Inflation Targeting under Fiscal Fragility

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Abstract

We model the intertemporal trade-off between fiscal and monetary policy under an inflation-targeting regime. An indebted and altruistic policymaker chooses public expenditure and current inflation. Private agents choose consumption, debt purchase, and form expected inflation. Debt level determines target credibility: low debt levels make the target fully assured, and high levels make target announcement innocuous. For an endogenous interval of intermediate debt level, named fiscal fragility zone (FFZ), there are multiple equilibria, expected inflation is higher than the announced target, and debt rollover is expensive. We show that, within FFZ, policymakers should (i) do fiscal austerity to reduce debt gradually (ii) increase the inflation target and the share of non-indexed bonds to raise the lower bound of the FFZ. High public debt limiting low inflation is a common fact in emerging-market crises. However, it may become a global fact after an adverse global shock such as a world war or health pandemics.

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1 Introduction

In a seminal paper, Kydland and Prescott (1977) post a general claim against discretionary policies arguing that rules are a better way to coordinate expectations. Regarding monetary policy, they conclude that a policymaker "doing what is best, given the current situation, results in an excessive level of inflation, but unemployment is no lower than it would be if inflation (possibly deflation or price stability) were at the socially optimal rate." During the 1980s and 90s, several countries adopted their prescription, and the inflation targeting regime became the cornerstone of central bank coordination of inflation expectations. However, inflation target ranges are missed frequently, both in advanced and emerging economies. There are common episodes of coordination failures in which inflation expectations suddenly lose their anchor and diverge from the announced targets.[1] Most of these episodes lack sizable changes in fundamentals that would explain the shift in expectations, raising questions on the limits of inflation targeting to anchor short-term inflation expectations.

We propose a model that rationalizes these observable episodes of coordination failures and self-fulfilling inflation. The heart of our argument is that the economy's fiscal side is fundamental to understanding the capacity of the inflation-targeting regime to coordinate inflation expectations. We model a closed economy in which two types of agents, an altruistic policymaker and private agents, act rationally in an environment with complete information. The policymaker acts jointly as a fiscal authority and central bank, targeting the inflation target by choosing current inflation and financing government expenditures by selling debt. We assume that the policymaker is not perfectly committed to the inflation target and might deviate from it to make fiscal room for spending. Its decision is the solution to the trade-off between inflating public debt away and keeping inflation on target to avoid the economic costs of deviating. Private agents choose how much debt to hold and form expectations about next period inflation. Our framework builds on Cole and Kehoe (1996, 2000) and Araujo, Leon, and Santos (2013) expanding the analysis to a monetary policy setting.

In our model, target failures happen when the public debt level exceeds an endogenous threshold limit and enters the fiscal fragility zone (FFZ). When debt is low enough, the interest burden is low, and therefore government spending is high. When debt is above the endogenous cutoff and is within the fiscal fragility zone, the expected inflation rate is higher than the inflation target, generating a higher cost of debt service and lower government spending. If the benefit of abandoning the inflation target is higher than the cost of keeping it, the policymaker will inflate the public debt and increase public spending. Within the fiscal fragility

[1] Roger and Stone (2005) notice that targets are often missed (40% in their sample) and sometimes "by substantial amounts and for prolonged periods." Based on our updated data set, used in Appendix B, we concluded targets are still frequently missed (26%).

zone, the policy maker is subject to confidence crises, so the optimal policy may be gradually reducing the public debt to shut down the door to confidence crises and supports the inflation targeting regime. Moreover, while inside this region, keeping higher targets and reducing the share of inflation-indexed bonds are valuable tools to anchor expectations and mitigate the risk of inflation overshooting.

The intuition of why a higher inflation target helps to anchor expectations is simple. We show that the amount of partial default available, the difference between the inflation chosen by the policy maker when she deviates and the inflation target, decreases at the target level. Higher targets lower the benefit of a higher inflation level for indebted policymakers, increasing their credibility. Consequently, the endogenous threshold limit that characterizes the FFZ increases with the inflation target level. The intuition for avoiding indexed bonds is similar. The share of non-indexed bonds provides room for partial default and reduces the ex-post inflation level, which in turn lowers the level of expected inflation.[2]

Our results have implications for the conduct of monetary policy. It seems naïve to choose a 2% inflation target without considering fiscal fundamentals as countries eventually do. Our model suggests raising the inflation target to help coordinate expectations in the short term. The call for indebted economies to not support low target levels and seek targets compatible with their fundamentals also holds under imperfect information when private agents disagree about inflation forecast (Araujo, Berriel, and Santos, 2016). In addition, pre-fixed interest rate securities improve the credibility of the central bank by reducing the level of inflation necessary to restore fiscal space during the crisis.

Our policy prescriptions reflect some emerging economies' practices as they are more prone to crises and usually have higher inflation targets than advanced economies. Recently, the fiscal limits of inflation targets have been tested by developed and emerging countries as they increase their debt levels to provide fiscal response against the Covid-19 pandemic and shutdowns.[3] Taylor, Cogan, and Heil (2020) highlight that the US debt level is expected to continue growing, and it should reach 192% in 2050. The fiscal deficits in the US are a structural problem and a challenge to inflation expectation coordination. Sims (2020) argues that the ratio of debt service cost to total tax receipts is critical to understanding the temptation to inflate the debt away. Debt services increase with the debt and/or the interest rate. Sims

[2] In the Appendix we test our model predictions, using a panel dataset of 20 countries with at least 15 years of inflation targeting. We find evidence that deviations from the target and the probability of overshooting are negatively related to the target level. We also find evidence that deviations from the target are positively related to the debt level.

[3] Hall and Sargent (2022) described the US government's response to the Covid-19 as a "War on Covid-19", comparing the economic policies employed in this period with the two twentieth-century World Wars, which significantly increased federal government expenditures and debt. The authors are concerned that the "War on Covid-19" may entail higher inflation rates in the future, following the example of the previous World Wars.

notices that the interest rate is a positive function of the debt to GDP ratio with no guarantee that it will be forever low. A sudden increase in inflation expectation and consequently in the nominal interest rate, as in our model, could trigger an inflation episode.

Our model also rationalizes the response to the inflationary pressures in Brazil at the end of 2002. In that period, it became clear that the presidential candidate who would win the election could arrive with a new policy framework. As a result, inflation expectations exceeded the upper bound of the target, as seen in Figure 1, indicating a target confidence crisis. In response to rising inflation expectations, Brazilian policymakers twice increased the target for 2003, first at an extra meeting held in June 2002 and again in January 2003, and gradually reduced the share of inflation-indexed bonds in the public debt. This response of policymakers is in line with the prediction of our model.

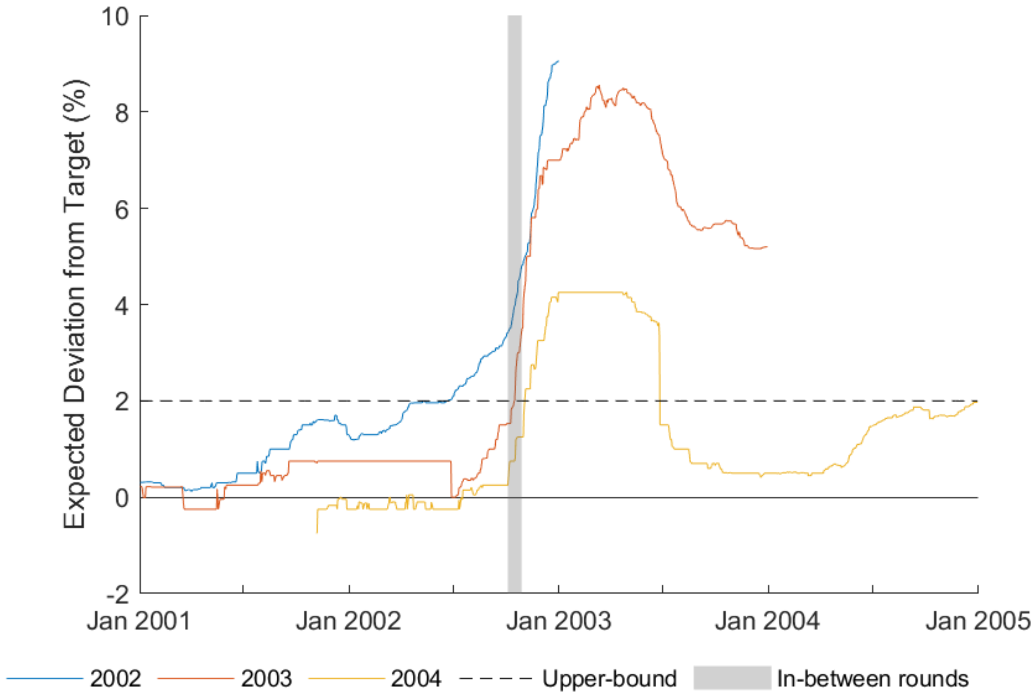

Figure 1: Expectation Crisis in Brazil

This figure shows the inflation expectation crisis that happened in Brazil in 2002. In the y-axis, we plot the expected inflation for the end of the year minus the inflation target for that year. Expected inflation is the mean expected inflation by professional forecasters, collected by the Central Bank of Brazil and available at The Focus – Market Readout. In the x-axis, we plot the date when the expected inflation was formed. Until October 2002, the expected inflation was within the inflation target bands. However, in the between rounds of the Presidential election – shaded grey region –, the inflation expectations overshoot the target's upper bounds at all horizons relevant to the central bank (current year, 1-year ahead, and 2-years ahead).

Related Literature: The message on the fiscal limits of monetary policy achievements and the interdependence between fiscal discipline and price stability is amply addressed in the literature. Sargent and Wallace (1981) show the importance of the fiscal side to understand inflation control, followed by Leeper (1991), Sims (1994, 2011), Woodford (1995), Araujo and Leon (2002), Leeper and Leith (2016), Araujo, Berriel, and Santos (2016), and Cochrane (2018). We are adding to this literature by formalizing some policy prescriptions for the indebted policymakers in an inflation-targeting regime. Our policy prescription for bond-type selection has been examined in a different framework by Fischer (1983).

We believe our messages are novel to the inflation-targeting literature. We innovate by bringing to a simple DSGE a strategic policymaker who may use inflation to default partially. Our approach closely follows papers on confidence crises in debt markets as in Cole and Kehoe (1996, 2000), Calvo (1988), and Arellano, Mihalache, and Bai (2019). However, we detach from a policymaker modeled by fiscal and monetary rules as in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Papers exploring debt crises and their relation to monetary policy include Uribe (2006), Aguiar, Amador, Farhi, and Gopinath (2013), Corsetti and Dedola (2016), Bacchetta, Perazzi, and van Wincoop (2018), and Arellano, Mihalache, and Bai (2019). However, none of these consider the relation with inflation target coordination. To the best of our knowledge, the literature combining inflation targeting and debt crises is scarce.

Next Sections: In section 2, we set out the model and derive the recursive form defining the equilibrium. In section 3, we specify functional forms and parameter values in a quantitative analysis to match the situation in Brazil in 2002. We then move to analyze the results from our model. In section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses. Finally, the last section presents some remarks.

2 Model

We consider a closed economy with two types of agents: a policymaker and private agents. Each agent lives infinite periods and forms rational expectations with complete information. The policymaker acts as a mix of fiscal and monetary authority, choosing current inflation and selling one-period debt to finance itself. In our setup, inflation choice reduces to a discrete choice each period whether to deviate or not from the target. We assume the policymaker is altruistic and maximizes private agent welfare. Private agents receive a stream of fixed endowments. In each period, they choose how much debt to hold and form expectations about next-period inflation taking into account the exogenous announced inflation target and the current debt level. When multiple equilibria are possible, a sunspot variable determines the equilibrium.

2.1 Basic Setup

Policymaker

We assume an altruistic policymaker who chooses both fiscal and monetary policies to maximize private agents' utility. As a monetary authority, the policymaker chooses the inflation rate π_t and as a fiscal authority next period's debt D_{t+1}:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{1}$$

where c_t is private agents consumption in period t, g_t is government spending on public goods, and β is the inter-temporal discount rate $0 < \beta < 1$. Consumption and public goods are non-negative. We define private agents utility as a weighted average of a linear consumption and government spending utility similar to Cole and Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as a relative preference for consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) v(g_t)$$

where v is a twice differentiable strictly increasing and strictly concave function of g satisfying $\lim_{g \to 0^+} v(g) = -\infty$.

Linearity in consumption is a strong assumption and deserves further comments. First, it allows us to define the real interest rate as a risk-neutral pricing formula, which approximates the equilibrium ex-post real interest rate. It also simplifies the problem by making the debt stationary outside the crisis zone that remains to be defined. Finally, it readily makes the marginal utility of public goods higher (lower) than the marginal utility of consumption for high (low) debt values. Given a constant marginal utility of consumption and a decreasing marginal utility of government goods, higher levels of debt, associated with higher interest burdens, have a higher marginal utility of public goods.

In each period, the policymaker finances the non-negative spending g_t and the repayments on previous period obligations through a fixed tax rate τ[4] on a deterministic endowment e and the issuance of new debt D_{t+1}. We assume that taxes level is not high enough so that

[4] The fixed tax rate hypothesis can be interpreted as a situation in which the policymaker has no additional space to increase taxes to reduce indebtedness without significantly affecting the output. This situation is close to what is observed in a middle-income economy with relatively high tax levels like Brazil.

total tax revenue is higher than the optimal amount of public spending, which is the level that equates to marginal spending between public and private consumption. Mathematically, we can write this hypothesis as $(1-\rho)v'(\tau e) \geq \rho$. The government's budget constraint is given by,

$$g_t + (1+r_t)D_t \leq D_{t+1} + \alpha_t \tau e \ , \tag{2}$$

where D_t is the last period debt.[5]

The fixed endowment is subject to a penalty α_t that depends on the policymaker's choice of inflation. Let π^a denote the exogenously set inflation target. The penalty function α is divided into two components, α^p and α^c. The first component, α^p, depends on the inflation level and reflects the productivity cost of the inflation level on output.[6] We assume a productivity cost of inflation function of the form of

$$\alpha^p(\pi) = (1-\kappa) + \kappa e^{-\lambda \pi^2} \ ,$$

where $1-\kappa$ is the lower limit on the inflation cost and λ is a fixed parameter. In this setup, $\alpha(0) = 1$, so the optimal inflation level considering only the productivity cost of inflation is zero. The second component of the penalty function α^c is a permanent fixed cost that affects the economy if the policymaker chooses to deviate from the target, reflecting the effect of a loss of credibility on the economy.[7] This fixed cost is of the form

$$\alpha_t^c = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \alpha_{t-1}^c = 0 \\ -\epsilon & \text{if } \pi_t \neq \pi^a, \alpha_{t-1}^c = 0 \\ \alpha_{t-1} & \text{otherwise} \end{cases}$$

This productivity factor α_t should be understood as a reduced form capturing both the impact of inflation on welfare and output and the cost of deviating from the inflation target on economic activity. Therefore, $\alpha_t = \alpha^p + \alpha^c = \alpha(\pi_t, \pi^a, \alpha_{t-1})$ is a function of current inflation

[5]We restrict our analysis to initial debt levels that leave the policymaker with a non-empty set of feasible choices, $D_t \in [0, D^{max}]$, where D^{max} is high enough. For very high initial debt levels, the policymaker could have no way of satisfying the positive constraint on c and g. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, which leaves no space for spending. Then, even if the policymaker were to default partially on debt payments, it would still be unable to meet future positive spending restrictions due to the high future debt servicing costs and the inability to use inflationary surprises again.

[6]See Bailey (1956) and Lucas (2000) for examples of models that find a role for inflation levels to affect welfare and output. Cysne (2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model, while Campos and Cysne (2018) estimate welfare costs of inflation for the Brazilian case.

[7]Our approach of assuming an exogenous function form for the "cost of deviating" is in line with the literature. Exogenous penalty functions are also assumed in self-fulfilling debt crisis models as in Cole and Kehoe (1996, 2000) and in sovereign default models as in Arellano (2008). We interpret the penalty function as a reduced and parsimonious form of capturing the negative impacts of inflation deviation on economic activity.

π_t, the inflation target π^a, and its past value α_{t-1}, and is differentiable for any $\pi \neq \pi^a$. Finally, we define $\alpha^a = \alpha^p(\pi^a)$, the productivity cost of committing to the inflation target.

Given a linear utility in c, we define the ex-post real interest rate by:

$$r_t = \frac{1+\pi_t^e}{1+\pi_t}\frac{1}{\beta} - 1 \,, \tag{3}$$

where $\pi_t^e = \mathbb{E}_{t-1}[\pi_t]$ is the expected inflation for period t formed by private agents in period $t-1$ and π_t is the current inflation.[8] In this risk-neutral formula for the real interest rate, an agent expects a real return over the government bonds of $1/\beta$ from period $t-1$ to period t.

In each period, the policymaker can satisfy the budget constraint by: i) adjusting expenditures, ii) issuing new debt D_{t+1}, and iii) partially defaulting on debt through an inflationary surprise ($\pi_t > \pi_t^e$) and rolling over the remaining debt. When the current inflation rate is equal to expectation $\pi_t = \pi_t^e$, the ex-post real interest rate will equal the inverse of the intertemporal discount rate, $1/\beta$. An inflationary surprise reduces the ex-post real interest rate and, consequently, the payments the policymaker makes on its debt. Such a partial default offers additional fiscal room for government spending.

Private-agents

We assume a continuum of infinitely lived private agents who choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \; \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \,, \tag{4}$$

Each period, private agents receive a deterministic endowment e and payments on their bond holdings. The endowment is taxed at a constant rate τ by the government. The private agents' budget constraint is given by:

$$c_t + d_{t+1} \leq (1+r_t)d_t + \alpha_t(1-\tau)e \tag{5}$$

[8]Using of the risk-neutral approximation 3 in the model allows us to prove some important mathematical properties throughout the text, while not affecting the numerical properties of the computational simulations. This is because, as it is shown in appendix E.1, the risk-neutral real pricing definition in 3 is a close approximation to the equilibrium real interest rate,

$$r_t^{eq} = \frac{1}{\mathbb{E}_{t-1}\left[\frac{1}{1+\pi_t}\right]}\frac{1}{1+\pi_t}\frac{1}{\beta} - 1 \,,$$

that comes out from the first-order condition to the private-agents problem.

where d_{t+1} is one-period bonds bought in t and d_t is the previous period bond holdings paying the interest rate $(1 + r_t)$. Private agents also form their inflation expectations π_t^e. The expectations formed will depend on the timing of actions assumed and will be properly defined further on.

Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker who might choose an inflation level higher than the inflation target as a way of transferring resources for increasing public spending.[9] In each period, the policymaker may choose to deviate from the exogenously set inflation target π^a, and private agents understand this when forming their expectations π_t^e. We call the inflation rate chosen by the policymaker when deviating from the target *discretionary inflation*. It is the result of a trade-off between increasing spending today against the costs of reducing consumption and the losses due to the costs of deviating from the inflation target. Let π_T^D be the endogenous and optimal level of discretionary inflation chosen at the time T of the deviation.

We assume that once the policymaker deviates from the inflation target, private agents lose confidence in the commitment of the policymaker to the target. Therefore, private agents update the probability of the policymaker deviating next period setting it equal to 1. Consequently, after the policymaker deviates, the economy enters a steady-state as there is no longer any uncertainty to be resolved. The optimal fiscal policy is to maintain constant debt, such as $D_t = D_T \; \forall \; t > T$, as shown below in Proposition 3. Finally, the penalty function takes the value $\alpha_T = \alpha(\pi_T)$ when deviating and remains so thereafter. The problem the policymaker resolves when defining the level of discretionary inflation can be written:

$$\pi_T^D = \underset{\pi}{\text{argmax}}\; u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g)$$

subject to

$$\begin{aligned} g_T &= \alpha(\pi)\tau e - D\left(\frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta} - 1\right) \\ g &= \alpha(\pi)\tau e - D\left(\frac{1}{\beta} - 1\right) \\ c_T &= \left(\frac{1+\pi_T^e}{1+\pi}\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e \\ c &= \left(\frac{1}{\beta} - 1\right) D + \alpha(\pi)(1-\tau)e. \end{aligned} \tag{6}$$

[9] We do not model mechanisms of partial default on local currency domestic debt other than inflation, although governments have opted for alternatives such as reduction of principal or lower coupons (Reinhart and Rogoff, 2008).

Observe that $u(c_T, g_T)$ is possibly increasing with π, but $\frac{\beta}{1-\beta}u(c, g)$ is necessarily decreasing in π. Given rational expectations, in equilibrium, π_T^D is optimal given π_T^e and vice versa.[10]

To gain intuition, consider the first-order condition of the policymaker problem for choosing discretionary inflation. The first-order condition is,

$$\rho\left(-\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'(1-\tau)e\right) + (1-\rho)v'(g_T)\left(\frac{1+\pi_T^e}{(1+\pi)^2}\frac{1}{\beta}D + \alpha'\tau e\right) \\ + \frac{\beta}{1-\beta}\left(\rho\alpha'(1-\tau)e + (1-\rho)v'(g)\alpha'\tau e\right) = 0\,. \tag{7}$$

The first term represents the short-term marginal loss from reduced consumption caused by the inflationary surprise and the productivity shock to endowments. The second term, which may be positive, is the marginal benefits of higher spending through the inflationary surprise after discounting marginal losses due to lower tax revenues. Finally, the last term illustrates the lasting effects of the productivity penalty on the economy, causing lost consumption and government spending.

We can characterize the discretionary inflation chosen by the policymaker through the following properties:

Proposition 1 ***(Discretionary inflation is increasing on debt)****: If π^D is an interior solution to the problem (6), $\alpha''(\pi^D) < 0$, and α' is sufficiently bounded, then the discretionary inflation level π^D is increasing on the debt level D: $\frac{\partial \pi^D}{\partial D} > 0$.*

Proof: see Appendix A.1.

Higher debt levels increase interest spending, reducing available funds for government spending, which causes the policymaker to increase the discretionary inflation level to compensate for the reduced public spending. This is true as long as the penalty function is not too steep since this would cause spending levels to drop due to the higher penalty.

Proposition 2 ***(Discretionary inflation deviation reduces with target level):*** *If π^D is an interior solution to the problem (6), $\alpha''(\pi^D) < 0$, and α' is sufficiently bounded, then there exists an interval $[0, \hat{D}]$ such that $\frac{\partial \pi^D}{\partial \pi^a} < 1$.*

Proof: see Appendix A.1.

[10] We numerically solve this problem by writing it as a fixed point. First, we assume an initial $\pi_{T,0}^e = \pi^a$ and then find the optimal $\pi_{T,1}^D$. We update $\pi_{T,1}^e$ using $\pi_{T,1}^D$ according to the inflation expectation formation process of the private agents that will be explained later. If $\pi_{T,1}^e \neq \pi_{T,0}^e$, the problem is iterated to find the new optimal $\pi_{T,2}^D$ given $\pi_{T,1}^e$. We continue this process until $|\pi_{T,i-1}^e - \pi_{T,i}^e| < \epsilon$, where ϵ is a small number. The existence of a rational expectation inflation π^e given an optimal discretionary inflation π^D chosen by the policymaker is shown in the online appendix C.

If $\frac{\partial \pi^D}{\partial \pi^a} < 1$,[11] then the deviation from the target $\pi^D - \pi^a$ decreases as the target level π^a rises. The intuition for this result is simple: a higher target induces the policymaker to raise π^D in order to attain the same level of reduction on interest payment for a given level of debt. However, since the penalty is increasing on inflation, raising the discretionary inflation causes total output to fall, which in turn reduces tax revenue and therefore spending. The policymaker balances these two effects, reducing deviation from the target.

Timing

Rational expectations govern the strategic interactions between the policymaker and private agents. As in Cole and Kehoe (1996, 2000), self-fulfilling multiple equilibria may occur. Conditional on the debt level, the best response from the policymaker's perspective may depend on the expectations of private agents. If private agents expect a deviation from the target, the best response will be to deviate. If they expect no deviations, the best response will be to keep inflation on target. In this case, we consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium. The sunspot variable determines which of the possible inflation rates will be the actual inflation rate π_t implemented by the government when there are two equilibrium rates: the inflation target π^a and the discretionary inflation π_t^D.

At the beginning of each period, uncertainty is resolved through the realization of the sunspot variable ζ_t. The policymaker, considering the sunspot variable previously drawn, chooses how much debt D_{t+1} to sell and the inflation rate π_t, which will either be the target π^a or the discretionary inflation rate π_t^D. Finally, private agents form their expectations about the next period's inflation rate π_{t+1}^e and decide how much debt d_{t+1}. In summary, the timing of the model is:

- 1^{st} Sunspot variable ζ_t is realized.
- 2^{nd} Policymaker chooses actual inflation π_t, given sunspot ζ_t;
- 3^{rd} Policymaker chooses next debt level D_{t+1};
- 4^{th} Private agents form next-period inflation expectation π_{t+1}^e and choose the amount of next period debt d_{t+1} to hold;

Given this timing, private agents may face uncertainty about which equilibrium will be selected next period when forming their inflation expectations. They will form expectations

[11] While we cannot analytically determine the upper bound $\hat{D}$ in proposition 2, numerically the interval in which the proposition is valid covers all relevant debt levels for the calibration.

over the probability of each outcome, considering the exogenous distribution of the sunspot variable that determines the actual inflation rate. Inflation expectations will therefore be $\pi_t^e = f\pi_t^D + (1-f)\pi^a$ where f is the exogenously determined probability of the policymaker deciding to deviate from the inflation target due to an adverse situation, a negative sunspot.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private agents choose their actions sequentially. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \alpha_{-1})$ is public since the aggregate debt D, the expected inflation for the current period π^e, the realization of the sunspot variable ζ, and the past penalty α_{-1} have all been determined in the previous period. The policy choices, π and D', the expected inflation for next period $\pi^{e\prime}$, and the individual debt holdings for next period d' determine the equilibrium jointly with s. We denote by $\pi(\cdot)$ and $D(\cdot)$ the inflation and debt policy functions, by $r(\cdot)$ the real interest rate function, and by $\pi^e(\cdot)$ the inflation expectation function, all yet to be defined.

To define a recursive equilibrium, we work backward on the timing of actions in each period. We start the definition of a recursive equilibrium with private agents as they move last. When forming expectations $\pi^{e\prime}$ at the end of any period, private agents know all their public debt holding d, the aggregate state s, the policymaker's offer of new debt D', the current period inflation π, and the policymaker's optimal policy functions. The following functional equation defines the private agent's value function:

$$V^{pa}(s, d, \pi, D') = \max_{c,d'} u(c, g) + \beta \mathbb{E} V^{pa}(s', d', \pi', D'')$$

subject to

$$\begin{aligned}
& c + d' \leq (1 + r(s, \pi))d + \alpha(\pi, \pi^a, \alpha_{-1})(1-\tau)e \\
& s' = \Big(D', \pi^e(s, d, \pi, D'), \alpha(\pi, \pi^a, \alpha_{-1}), \zeta'\Big) \\
& \pi' = \pi(s') \\
& D'' = D(s') \\
& c \geq 0 \\
& d' \geq 0
\end{aligned} \tag{8}$$

in which we assume that private agents cannot sell public debt. The penalty function $\alpha(\cdot)$ is a function of its previous value α_{-1}, the inflation target π^a, and current inflation π.

Each period after the policymaker decides how much debt D' to offer and the inflation rate π, private agents decide how much debt to hold. Let $d'(s, d, \pi, D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, they perfectly anticipate π and the real return is always $1/\beta$. If multiple equilibria are possible, private agents do not know what the policymaker will opt to do.

When forming inflation expectations, private agents look at what the policymaker could do next period. Their expectations are defined as $\pi^e(s,d,\pi,D') = \mathbb{E}\pi(s')$, where the expectation is conditional on all information available to the agent at the moment. When forming expectations, the set $(D', \pi^{e\prime}, \alpha) \in s'$ is known to private agents. Hence, the only unknown variable on which private agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable commonly known distribution, we have

$$\mathbb{E}\pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \alpha) + (1-f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \alpha) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{9}$$

where f is the exogenous probability of the adverse equilibrium occurring and $\pi^D(D', \pi^{e\prime}, \alpha)$ is the discretionary inflation chosen by the government when deviating given $(D', \pi^{e\prime}, \alpha) \in s'$.

The policymaker chooses, at the beginning of the period, inflation π and debt issuance D', given state s. The policymaker knows that the next period's debt level affects the private agents' inflation expectations and resolves the following problem:

$$\begin{aligned} V^p(s) = & \max_{\pi, D'} u\Big(c(s,d,\pi,D'), g\Big) + \beta \mathbb{E} V^p(s') \\ & \text{subject to} \\ & \qquad g + (1 + r(s,\pi))D \leq D' + \alpha(\pi, \pi^a, \alpha_{-1})\tau e \\ & \qquad s' = (D', \pi^e(s,d,\pi,D'), \alpha(\pi,\pi^a,\alpha_{-1}), \zeta') \\ & \qquad g \geq 0 \end{aligned} \tag{10}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions for the representative private agent V^{pa} and for the policymaker V^p; functions $c(\cdot)$ and $d'(\cdot)$ for the private agents' consumption and saving decisions; functions $\pi(\cdot)$ and $D'(\cdot)$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e(\cdot)$; a real interest rate function $r(\cdot)$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private agents' problem with c, d' and $\pi^{e\prime}$ the maximizing choices when $d' = D'$;

- Given π^e, V^p is the value function for the solution to the policymaker problem, and both D' and π are the maximizing choices;

- $D'(s)$ equals $d'(s, d, \pi, D')$.

Our definition of an equilibrium is similar to that of Cole and Kehoe (1996) and Cole and Kehoe (2000) and is restricted to a Markov equilibrium. Future conditional plans of the agent can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

The ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming inflation has always been on target, three different scenarios can be drawn according to the debt level D[12]:

- The *no crisis zone*: D such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D) \rightarrow \pi_t = \pi^a = \pi^e$;

- The *fiscal fragility zone*: D such that $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot;

- The *fiscal dominance zone*: D such that $V^p(D, \pi_t = \pi^D, \pi^e = \pi^a) \geq V^p(D, \pi_t = \pi^a, \pi^e = \pi^a) \rightarrow \pi_t = \pi^D = \pi^e$.

In the first case, the policymaker finds it better to keep inflation on target even when private agents think it will not. Consequently, only one equilibrium is possible where private agents have faith in the policymaker delivering on the target inflation. Since there is only one optimal choice for the policymaker regardless of the private agents' expectations, the sunspot ζ is disregarded, and the only important variable defining the policymaker value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

Whenever the policymaker is in the no-crisis zone or the fiscal dominance zone, it will always choose a stationary debt policy, as it is shown in Proposition 3:

Proposition 3 ***(Stationary Policy Outside of the Fiscal Fragility Zone):*** *The optimal debt policy chosen by the policymaker outside of the fiscal fragility zone at period T is stationary, that is, $D_t = D_T$ for all $t \geq T$.*

[12]With a slight abuse of notation, we denote as $V^p(D, \pi_t, \pi^e)$ the total intertemporal utility attained by the policymaker by choosing inflation level π_t, given debt level D and private agents expected inflation π^e.

Proof: see appendix A.2.

The more interesting scenario is multiple equilibria akin to self-fulfilling target failures. If private agents believe the target will be delivered, then the policymaker will indeed prefer to do so. On the contrary, in the face of adverse expectations, the policymaker chooses to deviate. In this zone, private agents have doubts about the commitment of the monetary authority to the target. The equilibrium is chosen by the realization of a sunspot, something the government binds its choice to but is unrelated to any observable fundamentals. In the rest of the article, we will interpret it as a deterioration in inflation expectations. That is, inflation expectations go from π_t^e to π_t^D for some reason unrelated to fundamentals.

When government debt is within the fiscal fragility zone, the expected inflation rate is higher than the inflation target, generating a higher cost of debt service and lower government spending. Thus, the government has an incentive to raise spending, which it can do through inflation or increasing debt. To see this, let us recall the real interest rate on bonds from equation (3), $r_t = (1+\pi_t^e)/\beta(1+\pi_t) - 1$. In the fiscal fragility zone, inflation expectations will be given by $\pi_t^e = f\pi_t^D + (1-f)\pi^a$. The real interest rate in the fiscal fragility zone when the policymaker delivers the target will be given by:

$$r_t = \frac{(1 + f\pi_t^D + (1-f)\pi^a)}{1+\pi^a}\frac{1}{\beta} - 1 \tag{11}$$

which is obviously higher than the interest rate outside this zone, $1/\beta$.

Whether an economy exhibit all of these zones will depend on the credibility cost α^c, as the following proposition shows:

Proposition 4 ***(Existence of the no-crisis and fiscal dominance zones):*** *For a β a sufficiently close to 1, we can state:*

- *If $\alpha^c = 0$, then every debt level $D \in [0, D^{max}]$ is in the fiscal dominance zone;*
- *For any $\alpha^c < \alpha^a - 1$, there is an interval $[0, D_-]$ in the no-crisis zone, with $D_- > 0$;*
- *There is a sufficiently low $\alpha^c > -1$ such that every $D \in [0, D^{max}]$ is in the no-crisis zone.*

Proof: see appendix A.3

A zero credibility cost means that it is costless to deviate from the target, and so the policymaker always deviates regardless of debt level D. On the other hand, a too high credibility cost means that deviating from the target is never optimal since the high penalty greatly reduces government spending through lower tax revenues.

Proposition 5 shows that whenever the economy is in the fiscal fragility zone, the policymaker will choose discretionary inflation levels above the inflation target, as long as there is a marginal utility for government spending that is higher than the marginal utility for private spending. This condition is always satisfied when debt is positive, given the linearity hypothesis for private consumption and strict concavity of public consumption.

Proposition 5 ***(Conditions for Positive Deviation from Target):*** *Suppose the utility function, penalty, and initial debt level satisfy the stated assumptions, and that $\alpha^c < \alpha^a - 1$. Then, in the fiscal fragility zone, the optimal deviation is always positive.*

Proof: see appendix A.4.

An altruistic policymaker maximizing private agent welfare may choose to deviate from the inflation target when it has limited fiscal room to finance public spending.

The Inflation Target Coordination Role

The marginal ability of the policymaker to transfer resources through inflation decreases as the target π^a increases, changing the tradeoff determining discretionary inflation. The marginal benefit of discretionary inflation will be reduced given the lower marginal capacity to transfer resources, which is a consequence of proposition 2.

A policymaker with a higher inflation target will choose a smaller deviation from the target when private agents start doubting the target. Consequently, as deviations decrease, the policymaker will face a lower real interest rate on its bonds in the fiscal fragility zone. To see this last point, let us look at the implication of the real interest rate in the fiscal fragility zone. The real interest rate on bonds is given by:

$$r = \frac{1 + \pi^e}{1 + \pi} \frac{1}{\beta} - 1$$

where $\pi = \pi^a$ and $\pi^e = f\pi^D + (1 - f)\pi^a$ in the fiscal fragility zone. By proposition 2, we know that $\frac{\partial \pi^D}{\partial \pi^a} < 1$. Therefore, it is also true that $\frac{\partial r}{\partial \pi^a} < 0$,

$$\begin{aligned}
\frac{\partial r}{\partial \pi^a} &= \frac{1}{\beta}\left(\frac{f\frac{\partial \pi^D}{\partial \pi^a} + (1-f)}{1+\pi^a} - \frac{1 + f\pi^D + (1-f)\pi^a}{(1+\pi^a)^2}\right) \\
&< \frac{1}{\beta}\left(\frac{1}{1+\pi^a} - \frac{1 + f\pi^D + (1-f)\pi^a}{(1+\pi^a)^2}\right) \\
&= \frac{1}{\beta}\left(\frac{f(\pi^a - \pi^D)}{(1+\pi^a)^2}\right) \\
&< 0
\end{aligned}$$

since $\pi^a - \pi^D < 0$.

2.4 Inflation-Indexed Debt

It is not unusual for governments to issue inflation-indexed bonds. We will look at the implications of changing the nature of the bonds. To achieve such indexed bonds within the framework of our model, we change the action timing to give private agents all the needed information to anticipate policymaker decisions perfectly. By allowing private agents to know the realization of the sunspot variable when forming their inflation expectations, bonds will pay a real interest rate $1/\beta$ in all states of nature.

1^{st} Policymaker chooses actual inflation π_t;

2^{nd} Policymaker chooses next debt level D_{t+1};

3^{rd} Next period sunspot variable ζ_{t+1} is realized;

4^{th} Private agents form next period inflation expectation π^e_{t+1} and choose the amount of next period debt d_{t+1} to hold.

With this new timing, private agents' information sets are given by $(s, d, \pi, D', \zeta') = s'$. Inflation expectations π^e given information set s' will be such that $\pi^e(s') = \pi(s')$ is the policymaker's choice of inflation for the next period.[13] As policymaker's choices are anticipated, it is no longer possible to transfer resources from private agents in the event of a bad sunspot. In equilibrium, the policymaker would choose $\pi^D = \pi^e$ in the discretionary equilibrium since π^D is optimal given π^e and vice versa. It is important to notice that, in equilibrium, the discretionary inflation could be different from the announced target $\pi^D \neq \pi^a$. The only way the discretionary inflation could equal the announced target $\pi^D = \pi^a$ would be if the inflation

[13] The different timing only change the problem (6) by how we update inflation expectation π^e.

expectation equals the inflation target $\pi^e = \pi^a$. The next section will exploit the differences between indexed and nominal bonds.

3 Quantitative Analysis

In this section, we calibrate the model based on the 2002 confidence crisis in Brazil. The presidential election of 2002 is an interesting case study in that the candidate most likely to win was running on a platform seem to deteriorate the fiscal situation. Professional forecasters surveyed by the central bank saw inflation overshooting the target for all horizons. This loss of credibility of the inflation target in the face of a perceived fiscally fragile situation is the type of event our model aims to capture.

3.1 Functional Forms and Calibration

Our model is calibrated on yearly frequency to match the usual time frame targeted by central banks, and almost all parameters correspond to observable values during the 2002 confidence crisis in Brazil. First, we set the inflation target, π^a, to 3.5%, the official target that prevailed in 2002. Second, the discount factor, β, is $1/1.0928$ to match the historical average of the ex-post real interest rate between 1996 and 2019.[14] Third, the tax rate on the endowments, τ, equals the 2002 general government revenue over GDP, 0.35. Fourth, the exogenous crisis probability, f, matches the country risk captured by the EMBI + Brazil around October 2002. Fifth, we set the endowments, e, to 1.5 so that the public spending marginal utility of the total tax revenues is not lower than the private consumption marginal utility: $(1-\rho)v'(\tau e) \geq \rho$. Finally, we choose a neutral value for consumption preference $\rho = 1/2$ for the baseline exercises. We will use this parameter to do some static comparative results later.

The parameter λ of the productivity cost of the inflation penalty is set according to Campos and Cysne (2018)'s estimation of a 0.35% of GDP cost for an inflation of 10% in recent Brazilian experience. The fixed cost of deviating ϵ equals 0.002, meaning a permanent 0.2% of GDP penalty for deviating from the target, and it is set to jointly match the gross debt level and the inflation index observed in Brazil in 2002. The crises zone starts approximately in 70% of the debt ratio, the debt observed in 2002. The lower bound for the penalty, κ, is set to 20%. Table 1 summarizes the chosen values. For the calibrated model, we assume that the government spending utility function $v(g) = \log(g)$.

[14] Using inflation-indexed bonds, such as Brazilian Bonds NTN-C or NTN-B, around 2002 would give similar results.

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
κ	20%	Limit to TFP cost	Brazilian 2002 crisis
ϵ	.002	Fixed cost	Brazilian 2002 crisis
λ	1.77	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table 1: Parameters of the Baseline Model

3.2 Results

An indebted and altruistic policymaker optimally choosing inflation may deviate from the target in the event of an expectation shock. In our calibrated model, the policymaker becomes subject to such shocks after reaching a debt to GDP ratio of 70%. Below the 70% of the debt ratio, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower bound, the equilibrium depends on the private agent's expectation, and the policymaker may decide to deviate given a negative sunspot shock. Taking this probability into account, private agents will demand higher nominal interest rates on government bonds once the policymaker exceeds this lower bound debt level. Finally, for debt levels exceeding 105% of GDP, the policymaker will always deviate from the target.

Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the fiscal fragility zone, it prefers to maintain debt levels constant, as shown in Proposition 3. Within the fiscal fragility zone, it might either: i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain constant debt levels; or iii) increase its debt to maintain a given spending level. In Figure 2, we plot next the period's debt as a function of current debt. The three possible responses of the policymaker are seen within the fiscal fragility zone. Those results are similar to Cole and Kehoe (1996).

For a moderate initial debt level inside the FFZ, the policymaker chooses a fiscally responsible debt path to avoid the expected endowment loss from deviating from the inflation target in the eventuality of adverse inflation expectation. In this region, expected inflation is higher than the target rate, which means that the policymaker faces a higher real interest

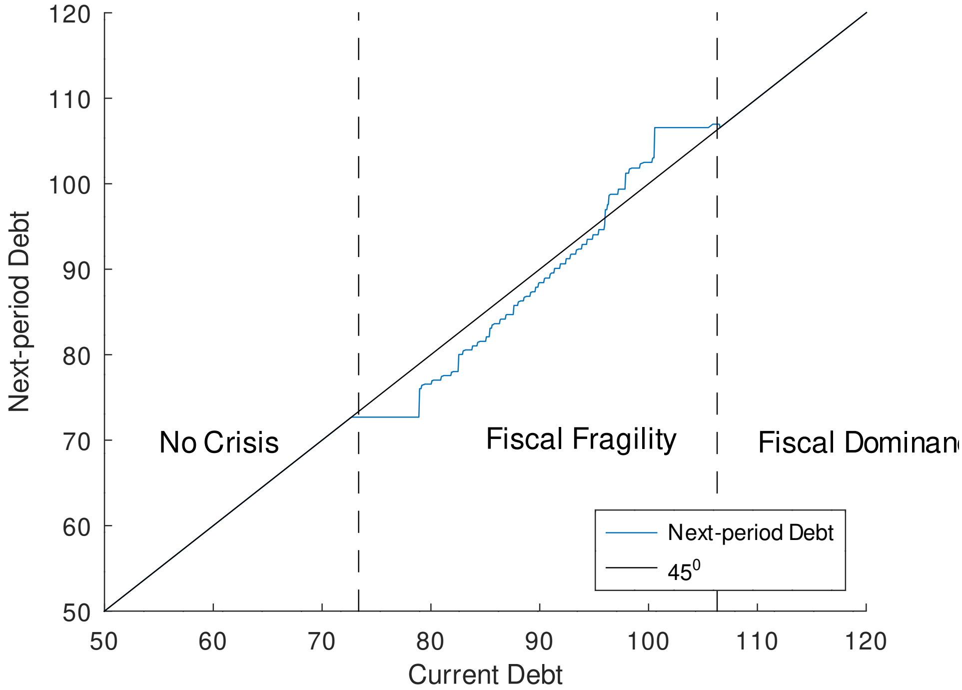

Figure 2: Debt Policy Function

rate when compared to the no crisis zone. However, as long as a negative sunspot shock that removes the credibility of the policymaker does not hit the economy, the optimal fiscal policy is to gradually reduce the debt-to-GPD ratio until the economy exits the FFZ. As the policymaker follows this austerity policy, expected inflation gradually lowers, reducing the real interest rate burden and making it easier for the economy to exit the FFZ. This can be noted by observing that the slope of the policy function decreases as debt-to-GDP approaches the lower bound of the FFZ. Table 2 presents the expected inflation rates and the corresponding number of periods required to exit the FFZ for different levels of initial debt:

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4%	1 year
90%	Fiscal Fragility	4.5%	6 years
110%	Fiscal Dominance	10%	-

Table 2: Expected inflation rates and Exiting Time of the FFZ in the calibrated model for the Brazilian case

For a high initial debt level and fixed inflation target, the policymaker gradually reduces debt to return to the no-crisis zone. However, it takes a significant number of periods for the policymaker to regain credibility, during which it faces expected inflation rates higher than the target. The optimal policy for stabilizing inflation expectations in an environment of high indebtedness results in higher inflation expectations for a significant amount of time. Gradual exiting as an optimal decision is also present in Cole and Kehoe (1996, 2000) and in Sims (2020). The latter explicitly makes this prescription when claiming that it is not wise to reduce debt quickly through a shrinking fiscal room. This result is in line with many episodes in which countries that experienced sudden increases in their debt-to-GDP ratios needed to stabilize their economies through higher temporary inflationary rates. Hall and Sargent (2022) describe the US post-war experience and attribute an important role in reducing the debt real-value to the increases in price levels. They argue that a similar scenario may happen after the rise in the debt-to-GDP ratio that followed the Covid-19 pandemic.

Nevertheless, as the debt level grows, the fiscal room available to the policymaker shrinks due to the increased interest burden. Eventually, it is more interesting to run up debt to maintain spending. This situation happens above debt levels of 95%, as seen in Figure 2. An austerity policy to exit the FFZ is not optimal, and the policymaker eventually suffers an adverse shock and loses credibility. By opting to run up debt, the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

Higher inflation targets may improve the credibility of monetary policy and help coordinate private agents' expectations by increasing the costs of deviating to attain a given inflationary transfer of resources. Private agents use the inflation target to form expectations in the fiscal fragility zone, and the target functions as a nominal anchor for expectations. In Figure 3, we show a sensitivity analysis of the deviations to changes in the inflation target. We plot $\pi^D - \pi^a$ for three different inflation targets (0%, 3.5%, and 7.5%), keeping the other parameters at their baseline.

A higher inflation target improves coordination by the policymaker by reducing the discretionary deviation from the target rate and by reducing the real fiscal burden of debt through a lower real interest rate. Firstly, a higher target rate reduces the marginal capacity of the policymaker to transfer resources, which implies a lower marginal benefit for discretionary inflation. Secondly, a policymaker with a higher inflation target chooses a smaller deviation from it and, consequently, faces a lower real interest rate on its bonds in the fiscal fragility zone.

For baseline parameters, deviations $\pi^D - \pi^a$ decrease in the inflation target, reducing the ex-post real interest rate in the fiscal fragility zone. Denote as $\underline{\mathrm{D}}$ the lower bound of the fiscal fragility zone. For initial debt levels below $\underline{\mathrm{D}}$, the policymaker will have a perfectly credible target, preferring to keep inflation on target regardless of the private agent expectations.

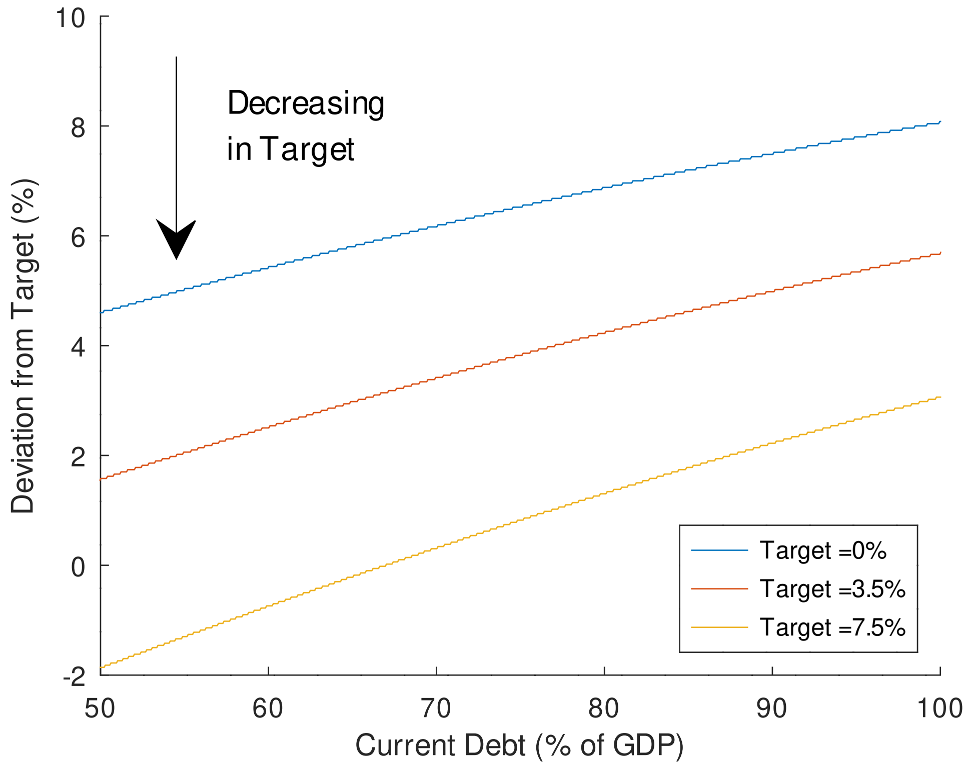

Figure 3: Sensitivity: Deviations to Inflation Target

Above the lower bound, private agents may doubt its commitment. As deviations decrease in the target, it becomes less costly to keep inflation on the target for a given debt levels. This effect increases the credibility of the inflation target as it remains fully assured up to higher levels of debt, as shown in Figure 4, which plots next-period debt for the different inflation targets.

The lower bound $\underline{D}$ increases as the target rate raises. This result implies that policymakers should consider current debt levels and fiscal conditions when deciding to decrease the inflation target. This reduction can cause a loss of credibility for the government commitment as it enters the fiscal fragility zone. While in the fiscal fragility, expected inflation is higher than the inflation target rate, and choosing a low inflation target is costly instead of optimal. This result is in line with Araujo, Berriel, and Santos (2016), where a lower inflation target might reduce the policymaker's coordination ability due to a loss of credibility in his commitment and result in a worse equilibrium outcome.

The above analysis suggests a trade-off when defining the inflation target rate for an economy with poor fiscal conditions. A lower target means a reduced welfare cost of inflation in the no-crisis zone and reduced discretionary inflation in the fiscal fragility and dominance zones, which is desirable to the policymaker. However, reducing the target also causes lower debt levels to be in the fiscal fragility zone, which greatly reduces welfare since there is a positive probability that a sunspot shock that causes a permanent credibility loss will hit the

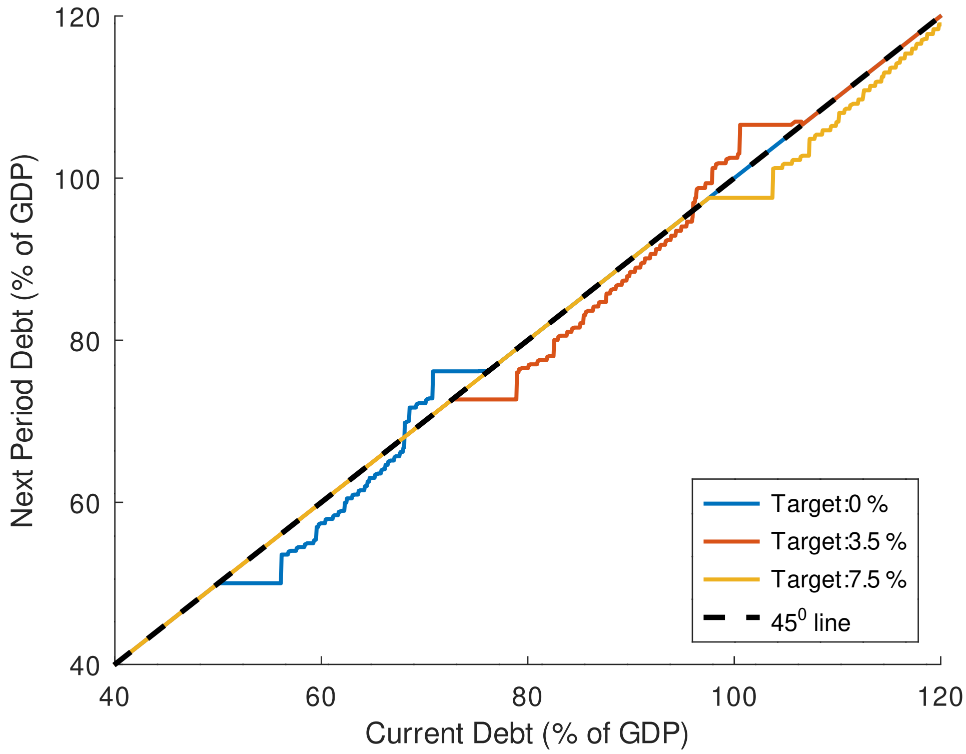

Figure 4: Sensitivity: Inflation Target and the Fiscal Fragility Zone

economy. By computing the inflation target that maximizes total intertemporal utility for each initial debt level, we see that the optimal target is the lowest target possible such that the current debt level is in the no-crisis zone, as is shown in Table 3:

Initial Debt Level	Optimal Inflation Target
60%	1.5%
75%	4%
90%	6.5%

Table 3: Optimal inflation target for each debt level in the calibrated 2002 Brazilian case

Inflation Indexed Debt

Indexed debt was defined by taking away the uncertainty about which equilibrium would be selected next period and, consequently, revealing the sunspot variable to private agents. As a result, private agents are able to correctly anticipate inflation and obtain a constant real interest rate on their bond holdings. We show that indexed debt, so defined, comes with higher inflation.

Recall that we find discretionary inflation solving for the discretionary policymaker's optimal inflation given expectations; that is, given π^e, we find the optimal π^D. The difference between both timing assumptions is in the formation of inflation expectations. In the fiscal fragility zone with nominal debt, the inflation expectation π^e is equal to $f\pi^D + (1-f)\pi^a$. Agents form expectations accounting for the probability of the policymaker delivering the target. With indexed debt, the inflation expectation π^e is equal to π^D. Agents do not form expectations accounting for the probability of the policymaker delivering the target. Intuitively, the policymaker attempts to transfer resources. However, he is unable to use inflation to partially default when subjected to a negative expectations shock since private agents adapt their expectations. This dynamic leads to higher levels of discretionary inflation. The optimal inflation chosen by the policymaker when its debt stock is higher nominal versus inflation-indexed is depicted in Figure 5.

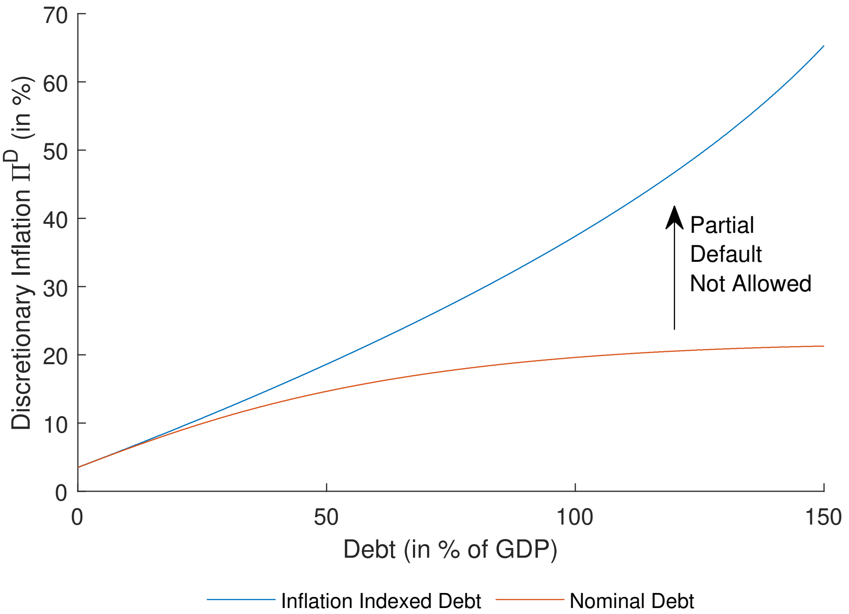

Figure 5: Discretionary Inflation

The higher discretionary inflation resulting from this timing may change the credibility of the inflation target for an initial debt stock, as the cost of maintaining the target increases in discretionary inflation under adverse expectations. Debt levels $D \in \{D : \pi(D, \pi^e = \pi^a, \zeta, \alpha_{-1}) = \pi(D, \pi^e = \pi^D, \zeta, \alpha_{-1}) = \pi^a\}$ support the inflation target with certainty. Intuitively, the policymaker facing higher inflation expectations would have to pay a higher real interest rate to keep inflation on target. As a result, the cost of keeping inflation on target when private agents expect a deviation is much higher. The reduced fiscal room with the policymaker facing higher debt servicing costs results in lower spending. Given the setup and calibration, we have higher marginal utility for spending than for consumption when the policymaker deviates, which implies that the value function decreases more when the policymaker attempts to maintain credibility by setting $\pi = \pi^a$ than when deviating and facing a productivity penalty by setting $\pi = \pi^D$. As a result, the lower bound of the fiscal fragility zone $\underline{D}$ decreases when debt is indexed, indicating reduced credibility of the target as a smaller set of initial debt levels fully supports it.

Preference for Spending

A shock to preferences can connect our model to the situation observed in Brazil during the 2002 confidence crisis. Suppose policymaker preferences shift towards giving more weight to public spending. Decreasing ρ would be tantamount to increasing the weight of public spending. This shift changes marginal utilities and the optimal allocation of resources, increasing the share going to public spending. The altruistic policymaker chooses higher discretionary inflation levels.

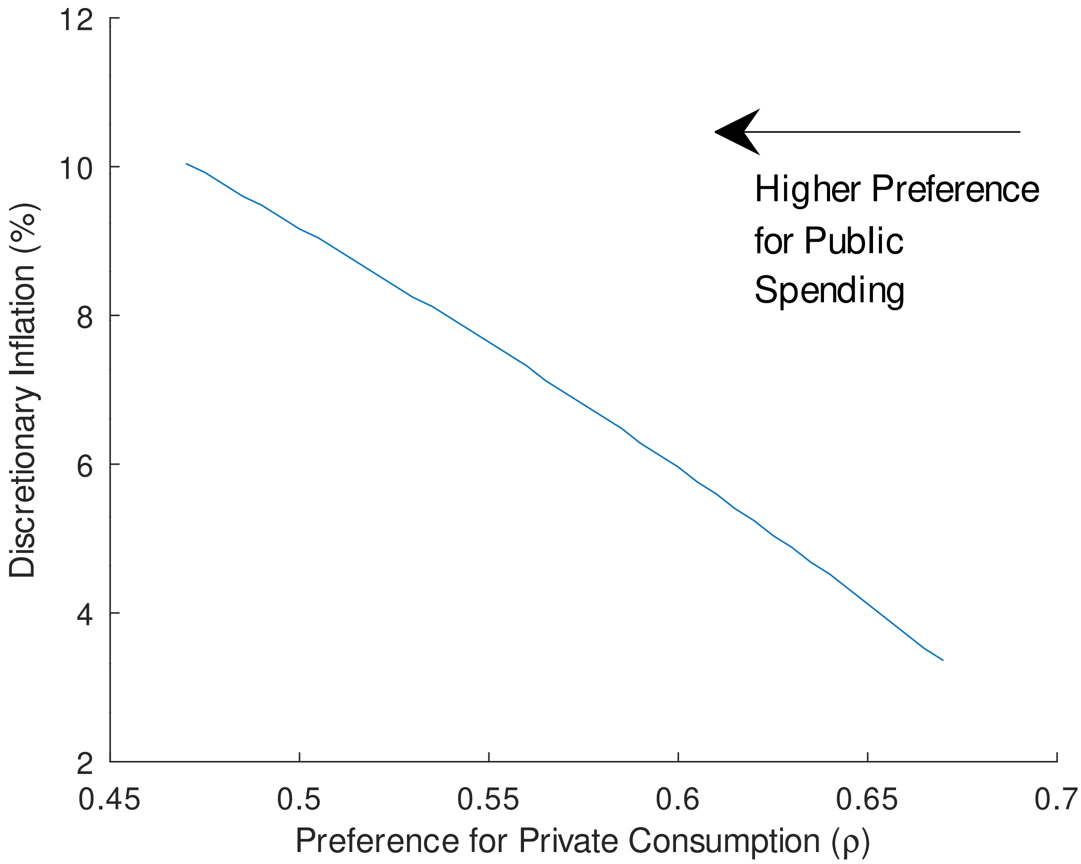

Figure 6: Sensitivity: Preference for Public Spending

Given a debt level, a relatively higher preference for public spending increases the level of discretionary inflation. For initial debt, a preference shock could push the policymaker into the fiscal fragility zone. A sufficiently big shock to ρ could result in the loss of credibility of the target under adverse expectations. Private agents would adapt their inflation expectations.

A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured. Such a preference shock explains how expectations can suddenly overshoot the target, as happened in Brazil in 2002.

4 2002 Confidence Crisis in Brazil

In 2002 and 2003, Brazilian policymakers faced inflationary pressures when it became clear the left presidential candidate would win. The perception was that his victory would mean implementing a new policy framework that could undermine the previous inflation reduction. Consequently, inflation expectations overshoot the target's upper bounds at all horizons relevant to the central bank, as shown in Figure 1. We map this event in our model as a shock to the preference for spending in the parameter ρ. Sensitivity analysis in Section 3.2 shows that, for a given initial debt stock, the target could lose credibility after a preference shock. By favoring more public spending, the policymaker could become vulnerable to adverse shocks that would make it deviate from the inflation target. Private agents taking this probability into account when forming expectations would increase their forecasts of future inflation, precisely as observed in 2002.

In response to rising inflation expectations, the outgoing and new administrations took several steps. First, to coordinate inflation expectations in the short run, they increased the target for 2003 in an extra meeting held in June 2002 and unofficially again in January 2003. Secondly, during 2003, public debt reduction sustained responsible macroeconomic policies. For example, the new administration changed the debt mix away from indexed bonds, mainly foreign exchange ones. Ultimately inflation expectations converged back to the target. These policy responses closely mirror the prescriptions suggested by our model. We will consider each of these policies in further detail.

Fiscal Policy

After the 2002 election, the government gradually reduced the gross public debt. The gross debt went down from almost 80% of GDP in 2002 to nearly 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations in a signal of fiscal responsibility. The primary surplus went up from 2.16% of GDP in 2001 to 2.70% in 2004. From the perspective of our model, such fiscal policy is compatible with the policymaker trying to exit the fiscal fragility zone and give the needed fiscal support to its inflation target.

Inflation Target

Before the October elections, the 2003 target was exceptionally revised upwards from a previously announced 3.25% to 4%. Similarly, the upper and lower bounds widened from -/+ 2% to 2.5%. In January 2003, the Ministry of Finance sent a letter stating that the adjusted target would be 8.5% in 2003 and 5.5% in 2004. The latter was confirmed by the National Monetary Committee as the inflation target for 2004 in June 2003, as we can see in Table 4. From the perspective of our model, an indebted policymaker with a higher inflation target might be more credible. The higher and more credible inflation target serves as a nominal anchor, making private agents readjust their inflation expectations.

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.50	2.0
2003	28/6/2001	3.25	2.0
	27/6/2002	4.00	2.5
	21/1/2003	8.50	
2004	27/6/2002	3.75	2.5
	21/1/2003	5.50	
	25/6/2003	5.50	2.5
2005	25/6/2003	4.50	2.5

Table 4: Brazil - Official Inflation Targets

Debt Management

The largest portion of Brazilian public securities was indexed to some benchmark prior to 2002. Such benchmarks include consumer price inflation, exchange rates, and the targeted policy interest rate. On one hand, debt indexed to inflation and exchange rates accounted for 56% of outstanding debt in 2002 and were gradually reduced to 33% in 2005. On the other hand, pre-fixed securities and those indexed to the overnight interest rate (Selic) were close to 44% in 2002 and increased to over 67% in 2005. The latter type of debt can be thought of as debt on which partial defaults are possible.[15] Using this classification as a proxy for the nominal and indexed debt denominations in the model, we suggest that reducing indexed debt could provide support to the inflation target.

[15]Pre-fixed securities when up from 1.5% up 2002 to 23.6% in 2005. Selic indexed securities were 42.4% in 2002 and 43.9% in 2005.

5 Remarks

High public debt opens the door to inflation due to target coordination failure, depressing private consumption and GDP. We propose a model to describe the intertemporal tradeoff between fiscal and monetary policy when forward-looking and rational private agents finance an altruistic policymaker. Indebted policymakers have a limited budget and are subject to expectation shocks forcing them to accept a higher interest rate with inflation on the pre-announced target or accept higher inflation. Our results endorse fiscal austerity to gradually lower the public debt to prevent coordination failure and self-confirmed inflation. However, if the debt is high, the policymaker should avoid an excessively low inflation target and inflation-indexed securities, as the possibility of partial default limits inflation.

In recurrent episodes of emerging-market crises, high public debt is associated with the difficulty of achieving low inflation. However, high public debt has also led to high inflation in advanced economies after the world wars, and it may lead again after the Covid-19 pandemic. In these cases, we suggest a set of tools based on our model that was successfully implemented in Brazil during the 2002 confidence crisis and can be used by central bankers who might face doubts about their credibility to sustain an inflation target.

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A Proofs

A.1 Characterization of the Discretionary Inflation

A.1.1 Discretionary Inflation is Increasing on Debt Level

Suppose that π^D is an interior solution to the discretionary inflation problem. To obtain $\partial\pi^D/\partial D$ we differentiate the first-order condition (7) with respect to the debt level D, to obtain:

$$\begin{aligned} \frac{\partial(7)}{\partial\pi^D}\frac{\partial\pi^D}{\partial D} &= (1-\rho)v''(g_T)\left(\frac{1+\pi_T^e}{1+\pi^D}\frac{1}{\beta}-1\right)\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\frac{D}{\beta}+\alpha'(\pi^D)\tau e\right) \\ &\quad - [(1-\rho)v'(g_T)-\rho]\frac{1+\pi_T^e}{(1+\pi^D)^2}\frac{1}{\beta}+\alpha'(\pi^D)(1-\rho)v''(g)\tau e \end{aligned} \tag{12}$$

where

$$\begin{aligned} \frac{\partial(7)}{\partial\pi^D} &= (1-\rho)v''(g_T)\left[(1-f)\frac{(1+\pi^a)}{(1+\pi^D)^2}\frac{D}{\beta}+\alpha'(\pi^D)\tau e\right]\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\frac{D}{\beta}+\alpha'(\pi^D)\tau e\right) \\ &\quad - [(1-\rho)v'(g_T)-\rho]\left\{\frac{f}{(1+\pi^D)^2}+\frac{2(1-f)(1+\pi^a)}{(1+\pi^D)^3}-\alpha''(\pi^D)\tau e\right\} \\ &\quad + \frac{\beta}{1-\beta}\alpha''(\pi^D)\left[\rho(1-\tau)+(1-\rho)v'(g)\tau+\frac{1-\beta}{\beta}\rho\right]e \\ &\quad + \frac{\beta}{1-\beta}(1-\rho)v''(g)(\alpha'(\pi^D)\tau e)^2 \end{aligned} \tag{13}$$

is the derivative of the FOC with respect to discretionary inflation π^D. To prove that $\partial\pi^D/\partial D > 0$ we show that the term in the right hand side of (12) and the term in (13) are both negative.

First, by (7) and the fact that $\alpha' \leq 0$ it is straightforward that

$$\frac{1+\pi_T^e}{(1+\pi^D)^2}\frac{D}{\beta}+\alpha'(\pi^D)\tau e \geq 0$$

for all D. Also, $v'' < 0$ and $[(1-\rho)v'(g)-\rho] > 0$ for any g by hypothesis. This means that the first two terms in (12) are nonpositive, and the second term is negative for every D. The last term in (12) is zero for $D = 0$, and we can assume that α' does not grow too fast, so that the whole sum is negative.

Now, in (13) we can analyse term-by-term to check that the only term that is ambiguous in sign in the sum is the first one. Now, if we again assume that α' is bounded so that

$$-\alpha'(\pi^D) \leq \frac{1-f}{\tau e}\frac{1+\pi^a}{1+\pi^D}\frac{D}{\beta}$$

then all terms are nonpositive, while the third term is strictly negative for all D, and the proposition is proved.

A.1.2 Discretionary Inflation Deviation is Decreasing on Inflation Target

To prove this proposition, we again assume that π^D is an interior solution to the maximization problem and differentiate the FOC (7) now with respect to π^a, to obtain:

$$\frac{\partial(7)}{\partial\pi^D}\frac{\partial\pi^D}{\partial\pi^a} = \left\{(1-\rho)v''(g_T)\left(\frac{1+\pi^e_T}{(1+\pi^D)^2}\frac{D}{\beta} + \alpha'(\pi^D)\tau e\right) - \frac{(1-\rho)v'(g_T)-\rho}{1+\pi^D}\right\}\frac{1-f}{1+\pi^D}\frac{D}{\beta}. \tag{14}$$

We can immediately see that the right hand side of equation (14) is negative, by the same analysis done in the previous proof. We want to show that $\partial\pi^D/\partial\pi^a < 1$. Rewrite equation (14) as

$$(\text{term } 1)\frac{\partial\pi^D}{\partial\pi^a} = \text{term } 2.$$

We need to show that term 2 - term 1 > 0, since this implies that term 1 is negative and that term 2/term 1< 1. Note that

$$\begin{aligned}
\text{term } 2 - \text{term } 1 = &\left\{\frac{1-f}{(1+\pi^D)^2}\frac{D}{\beta}(\pi^D-\pi^a) - \alpha'(\pi^D)\tau e\right\}(1-\rho)v''(g_T)\left(\frac{1+\pi^e_T}{(1+\pi^D)^2}\frac{D}{\beta} + \alpha'(\pi^D)\tau e\right)\\
&+ [(1-\rho)v'(g_T)-\rho]\left\{\left(\frac{2(1+\pi^a)}{(1+\pi^D)^3} + \frac{f}{(1+\pi^D)^2}\right)\frac{D}{\beta} - \alpha''(\pi^D)\tau e\right\}\\
&- \frac{\beta}{1-\beta}(\alpha'(\pi^D)\tau e)^2(1-\rho)v''(g)\\
&- \frac{\beta}{1-\beta}\alpha''(\pi^D)\left\{\rho(1-\tau) + (1-\rho)v'(g)\tau + \frac{1-\beta}{\beta}\rho\right\}e,
\end{aligned} \tag{15}$$

and we can inspect the last three terms of the sum and conclude that they are all nonnegative, with the last term being strictly positive for all D. The first term of the sum is negative whenever $\pi^D > \pi^a$, since the term in braces is positive in this case. However, we can assume that the marginal penalty α' does not grow too fast so that the positive term involving α' is dominated by the rest of the terms of the sum. Since (15) is strictly positive when $D = 0$ and by the stated assumptions it remains strictly positive as long as $\pi^D \leq \pi^a$, we can conclude, by continuity of the functions v and α with respect to D, that there is an interval $[0, \hat{D}]$, which contains, but is no restricted to, all D such that $\pi^D \leq \pi^a$. This proves the stated proposition.

A.2 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the fiscal fragility zone there is only a unique inflation equilibrium making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\max_{D_{t+1}} \sum_t \beta^t u(c_t, g_t)$$
$$\text{s.t. } c_t = \frac{1}{\beta} D_t + \alpha(1-\tau)e - D_{t+1}$$
$$g_t = D_{t+1} + \alpha\tau e - \frac{1}{\beta} D_t$$

The first order condition for D_{t+1} gives:

$$(1-\rho)v'(g_t) - \rho = (1-\rho)v'(g_{t+1}) - \rho$$

which implies $v'(g_t) = v'(g_{t+1})$. Given v strictly concave in g, we must have $g_{t+1} = g_t$. Replacing g_t and g_{t+1} by the government budget equation, iterating forward and taking limits we obtain:

$$\lim_{t\to\infty} D_t = \sum_{i=0}^{\infty} \left(\frac{1}{\beta}\right)^i (D_{t+1} - D_t) + D_{t+1}$$

Suppose that $D_{t+1} \neq D_t$, then the policymaker will either run up infinite debt or credit. The transversality condition for this problem states that

$$\lim_{t\to\infty} \beta^t D_{t+1} = 0,$$

so that if $D_{t+1} \neq D_t$ this condition is violated. This means that the only optimal trajectory for debt outside of the fiscal fragility is the stationary state such that $D_{t+1} = D_t$ for all t.

A.3 Existence of the no-crisis and fiscal dominance zones

To prove the first statement, assume that $\alpha^c = 0$. Fix a debt level D, a deviation time T and define

$$
\begin{aligned}
g_T^e &= \alpha(\pi^D)\tau e - \left(\frac{1+\pi^e}{1+\pi^D}\frac{1}{\beta} - 1\right) D, \\
g_T^D &= \alpha(\pi^D)\tau e - \left(\frac{1+\pi^a}{1+\pi^D}\frac{1}{\beta} - 1\right) D, \\
g^D &= \alpha(\pi^D)\tau e - \left(\frac{1}{\beta} - 1\right) D, \\
g^a &= \alpha(\pi^a)\tau e - \left(\frac{1}{\beta} - 1\right) D.
\end{aligned}
$$

Here, g_T^e corresponds to public spending at time T when the policymaker deviates from the expected level of inflation π^e; g_T^D considers the level of pubic spending at deviation when the private sector expects the target level π^a; g^D is the steady-state level of public spending after deviating; g^a is the steady-state level of public spending if the policymaker is always committed to the inflation target π^a.

We want to prove that $V^p(D, \pi_t = \pi^D, \pi^e = \pi^a) \geq V^p(D, \pi_t = \pi^a, \pi^e = \pi^a)$, which means that the policymaker finds it optimal to deviate even if the private sector expects the target. There are two cases to be considered: i) if $\pi^D \geq \pi^a$; ii) if $\pi^D < \pi^a$.

- In the first case, $\pi^D \geq \pi^a$ and the fact that π^D is a solution to the maximization problem 6 implies that

$$
u(\alpha(\pi^D)e - g_T^e, g_T^e) + \frac{\beta}{1-\beta} u(\alpha(\pi^D)e - g^D, g^D) \geq \frac{1}{1-\beta} u(\alpha(\pi^a)e - g^a, g^a). \quad (16)
$$

Since $\pi^D \geq \pi^a$, we have that $\pi^e \geq \pi^a$ and therefor $g_T^D \geq g_T^e$. It is straightforward from the strict concavity of u and the assumption that the marginal utility of public goods is higher than that of private goods that

$$
u(\alpha(\pi^D)e - g_T^D, g_T^D) \geq u(\alpha(\pi^D)e - g_T^e, g_T^e).
$$

By the inequality (16), we have

$$
u(\alpha(\pi^D)e - g_T^D, g_T^D) + \frac{\beta}{1-\beta} u(\alpha(\pi^D)e - g^D, g^D) \geq \frac{1}{1-\beta} u(\alpha(\pi^a)e - g^a, g^a), \quad (17)
$$

in which the left-hand side corresponds to total utility attained by deviating from target whenever the private sector expects the target inflation, and the right-hand side is total utility attained by delivering the target when the private sector expects it. This proves that D is in the fiscal dominance zone.

- If $\pi^D < \pi^a$ we no longer have that $g_T^D \geq g_T^e$. In this case, to show that the inequality (17) is true we must show that the long-run benefit of deviating and reducing the penalty is higher than the short-run benefit of committing to target and, in this case, having higher public spending, since g^a may be higher than g_T^D due to less interest spending. Mathematically, we need to show that

$$\frac{\beta}{1-\beta}\left[u(\alpha(\pi^D)e - g^D, g^D) - u(\alpha(\pi^a)e - g^a, g^a)\right] \\ \geq u(\alpha(\pi^a)e - g^a, g^a) - u(\alpha(\pi^D)e - g_T^D, g_T^D). \tag{18}$$

  The term in braces is positive, since $\alpha(\pi^D) < \alpha(\pi^a)$, and both the term in braces and in the right-hand side are bounded above, since $u(c, g) < u((1-\tau)e, \tau e)$ for all pairs (c, g) that are feasible. Then, inequality (18) is true if we consider a level of β close enough to 1, since $\beta/(1-\beta)$ diverges to infinity.

To prove the second statement we need to show that there exists a debt level D_- such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$ for all $D \in [0, D_-]$. For an arbitrary debt level D, we can consider the utility attained by the stationary policy of committing to the target when the private sector expects the deviation π^D and compare it to the total utility of deviating from the target. D is going to be in the no-crisis zone if

$$\frac{1}{1-\beta}u\left(\alpha^a(1-\tau)e + \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta} - 1\right)D, \alpha^a\tau e - \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta} - 1\right)D\right) \\ \geq \frac{1}{1-\beta}u\left(\alpha(\pi^D)(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D, \alpha(\pi^D)\tau e - \left(\frac{1}{\beta} - 1\right)D\right). \tag{19}$$

If $\pi^D < \pi^a$ we know that D is in the no-crisis zone by proposition 5. Assume that $\pi^D = \pi^a$, then inequality (19) becomes

$$\frac{1}{1-\beta}u\left(\alpha^a(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D, \alpha^a\tau e - \left(\frac{1}{\beta} - 1\right)D\right) \\ > \frac{1}{1-\beta}u\left(\alpha(\pi^a)(1-\tau)e + \left(\frac{1}{\beta} - 1\right)D, \alpha(\pi^a)\tau e - \left(\frac{1}{\beta} - 1\right)D\right), \tag{20}$$

where the strict inequality is true since $\alpha(\pi^a) = \alpha^a + \alpha^c < \alpha^a$, by hypothesis. By continuity, we can conclude that the strict inequality (19) is valid for an interval $[0, D_-]$, which gives our stated result.

To prove the last statement we need to show that, for a sufficiently high cost α^c, the condition (19) is valid for all D. We can rewrite this condition, using the mean value theorem, as

$$[\rho(1-\tau)+(1-\rho)v'(g_\theta)\tau]\,(\alpha^a-\alpha(\pi^D))e$$
$$\geq [(1-\rho)v'(g_\theta)-\rho]\left(\frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D\right), \tag{21}$$

where g_θ is a value between the spending levels in the right-hand and left-hand side in (19). By proposition 5, we know that if the fixed cost is high enough and $\pi^D < \pi^a$, D is in the no-crisis zone, and we only need to consider the case where $\pi^D \geq \pi^a$. For (21) to be valid for all D it is sufficient that the condition

$$(\alpha^a-\alpha(\pi^D))\tau e \geq \frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D \tag{22}$$

to be valid for all D. Since the right-hand side of equation (22) is total interest spending when the government commits to the target, and public spending is always non-negative, we have that

$$\frac{\pi^D-\pi^a}{1+\pi^a}\frac{1}{\beta}D \leq \tau\alpha^a e-\left(\frac{1}{\beta}-1\right)D \leq (\alpha^a-\alpha(\pi^D))\tau e$$

as long as the condition

$$\left(\frac{1}{\beta}-1\right)D \geq \alpha(\pi^D)\tau e$$

is satisfied for all D. But since we only need to consider $D \geq D_-$, it suffices to take a fixed cost α^c sufficiently negative so that

$$\left(\frac{1}{\beta}-1\right)D_- \geq \alpha(\pi^D)\tau e$$

is valid, and the proof is complete for all D.

A.4 Above-Target Discretionary Inflation

The intuition for the proof is simple: we show that if it is the case that $\pi^D < \pi^a$, then there exists a feasible policy that does not deviate from the target and attains a higher intertemporal utility then deviating, even if private agents believe that the policymaker will deviate from target. This implies that the economy is in the no-crisis zone. Therefore, whenever the economy is in the fiscal fragility zone, deviation from target must be positive.

For a given debt level D, assume that $\pi^D < \pi^a$, that is, the discretionary inflation rate is lower than the target rate. We want to prove that in this case $V^p(D,\pi_t=\pi^a,\pi^e=\pi^D) \geq V^p(D,\pi_t=\pi^D,\pi^e=\pi^D)$, that is, the policymaker follows the inflation target even when the private agents expect the policymaker to deviate. Let T be the time of deviation, and assume that private agents expect the policymaker to deviate, then the according to equations (6) total government spending both in period T and in the stationary long-run will be equal to

$$g^D = \alpha(\pi^D)\tau e - \left(\frac{1}{\beta} - 1\right) D, \tag{23}$$

since agents expect the deviation.

As an alternative, the government can choose the feasible path of not deviating from the target and following a stationary spending policy:

$$g^a = \tau\alpha^a e - \left(\frac{1+\pi^D}{1+\pi^a}\frac{1}{\beta}\right) D. \tag{24}$$

Since $\pi^D < \pi^a$, we have that $g^D < g^a$, since

$$g^a - g^D = (\alpha^a - \alpha(\pi^D))\tau e + \frac{\pi^a - \pi^D}{1+\pi^a}\frac{1}{\beta} D > 0.$$

To compare the total intertemporal utility of both policies we only need to compare which one of the allocations attain a higher utility at any period, since they are stationary allocations. Let $c^D = \alpha(\pi^D)e - g^D$ and $c^a = \alpha^a e - g^a$ be the market-clearing private consumption in each scenario. By the concavity of the utility function we have that

$$\begin{aligned} u(c^D, g^D) - u(c^a, g^a) \leq& \rho(c^D - c^a) + (1-\rho)v'(g^a)(g^D - g^a) \\ =& \rho(\alpha(\pi^D) - \alpha^a)e + ((1-\rho)v'(g^a) - \rho)(g^D - g^a) < 0 \end{aligned} \tag{25}$$

since, by the assumption, we have that $(1-\rho)v'(g^a) - \rho) \geq 0$.

Now, since there is a feasible policy trajectory in which the policymaker follows the inflation target and its intertemporal utility is greater than the attained by deviating to the inflation rate π^D, this means that the optimal policy chosen by the policymaker when following the target must also attain a higher utility than the attained by deviating, that is, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$. But this means that the policymaker chooses to follow the target even when private agents expect it to deviate, so that debt level D is in the no-crisis zone.

B Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability to overshoot the target would increase with debt and decrease with higher target levels. The present section questions whether there is empirical evidence for the predictions based on our model. We construct a dataset includes 20 countries with at least 15 years of inflation targeting[16] covering the period 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regards to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table 5. The variables present both inter and intra-country variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle income countries.[17]

	Debt/GDP	Revenue/GDP	CPI EOY	CPI target
Average	45.2	32.9	3.9	3.2
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table 5: Data Description

Real Effective Exchange Rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[18] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalent are seasonally adjusted with the Arima X-11 procedure.[19] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

Deviations from the Target

The first order condition of the discretionary inflation problem from 6 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^a_{i,t}$ to observable and latent variables for each country

[16] The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[17] We used the World Bank classification.

[18] BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[19] This was the case for Peru and Turkey.

i. We estimate the following model,[20]

$$\pi_{i,t} - \pi^a_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^a_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{26}$$

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$ with $X_{i,t}$ being a vector of the observable regressors at time t and for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map the model variables D, τe, and π^a to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservables variables e, f, c_1, c_2 are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator as it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0\ \forall t$ as required for a random effect estimator.[21]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in (26) one would have to look at the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative as the policymaker could help coordinate private agents expectations by adopting a more credible (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table 6 is the basic model from (26). The remaining estimations, II-V, are robustness checks.

In estimation I, deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation, the coefficient should not be statistically different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of

[20] In the online appendix D we show how (26) is related to the first order condition of the discretionary inflation problem from (7).

[21] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

	I	II	III	IV	V
Revenue	0.171^{**}	0.098	0.063	0.125^{*}	0.087
	(0.076)	(0.076)	(0.078)	(0.070)	(0.072)
Debt	0.069^{*}	0.074^{**}	0.073^{**}	0.062^{**}	0.058^{*}
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt * Revenue/100	-0.194^{*}	-0.168^{*}	-0.149	-0.163^{*}	-0.136
	(0.099)	(0.096)	(0.096)	(0.088)	(0.089)
Target	-0.403^{***}	-0.458^{***}	-0.441^{***}	-0.360^{***}	-0.342^{***}
	(0.063)	(0.062)	(0.062)	(0.059)	(0.058)
GDP Gap			0.363^{***}		0.342^{***}
			(0.102)		(0.095)
Reer YoY				-13.956^{***}	-13.645^{***}
				(1.648)	(1.653)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 6: Results - Deviations from the Inflation Target

GDP, the net impact of debt is positive. This result applies to the middle income countries in our sample. The result goes in the direction of what the theoretical model predicted as both the probability of deviating and deviations from the target are positively related to debt levels. On average countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of 88%, above the maximum in our sample. Hence the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated to preferences for public spending that in turn could lead to inflationary pressure.

The results remain accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect in order to account for global shocks such as commodity prices. In our sample 2008 stands out as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global co-movements in inflation in account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Overshooting the Target

The policymaker overshoots the inflation target when end-of-year inflation exceeds the upper bound of the target.[22] In the theoretical model, the policymaker had more incentive to overshoot the target when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to (26), but with regard to the probability of overshooting the target:

$$I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \quad (27)$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ overshoots the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1}, ..., X_{i,T}, c_i) = 0,\ t = 1, ..., T$. The probability of overshooting the target will then be a logistic function:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1 + e^{-X'_{i,t}\beta - c_i}}, \quad t = 1, ..., T \quad (28)$$

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of overshooting the target. Each year in the sample at least two countries overshoot their respective inflation target. The years 2007 and 2008 stand out as over half of of the countries overshoot their inflation target. A time dummy is likely to capture this effect. Also, virtually all countries except two overshoot their target at least once with some countries such as Turkey close to being serial over shooters. Overall, middle income countries overshoot the target more often than high-income countries. Nevertheless, high income countries overshoot the target 39 times.

The first column of Table 7 is the baseline model while the remaining columns represent robustness checks similar in spirit to the previous section. When looking at the net impact of debt on the probability to overshoot the target the coefficients have similar signs as the previous estimates with regards to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

The net impact of revenue remains positive for debt levels in the sample, not in the same

[22] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case of the UK and Norway. In such cases we used the average upper tolerance limit from the rest of the sample (1.2%).

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	−178.526	−151.367	−149.281	−139.619	−137.954

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: Results - Probability of Overshooting the Target

direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, decreasing incentive to use discretionary inflation to transfer resources away from private agents debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, what would increases incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of overshooting the target is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, leaving the door open to overshoot the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to overshoot their inflation target. The output gap is not significant.

C Online Appendix: Solution to the Discretionary Inflation

Proposition 6 *Let the the utility function $u(c, g)$ and the penalty function $\alpha(\pi)$ be such that they satisfy the already stated assumptions. If the universe of possible inflation choices is defined on the compact set $[0, \overline{\pi}]$ where $\overline{\pi} > 0$ is some upper limit. Then there exists a discretionary inflation level π^D such that π^D is optimal given private agents' inflation expectations π^e and vice versa.*

Proof: In order to prove that there exists a discretionary inflation level π^D such that π^D is optimal given π^e, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in equation 6.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi(f\pi^D + (1 - f)\pi^a) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by hypothesis we know that the penalty function $\alpha : [0, \overline{\pi}] \to (0, 1)$ mapping discretionary inflation into total factor productivity is continuous. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by hypothesis.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is easy to see that the map of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous. Which is what we wanted to show.

D Online appendix: Testing the FOC

The first order condition of the discretionary inflation problem from equation 7 when assuming linear utility in consumption is given by:

$$\frac{1+\pi_T^e}{(1+\pi_T^D)^2}\frac{1}{\beta}D(u_g^T-1)+u_g^T\alpha'\tau e+\frac{1}{1-\beta}\alpha'(1-\tau)e+\frac{\beta}{1-\beta}u_g\alpha'\tau e=0$$

where u_g^T is the marginal utility of spending when deviating at time T and u_g is the ensuing steady state marginal utility. π_T^e are the private agents' expectations at time T, π_T^D is the optimal discretionary inflation chosen by the policymaker when deviating from the target at time T, D is the level of debt, τe the policymaker's revenues and α' is the marginal productivity shock when deviating.

The equation can be rewritten as:

$$D\left(u_g^T-1\right)\left(\frac{1+\pi_T^e}{(1+\pi^D)^2}\right)=\tau e\alpha'\left[\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right]$$

Taking logs we obtain:

$$\begin{aligned}&d+\log(u_g^T-1)+\log(1+\pi_T^e)-2\log(1+\pi^D)\\&=\log(\tau e)+\log(\alpha')+\log\left(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T\right)\end{aligned}$$

where $d=\log(D)$. Replacing expectations by $\pi_T^e=f\pi^D+(1-f)\pi^a$ and using the approximation for $\log(1+x)\simeq x$ for small x, we have $\log(1+\pi_T^e)-2\log(1+\pi^D)=(2-f)(\pi^a-\pi^D)-\pi^a$. Hence:

$$\pi^D-\pi^a=-\frac{\log(\tau e)}{2-f}+\frac{d}{2-f}-\frac{\pi^a}{2-f}-\frac{c}{2-f}$$

Where $c=\log(\alpha')-\log(u_g^T-1)+\log(\frac{1}{1-\beta}\frac{1-\tau}{\tau}+\frac{\beta}{1-\beta}u_g+u_g^T)$ will also capture effects of debt levels d and revenue τe through the marginal utility of government spending. This unfortunately makes the coefficients less straightforward to interpret without any prior calibration and initial conditions. We propose to model the relationship for country i as follows:

$$\pi_{i,t}-\pi_{i,t}^a=\beta_{0,i}+\beta_1\text{revenue}_{i,t}+\beta_2\text{debt}_{i,t}+\beta_3\text{target}_{i,t}+\beta_4\text{revenue}_{i,t}*\text{debt}_{i,t}+u_{i,t}$$

where the interaction term between revenue and public debt is meant to capture the dynamics of the marginal utility of government spending at time t. The idiosyncratic error term $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0, t = 1,...,T$. Coefficient $\beta_{0,i}$ captures a country fixed effect while all other coefficients are common to all countries.

E Online appendix: Proofs

E.1 Definition of the Real Interest Rate

The first-order condition of the utility maximization problem for the consumer gives the following ex-post equilibrium real interest-rate

$$1 + r_t^{eq} = \frac{1}{\mathbb{E}\left[\frac{1}{1+\pi_t}\right]} \frac{1}{1+\pi_t} \frac{1}{\beta}$$

which differs from the ex-post real interest rate defined in the text of

$$1 + r_t = \frac{1+\pi_t^e}{1+\pi_t} \frac{1}{\beta}.$$

We show that the definition used in the model is a good approximation to the equilibrium ex-post real interest rate, in the sense that the difference between them is negligible, as long as the inflation rate is not far away from zero. To see that, consider the following Taylor expansion to the function $\frac{1}{1+\pi_t}$ around the expected inflation rate π_t^e:

$$\frac{1}{1+\pi_t} = \frac{1}{1+\pi_t^e} - \frac{1}{(1+\pi_t^e)^2}(\pi_t - \pi_t^e) + \sum_{j=2}^{\infty}(-1)^{j+1} \frac{1}{(1+\pi_t^e)^{j+1}} \frac{(\pi_t - \pi_t^e)^j}{j!}.$$

Taking expectation and considering that the random variable π_t has bounded support we obtain

$$\mathbb{E}\frac{1}{1+\pi_t} = \frac{1}{1+\pi_t^e} + \sum_{j=2}^{\infty}(-1)^{j+1} \frac{1}{(1+\pi_t^e)^{j+1}} \frac{m_j(\pi_t)}{j!}$$

where $m_j(\pi_t)$ is the j-th moment of the random variable π_t. Now, let ϵ_t be the maximum absolute value the random variable $\pi_t - \pi_t^e$ assumes, which is either $\pi_t^e - \pi^a$ or $\pi_t^D - \pi_t^e$, where π_t^D is the discretionary inflation rate chosen by the policy maker. It is trivial that $|m_j(\pi_t)| < (\epsilon_t)^j$, and if we consider only parameter specifications such that $\epsilon_t < 0.5$, we can use the inequality $e^x - 1 - x < x^2$ for $x < 0.5$ to conclude that

$$\mathbb{E}\frac{1}{1+\pi_t} - \frac{1}{1+\pi_t^e} \leq \frac{(\epsilon_t)^2}{(1+\pi_t^e)^3},$$

which is the same as

$$(1+\pi_t^e)\mathbb{E}\frac{1}{1+\pi_t} = 1 + \xi_t$$

where ξ_t is the estimation error which is bounded by $\left(\frac{\epsilon_t}{1+\pi_t^e}\right)^2$.

We can now estimate the difference between the equilibrium real ex-post interest rate and the definition used in the text by

$$\frac{1+r_t^{eq}}{1+r_t} = 1 + \xi_t$$

and by taking logs this relation approximates to $r_t^{eq} - r_t = \xi_t$. The error ξ_t is smaller than the square of the maximum deviation from the expected inflation, which will be numerically close to zero in any reasonable calibration of the model.