XIII SIMMEC
Simpósio de Mecânica Computacional
29 de Outubro a 1º de Novembro de 2018
UFES - Campus Goiabeiras - Vitória, ES

MODELO PROBABILÍSTICO BASEADO NO MÉTODO DOS ELEMENTOS DE CONTORNO APLICADO À ESTRUTURAS DE CONCRETO

Probabilistic model based on the Boundary Element Method applied to concrete structures

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Resumo:

A modelagem estrutural é uma atividade essencial durante a fase de projeto. Nesta fase, o engenheiro é obrigado a prever o comportamento dos elementos estruturais que ainda se quer foram construídos. Existem várias incertezas que estão presentes em todas as etapas do projeto, em particular, na previsão de carregamentos efetivos ao longo da vida útil, no controle da geometria e também na estimativa das propriedades do material. É consenso atual que os modelos mais realistas devem levar esses aspectos em consideração. O presente estudo se concentra no desenvolvimento de um novo modelo numérico para prever a resposta probabilística de estruturas ao longo do tempo, considerando as aleatoriedades nos dados de entrada. O modelo utiliza o Método dos Elementos de Contorno (MEC) para resolver as equações integrais mecânicas que descrevem o comportamento de um sólido bidimensional. O material constitutivo pode sofrer fluência, e tal comportamento é modelado utilizando as leis viscoelásticas lineares sob a validade das hipóteses de Boltzmann. Os parâmetros materiais, os carregamentos e a geometria estrutural são descritos por uma lei de probabilística conhecida. Neste caso, um critério de falha probabilístico é proposto com base no método de superfície de resposta (MSR). As aplicações numéricas são inspiradas no mundo real e evidenciam que pequenos distúrbios geométricos não afetam consideravelmente a confiabilidade estrutural. Entre outras aplicações, o modelo numérico proposto é útil para os engenheiros no processo de tomada de decisão. Além disso, demonstra-se a versatilidade do MEC para resolver problemas reais de engenharia estrutural.

Palavras chaves: MEC; MSR; viscoelasticidade; confiabilidade.

Abstract:

Structural modeling is an essential activity during the design phase. At this stage, the engineer is required to predict the behavior of elements that have not yet been built. There are several uncertainties present in all design phase, in particular for prediction of effective loads along the lifespan, the geometry controlling and also the material properties estimation. It is consensus that more realistic models should take these aspects into consideration. The present study focuses on the development of a new framework for predicting the probabilistic response of structures varying with time considering the randomness in the input data. The model utilizes the Boundary Element Method (BEM) for solving the mechanical integral equations defined on two-dimensional solid. The constitutive material is admitted to creep, and such behavior is modeled utilizing the linear viscoelastic laws under the validity of Boltzmann hypotheses. The material parameters, loads and geometry are considered to follow *a priori* probabilistic distribution. In this case, a probabilistic failure criterion is proposed based on the Response Surface Method (RSM). The numerical applications are inspired from the real world and let evident that small geometric disturbances do not impact considerably the structural reliability. Among other applications, the proposed numerical framework is useful for engineers in the decision-making process. In addition, it demonstrates the versatility of the BEM for solving real based engineering problems.

Keywords: BEM; RSM; viscoelasticity; reliability.

1 INTRODUCTION

Several challenges are present into the daily practice of civil engineering constructions. Some of them are chosen to receive attention in the present study, especially the uncertainties quantification and propagation. Particularly, the influence of the geometrical deviations into the global mechanical behavior. In loco structures are often subjected to such a problem, depending on the construction quality control. If this is the case, the referred structural member must be carefully inspected by the specialized technical staff. Two options are readily available. The deviations can be considered minors and do not affect the global structural behavior. On the other hand, the deviations can imply into major effect on the global behavior and do require additional actions or even reconstruction. The present study focuses on the cases where the geometric deviations has minor influence.

The creep has major importance in the serviceability state of concrete structures, especially in cases of prestressing. The gross estimates of creep deformations lead to the undesirable consequences for designers, builders, owners, users, insurers, etc. Among such consequences, the material cracking and high strains values affect the structural behavior, for instance. In spite of the major importance of these problems, studies involving integrity, durability, and structural reliability involving time-dependent effects are not numerous, which justifies the development of the present study.

Structural engineering can be seen as a discipline that applies the laws of mechanics for the safe design of structural systems. This task necessarily requires the elaboration of prediction models of displacements, deformations and stresses. Normally, these structural models are based on variables that represent the properties of resistance, the physical dimensions, and the forces involved. These variables cannot be known with absolute certainty, and can be distincted into, at least, three research trends. The first line of investigation concerns the influence of uncertainties on resistance properties (Chateauneuf, Raphael, & Moutou Pitti, 2014; Jordaan, 1980). The second is concerned with investigations involving loading uncertainties (De Lima, Lambert, Rade, Pagnacco, & Khalij, 2014; Philpot, Fridley, & Rosowsky, 1994). The third, object of the present study, deals with the effects of uncertainties related to physical geometry. One mentions that the division has only the didactic purpose of localization of the proposed theme.

The consideration of the effects of creep on civil structures constituted of structural concrete is a practice already adopted by international normative codes. Moreover, the loss of structural prestress is manifested as a consequence of late deformations. Investigations point out that expressions in normative codes can provide deformations that do not reproduce accurately the experimental results (Chateauneuf et al., 2014). From an extensive database, the authors note that as the concrete ages, there is an increasing dispersion between the values verified and estimated for creep. The authors then suggest that a statistical model can be utilized to characterize the data set, giving rise to a phenomenological creep model. From the probabilistic creep model, it is possible to estimate the possible loss of prestress. The study shows that when the uncertainties involved are taken into consideration in both the mathematical model and the material model, it is possible to estimate more realistically such losses (for creep). The present study can be seen as an extension of that research, since it considers the effects of geometric variability on long-term deformations. A crucial improvement is the use of the BEM, which expands the possibilities of application, once complex structural systems can be considered as opposed to more restrictive analytical models.

Some studies have considered the effects of random loads on wood beams (Philpot et al., 1994). The wood was assumed as following the rheological model of Burges (an extension of the model of Boltzmann). Keeping the remaining deterministic characteristics, the authors propose a methodology to generate random loading histories, and then calculate the probability of prohibitive displacements. The authors suggest that simulation methods, such as Monte Carlo, are more suitable for calculating probability of failure in systems composed of viscoelastic materials. Note that, one possible way to introduce load histories is generating a succession of constant loads over time intervals. This possibility is guaranteed by the Boltzmann superposition principle. The investigations of these authors inspired the Monte Carlo method for the present study, as will be presented below. In their studies, the authors (Philpot et al., 1994) used simplified analytical models that enable the Monte Carlo simulation directly. The idea is extensible to other numerical techniques when the problem in hands are not reducible to an analytical model.

Viscoelastic materials can be used to attenuate vibrations in sandwich panels, for instance. Additionally, the use of viscoelastic materials can be beneficial from the point of view of structural reliability. Some methodologies have been developed to calculate the probability of failure of sandwich panels including the uncertainties associated with the loading (De Lima et al., 2014). The authors do not consider the uncertainties associated with the material parameters. They suggest instead the temperature effects should be investigated. This idea comes from the fact that temperature increasing usually decreases damping effects, which can lead to a decrease in system reliability.

Recent works have appeared for treating the uncertainties in a sophisticated manner (Capillon, Desceliers, & Soize, 2016; Wu, Wu, Gao, & Song, 2016). However, no investigations have been reported applying the BEM to the study of effects caused by the geometric variability of viscoelastic structural members, which also serves as additional motivation of the present study. The present communication is a brief presentation of the recently published paper by the authors (Oliveira, Chateauneuf, & Leonel, 2018). For more deep insights and examples, the reader is referred to that monograph.

In the following section, the theoretical aspects that are used to formulate the problem will be briefly discussed. The concepts are borrowed from several disciplines of applied sciences and engineering.

2 THEORETICAL BACKGROUND

2.1 Elements of Reliability Theory

Some important notions borrowed from the structural reliability field are utilized here for the definition of structural performance. These notions concern the concepts of Limit

State and Failure. In the following, these concepts are briefly described, as well as some particular reliability techniques of interest for the present study.

2.1.1 Limit State and Failure

The concept of limit state is used to define failure in the context of structural reliability. A limit state is the boundary between the desired and the undesired behavior of a structure. This boundary is often represented by a mathematical equation, that is, the limit state function, or performance function. This undesired state can occur due to the various failure modes: cracking, corrosion, excessive displacements, excessive vibration, local buckling, excessive stresses, among others. In the traditional approach, each failure mode is considered separately, and each mode is defined using the concept of limit state.

In structural reliability, two types of limit state are identified (Nowak & Collins, 2012). The first of these is the Ultimate Limit State. This state is related to the loss of bearing capacity of the structure. Examples of this limit state include the formation of plastic hinges, excessive inelastic strains, loss of stability, yielding of reinforcement, among others. The second type is the Service State Limit. This condition is related to the gradual deterioration, user comfort or loss of serviceability. They are not necessarily associated with structural integrity. Some examples are: excessive displacement, excessive vibration, permanent deformation, cracking and fatigue. Fatigue is associated with loss of resistance caused by repetitive stresses with failure mechanism usually occurring in the presence and propagation of cracks.

The classical notion of safety margin serves as the inspiration for the definition of a limit state equation. Consider that R represents the overall strength of the structure, whereas Q represents the effects caused by the loads. The limit state function defined with these terms is as follows:

$$G(R,Q) = R - Q \tag{1}$$

Note that failure occurs whenever the load effects are greater than the strength of the structure. The limit state corresponding to the boundary between desired and undesired performances occurs when $G(R,Q) = 0$. If $G(R,Q) > 0$, the structure is safe (expected performance); If $G\,(R,Q) < 0$, the structure is not safe (undesired performance). The probability of failure, P_f, is defined as the probability that the unwanted behavior occurs. Mathematically, the probability of failure can be expressed as follows:

$$P_f = \mathbb{P}(R - Q < 0) = \mathbb{P}(G < 0) \tag{2}$$

If a set of Random Variables (RVs) is generically represented by $\boldsymbol{Y}$, an alternative way of writing the expression of the failure probability is the following:

$$P_f = \mathbb{P}[G(\boldsymbol{Y}) < 0] = \int_{G(\boldsymbol{Y})\leq 0} f_Y(\boldsymbol{y})d\boldsymbol{y} \tag{3}$$

In the equation (3), the term f_Y is the joint Probability Density Function (PDF). Although the equation (3) seems simple, in general it is not possible to evaluate this integral directly. The reason is that often the function f_Y is not known explicitly. In addition, the failure domain (region where $G\,(\boldsymbol{Y}) \leq 0$) has a nonlinear boundary or even discontinuous boundary definition. Integration requires special techniques that ensure high accuracy. Therefore, in practice, the probability of failure is calculated indirectly using other procedures such as Approximate Techniques and Simulation Techniques. The present study utilizes the simulation technique that is further explained in the following.

2.1.2 Monte Carlo simulation technique

Simulation techniques are advantageous from the theoretical point of view, since they enable the direct consideration of the nonlinearities involved in both the mechanical model and the limit state function. The literature presents several simulation techniques. The simplest of them is the Monte Carlo method, which will be used to calculate the probability of failure in the present study. This technique is resumed in the following steps:

Step 1: A real number $y^{(r)}$ is selected in the interval [0 , 1]. Let $\boldsymbol{X} = (X_1, X_2, \dots, X_m)^T$ be the random design variables. Because $\boldsymbol{X}$ is known a priori, its cumulative probability functions, F_{X_i}, are also known. Therefore, one obtains the sample vector $x^{(r)} = (x_1^{(r)}, x_2^{(r)}, \dots, x_m^{(r)})$, which represents the vector $\boldsymbol{X}$, thus:

$$x_i^{(r)} = F_{X_i}^{-1}(y^{(r)}) \tag{4}$$

Step 2: Evaluation of the limit state equation. Failure, $G(x^{(r)}) \leq 0$, or non-failure, $G\left(x^{(r)}\right) > 0$, are assigned to each sample. $G\left(x^{(r)}\right)$ represents a realization of the random variable $G(\boldsymbol{X})$.

Step 3: After N_t simulations, it is possible to estimate the probability of failure by the following expression:

$$P_f = \frac{1}{N_t} \sum_{r=1}^{N_t} I[G\left(x^{(r)}\right) \leq 0] \tag{5}$$

The index function is defined as follows:

$$I[G\left(x^{(r)}\right) = \begin{cases} 1, \text{if } G\left(x^{(r)}\right) \leq 0 \\ 0, \text{if } G\left(x^{(r)}\right) > 0 \end{cases} \tag{6}$$

The estimated P_f obtained is accurate when N_t tends to infinity.

2.2 Mechanical Modeling

Let u_i represent the displacement field components. Let λ and G be the Lamé material parameters. The classical formulation of linear elasticity assumes that the displacement field must respect the strain-displacement relation represented by equation (7), and the constitutive equation expressed by equation (8). Equation (8) is known as Hooke constitutive model. If the solid is in equilibrium, the displacement field is obtained from the solution of the differential equation (9).

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{7}$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{8}$$

$$Gu_{i,kk} + (\lambda + G)u_{k,ki} + b_i = 0 \tag{9}$$

The equation (9) is known as Navier equation. It is a coupled system of differential equations that do not have analytical solutions for general boundary conditions. However, alternative techniques are available for simplifying the formulation, proposing approximative solutions. One of such techniques is substituting the strong form from equation (9) by an integral version of the same field. This is the essence of the approximative numerical methods including the BEM. The classical Betti's theorem can be used for uniting equations (7, 8, 9) into only one, equation (10). This equation is known as Somigliana's identity and it can predict the displacement field of an elastic solid in equilibrium. Equation (10) is the classical starting point for the BEM computational procedures (Aliabadi & Wen, 2010). It is worth mentioning that BEM is intrinsically formulated in terms of boundaries. This makes it suitable for formulations involving geometric variability and loading. These observations motivate the coupling procedure proposed in the present study.

$$u_j(y) + \int_\Gamma u_i(x)p_{ij}^*(x,y)d\Gamma = \int_\Gamma t_i(x)u_{ij}^*(x,y)d\Gamma + \int_\Omega b_i u_{ij}^*(x,y)d\Omega \tag{10}$$

In equation (10), the body forces are represented by b_i, which is assumed not to influence the final response in this study. y is the point where the displacement is being calculated. It can be inside or outside the structural domain. t_j are tractions field components. The u_{ij}^* and p_{ij}^* are fundamental solutions calculated as follows:

$$u_{ij}^* = \frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)ln\left(\frac{1}{r}\right)\delta_{ij} + r_{,i}r_{,j}\right] \tag{11}$$

$$p_{ij}^*(x,y) = -\frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}\right] + (1-2\nu)(n_i r_{,j} - n_j r_{,i})\right\} \tag{12}$$

As usual with BEM, y is associated to the source point and x to the field point. Let $r_i = x_i - y_i$, $r = \sqrt{r_i r_i}$, μ the material shear modulus and ν the Poisson's ratio.

In the present study, the constitutive material is assumed to creep. Therefore, the Hooke equation is not enough for the formulation, and a more complex constitutive model needs to be adopted. The time-dependent behavior can be modelled according to the linear theory of viscoelasticity. There are two available forms for describing material response in this discipline. One of them is the integral form and the second is the differential form (utilized here). Differential forms are flexible in terms of predicting discontinuous responses due to the discontinuous load conditions. There are several constitutive models available in the literature, being the Boltzmann model one of them. The Boltzmann constitutive model has demonstrated to be efficient enough to approximate accurately the solution of several viscoelastic systems (Oliveira & Leonel, 2017). In addition, this model can be used to represent accurately the macroscopic behavior of concrete without temperature effects.

In the present study, the creep phenomenon is allowed to occur at constant temperature and humidity. This assumption makes the stochastic behavior of the Boltzmann model function exclusively of the uncertain material parameters. Such simplification preserves the simplicity of the formulation and at the same time permits the modelling of the random nature of the phenomenon. There are models available for considering temperature and humidity variation (Jordaan, 1980), for instance, utilizing the product between aging functions and loading duration. However, they are not considered in the following formulations.

It is illustrated in Figure 1 a one-dimensional schematic representation of this constitutive model. The global response is assumed to possess two components. One of them is elastic and instantaneous. The second part is followed by a viscous response time dependent. E_1, E_2, and η are material parameters that can be defined from experimental tests (see item 5.2). E_1 is responsible for the instantaneous elastic response, so it corresponds to the classic Young modulus. The stress response obtained for a given strain history is expressed by equation (13) (Oliveira & Leonel, 2017).

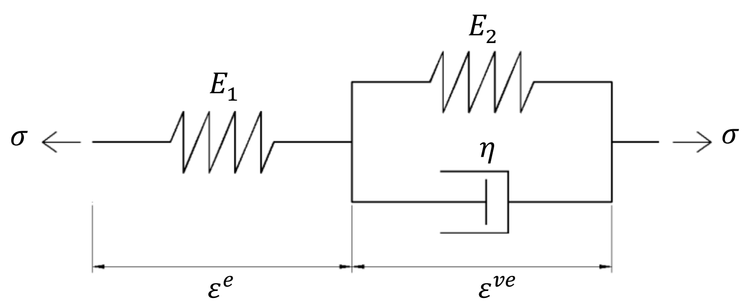

Figure 1 Boltzmann Constitutive Model

$$\sigma_{ij}(t) = \int_{\tau_0}^{t} \left[E_1 - \frac{E_1^2}{E_1 + E_2} (1 - e^{-\frac{E_1+E_2}{\eta}(t-\tau)}) \right] \dot{\varepsilon}_{ij}(\tau) d\tau \tag{13}$$

Substituting equation (8) by equation (13) origins a new fundamental boundary value problem. Thus, the new version of the Somigliana's identity is obtained by the Betti's theorem. This new version is expressed by equation (14), which in turn will be the new starting point for the BEM methodology. Equation (14) is the integral formulation of the problem considering the time-dependence of the Boltzmann constitutive model. By solving this equation, it is possible to obtain the displacement field as function of time. Consequently, it is possible to assess predictions of the future mechanical behavior of the solid.

$$\begin{aligned} u_j(y) = \frac{E_1 + E_2}{E_2} \int_{\Gamma} t_i(x) u_{ij}^*(x,y) d\Gamma - \int_{\Gamma} u_i(x) p_{ij}^*(x,y) d\Gamma \\ - \frac{E_1}{E_2} \int_{\Omega} u_{ji,k}^*(x,y) \theta_{lmik} \dot{\varepsilon}_{lm} d\Omega \\ + \gamma \left[\int_{\Gamma} u_{ij}^*(x,y)\, \dot{t}_j d\Gamma + \int_{\Omega} \dot{b}_i u_{ij}^*(x,y) d\Omega \right] \\ + \frac{E_1 + E_2}{E_2} \int_{\Omega} b_i u_{ij}^*(x,y) d\Omega \end{aligned} \tag{14}$$

In equation (14), $\theta_{lmik} = \gamma C_{lmik}$. Frequently it is not possible to find boundary fields u_j and t_j that can represent all possible combinations of fixing and load conditions. Nevertheless, equation (14) is used for generating a computational procedure for finding boundary unknowns. This is the essence of BEM. To calculate the integrals along the boundary, it is convenient to divide it into finite sized elements. Using lagrangean polynomials of order o follows:$u_k = \phi_m(\xi) u_k^m$, $p_k = \phi_m(\xi) p_k^m$. The isoparametric approach is adopted. Thus, the boundary fields are approximated by the same set of basis functions used for approximate geometry. It is possible to write the algebraic representation of the problem by substituting the approximate field in equation (14), considering the material response from equation (13), admitting η proportional to E_2 ($\eta = \gamma E_2$), as follows (Mesquita & Coda, 2001):

$$\left(1 + \frac{\gamma}{\Delta t}\right) HU^{s+1} = \left(\frac{\gamma}{\Delta t} + \frac{E_1 + E_2}{E_2}\right) G + \frac{\gamma}{\Delta t} (HU^s - GP^s) \tag{18}$$

The current iteration is symbolized by s, Δt is the time interval. It is worth mentioning that equation (18) represents in fact a system of equations which is obtained after numerical integration over all source points according to classical BEM procedures. U and P is the displacement and traction column matrix respectively. H and G are the classical matrix BEM kernels.

It is worth mentioning that equation (18) provides the nodal displacements along the boundary implicitly as function of geometrical, load, and material parameters. This finding will be useful in the item 5.3. Let $\boldsymbol{Q}$ comprises all load conditions. In mathematical terms, this fact can be explicit as follows:

$$u = u(\Gamma, E_1, E_2, \gamma, \boldsymbol{Q}) \tag{19}$$

Solving the system of equations (18) provides the displacements and tractions over the boundary at each instant of time. However, to solve the probabilistic problem, a high number of repetitive solutions is required. Thus, the computational time consuming may become prohibitive the direct use of BEM in the problem. Therefore, another numerical strategy is associated in the framework for reducing the overall analysis time, namely the metamodeling, which is next explained.

2.3 Response Surface Method

The mechanical response will be globally expressed in terms of the Response Surface Method. In practice, it is utilized a second-order response surface expressed by the following equation:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i \leq j=1}^{k} \beta_{ij} x_i x_j + \epsilon \tag{20}$$

In equation (20), the value k represents the number of RVs to be utilized, x_i is the value of each realization of the corresponding RV. β_i and β_{ij} are real coefficients to be determined from the numerical BEM experiments using nonlinear regression. The parameter ϵ indicates the approximating error due to the difference between the real response and the value predicted by the polynomial. It is assumed that the distribution of ϵ is Gaussian with zero mean (Das, 2014).

2.4 Proposed Algorithm

The algorithm developed for the present study consists of the following steps:

- Definition of geometry, loads, and support conditions. This step is commonly adopted as initial procedure for estimating structural behaviour;
- Choice of material model. In this step, it is important to know the parameters of the model that are more representative of the global behaviour;
- Choice of RVs. Step in which the uncertainties are included in the computational model. Note that in the present application, one of the RVs relates to the geometry. This implies that for each sample of this variable, a new mesh of boundary elements must be generated;
- Choice of response of interest. Define the variable of interest for the study;
- Construction of the Response Surface. See item 2.3. In the present study, a second-order polynomial was chosen. Samples of the chosen RVs are generated

(vector $x_i^{(n)}$). For each sample, it is necessary to perform a BEM analysis in order to obtain the answer $y_i^{(n)}$. Then, a nonlinear regression process is performed for finding the values of the polynomial coefficients (Equation (20));

- Monte Carlo simulation. From the defined polynomial, it is possible to carry out the simulations according to item 2.1.2.

3 APPLICATION

The material that constitute the structure under focus is the concrete. The creep phenomenon is considered through the Boltzmann model. The parameters need to be stipulated from experimental observation. For this application, the data basis collected for several types of concrete is utilized (Wassin, 2002). The good correlation between numerical and experimental is shown in Figure 2. The Boltzmann model has an important feature. It enables the instantaneous deformation (often elastic) and also time-dependent deformations (viscous). The mean values obtained for the Boltzmann parameters are presented in Table 1.

Table 1 Boltzmann parameters obtained (mean values) [Source:(Oliveira et al., 2018)**]**

$E_1(GPa)$	$E_2(GPa)$	$\gamma(days)$
22.30	5.60	450.00

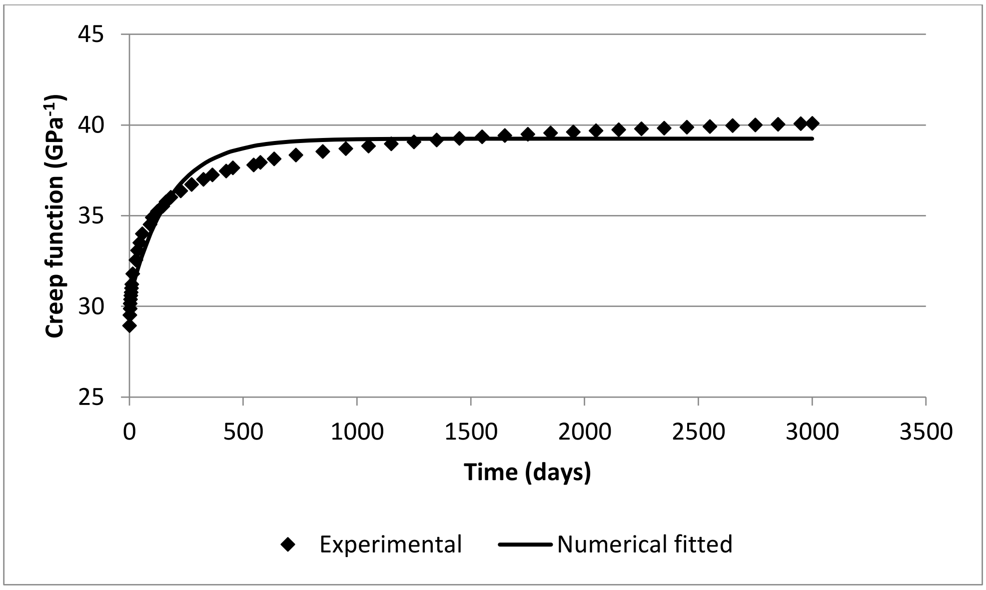

Figure 2 Correspondence numerical x experimental data [Source:(Oliveira et al., 2018)**]**

3.1 Numerical investigation: controlled test

Consider the following test case is a clamped bar under traction (Figure 3). The displacements of the right edge are given by the analytical expression from equation (21). The plane stress hypothesis is assumed with arbitrary thickness $1mm$.

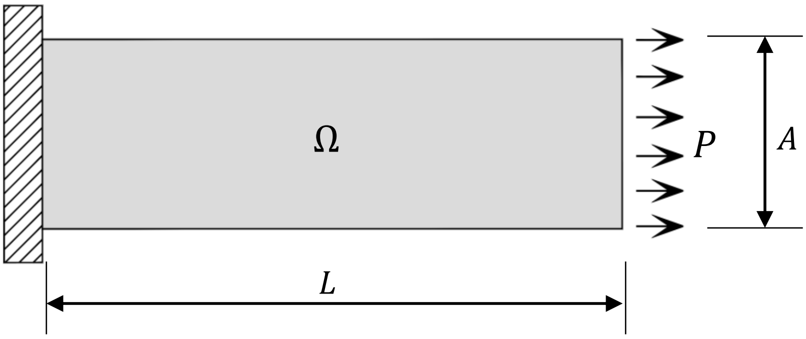

Figure 3 Structure : loading and geometrical aspects [Source:(Oliveira et al., 2018)**]**

$$\Delta(t) = \frac{PL}{A}\left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{E_2}e^{-t/\gamma}\right) \tag{21}$$

The randomness associated to each RVs is determinant for the problem. There is no fixed rule for this choice, but in general, minor variability lead the model to show deterministic responses (Nowak & Collins, 2012). The analyst is responsible for finding the variables set most appropriate for each case. In the present application, the following gaussian variables are chosen: the Boltzmann material parameters, E_1, E_2 and γ, the estimated service load Q_1. Three cases of variations are compared (SD1, SD2 and SD3) according to data of Table 2.

Table 2 RVs parameters [Source:(Oliveira et al., 2018)**]**

Gaussian RV	Mean Value	Standard Deviation 1 (SD1)	Standard Deviation 2 (SD2)	Standard Deviation 3 (SD3)
E_1	22.30 GPa	6.7 GPa	4.5 GPa	1.11 GPa
E_2	5.60 GPa	1.7 GPa	1,1 GPa	0.28 GPa
γ	450 $days$	120 $days$	90 $days$	6 $days$
P	50 kN/m	15 kN/m	10 kN/m	2.5 kN/m

The proposed numerical algorithm is used for obtaining the results shown in Figure 4. It is possible to distinguish the confidence interval (95%) for a period of 10 years. As expected, the decrease in the overall variability imply on the narrowing of consecutive strips. In the limit, the results tend to the mean response, which also coincides with the deterministic response. Note that the dispersion is function of time, and for any particular instant of time, the typical graph shown in Figure 5 is observed. The displacements dispersion become insignificant as the variability on the input data decreases. In consequence, the instantaneous mean value converges to the instantaneous deterministic displacement predicted by equation (21). This particular numerical experiment let evident the direct influence of the parameter variabilities. Also, the stability that the BEM manifest when coupled with the different numerical methods.

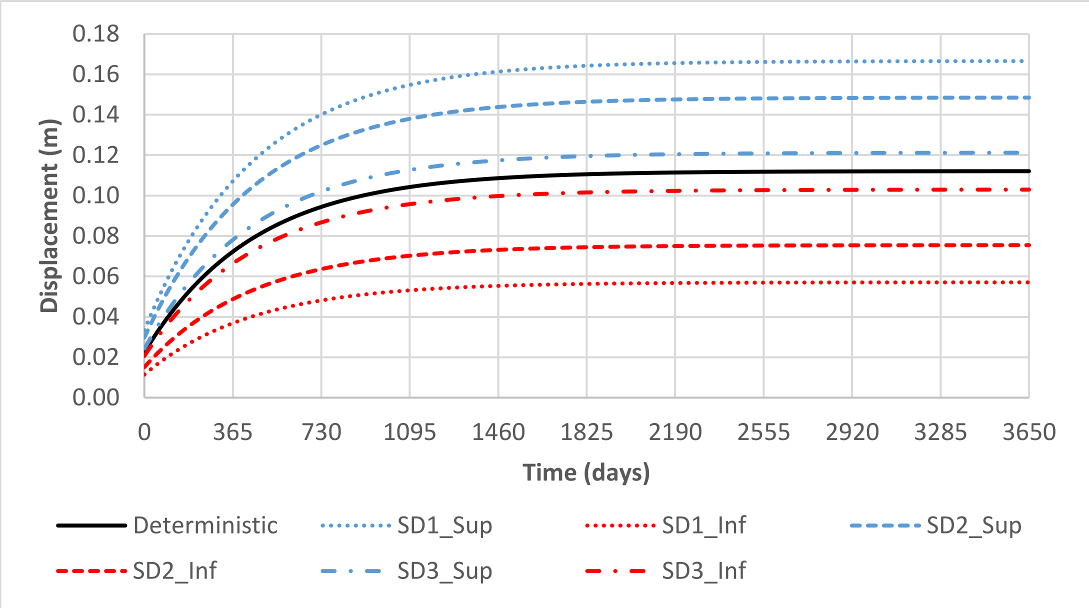

Figure 4 Confidence interval convergence - Inferior limit (Inf) and Superior Limit (Sup) [Source:(Oliveira et al., 2018)**]**

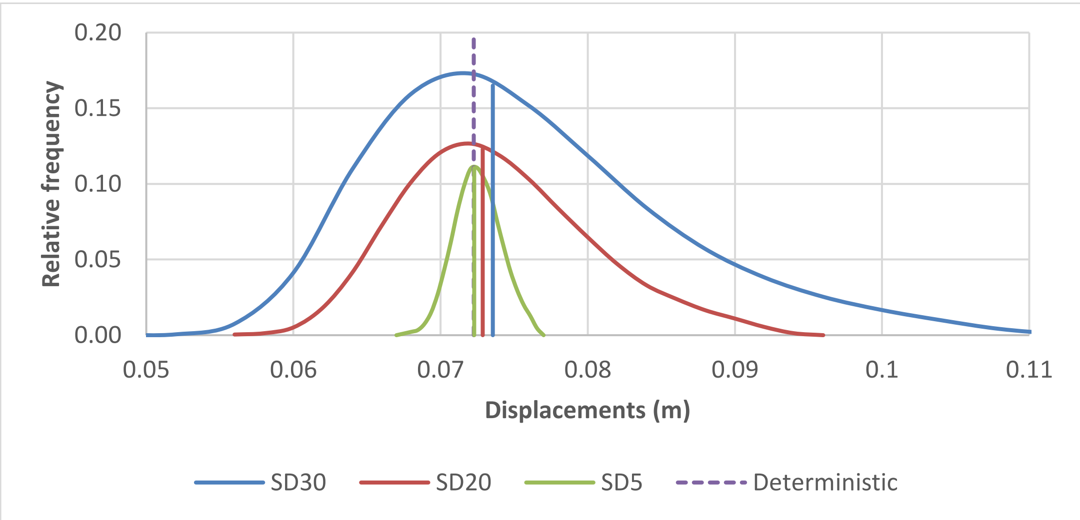

Figure 5 Dispersion of the displacements for $t = 365\ days$ (Continuous vertical line indicate the corresponding mean value) [Source:(Oliveira et al., 2018)**]**

4 CONCLUDING REMARKS

The present communication presented an algorithm able to predict the probabilistic response as function of time. This algorithm is built from the coupling of the viscoelastic BEM, the RSM and reliability theory. The material was allowed to creep following the Boltzmann model. The particular numerical experiment shown the ability of the algorithm to recover the deterministic response. The numerical coupling showed stability and accuracy. This numerical tool can be relevant for engineers at the concept phase, when exploring the alternatives are necessary to better take decisions.

As perspectives, the present research can be extended to include new material models without important changes in the core formulation.

ACKNOWLEDGEMENTS

This research was supported by the São Paulo Research Foundation (FAPESP; grant nº 2012/24944-5 and 2015/07931-5). The authors would like to thank Capes (Coordination for the Improvement of Higher Level Education) for supporting the present scientific communication.

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