enabling a comprehensive assessment of the interactions between the stock market and macroeconomic expectations. Furthermore, DBNs allow for the identification of partial linear relationships, which frequently remain undetected by conventional methods.

Our main findings reveal three key results. First, both year-end and three-year-ahead macroeconomic expectations and sectoral stock indices operate independently, whether for the full sample or across the five non-crisis and four crisis subperiods. This independence suggests that stock market movements in Brazil respond more strongly to firm-level and sector-specific fundamentals, which is consistent with the structure of a relatively small stock market dominated by large and efficient firms. Second, when we examine year-end expectations and stock indices jointly, we find that economic activity (GDP) and external sector expectations (CA) emerge as the main drivers within the expectations cluster, receiving the highest number of associations. The stock index cluster displays dense and predominantly positive unidirectional associations in both non-crisis and crisis periods, except during the Acute Phase of COVID-19. Third, in the network combining three-year-ahead expectations and stock indices, we observe a consistent two-cluster pattern: one composed exclusively of macroeconomic expectation variables and another formed by stock indices. Depending on the period, inflation (CPI) and external sector expectations (CA) act as the primary receivers of associations within the macro expectations cluster. This pattern indicates that medium- to long-term expectations depend on price stability and the sustainability of the current account. Meanwhile, the connections within the stock index cluster become less dense, although a recurrent inverse association between IMAT and ICON suggests a potential hedge relationship.

This study contributes to the literature in several respects. First, we advance the empirical analysis of the interaction between financial markets and macroeconomic expectations by applying Dynamic Bayesian Networks, a linear method based on putative partial correlations, to model the complex network of relationships linking the stock market and the real economy. The temporal analysis of networks across crisis and non-crisis periods allows us to identify how these interactions evolve under distinct macroeconomic contexts. Second, by using macroeconomic expectations collected by the Central Bank of Brazil, we evaluate whether current financial market conditions possess predictive content for short- and long-term macroeconomic variables. The results show significant associations only within the group of macroeconomic expectations or within the group of stock indices, confirming the independence of these two systems. Third, given the weak dynamic relationship between stock indices and macroeconomic expectations, policymakers seeking to influence short-term expectations should prioritize measures that affect economic activity (GDP) and the external sector through the current account. For medium- and long-term expectations, policies that reinforce price stability, anchor expectations, and maintain a sustainable current account appear more effective. Regarding the stock market, policymakers should act cautiously, as the unidirectional positive associations observed within the stock index cluster may amplify adverse shocks.

The remainder of this paper is structured as follows. Section 2 provides a focused review of the relevant literature. Section 3 outlines the theoretical framework and methodological procedures for estimating the Dynamic Bayesian Networks. Section 4 presents and discusses the empirical results obtained from the estimated networks. Finally, Section 5 concludes the study and highlights avenues for future research.

2. LITERATURE REVIEW

The theoretical framework studying the relationship between the financial market and macroeconomics has been extensively explored by researchers throughout economic history. Starting with Keynes' model, which assumes that individuals hold money for three motives: (i) the transaction motive; (ii) the precautionary motive; and (iii) the speculative motive. When interest rates fall below the normal level, people expect them to rise in the future. Consequently, interest rates remain constant if the money supply increases, a phenomenon known as the "liquidity trap," which has implications for the equilibrium level of output. On the other hand, investment is determined solely by the real interest rate. If the real interest rate rises, investments fall below savings at the full-employment level within the liquidity trap, resulting in inventory accumulation. To restore equilibrium, aggregate output will decline. Therefore, high interest rates do not constitute a continuous driving force for economic growth in Keynes' model. However, the model has been criticized for its short-term orientation and price rigidity (Pan & Mishra, 2018).

Moving from the classical model, we arrive at the Arbitrage Pricing Theory (APT), which employs multiple risk factors to explain asset returns, including macroeconomic factors (Ross, 1976). The APT model is based on systemic risk factors that can be influenced by macroeconomic variables, thereby affecting stock prices. Macroeconomic dynamics related to economic activity, as well as monetary and fiscal policy, can impact corporate stock returns. Conversely, fluctuations in stock prices can influence macroeconomic variables. In summary, firms and consumers may increase their spending or investment in response to rising stock prices, thereby affecting economic growth.

Shiller (1981) proposed the Present Value Model (PVM) to explain the connection between the stock market and macroeconomics. The PVM posits that the intrinsic value of a stock results from the present value of the firm's future cash flows. Therefore, if macroeconomic variables affect the firm's future cash flows or the discount rate, this will impact the stock's value. Conversely, empirical studies by Baek et al. (2005) and Long et al. (1990) found evidence that the stock market may fail to reflect information when stocks are speculative. Finally, the field of behavioral finance assumes that investors are influenced by behavioral biases, which may explain the disconnection between macroeconomic variables and the stock market (Kahneman, 2003; Shleifer & Summers, 1990).

In the field of empirical studies, there is an extensive literature examining the relationship between the financial market and macroeconomics, with mixed results. Levine (1997) argued that this relationship does not necessarily imply that financial market development is always exogenous to economic growth. Goldsmith (1969) found that financial development stimulates economic growth. However, his study, which used a dataset of 35 countries spanning the period from 1860 to 1963, has been criticized for lacking controls over several relevant factors and for failing to draw conclusions regarding causality or the relative importance of various transmission channels.

King and Levine (1993) used data from 80 countries over the period of 1960–1989, providing evidence in line with the Schumpeterian view that the financial system can promote economic growth. Their study found that the level of financial development is strongly associated with real per capita GDP growth, the rate of physical capital accumulation, and improvements in the efficiency with which economies utilize physical capital. This study subsequently gave rise to an empirical research stream aimed at determining whether the financial market can serve as a reliable barometer for macroeconomic conditions or the real economy.

Hamilton and Lin (1996) found that volatility in the U.S. stock market is preceded by the economic cycle, suggesting that it can be used to forecast fundamental economic activity and turning points in the business cycle. Additionally, the macroeconomic cycle during economic recessions has a stronger impact on stock market volatility, as it can affect firms' cash flows.

Mukherjee and Naka (1995) employed a VECM framework in a seven-equation system and demonstrated that the Japanese stock market is cointegrated with macroeconomic variables. Chauvet (1999) investigated the potential to forecast business cycle turning points using financial variables, showing that stock market fluctuations and business cycles can be represented by nonlinear dynamic factors at a monthly frequency for U.S. data.

The relationship between stock market volatility and macroeconomic activity has been studied by researchers aiming to better understand and formalize this association. Engle et al. (2013) proposed models incorporating a long-term component driven by inflation growth and industrial production, which proved more effective for long-horizon forecasts (beyond one quarter), outperforming traditional time-series volatility models.

However, some studies point to a disconnection between the financial market and the real economy, highlighting ambiguity in empirical results. Binswanger (2000) found that, since the early 1980s, U.S. stock market returns have become detached from measures of real economic activity. This finding holds regardless of whether monthly, quarterly, or annual real stock returns are used, and whether real activity is measured by industrial production growth rates or real GDP growth rates. Similarly, Quadir (2012) examined the relationship between the Bangladesh stock market (DSE – Dhaka Stock Exchange) and macroeconomic variables for the period of January 2000 to February 2007 using ARIMA models. The study found a positive relationship between treasury bond interest rates and industrial production with stock market returns, although the coefficients were statistically insignificant.

With the evolution of computational power, the social sciences have incorporated new analytical approaches, such as complex networks and dynamic Bayesian networks. Technological innovation in the field of economics has thus enabled scientific research on the behavioral dynamics among governments, countries, markets, and individuals, particularly through complex networks, generating novel insights.

Kim and Sayama (2017) applied network-based methods to the U.S. stock index S&P 500, demonstrating that changes in the network strength distributions (centrality measures) provide valuable information on future network movements. Predictive ARIMA models incorporating these network measures were estimated, resulting in improved forecast accuracy.

Ballester et al. (2023) analyzed systemic credit risk in the European financial system using Dynamic Bayesian Networks, finding that between 5% and 40% of the variations in European sectoral CDS (Credit Default Swaps) were explained by network relationships. Sener et al. (2019) employed Bayesian Networks to detect the effects of the July 15, 2016 coup attempt in Turkey on the financial market.

In emerging economies, several studies have investigated networks formed by financial assets. Carvalho & Chiann (2013) applied the Bayesian network framework to identify financial contagion among stock exchanges in countries such as Brazil, Argentina, Mexico, Malaysia, and Russia, finding that the U.S. stock market emerged as an authority node within the resulting networks. Bouri et al. (2018) examined implied volatility in BRICS equity markets using a Bayesian Graphical Structural Vector Autoregressive (BGSVAR) model and showed that it is influenced by the implied volatility of commodity markets and the equity markets of major developed economies. Aragón et al. (2021) employed a Bayesian network approach to assess the probability of financial stability among Mexican households, concluding that credit management and household composition are the most decisive variables.

China stands out within the Asian continent with several network-based studies unveiling the dynamics between the stock market and macroeconomic conditions. Wang and Li (2020) employed continuous wavelet analysis for the period from January 1995 to April 2018, and their results suggest structural breaks in the linkage between the stock market and the macroeconomy. In this context, stock returns cannot be used as leading indicators of macroeconomic activity, nor can real economic conditions predict bull market[4] phases in the Chinese equity market. Liu et al. (2019) applied Dynamic Bayesian Networks and found that changes in the stock market during bull market periods affect macroeconomic factors; however, this effect weakens as the stock market transitions into a bear[5] phase.

[4] It is the condition of a financial market in which prices are rising or are expected to rise. The term "bull market" is most commonly used to refer to the stock market, but it may be applied to any traded asset, such as bonds, real estate, currencies, or commodities.

[5] A time when stock prices are declining and market sentiment is pessimistic. Generally, a bear market occurs when a broad market index falls by 20% or more over at least a two-month period.

Similar findings, indicating a disconnection between the stock market and macroeconomic variables in China, were reported by Liu et al. (2021). Using Dynamic Bayesian Networks for the period from January 2007 to July 2020, the authors found that the correlation between the two domains was unstable over time. Consequently, they concluded that the stock market's role as an economic "barometer" is limited and highly susceptible to behavioral biases among economic agents.

Jin and Guo (2021) employed the Thermal Optimal Path (TOP) Method to analyze the relationships between the stock market and macroeconomic variables in China and the United States. Their results indicate that, for a mature economy such as the United States, certain stock indices have led the corresponding macroeconomic variables since 2013. In contrast, Chinese sectoral stock indices exhibit different lead-lag relationships across sectors, highlighting that the Chinese capital market remains relatively weak and more susceptible to behavioral biases among market participants compared to the U.S. market.

To synthesize the literature and methodological approaches used to examine the link between macroeconomics and financial markets, Table 1 presents a chronological overview of the studies and the strategies employed to explain this relationship.

Table 1 - Chronology of Studies

Year	Author(s)	Methodology / Model	Country / Region Studied
1969	Goldsmith	Financial development analysis	Multi-country (35 countries, 1860–1963)
1976	Ross	Arbitrage Pricing Theory (APT)	Theoretical / General
1981	Shiller	Present Value Model (PVM)	Theoretical / USA
1990	Long et al..	Empirical studies on stock market speculation	USA
1993	King & Levine	Panel data, Schumpeterian evidence	80 countries, 1960–1989
1995	Mukherjee & Naka	VECM (seven-equation system)	Japan
1996	Hamilton & Lin	Stock market volatility analysis	USA
1997	Levine	Empirical review of financial development	Multi-country
1999	Chauvet	Nonlinear dynamic models for business cycle forecasting	USA
2000	Binswanger	Historical stock return analysis	USA
2003	Kahneman	Behavioral finance (theoretical)	General
2005	Baek et al.	Empirical studies on stock speculation	USA
2012	Quadir	ARIMA models	Bangladesh (DSE)
2013	Engle et al.	Volatility models with long-term component	USA
2013	Carvalho & Chiann	Bayesian networks	Brazil, Argentina, Mexico, Malaysia, Russia
2017	Kim & Sayama	Network-based methods + predictive ARIMA	USA (S&P 500)
2018	Bouri et al..	Bayesian Graphical Structural VAR (BGSVAR)	BRICS
2019	Liu et al.	Dynamic Bayesian Networks	China
2019	Sener et al.	Bayesian networks	Turkey
2020	Wang & Li	Continuous wavelet analysis	China
2021	Liu et al.	Dynamic Bayesian Networks	China
2021	Jin & Guo	Thermal Optimal Path (TOP) Method	China and USA
2021	Aragón et al.	Bayesian Networks	Mexico
2023	Ballester et al.	Dynamic Bayesian Networks	Europe (sectoral CDS)

Given the relevance of the topic, this paper constructs Dynamic Bayesian Networks based on sectoral stock indices from the Brazilian stock exchange (B3) and macroeconomic expectations for year-end and three-year-ahead variables provided by market participants. The contribution to the literature lies in linking stock indices with macroeconomic expectation variables, since it is the agents' future expectations that, in theory, influence stock trading values. Finally, the results of this study shed light on the interaction between the Brazilian stock market and macroeconomic conditions using a linear putative partial correlation model.

3. METHODOLOGY

3.1. Primary Concept of Bayesian Network (BN)

The seminal work of Liu, Feng, and Guo (2021) applied the Dynamic Bayesian Network approach to investigate the associations between macroeconomics and the financial market in the Chinese context. From a theoretical perspective, a Bayesian Network (BN) can be defined as a graphical structure that enables the representation of an uncertain domain and reasoning about it. A BN consists of nodes and edges (arcs or links). The nodes represent a set of random variables within the domain, while the set of edges connects pairs of nodes, indicating direct dependencies among the variables. Assuming that the variables are discrete, the strength of the relationships is quantified through conditional probability distributions associated with each node. The only restriction on the edges allowed in a BN is that there must be no directed cycles, meaning that it is not possible to return to a node by following directed edges. Therefore, such networks are referred to as Directed Acyclic Graphs (DAGs) (Korb & Nicholson, 2004).

Let $G = (V, E)$ be a directed acyclic graph (DAG), where V is a finite set of nodes, and E is a finite set of directed edges between the nodes. Each node $v \in V$ in this graph represents a random variable X_v and comprises the set of variables in G. Given any pair of nodes X and $Y \in V$, if there is a directed edge from X to Y, X is called the parent node of Y. For each parent node of v, the notation $pa(v)$ is adopted. Additionally, a conditional probability function between v and $pa(v)$ is defined as $p\left(x_v \middle| x_{pa(v)}\right)$. The set of relational probability functions in the network is denoted as P. A BN for a given set of random variables is the pair (G, P).

It is important to highlight that the theory of Bayesian Networks (BN) includes the concept of *d-separation*, which guarantees the conditional and directional independence of a set of random variables (Carvalho & Chiann, 2013). The Markov condition implies that all *d-separations* are conditional independences, and all conditional independences implied by the Markov condition are identified by *d-separation*. In other words, if $(\mathbb{G}, P)$ satisfies the Markov condition, every *d-separation* in $\mathbb{G}$ is a conditional independence in P. Furthermore, every conditional independence that is common to all probability distributions satisfying the Markov condition with DAG $\mathbb{G}$ is identified by *d-separation* (Korb & Nicholson, 2004; Neapolitan, 2004).

3.2. Bayesian Network Learning

Analyzing from a statistical perspective, learning a network represents the estimation of model parameters based on a certain criterion and having knowledge of a specific dataset (Carvalho & Chiann, 2013).

The Bayesian approach is used to estimate parameters within the network, aiming to encode uncertainty about the parameters θ into a $prior$ distribution $p(\theta)$ and applying the data $\bar{d}$ (through the likelihood function). By applying Bayes' theorem, the uncertainty is updated with the $posterior$ distribution $p(\theta|\bar{d})$, as per Equation 1:

$$p(\theta|\bar{d}) = \frac{f(\bar{d}|\theta)h(\theta)}{\int_{\theta\in\Theta} f(\bar{d}|\theta)h(\theta)d\theta}, \theta \in \Theta, \tag{1}$$

given Θ as the parameter space, $\bar{d}$ as the random sample from the probability distribution $p(x|\theta)$, and $p(\bar{d}|\theta)$ as the likelihood function.

3.3. Concept of Dynamic Bayesian Network (DBN)

Dynamic Bayesian Network (DBN) is a temporal extension extensively used to model temporal relationships among variables over different time periods (Murphy, 2002). The unfolding of an interaction graph over time is highly beneficial for accommodating potential loops and feedbacks in the network's topology, as required by definition for a Bayesian Network (Nagarajan et al., 2013).

In a DBN, it is assumed that the dependency relationships are represented by a Vector Autoregressive Process (VAR), where it is important that the time series are stationary. In a VAR(p) process of order p, the observed variables at any time $t \geq p$ satisfy Equation 2:

$$X(t) = A_1X(t-1) + \cdots + A_iX(t-i) + \cdots + A_pX(t-p) + B + \varepsilon(t). \tag{2}$$

Where:

$X(t) = (X_i(t)), i = 1, \ldots, k,$ is the vector of k observed variables at time t;

$A_i, i = 1, \ldots, p$ are coefficient matrices of size $k \times k$;

B is a vector of size k that represents the baseline measurement for each variable, and is invariant as a function of time;

$\varepsilon(t)$ is a vector of white noise of size k, with zero mean $E(\varepsilon(t)) = 0$ and time-invariant positive definite covariance matrix $Cov(\varepsilon(t)) = \sum$.

Thus, if we assume a VAR(p) of order 1, as in Equation 3:

$$X(t) = AX(t-1) + B + \varepsilon(t), \quad com \quad \varepsilon(t) \sim N(0, \textstyle\sum), \tag{3}$$

the edges are defined between two successive time periods, and this set of edges is determined by all nonzero coefficients in matrix A (see Equation 4). If an element $a_{ij}, i \neq j$ is non-zero, then the network includes an edge from $X_i(t-1)$ to $X_j(t)$. Additionally, we assume that the error term for each variable X_i is independent of the other variables and their respective error terms, meaning the elements off the diagonal in $\sum$ can be set to 0.

A VAR(1) process, with a diagonal error covariance matrix $\sum$, can be represented as a Dynamic Bayesian Network, where the edges of the network correspond to the non-zero elements of the coefficient matrix A. Specifically, the elements a_{ij} of matrix A indicate the non-zero coefficients, which define the dependencies between the variables. This structure helps visualize the direct relationships between the variables over time, with the diagonal elements of $\sum$ representing the variances of the error terms for each variable:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{bmatrix}. \tag{4}$$

Thus, the Dynamic Bayesian Network can be represented as shown in the Appendix A, Figure 6.

3.4. Dynamic Bayesian Network Learning Algorithms

Classical ordinary least squares (OLS) estimate of the regression coefficients A and B from Equation 2 or 3, in the case of multiple individuals, can only be calculated when $n \gg k$, ensuring that the sample covariance matrix has full rank. For real-world data, regularized estimators are necessary in almost all cases.

According to Nagarajan et al. (2013), regularized estimators can be divided into four approaches:

The Least Absolute Shrinkage and Selection Operator or LASSO (Tibshirani, 1996): the estimation procedure tends to produce some coefficients exactly equal to zero by applying an L_1-norm penalty to the sum of the coefficients. Variable selection thus becomes straightforward: only the non-zero coefficients define significant dependency relationships.

James–Stein Shrinkage Approach: An efficient estimator of the covariance matrix can be obtained by "shrinking" the empirical correlation coefficients toward zero and the empirical variances toward their median. James and Stein (1961) and Stein (1956) demonstrated that the resulting correlation matrix dominates the empirical one, meaning its mean squared error is never worse than that of the empirical correlation matrix. Opgen-Rhein and Strimmer (2007b) proposed an application of the James–Stein shrinkage approach to the VAR process, achieving better results than many classical approaches. These improved estimates of the regression coefficients, which are essentially a function of the covariance matrix of X, are attributed to this method. The network structure is determined by the inclusion of edges in descending order of the coefficients. To control the False Discovery Rate (FDR), the local FDR approach suggested by Schäfer and Strimmer (2005) can be employed for multiple testing correction (see Significance Measures: local false discovery rate (fdr) and q-value). Finally, in static Bayesian networks, James–Stein shrinkage is used to compute regularized partial correlations and conditional probabilities, which are employed in conditional independence tests.

First-Order Conditional Dependencies Approximation: This approach is based on first-order conditional dependencies. The central concept of this method is the low-order conditional dependency graph, which originated within the context of graphical modeling theory with directed acyclic graphs. The directed acyclic graph defining the dynamic Bayesian network is approximated by first-order conditional dependencies. Lèbre (2009) proposes the G1DBN algorithm, which implements dynamic Bayesian network learning through a two-step procedure. First, it learns a directed acyclic graph that encodes first-order partial dependency relationships. Then, it infers the actual structure of the dynamic Bayesian network using the graph from the previous step.

Modular Networks: Chiquet et al. (2009) developed learning algorithms based on LASSO[6], adding a clustering effect for multiple data sets. The algorithm was named SIMoNe (Statistical Inference for MOdular Networks) and applies a score-based approach, seeking a latent clustering of the network to guide edge selection through an adaptive L_1 likelihood penalty, particularly for VAR(1) processes.

The approach employed in this study follows Opgen-Rhein and Strimmer (2007b), which achieved better results for the VAR process by reducing empirical correlation coefficients compared to classical models. For determining the inclusion of network edges, we apply the local FDR approach suggested by Schäfer and Strimmer (2005) for multiple testing correction. Thus, all estimations of the Dynamic Bayesian Networks were carried out using the GeneNet package in RStudio. In the next subsection, we present the theoretical rationale of the GeneNet package for network estimation.

[6]Least Absolute Shrinkage and Selection Operator.

3.4.1. GeneNet Package in R

The GeneNet estimation package in R implements a linear shrinkage estimator for a covariance matrix and the selection of a Gaussian Graphical Model (GGM) based on partial correlation obtained from the shrinkage estimator suggested by Opgen-Rhein and Strimmer (2007b). The package applies multiple testing using the local false discovery rate (fdr) approach through the method of Schäfer and Strimmer (2005), where the GGM selection controls the false discovery rate at a predetermined α level. Specifically, one of the most commonly used linear shrinkage estimators, S^*, for the covariance matrix $\sum$ is:

$$S^* = \lambda^* T + (1 - \lambda^*)S, \tag{5}$$

where $S = \left(s_{ij}\right)_{1 \leq ij \leq p}$ is the sample covariance matrix, $T = diag\left(s_{11}, s_{22}, \dots, s_{pp}\right)$ is the target shrinkage matrix, and $\lambda^* = \frac{\sum_{i \neq j} \widehat{Var}(s_{ij})}{\left(\sum_{i \neq j} s_{ij}^2\right)}$ is the optimal shrinkage intensity.

This estimator S^*, the partial correlation matrix $P = \left(\hat{\rho}^{ij}\right)_{1 \leq i,j \leq p}$, is defined as:

$$\tilde{r} = \hat{\rho}^{ij} = -\frac{\widehat{\omega}_{ij}}{\sqrt{\widehat{\omega}_{ii}\widehat{\omega}_{jj}}}, \tag{6}$$

where $\widehat{\Omega} = \left(\widehat{\omega}_{ij}\right)_{1 \leq i,j \leq p} = (S^*)^{-1}$.

To identify the significant edges, we apply the approach of Schäfer and Strimmer (2005), which assumes the distribution of partial correlations as a mixture:

$$f(\tilde{r}) = \eta_0 f_0(\tilde{r}; k) + (1 - \eta_0) f_A(\tilde{r}), \tag{7}$$

where f_0 is the null distribution, f_A is the alternative distribution corresponding to the true edges, and n_0 is the unknown mixture parameter. Using the algorithm developed by Opgen-Rhein and Strimmer (2007a), the GeneNet package identifies the significant edges that have the local false positive rate:

$$\text{Prob}(\text{null edge}|\tilde{r}) = \text{fdr}(\tilde{r}) = \frac{\hat{\eta}_0 f_0\left(\tilde{r};\ \hat{k}\right)}{\hat{f}(\tilde{r})}, \tag{8}$$

smaller than the pre-determined level α, where $|\tilde{r}|\text{Be}\left(\tilde{r}^2; \frac{1}{2}, \frac{k-1}{2}\right)$, and $\text{Be}(x; a, b)$ is the Beta distribution density, with k being the reciprocal variance of the null $\tilde{r}$.

For further details on the approach used, including the local false discovery rate (FDR) and the significance measure q-value (α), see Appendix B and C.

4. EMPIRICAL RESULTS

4.1. Data

The closing price series of sectoral stock indices from the Brazilian stock exchange were obtained from the Bloomberg financial terminal. The sectoral stock indices analyzed are: 1-IMAT: Basic Materials Index; 2-IMOB: Real Estate Index; 3-ICON: Consumer Index; 4-INDX: Industrial Sector Index; 5-IEE: Electric Energy Index; 6-IFNC: Financial Index; and 7-UTIL: Utilities Index.

The selected time frame spans from January 2, 2007, to December 6, 2023, with the IMOB series starting on January 2, 2008.

Daily returns were calculated based on closing prices using the natural logarithm, as specified in Equation 9.

$$R_{i,t} = \left[\ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)\right] \times 100. \tag{9}$$

The macroeconomic expectation series were obtained from the Central Bank of Brazil (Bacen) website. These series are collected through surveys conducted by the Central Bank with market participants, considering information from the past 30 days to calculate the average expectation. The selected time frame spans from January 2, 2007, to December 6, 2023.

The series analyzed were: 1-GDP: Gross Domestic Product; 2-CPI: Consumer Price Index; 3-GPI: General Price Index; 4-FDI: Foreign Direct Investment; 5-TB: Trade Balance; 6-CA: Current Account; 7-SELIC: Central Bank Policy Rate; 8-ER: Exchange Rate in relation to the US dollar; 9-NPSD: Net Public Sector Debt; and 10-PB: Primary Balance.

We used both end-of-year and three-year-ahead expectations to assess whether the financial market exhibits greater or lesser sensitivity to short- or long-term macroeconomic variables. Table 2 presents the descriptive statistics of the macroeconomic expectations and the daily log-returns of the stock indices.

Table 2 - Descriptive Statistics of the Variables

Variáveis	Nº	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
GDP_Y0_Mean[1]	4,190	1.7017	2.8636	-6.5500	0.3800	1.9562	3.4700	7.6400
CPI_Y0_Mean[1]	4,190	5.3495	1.6828	1.5500	4.0600	5.2900	6.3200	10.7000
GPI_Y0_Mean[1]	4,163	6.4411	4.4450	-3.7691	4.4500	5.8300	8.1250	24.0100
FDI_Y0_Mean[2]	4,190	55.7268	18.0628	16.2100	42.5050	58.6000	67.7600	86.2200
TB_Y0_Mean[2]	4,190	35.3863	21.8041	-1.8400	15.7025	38.5100	54.7975	82.6255
CA_Y0_Mean[2]	4,190	-36.2981	26.2469	-86.0500	-59.1975	-27.0750	-16.8025	11.7100
SELIC_Y0_Mean[1]	3,932	9.8434	3.2574	1.9400	7.3200	10.8100	12.5213	15.1500
ER_Y0_Mean[3]	4,189	3.1709	1.3047	1.5900	1.9400	3.2100	4.0600	5.6201
NPSD_Y0_Mean[4]	4,190	47.0058	10.2834	34.0100	37.9225	43.3800	56.6300	68.4900
PB_Y0_Mean[4]	4,190	0.1911	3.1426	-12.2300	-1.5400	0.8600	2.7100	4.3300
GDP_Y3_Mean[1]	4,190	3.1033	0.9577	1.6600	2.4000	2.6400	4.1700	4.7400
CPI_Y3_Mean[1]	4,190	4.2573	0.6563	3.1654	3.7200	4.3200	4.8200	5.4400
GPI_Y3_Mean[1]	4,190	4.4246	0.4276	3.6300	4.0214	4.5000	4.7500	5.2700
FDI_Y3_Mean[2]	4,190	61.0962	19.7829	17.1600	47.8800	61.2700	78.9375	89.0400
TB_Y3_Mean[2]	4,190	29.3676	18.1010	0.7500	11.8350	30.0100	42.7950	74.7500
CA_Y3_Mean[2]	4,190	-47.7620	20.1407	-83.6600	-67.9650	-47.2525	-34.5012	-0.1600
SELIC_Y3_Mean[1]	4,190	8.8984	1.4613	5.5000	8.0600	9.1500	10.1300	11.5000
ER_Y3_Mean[3]	4,190	3.2575	1.2208	1.7500	2.0200	3.3900	4.1500	5.3493
NPSD_Y3_Mean[4]	4,190	48.5391	14.3558	30.6900	35.4700	40.7850	61.8000	74.6000
PB_Y3_Mean[4]	4,190	1.3200	1.5550	-1.5300	-0.1900	1.8800	2.7200	4.1100
IMAT[1]	4,189	0.0335	2.0075	-17.2092	-1.0251	0.0590	1.1418	13.3859
IMOB[1]	3,944	-0.0009	2.2557	-19.4627	-1.1131	-0.0053	1.1539	17.7755
ICON[1]	4,189	0.0256	1.5521	-17.6174	-0.7329	0.0775	0.8508	11.8287
INDX[1]	4,189	0.0262	1.5390	-18.3383	-0.7084	0.0713	0.8399	11.8296
IEE[1]	4,189	0.0445	1.3410	-12.3209	-0.6301	0.0768	0.7745	11.5989
IFNC[1]	4,189	0.0342	1.9661	-14.2505	-0.9792	0.0610	1.0546	18.9978
UTIL[1]	4,189	0.0487	1.5019	-14.6001	-0.7365	0.0877	0.8725	11.6908

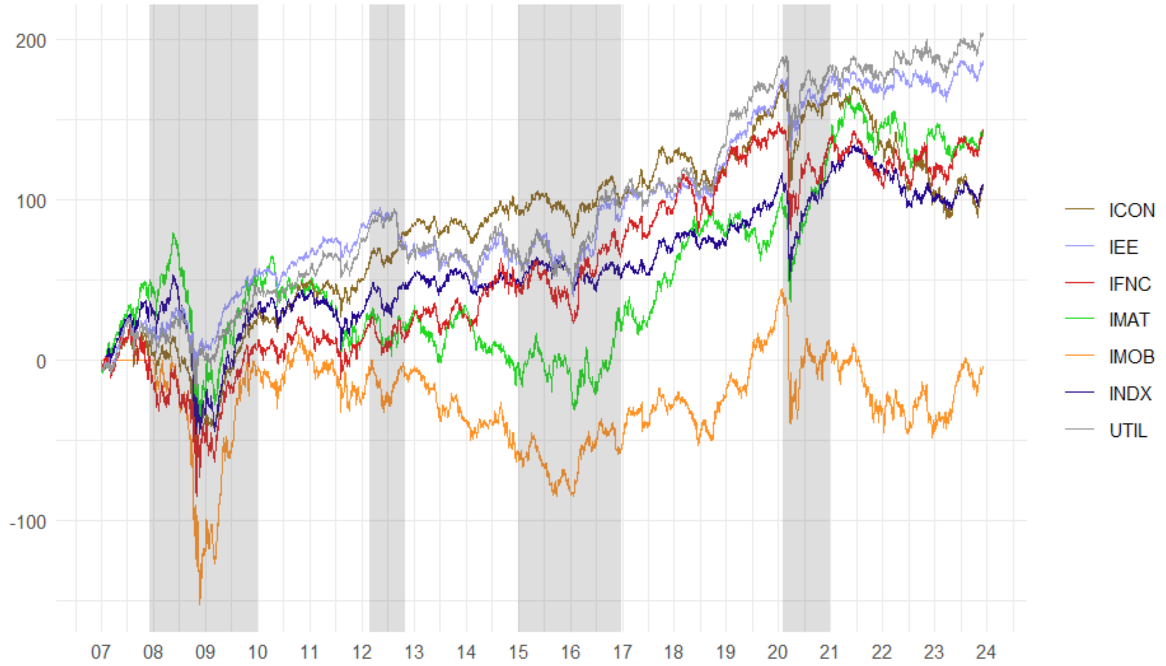
[1]Values in percentage (%), where Y0 represents the end-of-year expectation and Y3 represents the three-year-ahead expectation. | [2]Values in US$ billions, where Y0 represents the end-of-year expectation and Y3 represents the three-year-ahead expectation. | [3]Exchange rate in Brazilian reais (R$) per US dollar, where Y0 represents the end-of-year expectation and Y3 represents the three-year-ahead expectation. | [4]As a percentage (%) of GDP, where Y0 represents the end-of-year expectation and Y3 represents the three-year-ahead expectation.

Source: Central Bank of Brazil and Bloomberg.

The temporal delimitation of crisis and non-crisis periods in this study was established based on a literature review, drawing on works such as Chen et al. (2018), Korkusuz et al. (2022), So et al. (2021), and Zhang et al. (2020), who employed time frames comparable to those adopted here. In addition, a retrospective temporal analysis was conducted to complement and extend the existing literature, ensuring a more comprehensive identification of systemic shocks within the context of the Brazilian economy.

Accordingly, we identified four crisis periods that significantly affected the financial assets under analysis: (1) The Subprime Crisis (12/01/2007-12/31/2009); (2) The European Sovereign Debt Crisis (03/01/2012-10/31/2012); (3) The Great Brazilian Recession of the 21st Century (01/01/2015-12/31/2016); and (4) The Acute Phase of COVID-19 (02/02/2020-12/31/2020). The non-crisis periods correspond to the intervals between these crisis episodes. Figure 1 presents the cumulative returns of the sectoral stock indices, while Figure 2 and Figure 3 display the expectations for the macroeconomic variables.

Figure 1 - Cumulative Return of Sectoral Stock Indices (%)



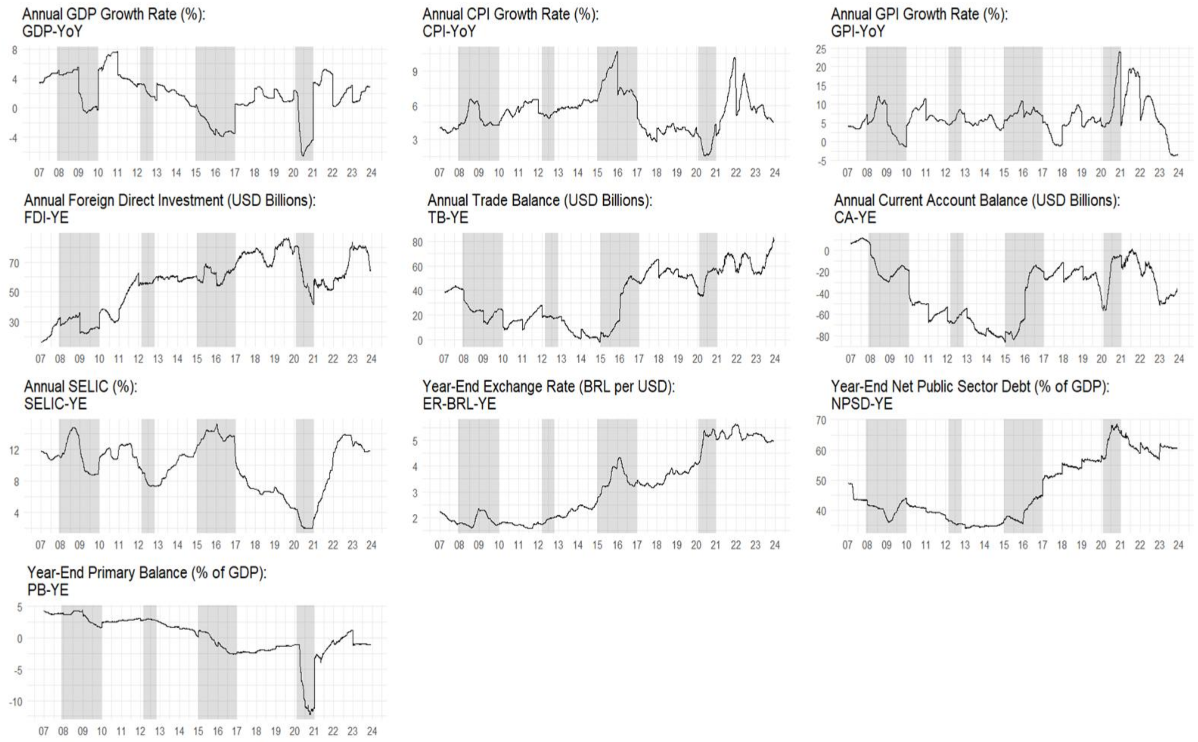
Note: Areas shaded in gray represent crisis/stress episodes.

Source: Bloomberg.

In Figure 1, we observe that the cumulative returns of the sectoral stock indices were positive, with the exception of the IMOB index, which recorded a negative return of 3.60%. This suggests that high long-term interest rates, instability in the domestic civil construction sector, and periods of market stress may have exerted downward pressure on the index.

Analyzing the crisis periods, we observe that the Subprime Crisis (12/01/2007-12/31/2009) had the greatest impact on the sectoral indices. In the other three crises, the effects differed across indices depending on the characteristics of each crisis. However, during the Acute Phase of COVID-19 (02/02/2020-12/31/2020), there was a sharp decline followed by a relatively rapid recovery. The monetary easing implemented by the Central Bank of Brazil redirected flows toward equity assets, as the benchmark interest rate fell to 2% per year.

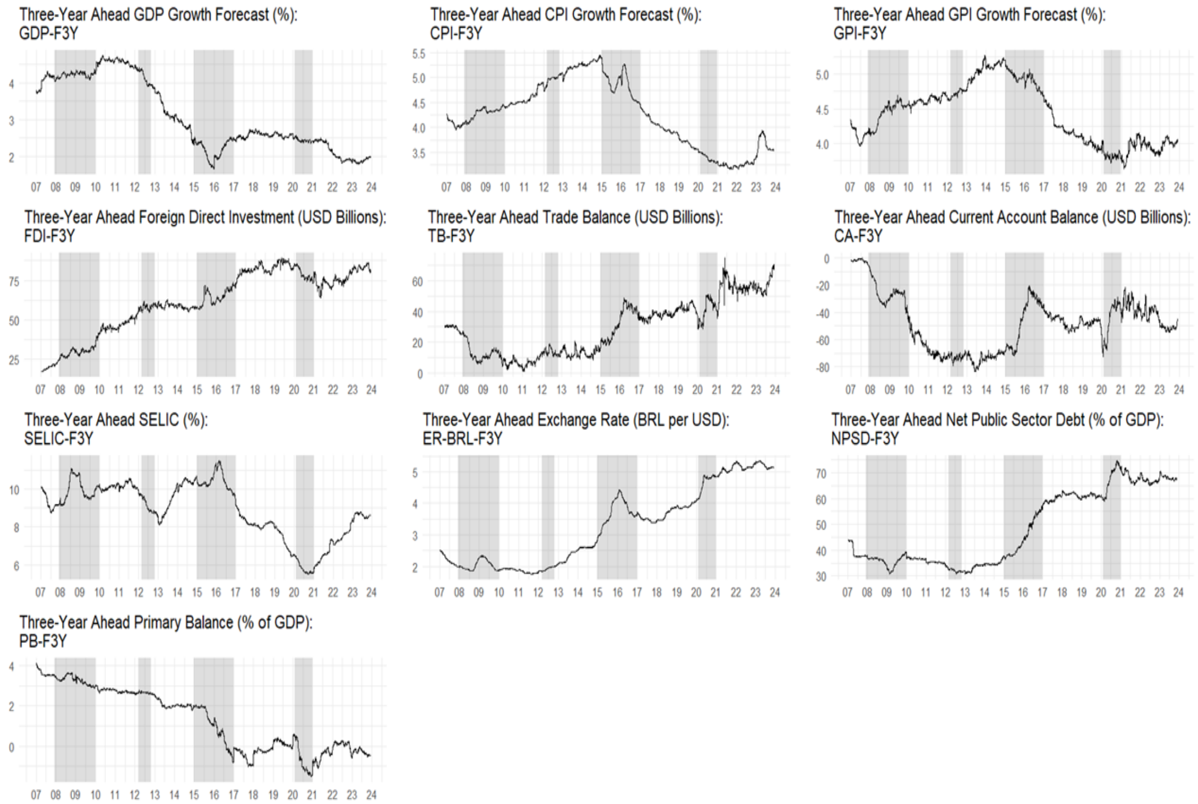
Figure 2 - Year-End Macroeconomic Expectations



Note: The gray-shaded areas refer to crisis/stress periods.

Source: Central Bank of Brazil and Bloomberg.

Figure 3 - Three-Year Ahead Macroeconomic Expectations



Note: The gray-shaded areas refer to crisis/stress periods.

Source: Central Bank of Brazil and Bloomberg.

The end-of-year and three-year-ahead macroeconomic expectations (Figure 2 and Figure 3) exhibit broadly similar dynamics. Nevertheless, the end-of-year expectations demonstrate higher volatility, reflecting the greater sensitivity of short-term forecasts to recent economic developments and market conditions incorporated into analysts' projections.

Analyzing the series, we observe that economic activity (GDP) declined across all crisis periods, with Acute Phase of COVID-19 showing the sharpest drop in GDP expectations, particularly for the end-of-year horizon.

In the price index series (CPI and GPI), it is worth highlighting that during the Great Brazilian Recession of the 21st Century (01/01/2015-12/31/2016), end-of-year CPI expectations increased until mid-2016 due to the adjustment of administered prices that took place in 2015. Around mid-2016, the impeachment process and the subsequent shift in economic policy guidelines influenced the price dynamics of the economy, leading to a decline in both end-of-year and three-year-ahead inflation expectations.

Furthermore, during the Acute Phase of COVID-19, end-of-year CPI expectations dropped to their lowest levels in the entire sample period, as the crisis strongly constrained aggregate demand. In contrast, GPI expectations rose, given that exchange rate fluctuations have a substantial weight in its calculation, and the domestic currency was highly depreciated during that period.

The expectations for Foreign Direct Investment (FDI) and the Trade Balance (TB) exhibit similar behavior, that is, as crises intensify, expectations deteriorate, and they begin to improve toward the end of the crisis period. Conversely, Current Account (CA) expectations tend to worsen during the first half of the crisis and recover in the latter half–a pattern also observed for the Exchange Rate (ER).

It is worth noting that, across all four series, the Acute Phase of COVID-19 led to a faster deterioration of expectations, as the globalized world had not previously experienced a health crisis with such a severe impact on economic activity.

Fiscal expectation — measured by the Net Public Sector Debt (NPSD) and the Primary Balance (PB) — follow the pattern of deteriorating expectations throughout the crisis periods, with improvements becoming evident toward their end. An exception to this pattern is observed during the Great Brazilian Recession of the 21st Century (2015–2016), when GDP contracted by 3.5% and 3.3%, respectively. Consequently, the sharp decline in economic activity severely affected expectations for both fiscal variables.

The behavior of the Central Bank Policy Rate (SELIC) varied across crises. During the Subprime Crisis and the Great Brazilian Recession of the 21st Century, risk perception increased substantially, resulting in higher interest rates. Conversely, during the European

Sovereign Debt Crisis and the Acute Phase of COVID-19, interest rates declined as a countercyclical policy measure aimed at stimulating domestic economic activity.

Finally, during non-crisis periods, we observe lower volatility and/or a clearer trend in expectations.

4.2. Estimation of Dynamic Bayesian Networks

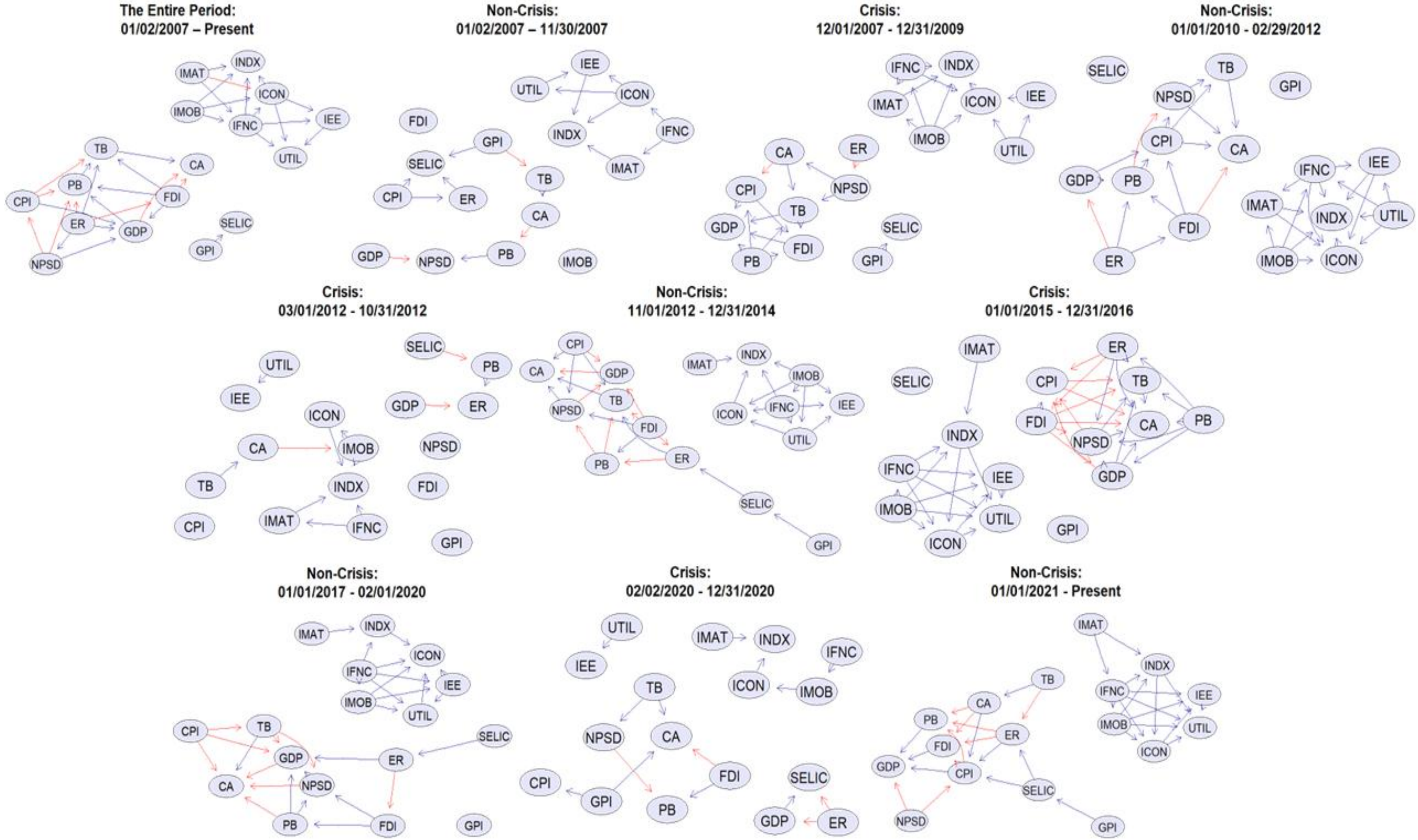
We employed the daily log returns of the sectoral stock indices to estimate the Dynamic Bayesian Networks. For the series of macroeconomic expectations, the first difference was applied[7], as all series were found to be non-stationary. Stationarity was verified using the Augmented Dickey-Fuller and Phillips-Perron tests at the 1% significance level. All time series analyzed proved to be stationary, which is an essential requirement for the estimation of Dynamic Bayesian Networks, as highlighted by Nagarajan et al. (2013).

Efron (2007) suggests q-values between 0.05 and 0.15 as feasible choices for estimating networks, with these q-value limits interpreted as a conservative Bayesian factor for interpreting the FDR. To assess result sensitivity, all networks were generated with q-values equal to 0.05, 0.1, and 0.15, and the networks remained unchanged, demonstrating estimation robustness. Finally, the edges represent the direction of partial correlation, with blue indicating positive and red indicating negative associations.

In Figure 4 and Figure 5, we present all the networks estimated throughout the analyzed period. In the first network of each figure, we estimate the structure considering the entire sample period, allowing for comparisons with both non-crisis and crisis periods. This approach enables us to identify whether there is a consistent pattern of association between the sectoral stock indices and the macroeconomic expectation variables. Appendix D presents the values of the partial correlations obtained from the estimated dynamic Bayesian networks.

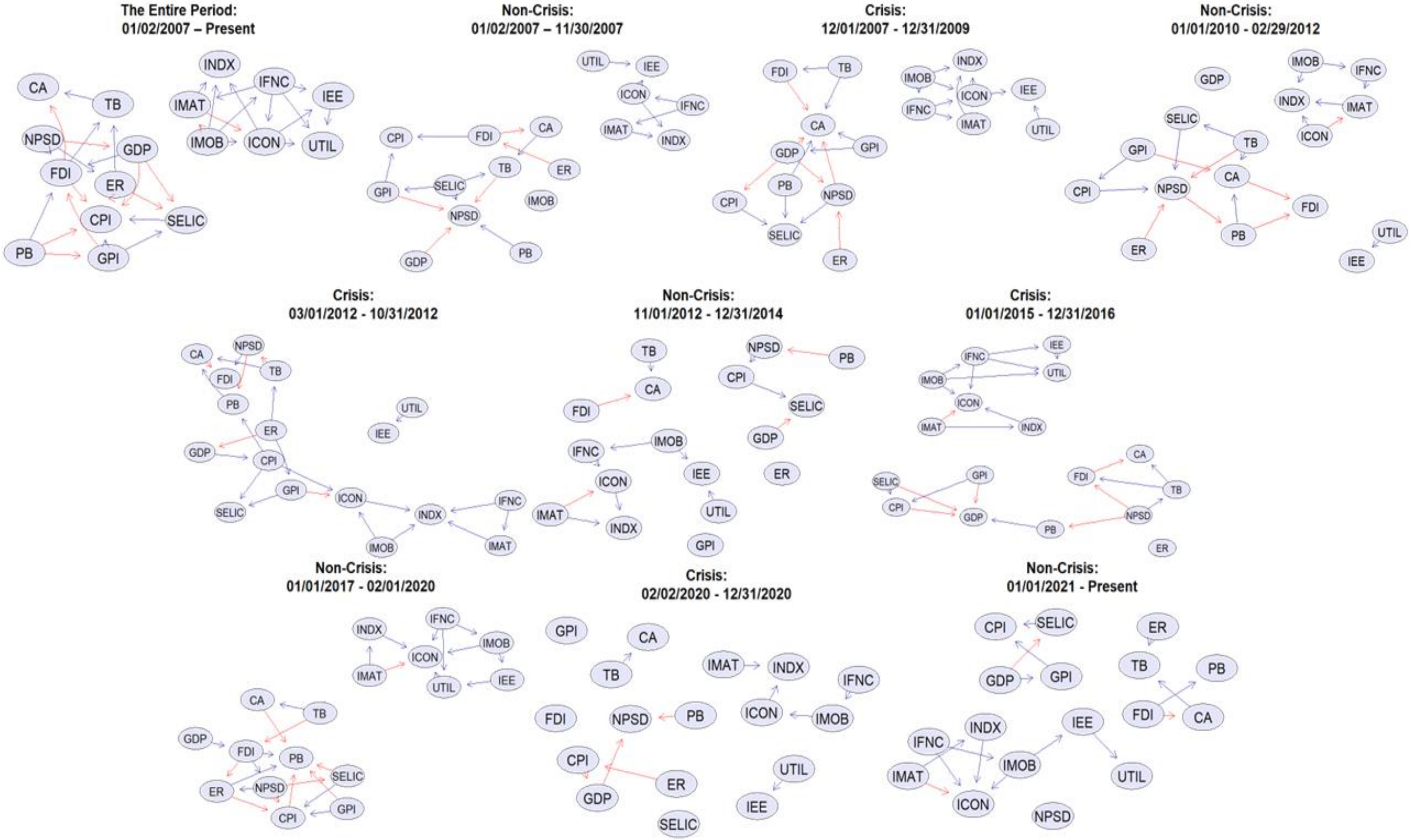
[7] $Diff(Y_t) = Y_t - Y_{t-1}$.

Figure 4 - Dynamic Bayesian Networks: Considering End-of-Year Macroeconomic Expectations



The blue edges indicate positive partial correlations, while the red edges indicate negative partial correlations. | To check the partial correlation values, see Appendix D | The acronyms correspond to the sectoral stock indices and macroeconomic expectation variables. | For the full description of each acronym, refer to Section 4.1, paragraphs 1 and 5.

Figure 5 - Dynamic Bayesian Networks: Considering Three-Year-Ahead Macroeconomic Expectations



The blue edges indicate positive partial correlations, while the red edges indicate negative partial correlations. | To check the partial correlation values, see Appendix D | The acronyms correspond to the sectoral stock indices and macroeconomic expectation variables. | For the full description of each acronym, refer to Section 4.1, paragraphs 1 and 5.

Analyzing the first estimated network for the entire period (Figure 4 and Figure 5: 01/02/2007 to The Present[8]), we find that the stock indices and macroeconomic expectations — both year-end and three-year-ahead — do not exhibit any associations. In other words, macroeconomic expectations do not influence stock indices. Moreover, the stock indices display predominantly positive associations (partial correlations) among themselves, indicating that, in the event of either positive or negative shocks, the indices tend to move in the same direction.

We find that, among the year-end macroeconomic expectation variables, the Primary Balance (PB) emerges as the main variable, that is, other economic variables are positively or negatively associated with the PB. This indicates that the balance of public accounts is the primary driver influencing other expectation variables. Secondarily, GDP and the Trade Balance (TB) also play important roles, capturing the dynamics of economic activity as a key short-term factor.

In contrast, among the three-year-ahead expectation variables, the most influential variable is the CPI (Consumer Price Index). In other words, most other variables are inversely associated with inflation. We infer that, in the medium to long term, price stability in the economy constitutes the main driving force influencing other macroeconomic expectation variables.

During the non-crisis period preceding the Subprime Crisis (Figure 4 and Figure 5: 01/02/2007 - 11/30/2007), the year-end and three-year-ahead macroeconomic expectations showed no association with the stock indices. Despite robust GDP growth of approximately 6.1%[9], controlled inflation, low unemployment, a positive trade balance, and an appreciated exchange rate[10], these factors were not sufficient to generate associations between the two markets. Within the cluster of macroeconomic expectation variables, the SELIC rate and the Net Public Sector Debt (NPSD) stand out as the most connected variables in the short and long term, respectively. It is noteworthy that, even amid strong economic growth, concerns about the balance of public accounts were already emerging as a significant issue.

During the Subprime Crisis period (Figure 4 and Figure 5: 12/01/2007 - 12/31/2009), we observe that the stock indices and the year-end and three-year-ahead macroeconomic expectations remained unassociated. The stock indices exhibited positive associations among themselves, indicating propagation in the same direction. In other words, asset prices

[8] We consider the present date as December 6, 2023.
[9] According to data from IBGE (Brazilian Institute of Geography and Statistics).
[10] According to IPEA (Institute of Applied Economic Research), the average exchange rate in 2007 was 1.9479.

experienced a sharp decline due to the intensification of the crisis; however, government support through fiscal and monetary policies provided stability to the markets.

Regarding the year-end expectations, GDP emerged as an influential variable affecting several other macroeconomic indicators, reflecting the fiscal stimulus implemented by the government. In the three-year-ahead macroeconomic expectations, the Current Account (CA) was identified as the variable receiving the most positive and negative associations. This suggests that concerns about a global economic slowdown and its longer-term effects on the Brazilian economy were primarily captured through movements in the current account.

In the post-Subprime Crisis period (Figure 4 and Figure 5: 01/01/2010 - 02/29/2012), when analyzing the network based on year-end expectations, we observe that the Current Account (CA) and the Consumer Price Index (CPI) are the two variables receiving the greatest number of connections, that is, they exert influence over other variables. We infer that, during this period, Brazil experienced strong economic growth; however, the exhaustion of its growth model led to a deterioration in price stability and a downward trend in the Current Account balance (see Figure 2).

In the three-year-ahead expectations, the Net Public Sector Debt (NPSD) emerges as the variable exerting the strongest influence on other macroeconomic expectations, reflecting increasing uncertainty regarding the country’s fiscal position.

When examining the stock indices in relation to the year-end expectations, we identify a cluster with several positive associations (partial correlations), in which the INDX index stands out as the main influencer within the network, consistent with the strong performance of the industrial sector during the period. A similar dynamic is observed when the stock indices are analyzed alongside the three-year-ahead macroeconomic expectations, albeit with lower intensity due to the reduced degree of predictability.

During the European Sovereign Debt Crisis (03/01/2012 - 10/31/2012), when analyzing the network estimated with the year-end macroeconomic expectations (Figure 4), we identify a negative association between the Current Account (CA) and the real estate stock index (IMOB). This inverse relationship can be explained by the fact that, during this period, the deterioration in economic activity expectations and the European crisis compelled the government to support the economy through monetary easing (i.e., a reduction in the SELIC rate).

Interest rate cuts positively affected the construction sector, which is highly sensitive to lower borrowing costs. However, the heightened global risk aversion slowed down global growth, negatively impacting the Current Account balance. The networks composed exclusively of stock indices exhibited fewer connections.

When analyzing the European Sovereign Debt Crisis (03/01/2012 - 10/31/2012) with respect to the three-year-ahead macroeconomic expectations (Figure 5), we observe that the inflation-related variables, CPI and GPI, are positively and negatively associated with the consumer sector stock index (ICON), respectively.

The positive relationship between CPI and ICON can be explained by the fact that CPI is a price index that does not directly incorporate exchange rate fluctuations. The deterioration in inflation expectations, driven by the low unemployment rate at the time[11], reflected an increase in household income and consumption capacity, benefiting consumer-oriented companies represented by the ICON index.

Conversely, the negative relationship between GPI and ICON arises because the GPI price index is more sensitive to exchange rate variations[12]. The crisis contaminated medium- to long-term expectations, leading to an appreciation of the U.S. dollar (USD) and a consequent erosion of households' purchasing power. Finally, both the stock index cluster and the macroeconomic variable cluster did not display a clear or consistent pattern.

The non-crisis period from 11/01/2012 to 12/31/2014 (Figure 4 and Figure 5) reflects the exhaustion of Brazil's demand-driven economic growth model. The year-end macroeconomic expectations and stock indices show no association. We find that the network formed by macroeconomic variables indicates that the Current Account (CA) and economic activity (GDP) are the variables receiving the most connections, capturing concerns about the deterioration of the country's economic fundamentals.

The stock indices form a network in which the real estate index (IMOB) is influenced by five other indices, meaning its performance depends on that of other sectors. When analyzing the network based on the three-year-ahead macroeconomic expectations, we observe that the prevailing uncertainty regarding price levels in the economy highlights the SELIC rate and the Current Account as the most influential variables.

On the side of the stock indices, an inverse relationship between the industrial materials index (IMAT) and the consumer sector index (ICON) emerges, indicating a potential hedge position, as the decline in domestic consumption tends to benefit assets that depend more heavily on external market performance[13], such as IMAT.

[11] According to data from IBGE, the average unemployment rate in the first 10 months of 2012 was 7.51%.

[12] Approximately half of the prices included in the Wholesale Price Index (IPA), which accounts for 60% of the GPI, are directly influenced by exchange rate fluctuations. Consequently, when agricultural commodities such as soybeans and wheat experience price increases driven by a surge in the U.S. dollar, the GPI is significantly affected.

[13] According to World Bank data, global growth in 2013 and 2014 was 2.8% and 3.1%, respectively. In Brazil, growth during the same period was 3.0% in 2013 and 0.5% in 2014, according to the Brazilian Institute of Geography and Statistics (IBGE).

During the Great Brazilian Recession of the 21^{st} century (01/01/2015 - 12/31/2016), the network estimated using year-end macroeconomic expectations (Figure 4) reveals that the cluster of stock indices exhibits a unidirectional positive movement, with the utilities index (UTIL) standing out as the most influential. This behavior reflects the fact that, although the crisis significantly reduced household income, the utilities sector, being defensive and characterized by predictable cash flows, performed relatively well during the period.

Overall, the companies comprising the stock indices were larger, more efficient, and had better access to funding, which allowed them to endure the crisis more effectively and even gain market share as smaller firms exited the market.

The cluster of macroeconomic expectation variables shows that the Current Account (CA) and the Consumer Price Index (CPI) were the most influential variables. The former is explained by the exchange rate depreciation, while the latter reflects the adjustment of administered prices, which had a substantial impact on inflation. It is noteworthy that the SELIC rate exhibited no associations, consistent with the period of fiscal dominance experienced in the country.

When analyzing the Great Brazilian Recession of the 21^{st} century (01/01/2015 - 12/31/2016) through the network estimated using the three-year-ahead macroeconomic expectations (Figure 5), we observe no associations between macroeconomic expectations and the stock market. In the network formed solely by macroeconomic expectation variables, GDP emerges as the central variable, receiving several connections. This indicates that the economic contraction experienced during 2015-2016 created room for a potential recovery in medium- to long-term growth.

The inverse associations (negative partial correlations) between the expectations for the SELIC rate, CPI, and GPI with GDP suggest that the economy's idle capacity was substantial, implying that the return to growth would be gradual and would not exert significant inflationary pressure, thus reducing the need for a highly contractionary monetary policy.

On the side of the network composed of stock indices, we observe several interconnections among all indices, capturing the dynamics of improving medium- to long-term macroeconomic expectations.

The non-crisis period (01/01/2017 - 02/01/2020), following the Great Brazilian Recession of the 21^{st} century, was marked by an economic policy stance focused on reducing government spending, which led to lower interest rates. However, economic growth remained below expectations[14]. The decline in interest rates, nonetheless, allowed variable-income financial assets to experience their strongest performance across the entire sample period.

Analyzing the network estimated with year-end macroeconomic expectations (Figure 4), we find no associations between the stock market and macroeconomic expectations. The network formed by stock indices shows that the financial index (IFNC) is positively associated with ICON, UTIL, IEE, INDX, and IMOB. This indicates that lower interest rates benefited the utilities, consumer, industrial, and construction sectors by fostering higher revenues and investment, while the financial sector expanded credit and capital market operations to support these industries.

In the network formed by macroeconomic expectations, GDP emerges as one of the key variables, with two positive associations standing out: (1) PB→GDP: The Primary Balance (PB) improves as GDP grows, suggesting that stronger economic activity enhances government revenues, helping to cover expenditures excluding debt service; (2) NPSD→GDP: This relationship is particularly noteworthy because, despite higher revenues, expectations for the Net Public Sector Debt (NPSD) deteriorated due to the burden of debt interest payments, since a relatively high SELIC rate significantly affected debt expectations.

Analyzing the same non-crisis period (01/01/2017 - 02/01/2020) from the perspective of the three-year-ahead macroeconomic expectations (Figure 5), we observe that the macroeconomic variables and the stock market indices do not exhibit any connections. In the cluster formed by macroeconomic expectations, the Primary Balance (PB) and the Consumer Price Index (CPI) emerge as the two most influential variables. This indicates that the shift in economic policy toward tighter control of public expenditures and the maintenance of price stability were the key drivers in the medium to long term. In the network composed solely of stock indices, the associations among all indices persist, reflecting a generally positive outlook regarding macroeconomic conditions.

In The Acute Phase of COVID-19 (Figure 4 and Figure 5: 02/02/2020 - 12/31/2020), we observe that the networks estimated using both the year-end and three-year-ahead macroeconomic expectations exhibit few connections and no links between stock indices and macroeconomic expectations. The health crisis led to a sharp reduction in the movement of people and the breakdown of global supply chains, severely affecting household income and corporate cash flows.

[14] According to data from the Brazilian Institute of Geography and Statistics (IBGE), the cumulative decline in GDP during the 2015-2016 biennium was 6.71%. The cumulative growth over the 2017–2019 triennium reached 4.39%, which fell short of the 7.19% growth required to return to the 2014 GDP level.

This event was unprecedented in a globalized economy characterized by free flows of capital and people, meaning that market participants had no clear reference or framework for how to react. Consequently, the heightened uncertainty surrounding the event acted as the key catalyst reducing the number of associations within the estimated networks.

In the Post-Acute Phase of COVID-19 (01/01/2021 - Present[15]), the fiscal and monetary stimuli implemented by the government to support the real economy and financial markets resulted in a rapid and robust recovery, with GDP expanding by 4.8% in 2021 and 3% in 2022. The networks estimated based on both year-end and three-year-ahead macroeconomic expectations (see Figure 4 and Figure 5, respectively) show no associations between macroeconomic expectations and the stock market. This indicates that during a period of strong economic rebound, these two segments operated independently, without mutual influence.

Analyzing the Post-Acute Phase of COVID-19, the network estimated with year-end macroeconomic expectations (Figure 4) reveals that the network formed solely by stock indices is dense and displays only positive associations, indicating a unidirectional movement with no potential for hedging among assets. The indices IEE, UTIL, and ICON received the highest number of connections, suggesting that the electricity and utilities sectors — characterized by stable and predictable revenues-remained resilient even in a high-interest-rate environment[16]. Meanwhile, the consumer sector benefited from fiscal expansion policies that increased household savings.

In the network composed exclusively of macroeconomic expectations, GDP emerges as a central variable, receiving four associations — three positive and one negative. The positive links indicate that higher GDP levels tend to improve the Primary Balance (PB), attract greater Foreign Direct Investment (FDI), and accelerate inflation (CPI). The negative association implies that higher GDP levels are related to a lower Net Public Sector Debt (NPSD), reflecting increased government revenue to cover expenditures. Two other macro variables of significant influence are CPI and PB, indicating that the fiscal stimulus implemented by the government to sustain the economy during The Acute Phase of COVID-19 led to higher expectations for both inflation and the primary fiscal balance.

When considering the network estimated for the Post-Acute Phase of COVID-19 using three-year-ahead macroeconomic expectations (Figure 5), we observe no associations between the stock market and medium- to long-term macroeconomic expectations. The network composed solely of financial assets becomes less dense, but all assets remain connected, with

[15] We consider December 6th, 2023 as the current date.
[16] The SELIC target rate rose from 2% p.a. to 13.75% p.a. over the period from January 1, 2021, to December 31, 2022.

a notable inverse association (negative partial correlation) between IMAT and ICON. This indicates that the basic materials sector, which is primarily export-oriented, can serve as a hedge against the consumer sector, which is more focused on the domestic market.

On the macroeconomic side, two distinct networks emerge: (1) External and fiscal variables: Trade Balance (TB), Foreign Direct Investment (FDI), Current Account (CA), Exchange Rate (ER), and Primary Balance (PB); (2) Price level, activity, and interest rate variables: Consumer Price Index (CPI) and General Price Index (GPI), GDP, and SELIC.

The trade balance is expected to remain structurally higher than observed up to 2022 due to significant growth in oil production and increased agricultural productivity, serving as an important driver for economic activity and positively impacting the primary fiscal balance.

Finally, considering the micro-reforms implemented over the past eight years and the closer alignment of monetary and fiscal policies, GDP is negatively associated with the Selic rate, which in turn is positively associated with CPI. This reflects a monetary policy framework that is more firmly anchored to control potential economic overheating.

5. CONCLUSION

The main objective of this study was to examine the presence of associations between macroeconomic expectation variables and the main Brazilian sectoral stock indices over the period from 01/02/2007 to 12/06/2023. We employed a dynamic Bayesian network approach in an effort to identify these associations. This method is more flexible and sophisticated compared to traditional linear techniques, aiming to better capture the potential dynamic relationships between macroeconomic expectation variables and sectoral stock indices within the Brazilian context.

The results show that year-end and three-year-ahead macroeconomic expectations and sectoral stock indices behave independently across the full sample and within both the five non-crisis periods and the four crisis periods. Only three associations emerged between macroeconomic expectations and the stock market, all during the European Sovereign Debt Crisis (03/01/2012-10/31/2012), as shown in Figure 4 and Figure 5. These findings indicate that stock market movements in Brazil respond more strongly to firm-level economic and financial fundamentals and sector-specific characteristics, particularly because listed firms are typically the largest and most efficient in their respective sectors. As a result, they tend to be less sensitive to economic cycles and often increase market share once crises dissipate.

The Brazilian stock market remains relatively small compared with those of advanced economies. Market capitalization of firms listed on B3 totals approximately BRL 4.8 trillion[17], while the combined stock of public and corporate debt reaches BRL 8.12 trillion[18], nearly twice as large, according to data from B3 and the National Treasury Secretariat (STN).

Additional insights arise from the clusters formed solely by macroeconomic expectation variables and solely by stock indices. When year-end expectations are considered alongside stock indices, the macroeconomic cluster consistently identifies economic activity (GDP) and the external sector (CA) as the variables exerting the strongest influence on other macroeconomic indicators, as they receive the highest number of associations across all estimated networks.

The cluster composed of stock indices displays dense and predominantly unidirectional positive associations among assets during both non-crisis and crisis periods, with the notable exception of the acute phase of the COVID-19 crisis. When the network is estimated using three-year-ahead macroeconomic expectations alongside stock indices, a clear pattern emerges. In the macroeconomic cluster, inflation (CPI) and the external sector (CA) consistently appear as the variables receiving the most associations, depending on the period analyzed. This suggests that, in the medium to long run, price stability and a Current Account with limited deficits play a central role in shaping agents' macroeconomic expectations.

In the stock index cluster, positive associations become less dense. However, a recurrent inverse association between IMAT and ICON appears across multiple periods. This inverse relationship may reflect a potential hedge position: when domestic consumption weakens, assets more exposed to external markets, such as IMAT, tend to perform relatively better. For policymakers, the main implication is that economic activity and the Current Account balance are the key macroeconomic expectations to monitor in the short run, whereas price stability, expectation anchoring, and maintaining a sustainable Current Account become more influential in the medium to long run. The hypothesis that the stock market functions as a "barometer" for the real economy is not supported by the evidence, although unidirectional positive associations within the stock index cluster may amplify adverse shocks once they occur.

Future research could apply machine learning techniques and other nonlinear models to further explore potential associations between macroeconomic expectations and the stock market. Such approaches may uncover additional insights into the dynamics linking Brazil's macroeconomy and equity market.

[17] Position as of December 2023.
[18] As of December 2023, public debt stood at BRL 6.52 trillion, while corporate debt totaled BRL 1.6 trillion.

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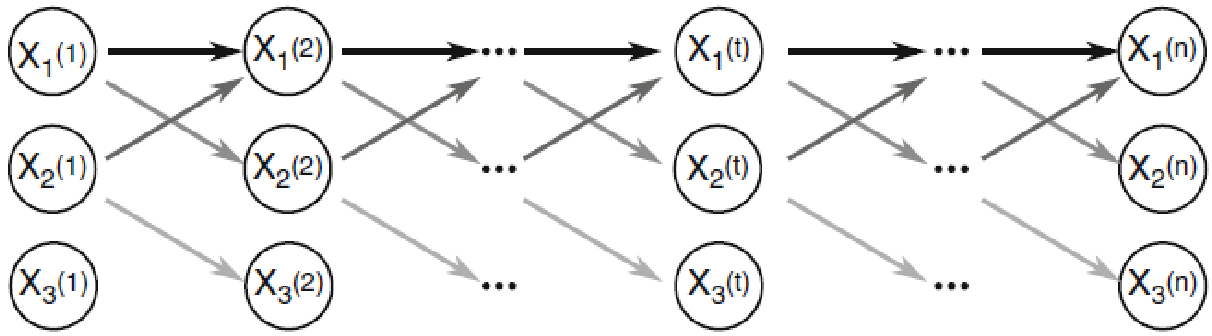
APPENDIX

A. Graphical Structure of a Dynamic Bayesian Network

We present the structure of a dynamic Bayesian network over multiple time periods:

Figure 6 - Graphical Representation of a Dynamic Bayesian Network with Temporal Variation



If the information of the variables is available over time, it is possible to map the feedback cycles and loops across temporal points. Therefore, we can define these interactions as a time-homogeneous dynamic Bayesian network, assuming that, at each time period t, all the parents of each node are measured at the previous time point $t-1$. It is important to emphasize that the dynamic Bayesian network at a given time period cannot have acyclic forward-backward movements.

Source: Nagarajan et al. (2013).

B. Significance Measures: local false discovery rate (fdr) and q-value

An efficient estimator of the covariance matrix can be obtained by setting the empirical correlation coefficients to zero and the empirical variances to their medians (Nagarajan et al., 2013).

In this context, the work conducted by Opgen-Rhein and Strimmer (2007a) and Schäfer and Strimmer (2005) present an algorithm that enables robust estimation of VAR(1) coefficients for Dynamic Bayesian Networks. Gaussian Graphical Models (GGM) are estimated for each Directed Acyclic Graph (DAG) based on applying shrinkage estimators to estimated covariances and partial correlation matrices, which represent the interactions between variables. The network structure is determined by including edges in descending order of coefficients and employing multiple tests of the local false discovery rate (local fdr), which tests the existence of false positives (edges with null probability) and eliminates them. Thus, only significant edges remain.

In Equation 10, we define the observed partial correlations $\tilde{r}$ in the edges:

$$f(\tilde{r}) = \eta_0 f_0(\tilde{r}; k) + (1 - \eta_0) f_A(\tilde{r}), \tag{10}$$

where f_0 is the null distribution, η_0 is the (unknown) proportion of "null edges", and f_A is the distribution of observed partial correlations assigned to actually existing edges. The null density f_0 is given by:

$$f_0(\tilde{r}; k) = (1 - \tilde{r}^2)^{(k-3)/2} \frac{\Gamma\left(\frac{k}{2}\right)}{\pi^{1/2}\Gamma\left(\frac{(k-1)}{2}\right)} = |\tilde{r}|\mathrm{Be}\left(\tilde{r}^2; \frac{1}{2}, \frac{k-1}{2}\right), \tag{11}$$

where $\mathrm{Be}(x; a, b)$ is the Beta distribution and k is the degrees of freedom, equal to the reciprocal variance of the null $\tilde{r}$. Fitting this mixture density allows k, η_0, and f_A to be determined. Subsequently, we can calculate the specific edge's false discovery rate (fdr) using Equation 12:

$$\mathrm{Prob}(\text{null edge}|\tilde{r}) = \mathrm{fdr}(\tilde{r}) = \frac{\hat{\eta}_0 f_0(\tilde{r}; \hat{k})}{\hat{f}(\tilde{r})}. \tag{12}$$

The Equation 12 highlights the local fdr as the posterior probability of an edge being null given $\tilde{r}$. In the estimation process, the q-values associated with each edge and the probabilities of an edge being non-null $(1 - \mathrm{fdr})$ are calculated, and these quantities can be used to define the significance level of the edges.

The q-value is a measure of statistical significance adjusted by the FDR (False Discovery Rate) and is used to address multiple testing statistical problems, where q-values represent the expected percentage of false positives among significance tests. On the other hand, p-values only indicate the significance level considering the overall number of performed tests (Storey, 2002).

Storey (2002) defines the FDR as the expected proportion of false positives among all rejected hypotheses multiplied by the probability of at least one rejection occurring. Storey (2003) establishes the pFDR (positive false discovery rate) to demonstrate that we are conditioned on at least one positive finding occurring:

$$\mathrm{pFDR} = \mathbb{E}\left(\frac{V}{R}|R > 0\right), \tag{13}$$

where V is the number of type I errors (or false positive results), and R is the number of rejected hypotheses. For a set of rejection regions $\{\Gamma\}$, these could be all sets of the form $[c, \infty)$ for $-\infty \leq c \leq \infty)$, where the p-value of an observed statistic $T = t$ is defined as:

$$p - \mathrm{value}(t) = \min_{\{\Gamma: t\in\Gamma\}} \{\Pr(T \in \Gamma|\, H = 0)\}. \tag{14}$$

According to Storey (2002), the p-value provides a measure of the strength of the observed statistic in terms of committing a type I error, i.e., it is the minimum rate of type I error that can occur when rejecting a statistic with value t for the set of rejection regions. On the other hand, the q-value is a measure of the strength of an observed statistic in relation to pFDR, being the minimum pFDR that can occur when rejecting a statistic with value t for the set of significance regions (Storey, 2003). Thus, for an observed statistic $T = t$, the q-value of t is defined as:

$$q(t) = \inf_{\{\Gamma: r \in \Gamma\}} \{\text{pFDR}(\Gamma)\}. \tag{15}$$

When statistics are independent p-values, the definition is simplified, and the set of rejection regions takes the form $[0; \gamma]$ and pFDR can be written more simply. Therefore, for a set of hypothesis tests conducted with independent p-values, the q-value of the observed p-value p is:

$$q(p) = \inf_{\gamma \geq p} \{\text{pFDR}(\gamma)\} = \inf_{\gamma \geq P} \left\{ \frac{\pi_0 \gamma}{\Pr(P \leq \gamma)} \right\}. \tag{16}$$

According to Schäfer and Strimmer (2005) , the q-value is intrinsically related to local Bayesian Fdr statistics. Efron (2005) asserts that using local Fdr is more appropriate because it naturally fits the mixture model setup and considers dependencies among estimated correlation coefficients. Therefore, we choose to use it as our measure of significance.

Efron (2007) asserts that the Bayesian posterior probability that a case is null given z, by definition, is the local false discovery rate:

$$\text{Fdr}(z) \equiv \Pr\{\text{null}|z\} = \frac{p_0 f_0(z)}{f(z)} = \frac{f_0^+(z)}{f(z)}, \tag{17}$$

where $p_0 = \Pr\{\text{null}\}, f_0(z) = \text{density if null},$

$p_1 = \Pr\{\text{non} - \text{null}\}, f_1(z) = \text{density if non} - \text{null}.$

Benjamini and Hochberg (1995) theorized about the discovery rate that relies on tail areas rather than densities. Assuming $F_0(z)$ and $F_1(z)$ as the cumulative distribution functions corresponding to $f_0(z)$ and $f_1(z)$, they define $F_0^+(z) = p_0 F_0(z)$ and $F_0(z) = p_0 F_0(z) + p_1 F_1(z)$. Consequently, we can infer that the posterior probability of a case being null given its z-value, Z, is less than some value z is:

$$\text{Fdr}(z) \equiv \Pr\{\text{null}|Z \leq z\} = \frac{F_0^+(z)}{F(z)}. \tag{18}$$

Therefore, $\text{Fdr}(z)$ corresponds to the q-value defined by Storey (2002) and to the false discovery rate value of the tail area achieved at a specific observed value $Z = z$. Analytically, Fdr is a conditional expectation of fdr:

$$\text{Fdr}(z) = \frac{\int_{-\infty}^{z} \text{fdr}(Z) f(Z)\, dZ}{\int_{-\infty}^{z} f(Z)\, dZ} = E_f\{\text{fdr}(Z)|Z \leq z\}, \tag{19}$$

E_f indicates the expectation concerning $f(z)$. In other words, $\text{Fdr}(z)$ is the average of $\text{fdr}(Z)$ for $Z \leq z$. $\text{Fdr}(z)$ will be lower than $\text{fdr}(z)$ in the usual situation where $\text{fdr}(z)$

decreases as $|z|$ increases. In the works of Opgen-Rhein and Strimmer (2007a) and Schäfer and Strimmer (2005), the authors detail the construction of the algorithm.

Finally, we can express that each new DAG ($posterior$ distribution) is composed of the previous DAG's data ($prior$ distribution) plus the data from that period (likelihood). Therefore, the parameters are sequentially updated over time, so that the past parameters of dependency influence the estimation of future dependency. It is important to mention that the initial distribution ($prior$) is uniform. By analyzing the networks formed over the periods, we will observe changes in interactions between assets through the direction of edges.

C. How GeneNet Uses the VAR Model

The dynamic procedure in the GeneNet package (invoked with *ggm.estimate.pcor(method = "dynamic")*) performs an internal pre-processing step based on a VAR(1) model. This VAR is not used for forecasting or for economic interpretation; its sole purpose is statistical filtering to remove serial autocorrelation before estimating the network.

1. Preliminary temporal modeling

For each variable (gene or, in this context, financial asset), GeneNet internally estimates:

$$X(t) = AX(t-1) + \varepsilon(t), \tag{20}$$

where:

A is the VAR(1) coefficient matrix; and

$\varepsilon(t)$ denotes the residuals, representing the component of the series not explained by its immediate past.

This step is executed automatically by the method, requiring no manual VAR estimation from the user.

2. Why GeneNet applies a VAR before network estimation

Time series — especially financial ones — exhibit autocorrelation, meaning each variable is influenced by its own past.

If GeneNet were to compute partial correlations directly from the raw series, many of the detected connections would be spurious, driven only by the temporal structure of the individual series.

The internal VAR step serves to:

- Remove autocorrelation;
- Disentangle the endogenous temporal component from the cross-sectional relational component; and

- "Clean" the series by producing residuals free from their own past dependence.

3. The VAR residuals are used to estimate the network

After the VAR step, GeneNet does not use the original series. It relies exclusively on:

$$\varepsilon(t) \tag{21}$$

These residuals capture what remains after removing the temporal effect, isolating:

- Contemporaneous interactions;
- Pure conditional dependencies among assets;
- Variability attributable to other variables rather than to time.

4. Estimation of dynamic partial correlations

Using the residuals, GeneNet estimates:

$$\mathrm{PCOR_{ij}} \\ = \text{partial correlation between } \varepsilon_{\mathrm{i}}(\mathrm{t}) \text{ and } \varepsilon_{\mathrm{j}}(\mathrm{t}) \text{ conditional on all other variables.} \tag{22}$$

These partial correlations undergo shrinkage and are subsequently subjected to significance testing.

5. Determination of putative direction

The dynamic method assumes a temporal structure in which:

- effects at $t-1$→ outcomes at t.

Thus, when the dynamic partial correlations suggest dependence between filtered series, GeneNet infers a putative direction:

$$X_i(t-1) \to X_j(t) \tag{23}$$

This direction is putative–i.e., based on temporal ordering rather than confirmed causal relations.

6. Summary

GeneNet uses a VAR(1) solely as a filtering step to remove serial autocorrelation; the network itself is estimated from the partial correlations of the VAR residuals, which represent the pure conditional dependencies among the variables.

Table 3 provides an overview of the network estimation procedures implemented in RStudio.

Table 3 - Procedures Performed by GeneNet to Estimate Dynamic Bayesian Networks in RStudio

Order	What Happens	R Function	Input	What Is Generated	Why Is It Necessary?	Output Format
1	Load and organize time-ordered data	Import and structuring	Original expression matrix	Time × financial asset matrix	Preserve temporal ordering	Observational matrix
2	Fit a VAR(1) from financial asset to financial asset	Internal step within *ggm.estimate.pcor(method = "dynamic")*	Original expression matrix	VAR residuals	Remove temporal autocorrelation	Residual matrix and
2.1	Estimate dynamic partial correlations (shrinkage)	*ggm.estimate.pcor(..., "dynamic")*	Residual matrix	Dynamic partial-correlation matrix	Capture conditional dependence among financial assets	Matrix (p × p)
3	Test the significance of edges	*network.test.edges()*	Partial-correlation matrix	Test statistics, p-values, local FDR	Verify that an edge is not noise	Table of financial-asset pairs
4	Edge selection	*extract.network(cutoff = α)* Observation: Efron (2007) suggests q-values between 0.05 and 0.15 as feasible choices for estimating networks, with these q-value limits interpreted as a conservative Bayesian factor for interpreting the FDR.	local FDR	Selected bipartite network	Control false positives	List or adjacency matrix
5	Determination of (putative) direction	Implicit in the dynamic method	Selected network	Arrows A → B	Represent temporal influence	Directed network
6	Visualization and export	*network.make.graph() / edge.info()* Additionally, the packages *Rgraphviz* and *Igraph* are employed.	Final network	Graph	Interconnections	DOT file / image

D. Partial Correlations of the Dynamic Bayesian Networks

a. Partial Correlations: End-of-Year Macroeconomic Expectations and Daily Log-Returns of Sectoral Stock Indices

Table 4 - The Entire Period: 02/01/2007 - Present

GDP→CA	GDP→PB	CPI→TB	CPI→PB	CPI→GDP	GPI→SELIC
-0.14144	0.56482	-0.11427	-0.21587	0.21815	0.15431
FDI→GDP	FDI→TB	FDI→PB	FDI→CA	TB→CA	ER→TB
0.08641	0.13663	0.27459	-0.31046	0.67518	0.07719
ER→GDP	ER→PB	ER→NPSD	ER→FDI	NPSD→PB	NPSD→GDP
0.09693	-0.10669	0.11074	-0.11514	-0.16927	0.18433
NPSD→CPI	PB→TB	IMAT→ICON	IMAT→IFNC	IMAT→INDX	IMOB→INDX
-0.3645	0.08229	-0.11301	0.12853	0.56486	0.12985
IMOB→ICON	IMOB→IFNC	ICON→IEE	ICON→UTIL	ICON→INDX	IEE→UTIL
0.23462	0.23903	0.10223	0.10924	0.42798	0.6703
IFNC→IEE	IFNC→INDX	IFNC→UTIL	IFNC→ICON		
0.09202	0.09311	0.10631	0.16693		

Table 5 - Non-Crisis: 01/02/2007 - 11/30/2007

GDP→NPSD	CPI→ER	CPI→SELIC	GPI→TB	GPI→SELIC	TB→CA
-0.42804	0.18134	0.2144	-0.15609	0.25005	0.44108
CA→PB	ER→SELIC	PB→NPSD	IMAT→INDX	ICON→UTIL	ICON→IEE
-0.19341	0.16034	0.40508	0.53329	0.12761	0.14455
ICON→INDX	IEE→INDX	IFNC→IMAT	IFNC→ICON	UTIL→IEE	
0.37101	0.13445	0.20249	0.21974	0.57939	

Table 6 - Crisis: 12/01/2007 - 12/31/2009

CPI→FDI	CPI→GDP	GPI→SELIC	FDI→GDP	TB→GDP	TB→FDI
0.18805	0.2354	0.14552	0.26189	0.21155	0.25301
CA→CPI	CA→TB	ER→NPSD	NPSD→TB	NPSD→CA	PB→CPI
-0.15637	0.27059	-0.1623	0.21609	0.28638	0.15607
PB→TB	PB→GDP	PB→FDI	IMAT→INDX	IMOB→IMAT	IMOB→ICON
0.17139	0.17401	0.17414	0.317	0.14258	0.14501
IMOB→INDX	IMOB→IFNC	ICON→INDX	IEE→ICON	IFNC→INDX	IFNC→ICON
0.15446	0.17007	0.1985	0.16328	0.14309	0.14884
IFNC→IMAT	UTIL→ICON	UTIL→IEE			
0.16265	0.1455	0.37931			

Table 7 - Non-Crisis: 01/01/2010 - 02/29/2012

GDP→CPI	GDP→PB	CPI→NPSD	CPI→CA	CPI→TB	FDI→CA
0.10436	0.2115	0.152	0.16987	0.19471	-0.09624
FDI→CPI	FDI→PB	TB→CA	ER→PB	ER→GDP	ER→FDI
0.1875	0.27727	0.29589	0.08078	-0.14311	0.15167
NPSD→TB	NPSD→CA	PB→NPSD	PB→CPI	IMAT→ICON	IMAT→INDX
0.22682	0.24123	-0.08868	0.19349	0.09796	0.23383
IMOB→IMAT	IMOB→ICON	IMOB→IFNC	IMOB→INDX	ICON→INDX	IEE→INDX
0.10751	0.13691	0.15065	0.17055	0.21191	0.07929
IEE→ICON	IFNC→IEE	IFNC→ICON	IFNC→INDX	IFNC→IMAT	UTIL→INDX
0.10205	0.08215	0.12541	0.14807	0.15306	0.07687
UTIL→IFNC	UTIL→ICON	UTIL→IEE			
0.08196	0.09337	0.32344			

Table 8 - Crisis: 03/01/2012 - 10/31/2012

GDP→ER	TB→CA	CA→IMOB	SELIC→PB	PB→ER	IMAT→INDX
-0.18221	0.21917	-0.16691	-0.19386	0.1951	0.46762
IMOB→ICON	IMOB→INDX	ICON→INDX	IFNC→INDX	IFNC→IMAT	UTIL→IEE
0.19512	0.28673	0.49383	0.15542	0.27669	0.81868

Table 9 - Non-Crisis: 11/01/2012 - 12/31/2014

GDP→CA	CPI→CA	CPI→NPSD	CPI→GDP	CPI→TB	GPI→SELIC
-0.27876	0.0946	0.09563	-0.10453	0.10696	0.08798
FDI→NPSD	FDI→TB	FDI→PB	FDI→ER	FDI→GDP	TB→GDP
0.092	-0.14156	0.15332	-0.16841	-0.26033	-0.0967
TB→CA	SELIC→ER	ER→TB	ER→PB	NPSD→CA	NPSD→GDP
0.33252	0.10457	0.14731	-0.20475	0.16864	-0.34399
PB→NPSD	PB→TB	IMAT→INDX	IMOB→INDX	IMOB→UTIL	IMOB→IEE
-0.14141	-0.21291	0.34101	0.10894	0.11504	0.14423
IMOB→ICON	IMOB→IFNC	ICON→INDX	IFNC→IEE	IFNC→UTIL	IFNC→INDX
0.15644	0.22305	0.3654	0.08908	0.12001	0.12446
IFNC→ICON	UTIL→ICON	UTIL→IEE			
0.18836	0.08437	0.45149			

Table 10 - Crisis: 01/01/2015 - 12/31/2016

GDP→CA	GDP→NPSD	GDP→TB	GDP→CPI	CPI→TB	CPI→CA
0.11392	0.1159	0.1219	-0.14798	-0.15002	-0.15081
FDI→NPSD	FDI→GDP	FDI→TB	FDI→CPI	FDI→CA	TB→CA
-0.07113	-0.08071	-0.1135	0.11608	-0.13431	0.16336
ER→NPSD	ER→CPI	ER→FDI	ER→TB	ER→CA	NPSD→TB
0.08261	-0.08749	-0.10064	0.10179	0.10866	0.13306
NPSD→CPI	NPSD→CA	PB→NPSD	PB→CA	PB→ER	PB→TB
-0.13307	0.13934	0.08923	0.09506	0.09666	0.11014
PB→GDP	IMAT→INDX	IMOB→INDX	IMOB→IEE	IMOB→UTIL	IMOB→ICON
0.11654	0.21325	0.08119	0.11617	0.12266	0.1386
IMOB→IFNC	ICON→IEE	ICON→UTIL	INDX→UTIL	INDX→ICON	IEE→UTIL
0.18202	0.0963	0.09745	0.07491	0.23659	0.27902
IFNC→INDX	IFNC→IEE	IFNC→UTIL	IFNC→ICON		
0.08553	0.12698	0.12741	0.14471		

Table 11 - Non-Crisis: 01/01/2017 - 02/01/2020

GDP→CA	CPI→CA	CPI→TB	CPI→GDP	FDI→NPSD	FDI→PB
-0.17307	-0.25266	-0.29121	-0.29276	0.09261	0.13634
TB→GDP	TB→NPSD	TB→CA	SELIC→ER	ER→GDP	ER→FDI
-0.13918	-0.15573	0.32267	0.15799	0.11611	-0.16186
NPSD→CA	NPSD→GDP	PB→NPSD	PB→GDP	PB→CA	IMAT→INDX
-0.20861	0.4053	0.09478	0.17634	-0.19474	0.39407
IMOB→UTIL	IMOB→IEE	IMOB→ICON	INDX→ICON	IEE→ICON	IEE→UTIL
0.14018	0.15788	0.23114	0.35584	0.09082	0.43277
IFNC→INDX	IFNC→IEE	IFNC→UTIL	IFNC→ICON	IFNC→IMOB	UTIL→ICON
0.08772	0.12749	0.15883	0.18425	0.2098	0.12922

Table 12 - Crisis: 02/02/2020 - 12/31/2020

GDP→SELIC	GPI→CA	GPI→CPI	FDI→PB	FDI→CA	TB→NPSD
0.29198	0.18843	0.21315	0.18949	-0.34424	0.22757
TB→CA	ER→GDP	ER→SELIC	NPSD→PB	IMAT→INDX	IMOB→ICON
0.36591	-0.27775	-0.34337	-0.55731	0.52027	0.33276
ICON→INDX	IFNC→IMOB	UTIL→IEE			
0.46531	0.30702	0.63632			

Table 13 - Non-Crisis: 01/01/2021 - Present

CPI→PB	CPI→FDI	CPI→GDP	GPI→SELIC	FDI→GDP	FDI→PB
-0.08954	-0.20873	0.31346	0.24376	0.12391	0.2517
TB→ER	TB→CA	CA→CPI	CA→PB	CA→FDI	SELIC→CPI
-0.15905	0.19596	0.09838	-0.09906	-0.2132	0.07511
SELIC→ER	ER→CPI	ER→FDI	ER→PB	NPSD→GDP	NPSD→CPI
0.10299	0.09226	-0.0946	-0.10753	-0.13031	-0.27634
PB→GDP	IMAT→IFNC	IMAT→INDX	IMOB→INDX	IMOB→UTIL	IMOB→IEE
0.4581	0.0831	0.38455	0.08946	0.13456	0.14982
IMOB→ICON	ICON→IEE	ICON→UTIL	INDX→UTIL	INDX→ICON	IEE→UTIL
0.27105	0.11723	0.12615	0.07822	0.23875	0.41325
IFNC→IEE	IFNC→INDX	IFNC→UTIL	IFNC→ICON	IFNC→IMOB	
0.1079	0.11622	0.12688	0.15672	0.17857	

b. *Partial Correlations: Three-Year-Ahead Macroeconomic Expectations and Daily Log-Returns of Sectoral Stock Indices*

Table 14 - The Entire Period: 02/01/2007 - Present

GDP→FDI	GDP→CPI	GDP→SELIC	GPI→FDI	GPI→SELIC	GPI→CPI
0.0559	-0.08383	-0.14411	-0.05447	0.08119	0.17693
FDI→CPI	FDI→TB	FDI→CA	TB→CA	SELIC→CPI	ER→SELIC
-0.05274	0.06523	-0.28548	0.33741	0.28803	-0.05004
ER→CPI	ER→TB	NPSD→FDI	NPSD→ER	NPSD→GDP	PB→FDI
-0.06443	0.08914	0.04891	0.05052	-0.0514	0.06435
PB→CPI	PB→GPI	IMAT→ICON	IMAT→INDX	IMOB→IMAT	IMOB→INDX
-0.09432	-0.10378	-0.3826	0.75426	-0.06059	0.11131
IMOB→ICON	IMOB→IFNC	ICON→UTIL	ICON→IEE	ICON→INDX	IEE→UTIL
0.22328	0.27281	0.07333	0.07389	0.63231	0.85315
IFNC→IEE	IFNC→UTIL	IFNC→IMAT	IFNC→ICON		
0.05186	0.08475	0.13745	0.1696		

Table 15 - Non-Crisis: 01/02/2007 - 11/30/2007

GDP→NPSD	GPI→NPSD	GPI→CPI	FDI→CA	FDI→CPI	TB→NPSD
-0.35286	-0.15248	0.30773	-0.19207	0.19892	-0.20504
CA→TB	SELIC→NPSD	SELIC→TB	SELIC→GPI	ER→FDI	PB→NPSD
0.4387	0.15372	0.16405	0.17163	-0.16917	0.22833
IMAT→INDX	ICON→IEE	ICON→INDX	IFNC→IMAT	IFNC→ICON	UTIL→IEE
0.52715	0.15214	0.35658	0.18751	0.21383	0.5858

Table 16 - Crisis: 12/01/2007 - 12/31/2009

GDP→CA	GDP→CPI	GDP→NPSD	CPI→SELIC	GPI→GDP	GPI→CA
-0.13219	-0.13632	-0.18444	0.2684	0.10281	0.13732
FDI→CA	TB→FDI	TB→CA	ER→NPSD	NPSD→CA	NPSD→SELIC
-0.43511	0.17131	0.28842	-0.12395	-0.13564	0.30271
PB→CA	PB→SELIC	IMAT→INDX	IMOB→ICON	IMOB→INDX	IMOB→IFNC
0.11013	0.14066	0.70473	0.15308	0.15375	0.2264
ICON→IMAT	ICON→IEE	ICON→INDX	IFNC→IMAT	IFNC→ICON	UTIL→IEE
-0.11714	0.17515	0.38251	0.16824	0.17173	0.7388

Table 17 - Non-Crisis: 01/01/2010 - 02/29/2012

CPI→NPSD	GPI→CA	GPI→CPI	TB→NPSD	TB→SELIC	TB→CA
0.14667	-0.13352	0.13648	-0.14027	0.15424	0.24852
CA→FDI	SELIC→NPSD	ER→NPSD	NPSD→PB	PB→CA	PB→FDI
-0.33428	0.28887	-0.17446	-0.17751	0.14593	-0.26965
IMAT→INDX	IMOB→IFNC	IMOB→INDX	ICON→IMAT	ICON→INDX	IFNC→IMAT
0.64452	0.22935	0.31541	-0.24257	0.57012	0.19189
UTIL→IEE					
0.77503					

Table 18 - Crisis: 03/01/2012 - 10/31/2012

GDP→CPI	CPI→ICON	CPI→SELIC	CPI→PB	GPI→ICON	GPI→SELIC
0.13018	0.13361	0.13799	0.14626	-0.13329	0.20784
TB→NPSD	TB→CA	CA→FDI	ER→GPI	ER→GDP	ER→TB
-0.13386	0.23517	-0.43278	0.13579	-0.16427	0.16632
NPSD→PB	NPSD→FDI	PB→CA	PB→FDI	IMAT→INDX	IMOB→ICON
-0.17009	0.22086	0.17797	-0.25182	0.45816	0.21389
IMOB→INDX	ICON→INDX	IFNC→INDX	IFNC→IMAT	UTIL→IEE	
0.28712	0.48014	0.14942	0.27976	0.80894	

Table 19 - Non-Crisis: 11/01/2012 - 12/31/2014

GDP→SELIC	CPI→SELIC	FDI→CA	TB→CA	NPSD→CPI	PB→NPSD
-0.20376	0.37769	-0.30461	0.30814	0.217	-0.20252
IMAT→ICON	IMAT→INDX	IMOB→IEE	IMOB→IFNC	ICON→INDX	IFNC→ICON
-0.31196	0.61329	0.15881	0.31223	0.67028	0.20618
UTIL→IEE					
0.74969					

Table 20 - Crisis: 01/01/2015 - 12/31/2016

CPI→GDP	GPI→CPI	GPI→GDP	FDI→CA	TB→FDI	TB→CA
-0.18879	0.19133	-0.22686	-0.34658	0.18266	0.28619
SELIC→GDP	SELIC→CPI	NPSD→FDI	NPSD→TB	NPSD→PB	PB→GDP
-0.1862	0.23723	-0.10155	0.17162	-0.18757	0.26877
IMAT→ICON	IMAT→INDX	IMOB→UTIL	IMOB→ICON	IMOB→IFNC	INDX→ICON
-0.175	0.50393	0.12162	0.19226	0.33675	0.57782
IEE→UTIL	IFNC→UTIL	IFNC→IEE	IFNC→ICON		
0.67105	0.11543	0.12053	0.22656		

Table 21 - Non-Crisis: 01/01/2017 - 02/01/2020

GDP→FDI	CPI→PB	GPI→PB	GPI→CPI	FDI→PB	FDI→ER
0.11905	-0.22033	-0.12822	0.17251	0.12889	-0.14793
FDI→NPSD	TB→FDI	TB→CA	CA→PB	SELIC→PB	SELIC→CPI
0.18081	-0.14167	0.25871	-0.23425	-0.16246	0.18855
ER→CPI	ER→PB	NPSD→SELIC	NPSD→CPI	NPSD→ER	IMAT→ICON
-0.12526	0.25633	-0.141	-0.14202	0.16659	-0.20446
IMAT→INDX	IMOB→IEE	IMOB→ICON	INDX→ICON	IEE→UTIL	IFNC→UTIL
0.57196	0.1595	0.30417	0.55227	0.63498	0.16678
IFNC→ICON	IFNC→IMOB	UTIL→ICON			
0.20344	0.24468	0.12436			

Table 22 - Crisis: 02/02/2020 - 12/31/2020

GDP→NPSD	CPI→GDP	TB→CA	ER→CPI	PB→NPSD	IMAT→INDX
-0.19148	-0.22712	0.50588	-0.18425	-0.49737	0.49796
IMOB→ICON	ICON→INDX	IFNC→IMOB	UTIL→IEE		
0.32678	0.43876	0.28611	0.60534		

Table 23 - Non-Crisis: 01/01/2021 - Present

GDP→GPI	GDP→SELIC	GPI→CPI	FDI→PB	FDI→CA	CA→TB
0.14516	-0.21789	0.18164	0.1544	-0.23397	0.3326
SELIC→CPI	ER→TB	IMAT→ICON	IMAT→INDX	IMOB→IEE	IMOB→ICON
0.2081	0.23472	-0.24822	0.67373	0.134	0.45316
INDX→ICON	IEE→UTIL	IFNC→ICON	IFNC→IMOB		
0.46642	0.78685	0.16004	0.21522		