Challenges and Solutions in Column Rotation for Multi-Story Building Design

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Summary

Advancements in material strength, especially concrete, and architectural boldness have made structural solutions for multi-story buildings increasingly complex. This paper reviews discontinuity region concepts and presents structural solutions for 90° column rotation between floors, focusing on high-load scenarios. The methodology involves strut-and-tie models, validated through nonlinear analyses, in accordance with Eurocode 2 and NBR 6118.

1 INTRODUCTION

With advancements in construction technology, architectural designs have become increasingly complex regarding space utilization and occupancy. This evolution presents significant challenges for structural engineers, who must balance slenderness, positioning, and constraints of structural elements without compromising the safety and feasibility of buildings.

One of the most frequent challenges in building structural design is the allocation of columns, which can lead to discrepancies in sectional orientation between different floors. This issue is particularly evident when comparing parking levels, which have geometric constraints to accommodate parking spaces, with typical floors, where spatial optimization is essential to maximize room usage.

In this context, the present study aims to propose design and verification solutions for transition blocks used in columns subjected to a 90° rotation between floors (cross-shaped configuration). These elements, classified as discontinuity regions, also face the additional challenge of withstanding high loads, requiring rigorous analyses to ensure adequate structural performance.

2 DISCONTINUITY REGIONS AND THE STRUT-AND-TIE METHOD

In certain situations, structures present specific regions where the linear distribution of deformations across the section is no longer applicable [1], such as areas exhibiting geometric nonlinearity. These regions require detailed analysis to ensure the structural integrity of the element. In this context, columns subjected to orientation changes, with a 90° section rotation between floors (cross-shaped configuration), exhibit geometric nonlinearity and therefore become a specific subject of study, requiring a more rigorous evaluation.

According to NBR 6118 [2] (since 2014) and CEB-FIP [3] (since 1990), among the possible approaches for evaluating discontinuity regions, the Strut-and-Tie Method can be applied. The Strut-and-Tie Method allows for the analysis of discontinuity regions through an idealized truss model composed of three types of elements: struts, which represent concrete compression regions; ties, corresponding to the tensile reinforcement; and nodes, which transmit forces between struts and ties and must therefore be verified for strength. In general, the system should be statically determinate, with external forces and support reactions concentrated at the nodes, ensuring a self-equilibrated force system.

3 THE TRANSITION BLOCK MODEL USING THE STRUT-AND-TIE METHOD (MBT)

The methodology used in this article consists of two main stages. In the first stage, the model will be developed using the Strut-and-Tie Method, along with the calculation procedure, which includes the reinforcement design. In the second stage, the conceived model will be verified through the application of real examples, including the reinforcement detailing.

3.1 Critical Loading

To begin the verification of the transition block, it is necessary to define the most critical loading condition for the columns, also considering the loads induced by wind action. After defining the loading condition, the design force to be used in the verification is calculated.

3.2 Model Definition

In the transition block, two independent Strut-and-Tie models must be developed and verified: the first model considers the forces transferred from column P' to column P, while the second model analyzes the opposite direction, from column P to column P'. The models adopt a three-dimensional load distribution, which will be detailed later, ensuring greater accuracy in representing the real behavior. For the definition of the models, consider the sections of the columns as shown in Fig. 1 and (1).

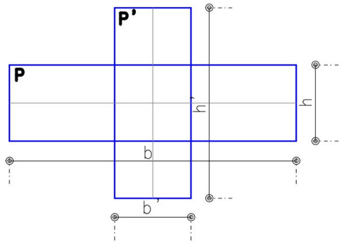

Fig. 1 Sections of Columns P' and P

$$b - b' \geq h - h' \tag{1}$$

3.2.1 Model 1

As mentioned earlier, the first model will consider the load path from P' to P, i.e., it will verify the most critical strut case. For the three-dimensional analysis, the sections of the columns will be divided into four parts, and the load paths will be defined based on proximity, as shown in Fig. 2 (left).

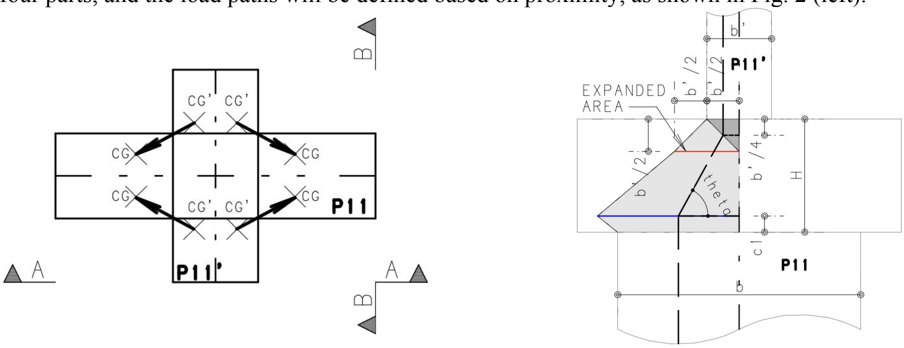

Fig. 2 Direction of the Load Path in Model 1 | Section AA: Model 1

The height of the block, H (2), was defined considering the most optimized case for the Strut-and-Tie Model, with parameters including the widths of the columns (b and b') and the distance from the tie reinforcement to the face of the block (c_1). Additionally, it is necessary to verify whether the block has sufficient height for the reinforcement anchorage (4).

$$H = \frac{b - b'}{2} + \frac{b'}{8} + c_1 \tag{2}$$

$$c_1 \geq \begin{cases} 10cm \\ \dfrac{b'}{4} \end{cases} \tag{3}$$

$$H \geq l_{0c} + c \tag{4}$$

It is important to note that for Model 1, Fig. 2 (right), to be consistent, the condition established in (5) must be satisfied. If this condition is not met, the model can be adjusted by modifying: the offset of the expanded area, which has an initial value of b'/2; the height H of the block; or the value of c_1.

$$\tan\theta \leq 2 \tag{5}$$

3.2.2 Model 2

Model 2 aims to verify the load path in the opposite direction, i.e., from P to P'. For this model, the same conditions as in Model 1 will be considered: the sections of the columns will be divided into four parts, and the load paths will be defined based on proximity, as illustrated in Fig. 3 (left). Fig. 3 (right) shows the corresponding Strut-and-Tie Model for this new arrangement.

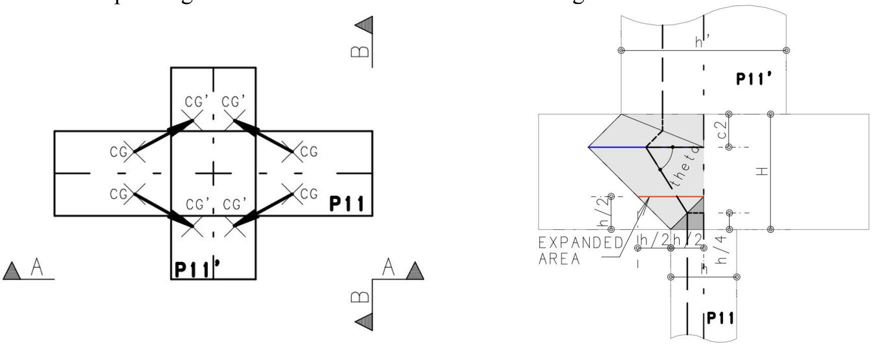

Fig. 3 Direction of the Load Path in Model 2 | Section BB: Model 2

It can be observed that, similar to c_1, c_2 will be the distance from the reinforcement (ties) to the block, defined by (6).

$$c_2 \geq \begin{cases} 10cm \\ \dfrac{h}{4} \end{cases} \tag{6}$$

Like Model 1, if (5) is not satisfied, the model should be adjusted by changing the value of c_2 so that the model meets the strut angle condition to be considered acceptable.

3.3 Verification of Nodes and Struts

The stresses at the nodes of Models 1 and 2 will be given by:

$$\sigma_{nó,1} = \frac{F_d}{A_{amp,P'} \cdot \sin^2\theta_1} \qquad \sigma_{nó,2} = \frac{F_d}{A_{amp,P} \cdot \sin^2\theta_2} \tag{7}$$

Where $A_{amp,P'}$ and $A_{amp,P}$ refer to the expanded areas of columns P' and P, respectively, and θ_1 and θ_2 refer to the angles of the struts in Models 1 and 2.

Considering that, due to the two models being inverted in the system, a tie passes through the compressed zone of both nodes, equation (8) will be used for the verification:

$$\gamma_n \cdot \sigma_{nó} < f_{cd,3} = 0.72 \cdot \alpha_{v2} \cdot f_{cd} \tag{8}$$

$$f_{cd,3} < \gamma_n \cdot \sigma_{nó} \tag{9}$$

According to NBR 6118 [1], in item 22.2, γ_n should be at least equal to 1.2. It is important to note that the verification must be performed for both models. If (9) occurs for any of the models, one of the following methods can be used for resolution:

- Adjust the model (item 3.2), provided that the strut inclination limits are respected.
- Provide concrete confinement as defined in item 3.6.
- Increase the f_{ck} of the concrete used and recheck the node stress.

3.4 Tie Calculation – Main Reinforcements

Once the model to be used is defined and the strut resistances are verified, the design of the upper and lower reinforcements can be carried out:

$$T_1 = \cot\theta_1 \cdot \frac{F_d}{2} \qquad T_2 = \cot\theta_2 \cdot \frac{F_d}{2} \tag{10}$$

$$T_{X,1} = T_1 \cdot \cos\alpha \qquad T_{X,2} = T_2 \cdot \cos\alpha \tag{11}$$

$$T_{Y,1} = T_1 \cdot \sin\alpha \qquad T_{Y,2} = T_2 \cdot \sin\alpha \tag{12}$$

$$A_{s,X,1} = \frac{T_{X,1}}{f_{yd}} \qquad A_{s,X,2} = \frac{T_{X,2}}{f_{yd}} \tag{13}$$

$$A_{s,Y,1} = \frac{T_{Y,1}}{f_{yd}} \qquad A_{s,Y,2} = \frac{T_{Y,2}}{f_{yd}} \tag{14}$$

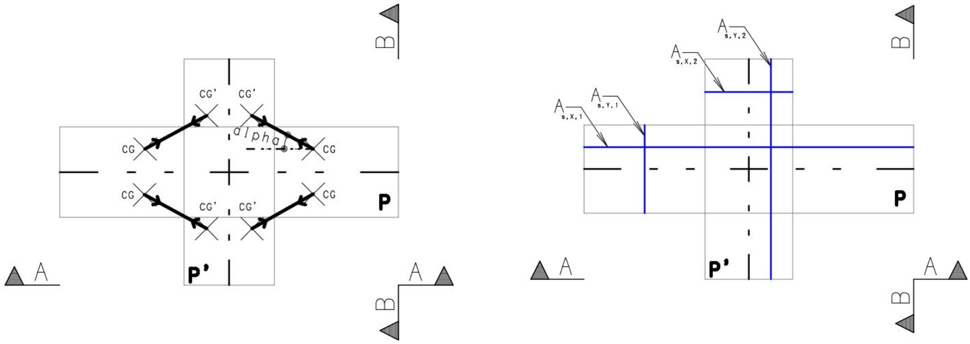

Fig. 4 Definition of α | Reinforcement Reference

3.5 Verification of Tensile Stresses on the Surface – Surface Reinforcement

Fusco [4], in Chapter 7, defines the methodology for the design of surface reinforcements that mitigate cracking caused by concentrated loads, through the calculation of the surface tensile stress. The author also states that, for practical design purposes, the surface reinforcement can be calculated based on the worst-case scenario:

$$R_{t0} = 0.04 \cdot F_d \tag{15}$$

$$A_{s,sup} = \frac{R_{t0}}{f_{yd}} \tag{16}$$

3.6 Verification of Concrete Strength – Confinement Reinforcement

If an increase in concrete strength is required, as mentioned in item 3.1.3, NBR 6118:2023 [1] allows the enhancement of strength (18) through concrete confinement, considering the multiaxial stress state described in item 8.2.6 of the standard. Fusco [4], in topic 5.3, defines that confinement can be achieved by using a mesh confinement reinforcement, defined by (19).

$$\sigma_1 \geq -f_{ctk} \tag{17}$$

$$\sigma_3 \leq f_{ck} + 4 \cdot \sigma_1 \tag{18}$$

$$1.4 \cdot f_{ck} = f_{ck} + 1.7 \cdot A_t / A_{ci} \cdot f_{yk} \tag{19}$$

Where:

- A_t is the fictitious confinement area, defined by the steel volume per unit length;
- A_{ci} is the area of the inscribed circle in the confined zone.

3.7 Example

For demonstration, the example of Column 11 will be used, which has sections P11' 40x100 cm and P11 150x40 cm and a design load F_d=16595.9kN. Models 1 and 2 of Struts and Ties define the load paths and can be observed below:

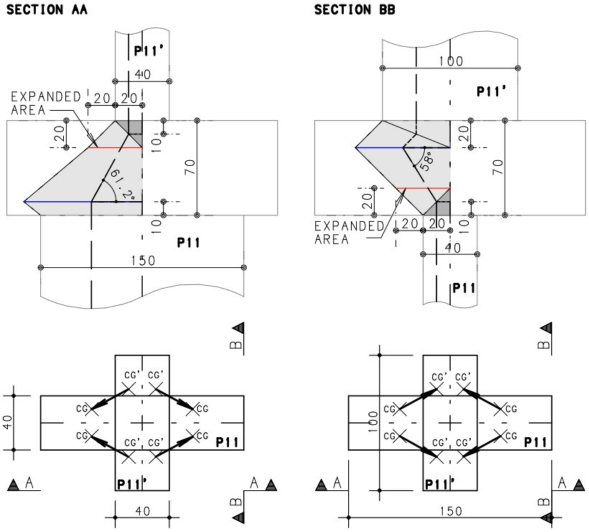

Fig. 5 Model 1 | Model 2

It can be observed that in Model 2, adjustments to c2 were necessary, as well as adjustments to the path of the struts to satisfy the θ condition. The verifications of the models can be seen in Table 1 and Table 2 below.

Table 1 Calculation Procedure for Model 1

Procedure	Formula	Model 1
Verification of θ	(5)	$\tan 61.2° = 1.82 \therefore 1.82 \leq 2 \therefore OK!$
Nodal Stress Verification	(7)	The nodal stress should be verified for the expanded area. $A_{amp,P'} = 0.8m \cdot 1.4m = 1.12m^2$ $\gamma_n \cdot \sigma_{nó,1} = 1.2 \cdot \frac{16595.9\text{kN}}{1.12m^2 \cdot \sin^2 61.2°} \therefore$ $\gamma_n \cdot \sigma_{nó,1} = 23.16MPa$
Node Verification	(8)	Considering f_{ck} = 50MPa, we have: $\alpha_{v2} = 1 - \frac{50}{250} \therefore \alpha_{v2} = 0.8$ $f_{cd} = \frac{50}{1.4} \therefore f_{cd} = 35.71MPa$ $f_{cd,3} = 0.72 \cdot 0.8 \cdot 35.71 \therefore f_{cd,3} = 20.57MPa$ $\gamma_n \cdot \sigma_{nó,1} > f_{cd,3} \therefore Promote\ Confinement!$
Force in the Tie	(10)	$T_1 = \cot 61.2° \cdot \frac{16595.9\text{kN}}{2} \therefore T_1 = 4561.8kN$
Calculation of α	-	$\alpha = \tan^{-1} \frac{25cm - 10cm}{37.5cm - 10cm} \therefore \alpha = 28.6°$
Tie Force in X	(11)	$T_{X,1} = 4561.8kN \cdot \cos 28.6° \therefore T_{X,1} = 4005.2kN$
Tie Force in Y	(12)	$T_{Y,1} = 4561.8kN \cdot \sin 28.6° \therefore T_{Y,1} = 2183.7kN$

Procedure	Formula	Model 1
Reinforcement in X de P11’	(13)	$A_{s,X,1} = \frac{4005.2kN}{43.5kN/cm^2} \therefore A_{s,X,1} = 92cm^2$
Reinforcement in Y de P11’	(14)	$A_{s,Y,1} = \frac{2183.7kN}{43.5kN/cm^2} \therefore A_{s,Y,1} = 50.2cm^2$

Table 2 Calculation Procedure for Model 2

Procedure	Formula	Model 2
Verification of θ	(5)	$\tan 58° = 1.60 \therefore 1.60 \le 2 \therefore OK!$
Nodal Stress Verification	(7)	The nodal stress should be verified for the expanded: $A_{amp,P} = 0.8m \cdot 1.9m = 1.52m^2$ $\gamma_n \cdot \sigma_{nó,2} = 1.2 \cdot \frac{16595.9\text{kN}}{1.52m^2 \cdot \sin^2 58°} \therefore$ $\gamma_n \cdot \sigma_{nó,2} = 18.22MPa$
Node Verification	(8)	Considering f_{ck} = 50MPa, we have: $\alpha_{v2} = 1 - \frac{50}{250} \therefore \alpha_{v2} = 0.8$ $f_{cd} = \frac{50}{1.4} \therefore f_{cd} = 35.71MPa$ $f_{cd,3} = 0.72 \cdot 0.8 \cdot 35.71 \therefore f_{cd,3} = 20.57MPa$ $\gamma_n \cdot \sigma_{nó,2} < f_{cd,3} \therefore OK!$
Force in the Tie	(10)	$T_2 = \cot 58° \cdot \frac{16595.9\text{kN}}{2} \therefore T_2 = 5185.13kN$
Calculation of α	-	$\alpha = \tan^{-1} \frac{25cm - 10cm}{37.5cm - 10cm} \therefore \alpha = 28.6°$
Tie Force in X	(11)	$T_{X,2} = 5185.13kN \cdot \cos 28.6° \therefore T_{X,2} = 4552.5kN$
Tie Force in Y	(12)	$T_{Y,2} = 5185.13kN \cdot \sin 28.6° \therefore T_{Y,2} = 2482.1kN$
Reinforcement in X de P11	(13)	$A_{s,X,2} = \frac{4552.5kN}{43.5kN/cm^2} \therefore A_{s,X,2} = 104.7cm^2$
Reinforcement in Y de P11	(14)	$A_{s,Y,2} = \frac{2482.1kN}{43.5kN/cm^2} \therefore A_{s,Y,2} = 57.1cm^2$

The design of the surface reinforcement and confinement required for the block will be described in Table 3:

Table 3 - Surface Reinforcement and Confinement

Procedure	Formula	Transition Block
Surface Tensile Stress	(15)	$R_{t0} = 0.04 \cdot 16595.9\text{kN} \therefore R_{t0} = 663.84kN$
Surface Reinforcement	(16)	$A_{s,sup} = \frac{663.84kN}{43.5kN/cm^2} \therefore A_{s,sup} = 15.26cm^2$
Verification of Confined Concrete	-	$1.4 \cdot f_{cd,3} > \gamma_n \cdot \sigma_{nó,1} \therefore 1.4 \cdot 20.57MPa > 23.16MPa$ $28.8MPa > 23.16MPa \therefore OK!$
Confinement Reinforcement	(19)	$1.4 \cdot 50MPa = 50MPa + 1.7 \cdot \frac{A_t}{0.8m \cdot 1.4m} \cdot 500MPa$

		$A_t = 0.0263 m^3/m$

Considering that the block has dimensions 200x200x70 cm, we have the following detailing:

Table 4 Block Detailing

Procedure	Transition Block
Main Reinforcement	$A_{s,X,1} = 92cm^2 \therefore A_{s,X,1} = 50\ \varnothing 16\ c/\ 7\ (2cam)$ (N2) $A_{s,Y,1} = 50.2cm^2 \therefore A_{s,Y,1} = 25\ \varnothing 16\ c/\ 7$ (N1) $A_{s,X,2} = 104.7cm^2 \therefore A_{s,X,2} = 54\ \varnothing 16\ c/\ 6\ (2cam)$ (N2) $A_{s,Y,2} = 57.1cm^2 \therefore A_{s,Y,2} = 29\ \varnothing 16\ c/\ 6$ (N1)
Surface Reinforcement	$A_{s,sup} = 15.26cm^2 \therefore A_{s,sup} = 20\ \varnothing 10\ c/\ 10$ (N3)
Confinement Reinforcement	Vertical spacing: $s = 0.04m$ Rebar length: $c_t = 22.04$m Area of the rebar diameter (A_{st}): $A_{st} = \frac{s \cdot A_t}{c_t} \therefore A_{st} = \frac{0.04m \cdot 0.0263m^3/m}{22.04\text{m}}$ $A_{st} = 0.48cm^2 \therefore \varnothing 8mm$ (N4 e N5)
Rib Reinforcement NBR 6118:2023 [1] item 22.7.2.4.1.2	According to NBR 6118 [1], the stirrup reinforcement is equivalent to 20% of the main reinforcement and must be uniformly distributed in both directions. For the calculation of the stirrup reinforcement, the larger main reinforcement will be considered, that is, $A_{s,máx} =$ 54 $\varnothing$16: $A_{s,stirrup} = \frac{0.2 \cdot A_{s,máx}}{2 \cdot h_{bloco}} \therefore A_{s,stirrup} = \frac{0.2 \cdot 54 \cdot 2cm^2}{2 \cdot 0.7m}$ $A_{s,stirrup} = 15.4cm^2/m \therefore A_{s,stirrup} = \varnothing 10\ c/\ 5$ (N6)

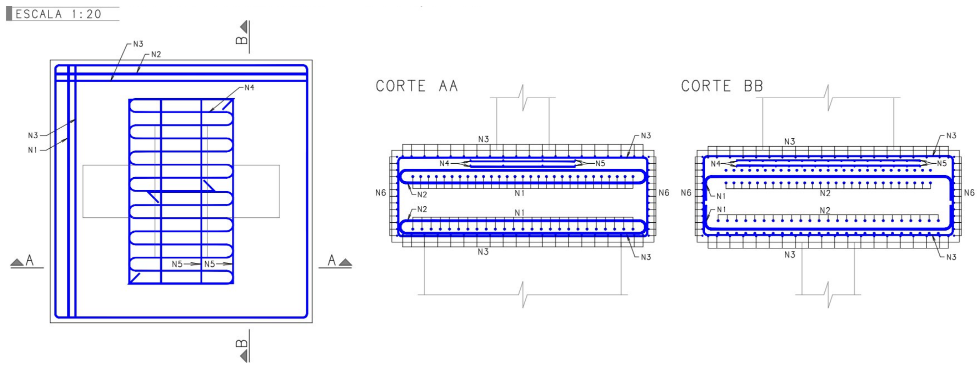

Fig. 6 Detailing of the Transition Block

3.8 Model Verification

The nonlinear analysis stands out as an essential and precise tool for understanding the behavior of materials with more complex constitutive properties, such as concrete, whether plain or reinforced. In this study, a nonlinear model will be developed, identical to the example in item 3.7, in order to validate the proposed methodology for dimensioning transition blocks.

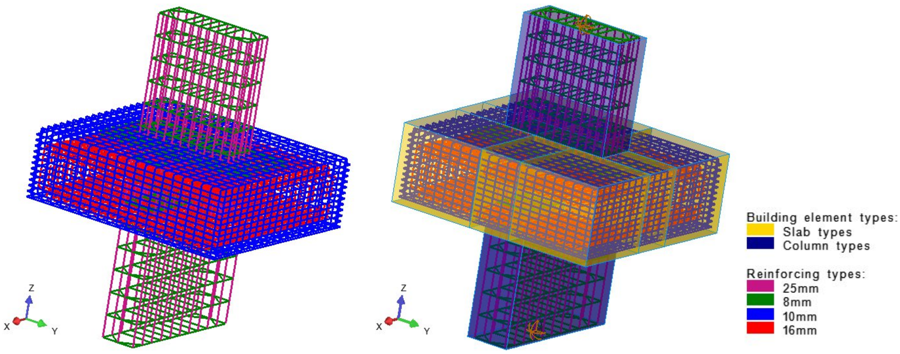

Fig. 7 Model Nonlinear

The modeling will be carried out using the ATENA v6.0.0 software, which has worldwide expertise in the nonlinear analysis of reinforced concrete structures. As is commonly done in nonlinear analyses, the loading will be incremental and based on the Newton-Raphson method, with 60 steps and 20 load increments, totaling an imposed displacement of 6 mm. The nonlinear behavior of reinforced concrete will be represented by the 'CC3DNonLinCementitious2' model implemented in ATENA. The uniaxial constitutive law of concrete combines a fracture model for cracking (tension) with a plasticity model for concrete crushing (compression).

The main input parameters for the constituent materials are listed below:

Table 5 Input Parameters

Input	Columns	Transition Block
f_{ck}	35 MPa	50 MPa
E_c	35 GPa	38 GPa
ν	0.20	0.20
f_t	3.21 MPa	4.07 MPa
f_c	43 MPa	58 MPa
G_f	149 N/m	149 N/m
ε_{cp}	0.00189	0.0021

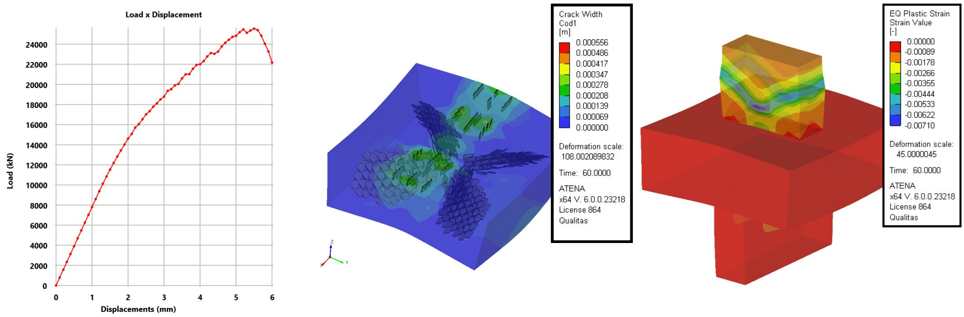

Fig. 8 Load x Displacement | Cracks and Deformations of the Nonlinear Model at Failure

Fig. 8 shows the entire loading process performed in the analysis, as well as the state of the elements at the end of the loading, demonstrating that the failure occurs in the upper pillar of the set. Furthermore, as observed in Fig. 9, the deformations that occurred in the main and in the block at the moment of failure are shown.

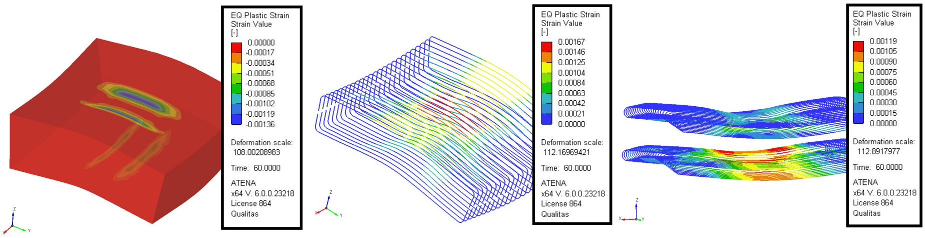

Fig. 9 Concrete and reinforcement deformations at failure

The nonlinear model allowed the analysis under specific conditions of the elements: the moment of the first crack opening, the service state, the design load (F_d), and the failure state. Table 6 shows the corresponding values:

Table 6 Results of the Nonlinear Model

Analyzed State	Step	Loading	Cracking
Opening of the First Crack	9	7033.4 kN	-
Service State	15	11854.2 kN	0.044 mm
Design Load	24	16595.9 kN	0.091 mm
Failure State	52	25471.0 kN	0.466 mm

4 CONCLUSION

Based on the obtained results, it is concluded that the Strut-and-Tie model and the structural details developed from it for the transition block show satisfactory performance in the nonlinear analysis, particularly in controlling cracking, displacements, and structural safety. The results also indicate a high degree of compatibility between the adopted theoretical model and the response obtained in the nonlinear analysis, validating its practical applicability.

Acknowledgments

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