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Spectral Analysis of Irregular Sea Waves

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Abstract: Accurate modeling of ocean wave behavior is a critical component in ensuring the safety, efficiency, and longevity of offshore structures and naval systems. The marine environment is inherently complex, characterized by highly irregular and dynamic wave patterns influenced by a wide range of environmental factors, including wind, tides, and ocean currents. These variables introduce significant challenges in predicting wave-induced forces and their impact on floating and fixed offshore platforms, vessels, and energy harvesting devices. To address these challenges, spectral wave analysis has emerged as a fundamental approach, offering engineers a robust framework for interpreting and simulating wave behavior with enhanced precision. Among the numerical techniques employed in spectral analysis, the Fast Fourier Transform (FFT) stands out due to its computational efficiency and ability to decompose complex, time-domain wave signals into their constituent frequency components. This transformation enables a clearer understanding of the temporal and spatial characteristics of wave motion, which is essential for the design of resilient and adaptive offshore systems. In this context, the present study investigates the application of Fourier series and FFT in modeling irregular sea wave patterns, emphasizing their effectiveness in representing periodic and quasi-periodic phenomena through the superposition of sinusoidal functions. The methodology of this study involves the acquisition of real-world wave data from the Rede Ondas project, followed by signal processing using FFT implemented in spreadsheet software. The resulting complex coefficients were analyzed to extract amplitude and frequency information, enabling the construction of a spectral profile of the wave field. Key wave parameters such as significant wave height and peak frequency were identified, providing insights into the dynamic loading conditions that offshore structures may encounter.
Keywords: Irregular Waves. Fast Fourier Transformation. Offshore.

1. Introduction

Spectral wave analysis is a fundamental tool in the design and evaluation of offshore structures, ships, and tidal energy generation systems. The oscillatory behavior of ocean waves induces dynamic loads that can significantly affect the structural integrity and operational stability of these systems. In addition to contributing to structural instability, wave-induced motions are critical for assessing oceanographic data relevant to engineering applications. Understanding these dynamics through spectral methods enables more accurate modeling and enhances the reliability of marine infrastructure [1-3].

Marine waves are typically considered random in nature. However, large wave formations can often be approximated as periodic and regular. In general, wave behavior is modeled using combinations of harmonic oscillations. In simplified scenarios, a single harmonic function may sufficiently represent the wave phenomenon, leading to what are known as regular wave models. Conversely, irregular waves are characterized by the superposition of multiple harmonic components, allowing for a more accurate representation of their complex and stochastic behavior.

Li et al. (2012) [4] proposed a study on nonlinear wave-structure interaction using a two-dimensional finite volume approach to assess ship stability. Davidson et al. (2015) [5] explored the development of a numerical wave tank through computational fluid dynamics to simulate marine energy generation using floating devices. Wang et al. (2015) [6] investigated the behavior of energy-harvesting buoys under the influence of irregular waves by applying the Fast Fourier Transform (FFT). Tabeshpour et al. (2023) [7] examined the role of the superposition principle of harmonic functions in modeling irregular wave patterns.

Ocean wave measurements are typically conducted using specialized instruments known as wave gauges or wave buoys, which may be equipped with accelerometers or acoustic profilers [8].

Accelerometer-based wave buoys, such as the directional Waverider model, record time series of horizontal and vertical displacements of the buoy, enabling the reconstruction of sea surface motion. These data are processed by onboard microprocessors that compute the frequency spectrum every 30 minutes. Raw and spectral data are transmitted hourly via radio (operating between 25.5 and 35.5 MHz) to a land-based receiving station, which forwards them to a coastal engineering laboratory for processing and dissemination [8].

Alternatively, acoustic profilers such as the ADCP (Acoustic Doppler Current Profiler) utilize the Doppler effect to measure particle displacement within the water column, allowing for the estimation of wave height, direction, and period. These instruments typically operate in burst sampling mode at 2 Hz and can generate detailed directional spectra, which are crucial for accurately characterizing sea state conditions [8].

The processed data enable the calculation of key wave parameters, including the significant wave height (Hs)—defined as the average of the highest one-third of waves in a given record—and the peak period, which corresponds to the most energetic component of the directional spectrum. Wave direction is expressed in degrees, with 0° indicating North, 90° East, and so forth [8].

1.1. Theoretical Basis

In vessels, for example, the equation of motion is described from Equation 1 [1]:

$$(m + a_2)x_2'' + b_2 x_2' + c_2 x_2 = F_0 sin(\omega t) \qquad (1)$$

Where m is the mass of the vessel, a_2 is the mass of water displaced during movement, b_2 is the equivalent damping, and c_2 is the equivalent stiffness caused by buoyancy and gravity.

Ocean waves are generally random in nature; however, larger waves can often be approximated as regular and periodic. Typically, wave modeling involves a combination of harmonic oscillations. In simplified cases, a single harmonic function may be used to represent the phenomenon. These models are referred to as regular waves, characterized by a single harmonic oscillation, whereas irregular waves are represented by the superposition of multiple harmonic functions [9].

The Fast Fourier Transform (FFT) emerges as a powerful tool for characterizing these irregular wave patterns. Several studies in the literature employ this method to obtain frequency-domain responses of specific oscillatory systems.

The Fourier Transform is a mathematical technique that allows the decomposition of a non-harmonic periodic function into a linear combination of harmonic functions. In its analytical approach, the general motion function is equated to a Fourier series, which represents the periodic oscillation, as shown in Equation 2 [10].

$$F(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos(j\omega t) + \sum_{j=1}^{\infty} b_j \sin(j\omega t) \quad (2)$$

The Fast Fourier Transform (FFT) is a numerical method that requires the input data to consist of a number of ordered pairs in the form of 2n, where n is an integer. This structure ensures computational efficiency and allows the representation of the signal as a harmonic series, as illustrated in Equation 3 [11].

$$X_k = \sum_{j=1}^{\infty} x_{\mathrm{m}} e^{-\frac{i2\pi km}{n}} \quad (3)$$

Where n is the total number of data points and k is the index of each of the applied data points. It is also possible to note the presence of a complex exponential function that is related to a harmonic function as indicated by Euler's theorem represented in Equation 4 [11].

$$e^{ix} = \cos(x) + i\sin(x) \quad (4)$$

The Fast Fourier Transform (FFT) can be implemented using Excel®, where it is applied to displacement data to extract harmonic components in their complex form. From these complex values, the amplitude of each harmonic is determined by calculating the modulus of the complex number obtained through the Fourier Transform and dividing it by 2/n, where n represents the total number of data points.

To identify the corresponding frequency of each harmonic, the index k of the analyzed component is divided by the total duration of the dataset used in the analysis. This process enables the characterization of wave behavior in the frequency domain, which is essential for evaluating the dynamic response of marine systems.

This article aims to perform a spectral analysis of irregular waves using FFT and evaluate which frequencies have the highest amplitude.

2. Metodology

Using data provided by the Rede Ondas (2025) [8], it was possible to analyze measurements recorded by a

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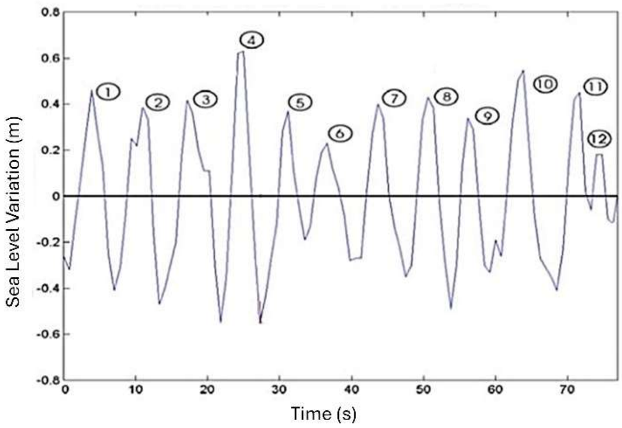
wave gauge over a 75-second period. These measurements enabled the identification of wave displacements on the ocean surface. Figure 1 illustrates the time-domain response of the recorded wave motion, providing insight into the dynamic behavior of the sea surface during the observation interval.

Figure 1. Analysed Graph. Source: Adapted from Rede Ondas (2025) [9]

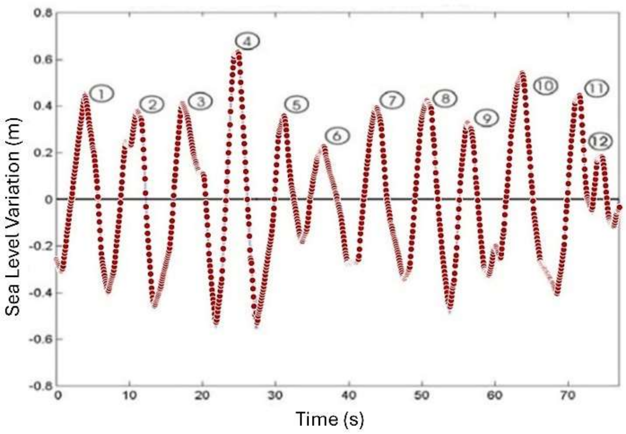


The data points used in the analysis were extracted from the original graph using the WebPlotDigitizer tool [12]. The image of the graph was first calibrated, allowing the software to accurately interpret the axes and scale. Following calibration, the data points were manually marked on the graph, enabling the tool to convert pixel positions into corresponding numerical values. This process facilitated the reconstruction of the dataset for further analysis.

To improve the accuracy of the extracted data, a filtering process was applied by limiting the RGB deviation to a maximum of 50 units from the selected color range of the graph. This constraint allowed for more precise identification of relevant data points by minimizing noise and inconsistencies in pixel interpretation.

To enable automatic selection of data points, values along the x-axis were sampled at regular intervals of 0.07 seconds. This uniform spacing is essential for the subsequent application of the Fast Fourier Transform (FFT), which requires evenly spaced time-domain data. As a result of this sampling process, a total of 1,101 data points were generated, as shown in Figure 2.

Figure 2. Selected Points



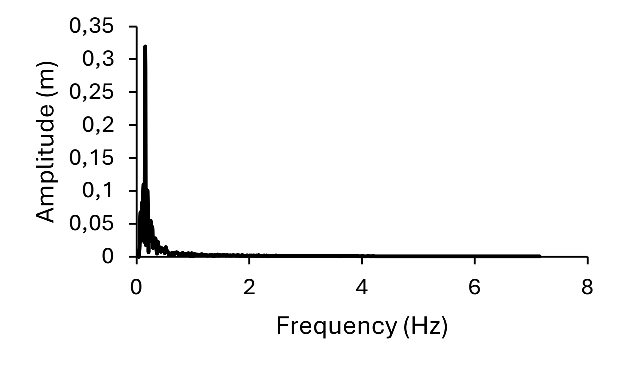
The data were initially imported into a spreadsheet in CSV format, enabling the application of the Fast Fourier Transform (FFT). To meet the computational requirements of the FFT algorithm, the time series was truncated to exactly 1024 data points, as the algorithm is optimized for datasets whose length is a power of two (2^n).

The FFT produced a series of complex coefficients representing the spectral components of the waveform, including their respective frequencies and amplitudes. To extract the magnitude of these components, the "IMABS" function was employed, which computes the modulus of each complex number. Subsequently, the resulting values were multiplied by a scaling factor (unspecified in the original excerpt) to adjust the amplitudes for the purpose of quantitative analysis or graphical visualization of the spectral data.

3. Results and Discutions

The spectral graph reveals a dominant harmonic component observed at frequencies of approximately 0.15 Hz exhibiting a peak amplitude of around 0.32 meters. This indicates that marine structures intended for deployment in such environments should avoid natural frequencies within this range to prevent resonance effects. Figure 3 illustrates the frequency-domain representation, highlighting the critical harmonic peaks.

Figure 3. Spectral Analysis



4. Conclusion

This study highlights the importance of spectral wave analysis in the design and evaluation of marine structures, as well as the effectiveness of the Fast Fourier Transform (FFT) as a method for analyzing wave phenomena in the frequency domain.

Through the spectral analysis of ocean waves, it was observed that the frequency with the highest amplitude is approximately 0.15 Hz. This finding suggests that the natural frequency of equipment intended for operation under such conditions should be carefully designed to avoid resonance within this range.

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