The welfare effects of adopting NDC pension schemes in developing economies

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Abstract

This paper studies the effect of implementing a notional defined contribution (NDC) pension scheme in a developing economy, using a 3-period overlapping generations model. We begin to calibrate our base economy to the Brazilian economy and test what happens in the steady state when we adopt as a counterfactual the social security reform that occurred in 2019, where the PAYG structure was maintained. We then compare the results we found with a range of NDC alternative pension schemes within a framework that allows for formal and informal work and age-dependent taxes. We are the first to model NDC pension schemes in an economy that allows the informal sector. Our conclusions are that adopting an NDC pension scheme results in welfare gains of at least 5.60% in our more general NDC alternatives.

Key words: Notional Defined Contributions, Social insurance, Informal labor.
J.E.L. codes: E6, H55, J26

1 Introduction

To respond to the pressures from an aging population, the Brazilian government enacted a pension reform in 2019 that changed important parameters of the pension system, such as: unification of public and private sector rules, changes in eligibility rules, specifically the increase in the minimum age to retire, and changes in the rule for calculating the pension benefits.[1]

Despite some important fiscal improvements, the current commitments that burden the sytem are bound to require further reforms. So, in this paper we focus on a type of reform that completely changes the logic of the current system. To understand its context, it is important to mention an important question that arose during the discussions to implement the reform: whether the pension system should continue to be a pay-as-you-go (PAYG) pension scheme or whether it should be changed to be capitalized in some way. This debate had already happened a long time ago, in the 1990s, where after the Chilean pension reform, some Latin America countries envisioned the possibility of also capitalizing their pensions. Barreto and Oliveira (2001) found for the Brazilian economy that the Fully-Funded systems increases the capital of the economy, lowers interest rates, and increases wage levels. The problem is that these same findings were conditioned in the form of the transition between the pension systems. This result they found in their paper was one of the main reasons why other countries in Latin America did not want to change their PAYG pension scheme to a Fully-Funded one, the cost of transition could be too high to go forward.

In addition to the issue of the fiscal cost of the transition and the uncertainty about the demographic transition, in developing economies there are many other questions that policy makers need to address when they think about pension reforms. In Brazil, the economy is very exposed to any type of external economic shock, have a large informal sector, a noisy political environment, and a very strong resistance from part of the population to privatizing any type of public service. Adding all this up, an alternative to the PAYG system needs to be a feasible one that takes these issues into consideration, and we argue that the Notional Defined Contribution (NDC) pension system can be that system.

The goal of this paper is to assess the welfare impact of adopting an NDC pension scheme in a developing economy where the share of infrequent contributors is generally large and compare it with the traditional PAYG already adopted in many

[1] The fiscal impact estimated by Afonso and Carvalho (2022) for the first ten years after the reform, compared to the case without the reform, is that there will be savings of approximately $US\$$ 190.000.000,00.

countries, including Brazil. To answer this question we run a three period overlapping generations model where we allow for two forms of heterogeneity: age and efficiency. Moreover, our model has a framework where the individual can divide his time working in formal and informal sector. We calibrate our model parameters for the Brazilian economy and use the Brazilian economy before implement their last social security reform as our baseline model. The 2019 pension reform changed many parameters of the system, but maintain the system as a PAYG pension scheme, where the link between pensions and contributions is almost nonexistent.

We start our counterfactual simulations running the 2019 reform pension scheme, and after that we run a series of NDC pension schemes comparing the results with our baseline findings. As in our model agents live only three periods, we can't capture all the elements brought by the last pension reform, including the increase in the minimum age and the unification of rules for calculating pensions for public and private workers, as we treat all workers as being in the private sector. What we can capture from the last reform is the definition of only one formula to calculate pension benefits, instead the three rules existent before the reform took place, so all comparisons we make take the other changes as given and we interpret our NDC simulations as what would happen if Brazil had switched from the actual PAYG pension scheme benefit calculation formula to an NDC pension scheme, where the link between future pensions and contributions are strong, the replacement rates are easier to manage, the pension benefit formula is easy to explain to the workers, and there are much more flexibility to change the parameters of the model in case of adverse economic situations.

One important innovation that we brought in this paper is the use of age-dependent taxes to finance the pension scheme of the economy. As in our model the wage rates between the individuals differs due their different productivity, we use the fact demonstrated by Akerlof (1978) that using tagging schemes to tax individuals in this situation is welfare improving. In our model, allowing for two types of productivity, aging and efficiency, we can capture not just the differences that are natural characteristics of the individual but also the learning curve through their life-cycle and the differences between their efficiency productivity.

Furthermore, the use of age-dependent taxes allows the individuals to consume and save more in a period of their lives where they are more credit constrained than in older periods of their lives, where they have not just more experience and higher earnings but also have had enough time to save more in assets to protect themselves against any type of shocks. Even with our model being a deterministic one, using age-dependent taxes resulted in a capital accumulation gain, and the agents changed their decisions compared to the baseline and other counterfactual models without age-dependent taxation.

The difference in using age-dependent taxes to finance social security compared with the use of the general labor income taxation is twofold: (1) pension benefits depends on individual history; (2) age plays a much larger role in pension determination than in tax determination (Diamond 2009). Furthermore, note that the design of the pension scheme can affect the welfare results of using age-dependent taxes. For example, the incentives in a Beveridge pension scheme where all agents receive just a minimum pension, even if they contribute just a few years to finance the system, are way different than in a Bismarck system where agents receive pension benefits based exactly on how much they contributed through their role working life.

We are not the first to use age-dependent taxes in social security contributions. da Costa and Santos (2024) implemented it for the NDC schemes in the US. The difference in our approach is that we allow for informal work here, therefore, encouraging the formalization of work in the first period of life ends up having an effect precisely in the worker's most critical period, as young people are those who supply most of the informal work in the economy. Moreover, unlike PAYG systems, where a minimum contribution time is required, in NDC schemes this is not necessarily the case, as the system flexibility allows for both cases, with and without a minimum contribution time requirement. In our exercises we test both cases.

In our simulations we found that if the last reform had implemented an NDC system rather than maintaining the current PAYG the welfare gains would be at least 5.60% compared to the baseline, and almost 4.8% when compared to the 2019 pension reform. Furthermore, the drop in replacement rates to the most productive workers would be much higher, decreasing the necessary consumption taxes to keep the social security budget balanced. Finally, the welfare gains of use age-dependent taxes to finance the social security system are of at least 2.87% in all pension schemes we tested.

The rest of the paper is organized as follows: section 2 describes the Brazilian pension system and some of its history, section 3 describes our model, section 4 continues the model presentation defining the pension system and our counterfactual pension schemes, section 5 describes data and calibration procedures, section 6 shows our results, and section 7 concludes.

2 The Brazilian pension system

The Brazilian social security system was created in 1923, by the Eloy Chaves law, which established the beginning of the Brazilian social security system. Since then a lot of changes have been made in the system, and today its coverage is generalized over the country. This section aims to explain in two subsections the Brazilian pension system in a brief and didactic way. The first subsection shows what changes have been made by the social security reforms that occurred before the 2019 pension reform, the last one. This introduction is necessary to understand the benefit calculation rules which we modeled in the section 4 and the results we reported in section 6 about the 2019 pension reform counterfactual. The second subsection explain what were the modifications made by the 2019 pension reform in Brazil.

2.1 Pension scheme before the 2019 reform

The Brazilian PAYG pension scheme that is divided in two regimes, one for the private workers and other for public workers. The private workers public pension system is the INSS (National Institute of Social Security) covers the majority of the retirement transfers made for the Brazilian workers, while the public workers are covered by the Regime Próprio de Previdência Social (RPPS), which is the public sector social security authority. These public sector pensions are offered at the federal, state, and local levels of government. In this paper we will only mention pension reforms at the federal level, so the pension reforms mentioned here does not necessarily affect another level of government. In practice, if any other state or city in Brazil wants to change their public pension system rules they need to approve their own pension reform independently.

The first pension reform that we mention here is the reform carried out during the government of President Fernando Henrique Cardoso, approved in 1998 by the constitutional amendment 20 (EC 20/1998) which only affected the private workers covered by the INSS. The main points changed by this reform were[2]:

1. Deconstitutionalization of the retirement adjustment formula. Before this change, all the pension benefits had their value defined by the last 36 months of contributions made by the employee, so for the benefit calculation it did not matter any of their $N-36$ contributions made before their last 36. This encouraged the under-declaration of income by the employee and increased the value of benefits paid by the government;

2. Established the minimum age for retirement, being 53 years old for men and 48 years old for women;

3. It opened the doors for the approval of law 9.876, the law which created the Social Security Factor (Fator Previdenciário (FP)), a factor encouraging the postponement of early retirement, which in practice reduces the pension benefits of workers who wish to retire early.

The second pension reform was carried out during the first year of the government of President Lula in 2003 by the constitutional amendment 41 (EC 41/2003). Unlike the reform carried out during the previous government, this reform focused on the federal civil service pension system. The main modifications made were:

1. Established the minimum age for retirement of federal public employees, being 60 years old for men and 55 years old for women;

2. Established a maximum pension benefit payment (Teto do INSS);

[2]For the 1998 and 2003 pension reforms we are following Giambiagi and Estermínio (2006).

3. Established the collection of social security contributions for retirees and pensioners, so even after they have retired, retirees and pensioners continue to pay social security taxes.

The third pension reform that occurred before 2019 was under the government of President Dilma Rousseff. This reform was called the "bomb agenda" (Law 13.183/2015), this agenda was called that way due to the fact that some of the parliamentarians wanted to approve it just to wear down the government that was already on the verge to suffer the impeachment at that time. The only point we mention here is a third alternative to achieve retirement, the factor 85/95. This factor allowed the retirement with full replacement rate, without the impact of the social security factor on the calculation of the pension benefit. The name 85/95 is due to the fact that if the sum of the worker's age plus contribution time at the time of the benefit claiming is equal to 85, in the case of a woman, or 95, in the case of a man, then the worker would receive the benefit with 100% replacement rate. After this reform, there were three ways to claim retirement: contribution time, age and the 85/95 factor.

In practice, if the worker wanted to claim the retirement benefit by age or contribution time, the rules were as follows:

1. Retirement by age: the worker had to have contributed to the system for at least 15 years working in the formal sector and be 65 years old, if a man, and 60 years old, if a woman;

2. Retirement by contribution time: the worker was eligible for a pension if he had contributed for the social security system working in the formal sector for at least 35 years. For the workers who started to contribute to the system before 1998 there was a proportional retirement possibility, but after the reforms, the number of people eligible to claim is decreasing fast so we did not take this into account in our modelling.

Finally, its worth to mention some other points here. The first is that is possible to the public worker to engage in a supplementary pension system which consist of a private pension system where the employee pays a higher social security tax rate to increase its future pension benefits. As the number of people in this program is not relevant we not even explained it before. The second point is that the social security pension system in Brazil pays several other types of assistance benefits in addition to the pensions mentioned here, but as our focus is on pensions paid to private sector workers we will not go into the details of these other transfers. Finally, the Brazilian pension distinguish between urban and rural pension benefits, where the eligibility criteria are different between the two types. In this paper we are working only with urban pensions.

2.2 Pension scheme after the 2019 reform

The 2019 pension reform (EC 103/2019) was approved under the government of President Jair Bolsonaro in an emergency fiscal scenario, where public accounts were being suffocated by pension spending. Here to present the main changes made by the reform we follow Afonso and Carvalho (2022):

1. Unification and increase in the progressivity of the social security tax rates in the INSS and RPPS systems;
2. Unification of the eligibility conditions to claim the retirement benefits in the INSS and RPPS systems;
3. Unification of rules for requesting retirement. A new rule was created for requesting retirement, where the way to calculate the pension benefit now depends on the average lifetime income and contribution time. Furthermore, the minimum contribution period is now 20 years. The older paths, 85/95 factor, age and contribution time were eliminated. Note that the 85/95 factor served to prevent the social security factor from reducing the value of pensions, so when the latter is extinguished, the 85/95 factor is also eliminated. Workers who were already in the job market during the approval of the reform will follow transition rules.

In the next section we present our model in detail.

3 The Model

The model is a three-period overlapping generations (OLG) model where the economy is populated by heterogeneous households that differ by their age, age-specific productivity and efficiency type productivity. In the first two-periods the households work and in the third-period they retire. Each period length in the model represents 20 years, so the household enters in the labor force at age of 21 years and retire at the age of 60 years old.

3.1 Demographics and timing

At each period t we have three living generations, two working generations and a retired one. There are three types of households represented by their inherent productivity. In our model there is no survival uncertainty, and all households die after the third period.

Each skill type of household is represented by a invariant to time measure, μ_j, where j the efficiency productivity type of the household, with $\sum_j \mu_j = 1, \forall s \in \{1,2,3\}$, where s represents the age. The population grows at a constant rate n, so that the dynamics of the population growth is given by:

$$N_t = (1+n)N_{t-1} \tag{1}$$

3.2 Households

Our economy is populated by a continuum of mass one agents in each period who lives for three periods. Each agent has a time endowment equal to one. The agent lifetime utility is given by:

$$U = \sum_{s=1}^{3} \beta^{s-1} u(c_{t+s-1}^{s}, 1 - n_{t+s-1}^{s}) \tag{2}$$

where $\beta > 0$ is the discount factor, c_t^s and n_t^s denote consumption and labor supply of the s-year old in period t. Per-period utility, $u(c, 1-n)$, is given by a CRRA utility function:

$$u(c, 1-n) = \frac{c^{1-\sigma}(1-n)^{\iota(1-\sigma)}}{(1-\sigma)} \tag{3}$$

where σ is the intertemporal elasticity of substitution between consumption and leisure.

The leisure input is defined below:

$$l^s_{\varepsilon_j} = 1 - n^s_{\varepsilon_j} \tag{4}$$

where for the definition of the labor input we follow the approach used by McKiernan (2022) and define the leisure input of an individual with age s as a linear function of hours spent working in the market, $n^s_{\varepsilon_j}$. This time working in the market is divided between work in formal and informal sector. The difference is that in the formal sector the agent pays the contributions for the pension system, and also the firm, and receives the pension payment in his retirement period, while in the time he spends working in the informal sector he does not pays any contribution, but it also does not count anything for his retirement pension benefits. We define the labor input as a function of time working in both sectors:

$$n^s_{\varepsilon_j} = \{a_{\varepsilon_j}(n^s_{f,\varepsilon_j})^{b_{\varepsilon_j}} + (1-a_{\varepsilon_j})(n^s_{i,\varepsilon_j})^{b_{\varepsilon_j}}\}^{1/b_{\varepsilon_j}} \tag{5}$$

where n^s_{i,ε_j} represents the informal labor and n^s_{f,ε_j} the formal labor of the agent with age s with efficiency type ε_j. The parameter a_{ε_j} governs the share of formal and informal work and b_{ε_j} will determine the substitution between the formal and informal sectors. The elasticity of substitution between sectors is given by $\epsilon_j = \frac{1}{1-b_{\varepsilon_j}}$

We assume that agents are heterogeneous in age, s, and in individual labor productivity. The labor productivity has a common component of age given by η_s, and an individual efficiency component, which we call the efficiency type productivity and define as ε_j. Let w_f and w_i represent the formal and informal wages paid by formal and informal firms, respectively. All agents pay consumption taxes, τ_c, and social security taxes on hours worked in the formal sector for the economy's pension system, $T(y^s_{f,t})$, where $y^s_{f,t}$ is the formal income of the agent of age s in period t. The consumption tax is necessary to adjust the pension system deficit or surplus. Finally, the agents can save in a risk-free asset, $k^s_{\varepsilon_j}$ which pays an interest rate, r. We assume they born without any savings, so that $k^1_t = 0$ for all agents and they cannot have negative assets in any period of their lives. Furthermore, they consume all their assets in the last period of life, so $k^4_{t+1} = 0$.

The budget constraint of an agent with efficiency type ε_j and asset k^s_t in period t with age $s \in \{1,2,3\}$ is given by:

$$c^s_t + k^{s+1}_{t+1} = \begin{cases} A_t \varepsilon_j \eta_s (w_{f,t} n^s_{f,\varepsilon_j,t} + w_{i,t} n^s_{i,\varepsilon_j,t}) - T(y^s_{f,t}) - k^s_{\varepsilon_j,t}, & \text{for } s \in \{1,2\} \\ pen_t + k^3_t, & \text{for } s = 3 \end{cases} \tag{6}$$

where A_t is the labor-augmenting technological progress and it grows at the exogenous rate γ:

$$A_t = (1+\gamma)A_{t-1} \tag{7}$$

3.3 Technology

We assume here that there are two representative firms in the economy, one for the formal sector and other for the informal sector, and that they produce substitute goods. Both produce using a Cobb-Douglas production function so that in every period t the production of each sector is presented below:

$$Y_{ft} = B_1 K^{\alpha}_{ft} (A_t N_{ft})^{1-\alpha} \tag{8}$$

$$Y_{it} = B_2 K^{\rho}_{it} (A_t N_{it})^{1-\rho} \tag{9}$$

where K_{ft} and N_{ft} are the aggregate formal capital and labor inputs, while K_{it} and N_{it} are the aggregate informal capital and labor inputs, α is the capital share of the formal firm and ρ the informal one. Moreover, B_1 and B_2 represents the formal and the informal sector total factor productivity (TFP), respectively.

Both, formal and informal capital, are assumed to depreciate in every period at an exogenous and constant rate δ. Furthermore, the formal firm pays social security taxes, θ, while the informal does not.

Every period, the profit maximization problems of formal and informal firms are given, respectively, by:

$$\Pi_{ft} = B_1 K_{ft}^{\alpha}(A_t N_{ft})^{1-\alpha} - r_{ft}K_{ft} - w_{ft}(1+\theta)A_t N_{ft} - \delta K_{ft} \tag{10}$$

$$\Pi_{it} = B_2 K_{it}^{\rho}(A_t N_{it})^{1-\rho} - r_{it}K_{it} - w_{it}A_t N_{it} - \delta K_{it} \tag{11}$$

where r_{ft} and r_{it} are the rental rate of physical capital for the formal and for the informal sector, and w_{ft} and w_{it} are the rental rate of efficiency units of labor for the formal and informal sector, respectively. Note that because we have two interest rates, formal and informal sector, we will need a no-arbitrage condition in our equilibrium conditions to make both equal.

3.4 Social security budget

We define the social security budget in a general form, where all pension schemes presented in the paper will respect it. From the revenue side, the social security authority collects contributions only from the formal hours supplied by the workers, and a consumption tax is levied on everyone to adjust the social security budget in every period t. Pension expenses are the sum of all pensions paid in each period for agents of all efficiency types:

$$T_c + T_{ss} = Pen_t \tag{12}$$

The total contributions to the pension scheme are given by the contributions paid by agents in their working life periods and formal firms.

3.5 Stationary equilibrium

In stationary equilibrium, given the pension scheme framework, prices, and allocations we have that:

1. Individual behavior is consistent with the aggregate behavior of the economy and households maximize lifetime utility subject to the budget constraint

2. Formal and informal firms maximize their profits
3. The budget of the social security system is balanced
4. All markets clear

4 The pension system

In this section we will define three different pensions schemes that we will simulate. The first is our baseline model, the Brazilian pension scheme before the 2019 pension reform, the second is the pension scheme implemented by the 2019 pension reform, and the last one is a general NDC pension scheme which we will use to simulate our other counterfactuals.

1. **The baseline tax and pension scheme - Before the 2019 pension reform**

   In both Brazilian PAYG systems there is a minimum time required working in the formal sector to require a certain type of pension formula benefit. Furthermore, there are minimum and maximum pension payments defined in law, where the minimum payment needs to be equal to the minimum wage of the economy. We will define below our formulas to calculate this working time in the formal sector, the minimum pension benefit.

   TFS: As we have a 3 period model, we do not have how to model the working period in years just summing the periods t. So, I will calculate the working time in the formal sector (TFS) using the formula below:

$$TFS_{\varepsilon_j} = \left(\sum_{s=1}^{2} \left\{ \frac{a_{\varepsilon_j} (n^s_{f,\varepsilon_j})^{b_{\varepsilon_j}}}{a_{\varepsilon_j} (n^s_{f,\varepsilon_j})^{b_{\varepsilon_j}} + (1 - a_{\varepsilon_j})(n^s_{i,\varepsilon_j})^{b_{\varepsilon_j}}} \right\} \right) \times 20 \tag{13}$$

   where TFS_{ε_j} is measured in years.[3]

   Minimum and maximum pension payments: The minimum pension payment is defined to be equal to the minimum wage, which was equal to $R\$788,00$ in

[3] By example, if the time spent in formal sector was 60% in each period, then we have $(0,6+0,6)\times 20 = 24$ and the additional years will be $24-20=4$. If his average wage were R$ 1.000, his pension payment

2015. The maximum payment of the pension system (the ceiling) was equal to $R\$4.663,75$, but just for the private workers. As the majority of the Brazilian workers work in the private sector we ignore this distinction here.

We define the pen_{min} as being equal to the minimum wage. We model this estimating the share of each efficiency type in the economy, μ_i, using the PNAD 2015 again[4]. It's important to make a point here: in all the equilibrium conditions we will use these same shares for each agent efficiency type. The pen_{min} is defined as $pen_{min} = \hat{\beta}\overline{Y}_f$, where:

$$\overline{Y}_f = \mu_1 \left(\frac{y^1_{f,\varepsilon_1}}{2} + \frac{y^2_{f,\varepsilon_1}}{2(1+n)} \right) + \mu_2 \left(\frac{y^1_{f,\varepsilon_2}}{2} + \frac{y^2_{f,\varepsilon_2}}{2(1+n)} \right) + \mu_3 \left(\frac{y^1_{f,\varepsilon_3}}{2} + \frac{y^2_{f,\varepsilon_3}}{2(1+n)} \right) \quad (14)$$

where y^s_{f,ε_j} is the formal income of the agent with efficiency type j in the $s-$period of their life[5]. We found via PNAD data that $\hat{\beta} \approx 0.5$ is the value that equalize pen_{min} to the minimum wage.

As the minimum wage in 2015 was equal to $R\$788,00$, we define our tax schedule based on the threshold values using the minimum wage as our measure variable[6]:

$$T(y_{f,\varepsilon_j,t}) = \begin{cases} 0,08 \cdot y_{f,\varepsilon_j,t} & \text{if } y_{f,\varepsilon_j,t} \leq 1,78 \cdot pen_{min} \\ 0,09 \cdot y_{f,\varepsilon_j,t} & \text{if } y_{f,\varepsilon_j,t} \in (1,78 \cdot pen_{min}, 2,96 \cdot pen_{min}] \\ 0,11 \cdot y_{f,\varepsilon_j,t} & \text{if } y_{f,\varepsilon_j,t} \in (2,96 \cdot pen_{min}, 5,92 \cdot pen_{min}] \\ 0,11 \cdot (5,92) \cdot pen_{min} & \text{if } y_{f,\varepsilon_j,t} > 5,92 \cdot pen_{min} \end{cases} \quad (15)$$

will be given by:

$$Pension = 0,6 \times 1.000 + 0,02 \times 4 \times 1.000 = 600 + 80 = 680$$

It represents a 68% replacement rate.

[4]We found that $\mu_1 = 0,72$, $\mu_2 = 0,20$, and $\mu_3 = 0,08$.

[5]In the algorithm, these variables are all at stationary values.

[6]The intervals are calculated based on table presented above with data from PNAD 2015. For example: 1.399,12/788,00 = 1,78; 2.331,88/788,00 = 2,96.

where $y_{f,\varepsilon_j,t}$ is defined as the total formal income from the individual at period t.

(a) **Contribution time:**[7]

Defining the general pension formula below as:

$$pen_{\varepsilon_j} = \overline{y}_{f,\varepsilon_j} \cdot \underbrace{\left[\frac{0,31 \cdot TFS}{20}\left(1 + \frac{tr + 20 + 0,31 \cdot TFS}{100}\right)\right]}_{\text{=Fator previdenciário (FP)}} \tag{16}$$

where tr means age of retirement, which is tr = 40 in our model[8] and

$$\overline{y}_{f,\varepsilon_j,t} = \frac{w_f \varepsilon_j A_t (n^1_{f,\varepsilon_j}\eta_1 + n^2_{f,\varepsilon_j}\eta_2)}{2} \tag{18}$$

The Brazilian federal legislation requires that the amounts paid be updated according to inflation. Therefore, we have already included an implicit rate of return on the pensions received. Note that if we didn't have do this

[7]After 1998 we had the creation of the social security factor (Fator previdenciário (FP)). From then until 2019 we had two types of retirement possibility, by age and by the contribution time. We decide to assume two important things in our modelling here. The first is to assume the minimum TFS to retirement to be ≥ 35, which the TFS for the men. For the women the TFS need to be ≥ 30. The second assumption was to not consider the third possibility to retirement implemented in 2015 in the Dilma Roussef's government period where was implemented the 95/85 factor. This factor was a possibility in which if you are a man/woman which the sum of years of age and TFS are 95/85 you can get retired without take the FP in consideration and get retired with full pension, which in our model would mean that $pen_{\varepsilon_j} = \overline{y}_f$. As our model get just 3 periods and the 2019 pension reform finished this possibility for new entrants in working force, then we decided to avoid what we considered an unnecessary complication.

[8]The general formula is given by:

$$pen_{\varepsilon_j} = \overline{y}_{f,\varepsilon_j} \cdot \underbrace{\left[\frac{0,31 \cdot TFS}{LE}\left(1 + \frac{tr + j_0 + 0,31 \cdot TFS}{100}\right)\right]}_{\text{=Fator previdenciário (FP)}} \tag{17}$$

where LE is the life expectancy of the individual in the time of retirement, which in our model is of one period length, 20, and j_0 is the age at which the agent enters in the labor force, also 20 years old. Although our formulation is slightly different because the model is three-period, McKiernan (2022) also modeled the pre-reform formula this way.

we would have the pension received in the period t, $pen_{\varepsilon_j,t}$, by the same formula above, but with mean income given by:

$$\overline{y}_{f,\varepsilon_j,t} = \frac{w_f \varepsilon_j (n^1_{f,\varepsilon_j} \eta_1 A_{t-2} + n^2_{f,\varepsilon_j} \eta_2 A_{t-1})}{2} \tag{19}$$

Hence, if $TFS \geq 35$ the pension benefit is defined below:

$$pen_{\varepsilon_j} = \begin{cases} pen_{min}, & \text{if } pen_{\varepsilon_j} \leq pen_{min} \\ pen_{\varepsilon_j}, & \text{if } pen_{min} < pen_{\varepsilon_j} < 5,92 \cdot pen_{min} \\ 5,92 \cdot pen_{min}, & \text{if } pen_{\varepsilon_j} \geq 5,92 \cdot pen_{min} \end{cases} \tag{20}$$

(b) **Age:**

For the age retirement case the pension general formula is given by:

$$pen_{\varepsilon_j} = \max\{0,7 + 0,01 \cdot tr; 1\} \cdot \overline{y}_f \cdot \max\{1, FP\} \tag{21}$$

as tr = 40 and even when $TFS = 35$ we have that $FP = 0,927 < 1$, then

$$pen_{\varepsilon_j} = 1,1 \cdot \overline{y}_f \tag{22}$$

So, the pension benefit is defined by:

$$pen_{\varepsilon_j} = \begin{cases} pen_{min}, & \text{if } pen_{\varepsilon_j} \leq pen_{min} \text{ or } TFS < 15 \\ pen_{\varepsilon_j}, & \text{if } pen_{min} < pen_{\varepsilon_j} < 5,92 \cdot pen_{min} \\ 5,92 \cdot pen_{min}, & \text{if } pen_{\varepsilon_j} \geq 5,92 \cdot pen_{min} \end{cases} \tag{23}$$

2. **Pension scheme after the 2019 reform - The current pension system**

In the 2019 pension reform the legislators' intention was to make the system more progressive. To do this, they increased the number of bands in the system and changed the value of tax rates. The tax schedule was defined in the following way:[9]

$$T(y_{f,\varepsilon_j,t}) = \begin{cases} 0,075 \cdot y_{f,\varepsilon_j,t} & \text{if } y_{f,\varepsilon_j,t} \leq pen_{min} \\ \\ 0,09 \cdot (y_{f,\varepsilon_j,t} - pen_{min}) & \text{if } y_{f,\varepsilon_j,t} \in (pen_{min},\ 2pen_{min}] \\ \quad +0,075 pen_{min} & \\ \\ 0,12 \cdot (y_{f,\varepsilon_j,t} - 2pen_{min}) & \text{if } y_{f,\varepsilon_j,t} \in (2pen_{min},\ 3pen_{min}] \\ +(0,09+0,075) pen_{min} & \\ \\ 0,14 \cdot (y_{f,\varepsilon_j,t} - 3pen_{min}) & \text{if } y_{f,\varepsilon_j,t} \in (3pen_{min},\ (5,84) pen_{min}] \\ +(0,12+0,09+0,075) pen_{min} & \\ \\ 0,14 \cdot (2,84) pen_{min} & \text{if } y_{f,\varepsilon_j,t} > 5,84 pen_{min} \\ +(0,12+0,09+0,075) pen_{min} & \end{cases} \tag{24}$$

As the benefit formula also change[10], we need to define a new pension formula. Defining the pension formula as $PenForm = 0,6 \cdot \overline{y}_{f,\varepsilon_j} + 0,02 \cdot (TFS - 20) \cdot \overline{y}_{f,\varepsilon_j}$, we have now that the pension payments are defined below:

$$pen_{\varepsilon_j} = \begin{cases} pen_{min}, & \text{if } TFS < 20 \text{ or if } (TFS \geq 20 \text{ and } PenForm \leq pen_{min}) \\ PenForm, & \text{if } TFS \geq 20 \text{ and } pen_{min} < PenForm < 5,84 \cdot pen_{min} \\ 5,84 \cdot pen_{min}, & \text{if } TFS \geq 20 \text{ and } PenForm \geq 5,84 \cdot pen_{min} \end{cases} \tag{25}$$

It is worth to note that we assume here the TFS for the men again, which is

[9] The pension reform was approved in 2019, but the rules only came into effect in 2020, therefore, we used the values of the minimum wage and social security thresholds from 2020 to define our tax schedule. In 2020 the minimum wage was $R\$1.045,00$.

[10] See section about the Brazilian Pension Scheme above in this paper.

greater or equal to 20. The women requirement is $TFS \geq 15$. Moreover, the minimum age to retire was defined in 65 to the men and 62 for the women. In our model we assume everybody gets retired in the third period, which corresponds to 60 years old.

3. **NDC pension scheme - The general formula**

   We propose an NDC pension scheme as an alternative to the current PAYG system in Brazil. The formulation we chose was as general as possible, so that we can make several counterfactuals.

   We will test three social security tax schedules in our exercises, which we present below:

   (a) The baseline tax schedule before the 2019 pension reform;

   (b) The 2019 pension reform tax schedule;

   (c) A linear tax rate where all agents pay the same tax rate in each period:

$$T(y^s_{f,\varepsilon_j,t}) = \tau_s \cdot y^s_{f,\varepsilon_j,t} \tag{26}$$

   where $s \in \{1,2\}$ and τ_1 and τ_2 does not need to be the same.

   The general pension formula for our NDC pension scheme is given by:

$$Pen_{NDC,\varepsilon_j} = pen_{min} + \zeta_{pen}[(T(y^1_{f,\varepsilon_j}) + \theta y^1_{f,\varepsilon_j})(1+d)^2 + (T(y^2_{f,\varepsilon_j}) + \theta y^2_{f,\varepsilon_j})(1+d)] \tag{27}$$

   where $T(y^1_{f,\varepsilon_j}), T(y^2_{f,\varepsilon_j})$ are paid by the worker and θ paid by the firms, while d represents the return rate from the NDC pension system[11]. Although in our modeling we have allowed for a rate of return d, in our simulations we will assume $d = 0$ unless stated otherwise. As presented before, y^s_{f,ε_j} is the formal income earned by the ε_j type efficiency agent in the s period of his life.

   For the first two tax schedules the pension benefit formula is presented below:

[11] Again, here the value received will already be updated, as pointed before. Hence, any $d > 0$ represents a return rate greater than it happens in the actual system.

$$pen_{\varepsilon_j} = \begin{cases} pen_{min}, & \text{if } TotalContrib \leq pen_{min} \text{ or } TFS < 15 \\ Pen_{NDC,\varepsilon_j}, & \text{if } Pen_{NDC,\varepsilon_j} < MaxPen_{INSS} \\ MaxPen_{INSS}, & \text{if } Pen_{NDC,\varepsilon_j} \geq MaxPen_{INSS} \end{cases} \tag{28}$$

where $TotalContrib$ represents the sum

$$TotalContrib = (T(y^1_{f,\varepsilon_j}) + \theta y^1_{f,\varepsilon_j})(1+d)^2 + (T(y^2_{f,\varepsilon_j}) + \theta y^2_{f,\varepsilon_j})(1+d),$$

$MaxPen_{INSS}$ is the maximum pension benefit paid by the INSS, and pen_{min} is defined ex-ante for each counterfactual simulation.

For the last tax schedule the pension benefit is slightly different:

$$pen_{\varepsilon_j} = \begin{cases} pen_{min}, & \text{if } TotalContrib \leq pen_{min} \\ Pen_{NDC,\varepsilon_j}, & \text{if } TotalContrib > pen_{min} \end{cases} \tag{29}$$

where there is no $MaxPen_{INSS}$ in this case.

In our theoretical exercises, we can define pen_{min} as we want, even though the Brazilian's law does not allow for $pen_{min} <$ minimum wage. So, we will test the following cases:

(a) **Beveridge system**: $pen_{min} > 0$ and $\zeta_{pen} = 0$, with pen_{min} assuming a previously defined value.

(b) **Bismarck system**: $pen_{min} = 0$ and $\zeta_{pen} = 1$.

(c) **NDC**: $pen_{min} > 0$ and $\zeta_{pen} \in (0,1)$.

The Beveridge system is the case where all agents receive the same minimum pension benefit, independently of their past contributions. In the other hand, the Bismarckian system represents a case without minimum pension benefit, so every agent receives a benefit related with their past contributions only, without any intragenerational redistribution. What we call here NDC, is the intermediate case, where there is a minimum benefit, but there is also a pillar relating the agents future benefits to their past contributions while working in the formal sector[12].

In the PAYG formulations we presented before, the system always required a minimum time working in the formal sector to receive a pension benefit, while here it does not need to happen. When we are in the Bismarck system, there is no need for require any contribution time to the agent receive the benefit, because there is no minimum benefit depending on it. In Brazil, this discussion happens mainly due to the fact that many women do not reach this minimum contribution time in practice. Note that we are convexifying our pension system compared to the others frameworks presented before, as the pension payment comes closer to the individual past contributions.

5 Data and calibration

5.1 Data

The data source we use to to estimate our parameters is from Pesquisa Nacional por Amostra de Domicílios (PNAD) 2015. PNAD is a national representative survey of Brazilian households that contains information about demographic, employment and other caracteristics. Using this data set we can verify if the households are working formally, informally, and if they are splitting their time between these sectors. We will use this data set to estimate productivity parameters and to define the moments for our calibration procedure of other parameters.

5.2 Parameters

The parameters $\{\gamma, n\}$ were defined using the government statistics from the last 20 years. For $\{\alpha, \rho\}$ we use the Giambiagi, Ferreira, et al. (2013). The parameters $\{\beta, \iota, \sigma\}$ are standard in the literature. To estimate the productivity we divide our sample from PNAD by formal income in certain intervals, using the official 2015 minimum

[12]Note that the Bismarck is an NDC pension system too.

wage as a measure and estimated both, age and efficiency productivity, using the same regression presented below:

$$\ln\left(\frac{y_f}{n_f}\right)_i = \beta_0 + \beta_1 D_{\varepsilon_2,i} + \beta_2 D_{\varepsilon_3,i} + \beta_4 \eta_i + u_i \tag{30}$$

where i represents the individual, y is the total income, n_f the formal work hours, β_0 represents the reference to individual who is efficiency type 1, D_{ε_j} the dummy for efficiency type when $j \in \{2,3\}$, and η is a dummy variable that represents the age productivity.

We present the results in the table below:

Parameter	Description	Value
Efficiency type		
ε_1	$y_f \leq 2$ min. wages	1.0
ε_2	$y_f \in (2\text{min. wages}, 5\text{min. wages})$	2.17
ε_3	$y_f \geq 5$min. wages	6.14
Age productivity		
η_1	age $\in [21, 40]$	1.0
η_2	age $\in [41, 60]$	1.128

Notes: y_f represents formal income.

Table 1: Parameter Values

The other parameters were calibrated using a minimum distance method or we took from related literature:

$$\vec{X} = \{B_2, a_{\varepsilon_1}, b_{\varepsilon_1}, a_{\varepsilon_1}, b_{\varepsilon_1}, a_{\varepsilon_1}, b_{\varepsilon_1}\} \tag{31}$$

The table including all the parameters and its values found is presented below[13]:

[13] $n = 0.20$ represents the average population growth from 2000 until 2019, γ represents the average GDP per capita growth from 2000 until 2019, $\beta = 0.67$ is equivalent to 0.98^{20}, while $\delta = 0.7$ comes from $1 - \delta = (1 - 0.06)^{20}$. The parameters α and ρ we defined based on the 0.4 capital share estimated for Brazil presented in Giambiagi, Ferreira, et al. (2013). As in the PNAD nearly 80% of agents income comes from the formal sector we made a simplification to adjust the capital share parameters in our

Parameter	Description	Source	Value
Economy			
n	Population growth	IBGE	0.20
γ	GDP per capita growth	IBGE	0.30
Utility function			
β	Discount factor	Standard	0.67
σ	IES	Standard	2.0
ι	Leisure parameter	Standard	2.0
Production			
B_1	Formal firm TFP	Calibrated	1.0
B_2	Informal firm TFP	Calibrated	0.1021
α	Capital share of the formal firm	Standard	0.45
ρ	Capital share of the informal firm	Standard	0.20
δ	Capital depreciation rate	Standard	0.7
θ	Formal firm tax rates	Legislation	0.20
Labor input			
a_{ε_1}	Share of formal work type ε_1	Calibrated	0.3958
a_{ε_2}	Share of formal work type ε_2	Calibrated	0.9364
a_{ε_3}	Share of formal work type ε_3	Calibrated	0.9159
b_{ε_1}	Substitution parameter type ε_1	Calibrated	−0.3167
b_{ε_2}	Substitution parameter type ε_2	Calibrated	−0.0835
b_{ε_3}	Substitution parameter type ε_3	Calibrated	−0.9270

Table 2: Parameter Values

5.3 Model fit

In this section we will show how the model fitted with data.

The targeted moments were all took from PNAD 2015 data. Note that in the table 3, y_f and y_i represent the formal and informal income of agents, respectively,

two sector economy: $0.8 \times 0.45 + 0.2 \times 0.2 = 0.4$. Finally, $B_1 = 1$ was defined as the basis while B_2 was internally calibrated by the model comparatively to B_2.

Variable	Data	Model	Source
Targeted moments			
$\sum_s \sum_{\varepsilon_j} y_f / \sum_s \sum_{\varepsilon_j} y_{total}$	0.81	0.7215	PNAD
$\sum_s y_{total,\varepsilon_1} / \sum_s \sum_{\varepsilon_j} y_{total}$	0.41	0.4125	PNAD
$\sum_s y_{total,\varepsilon_2} / \sum_s \sum_{\varepsilon_j} y_{total}$	0.25	0.2660	PNAD
$\sum_s y_{total,\varepsilon_3} / \sum_s \sum_{\varepsilon_j} y_{total}$	0.33	0.3215	PNAD
$\sum_s y_{f,\varepsilon_1} / \sum_s \sum_{\varepsilon_j} y_f$	0.28	0.2807	PNAD
$\sum_s y_{f,\varepsilon_2} / \sum_s \sum_{\varepsilon_j} y_f$	0.31	0.3435	PNAD
$\sum_s y_{f,\varepsilon_3} / \sum_s \sum_{\varepsilon_j} y_f$	0.41	0.3758	PNAD
Untargeted moments			
Capital investment	15.52%	8.72%	IBGE
Replacement rate ε_1	101%	123%	Silva Filho et al (2021)
Replacement rate ε_2	72.9%	99%	Silva Filho et al (2021)
Replacement rate ε_3	52.7%	58%	Silva Filho et al (2021)
Contr. density Age Ret.	43.3%	51.47%	Silva Filho and Sidone (2022)
Contr. density Cont.Time Ret.	89.4%	92.8%	Silva Filho and Sidone (2022)

Table 3: Model fit

and $y_{total} = y_f + y_i$ the total income of agents. So, $\sum_s \sum_{\varepsilon_j} y_f / \sum_s \sum_{\varepsilon_j} y_{total}$ is the ratio between the total formal income of the economy held by agents over the total income of the economy held by agents. It is important to mention that those are not ratios between formal output over total output, but ratios between the income held by agents separated by their efficiency type ε_j and age s.

For the untargeted moments of pension measures we defined the replacement rates and the contributory densities[14] based on two government studies, Silva Filho et al (2021) and Silva Filho and Sidone (2022), who took a microdata sample of

[14]The contributory density is defined as:

$$ContDensity = \frac{\text{\#Years contributing}}{\text{Age of retirement} - 16} \tag{32}$$

workers who get retired in 2018 in Brazil. We focused on male observations of the these studies. The replacement rate measure they use for each retired worker i is given by:

$$RepRate_{iT^*} = \frac{B_{iT^*}}{\left[\frac{\sum_{T^*-n}^{T^*-1} R_{it}}{n}\right]}, \tag{33}$$

where B_{iT^*} is the pension benefit value of the retired worker when he retired in the period T^*; R_{it} is the formal income of the worker i in the moment t, with $t < T^*$, and n is the number of months took in consideration to calculate the average formal income of the worker, where in the study they use $n = 36$.

For the replacement rates statistics we presented here we filtered the data found by them to take only the age retirement pensions for ε_1 and ε_3 efficiency types and contribution time for ε_2 efficiency type, as in our model the TFS_{ε_j} was greater or equal 35 for the ε_2 efficiency type only. We only report the numbers for the age retirement pensions for the normal case, but there is the possibility to get the age retirement pension by disability retirement, which is also considered age retirement but by health issues. If we add this case the replacement rates data for ε_1 and ε_3 goes to 110% and 57,1%. The replacement rates calculated by us using our model were all based on the average lifetime formal income only, while Silva Filho et al (2021) calculated the replacement rates using only the last 36 month contributions made by the workers. As in our model each period represents a 20 year period length, we do not have how to create a measure exactly equal of their studies. As in our model the agents offer less labor supply in the second period, if we have used the second period formal income as our denominator the replacement ratio generated by the model would increase considerably.

The contributory density is calculated slightly for them, as they have the total number of working years of each worker, while in our model we considered that every agent works for 40 years, so the numbers are again an approximation only. In our baseline model the only efficiency type agent who retired by contribution time possibility was the ε_2, while ε_1 and ε_3 retired by age eligibility criteria. So to calculate the contributory density for the age retirement case we just took the ratio $TFS_{\varepsilon_2}/40$, while for the contribution time we took the weighted average ratio

$$\mu_1 \cdot \frac{TFS_{\varepsilon_1}}{40} + \mu_3 \cdot \frac{TFS_{\varepsilon_3}}{40} = 51.47\%.$$

If we only take the ratio for the efficiency types separately we have that

$$\frac{TFS_{\varepsilon_1}}{40} = 19.09/40 = 47.73\% \text{ and } \frac{TFS_{\varepsilon_3}}{40} = 34.03/40 = 85.01\%.$$

Even though we know our model statistics for the untargeted moments are just rough approximations, we think these moments are interesting to compare to understand where our model results are when comparing with real data.

6 Results

In this section we will present the results obtained with our model. We will make all our comparisons based on steady state results and take the consumption equivalent change (CEC) as our welfare measure to define what pension scheme it is the best for the Brazilian economy based on our model. In the first subsection we will discuss the differences between our baseline results, the economy before the 2019 pension reform, and the economy after the 2019 pension reform. In the following subsection we present the results for three possibilities using the NDC model: bismarckian, beveridge and an intermediate NDC system between them. We discuss the differences between implementing a pension reform using an NDC pension scheme compared to our baseline and the 2019 reform. In the third subsection, we focus on run the same models already analyzed but using age-dependent taxation to finance the pension benefits. We implement this by reducing the taxation of the first period by 40% and increasing that of the second by 40% as well. Finally, we discuss some more NDC models that may be proposed for the Brazilian economy by policy makers in the future and compare them with the others already presented.

6.1 2019 pension reform

The analysis of the effects of the 2019 pension reform must to be made with caution here. The main changes made by the 2019 pension reform were: the unification of retirement rules for public and private workers, the increasing of the minimum age to apply for retirement, and the change in the retirement benefit calculation rule. In our model we only take into account the third system change. In the model all the workers retire in the third period of life and we do not make distinctions between private and public workers. Furthermore, we do not model the 85/95 rule here, so this is another point that diminishes the welfare gains we found. Hence, all comparisons in terms of welfare gains need to be made taking into account only the effect of the retirement benefit calculation rule on the economy, so the interpretation should be as follows: "What would be the differences if the rule adopted was not the same as the 2019 reform, but rather one of the NDC models we defined?".

The welfare measure we will use is the CEC. Let the U_0 be the lifetime utility of our benchmark, we define the CEC through the Δ below:

$$\Delta = \left(\frac{U_1}{U_0}\right)^{1/(1-\sigma)} - 1 \tag{34}$$

where U_1 is the counterfactual lifetime utility. We want to know what increase in consumption is necessary for the agent to obtain the same lifetime utility as the counterfactual in the benchmark pension scheme, so that:

$$U_0((1+\Delta)C, 1-N) = U_1(C', 1-N') \tag{35}$$

The main results found are presented in table 4[15]. With the implementation of the 2019 pension reform the aggregate variables of capital stock, output, consumption, and labor supply will increase in the steady state, while the interest rate will decrease, and the replacement rates of the most productive workers will, but not much. The more productive workers will continue to receive the maximum pension benefit allowed by the system, while the intermediate productivity worker will receive pension benefits of 3.47 minimum wages instead of 3.70. The less productive worker will continue to receive the minimum pension benefit. In terms of welfare gains, the

[15] In all tables the variables are presented in stationary values. $Pen^{w}_{\varepsilon_j}$ means the value of the pension benefit received by the efficiency type j in terms of minimum wages. So if $Pen^{w}_{\varepsilon_j} = 3.70$ it means the agent receives a pension worth 3.70 minimum wages. The variable N_{k,ε_j} represents the total effective labor supply in the sector k by the efficiency type j agents.

general gain will be of 0.83%, with the most benefited type being the intermediate. Notice that the drop in consumption tax is small. This happens because the number of people receiving the minimum benefit in the economy is more than 70%, and the new benefit calculation rule is not linking the benefits received to the contributions made.

6.2 NDC pension schemes

In the implementation of the NDC pension schemes we tested many pension designs, and we will analyze each them from now on. The first models tested were: the Beveridge system with minimum pension payment being equal to one minimum wage, the NDC with a minimum wage minimum payment and $\zeta_{pen} = 0.5$, and finally the Bismarck system, where there is no minimum pension and $\zeta_{pen} = 1$. Remember that this last pension scheme is only possible in theoretical exercises, because the Brazilian's law would not allow this to be implemented. We will use the same tax schedule actually present in the Brazilian pension system to let it more comparable to the 2019 pension reform, and because it is unlikely that it would be reverted by the Brazilian parliamentarians. We use the subscript 1 in these NDCs to differentiate them from the other NDCs we will analyze later in which we use other tax structures.

1. **Beveridge 1**

   In this scenario we would have the biggest increases in the aggregate variables of the economy. The formal capital stock would increase more than 22%, the formal output 9.59%, the informal output 7.83%, while the aggregate consumption more than 8%. In the price side, we would have an increase of 9% in formal wages, 2.5% in informal wages, and a drop of more than 7% in the interest rates. In the labor side, we have an increase of almost 6% in the aggregate informal labor supply, driven by the ε_1 and ε_3 efficiency types agents. This happens because the Beveridge system represents a disincentive for agents to seek formal work in the economy, since now the gain of becoming formal is low in terms of future pension benefits.

   In terms of pension benefits, the replacement rates of the more productive workers dropped much more compared to the baseline or the 2019 reform, while the less productive workers continue to receive a minimum wage as a pension. The formal labor supply of the ε_3 agent increase 3.4%, while the informal increase more than 13%. The less productive increase its informal labor supply by 4%. Note that while the intermediate agent goes from a scenario in which he receives a pension worth 3.70 minimum wages to 1 minimum wage, the most productive goes from 5.92 minimum wages to 1 minimum wages. Still about pensions, note that with the increase in the economy total output, the minimum pension of the economy increase by more than 10%, going from a value of 0.0118 to 0.0130.

The welfare effects are huge. The model predicts welfare gains for efficiency types, even with the fall in the replacement rates for ε_2 e ε_3. Comparing with the 2019 pension reform rule the welfare gains are more than 13 times. This result was driven by the drop in the consumption taxes, that goes from 0.021 in the baseline to -0.045 in the Beveridge case, the increase in the steady state capital stock, and in the increases in the formal labor supply of the more productive workers, N_f, ε_3, and total informal labor supply, N_i.

2. **NDC 1**

   The NDC alternative we propose returns results that go in the same direction of we already saw in the Beveridge system. The differences are the magnitudes of the results we found. The increases in the steady state capital stock are nearly 11.4% instead of the 22.78% of the Beveridge scheme, while the increases in the formal and informal output, total consumption are high but not as in the Beveridge, with the same been valid to the drop in the interest rates. In the labor side we found an increase in the formal and informal labor supply of the most productive workers.

   The replacement rates dropped for the most productive workers only, but they still receive 2.1 and 3.8 minimum wages in pensions, respectively, which is between 2/3 of what they would receive in the baseline case. The consumption taxes are negative again, as in the Beveridge scheme and the minimum pension also increase as a result of the personal income increase for all agents. Furthermore, the welfare gains are approximately half of what we found in the later case.

3. **Bismarck 1**

   In the Bismarck pension scheme we have an extreme case in which the agent receives benefits only based on their past contributions, then the incentives structure changes compared to previous cases. We observe bigger increases in the aggregate variables compared to the NDC case, but smaller then the Beveridge scheme. The increases are in the capital stock, output and consumption as before, but here we have the biggest value for the total informal labor supply, mainly driven by N_{i,ε_1} and N_{i,ε_3}.

   In the pension benefits side we have the only case in which the less productive worker do not receive a pension benefit worth one minimum wage, instead he receives 0.45 minimum wage as a pension. The replacement rate dropped from 123% of the baseline case to 55%. Even though it seems to be bad to the agent, the welfare gains here are higher for all efficiency type agents than what we observed in the 2019 reform and in the NDC system. This is mainly driven by the fall in consumption taxes, which allows for the agents to accumulate capital from the first to the second period of life. Comparing with the other two NDC alternatives, this is the second best scenario.

6.3 Use of age-dependent taxation in pension schemes

In this subsection we will discuss the results we found after estimating the same models presented in table 4, but now using age-dependent taxation in the tax schedule. We implement this by reducing the social security tax rates for the first period of life by 40% and increasing in 40% in the second period. The results are presented in table 5. In all the scenarios the comparison was made against the baseline presented in table 4.

Analyzing our results we see that using age-dependent taxation improves the welfare in all the pension schemes estimated in at least 2.87%, going until 14.40% in the Beveridge case. Comparing the same models with age-dependent against the case with age-dependent gives to us an improvement in aggregate variables also, with the capital stock, output and consumption increasing in all scenarios. The formal wages increased more than 3% in all scenarios, the formal labor supply, N_f^{adt}, is smaller or equal than in the normal case, N_f, while the informal, N_i^{adt}, is greater or equal to N_i.

The mechanism by which the age-dependent taxation act in our pension schemes is as follows: the drop in the first period social security tax rates increases formal income in the first period, which increases not only consumption in the first period, but also capital accumulation. This greater capital accumulation increases consumption in the second period even with a lower equilibrium interest rate in the age-dependent case than in the base scenario. Therefore, we have an increase in aggregate consumption in the first two periods, although not necessarily for all types of agents. The increase in the second period tax rates levies on a lower formal income, as in the model agents offer a lower labor supply in the second period of life. Therefore, as a result we have agents paying less labor income taxes throughout their lives, where even with a higher consumption tax rate than in the base scenario leads to welfare gains for all types of agents. In the aggregate we have a lower tax burden on income and a slightly higher on consumption.

These results we found are that in an environment in which agents born without assets but can save from a period of life to another, the young agents should face lower income social security tax rates than older workers, but that is different from the results found by Weinzierl (2011) in which the agents cannot borrow or save in a dynamic Mirrleesian framework.

6.4 Pension schemes for policy making

In this final subsection we discuss a few other NDC pension schemes which can be interesting for future public policy in Brazil. We run six other models we present below and present in table 6:

1. **Beveridge$_2$, NDC$_2$ and Bismarck$_2$**: the same models as we show in table 4 but keeping the social security tax schedule of the Brazilian pension system before the 2019 reform.

2. **NDC$_3$ and NDC$_3^{adt}$**: this model is an NDC with a minimum pension equal to one minimum wage and assuming $\zeta_{pen} = 0.5$, but with agents paying the same tax rate in both periods independently of their efficiency types, where $\tau_1 = \tau_2 = 0.10$ and there is no $MaxPen_{INSS}$.

3. **Beveridge$_1^2$**: in this case we simulate a scenario in which the minimum pension is equal to two minimum wages.

In the NDC models in which we use the older social security tax schedule we found welfare improvements compared to the baseline, but the welfare gains were not as high as in the NDC models we estimated using the actual tax schedule. All the results we found go on the same direction, but with an small impact on welfare.

In the case with an equal linear taxation to all agents we found this system is dominated by all other NDCs we tested. Furthermore, when we apply age-dependent taxes on this framework there is a huge drop in the formal and informal labor supply of the more productive worker of the economy and the consumption taxes are bigger than in the other NDC models.

Finally, when we test the Beveridge system providing an minimum pension of 2 minimum wages we have the only case in which we found welfare loss for some of the efficiency types of agents. The efficiency types ε_2 and ε_3 suffer a welfare loss in this case., while the less productive worker have the minor welfare gain compared to the baseline in all cases tested, even though the pension replacement rates increases from 123% to 246%. Comparing with the baseline case, we have decreases in output, consumption, and formal and informal labor supply. Furthermore, this is the scenario in which we have the highest consumption tax.

Variable	Baseline	Reform	Beveridge$_1$	NDC$_1$	Bismarck$_1$
$\tilde{k}_f$	0.0079	0.008	0.0097	0.0088	0.0091
$\tilde{k}_i$	$7.42e-04$	$7.53e-04$	$8.9e-04$	$8.14e-04$	$8.63e-04$
$\tilde{y}_f$	0.1032	0.1034	0.1131	0.1078	0.1104
$\tilde{y}_i$	0.0217	0.0218	0.0234	0.0226	0.0236
$\tilde{c}$	0.1140	0.1141	0.1232	0.1183	0.1214
w_f	0.0562	0.0567	0.0614	0.059	0.059
w_i	0.0199	0.0199	0.0204	0.0201	0.0202
N_f	0.84	0.83	0.84	0.84	0.85
N_i	0.87	0.88	0.92	0.90	0.93
τ_c	0.021	0.019	-0.045	-0.012	-0.027
r	8.53%	8.47%	7.87%	8.20%	8.12%
Δ_{total}	–	0.83%	11.40%	5.60%	6.72%
Δ_{ε_1}	–	0.82%	11.38%	5.58%	6.46%
Δ_{ε_2}	–	0.99%	11.55%	5.71%	8.55%
Δ_{ε_3}	–	0.52%	11.29%	5.50%	8.56%
$rep.rate_{\varepsilon_1}$	123%	123%	123%	123%	55%
$rep.rate_{\varepsilon_2}$	99%	94%	28%	60%	64%
$rep.rate_{\varepsilon_3}$	58%	57%	9.5%	36%	54%
$Pen^w_{\varepsilon_1}$	1.0	1.0	1.0	1.0	0.45
$Pen^w_{\varepsilon_2}$	3.70	3.47	1.0	2.16	2.32
$Pen^w_{\varepsilon_3}$	5.92	5.84	1.0	3.80	5.60
pen_{min}	0.0118	0.0118	0.0130	0.0124	0.0126
N_{f,ε_1}	0.32	0.32	0.32	0.32	0.32
N_{i,ε_1}	0.99	0.99	1.03	1.01	1.06
N_{f,ε_2}	1.40	1.37	1.36	1.36	1.37
N_{i,ε_2}	0.32	0.31	0.31	0.31	0.32
N_{f,ε_3}	3.83	3.84	3.96	3.93	3.96
N_{i,ε_3}	1.56	1.58	1.77	1.72	1.80

Table 4: Comparison of pension schemes

Variable	Baseline^{adt}	Reform^{adt}	Beveridge$_1^{adt}$	NDC$_1^{adt}$	Bismarck$_1^{adt}$
$\tilde{k}_f$	0.0085	0.0086	0.0104	0.0094	0.0098
$\tilde{k}_i$	$7.93e-04$	$8.02e-04$	$9.52e-04$	$8.72e-04$	$9.27e-04$
$\tilde{y}_f$	0.1056	0.1058	0.116	0.1105	0.1135
$\tilde{y}_i$	0.0222	0.0223	0.0239	0.0231	0.0242
$\tilde{c}$	0.1161	0.1163	0.1254	0.1207	0.1243
w_f	0.0583	0.0586	0.064	0.061	0.062
w_i	0.0201	0.0201	0.0206	0.0204	0.0204
N_f	0.83	0.83	0.83	0.83	0.84
N_i	0.88	0.88	0.93	0.91	0.95
τ_c	0.0241	0.022	−0.0414	−0.0094	-0.0275
r	8.26%	8.22%	7.6%	7.92%	7.84%
Δ_{total}	2.87%	3.48%	14.40%	8.60%	9.86%
Δ_{ε_1}	2.49%	3.09%	14.0%	8.20%	9.21%
Δ_{ε_2}	5.85%	6.64%	17.8%	11.78%	14.85%
Δ_{ε_3}	3.52%	3.89%	15.07%	9.33%	12.78%
$rep.rate_{\varepsilon_1}$	124%	123%	123%	123%	54%
$rep.rate_{\varepsilon_2}$	99%	94%	28%	57%	59%
$rep.rate_{\varepsilon_3}$	57%	56%	9.0%	36%	53%
$Pen^w_{\varepsilon_1}$	1.0	1.0	1.0	1.0	0.44
$Pen^w_{\varepsilon_2}$	3.64	3.44	1.0	2.04	2.12
$Pen^w_{\varepsilon_3}$	5.92	5.84	1.0	3.84	5.64
pen_{min}	0.0121	0.0121	0.0133	0.0127	0.0130
N_{f,ε_1}	0.31	0.31	0.31	0.31	0.32
N_{i,ε_1}	1.0	1.0	1.05	1.02	1.07
N_{f,ε_2}	1.37	1.35	1.33	1.33	1.36
N_{i,ε_2}	0.33	0.32	0.32	0.31	0.33
N_{f,ε_3}	3.83	3.84	3.96	3.93	3.95
N_{i,ε_3}	1.61	1.62	1.82	1.77	1.85

Table 5: Comparison of pension schemes using ADT

Variable	Beveridge$_2$	NDC$_2$	Bismarck$_2$	NDC$_3$	NDC$_3^{adt}$	Beveridge$_1^2$
$\tilde{k}_f$	0.0097	0.0088	0.0091	0.0085	0.0093	0.008
$\tilde{k}_i$	$8.9e-04$	$8.13e-04$	$8.6e-04$	$7.8e-04$	$8.55e-04$	$7.4e-04$
$\tilde{y}_f$	0.1135	0.1083	0.1110	0.1063	0.1097	0.1019
$\tilde{y}_i$	0.0234	0.0225	0.0236	0.0219	0.0226	0.0215
$\tilde{c}$	0.1235	0.1187	0.1220	0.1165	0.1194	0.1124
w_f	0.0613	0.0587	0.0593	0.0582	0.061	0.06
w_i	0.0204	0.0201	0.0202	0.0201	0.0204	0.02
N_f	0.85	0.85	0.86	0.84	0.82	0.82
N_i	0.92	0.90	0.93	0.87	0.89	0.86
τ_c	−0.046	−0.0134	−0.03	−0.019	-0.015	0.03
r	7.9%	8.2%	8.13%	8.3%	7.9%	8.5%
Δ_{total}	11.17%	5.46%	6.6%	4.7%	8.5%	1.5%
Δ_{ε_1}	11.16%	5.47%	6.38%	4.5%	7.95%	1.9%
Δ_{ε_2}	11.14%	5.41%	8.17%	6.30%	12.97%	−1.34%
Δ_{ε_3}	11.54%	5.81%	8.86%	1.85%	7.62%	−1.43%
$rep.rate_{\varepsilon_1}$	124%	124%	56%	121%	122%	246%
$rep.rate_{\varepsilon_2}$	27.40%	58%	62%	57%	56%	56%
$rep.rate_{\varepsilon_3}$	9.56%	36%	53%	40%	39%	19%
$Pen^w_{\varepsilon_1}$	1.0	1.0	0.45	1.0	1.0%	2.0
$Pen^w_{\varepsilon_2}$	1.0	2.13	2.29	2.10	2.1	2.0
$Pen^w_{\varepsilon_3}$	1.0	3.7	5.45	4.1	3.98	2.0
pen_{min}	0.0130	0.0124	0.0127	0.0122	0.0126	0.0117
N_{f,ε_1}	0.32	0.32	0.32	0.32	0.32	0.31
N_{i,ε_1}	1.03	1.01	1.05	1.0	1.01	0.98
N_{f,ε_2}	1.38	1.38	1.40	1.37	1.35	1.32
N_{i,ε_2}	0.32	0.32	0.33	0.32	0.33	0.29
N_{f,ε_3}	3.95	3.92	3.95	3.82	3.77	3.87
N_{i,ε_3}	1.76	1.72	1.79	1.57	1.49	1.60

Table 6: Comparison of pension schemes with others NDC alternatives

7 Conclusion

The aging of economies has been a cause for great concern in developed countries, but even in developing economies this problem has emerged more quickly than expected and Brazil is a well-known example of this. In this paper we described the entire history of the pension reform process that took place in the Brazilian economy and model the issue using a three-period OLG model for the Brazilian economy before and after the last pension reform. Our main contribution was to design NDC pension schemes to the Brazilian economy and compare the welfare effects of adopting these NDC pension schemes and also using age-dependent taxation in a social security scheme.

The main results we found were that the change in the retirement benefit calculation rule made in the last pension reform gives a welfare improvement to all types of agents and increase the main aggregate variables of the economy. As we made clear in the last section, we focus our modeling in the change of the pension benefit rule, so comparing the benchmark and the reform with the NDC schemes we have that if instead the last pension benefit rule an NDC design had to been implemented the welfare would have improved much more than in the last reform, where the main mechanism is the reduction in the replacement rate of agents' pensions.

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