Exploring time-varying efficient price of Bitcoin and Ethereum prices*

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Abstract

We investigate whether Bitcoin (Ethereum) price has a cointegration relationship with cryptocurrencies prices, gold price, and VIX. We analyze that the possible cointegration is time-varying due to the development that has taken place in the cryptocurrency market over time and evaluate the effects of this possible change in behavior between cryptocurrency price and gold price. Our procedure is based on vector error-correction model with time-varying parameters and volatility stochastic from Koop et al. (2011). Our sample is daily between 08/06/2015 and 01/31/2023 in general. We use traded and non-traded data. The use of non-traded data leads to fewer long-term relationships between variables. Tether as a stablecoin does not have a cointegration relationship with Ethereum and Bitcoin. Comparing results between traded and non-traded data, the use of traded data leads to a result where Bitcoin and Ethereum prices adjust to deviations in the long-term relationship over an 18 times greater number of periods, for example. The use of non-traded data tends to underestimate the coefficient of Ethereum's price in the long-term relationship.

Keywords: Ethereum price, Bitcoin price, Efficient price, Time-varying cointegrating relationship, Cryptocurrencies.

1 Introduction

Cryptocurrencies have been gaining the attention of investors around the world in a context of decentralisation and low transaction costs. Despite the security of a blockchain network, the value of these coins is not tied to any tangible asset in general, but to an algorithm capable of consolidating trades. Thus, a market price is defined based on the relationship between supply and demand.

A field in the literature seeks to explain the mechanisms that influence the pricing of cryptocurrencies which is the efficient price. The efficient price can be understood as the long-term behaviour of the asset price. This long-term behaviour subtracts from short-term swings, the presence of outliers, or some transitory disruptions in the market (Kapar and Olmo, 2021).

The reason for finding the efficient price of Bitcoin or Ethereum is that investors can use it in strategic trading decisions. For example, consider a situation where there is a long-term relationship, which would be the efficient price of Bitcoin. A deviation from long-term equilibrium can trigger investor

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decisions to buy or sell in the belief that the price of Bitcoin will return to long-term equilibrium. Also cryptocurrencies have the potential to affect sectors on the real side of the economy, which generates economic and social concern with digital asset markets (Liu and Tsyvinski, 2021).

Our aim is to investigate how the criptocurrency price behaves in the long term. The emphasis is on the relationship that may exist between the price of Bitcoin (Ethereum) and the price of gold, VIX, and other cryptocurrencies (Tether and BNB). We consider the case that cointegration is time-varying due to the development that has taken place in the cryptocurrency market over time and evaluate the effects of this possible change in behavior. We estimate the time-varying version of vector error-correction model (VECM) with stochastic volatility from Koop et al. (2011). We use daily data between 01/01/2018 and 01/31/2023 in general. We have daily data for only Bitcoin prices since 2013 among all cryptocurrencies used. Alexander and Dakos (2020) point out that there are differences between using traded and non-traded data. We use traded and non-traded data and we compare results. We have the same sample period for traded and non-traded data because we have a longer time period available for non-traded data.

Our research proposal is directly related to Adebola et al. (2019). Adebola et al. (2019) analyze the relationship between cryptocurrencies and gold prices through cointegration, if any. They use fractional cointegration as a methodology. Our contribution is the difference from Adebola et al. (2019) because we analyze the presence of cointegration with time-varying parameters (TVP) based on Koop et al. (2011). Our motivation for the time-varying behavior for cointegration comes from the presence of pronounced increases and decreases of cryptocurrency prices. For example, Yaya et al. (2019) analyze the behavior of cryptocurrency prices before and after the 2017 and 2018 crash in the cryptocurrency market. Also cointegration refers to variables moving toward a long-term equilibrium. The time-varying cointegration relation can be understood as a slowly changing equilibrium, in which variables are attracted at any point in time, but not necessarily at all points in time (Saikkonen and Choi, 2004; Bierens and Martins, 2010; Koop et al., 2011). Thus, moments of pronounced rises in the price of the cryptocurrency could be associated with periods in which there is no attraction towards equilibrium.

Furthermore, we explain why we use the price of gold as a variable for the relationship. Gold is considered a hedge asset under periods of market turmoil. At the same time, Bitcoin has similar characteristics to gold for hedging for example, which was corroborated by the positive reaction for both assets during stock price shocks according to Kapar and Olmo (2021). Because of this, we analyze the relationship between Bitcoin (and Ethereum) and gold prices.

We have only two cases without cointegrating relationship using traded data: between Bitcoin (Ethereum) and Tether prices. Using non-traded data, there is one additional case without cointegration: between Ethereum and BNB prices. Thus, non-traded data leads to an additional non-cointegration case.

2019 and beggining of 2020 presents a cointegrating relationship with more effect between Bitcoin and Ethereum prices. The use of traded data leads to a posterior mean of the Ethereum price coefficient that has a smaller 90% confidence interval than the use of non-traded data. Ethereum price responds more to adjust to deviations to the long-term relationship. Comparing results between traded and non-traded data, the use of traded data leads to a result where Bitcoin and Ethereum prices adjust to deviations in the long-term relationship over an 18 times greater number of periods. The use of non-traded data tends to underestimate the coefficient of Ethereum's price in the long-term relationship.

The long-term trend between Bitcoin and gold prices is an increase in the relationship based on the long-term coefficient estimate. Over the analyzed period, the gold price coefficient for 2021 has the highest estimate. The variable that responds - when it responds - to maintain long-term equilibrium is the Bitcoin price.

We have a statistically significant coefficient of gold price at 10% in 122 periods of 1293 for the cointegrating relationship between Ethereum and gold prices. When one variable adjusts to deviations from the long-term relationship, this variable is Ethereum price. The use of non-traded data leads to a posterior mean of the Ethereum price adjustment speed that is statistically significant at 10% over a greater number of days than the use of traded data.

The coefficient estimate of BNB price was higher in 2018 and early 2019 for the cointegrating relationship between Bitcoin and BNB prices. The estimate has stabilized near two since 2021. BNB price adjusts more to deviations from the long-term relationship over the period than Bitcoin price. But even the BNB price does not respond to deviations in the long-term relationship in much of the sample. Most of the adjustment to deviations in the long-term relationship for BNB price was during the pandemic period - between 2019 and 2020. Basically, the use of non-traded data tends to adjust to maintain the long-term relationship much less frequently. The pattern of cointegrating relationship is similar between Bitcoin and BNB prices or Ethereum and BNB prices.

The pandemic period in the year 2020 has a long-term relationship with a lower estimate of the coefficient of VIX for the relationship between Bitcoin price and VIX. During this period, there is a large increase in the VIX without a corresponding movement in the Bitcoin price. An excessive price increase in the Bitcoin price - compared to the long-term relationship with VIX - tends to be accompanied by an increase in the VIX to maintain the cointegration relationship. The VIX only responds to deviations in the long-term relationship in just 10% of the sample period, although it responds to deviations more than the Bitcoin price. In the case of the relationship between the Ethereum price and VIX, the VIX responds to deviations in the long-term relationship in about 20% of the sample period, a greater number of periods than in the relationship between Bitcoin and Ethereum prices.

This paper is organised in five sections besides this introduction. The second section provides an overview of the literature on the efficient price of cryptocurrencies. The third section covers the methodology we apply. We describe data source in the fourth section. In the fifth section, we report the results of the estimation. Finally, we present our concluding remarks.

2 Literature Review

The growth of cryptocurrencies led to studies that were concerned with explaining their behaviour, especially Bitcoin. In addition to the factors that determine the price of this asset, Kristoufek (2013), Bouoiyour et al. (2014), and Kapar and Olmo (2021) analyse its efficient price, for example.

Kapar and Olmo (2021) seek to explain the cryptocurrency price dynamics by adding other variables such as gold price, which is a hedge asset for turbulent moments in the market. Based on Corbet et al. (2019), the demand for gold increased significantly during the Bitcoin crash in January 2018. Klein et al. (2018) study the two assets together and conclude that their behaviors are exactly the opposite in times of distress. The price of gold may have similar characteristics to Bitcoin according to Kapar and Olmo (2021).

Adebola et al. (2019) study the relationship between cryptocurrency and gold prices. One of the methodologies is fractional cointegration. The authors considered the 12 cryptocurrencies with the highest market capitalizations. They analyze whether the price of each cryptocurrency cointegrates (fractionally) with the price of gold, that is, they consider cointegration only between two variables. We rely on Adebola et al. (2019) to study the cointegration of the cryptocurrency price with the gold price considering only two variables in the cointegration test and in the model estimation. They find cointegration between cryptocurrency and gold prices only for Bytecoin and Bitcoin and partially for Dash and Stellar.

Yaya et al. (2019) discuss the crash in the cryptocurrency market between 2017 and 2018. For example, Cheung et al. (2015), Cheah and Fry (2015), Corbet et al. (2018), Su et al. (2018), and Chaim and Laurini (2019) find evidence of explosive behavior in the price of Bitcoin, that is a temporary pattern.

For example, Bitcoin price reached 19,000 USD before the crash of December 2017 (Yaya et al., 2019). But, at 25 November 2018, Bitcoin price was 4,141 USD. That is, cryptocurrency prices had skyrocketed and had pronounced drops as well. This motivates the study of cointegrating relationships that vary over time. In addition to being a market that has been evolving over time. Therefore, we choose to use time-varying cointegration based on Koop et al. (2011).

We will also analyze the fear index that is the VIX. Based on trading in S&P 500 index options, VIX serves as an indicator of expected volatility in the index over the coming 30-day period. The fear index captures market sentiments by the Fed's financial stress index. Kapar and Olmo (2021) use the Federal Reserve (Fed) financial stress index to capture market sentiment and analyze whether there is a cointegration relationship with the price of Bitcoin. That is, whether market sentiment affects the price of Bitcoin. Unlike Kapar and Olmo (2021), we will use the VIX because it is a measure that comes directly from market negotiations.

Our contribution is to use the VEC model with time-varying parameters and stochastic volatility following Koop et al. (2011). We consider the possibility of time-varying cointegration, which the literature do not study. In addition, we seek to cover a cryptocurrency Ethereum that has been less studied than Bitcoin and therefore we can bring reflections on it.

According to Liu and Tsyvinski (2021), cryptocurrency returns have low exposure to equity, currency and commodity markets, which could be a sign of influence of specific factors in digital currency markets. But Ciaian et al. (2018) study the interdependence between the price of Bitcoin and the price of other cryptocurrencies. Their result is that the interdependence between other cryptocurrency and Bitcoin prices is more present in the short term than in the long run. They obtain the presence of Bitcoin price cointegration with four other cryptocurrencies out of the 16 considered. This indicates that although Bitcoin is the virtual dominant currency, there is no price interdependence between Bitcoin and most cryptocurrencies in the long run, indicating that the factors that affect cryptocurrency prices might not be the same in the long run with respect to the price of Bitcoin. That is, along these lines, we can expect that a possible cointegration relationship (if any) between Bitcoin and gold prices may not be representative of the cryptocurrency market. We will take whether this occurs in our study comparing with Ciaian et al. (2018). In this version of the article, we estimate the relationship between the price of gold and Ethereum.

3 Methodology

Basically, we will analyze whether there is cointegration between Ethereum ($Peth_t$) and gold prices ($Pgold_t$). Our procedure will be to specify the VECM model if there is presence of cointegration. Furthermore, we allow the short-term dynamics, speed of adjustment term and cointegrating vectors to be time-varying or time-invariant. Our procedure is based on Koop et al. (2011). We choose the model specification based on the information criterion to consider whether there is a cointegration and whether the parameters are time-varying. Consider vector y_t for $t = 1, ..., T$, we write TVP-VECM as

$$\Delta y_t = \Phi_t + \alpha_t \beta_t' y_{t-1} + \sum_{h=1}^{l} \Gamma_{h,t} \Delta y_{t-h} + \varepsilon_t \tag{1}$$

where $y_t = (Peth_t, Pgold_t)'$, ε_t are independent $N(0, \Omega_t)$, $n \times r$ matrices α_t and β_t are full rank, r is the number of cointegrating relationships. We follow Koop et al. (2011) that only the cointegrating space is identified, and not the cointegrating vectors. The notation for the space spanned by β is $p_t = sp(\beta_t)$. We specify β_t to be semi-orthogonal to achieve identification according to Strachan and Inder (2004).[1] So, the cointegrating space at time t has a distribution centered over the cointegrating space at time $t-1$. Also the evolution of cointegrating space p_t is gradual similar to TVP-VAR models. Also we can express prior beliefs about the marginal distribution of the cointegrating space at time t.

The parameters $\gamma_t = (\alpha_t, \Gamma_{1,t}, ..., \Gamma_{l,t}, \Phi_t)$ follow a standard state equation such as

$$\gamma_t = \rho \gamma_{t-1} + \vartheta_t \tag{2}$$

where ϑ_t is a vector of states. Equation (2) is commonly used as a state equation in TVP-VAR models. We use the same specification of multivariate stochastic volatility model for Ω_t as Primiceri (2005).

We can replace $\alpha_t \beta_t^{'}$ in (1) by $\alpha_t^* \beta_t^{*'}$, where $\alpha_t^* = \alpha_t \kappa_t^{-1}$. We restrict β_t to achieve identification and we consider β_t^* the unrestricted matrix of cointegrating vectors without identification imposed. β_t^* is related to the restricted β_t as

$$\beta_t = \beta_t^* (\kappa_t)^{-1} \tag{3}$$

where $\kappa_t = (\beta_t^{*'} \beta_t^*)^{1/2}$. Our state equation for the time variation in the cointegrating space is based on $b_t^* = vec(\beta_t^*)$ for $t = 2, ..., T$ as

$$b_t^* = \overline{\rho} b_{t-1}^* + \eta_t \tag{4}$$

where $\eta_t \sim N(0, I_{nr})$ for $t = 2, ..., T$, and $b_1^* \sim N\left(0, I_{nr} \frac{1}{1-\overline{\rho}^2}\right)$, $\overline{\rho}$ is a scalar and $|\overline{\rho}| < 1$ assures the cointegrating space today has a distribution that is centered over last period's cointegrating space. Also we normalize the error covariance matrix in the state equation (4) to the identity. We consider a single vector $Tnr \times 1$ $b^* = (b_1^{*'}, ..., b_T^{*'},)$. From the conditional distribution in Equation (4), we can infer that the joint distribution of b^* is normal with zero mean and the standard covariance matrix of a stationary AR(1) process.

We allow for multivariate stochastic volatility and all the parameters in a_t evolve according to random walks, where $A_t = (\alpha_t^*, \Gamma_{1,t}, ..., \Gamma_{l,t}, \Phi_t)$ and $a_t = vec(A_t)$. As we mention before, we also allow variants of this model that restrict the parameters of this model to be time-invariant. If $r = 0$, we have TVP-VAR model.

We have a four sets of state equations: two associated with the measurement error covariance matrix, one for the cointegrating space given in Equation (4), and one for the conditional mean coefficients. The prior for the initial condition for the cointegration space implies a uniform for p_1. The prior for $\overline{\rho}$ is uniform over the interval (0.999, 1).[2] We discard the first 1,000 iterations as burn-in, and we use 20,000 consecutive iterations with the MCMC algorithm.

4 Data

We use two types of database. The first type is with traded data and the second is with non-traded data. We use Bitcoin and Ethereum prices in dollars traded on Kraken, which is a centralized crypto exchange. We extract BNB prices traded on DAR in dollars. According to Alexander and Dakos (2020), some crypto exchanges have problems. However, we get the Tether prices in dollars from the crypto exchange Bitfinex.

We use most of the cryptocurrency prices from Kraken. We avoid cryptocurrency prices traded on Bitfinex. The only exception is the Tether price. The reason for this is that the prices traded on this cryptocurrency exchange deviate from the prices of other exchanges. The reason is that the cryptocurrency

[1] I.e., $\beta_t^{'} \beta_t = I_r$. This identification constraint does not restrict the estimable cointegration space. If we assume a uniform prior for β_t, the implication is a uniform prior on p_t according to Strachan and Inder (2004).

[2] $\overline{\rho} = 0.99$ leads to unrealistically huge changes in the cointegrating space according to Koop et al. (2011).

prices on Bitfinex are based on the price of Tether, which is a stablecoin and not in dollars. There is supposed to be a price ratio of 1 Tether to 1 dollar. This should not affect cryptocurrency prices on Bitfinex, but Alexander and Dakos (2020) observe differences in prices for this reason.

We extract non-traded cryptocurrencies prices from Coincodex which calculates a volume-weighted average of its dollar price on different crypto exchanges.

The VIX series comes from Federal Reserve Economic Data. We obtain the gold price variable in dollars per troy ounce from Bloomberg.

We use the same sample period for traded and non-traded data because we have a longer time period available for non-traded data. This is to compare the difference in results between traded and non-traded data. We use daily data. The data available for traded data is from 2018 to 2023 in general. Bitcoin price is the only series with traded data from 2013 to 2023. As we only have the gold price series since 2013, the analysis of the relationship between gold price and bitcoin price considers the sample from 2013 to 2023. All other price relationships are from 2018 to 2023. The other series start in different months of 2018 depending on availability. But the sample period ends on January 31, 2023 for all series. This means that the relationship between Gold and Bitcoin prices contains 2421 observations. The remaining relationships are analyzed between 1296 and 1819 observations depending on the variables.

5 Results

We analyze in this section whether there is a long-term relationship between variable prices, whether such a relationship is time-varying, if any. One point is that we only consider the relationship between two variables because the code takes considerable computational time to estimate one model with time-varying parameters.[3] The first step is for us to choose the number of lags of the VAR model in level by the information criterion. We choose the number of lags using the BIC criterion according to Koop et al. (2011). Most models of relationships between variables have two lags according to the BIC criterion for traded and non-traded data. There are three exceptions with non-traded data. The VAR model in level for the relationships between Bitcoin and Ethereum prices, between Ethereum and Tether prices, and between Bitcoin and Tether prices contains three lags. There is higher persistence in these non-traded series with volume-weighted averages than with traded prices.

Next, we analyze the specifications and whether there is a long-term relationship between the variables and whether the parameters vary over time based on the information criterion. In Appendix, Table 1 presents the information criterion for the different specifications of the VEC model between traded variables with $r = 0, 1$ and with TIP or TVP using traded data. We highlight the lowest BIC value in blue and bold for each model between the variables. Based on this table, we have two cases without cointegration using traded data with the lowest BIC value: (i) between Bitcoin and Tether prices, and (ii) between Ethereum and Tether prices. The common element is Tether. There is no stable long-term behavior (or even subperiods) of this cryptocurrency price with that of Bitcoin or Ethereum. The presence of cointegration, time-varying parameters and the deterministic specification of restricted or unrestricted constant represents the majority of cases (seven cases) of the relationship between variables with traded data.

In the case of non-traded series, Table 2 of the Appendix presents the information criteria for the different specifications of the VEC model between variables with $r = 0, 1$ and with TIP or TVP using non-traded data. Based on this table, we have three cases without cointegration using non-traded data with the lowest BIC value: (i) between Ethereum and BNB prices, (ii) between Ethereum and Tether

[3] It takes at least 12 days to estimate one model with cointegrating relationship and time-varying parameters between two variables on a Linux server. This is after we found the specification which takes a long computational time. We used parallel computing with the Linux operating system, which speeds up the process.

prices, and (iii) between Bitcoin and Tether prices. The use of non-traded series leads to an additional case of no cointegration, which is between Ethereum and BNB prices.

We analyze the α and β parameter estimates to understand the the time variability of the cointegrating relationship for these cases with cointegration. Below we have subsections for the analysis of each relationship between two variables that have the presence of cointegration.

5.1 Bitcoin and Ethereum

Figure 1 presents the posterior mean of the cointegrating vector associated with the Ethereum price and the 90% confidence interval using traded (on the left) or non-traded (on the right) data. Using traded data (on the left of this figure), the posterior mean of the cointegrating vector is statistically significant at 10% for all period. 2019 and beggining of 2020 presents a cointegrating relationship with more effect between Bitcoin and Ethereum prices.

Using non-traded data (on the right of this figure), the posterior mean of the cointegrating vector is statistically significant at 10% for 1142 periods of 1255. The use of non-traded data leads to periods with the posterior mean of the cointegration relationship statistically not significant at 10% compared to using traded data. The posterior mean of the Ethereum price coefficient in the cointegrating relationship reaches peaks with values above two and remains at levels above 1.8 using traded data for the years 2019 and early 2020. These levels are higher than the value of 1.8 that the coefficient tends to achieve using non-traded data in this period. The use of traded data leads to a posterior mean of the Ethereum price coefficient that has a smaller 90% confidence interval than the use of non-traded data.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Figure 2 reports time-varying posterior mean of Bitcoin price (Ethereum price) response to deviations from the long-term relationship on the left (on the right) using traded data. The coefficient of response of Bitcoin price to deviations is statistically significant at 10% only in 163 periods of 1816. These moments are most on April 2018, November 2018, March 2020, April 2020, and May 2021. The posterior mean of the coefficient of response of Ethereum price is statistically significant at 10% in only 181 periods. These moments are most on March 2018, November 2018, December 2018, April 2020, and May 2021. Most of the cases $\hat{\alpha}_2 < \hat{\alpha}_1$ and both are negative, this means that Ethereum price responds more to adjust to deviations to the long-term relationship.

Using non-traded data, Figure 3 shows time-varying posterior mean of Bitcoin price (Ethereum price) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 4 periods of 1255. The posterior mean of the Ethereum response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 10 periods of 1255. March 2020 (negative estimate for α) and December 2020 - positive estimate for α.

Comparing results between traded and non-traded data, the use of traded data leads to a result where Bitcoin and Ethereum prices adjust to deviations in the long-term relationship over an 18 times greater number of periods. Basically, the use of non-traded data tends to underestimate the coefficient of Ethereum's price in the long-term relationship and to adjust to maintain the long-term relationship much less frequently. In the next subsection we analyze the cointegration relationship between Bitcoin and Gold prices.

5.2 Bitcoin and Gold

Figure 4 presents the posterior mean of the cointegrating vector associated with the Ethereum price and the 90% confidence interval using traded (on the left) or non-traded (on the right) data. The posterior mean of the cointegrating vector is statistically significant at 10% for all period using traded data (on the

Figure 1: Posterior mean of the coefficient $\hat{\beta}_t$ associated with the price of Ethereum and the 90% confidence interval using traded (on the left) or non-traded (on the right) data

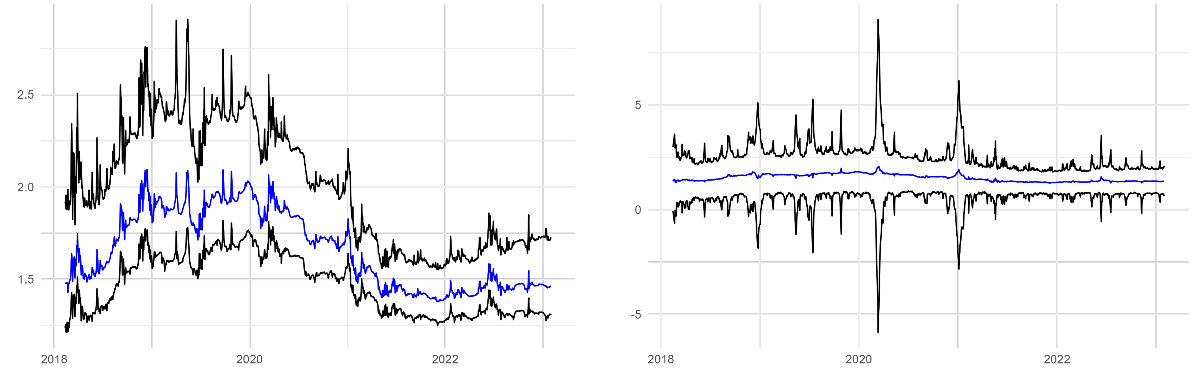
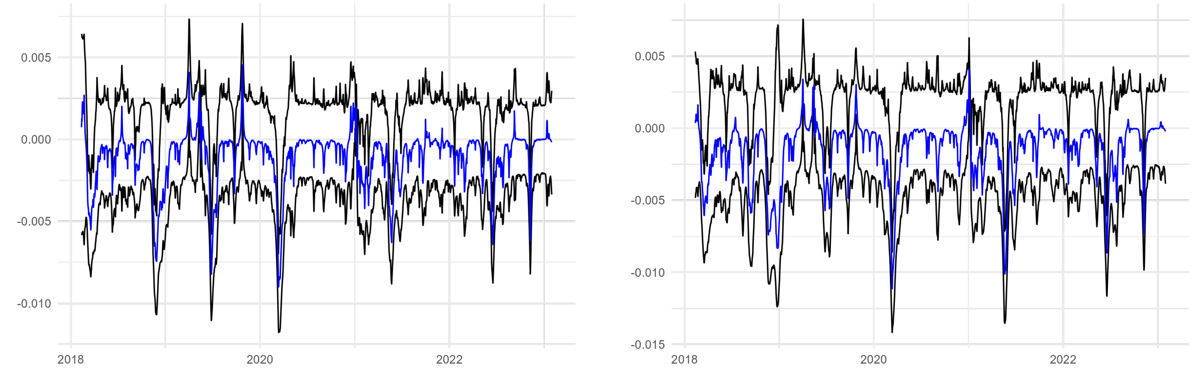


Figure 2: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and Ethereum Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data

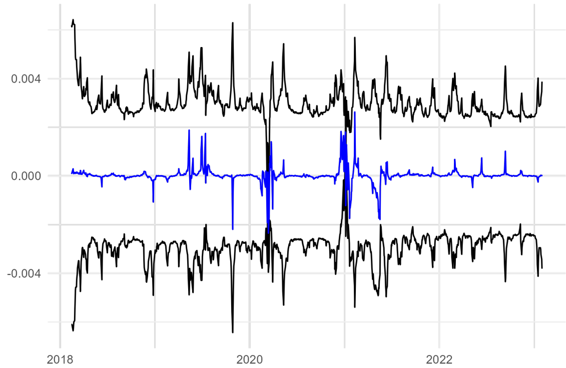


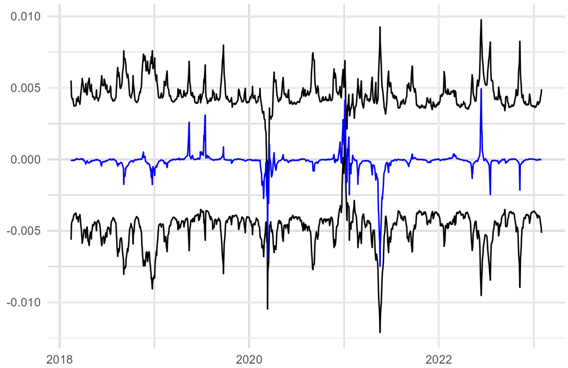
left in this figure). The long-term trend between Bitcoin and gold prices is an increase in the relationship based on the long-term coefficient estimate. The coefficient of gold price in the long term has been at a high level since 2018. Over the analyzed period, the gold price coefficient for 2021 has the highest estimate. The posterior mean of the cointegrating vector is similar using traded and non-traded data. The difference is that there are some sharp spikes in the posterior mean when using non-traded data (on the right in this figure). The highlight is the spike increase in the posterior mean of the cointegrating vector in March 2020 using non-traded data which is not of the same size as using traded data.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Using traded data, Figure 5 reports time-varying posterior mean of Bitcoin price (Gold price) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for 236 periods of 2421. This estimate is positive in almost all periods that it is statistically significant at 10%. The posterior mean of this coefficient is statistically significant at 10% for a longer period at the end of years 2013 and 2017, beginning of 2018, and between June and July 2019. The posterior mean of the Gold price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 7 periods of 2421. The variable that responds - when it responds - to maintain long-term equilibrium is the Bitcoin price.

Using non-traded data, Figure 6 reports time-varying posterior mean of Bitcoin price (Gold price) response to deviations from the long-term relationship on the left (on the right). The posterior mean

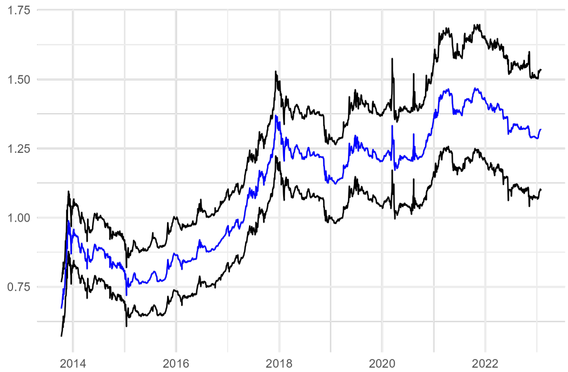
Figure 3: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and Ethereum Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data

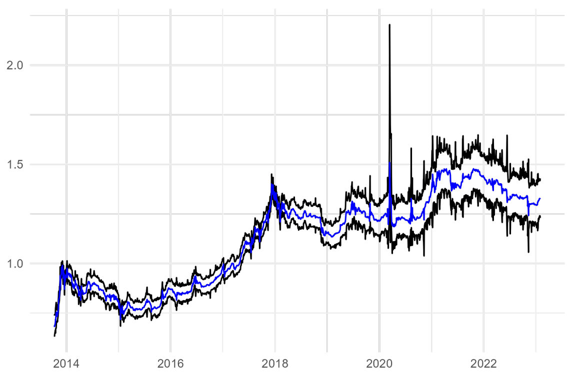




of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for 217 periods of 2421. The posterior mean of the speed of adjustment of the Bitcoin price to deviations in the long-term relationship is positive in almost all periods where it is statistically significant at 10%. The posterior mean of the Gold price response coefficient to deviations from the long-term relationship is statistically significant at 10% for five periods of 2421. The difference is that the posterior mean of Bitcoin's adjustment speed is statistically significant at 10% between June and July 2019 using traded data. But this period is not marked as an adjustment period for the Bitcoin price with non-traded data.

Figure 4: Posterior mean of the coefficient $\hat{\beta}_t$ associated with the price of Gold and the 90% confidence interval using traded (on the left) or non-traded (on the right) data





5.2.1 Bitcoin and BNB

Figure 7 presents the posterior mean of the cointegrating vector associated with the BNB price and the 90% confidence interval using traded (on the left) or non-traded (on the right) data. The posterior mean of the cointegrating vector is statistically significant at 10% for all period using traded data (on the left of this figure). The coefficient estimate was higher in 2018 and early 2019. The estimate has stabilized near two since 2021.

In general, the trend of the posterior mean of the cointegrating vector coefficient is the same using traded (on the left) or non-traded data (on the right) according to Figure 7. There are four differences that deserve comment. The 90% confidence interval is larger using non-traded data. At the beginning of the sample which is at the end of 2018 and beginning of 2019, the coefficient estimate reaches a higher

Figure 5: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and Gold Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data

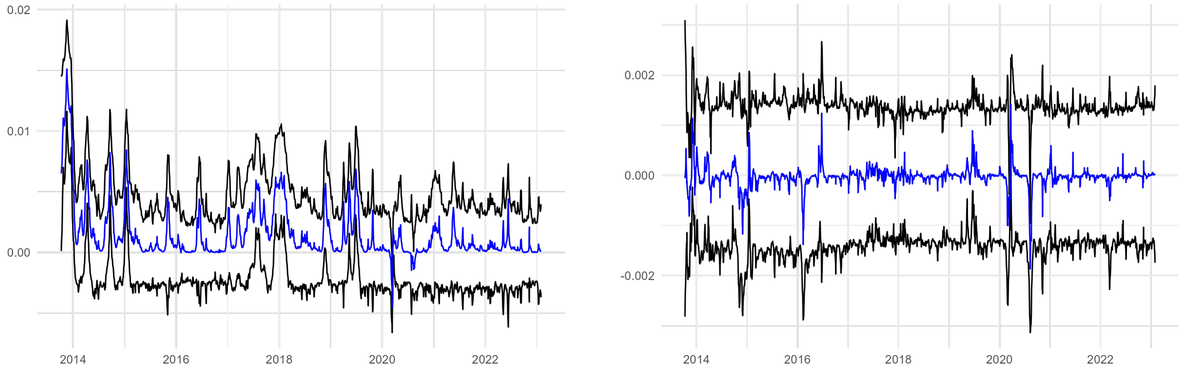
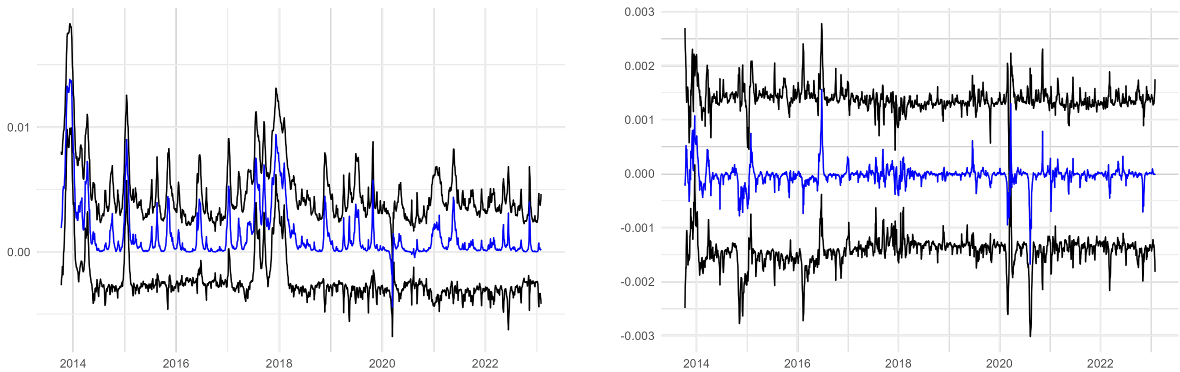


Figure 6: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and Gold Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data



value using traded data compared to non-traded data. Another point is that there is considerable variation in the estimate using non-traded data that does not appear in the estimate using traded data at the end of 2019 and beginning of 2020. Furthermore, the behavior at the beginning of 2021 is also different. The coefficient estimate for traded data shows a fall, while the coefficient estimate for non-traded data shows an increase to a considerable reduction in the sequence. Thus, this drop in the coefficient is delayed using non-traded data if we use the estimate using traded data as a proxy. Again, the 90% posterior mean confidence interval is smaller using traded data instead of non-traded data.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Using traded data, Figure 8 reports time-varying posterior mean of Bitcoin price (BNB price) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 58 periods of 1499. This estimate is negative when it is statistically significant at 10%. The posterior mean of the BNB price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 182 periods of 1499. BNB price adjusted for deviations in the long-term relationship in late 2019, during some periods in the year 2020, in the first half of 2021, and for a brief period in late 2022. Most of the adjustment to deviations in the long-term relationship for BNB price was during the pandemic period - between 2019 and 2020.

When the estimated Bitcoin price adjustment speed is statistically significant at 10%, the estimated response of BNB price to deviations from the long-term relationship will be statistically significant at the same level. In general, the BNB price adjusts more to deviations from the long-term relationship over the period based on statistically significant coefficients.

Using non-traded data, Figure 9 reports time-varying posterior mean of Bitcoin price (BNB price) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 24 periods of 1501. The posterior mean of the BNB price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 54 periods of 1501.

The use of traded data leads to a result where Bitcoin and BNB prices adjust to deviations in the long-term relationship over a three times greater number of periods using traded data than non-traded data. Still, the posterior mean of BNB price adjustment speed is negative when statistically significant at 10% using traded data. While there are negative (majority) and some positive values that are statistically significant at 10% for this adjustment speed using non-traded data. Basically, the use of non-traded data tends to adjust to maintain the long-term relationship much less frequently.

Figure 7: Posterior mean of the coefficient $\hat{\beta}_t$ associated with the BNB price and the 90% confidence interval

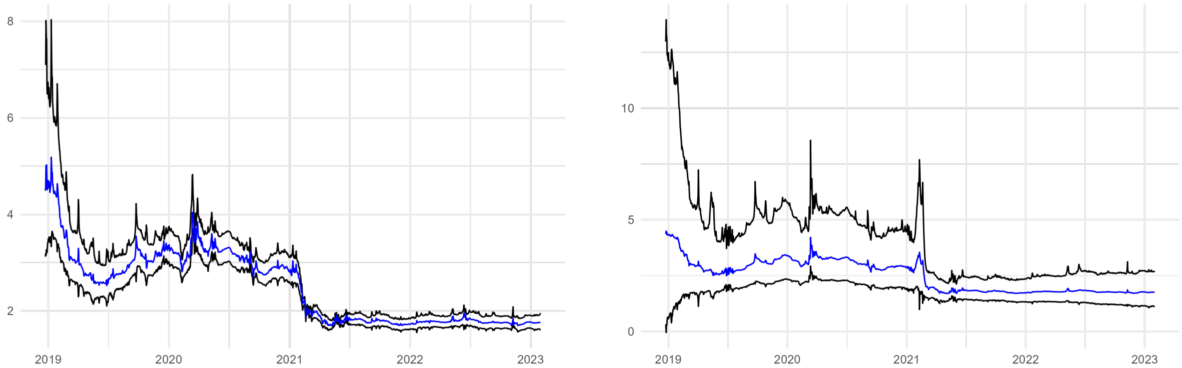


Figure 8: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and BNB Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data

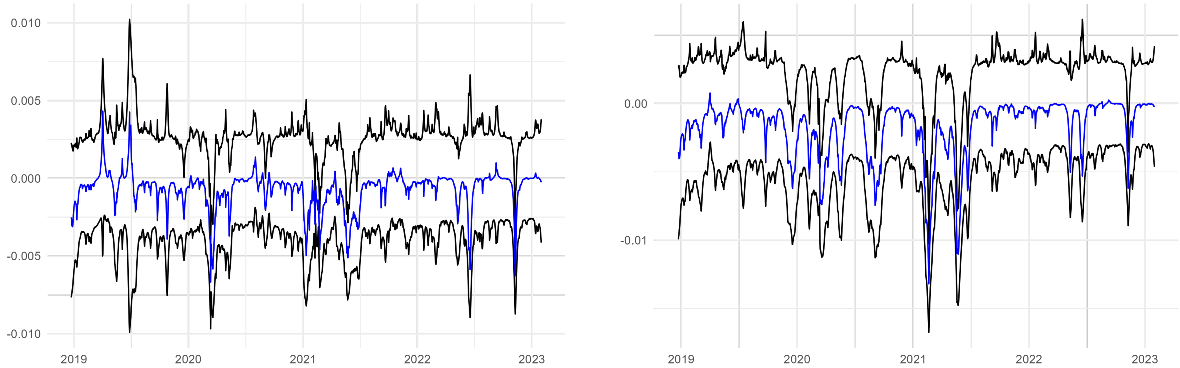
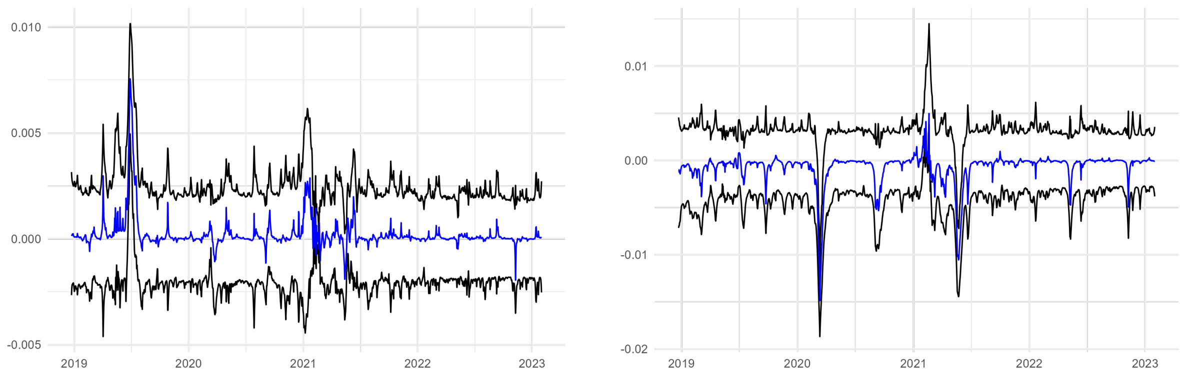


Figure 9: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and BNB Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data



5.2.2 Bitcoin and VIX

First we analyze the long-term coefficient for the relationship between variables and then the adjustment speeds of deviations in the long-term relationships. Figure 10 reports the posterior mean of the cointegrating vector associated with VIX and the 90% confidence interval using traded (on the left) or non-traded (on the right) data. The posterior mean of the cointegrating vector is statistically significant at 10% for all period using traded data (on the left of this figure). The pandemic period in the year 2020 has a long-term relationship with a lower estimate of the coefficient of VIX. During this period, there is a large increase in the VIX without a corresponding movement in the Bitcoin price.[4] This explains the change in the estimate of the coefficient of the cointegrating relationship. In general, the trend of the posterior mean of the cointegrating vector coefficient is the same using traded or non-traded data. One difference is that the decrease in the posterior mean of the coefficient of the cointegration relationship at the beginning of 2020 is higher using traded data than non-traded data.

Using traded data, Figure 11 reports time-varying posterior mean of Bitcoin price (VIX) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 29 periods of 1256. The posterior mean of the VIX response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 132 periods of 1256. The posterior mean of the VIX adjustment speed is positive when it is statistically significant at 10%. During this period, an excessive price increase in the Bitcoin price - compared to the long-term relationship with VIX - tends to be accompanied by an increase in the VIX to maintain the cointegration relationship. The consecutive periods in which the VIX adjustment speed was statistically significant at 10% were July 2019, between February and March 2020, between November and December 2021, and between April and May 2022.

When the estimated Bitcoin price adjustment speed is statistically significant at 10%, the estimated response of VIX to deviations from the long-term relationship tends to be statistically significant at the same level. In general, the VIX responds more periods to deviations in the long-term relationship with the Bitcoin price than to the Bitcoin price itself.

Using non-traded data, Figure 12 reports time-varying posterior mean of Bitcoin price (VIX) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Bitcoin price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 3 periods of 1256. The posterior mean of the BNB price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 110 periods of 1256. The use of non-traded data leads to a lower number of posterior mean of adjustment speed which is statistically significant compared to the use of traded data, but the difference is not that large compared to other cases. The estimate using non-traded data takes the VIX reaction to deviations in the long-term relationship in three different periods (such as 2018 and August 2019) in relation to the use of traded data. But the use of traded data indicates a reaction of the VIX to deviations in the long-term relationship in the 2020 pandemic period that the use of non-traded data does not.

5.2.3 Ethereum and Gold

Figure 13 presents the posterior mean of the cointegrating vector using traded (on the left) or non-trade (on the right) data. The posterior mean of the cointegrating vector is statistically significant at 10% between June 2018 and January 2019 using traded data (on the left in this figure). Basically, we have a statistically significant coefficient at 10% in 122 periods of 1293. Using non-traded data (on the right in this figure), the posterior mean of the cointegrating vector is statistically significant at 10% for all period.

[4] Comparing between January 9 and March 16, 2020, the VIX increased by 559%, while the Bitcoin price dropped by 32%.

Figure 10: Posterior mean of the coefficient $\hat{\beta}_t$ associated with VIX and the 90% confidence interval using traded (on the left) or non-traded (on the right) data

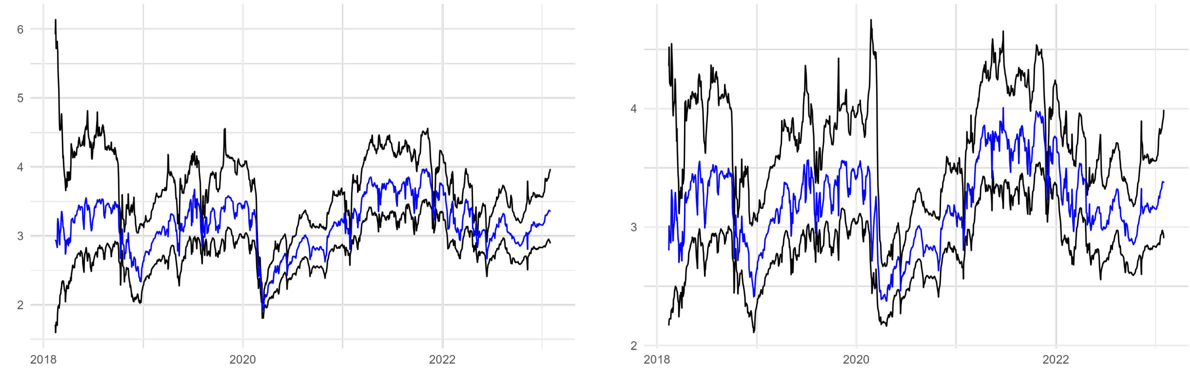
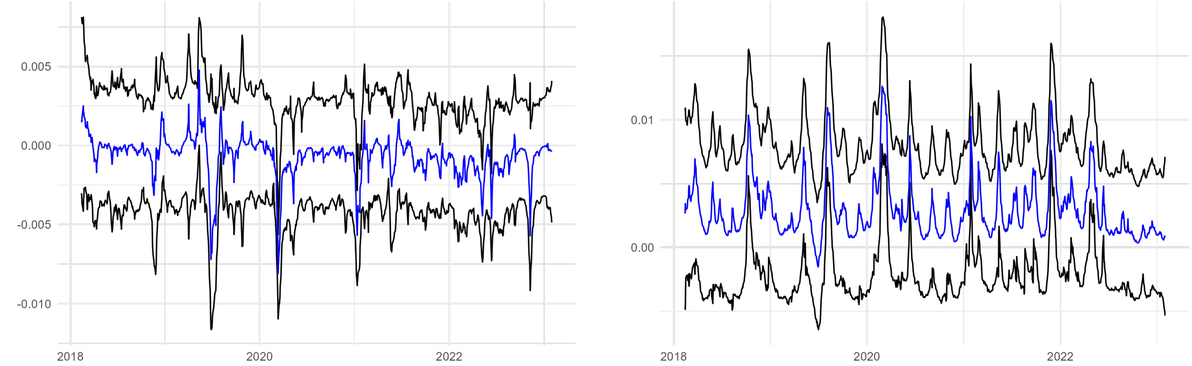


Figure 11: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and VIX (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data



This indicates a posterior mean that is statistically significant at 10% more often with non-trade data than traded data.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Using traded data, Figure 14 reports time-varying posterior mean of Ethereum price (Gold price) response to deviations from the long-term relationship on the left (on the right). The coefficient of response of Ethereum price to deviations is statistically significant at 10% only in 20 periods that are most in May 2021 - we also have six days in March 2020. When this coefficient is statistically significant at 10%, the posterior mean reaches -0.01265, which gives half-life of 54 days. The posterior mean of the coefficient of response of Gold price is statistically significant at 10% in only five periods. When one variable adjusts to deviations from the long-term relationship, this variable is Ethereum price.

Using non-traded data, Figure 15 presents time-varying posterior mean of Ethereum price (Gold price) response to deviations from the long-term relationship on the left (on the right). The coefficient of response of Ethereum price to deviations is statistically significant at 10% only in 37 periods that are in March 2020, May 2021, and June 2022. We have June 2022 with non-traded data that we do not have with traded data. The posterior mean of the coefficient of response of Gold price is statistically significant at 10% in only four periods. The use of non-traded data leads to a posterior mean of the Ethereum price adjustment speed that is statistically significant at 10% over a greater number of days than the use of traded data.

Figure 12: Posterior mean of the coefficient $\hat{\alpha}_t$ of Bitcoin Price (on the left) and VIX (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data

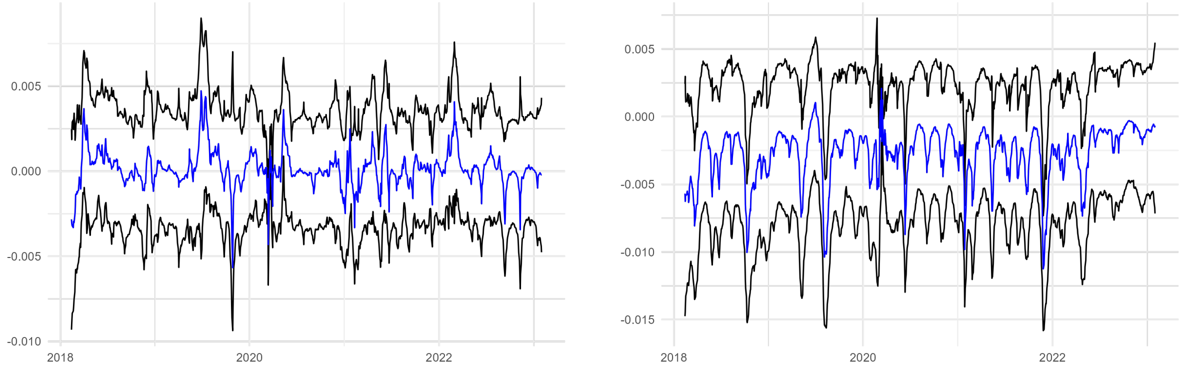
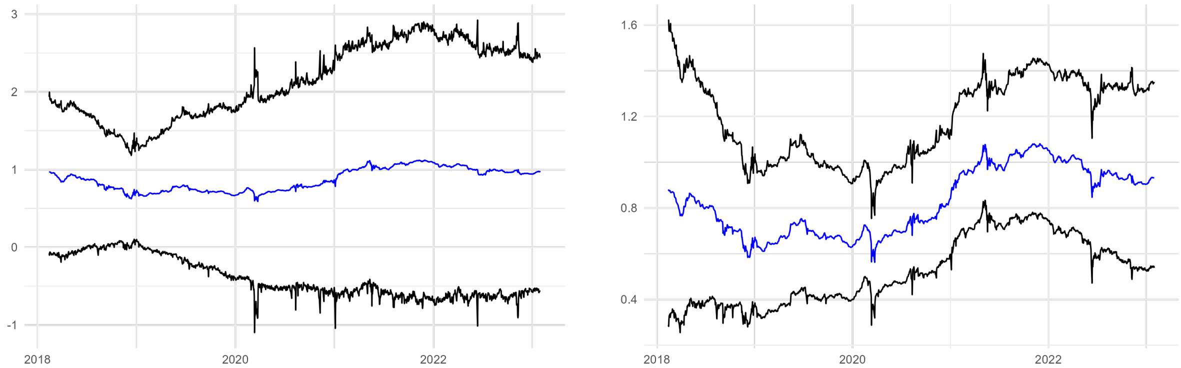


Figure 13: Posterior mean of the coefficient $\hat{\beta}_t$ associated with the Gold price and the 90% confidence interval using traded (on the left) or non-traded (on the right) data



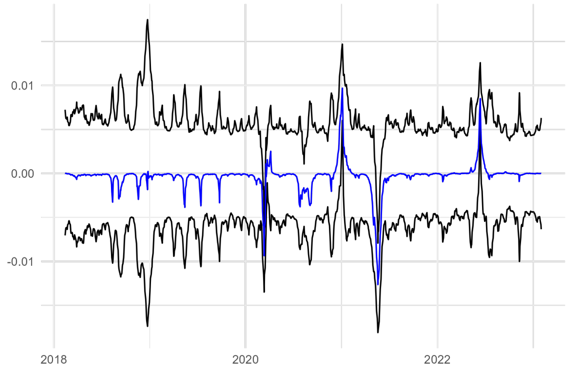
5.2.4 Ethereum and BNB

We present cointegrating vector in Figure 16 using traded data. The posterior mean of the cointegrating vector is statistically significant at 10% for all period. This pattern is similar to the BNB price coefficient for the long-term relationship between Bitcoin and BNB prices in Figure 7. The coefficient estimate was higher in 2018 and early 2019. The estimate has stabilized near two since 2021.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Figure 17 reports time-varying posterior mean of Ethereum price (BNB price) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Ethereum price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 48 periods of 1499. The posterior mean of the BNB price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 185 periods of 1499. The posterior mean of the BNB price adjustment speed is negative when it is statistically significant at 10%. The periods in which the BNB price adjustment speed was statistically significant at 10% were December 2019, between March and May 2020, between August and September 2020, between February and March 2021, and between May and June 2021.

When the estimated Ethereum price adjustment speed is statistically significant at 10%, the estimated response of BNB to deviations from the long-term relationship tends to be statistically significant at the same level. In general, the BNB price responds more periods to deviations in the long-term relationship with the Ethereum price than to the Ethereum price itself. There is no cointegration using non-traded

Figure 14: Posterior mean of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and Gold Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data



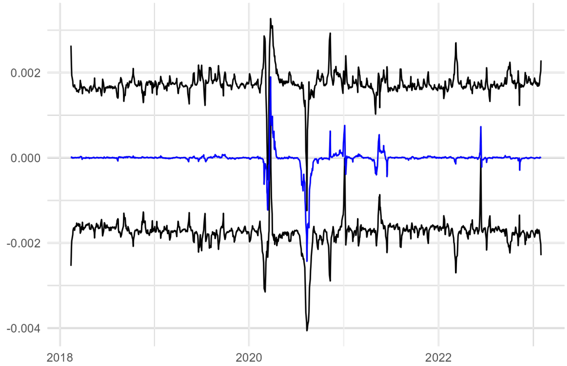
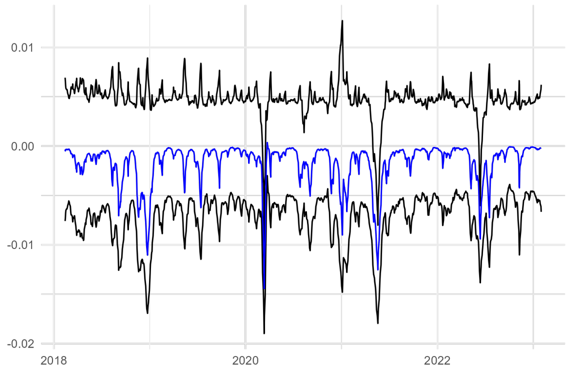
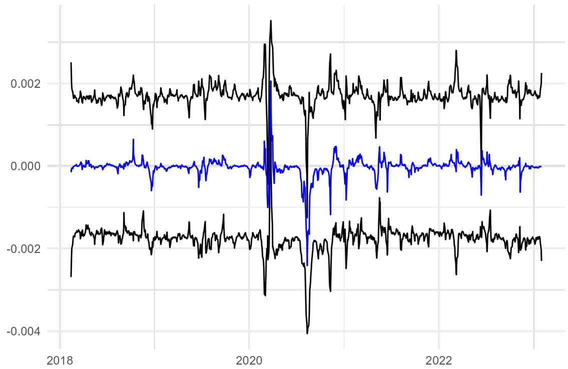


Figure 15: Posterior mean of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and Gold Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data





data based on BIC according to the Table 2 of Appendix. The use of non-traded data leads to evidence of the lack of cointegration between the variables, unlike the results with traded data.

Figure 16: Posterior mean of the coefficient $\hat{\beta}_t$ associated with the price of BNB and the 90% confidence interval using traded data

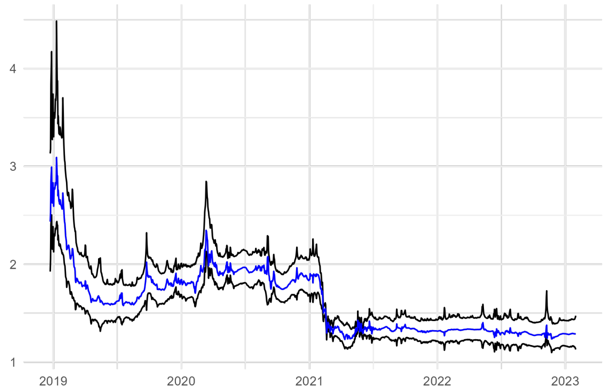
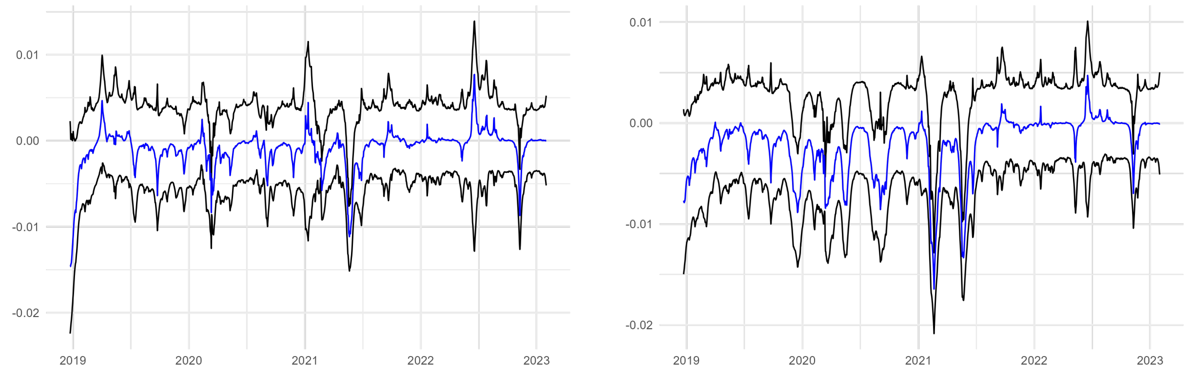


Figure 17: Posterior mean of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and BNB Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data



5.2.5 Ethereum and VIX

Figure 18 shows the posterior mean of the cointegrating vector using traded (on the right) or non-traded (on the left) data. Using traded data (on the left), the posterior mean of the cointegrating vector is statistically significant at 10% for all period. The posterior mean of the VIX coefficient of the long-term relationship has fallen in 2020 due to the large increase in the VIX. However, the posterior mean of the VIX coefficient returns to a higher level between 2021 and early 2022. As opposed to what we find with the VIX coefficient estimate in the relationship with Bitcoin price, which goes back to the same level in 2022 as it did before 2020. In general, the trend of the posterior mean of the cointegrating vector coefficient is the same using traded or non-traded data.

Next, we analyze the posterior mean of the speed of adjustment of variables to deviations in the long-term relationship. Using traded data, Figure 19 reports time-varying posterior mean of Ethereum price (VIX) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Ethereum price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 56 periods of 1256. The posterior mean of the VIX response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 219 periods of 1256. The periods in which the VIX adjustment speed was statistically significant at 10% were between October and November 2018, between July and August 2019, between January and March 2020, between January and March 2021, and between April and June 2022.

In half of the cases where the estimated Ethereum price adjustment speed is statistically significant at 10%, the estimated response of VIX to deviations from the long-term relationship is statistically significant at the same level. In general, the VIX responds more periods to deviations in the long-term relationship with the Ethereum price than to the Ethereum price itself. In the following subsection, we analyze the relationships between variables using non-traded data to understand the changes compared using traded data.

Using non-traded data, Figure 12 reports time-varying posterior mean of Ethereum price (VIX) response to deviations from the long-term relationship on the left (on the right). The posterior mean of the Ethereum price response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 63 periods of 1256. The posterior mean of the VIX response coefficient to deviations from the long-term relationship is statistically significant at 10% for only 216 periods of 1256. The number of periods in which the posterior mean of the adjustment speed of Ethereum price and the VIX is similar whether using traded or non-traded data. VIX has an adjustment to deviations in the long-term relationship in November and December 2021 considering non-traded data that does not have traded data.

Figure 18: Posterior mean of the coefficient $\hat{\beta}_t$ associated with VIX and the 90% confidence interval using traded (on the left) or non-traded data (on the right)

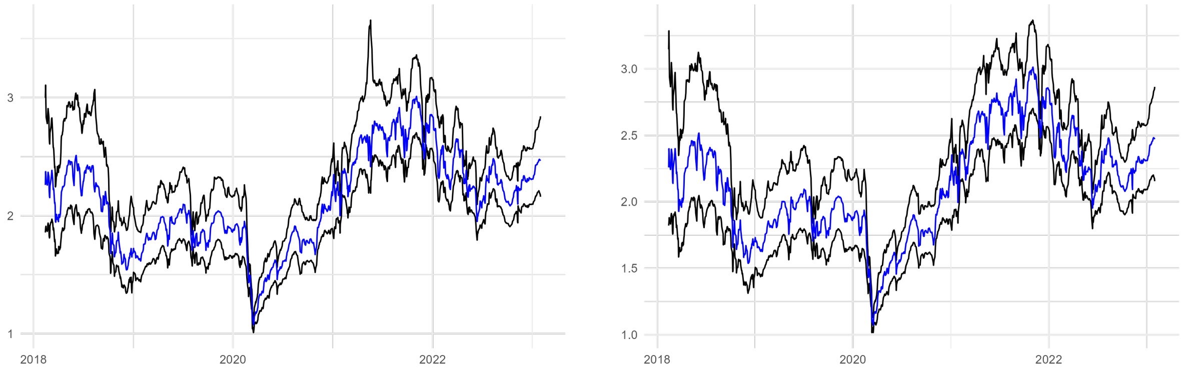


Figure 19: Posterior mean of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and VIX (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using traded data

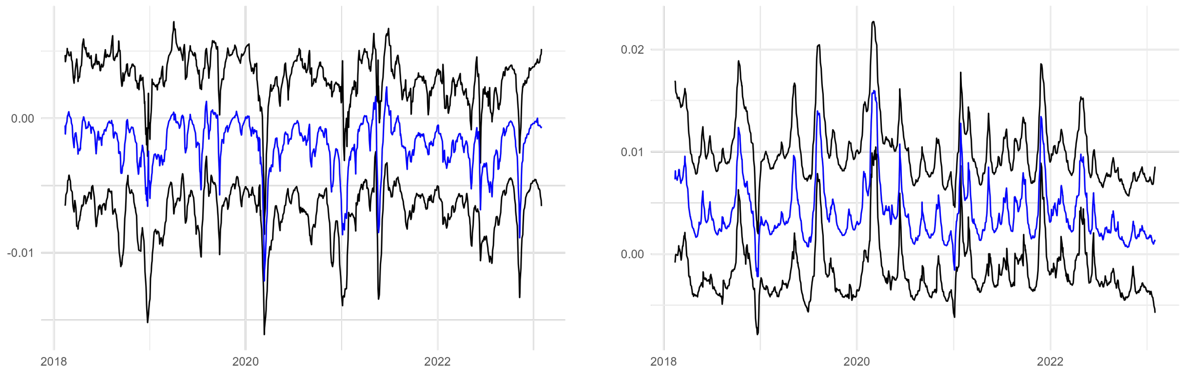
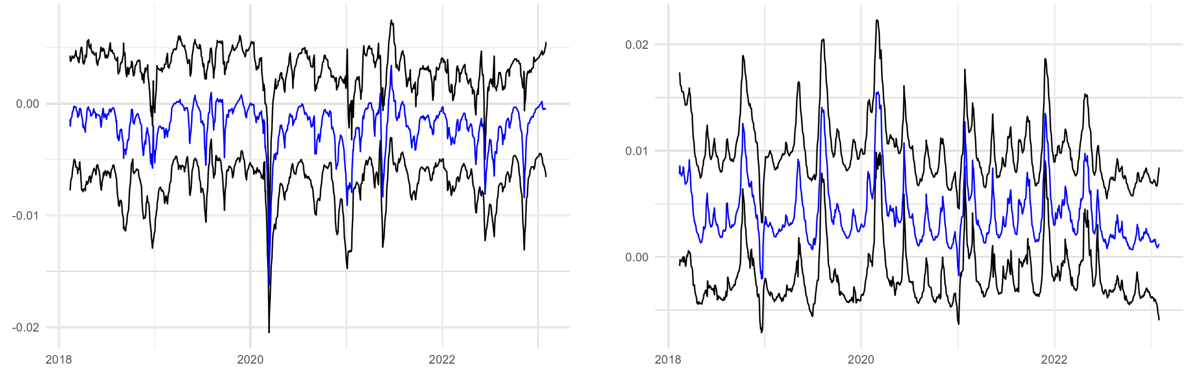


Figure 20: Posterior mean of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and VIX (on the right) response to deviations from the long-term relationship, and the 90% confidence interval using non-traded data



6 Conclusion

We analyze whether time-varying cointegration exists for the efficient price of Bitcoin or Ethereum prices with daily data. We estimate a VEC model with time-varying cointegration (for the long-term relation vector and for the loadings vector) with stochastic volatility based on Koop et al. (2011). We use traded and non-traded data to compare results based on the concerns of Alexander and Dakos (2020).

We find two cases without cointegrating relationship using traded data: between Bitcoin (Ethereum) and Tether prices. Using non-traded data leads to three cases without cointegration. Additionally, we do not find any cointegration between Ethereum and BNB prices using this data. Tether as a stablecoin does not have a cointegrating relationship with the cryptocurrencies Ethereum and Bitcoin prices, not even in sub-periods.

Comparing results between traded and non-traded data, the use of traded data leads to a result where Bitcoin and Ethereum prices adjust to deviations in the long-term relationship over an 18 times greater number of periods. The use of non-traded data tends to underestimate the coefficient of Ethereum's price in the long-term relationship.

The gold price coefficient of the cointegrating relationship with Bitcoin price has the highest estimate for 2021. The variable that responds - when it responds - to maintain long-term equilibrium is the Bitcoin price.

We have a statistically significant coefficient of gold price at 10% in 122 periods of 1293 for the cointegrating relationship between Ethereum and gold prices. When one variable adjusts to deviations from the long-term relationship, this variable is Ethereum price.

The coefficient estimate of BNB price for the cointegrating relationship with Bitcoin price stabilizes since 2021. BNB price adjusts more to deviations from the long-term relationship over the period than Bitcoin price. But even the BNB price does not respond to deviations in the long-term relationship in much of the sample. Most of the adjustment to deviations in the long-term relationship for BNB price was during the pandemic period. The pattern of cointegrating relationship is similar between Bitcoin and BNB prices or Ethereum and BNB prices.

The pandemic period in the year 2020 has a long-term relationship with a lower estimate of the coefficient of VIX for the relationship between Bitcoin price and VIX because of the large increase in VIX. The VIX only responds to deviations in the long-term relationship in just 10% of the sample period, although it responds to deviations more than the Bitcoin price. In the case of the relationship between the Ethereum price and VIX, the VIX responds to deviations in the long-term relationship in about 20% of the sample period, a greater number of periods than in the relationship between Bitcoin and Ethereum prices.

When there is cointegration, the estimated coefficient of adjustment to deviations in the long-run relationship are small in modulus. This indicates a low reversion to the mean by the variables.

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Appendix

Table 1: Choice of model specification and whether there is a cointegration relationship between the variables based on the BIC using traded data

			Traded data								
Specification	Parameters	r	Bitcoin and Ethereum	Bitcoin and Gold	Bitcoin and Tether	Bitcoin and BNB	Bitcoin and VIX	Ethereum and Gold	Ethereum and Tether	Ethereum and BNB	Ethereum and VIX
No constant	TIP	0	-13815.0	-24512.3	-17712.0	-11332.8	-7853.8	-12358.9	**-16907.7**	-10527.4	-7143.8
No constant	TIP	1	-13755.9	-24492.8	-17331.4	-11295.4	-7883.3	-12330.4	-16748.4	-10495.6	-7131.2
Unrestricted constant	TIP	0	-10632.4	-21717.0	-17716.1	-11337.1	-7875.8	-10408.9	-16902.8	-10523.1	-5329.2
Unrestricted constant	TIP	1	-13674.9	-24457.0	-16978.0	-11260.6	-7821.5	-12263.5	-16802.5	-10465.9	-7101.0
Restricted constant	TIP	0	-13815.0	-24538.4	-17707.5	-11316.3	-7850.2	-12376.2	-16894.1	-10518.2	-7140.0
Restricted constant	TIP	1	-13755.9	-24172.2	-17527.8	-11174.8	-7876.1	-12262.3	-16243.3	-10470.2	-7123.8
Unrestricted trend	TIP	0	-13781.8	-5897.1	-17717.3	-11316.2	-7847.7	-12347.7	-16896.6	-10507.7	-7136.1
Unrestricted trend	TIP	1	-13760.0	-24463.6	-17405.3	-11291.2	-7849.3	-12305.2	-16735.3	-10484.4	-7114.9
Restricted trend	TIP	0	-13814.0	-24518.7	**-17728.5**	-11335.4	-7861.0	-12362.1	-16897.3	-10524.2	-7114.6
Restricted trend	TIP	1	-13776.5	-24410.3	-17455.6	-11292.7	-7845.5	-12268.2	-16606.7	-10465.9	-7092.5
No constant	TVP	0	-13821.5	-24540.6	-17702.4	-11317.0	-7860.3	-12330.9	-16873.3	-10535.5	-7134.4
No constant	TVP	1	-15023.7	-25073.5	-17649.3	-11837.8	-8184.8	-12701.8	-16844.9	-10975.0	-7440.4
Unrestricted constant	TVP	0	-14086.0	-24244.7	-16828.7	-11544.1	-8017.5	-12227.0	-16062.3	-10789.2	-7335.1
Unrestricted constant	TVP	1	-14893.5	-24680.4	-16870.8	**-11839.3**	**-8227.2**	-12479.2	-16300.5	**-11001.6**	**-7546.7**
Restricted constant	TVP	0	-13821.5	-24525.6	-17694.7	-11331.4	-7858.6	-12355.2	-16865.7	-10513.2	-7136.7
Restricted constant	TVP	1	**-15023.7**	**-25082.6**	-17635.9	-11831.5	-8194.6	**-12712.0**	-16827.1	-10978.8	-7449.0
Unrestricted trend	TVP	0	-9133.6	-4853.2	-7946.3	-3796.2	-6200.2	-5541.7	-7911.3	-3378.7	-5192.6
Unrestricted trend	TVP	1	-12343.3	-21744.9	-12356.7	-9860.3	-7046.7	-11337.0	-11778.9	-9102.7	-6374.7
Restricted trend	TVP	0	-13806.1	-24258.9	-17681.3	-11322.0	-7848.2	-12208.4	-16875.4	-10550.3	-7141.0
Restricted trend	TVP	1	-14342.1	-24147.3	-17579.5	-11415.1	-8043.9	-12366.1	-16801.4	-10646.4	-7390.2

Note: Specification refers to the specification of the VECM model. Parameters can be time-invariant parameters (TIP) or time-varying parameters (TVP). r is the number of cointegration relationships. We highlight the lowest BIC value in blue and bold for each model between the variables.

Table 2: Choice of model specification and whether there is a cointegration relationship between the variables based on the BIC using non-traded data

			Non-Traded data								
Specification	Parameters	r	Bitcoin and Ethereum	Bitcoin and Gold	Bitcoin and Tether	Bitcoin and BNB	Bitcoin and VIX	Ethereum and Gold	Ethereum and Tether	Ethereum and BNB	Ethereum and VIX
No constant	TIP	0	-9464.288	-24648.5	-17490.9	-12349.6	-7857.7	-12317.5	-16654.3	-11993.1	-7087.0
No constant	TIP	1	-9424.162	-24617.4	-17474.1	-12323.8	-7872.1	-12279.9	-16630.6	-11945.0	-7090.1
Unrestricted constant	TIP	0	-9452.446	-24650.4	-17482	-12367.5	-7857.7	-11307.2	**-16665.6**	-11993.2	-7103.7
Unrestricted constant	TIP	1	-9294.026	-24580.2	-17422.8	-12301.0	-7808.7	-12226.2	-16619.8	-11927.4	-7071.8
Restricted constant	TIP	0	-9457.699	-24669.2	-17484.8	-12357.5	-7848.7	-12318.0	-16661.3	-11990.0	-7102.3
Restricted constant	TIP	1	-9445.127	-24102.3	-17446.9	-12202.3	-7848.6	-12266.6	-16641.4	-11913.2	-7104.7
Unrestricted trend	TIP	0	-9481.481	-24666.5	-17461.1	-12358.1	-7853.3	-12311.8	-16651.7	-11991.7	-7097.0
Unrestricted trend	TIP	1	-9397.889	-24632.9	-17456.9	-12316.2	-7857.9	-12270.7	-16639.0	-11943.2	-7093.0
Restricted trend	TIP	0	-9459.209	-24649.6	**-17491.2**	-12345.1	-7848.3	-12314.0	-16653.8	-12000.2	-7093.5
Restricted trend	TIP	1	-9402.531	-24572.3	-17456.4	-12288.7	-7843.3	-12272.7	-16640.3	-11892.8	-7076.9
No constant	TVP	0	-9531.673	-24681.2	-17437.6	-12650.8	-7913.2	-12300.0	-16600.8	-12113.2	-7090.3
No constant	TVP	1	-9866.066	**-25168.4**	-17435.1	-12868.4	-8174.9	-12616.1	-16584.1	-12240.3	-7386.8
Unrestricted constant	TVP	0	-9710.321	-24378.5	-16662.7	-12835.7	-8070.0	-12156.7	-15909.2	**-12311.6**	-7266.9
Unrestricted constant	TVP	1	**-9894.846**	-24750.6	-16939.1	-12831.3	**-8268.1**	-12408.5	-16250.1	-12245.6	**-7483.0**
Restricted constant	TVP	0	-9516.086	-24681.8	-17453.5	-12637.9	-7919.8	-12311.8	-16637.0	-12142.3	-7098.0
Restricted constant	TVP	1	-9850.21	-25120.7	-17436.1	**-12868.7**	-8175.9	**-12628.2**	-16594.5	-12256.3	-7396.8
Unrestricted trend	TVP	0	-4119.997	-7717.2	-8102.15	-7970.3	-5335.0	-7052.6	-5840.6	-5729.0	-3992.2
Unrestricted trend	TVP	1	-8439.752	-21736.1	-14324.7	-10864.7	-6972.8	-11235.2	-13536.0	-10202.6	-6289.1
Restricted trend	TVP	0	-9517.479	-24665.1	-17460.4	-12659.8	-7906.8	-12311.0	-16602.7	-12120.3	-7095.0
Restricted trend	TVP	1	-9584.442	-24567.2	-17372.1	-12356.2	-8033.8	-12495.1	-16487.6	-11725.7	-7306.4

Note: Specification refers to the specification of the VECM model. Parameters can be time-invariant parameters (TIP) or time-varying parameters (TVP). r is the number of cointegration relationships. We highlight the lowest BIC value in blue and bold for each model between the variables.