Predicting U.S. Recessions in Real Time in a Big-Data Setting*

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Abstract

We propose a novel approach for predicting recessions in real-time (or slightly *a posteriori*) in a *big data* environment. The context is one where we possess a large set of variables that are known to be coincident with the business cycle, another large set that is known to lead the business cycle, and also possess the dating of previous recessions. We employ principal component analysis to reduce the dimensions of the coincident and leading sets of variables, and use canonical-correlation analysis to establish the links between these two sets of variables, concentrating on their cyclical features rather than noise. These cyclical features are then linked to the recession indicator via a structural Probit model. In addition, we also employ the expectation-maximization (EM) algorithm to solve the jagged-edge problem allowing the real-time use of these techniques by means of vintages.

On the empirical implementation of this framework, we are careful to avoid using future information to forecast the future. This entails devising an updating rule for the vintages of data employed in forecasting, an updating rule for the estimates of the structural Probit model, and a real-time recursive technique for choosing the best model for forecasting based on past performance.

On evaluating the final forecasts for U.S. recessions, we resort to the receiver operating characteristic curve (ROC curve) – which maps true-positive rates (TPR) and

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false-positive rates (FPR) of forecasts for each cutoff used on deciding what state the economy is in: recession versus non-recession. All in all, depending on the cutoff chosen, we could achieve a true-positive rate of 94% coupled with a false-negative rate of 5% for predictions one-month *a posteriori*. In real time, we achieved a TPR of 94% coupled with a FPR of 8%. Also, we find little difference between forecasting using real-time information and forecasting using information slightly *a posteriori*, a sign that the EM algorithm is working properly: the updating lag of the data base is relatively short and we have several coincident series readily available to forecast missing data.

Keywords: Forecasting Recessions; Common Cycles; Canonical-Correlation Analysis; Principal-Component Analysis; Structural Probit Model.

JEL Classification: C14; C15; C22; C53; C55; E17; E31.

1 Introduction

In this paper we propose a novel framework to predict U.S. recessions using information available in real time or using information available slightly *a posteriori*. It is a *Big-Data* approach, characterized by the fact that the number of variables used in forecasting is greater than the number of time-series observations accessible to the econometrician to forecast recessions. Thus, some type of dimension-reduction technique must be used for feasibility. Our approach is based on the following ingredients.

First, as a dimension-reduction device, we employ principal-component analysis (PCA). This technique was proposed by Stock and Watson (2002) as a method for estimating common dynamic factors in a large set of variables. It is well-established that the PCA is a *dense*[1] technique capturing the common components of a given data base. In some cases, a small number of factors can capture close to or more than 50% of the variation of a large data base, which is remarkable. We use PCA to reduce the dimensions of a large set of coincident series used in Stock and Watson (2014) and a large set of leading series used in Costa et al. (2021).

Second, the modeling strategy to predict U.S. recessions follows closely the setup of Issler and Vahid (2006). Their idea, based on canonical correlations, is to use the information content in the NBER Business Cycle Dating Committee decisions to construct a coincident and a leading index of economic activity. The forecasting equation is a *structural* Probit model in which the probability of recession is a linear combination of the cycles of the coincident series (basis cycles), where the natural instruments are linear combinations of the leading series. Indeed, their key identification assumption imposes the restriction that there exists a common cycle between the unobserved state of the economy, modeled in the probit model, and the coincident series.

[1]See Giannone, Lenza, and Primiceri (2021).

In this context, the canonical-correlation framework: (i) computes the coincident cycles – the linear combinations of the coincident series that are statistically predicted by linear combinations of the leading series; (ii) computes these optimal linear combinations of the latter; and (iii) offers a statistical test of predictability, which is critical step to separate the U.S. business cycle from noise. Also, canonical correlation analysis does it all at the same time.

By contrast, principal-component analysis performs a similar task in two steps. First, computes a relevant set of predictors (factors) and then, using a bridge regression, forecasts the coincident series of interest using least squares. A possible drawback of using principal-component analysis is the fact that there is no guarantee that the factors and the lagged-dependent variables of the bridge model can capture all the serial correlation of the series being forecast. So, one may be forced to model the serial dependence of the error term in the bridge regression.[2]

Third, in order to deal with the jagged-edge problem, e.g., Giannone et al. (2008), we employ a well-known statistical method – the expectation-maximization (EM) algorithm. It uses least squares to fulfill missing data with a delay on their release date, conditional on series already released. More generally, it also fulfills discontinued series based on available ones. In the context of forecasting, since one cannot employ future information to predict the future, the EM algorithm has to be used recursively. Indeed, a new vintage for period t, $t = 1, 2, ...$, is generated where only information up to period t is used to predict series with missing data.

Fourth, probit forecasts of the probability of a recession occurring in period t have to be feasible in real-time. No matter who dates recessions and how they do it, recession indicators are only available with some delay. Therefore, in real time t, while data for all coincident and leading variables are available up to time t, the dependent variable is available only up to time $t - h$. We use the NBER business-cycle dating committee data as our dependent variable, and, to be on the safe side, we allow a delay of 24 months (the maximum delay for the NBER committee to declare a recession was 21 months for the forecasting sample). So, while the dynamic factors and basis cycles in the coincident series are estimated based on information up to time t, the parameters of the Probit model that links the dependent variable to the basis cycles are estimated based on information available at $t - 24$. Using these parameters and the value of the basis cycles at time t, we then predict the probability of a recession at time t.[3]

Fifth, we put in place a dynamic algorithm designed to perform real-time forecasting. It uses the EM algorithm to balance our databases of coincident and leading series, generating the vintages to be used for *pseudo*-out-of-sample forecasting. We allow a variety of choices for principal-component analysis, explaining from 30% to 80% of the variance of these databases,

[2]We discuss this issue further below.

[3]Regarding real-time analysis, we also do not deal with seasonal adjustment and data revisions, but this is true for the literature as well.

giving us a total of 36 diverse data groups. We then perform canonical-correlation analysis on them and estimate a *structural* Probit model for each of them, which generates a set of 36 different probabilities of recession for every out-of-sample period.

Using a burning sample, the dynamic algorithm compares these forecasts using a receiver operating characteristic curve (ROC curve) – which maps true-positive rates (TPR) and false-positive rates (FPR) of forecasts for each cutoff used on deciding what state the economy is in: recession versus non-recession. Once the dynamic algorithm is set in motion, for every out-of-sample observation t, it can establish which of the 36 models has had the best record in forecasting based on information provided by the ROC curve.[4] Then, it chooses the forecast of the best model as the forecast for period t, where models are compared using their cumulative "area under the curve" (AUC – ROC curve) performance for the forecasting sample; see our discussion below.

There is a vast literature on predicting U.S. recessions. A probabilistic setup was used by Stock and Watson (1989a and b) to build a coincident and leading index of economic activity, as well as an index of recessions. The performance of financial variables (interest rates, spreads, stock prices and monetary aggregates) in predicting the probability of recessions was evaluated by Estrella and Mishkin (1998). They found that stock prices were good predictors of recessions over the one to three quarter horizon, while the slope of the yield curve was a better predictor beyond one quarter. The predictive power of the term structure is also documented in Rudebusch and Williams (2009), who emphasize the fact that professional analysts do not adequately incorporate the information contained in the yield spread.

A dynamic probit model was used by Nyberg (2010) who found that in addition to the term spread, lagged stock return values and external spreads were important predictors of a recession. Nonlinear models and stochastic simulations were used by Anderson and Vahid (2001) to predict the probability of recession in the U.S. using the interest rate spread and the growth of the money stock (M2). Several probit models were estimated by Wright (2006), who found that adding the Fed Funds rate to the term spread outperformed the Estrella and Mishkin (1998) model. Christiansen, Eriksen and Miller (2013) found that sentiment variables have a superior predictive power compared to financial variables. Kotchoni and Stevanovic (2018) propose an ARX model for forecasting GDP growth, where X is the recession-probability. This indirectly involves having a real time forecast of the recession probability.

There is also a large literature on dating recessions following the seminal work of Burns and Mitchell (1946), including, inter alia, Bry and Boschan (1971), Harding and Pagan (2002), Chauvet and Hamilton (2006), Stock and Watson (2010b, 2014) and Camacho, et al. (2022). Since our paper is about developing a predictive model for the binary indicator generated by a dating mechanism, not the dating mechanism itself, we do not review this literature here. Even though a complete dynamic statistical model that specifies the evolution of the state

[4]In the ROC-curve literature, comparisons are made using the "area under the curve" (AUC). Here we employ different versions of AUC metrics to examine the performance of the 36 competing models.

of the economy, such as Hamilton's (1989) Markov-switching model, can be used both for predicting recessions and, with the passage of time, for determining the historical turning points with more and more accuracy, Hamilton (2011) discusses the difficulties in dating recessions in *real time*.

Finally, there exists an emerging literature applying machine learning (ML) methods to forecast economic variables. In general, these methods are known to not work well when the predictors are highly correlated, when the dependent variable is highly persistent, and sample size is small, which is the case for forecasting recessions. Nevertheless, Vrontos et al. (2021) apply these methods to predict U.S. recessions. They conclude that while some ML methods perform better than using only the yield curve to predict recessions, their performance is not consistently better over different horizons. Their best results are obtained when they use the nine predictors that were selected more than 90% of the time by all ML methods with a simple logit or probit link to predict recessions, and based on that they recommend ML methods as a method for eliminating irrelevant predictors.

We start with a set of variables that are believed, a priori, to be coincident with the business cycle and another set of variables that are believed to be leading the business cycle. Our use of principal-component analysis, canonical-correlation analysis, and a generalized linear model with Probit link function to connect a small number of basis cycles to predict recessions can all be interpreted as bespoke dimension reduction and regularization techniques to avoid overfitting and produce robust results. So we differ from most of the machine-learning methods by employing *dense* techniques instead of *sparse* ones; see the results in Giannone et al. (2021) on forecasting.

Our set of coincident series are the series used for dating business cycles in Stock and Watson (2014). Regarding leading series of economic activity, we employ the database in Costa et al. (2021), which combines several data bases in the macro-finance literature and use them to forecast oil prices.

When we take to the data our proposed method, empirical results show that it is possible to track the state of the U.S. economy using our model that produces probabilities of recessions. Not only its has a good in-sample fit, but also out-of-sample recession predictions have a surprising accuracy in real time (or slightly *a posteriori*). Employing ROC-curve analysis, depending on the cutoff chosen, we achieve a true-positive rate (TPR) of 94% coupled with a false-positive rate (FPR) of 5% for predictions one-month *a posteriori*. In real time, we achieved a TPR of 94% coupled with a FPR of 8%. This is a sign that the EM algorithm is working properly since the updating lag of the data base is relatively short and we have at our disposal a large group of current series readily available to solve the ragged-edge problem.

The outline of the paper is as follows. In Section 2, we present our methodology to predict recessions in real time using big data. Section 3 present a critical discussion of the methodology proposed here vis-a-vis the current literature. This discussion is complemented by our review of part of the literature in Appendix B. Section 4 shows the empirical results of recession/expansion predictions for the U.S. economy. Section 5 concludes.

2 Methodology

2.1 Issler and Vahid's (2006) methodology

Our methodology is an extension of Issler and Vahid (2006) that could be applied to a big-data environment. Appendix B presents a summary of the original techniques proposed there. Their goal was to construct a coincident index and a leading index of the business cycle for the U.S. They exploit the information in the NBER recession indicator to construct a coincident and a leading index of the U.S. economic activity. However, they argue that a simple logit or probit regression of the indicator on the coincident series can produce results that are influenced by the irregular components in the coincident series rather than their cyclical components. Hence they first extract the cyclical features of the coincident series, which they call the "basis cycles", and then use them as regressors with the NBER recession indicator as the dependent variable in a *structural* Probit model, where regressors and errors are correlated and instruments are needed. The procedure produces a linear combination of coincident series that is highly predictable from the past information set *and* explains the NBER recession indicator.

The basis cycles are constructed not solely based on their correlation with the lags of coincident series alone, but from analyzing their correlation with a larger set that includes lags of coincident and leading variables. By testing the statistical significance of the canonical correlations between the set of coincident variables and the union of lags of coincident and leading variables, Issler and Vahid (2006) determine linear combinations of coincident variables that cannot be predicted from the past, and those that can. They discard the unpredictable combinations (noise) and keep only those combinations that can be predicted as the common basis cycles that embody all cyclical signal in coincident variables. Using canonical correlation procedure to test for common serial correlation features (Engle and Kozicki, 1993) in multivariate settings was proposed by Vahid and Engle (1993), and has been used by Engle and Issler (1995) and Issler and Vahid (2001), among others.

The key identifying assumption in Issler and Vahid (2006) is that the latent coincident index representing the U.S. business cycle in the Probit model is solely a linear combination of the cyclical components (as opposed to noise) of the coincident variables. In turn, these cyclical components represent the predictable portion of the coincident series that are explained by the leading series. Indeed, there may be more than one predictable cyclical component and canonical-correlation analysis generates the cyclical components, their optimal predictor, and a test of predictability – all at once.

This contrasts with the view of a single latent dynamic index in the coincident variables, as proposed by Stock and Watson (1989) and Chauvet (1998). In their approach, business cycles are confined to a singular common cyclical factor shared by the coincident variables. To identify this common cycle, the single latent dynamic factor approach allows the coincident variables to have other idiosyncratic cycles, which yields no control over the intensity of these idiosyncratic cycles relative to the common cycle. Additionally, the optimization process involves two steps: first, estimating principal components (factors), and second, employing least-squares in a bridge equation to estimate factor loadings.

The goal of Issler and Vahid (2006) was to construct a coincident series based on the four variables that constituted the Conference Board's coincident index, and they used twelve leading series. Here, we have 177 coincident series and 294 leading series. Adding several lags of coincident and leading series, we may have in excess of 1,000 series in the predictor set. Canonical-correlation analysis between a set of 177 variables and a set close to 1,000 variables is infeasible when time series dimension is not very large. While maintaining the hypothesis that there is a linear index (of the cyclical parts) of the coincident series that has exactly the same pattern of correlation with previous information as the unobserved state of the economy, we devise a feasible procedure to estimate this linear index in real time in a big-data environment.

2.2 Adapting the approach of Issler and Vahid to a *big-data* environment

There are two critical issues related to adapting the approach in Issler and Vahid (2006) to a big-data environment.

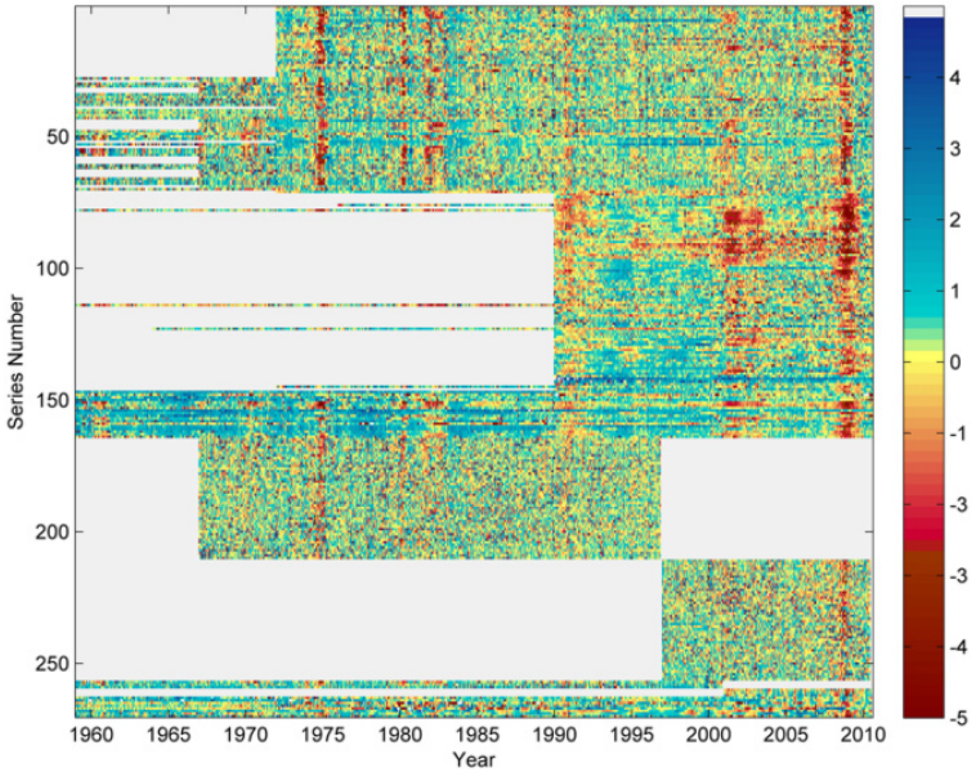
First, as discussed by Stock and Watson (2010, 2014), the data of the coincident series stacked in vector x_t, where we let n be the number of rows in x_t, and the leading series stacked in vector z_t, where $m \geq n$ is the number of rows in z_t, contain missing values due to the a "jagged-edge" problem. Of course, this is a problem for their real-time use, but not for their use slightly *a posteriori*. On top of the jagged-edge problem, some coincident series were discontinued for different reasons. In some cases, this prevents their use for out-of-sample prediction and discarding them from the database is recommended.

Second, to implement canonical-correlation analysis, the variance covariance matrix of $(x_t', z_t')'$, must be full rank. However, in a big-data setting, the dimension of the column vector $(x_t', z_t')'$ is greater than the number of observations through time, making the variance covariance matrix of $(x_t', z_t')'$ rank deficient.

2.2.1 Imputation of Missing Values

The first practical issue to tackle before implementing the methods discussed in previous section is the *missing data*. This occurs in databases for several reasons, but it is up to the final users of the data to treat the missing values using the information available. In the context of canonical correlations, we have two databases in a *big data* format: (i) the coincident series; and (ii) the leading series. The latter database was constructed by Costa et al. (2021), avoiding missing data at the beginning and middle of the sample. But, like any database with many series, there is a mismatch in the updates of the various series, which generates the so-called *ragged-edge* or *jagged-edge* problem. The ragged edge problem results in an incomplete database for the final periods of the dataset (unbalanced panel), causing a jagged edge there; see Wallis (1986). In turn, using the coincident series proposed by Stock and Watson (2014) poses additional problems, as there are more recent series with gaps at the beginning of the sample as well as old series that were discontinued at the end of the sample.

Figure 1 – Coincident Data Base (Stock and Watson, 2014)



Source: Stock and Watson (2014)

Figure 1 illustrates the challenges of working in a big-data environment. Overall, Stock and Watson (2014) employ a total of 270 coincident data series. Each of them falls into one of the four classes that have been traditionally used in business-cycle analysis: Employment, Income, Industrial Production and Sales. The heatmap allows us to spot U.S. recessions using the date-than-average method: recessions are the red portions of the available series, noting that the white spaces show missing data. Indeed, missing data is a major problem. Less so for the past, but for the future, since one wants to implement real-time forecasting and cannot rely on unavailable data.

Figure 1 misses the issue of jagged-edge, where different release dates for different series generate a jagged-edge problem, where the econometrician has to efficiently forecast the series with a late release employing those readily available (usually financial data).

A classic and efficient way to solve the missing data problem is to use the Expectation Maximization (EM) algorithm. There are several forms and versions of this algorithm available in the literature. In this paper, we use a modern form of the EM algorithm proposed in Schneider (2001), which also provides an easy and complete code in MATLAB. Schneider argues that the EM algorithm with Gaussian specification for the data is an iterative method both for the estimation of mean values and the variance-covariance matrix of a set of incomplete data, being considered the starting point for the development of a regularized EM algorithm. In contrast to the conventional EM algorithm, the regularized algorithm is applicable to datasets in which the number of variables normally exceeds the sample size, which is usually the case in a big data setup.

The regularized EM algorithm is based on iterated linear regressions of variables with missing values into variables with available values. The regression coefficients are estimated by *Ridge Regressions*, a classic regression method in which a continuous regularization parameter controls the filtering of noise in the data. The regularization parameter is determined by generalized cross-validation, in order to minimize the expected mean squared error of the imputed values. For the imputation of missing values, the regularized EM algorithm can estimate synchronous and diachronic covariance matrices, which may contain information about spatial covariability, stationary temporal covariability or cyclostationary temporal covariability. In this sense, the data to be completed must contain only weakly stationary series, which requires transformations in some original series of the database that are non-stationary, which is very common in the study of coincident and/or leading indicators of economic activity, such as the case of the current study.

2.2.2 Dimensionality reduction

Once we obtain a balanced panel of coincident and leading series, the next step is to implement the canonical-correlations analysis between these two groups of series. It is worth highlighting that this is done by adding lags of the coincident series into the leading database, since the past of the former might have good predictive power for the current coincident series.

In several practical cases, the number of series is greater than the number of observations, which is a constraint to the standard implementation of the canonical correlation analysis. Indeed, some of the *large* variance-covariance matrices have rank deficiency and cannot be inverted to compute canonical correlations.

A natural solution to deal with this problem is to extract from each database (coincident and leading) its common components, with much less series than the number of series in the original databases. A similar problem was solved by Bai and Ng (2021) using block and subblock factors. The classic way of extracting these factors is through Principal Components Analysis (PCA) – which are simply (orthogonal) linear combinations of the original series, with maximum variance sequentially computed.

In order to give an idea of the dimensionality reduction that we can achieve with this procedure, we note that the coincident database has originally 177 contemporaneous time series. To estimate the probit model proposed in Issler and Vahid (2006), we ended up using a maximum of 25 principal components (and a minimum of 1). These 25 principal components explain 75% of the variation in the data, which represents a large reduction in dimensionality, without much loss of information.

In the leading database, this reduction is much greater. There are 294 leading series. By adding lags of the leading series, and those of the coincident series, we can easily exceed $1,000$ series, depending on the lag specification adopted. Again, we managed to use a maximum of 35 principal components (also explaining 75% of the variance in the data) and a minimum of 8 components for the leading series.

Having solved the problem of missing data and dimensionality reduction in the databases, the next step in the method of Issler and Vahid is to compute canonical correlations: they solve the problem of signal extraction, making sure that the Probit model only conditions on the cyclical portions of the coincident data to explain the state of the economy – expansion *versus* recession.

2.2.3 Estimating the structural probit model

The next step of the empirical analysis is to estimate the Probit model proposed in Issler and Vahid (2006). Appendix B of this paper discusses the reasons why regressors and errors of the Probit model are correlated, as well as optimal instruments based on canonical-correlation analysis. We assume that the coincident series can be explained by N basis cycles $\{c_{1t}, c_{2t}, \cdots, c_{Nt}\}$. Let NBER_t denote the NBER indicator at time t. We employ a structural Probit model, which is estimated by conditional maximum likelihood in two stages (2SCML). We chose Rivers and Vuong's (1988) 2SCML technique. The first stage involves regressing the basis cycles $\{c_{1t}, c_{2t}, \cdots, c_{Nt}\}$ into k instruments – which are the main components of the leading series – and save the residuals, denoted by $\{\hat{v}_{1t}, \hat{v}_{2t}, \cdots, \hat{v}_{Nt}\}$. In the second stage, both the basis cycles $\{c_{1t}, c_{2t}, \cdots, c_{Nt}\}$ and the first-stage residuals $\{\hat{v}_{1t}, \hat{v}_{2t}, \cdots, \hat{v}_{Nt}\}$ are included as regressors in the probit model:

$$\Pr(\text{NBER}_t = 1) = \Phi\left(-\left(\beta_0 + \beta_1 c_{1t} + \cdots + \beta_N c_{Nt} + \beta_{N+1}\hat{v}_{1t} + \beta_{N+2}\hat{v}_{2t} + \cdots + \beta_{2N}\hat{v}_{Nt}\right)\right), \tag{1}$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function (CDF). The second-stage estimates of $\beta_0, \beta_1, \beta_2, \cdots, \beta_N$ in the Probit model (8) are the 2SCML estimates – denoted by $\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\beta}_2, \cdots, \widehat{\beta}_N$. Based on these estimates, we can predict in *real time* the probability of a recessions in period t using:

$$\Pr(\widehat{\text{NBER}_t} = 1) = \Phi\left(-\left(\widehat{\beta}_0 + \widehat{\beta}_1 c_{1t} + \widehat{\beta}_2 c_{2t} + \cdots + \widehat{\beta}_N c_{Nt}\right)\right). \tag{2}$$

There are a few issues regarding the use of this model in real time. As noted above, the first one is that the NBER Committee declares a recession in period t with a delay – say, in period $t+h$. Thus, if we are in period t, we would only have the dependent variable of the probit model (1) up to $t-h$. A simple solution to this problem is to look ahead into the forecasting sample and allow a sufficient slack as to always be able to count on data for the left-hand side of (1). Table 1 presents NBER dated peaks and throughs and their respective

Table 1 – NBER Peaks and Throughs Dating Delays

Turning point	Dated	Announcement	NBER Comm. Delay
peak	jul/1990	apr/1991	9
trough	mar/1991	dec/1992	21
peak	mar/2001	nov/2001	8
trough	nov/2001	jul/2003	20
peak	dec/2007	dec/2008	12
trough	jun/2009	sep/2010	15
peak	feb/2020	jun/2020	4
trough	apr/2020	jul/2021	15

Source: NBER Homepage

Note: Delay measured in months.

delays, which attains a maximum of 21 months. So, for the forecasting sample, if we employ $h \geq 21$, we will be certain that, in the forecasting sample, we will be able to run equation (1).

This is one instance where we employ partial future information in forecasting to implement a pseudo out-of-sample experiment. Indeed, there are two other cases as well: we are not able to deal with realistic seasonal adjustment and and data revisions. But, this is also true for the rest of the literature as well.

The second issue has to do with the basis cycles $\{c_{1t}, c_{2t}, \cdots, c_{Nt}\}$. As explained above, using sequentially the EM algorithm, the principal-component analysis, and the canonical-correlation analysis, we can create vintages of the basis cycles that employ information up to period t. So, there is no constraint on how far the information set can go to forecasts the probability of a recession in period t. Of course, the estimates $\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\beta}_2, \cdots, \widehat{\beta}_N$ employ information only up to $t - h$, but the basis cycles $\{c_{1t}, c_{2t}, \cdots, c_{Nt}\}$ employ information up to period t. Hence, the final forecast using (2) employs only information up to t to compute $\Pr(\widehat{\text{NBER}_t} = 1)$.

3 Alternative approaches – a discussion

This section presents a critical discussion of the techniques proposed here in light of the existing literature. For the sake of completion, Appendix B presents a relevant portion of this literature.

The U.S. experience in forecasting rare and extreme events, such as recessions, shows us that we should not be too optimistic when using them in real time. Indeed, as noted by Hamilton (2011), it is often observed that the fit of different models is very good in sample, but they often fail in their behavior out of sample. Since the NBER Committee usually dates recessions with a delay ranging from 6 months to a year (sometimes even more), the NBER-Committee decisions cannot be used *directly* as a real-time device for forecasting recessions. Issler and Vahid (2006) recognize this in the Introduction of their paper, but, they do not discard their *indirect* use, something we advocate and employ here:

> *"Suppose that we are asked to construct an index of the health status of a patient. Also, suppose that we know that the best indicator of the health of the patient is the results of a blood test. However, blood samples cannot be taken too frequently, and test results are only available with a lag, sometimes too long to be useful. Our index therefore must be a function of variables such as blood pressure, pulse rate and body temperature that are readily available at regular frequencies. In order to estimate the best way to combine these variables into an index, would we (i) use the historical data on these variables only, or (ii) use the historical blood test results as well? The answer is, obviously, the latter."*

Indeed, the fact that there are delays in the release of NBER-Committee decisions does not imply that we cannot use them in a forecasting model *ex-post*, since NBER-Committee decisions are an accurate description of the state of the U.S. economy, recognized by the literature.

Hamilton (2011), however, has a somewhat pessimistic tone regarding real-time (or slightly *a posteriori*) predictions of recessions:

> *"If people could predict recessions, they probably would not happen. Firms would not be stuck with inventories, labor, and capital they turn out not to need, and the Federal Reserve would probably ease its policy stance earlier. Economists are used to viewing magnitudes such as stock prices as difficult or impossible to predict if the market is functioning properly, and it may be that economic recessions, by their very nature, imply similar fundamental limitations for forecasting."*

Economists understand the difficulty of the task in hand of predicting recessions in real time. But, the rewards that a good prediction model would have is very high – which has certainly motivated this literature. Moreover, we live more and more in world where big-data, machine learning and artificial intelligence are commonplace, which challenges their use in solving the relevant problems of the day.

Stock and Watson (2014) compare two business cycle modeling techniques: *date then average* and *average then date*. The former is their preferred approach and the latter is the preferred technique of Chauvet and Hamilton (2006) and Hamilton (2011), who focus on aggregate series like GDP. Stock and Watson (2010, 2014) propose employing a *large* database of coincident series to date recessions, where dating used a generalized version of the technique proposed by Harding and Pagan (2006). Regarding the *average* portion of the technique, it entails a non-parametric definition of a turning point based on turning points of the coincident series. This allows constructing estimates of turning points (means, medians, etc.), their sampling distributions, and estimates of their respective standard errors.

Notwithstanding the construction of the *average* portion of the technique, from our point-of-view, the key message in Stock and Watson (2014) is about the usefulness of a large database of coincident variables in dating business cycles, where these dates can serve as a basis to get an average date of a distribution of dates. Indeed, they used a coincident database containing 270 series representing four categories of real monthly economic activity for the U.S. from 1959:M1 to 2010:M9: employment, industrial production, income, and sales. Apparently, the use of this large database by Stock and Watson alleviates the problem that their basic model had in detecting the 1990-91 recession. Hamilton (2011) writes:

> *"What went wrong [in the 1990-91 recession]? One of the intriguing new leading indicators that Stock and Watson discovered was the spread between the yield on commercial paper and Treasury bills ... this spread had a dramatic spike prior to each of the recessions in their original sample, but did very little out of the ordinary in the 1990-91 recession, for which their model was on real-time display."*

One way to avoid having a specific variable dictate the behavior of recession dating is to use a large database, employing *dense* methods in getting the average date. In multivariate statistics, there are two classic ways of implementing *dense* techniques in the context of forecasting recessions: (i) principal component analysis, proposed by Stock and Watson in several articles, and (ii) canonical correlation analysis, proposed by Issler and Vahid. The first extracts a reduced number of factors from the database of potential predictors and then uses a bridge regression to forecast the variable(s) of interest. The second generalizes the idea behind a least squares regression (or maximum likelihood, under Gaussian assumption). From two sets of data, coincident series and leading series, it recursively selects the linear combinations of the first set that have maximum correlation with linear combinations of the second. A test of predictability is also implemented to be able to exclude noise.

Comparing the two ways to extract common components we see an advantage for the use of canonical correlations. In addition to separating *noise* and *signal* from the series in each database (coincident and leading) it still makes a bridge between them, in a single optimization problem.

For exposition purposes, consider the small-scale Stock and Watson's (1989) factor model, where the four series y_{it}, $i = 1, 2, 3, 4$, in the vector $(\Delta \ln(Y_t), \Delta \ln(N_t), \Delta \ln(S_t), \Delta \ln(I_t))'$, represent, respectively, the growth rates of industrial production, employment, sales and income in period t. These are the coincident series whose cyclical behavior are thought to be synchronized with the U.S. business cycle – a latent variable. The common factors in f_t are obtained from principal-component analysis using a set of predictors, where the number of factors r are much smaller than the set of predictors N:

$$y_{it} = \alpha_i + \lambda_i f_t + \gamma_i z_{t-h} + u_{it}, \quad i = 1, 2, 3, 4, \tag{3}$$

$$f_t = \alpha_f + \phi_1 f_{t-1} + \phi_2 f_{t-2} + \varepsilon_t, \tag{4}$$

$$u_{it} = \rho_{i1} u_{it-1} + \rho_{i2} u_{it-2} + e_{it}, \tag{5}$$

$$\begin{bmatrix} \varepsilon_t \\ e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix} \sim i.i.d.\, \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon & 0 & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & 0 & \sigma_4 \end{bmatrix} \right). \tag{6}$$

Note that factor f_t is an $AR\,(2)$ process, which cyclical behavior could in principle capture the cyclical behavior of the series y_{it}, $i = 1, 2, 3, 4,$. However, the error term u_{it} also has a cyclical behavior (also a $AR\,(2)$).

Thus, this divides the cyclical behavior of the series $(\Delta \ln(Y_t), \Delta \ln(N_t), \Delta \ln(S_t), \Delta \ln(I_t))'$ between two components: one common and another idiosyncratic, which cannot control the importance of the common factor vis-à-vis the term idiosyncratic. For example, one can get an idiosyncratic component u_{it} that explains much of the variance of y_{it}, with low explanatory power for f_t. In such a context, the idea of a common component of business cycles, which has been the cornerstone of research in this area since Burns and Mitchell (1946), would fall apart.

The canonical-correlation approach we propose here avoids this problem. Only the cyclical behavior of the coincident series that are predicted by the leading series are used in the probit model, implementing the key identification assumption that the cyclical series have a common cycle with the latent series representing the U.S. business cycle. So, noisy idiosyncratic components are excluded. In Issler and Vahid, there is serial correlation for the error term in the Probit model. But, it is solely a function of the delay of the NBER Committee to date recessions and is related to the use of future information by the NBER Committee in dating recessions.

Next we provide a roadmap of our empirical experiment. First, given a large enough burning period for parameter-estimates to reach a stable value, probit-model estimates are implemented using information up to $t - h$, where we set $h = 24$ months. Second, we show that a *big-data* approach can be successfully implemented, where its key elements is to get a balanced data base using the EM Algorithm, allowing a continuous update of principal-component and canonical-correlation estimates in real time or slightly *a posteriori*. Based on these two elements, we forecast recessions using models of this kind:

$$\Pr\left(\widehat{\mathrm{NBER}_t} = 1\right) = \Phi\left(-\left(\widehat{\beta}_0 + \widehat{\beta}_1 \widehat{c_{1t}} + \widehat{\beta}_2 \widehat{c_{2t}} + \cdots + \widehat{\beta}_N \widehat{c_{Nt}}\right)\right), \tag{7}$$

where estimates $\widehat{\beta} = (\widehat{\beta}_0, \widehat{\beta}_1, \widehat{\beta}_2, \cdots, \widehat{\beta}_N)'$ use information up to $t-h$, but the $\widehat{c_{it}}$, $i = 1, 2, ..., N$

Table 2 – Probit model estimates — full sample (2SCML)

Regressor	β_i	Robust S.E.
Constant	-2.47***	(0.258)
c_{1t}	17.93***	(6.699)
c_{2t}	-113.10***	(18.928)
c_{3t}	8.61	(14.094)
c_{4t}	-8.86	(10.463)
c_{5t}	24.54**	(11.546)

Note: ** and *** denotes 5% and 1% significance levels, respectively.

use information up to period t, therefore feasible for real-time use.

4 Empirical experiment

4.1 Data and empirical strategy

For the U.S. economy, in order not to sacrifice too much the number of recessions, we decided to start the sample at 1970:M1 and end it at 2023:M1 (637 monthly observations), which gives us a total of 8 recessions in about 51 years, i.e., approximately one recession for every 6 and a half years.

The *coincident* series came from the large database put together by Stock e Watson (2010, 2014) discussed above. Their database was updated here until 2023:M1. From the 270 series, we decided not to use the *sales* series in the analysis, as they have a lot of missing values, either going forward or backward. Moreover, the weight computed by Issler and Vahid (2006) for sales is only 2%. So we do not foresee a major problem in ignoring sales data. This procedure resulted in 177 series to form our database of coincident series; See Appendix A for the full list of coincident variables.

Regarding the *leading* series, we relied mostly on the database put together by Costa et al. (2021), which was used to forecast *Brent* oil prices. It has a total of 294 variables, mostly U.S. based; see Appendix A for the full list of leading variables. Of course, on top of the lagged leading series, we add their lags and also the lags of the coincident series, which are good predictors of the coincident series. For example, with two lags of both we end up with 942 series overall. Since the total number of time-series observations is 617, we are in a *big-data* environment where dimension-reduction techniques are needed.

Our selections allowed us to apply the EM algorithm with a small number of missing data, which favors a healthy database from the point of view of collinearity between the final series. Before the out-of-sample forecasting experiment is implemented, we used the whole sample to fit the Probit model. This entailed the following steps:

1. Using the balanced database after employing the EM algorithm, compute the principal components of the coincident and of the leading series separately, to serve as a

dimension-reduction device. This procedure is applied in Stock and Watson (2014) for the for the coincident series.

2. Compute the canonical correlations between the principal components of the coincident and leading series to generate the basis cycles, $c_{1t}, c_{2t}, \cdots, c_{Nt}$, their optimal predictors $\gamma_i' z_t$, $i = 1, 2, ..., N$, discarding the bases cycles that are pure noise, i.e., that are statistically unpredictable by the leading series.

3. Estimate the structural probit model, obtaining the maximum likelihood (2SCML) estimates of $\beta_0, \beta_1, \beta_2, \cdots, \beta_N$, computing the in-sample probabilities of recession.

Next, in Table 2, we present the estimates of $\beta_0, \beta_1, \beta_2, \cdots, \beta_N$ for the full sample. This model was produced with 5 principal components for the coincident series and 7 principal components for the leading-series group, which was assembled and with two lags of the coincident and leading series. Note that there are two basis cycles that are not individually statistically significant at usual levels. Also, note the high relative importance of c_{2t} in the Probit model. In any case, full-sample estimates show a tremendous degree of parsimony, where recession probabilities are a function of only a handful of cycles.

We now turn to the issue of predicting recessions in real time. To do this, we have implemented the following procedure:

1. Using an initial *burning period*, recursively employ the EM algorithm proposed by Schneider (2001) to balanced the databases (coincident and leading) with only information up to period t. This creates period t vintages for the *pseudo*-out-of-sample period.

2. Using these period t vintages, compute recursively the principal components of the coincident and of the leading series.

3. Implement recursively canonical-correlation analysis with the principal components of the coincident and the leading series.

4. After a *burning period*, that runs from 1970:M1 to 1989:M12, estimate recursively the probit model to be used in the *pseudo*-out-of-sample predictions from 1990:M1 to 2023:M1.

5. The real-time probability of recession for period t employs regression coefficients using information up to $t - 24$ months, which guarantees that the dependent variable in the probit-model is observed for the *pseudo*-out-of-sample period,[5] and period t recursive basis cycles computed in the canonical-correlation round.

6. We now explain the dynamic algorithm to select the best model among many choices, which are associated with a different number of principal components (or the percentage

[5]See Table 1.

of variance of the data base that is explained by them). We consider models with principal components that explain from 30% to 80% of the variance of the respective database (coincident or leading). We let these models compete in real time, choosing the one with the best performance up to time t. This choice can change at any time, depending on the accuracy of the competing models. To measure it, we employ *receiver operating characteristic curve* – (ROC) curves. This curve is a graph that illustrates the diagnostic capability of a binary classifier system as its discrimination threshold is varied. Our accuracy for each model is measured as the "area under the curve," using distinct slices of the plane.

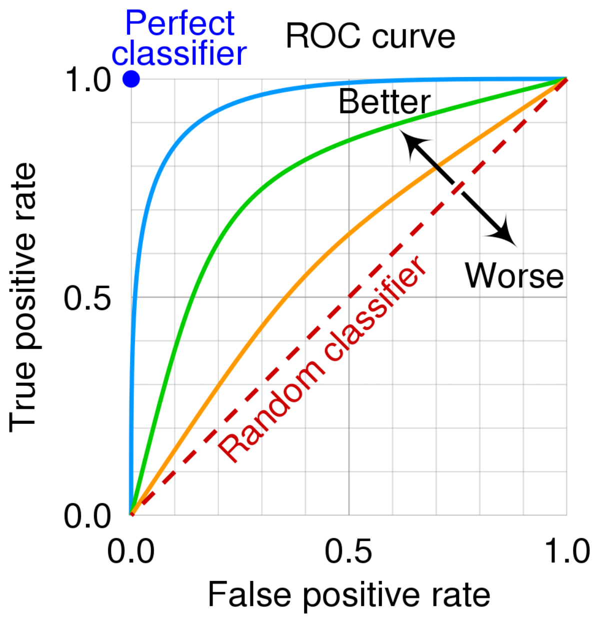
4.2 Receiver operating characteristic (ROC) Curves

A ROC curve plots the true positive rate (TPR) of classification and the respective false positive rate (FPR) as we vary the classification thresholds; see Fawcett (2006) for an introduction. These thresholds are used as a decision tool to determine an actual state: in our case here, recession *versus* non-recession. Thresholds have to lie in the interval [0,1]. If the forecast of the probability of a recession is above the threshold, then a recession *state* is chosen, otherwise, a non-recession *state* is chosen. In a *pseudo*-out-of-sample exercise, we can confront these choices to the state of the economy declared by the NBER. The true positive rate is also known as *recall* or *detection probability* in the machine learning literature. The false positive rate is also known as the *false alarm probability.*

The ROC curve can also be thought of as a graph of power versus Type I error of the decision rule. In short, if used correctly, the ROC curve is a powerful tool as a statistical measure of performance in detection/classification theory and hypothesis testing, as it allows having all relevant quantities on a single graph; See Figure 2 as follows:

As Figure 2 illustrates, preferred points in the ROC-curve space are to the Northwest of the plot, closer to a true-positive rate of one and a false-positive rate of zero. Here, in the real-time forecasting exercise, we employ ROC curves to evaluate models that explain a different proportion of the variance of the (coincident or leading) database in principal-component analysis. This is done recursively month-by-month, so the forecasting model could indeed be changing every month. Using a ROC curve for each model, we propose three different ways to select models. The first employs the *area under the curve* or AUC for short. It computes the integral of each ROC curve (averaged between the real-time and the 1-month-lagged prediction) and chooses the one with highest area. The second is a refinement of this *criterium*, since it computes the AUC only for the subset $[0, 0.2] \times [0.8, 1]$ of the plane, labeled AUC20. The third is an even finer criterion, which computes the AUC only for the subset $[0, 0.1] \times [0.9, 1]$, labeled AUC10.

Figure 2 – Receiver Operating Characteristic (ROC) curves



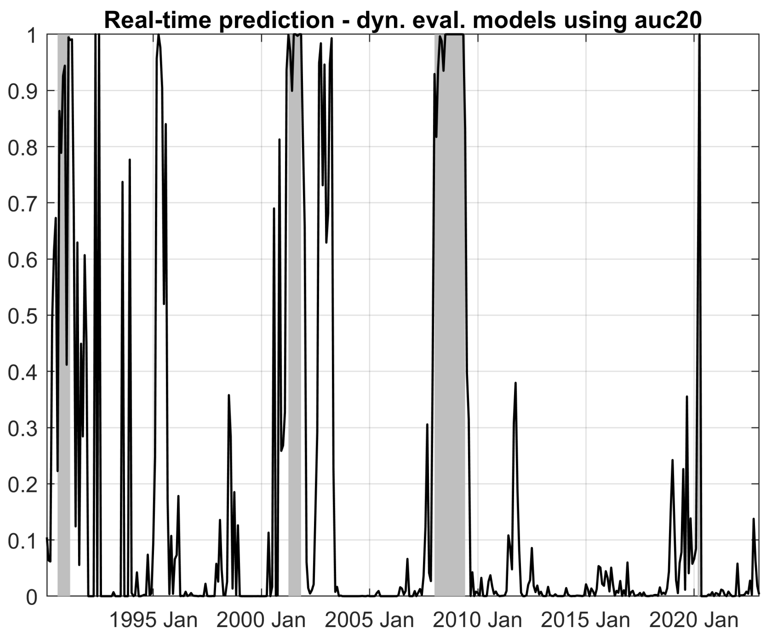
Source: Wikipedia

4.3 Empirical results

4.3.1 A first run

Empirical results of the *pseudo*-out-of-sample forecasting experiment will be presented in layers. First, Figure 3 presents the forecasts of dynamically selected models using the AUC20 *criterium* for the forecasting sample of 1990:M1 until 2023:M1. Gray areas represent the four recessions as dated by the NBER for the forecasting sample. Our chosen strategy usually picks up all four recessions, but false predictions of a recession may be a problem.

Figure 3 – Predictions of the dynamic evaluation exercise:
Real-time prediction using the *AUC20* criterion



Different models were selected based on the percentage of the variance of the database (coincident and leading) – from 30% to 80% – explained by their respective number of factors. This yields a total of 36 possible models, which compete to be chosen for forecasting in period t.

Figure 4 presents ROC curves for the dynamic selection program devised above. In real-time, one can choose between a true-positive rate (TPR) slightly above 90% coupled with a false-positive rate (FPR) slightly below 5% or, for example, a true-positive rate of about 95% coupled with a false-positive rate slightly below 7%. One-month *a posteriori* these numbers improve to a TPR of about 96% coupled with a FPR of about 6%.

Figure 4 – ROC of the dynamic evaluation exercise:
AUC20 criterion

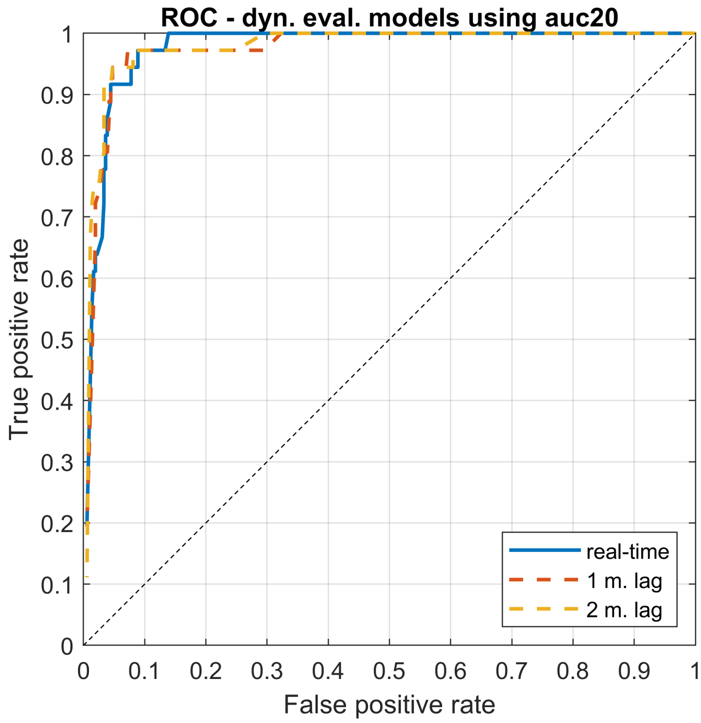
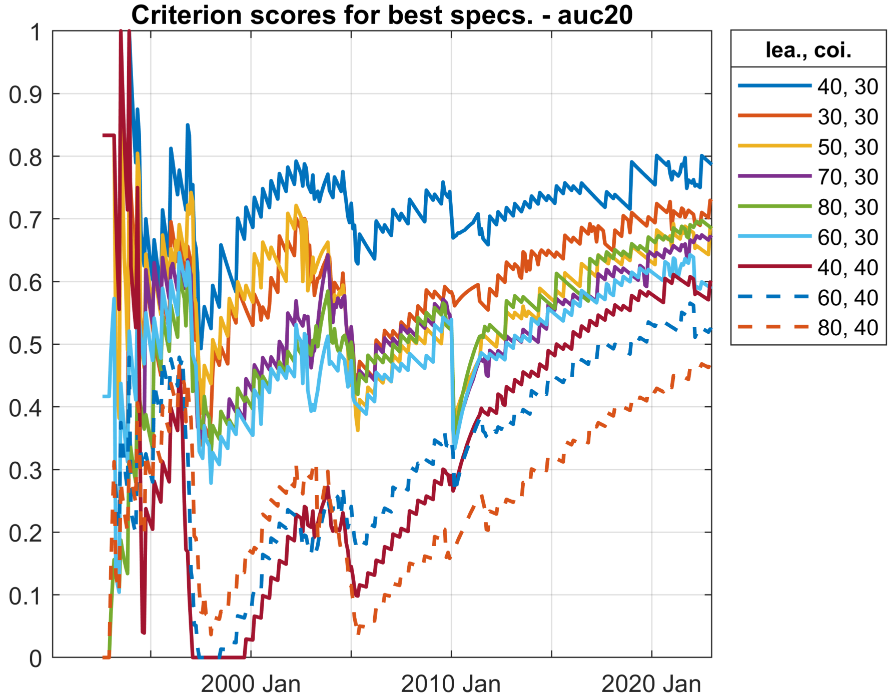


Figure 5 presents the area under the curve (AUC20) of the best 9 models, out of 36, i.e., the top *quartile* of the distribution. The model which employs principal components that explain 40% of the leading database and 30% of the coincident database scores the best area at the top "20 square" most of the time. On the other hand, models that explain a high proportion of the variance (either of the leading or coincident database) tend to do worse than those which explain a low proportion of the variance.

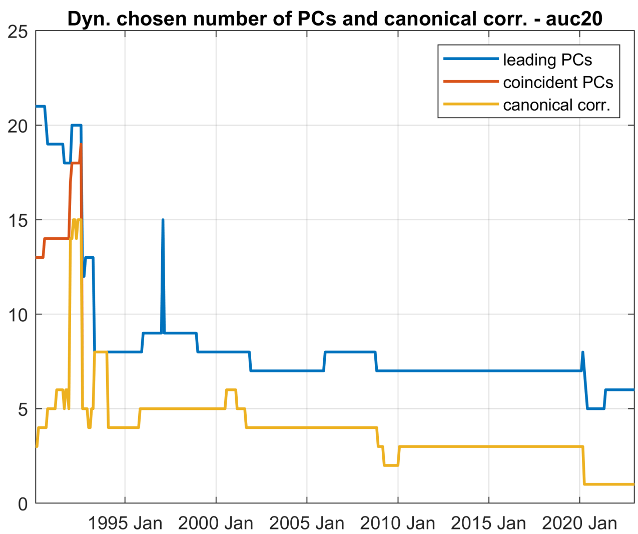
To better understand what this means, Figure 6 presents the number of principal components and canonical correlations chosen dynamically by the program above. So, results for Figures 5 and 6 go hand-in-hand. The first few periods, which can be thought of as a burning period, is clearly odd, where models chosen have a high number of principal components and of canonical correlations. About 10 years afterwards, leading principal components chosen amount to 7, which declines further to 6. After 2000, coincident principal components amount to 5, with occasional declines to 4, 3, and later to 1. Finally, the number of canonical correlations show an identical pattern after 2000.[6]

Figure 5 – Scores for each principal-component specification: *AUC20* criterium



We interpret these last results to mean that we need a small number of either principal components or of canonical correlations to capture the *commonality* of the coincident and leading databases and of the *cyclical* behavior of U.S. economy. Indeed, in our robustness checks, we found that models with a high number of principal components or of canonical correlations show *overfitting*, i.e., good performance in-sample, but a very poor performance out-of-sample.

Figure 6 – Number of principal components and statistically significant canonical correlations: *AUC20* criterion



[6]Note that, after 1995, the number of coincident principal components and of canonical correlations coincide.

Table 3 – Accuracy of predictions for different horizons and thresholds

	Real-time prediction accuracy (%)					
	AUC		AUC10		AUC20	
Threshold (%)	TPR	FPR	TPR	FPR	TPR	FPR
30	97.2	12.2	97.2	11.7	97.2	11.4
40	97.2	9.2	97.2	9.7	97.2	9.2
50	94.4	7.5	94.4	8.3	94.4	8.1
60	91.7	7.5	91.7	7.8	91.7	7.8
70	91.7	5	91.7	5.3	91.7	5.3
	1 month lag prediction accuracy (%)					
	AUC		AUC10		AUC20	
Threshold (%)	TPR	FPR	TPR	FPR	TPR	FPR
30	97.2	13.4	97.2	13.1	97.2	12.5
40	97.2	10.6	97.2	10.9	97.2	10.3
50	97.2	8.6	97.2	8.4	97.2	7.8
60	94.4	6.4	94.4	5.8	94.4	5.8
70	94.4	5	94.4	4.7	94.4	4.7
	2 month lag prediction accuracy (%)					
	AUC		AUC10		AUC20	
Threshold (%)	TPR	FPR	TPR	FPR	TPR	FPR
30	97.2	13.1	97.2	12.3	97.2	12.3
40	97.2	10.9	97.2	9.8	97.2	9.8
50	94.4	8.9	94.4	8.1	94.4	8.1
60	94.4	6.7	94.4	6.4	94.4	6.1
70	94.4	5.6	94.4	5.6	94.4	5.3

Finally, Table 3 presents a summary of the accuracy of our dynamic predictions for different horizons (real time, one-month lag, two-month lag) and thresholds: cutoff used to determine the state – *recession* versus *non-recession*. First, there is not much difference between the accuracy across horizons. This is probably due to a nice choice of a coincident database, for which release delays are relatively small, never exceeding 1 month; see Appendix A. Indeed, data for employment has a zero lag, according to our classification – since data for period t are available before the middle of the subsequent month, i.e., less than a 15-day delay – and data for income and industrial production has only one month delay. Second, increasing the threshold has the expected result of decreasing both the TPR and the FPR, showing the trade-off in accuracy. Third, usually AUC20 fares better than AUC and AUC10 in identifying the best model dynamically: TPR is identical in all cases, but FPR is frequently smaller for AUC20.

All in all, the results in Table 3 show that it is possible to predict recessions with approximately a 94% TPR coupled with a 6% FPR one month *a posteriori*. In real time, we can obtain approximately a 92% TPR coupled with a 5% FPR, or, approximately a 97% TPR coupled with a 9% FPR. These results are obtained employing different thresholds. Of course, it is the job of the final user of the ROC curve to determine the threshold that suits her/him best.

4.3.2 A closer look at results

In this section we consider potential improvements on our chosen strategy taking into account what we have learned in the *first run* of the empirical experiment. We also consider a critical analysis of the previous literature, to measure more precisely our marginal contributions to it. Our practical recommendations for the dating of recessions in real time using *big data*, can be summarized as follows:

(i) **Scope**: The techniques discussed in this paper can be immediately applied to countries or regions that have official business cycle dates. The most relevant today being the Euro Area;

(ii) **Coincident series**: We benefited from the *large* database put together by Stock and Watson (2010, 2014), which has a very small lag regarding current month. This helps substantially the implementation of the EM algorithm in *real time*.

(iii) **Initial conditions of the dynamic forecasting algorithm**: We now understand much better the benefits of *parsimony* in the context of *big-data* techniques. Best models explained about 30-40% of the variance of our databases. Increasing this percentage does not help forecasting recessions out of sample, although it may improve in-sample prediction. This is a classical *overfitting* result. To illustrate the potential gains, consider two alternative initial conditions and their respective results: one explained only 30% of the variance of both databases and the other explained 80%. Figures 7 and 8 below show a very different ROC curve for each of them respectively. Note that the big difference is on the top Northwest corner, where the "area under the curve" matters the most.

Figure 7 – ROC curve based on an initial condition where principal components explained 30% of the variance of both databases

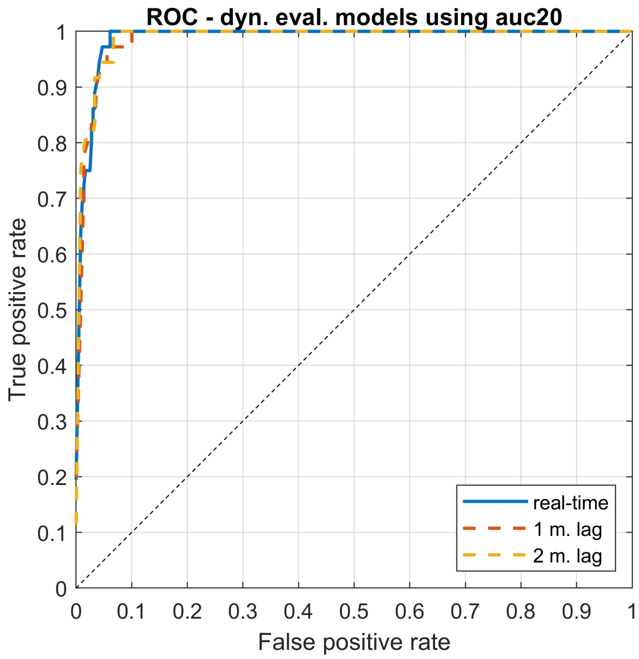
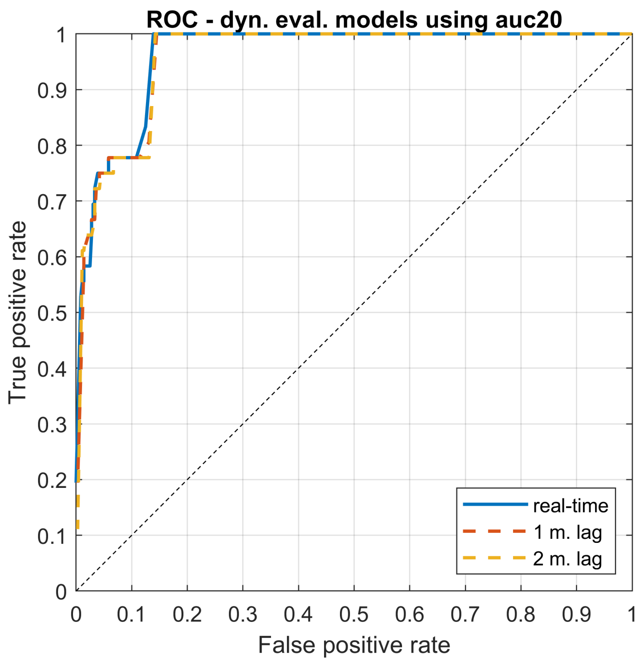


Figure 8 – ROC curve based on an initial condition where principal components explained 80% of the variance of both databases



(iv) **Matrix inversion in a big-data setting**: A big-data setting entails a large database of series that surpasses the number of time-series observations available for parameter estimation. To circumvent that problem, we used principal-component analysis (Stock and Watson, 2002) as a dimension reduction technique. However, we also experimented with using the Moore-Penrose generalized inverse matrix, which in principle allows using the whole database. Results were disappointing. But, alternative techniques could be tried in the future.

(v) **Date then average**: Stock and Watson (2014) use a large coincident database to implement the *date than average* technique, based on Harding and Pagan (2006), but ultimately also on Bry and Boschan (1971). There is a well-known endpoint problem for using these algorithms in real time, since you have to forecast the coincident series 6 months ahead to implement it. So, it is not really meant to be used in real time. So, as did Hamilton (2011), we were cautious to devise a a *big-data* technique that could be implemented in *real time*.

(vi) **Average than date**: Hamilton (2011) proposes the use of this technique, which entails using an aggregate for dating, e.g., real GDP. Regarding GDP, when forecasting peaks in real time, the average advantage over the NBER releases was 0.8 quarters and for throughs was 3.2 quarters – a much better performance. Hamilton notes: "I feel that the procedure performed well in terms of that objective. However, it is clearly worth taking a look at the realtime performances of various other procedures that were making use of alternative economic indicators." Here, we follow this suggestion, adding a twist of a big-data setting.

(vii) **Going forward**: Of course, in future analyses out of sample, it is valid to employ the lessons learned here to improve forecasting accuracy. Indeed, this is past information regarding the period 2023:M2 onward. As is the case with out-of-sample forecasting, avoiding overfitting is the most important lesson to be learned in our opinion. So, focusing on parsimonious models from the outset may produce perhaps even better results than the ones presented in this Section.

5 Conclusions

In this paper, we propose a novel approach to forecasting recessions in real time, or slightly *a posteriori*. In the *big-data* era, it applies modern techniques that involve the following ingredients. First, to have a balanced database for coincident and leading series of economic activity, solving the ragged-edge problem, we apply the EM algorithm creating vintages that are suitable for out-of-sample forecasting. Using them, we compute their principal components as a dimension-reducing device. Using the principal components of the coincident and leading series, we implement the cannonical-correlation approach of Issler and Vahid (2006) to solve a signal extraction problem and include in our forecasting model only signal. The forecasting structural *Probit* model explains the latent U.S. business cycle only with the cyclical part of the coincident series that are predictable from the leading series.

In our approach, we did not focus on a single model for *pseudo*-out-of-sample forecasting. On the contrary, we let a group of models compete and chose the one with the best performance based on an "area under curve" (AUC) metric employed in ROC-curve analysis. For every period t in the forecasting sample, a dynamic algorithm potentially chooses a different model based on AUC results for every period t.

Empirical results of applying this dynamic algorithm are encouraging. It is possible to predict recessions with approximately a 94% true-positive rate (TPR) coupled with a 6%

false-positive rate (FPR) one month *a posteriori*. In real time, we can obtain approximately a 92% TPR coupled with a 5% FPR, or, approximately a 97% TPR coupled with a 9% FPR.

References

[1] Andreou , E., Ghysels, E., Kourtellos, A., 2013. Should Macroeconomic Forecasters Use Daily Financial Data and How?, *Journal of Business and Economic Statistics* 31 (2), 240-251.

[2] Akaike, H., 1976. Canonical Correlation Analysis of Time Series and the Use of an Information Criterion. In R.K. Mehra and D.G. Lainiotis (Eds) *System Identification: Advances and Case Studies,* New York: Academic Press, 27-96.

[3] Anderson, T.W., 1984. *An Introduction to Multivariate Statistical Analysis.* Second edition. NewYork: John Wiley and Sons.

[4] Anderson, H.M., Vahid, F., 1998. Testing Multiple Equation Systems for Common Nonlinear Factors. *Journal of Econometrics* 84, 1-37.

[5] Bai, J., Ng, S., 2021. Matrix Completion, Counterfactuals, and Factor Analysis of Missing Data. *Journal of the American Statistical Association* 116 (536), 1746-1763.

[6] Baker, S.R., Bloom, N., Davis, S.J., 2015. Measuring Economic Policy Uncertainty. NBER Working Paper 21633, National Bureau of Economic Research.

[7] Balke, N.S., Fomby, T.B., 1997. Threshold Cointegration. *International Economic Review* 38, 627-647.

[8] Birchenhall, C., Jensen, H., Osborn, D., Simpson, P., 1999. Predicting US business cycle regimes. *Journal of Business and Economic Statistics* 17, 313-323.

[9] Bry, G., Boschan, C., 1971. *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, New York: National Bureau of Economic Research.

[10] Burns, A.F., Mitchell, W.C., 1946. *Measuring Business Cycles.* New York: National Bureau of Economic Research.

[11] Caldara, D., Iacoviello, M., 2018. Measuring Geopolitical Risk. FRB International Finance Discussion Paper n. 1222, Board of Governors - FED.

[12] Camacho, M., Perez-Quirós, G., 2010. Introducing the EUROSTING: short term indicator of Euro area growth. *Journal of Applied Econometrics* 25, 663-694.

[13] Camacho, M., Gadea, M.D., Loscos, A.G., 2022. A New Approach to Dating the Reference Cycle. *Journal of Business and Economic Statistics*, Vol. 40(1), 66–81.

[14] Campbell, J.Y., 2003. Consumption-based asset pricing. In *Handbook of the Economics of Finance*, vol.1(B), 803-887.

[15] Chauvet, M., 1998. An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching. *International Economic Review* 39, 969-996.

[16] Chauvet, M., Hamilton, J.D., 2006. Dating business cycle turning points. In C. Milas, P. Rothman, & D. van Dijk (Eds.), *Nonlinear time series analysis of business cycles*, 1-54. Amsterdam: Elsevier.

[17] Chevallier,J., Zhu, B., and Zhang, L., 2021. Forecasting Infection Points: Hybrid Methods with Multiscale Machine Learning Algorithms. *Computational Economics*, 57:537–575.

[18] Costa, A.B.R., Ferreira, P.C., Gaglianone, W.P., Guillén, O.T.C., Issler, J.V., Rodrigues, A.B., 2023, Predicting recessions in (almost) real time in a big-data setting. Technical report, Brazilian Central Bank.

[19] Costa, A.B.R., Ferreira, P.C., Gaglianone, W.P., Guillén, O.T.C., Issler, J.V., Lin, Y., 2021. Machine Learning and Oil Price Point and Density Forecasting. *Enegy Economics* 102, 105494.

[20] Davidson, R., MacKinnon, J.G., 1993. *Estimation and Inference in Econometrics.* Oxford: Oxford University Press.

[21] Dhrymes, P.J. (1974). Econometrics, Statistical Foundations and Applications. Springer, New York, NY.

[22] Engle, R.F., Kozicki, S., 1993. Testing for Common Features. *Journal of Business and Economic Statistics* 11, 369-395.

[23] Engle, R.F., Issler, J.V., 1995. Estimating Sectoral Cycles Using Cointegration and Common Features. *Journal of Monetary Economics* 35, 83-113.

[24] Estrella A., Mishkin, F.S., 1998. Predicting U.S. Recessions: Financial Variables as Leading Indicators. *Review of Economics and Statistics* 80, 45-61.

[25] Fawcett, T. (2006). An Introduction to ROC Analysis. *Pattern Recognition Letters*, 27(8): 861–874.

[26] Geweke, J., 1977. The Dynamic Factor Analysis of Economic Time Series. Chapter 19 in D.J. Aigner and A.S. Goldberger (eds.) *Latent Variables in Socio-Economic Models*, Amsterdam: North Holland.

[27] Giannone, D., Reichlin, L., and Small, D., 2008. Nowcasting: The real-time informational content of macroeconomic data, *Journal of Monetary Economics*, vol. 55(4), pp. 665-676.

[28] Giannone, D., Lenza, M., and Primiceri, G., 2021. Economic Predictions With Big Data: The Illusion of Sparsity, *Econometrica*, Vol. 89, pp. 2409-2437.

[29] Hall, R., Feldstein, M., Bernanke, B., Frankel, J., Gordon, R. and Zarnowitz, V., 2002. The NBER's Business-Cycle Dating Procedure. Downloadable from http://www.nber.org/cycles/november2001/recessions.pdf.

[30] Hamilton, J.D., 1994. *Time Series Analysis.* Princeton: Princeton University Press.

[31] Hamilton, J.D., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica* 57, 357-384.

[32] Hamilton, J.D., 2005. What's real about the business cycle? *Federal Reserve Bank of St. Louis Review*, 87(4), 435-452.

[33] Hamilton, J.D., 2011. Calling recessions in real time. *International Journal of Forecasting*, 27, 1006–1026.

[34] Hamilton, J.D., Owyang, M.T., 2009. The propagation of regional recessions. *The Review of Economics and Statistics*, 94(4), 935-947.

[35] Harding, D., Pagan, A.R., 2002. Dissecting the Cycle: A Methodological Investigation. *Journal of Monetary Economics*, 49, 365-381.

[36] Harding, D., Pagan, A.R., 2006. Synchronization of cycles. *Journal of Econometrics*, vol. 132(1), 59-79.

[37] Heckman, J.J., 1979. Sample Selection Bias as a Specification Error. *Econometrica* 47(1), 153-161.

[38] Hecq, A., Palm, F., Urbain, J.P., 2000. Permanent-Transitory Decomposition in VAR Models with Cointegration and Common Cycles. *Oxford Bulletin of Economics and Statistics* 62, 511-532.

[39] Hotelling, H., 1935. The Most Predictable Criterion. *Journal of Educational Psychology* 26, 139-142.

[40] Hotelling, H., 1936. Relations Between Two Sets of Variates. *Biometrika* 28, 321-377.

[41] Issler, J.V., Vahid, F., 2001. Common Cycles and the Importance of Transitory Shocks to Macroeconomic Aggregates. *Journal of Monetary Economics* 47, 449-475.

[42] Issler, J.V., and Vahid, F., 2006. The missing link: using the NBER recession indicator to construct coincident and leading indices of economic activity, *Journal of Econometrics* 132, pp. 281–303.

[43] Issler, J.V., Pimentel, L.M.M., 2019. Uma Medida de PIB Mensal Para o Brasil Usando o Term Spread. *Revista Brasileira de Economia* 73(1), 53-75.

[44] Kotchoni, R., Stevanovic, D., 2018. Forecasting U.S. Recessions and Economic Activity. Working paper, Université du Québec à Montréal.

[45] Lahiri, K., Moore, G., 1993. *Leading Economic Indicators: New Approaches and Forecasting Records.* New York: Cambridge University Press.

[46] McCracken, M.W., Ng, S., 2015. FRED-MD: A Monthly Database for Macroeconomic Research. Working Paper 2015-012B, Federal Reserve Bank of St. Louis.

[47] Mizon, G.E., 1984. The Encompassing Approach in Econometrics. Chapter 6 in D.F. Hendry and K.F. Wallis (eds.) *Econometrics and Quantitative Economics*, Oxford: Blackwell.

[48] Nalewaik, J.J., 2007. Estimating probabilities of recession in real time using GDP and GDI. Working paper, Federal Reserve Board.

[49] Newey, W., West, K., 1987. A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55, 703-708.

[50] Rivers, D., Vuong, Q.H., 1988. Limited Information Estimators and Exogeneity Tests for Simultaneous Probit Models. *Journal of Econometrics* 39, 347-366.

[51] Schneider, T., 2001. Analysis of incomplete climate data: Estimation of mean values and covariance matrices and imputation of missing values. *Journal of Climate* 14, 853-871.

[52] Stock J.H., Watson M.W., 1988a. A New Approach to Leading Economic Indicators. Mimeo, Harvard University, Kennedy School of Government.

[53] Stock J.H., Watson M.W., 1988b. A Probability Model of the Coincident Economic Indicator. NBER Working Paper # 2772.

[54] Stock J.H., Watson M.W., 1989. New Indexes of Leading and Coincident Economic Indicators. *NBER Macroeconomics Annual*, 351-95.

[55] Stock J.H., Watson M.W., 1991. A Probability Model of the Coincident Economic Indicators. In *"Leading Economic Indicators: New Approaches and Forecasting Records,"* K. Lahiri and G. Moore, Eds. New York: Cambridge University Press.

[56] Stock J.H., Watson M.W., 1993a. A Procedure for Predicting Recessions with Leading Indicators: Econometric Issues and Recent Experiences. In "*New Research on Business Cycles, Indicators and Forecasting,*" J.H. Stock and M.W. Watson, Eds., Chicago: University of Chicago Press, for NBER.

[57] Stock J.H., Watson M.W., 1993b. *New Research on Business Cycles, Indicators and Forecasting.* Chicago: University of Chicago Press, for NBER.

[58] Stock J.H., Watson M.W., 2002. Forecasting Using Principal Components From a Large Number of Predictors. *Journal of the American Statistical Association* 97, 1167-1179.

[59] Stock, J.H., Watson, M.W., 2010a. New Indexes of Monthly GDP. Available at http://www.princeton.edu/˜mwatson/mgdp_gdi.html

[60] Stock, J.H., Watson, M.W., 2010b. Indicators for dating business cycles: cross-history selection and comparisons. *American Economic Review: Papers and Proceedings*, Vol. 100, pp. 16–19.

[61] Stock, J.H., Watson, M.W., 2014. Estimating turning points using large data sets. *Journal of Econometrics* 178, 368-381.

[62] The Conference Board. 1997. Business Cycle Indicators. Mimeo, The Conference Board. Downloadable from http://www.tcb-indicators.org/bcioverview/bci4.pdf.

[63] Vahid, F., Engle, R.F., 1993. Common Trends and Common Cycles. *Journal of Applied Econometrics* 8, 341-360.

[64] Vahid, F., Engle, R.F., 1997. Codependent Cycles. *Journal Econometrics* 80 (2), 199-221.

[65] Vrontos, F.D., Galakis, J., Vrontos, I.D., 2021. Modeling and predicting US recessions using machine learning techniques. *International Journal of Forecasting* 37, 647-671.

[66] Wallis, K.F., 1986. Forecasting with an econometric model: The 'ragged edge' problem. *Journal of Forecasting* 5(1), 1-13.

[67] Watson, M.W., 1994. Business-Cycle Durations and Postwar Stabilization of the U.S. Economy. *American Economic Review* 84, 24-46.

[68] Wooldridge, J.M., 1994. Estimation and Inference for Dependent Processes. In "*Handbook of Econometrics IV,*" R.F. Engle and D. McFadden, Editors. Amsterdam: Elsevier Press.

[69] Zellner, A., Min, C., 1999. Forecasting Turning Points in Countries' Output Growth Rates. *Journal of Econometrics* 88, 203-206.

Appendix A. Database

Coincident Series – Stock and Watson (2014)

Table 4 – Coincident series of economic activity

	Description	Name	tcode	rlags
	Industrial production			
1	Manufacturing: Durable Goods: Wood Product (NAICS = 321)	IPG321S	5	1
2	Manufacturing: Durable Goods: Nonmetallic Mineral Product (NAICS = 327)	IPG327S	5	1
3	Manufacturing: Durable Goods: Primary Metal (NAICS = 331)	IPG331S	5	1
4	Manufacturing: Durable Goods: Fabricated Metal Product (NAICS = 332)	IPG332S	5	1
5	Manufacturing: Durable Goods: Machinery (NAICS = 333)	IPG333S	5	1
6	Manufacturing: Durable Goods: Computer and Electronic Product (NAICS = 334)	IPG334S	5	1
7	Manufacturing: Durable Goods: Electrical Equipment, Appliance, and Component (NAICS = 335)	IPG335S	5	1
8	Manufacturing: Durable Goods: Transportation Equipment (NAICS = 336)	IPG336S	5	1
9	Manufacturing: Durable Goods: Furniture and Related Product (NAICS = 337)	IPG337S	5	1
10	Manufacturing: Durable Goods: Miscellaneous (NAICS = 339)	IPG339S	5	1
11	Manufacturing: Non-Durable Goods: Food (NAICS = 311)	IPG311S	5	1
12	Manufacturing: Non-Durable Goods: Beverage (NAICS = 3121)	IPG3121S	5	1
13	Manufacturing: Non-Durable Goods: Tobacco (NAICS = 3122)	IPG3122S	5	1
14	Manufacturing: Non-Durable Goods: Textile Mills (NAICS = 313)	IPG313S	5	1
15	Manufacturing: Non-Durable Goods: Textile Product Mills (NAICS = 314)	IPG314S	5	1
16	Manufacturing: Non-Durable Goods: Apparel (NAICS = 315)	IPG315S	5	1
17	Manufacturing: Non-Durable Goods: Leather and Allied Product (NAICS = 316)	IPG316S	5	1
18	Manufacturing: Non-Durable Goods: Paper (NAICS = 322)	IPG322S	5	1
19	Manufacturing: Non-Durable Goods: Printing and Related Support Activities (NAICS = 323)	IPG323S	5	1
20	Manufacturing: Non-Durable Goods: Petroleum and Coal Products (NAICS = 324)	IPG324S	5	1
21	Manufacturing: Non-Durable Goods: Chemical (NAICS = 325)	IPG325S	5	1
22	Manufacturing: Non-Durable Goods: Plastics and Rubber Products (NAICS = 326)	IPG326S	5	1
23	Mining, Quarrying, and Oil and Gas Extraction: Oil and Gas Extraction (NAICS = 211)	IPG211S	5	1
24	Mining, Quarrying, and Oil and Gas Extraction: Mining (Except Oil and Gas) (NAICS = 212)	IPG212S	5	1
25	Mining, Quarrying, and Oil and Gas Extraction: Support Activities for Mining (NAICS = 213)	IPG213S	5	1
26	Utilities: Electric Power Generation, Transmission, and Distribution (NAICS = 2211)	IPG2211S	5	1
27	Utilities: Natural Gas Distribution (NAICS = 2212)	IPG2212S	5	1
28	Durable Consumer Goods: Automotive Products	IPB51110S	5	1
29	Durable Consumer Goods: Autos and Trucks, Consumer	IPB51111S	5	1
30	Durable Consumer Goods: Auto Parts and Allied Goods	IPB51112S	5	1
31	Durable Consumer Goods: Other Durable Goods	IPB51120S	5	1
32	Durable Consumer Goods: Computers, Video and Audio Equipment	IPB51121S	5	1
33	Durable Consumer Goods: Appliances, Furniture, and Carpeting	IPB51122S	5	1
34	Durable Consumer Goods: Miscellaneous Durable Goods	IPB51123S	5	1
35	Non-Durable Nonenergy Consumer Goods: Foods and Tobacco	IPB51211S	5	1
36	Non-Durable Nonenergy Consumer Goods: Clothing	IPB51212S	5	1
37	Non-Durable Nonenergy Consumer Goods: Chemical Products	IPB51213S	5	1
38	Non-Durable Nonenergy Consumer Goods: Paper Products	IPB51214S	5	1
39	Non-Durable Nonenergy Consumer Goods: Miscellaneous Non-Durable Goods	IPB51215S	5	1
40	Non-Durable Consumer Energy Products: Consumer Energy Products	IPB51220S	5	1
41	Non-Durable Consumer Energy Products: Fuels	IPFUELS	5	1
42	Non-Durable Consumer Energy Products: Residential Utilities	IPB51222S	5	1
43	Equipment: Transit Equipment	IPB52110S	5	1

Table 4 – Coincident series of economic activity (continued)

	Description	Name	tcode	rlags
44	Equipment: Information Processing and Related Equipment	IPB52120S	5	1
45	Equipment: Industrial and Other Equipment	IPB52130S	5	1
46	Equipment: Industrial Equipment	IPB52131S	5	1
47	Equipment: Other Equipment	IPB52132S	5	1
48	Equipment: Oil and Gas Well Drilling and Manufactured Homes	IPB52200S	5	1
49	Equipment: Defense and Space Equipment	IPB52300S	5	1
50	Durable Goods Materials: Consumer Parts	IPB53110S	5	1
51	Durable Goods Materials: Equipment Parts	IPB53120S	5	1
52	Durable Goods Materials: Computer and Other Board Assemblies and Parts	IPB53121S	5	1
53	Durable Goods Materials: Semiconductors, Printed Circuit Boards, and Other	IPB53122S	5	1
54	Durable Goods Materials: Other Equipment Parts	IPB53123S	5	1
55	Durable Goods Materials: Other Durable Materials	IPB53130S	5	1
56	Durable Goods Materials: Basic Metals	IPB53131S	5	1
57	Durable Goods Materials: Miscellaneous Durable Materials	IPB53132S	5	1
58	Non-Durable Goods Materials: Textile Materials	IPB53210S	5	1
59	Non-Durable Goods Materials: Paper Materials	IPB53220S	5	1
60	Non-Durable Goods Materials: Chemical Materials	IPB53230S	5	1
61	Non-Durable Goods Materials: Other Non-Durable Materials	IPB53240S	5	1
62	Non-Durable Goods Materials: Containers	IPB53241S	5	1
63	Non-Durable Goods Materials: Miscellaneous Non-Durable Materials	IPB53242S	5	1
64	Energy Materials: Primary Energy	IPB53310S	5	1
65	Energy Materials: Converted Fuel	IPB53320S	5	1
66	Construction Supplies	IPB54100S	5	1
67	Business Supplies	IPB54200S	5	1
68	Non-Energy Business Supplies	IPB54210S	5	1
69	Commercial Energy Products	IPB54220S	5	1
	Employment			
70	Logging	CES1011330001	5	0
71	Oil and Gas Extraction	CES1021100001	5	0
72	Mining (Except Oil And Gas)	CES1021200001	5	0
73	Support Activities for Mining	CES1021300001	5	0
74	Construction of Buildings	CES2023600001	5	0
75	Heavy and Civil Engineering Construction	CES2023700001	5	0
76	Specialty Trade Contractors	CES2023800001	5	0
77	Wood Product Manufacturing	CES3132100001	5	0
78	Nonmetallic Mineral Product Manufacturing	CES3132700001	5	0
79	Primary Metal Manufacturing	CES3133100001	5	0
80	Fabricated Metal Product Manufacturing	CES3133200001	5	0
81	Machinery Manufacturing	CES3133300001	5	0
82	Computer And Electronic Product Manufacturing	CES3133400001	5	0
83	Electrical Equipment, Appliance, And Component Manufacturing	CES3133500001	5	0
84	Transportation Equipment Manufacturing	CES3133600001	5	0
85	Furniture And Related Product Manufacturing	CES3133700001	5	0
86	Miscellaneous Manufacturing	CES3133900001	5	0
87	Food Manufacturing	CES3231100001	5	0
88	Nondurable Goods: Beverages and Tobacco Products	CES3231200001	5	0
89	Textile Mills	CES3231300001	5	0
90	Textile Product Mills	CES3231400001	5	0
91	Apparel Manufacturing	CES3231500001	5	0
92	Nondurable Goods: Leather and Allied Products	CES3231600001	5	0

Table 4 – Coincident series of economic activity (continued)

	Description	Name	tcode	rlags
93	Paper Manufacturing	CES3232200001	5	0
94	Printing and Related Support Activities	CES3232300001	5	0
95	Petroleum And Coal Products Manufacturing	CES3232400001	5	0
96	Chemical Manufacturing	CES3232500001	5	0
97	Plastics And Rubber Products Manufacturing	CES3232600001	5	0
98	Merchant Wholesalers, Durable Goods	CES4142300001	5	0
99	Merchant Wholesalers, Nondurable Goods	CES4142400001	5	0
100	Wholesale Trade Agents And Brokers	CES4142500001	5	0
101	Motor Vehicle and Parts Dealers	CES4244100001	5	0
102	Furniture And Home Furnishings Retailers	CES4244200001	5	0
103	Electronics And Appliance Retailers	CES4244300001	5	0
104	Building Material And Garden Equipment And Supplies Dealers	CES4244400001	5	0
105	Food And Beverage Retailers	CES4244500001	5	0
106	Health And Personal Care Retailers	CES4244600001	5	0
107	Gasoline Stations And Fuel Dealers	CES4244700001	5	0
108	Clothing, Clothing Accessories, Shoe, And Jewelry Retailers	CES4244800001	5	0
109	Sporting Goods, Hobby, Musical Instrument, Book, And Miscellaneous Retailers	CES4245100001	5	0
110	General Merchandise Retailers	CES4245200001	5	0
111	Other Miscellaneous Retailers	CES4245300001	5	0
112	Nonstore Retailers	CES4245400001	5	0
113	Air Transportation	CES4348100001	5	0
114	Rail Transportation	CES4348200001	5	0
115	Water Transportation	CES4348300001	5	0
116	Truck Transportation	CES4348400001	5	0
117	Transit and Ground Passenger Transportation	CES4348500001	5	0
118	Pipeline Transportation	CES4348600001	5	0
119	Scenic and Sightseeing Transportation	CES4348700001	5	0
120	Support Activities for Transportation	CES4348800001	5	0
121	Couriers and Messengers	CES4349200001	5	0
122	Warehousing and Storage	CES4349300001	5	0
123	Utilities	CES4422000001	5	0
124	Publishing Industries	CES5051100001	5	0
125	Motion Picture and Sound Recording Industries	CES5051200001	5	0
126	Broadcasting And Content Providers	CES5051500001	5	0
127	Telecommunications	CES5051700001	5	0
128	Computing Infrastructure Providers, Data Processing, Web Hosting, And Related Services	CES5051800001	5	0
129	Web Search Portals, Libraries, Archives, And Other Information Services	CES5051900001	5	0
130	Monetary Authorities-Central Bank	CES5552100001	5	0
131	Credit Intermediation and Related Activities	CES5552200001	5	0
132	Securities, Commodity Contracts, Funds, Trusts, And Other Financial Vehicles, Investments, And Related Activities	CES5552300001	5	0
133	Insurance Carriers and Related Activities	CES5552400001	5	0
134	Financial Activities: Funds, Trusts, and Other Financial Vehicles	CES5552500001	5	0
135	Real Estate	CES5553100001	5	0
136	Rental and Leasing Services	CES5553200001	5	0
137	Lessors Of Nonfinancial Intangible Assets (Except Copyrighted Works)	CES5553300001	5	0
138	Professional, Scientific, And Technical Services	CES6054000001	5	0
139	Management of Companies and Enterprises	CES6055000001	5	0
140	Administrative And Support And Waste Management And Remediation Services	CES6056000001	5	0
141	Private Educational Services	CES6561000001	5	0
142	Health Care	CES6562000101	5	0
143	Social Assistance	CES6562400001	5	0

Table 4 – Coincident series of economic activity (continued)

	Description	Name	tcode	rlags
144	Arts, Entertainment, and Recreation	CES7071000001	5	0
145	Accommodation	CES7072100001	5	0
146	Food Services and Drinking Places	CES7072200001	5	0
147	Federal	CES9091000001	5	0
148	State Government	CES9092000001	5	0
149	Local Government	CES9093000001	5	0
150	Durable Goods	DMANEMP	5	0
151	Nondurable Goods	NDMANEMP	5	0
152	Construction	USCONS	5	0
153	Private Education And Health Services	USEHS	5	0
154	Financial Activities	USFIRE	5	0
155	Government	USGOVT	5	0
156	Information	USINFO	5	0
157	Leisure and Hospitality	USLAH	5	0
158	Mining and Logging	USMINE	5	0
159	Professional and Business Services	USPBS	5	0
160	Other Services	USSERV	5	0
161	Retail Trade	USTRADE	5	0
162	Wholesale Trade	USWTRADE	5	0
163	Trans/Utilities (USTPU−USTRADE−USWTRADE)	USPU	5	0
	Personal Income			
164	Wages and salaries: Private industries: Goods-producing industries: Manufacturing	A552RC1M027SBEA	5	1
165	Wages and salaries: Private industries: Distributive industries	A212RC1M027SBEA	5	1
166	Wages and salaries: Private industries: Service industries	A218RC1M027SBEA	5	1
167	Wages and salaries: Private industries: Goods-producing industries: Manufacturing	N552RC1M027SBEA	5	1
168	Wages and salaries: Private industries: Services-producing industries: Trade, transportation, and utilities	N234RC1M027SBEA	5	1
169	Wages and salaries: Private industries: Services-producing industries: Other services-producing industries	N235RC1M027SBEA	5	1
170	Compensation of employees, received: Wage and salary disbursements: Government	B202RC1	5	1
171	Compensation of Employees, Received: Supplements to Wages and Salaries	A038RC1	5	1
172	Proprietors' income with inventory valuation and capital consumption adjustments: Farm	B042RC1	5	1
173	Proprietors' income with inventory valuation and capital consumption adjustments: Nonfarm	A045RC1	5	1
174	Rental income of persons with capital consumption adjustment	A048RC1	5	1
175	Personal Income Receipts on Assets: Personal Interest Income	PII	5	1
176	Personal Income Receipts on Assets: Personal Dividend Income	PDI	5	1
177	Personal current taxes	W055RC1	5	1

Source: All series were obtained from the FRED database. Industrial production series are provided by the Board of Governors of the Federal Reserve System; employment series by the U.S. Bureau of Labor Statistics; and personal income series by the U.S. Bureau of Economic Analysis.

Note: "Name" means the series' internal name usually provided in the release data files. "tcode" means the code of the transformation applied to the series. "rlags" means the approximated lag of the last observation available in relation to t. We consider the lag to be zero if the observation of $t-1$ is normally available by the 15th of month t. Transformation codes: (1) no transformation, (2) Δx_t, (3) $\Delta^2 x_t$, (4) $\log(x_t)$, (5) $\Delta \log(x_t)$, (6) $\Delta^2 \log(x_t)$, (7) $\Delta\left(x_t/x_{t-1} - 1\right)$.

Leading Series – Costa et al. (2021)

Table 5 – Leading series of economic activity

	Description	Name	Source	tcode	rlags
	Outcome and income				
1	Real Personal Income	RPI	FRED-MD	5	1
2	Real personal income ex transfer receipts	W875RX1	FRED-MD	5	1
3	IP Index	INDPRO	FRED-MD	5	1
4	IP: Final Products and Nonindustrial Supplies	IPFPNSS	FRED-MD	5	1
5	IP: Final Products (Market Group)	IPFINAL	FRED-MD	5	1
6	IP: Consumer Goods	IPCONGD	FRED-MD	5	1
7	IP: Durable Consumer Goods	IPDCONGD	FRED-MD	5	1
8	IP: Nondurable Consumer Goods	IPNCONGD	FRED-MD	5	1
9	IP: Business Equipment	IPBUSEQ	FRED-MD	5	1
10	IP: Materials	IPMAT	FRED-MD	5	1
11	IP: Durable Materials	IPDMAT	FRED-MD	5	1
12	IP: Nondurable Materials	IPNMAT	FRED-MD	5	1
13	IP: Manufacturing (SIC)	IPMANSICS	FRED-MD	5	1
14	IP: Residential Utilities	IPB51222S	FRED-MD	5	1
15	IP: Fuels	IPFUELS	FRED-MD	5	1
16	Capacity Utilization: Manufacturing	CUMFNS	FRED-MD	2	1
	Labor market				
17	Help-Wanted Index for United States	HWI	FRED-MD	5	2
18	Ratio of Help Wanted/No. Unemployed	HWIURATIO	FRED-MD	2	2
19	Civilian Labor Force	CLF16OV	FRED-MD	5	1
20	Civilian Employment	CE16OV	FRED-MD	5	1
21	Civilian Unemployment Rate	UNRATE	FRED-MD	2	1
22	Average Duration of Unemployment (Weeks)	UEMPMEAN	FRED-MD	2	1
23	Civilians Unemployed - Less Than 5 Weeks	UEMPLT5	FRED-MD	5	1
24	Civilians Unemployed for 5-14 Weeks	UEMP5TO14	FRED-MD	5	1
25	Civilians Unemployed - 15 Weeks& Over	UEMP15OV	FRED-MD	5	1
26	Civilians Unemployed for 15-26 Weeks	UEMP15T26	FRED-MD	5	1
27	Civilians Unemployed for 27 Weeks and Over	UEMP27OV	FRED-MD	5	1
28	Initial Claims	CLAIMSx	FRED-MD	5	1
29	All Employees: Total nonfarm	PAYEMS	FRED-MD	5	1
30	All Employees: Goods-Producing Industries	USGOOD	FRED-MD	5	1
31	All Employees: Mining and Logging: Mining	CES1021000001	FRED-MD	5	1
32	All Employees: Construction	USCONS	FRED-MD	5	1
33	All Employees: Manufacturing	MANEMP	FRED-MD	5	1
34	All Employees: Durable goods	DMANEMP	FRED-MD	5	1
35	All Employees: Nondurable goods	NDMANEMP	FRED-MD	5	1
36	All Employees: Service-Providing Industries	SRVPRD	FRED-MD	5	1
37	All Employees: Trade, Transportation& Utilities	USTPU	FRED-MD	5	1
38	All Employees: Wholesale Trade	USWTRADE	FRED-MD	5	1
39	All Employees: Retail Trade	USTRADE	FRED-MD	5	1
40	All Employees: Financial Activities	USFIRE	FRED-MD	5	1
41	All Employees: Government	USGOVT	FRED-MD	5	1
42	Avg Weekly Hours : Goods-Producing	CES0600000007	FRED-MD	1	1
43	Avg Weekly Overtime Hours : Manufacturing	AWOTMAN	FRED-MD	2	1
44	Avg Weekly Hours : Manufacturing	AWHMAN	FRED-MD	1	1

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
45	Avg Hourly Earnings : Goods-Producing	CES0600000008	FRED-MD	5	1
46	Avg Hourly Earnings : Construction	CES2000000008	FRED-MD	5	1
47	Avg Hourly Earnings : Manufacturing	CES3000000008	FRED-MD	5	1
	Housing				
48	Housing Starts: Total New Privately Owned	HOUST	FRED-MD	4	1
49	Housing Starts, Northeast	HOUSTNE	FRED-MD	4	1
50	Housing Starts, Midwest	HOUSTMW	FRED-MD	4	1
51	Housing Starts, South	HOUSTS	FRED-MD	4	1
52	Housing Starts, West	HOUSTW	FRED-MD	4	1
53	New Private Housing Permits (SAAR)	PERMIT	FRED-MD	4	1
54	New Private Housing Permits, Northeast (SAAR)	PERMITNE	FRED-MD	4	1
55	New Private Housing Permits, Midwest (SAAR)	PERMITMW	FRED-MD	4	1
56	New Private Housing Permits, South (SAAR)	PERMITS	FRED-MD	4	1
57	New Private Housing Permits, West (SAAR)	PERMITW	FRED-MD	4	1
	Consumption, orders, and inventories				
58	Real personal consumption expenditures	DPCERA3M086SBEA	FRED-MD	5	1
59	Real Manu. and Trade Industries Sales	CMRMTSPLx	FRED-MD	5	2
60	Retail and Food Services Sales	RETAILx	FRED-MD	5	1
61	New Orders for Consumer Goods	ACOGNO	FRED-MD	5	2
62	New Orders for Durable Goods	AMDMNOx	FRED-MD	5	1
63	New Orders for Nondefense Capital Goods	ANDENOx	FRED-MD	5	1
64	Unfilled Orders for Durable Goods	AMDMUOx	FRED-MD	5	1
65	Total Business Inventories	BUSINVx	FRED-MD	5	2
66	Total Business: Inventories to Sales Ratio	ISRATIOx	FRED-MD	2	2
67	Consumer Sentiment Index	UMCSENTx	FRED-MD	2	1
	Money and credit				
68	M1 Money Stock	M1SL	FRED-MD	5	1
69	M2 Money Stock	M2SL	FRED-MD	5	1
70	Real M2 Money Stock	M2REAL	FRED-MD	5	1
71	Monetary Base	BOGMBASE	FRED-MD	5	1
72	Total Reserves of Depository Institutions	TOTRESNS	FRED-MD	5	1
73	Reserves Of Depository Institutions	NONBORRES	FRED-MD	7	1
74	Commercial and Industrial Loans	BUSLOANS	FRED-MD	5	1
75	Real Estate Loans at All Commercial Banks	REALLN	FRED-MD	5	1
76	Total Nonrevolving Credit	NONREVSL	FRED-MD	5	2
77	Nonrevolving consumer credit to Personal Income	CONSPI	FRED-MD	2	2
78	Consumer Motor Vehicle Loans Outstanding	DTCOLNVHFNM	FRED-MD	5	2
79	Total Consumer Loans and Leases Outstanding	DTCTHFNM	FRED-MD	5	2
80	Securities in Bank Credit at All Commercial Banks	INVEST	FRED-MD	5	1
	Interest and exchange rates				
81	Effective Federal Funds Rate	FEDFUNDS	FRED-MD	2	1
82	3-Month AA Financial Commercial Paper Rate	CP3Mx	FRED-MD	2	1
83	3-Month Treasury Bill	TB3MS	FRED-MD	2	1
84	6-Month Treasury Bill	TB6MS	FRED-MD	2	1
85	1-Year Treasury Rate	GS1	FRED-MD	2	1
86	5-Year Treasury Rate	GS5	FRED-MD	2	1

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
87	10-Year Treasury Rate	GS10	FRED-MD	2	1
88	Moody's Seasoned Aaa Corporate Bond Yield	AAA	FRED-MD	2	1
89	Moody's Seasoned Baa Corporate Bond Yield	BAA	FRED-MD	2	1
90	3-Month Commercial Paper Minus FEDFUNDS	COMPAPFFx	FRED-MD	1	1
91	3-Month Treasury C Minus FEDFUNDS	TB3SMFFM	FRED-MD	1	1
92	6-Month Treasury C Minus FEDFUNDS	TB6SMFFM	FRED-MD	1	1
93	1-Year Treasury C Minus FEDFUNDS	T1YFFM	FRED-MD	1	1
94	5-Year Treasury C Minus FEDFUNDS	T5YFFM	FRED-MD	1	1
95	10-Year Treasury C Minus FEDFUNDS	T10YFFM	FRED-MD	1	1
96	Moody's Aaa Corporate Bond Minus FEDFUNDS	AAAFFM	FRED-MD	1	1
97	Moody's Baa Corporate Bond Minus FEDFUNDS	BAAFFM	FRED-MD	1	1
98	Trade Weighted U.S. Dollar Index	TWEXAFEGSMTHx	FRED-MD	5	1
99	Switzerland / U.S. Foreign Exchange Rate	EXSZUSx	FRED-MD	5	1
100	Japan / U.S. Foreign Exchange Rate	EXJPUSx	FRED-MD	5	1
101	U.S. / U.K. Foreign Exchange Rate	EXUSUKx	FRED-MD	5	1
102	Canada / U.S. Foreign Exchange Rate	EXCAUSx	FRED-MD	5	1
	Prices				
103	PPI: Finished Goods	WPSFD49207	FRED-MD	5	1
104	PPI: Finished Consumer Goods	WPSFD49502	FRED-MD	5	1
105	PPI: Intermediate Materials	WPSID61	FRED-MD	5	1
106	PPI: Crude Materials	WPSID62	FRED-MD	5	1
107	Crude Oil, spliced WTI and Cushing	OILPRICEx	FRED-MD	5	1
108	PPI: Metals and metal products	PPICMM	FRED-MD	5	1
109	CPI : All Items	CPIAUCSL	FRED-MD	5	1
110	CPI : Apparel	CPIAPPSL	FRED-MD	5	1
111	CPI : Transportation	CPITRNSL	FRED-MD	5	1
112	CPI : Medical Care	CPIMEDSL	FRED-MD	5	1
113	CPI : Commodities	CUSR0000SAC	FRED-MD	5	1
114	CPI : Durables	CUSR0000SAD	FRED-MD	5	1
115	CPI : Services	CUSR0000SAS	FRED-MD	5	1
116	CPI : All Items Less Food	CPIULFSL	FRED-MD	5	1
117	CPI : All items less shelter	CUSR0000SA0L2	FRED-MD	5	1
118	CPI : All items less medical care	CUSR0000SA0L5	FRED-MD	5	1
119	Personal Cons. Expend.: Chain Index	PCEPI	FRED-MD	5	1
120	Personal Cons. Exp: Durable goods	DDURRG3M086SBEA	FRED-MD	5	1
121	Personal Cons. Exp: Nondurable goods	DNDGRG3M086SBEA	FRED-MD	5	1
122	Personal Cons. Exp: Services	DSERRG3M086SBEA	FRED-MD	5	1
	Stock market				
123	S& P's Common Stock Price Index: Composite	S& P 500	FRED-MD	5	1
124	S& P's Common Stock Price Index: Industrials	S& P: indust	FRED-MD	5	1
125	S& P's Composite Common Stock: Dividend Yield	S& P div yield	FRED-MD	2	1
126	S& P's Composite Common Stock: Price-Earnings Ratio	S& P PE ratio	FRED-MD	5	3
127	VIX	VIXCLSx	FRED-MD	1	1
	Industrial production				
128	Production of Total Industry in Austria	AUTPROINDMISMEI	FRED	5	3
129	Production of Total Industry in Belgium	BELPROINDMISMEI	FRED	5	2
130	Production of Total Industry in Brazil	BRAPROINDMISMEI	FRED	5	2

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
131	Production of Total Industry in Canada	CANPROINDMISMEI	FRED	5	3
132	Production of Total Industry in Chile	CHLPROINDMISMEI	FRED	5	1
133	Production of Total Industry in Czech Republic	CZEPROINDMISMEI	FRED	5	2
134	Production of Total Industry in Denmark	DNKPROINDMISMEI	FRED	5	2
135	Production of Total Industry in Finland	FINPROINDMISMEI	FRED	5	3
136	Production of Total Industry in France	FRAPROINDMISMEI	FRED	5	2
137	Production of Total Industry in Germany	DEUPROINDMISMEI	FRED	5	2
138	Production of Total Industry in Greece	GRCPROINDMISMEI	FRED	5	2
139	Production of Total Industry in Hungary	HUNPROINDMISMEI	FRED	5	2
140	Production of Total Industry in Ireland	IRLPROINDMISMEI	FRED	5	2
141	Production of Total Industry in Israel	ISRPROINDMISMEI	FRED	5	3
142	Production of Total Industry in Italy	ITAPROINDMISMEI	FRED	5	2
143	Production of Total Industry in Japan	JPNPROINDMISMEI	FRED	5	1
144	Production of Total Industry in Korea	KORPROINDMISMEI	FRED	5	1
145	Production of Total Industry in Netherlands	NLDPROINDMISMEI	FRED	5	2
146	Production of Total Industry in Norway	NORPROINDMISMEI	FRED	5	2
147	Production of Total Industry in Poland	POLPROINDMISMEI	FRED	5	1
148	Production of Total Industry in Portugal	PRTPROINDMISMEI	FRED	5	1
149	Production of Total Industry in Russian Federation	RUSPROINDMISMEI	FRED	5	3
150	Production of Total Industry in Spain	ESPPROINDMISMEI	FRED	5	2
151	Production of Total Industry in Sweden	SWEPROINDMISMEI	FRED	5	2
152	Production of Total Industry in Turkey	TURPROINDMISMEI	FRED	5	2
153	Production of Total Industry in United Kingdom	GBRPROINDMISMEI	FRED	5	2
154	Industrial Production: Durable Manufacturing	IPDMAN	FRED	5	0
155	Industrial Production: Durable Goods: Iron and Steel Products	IPG3311A2S	FRED	5	0
156	Industrial Production: Durable Goods: Alumina and Aluminum Production and Processing	IPG3313S	FRED	5	0
157	Industrial Production: Durable Goods: Raw Steel	IPN3311A2RS	FRED	5	1
158	Industrial Production: Durable Goods: Automotive Products	IPB51110S	FRED	5	0
159	Industrial Production: Durable Goods: Cement and Concrete Product	IPG3273S	FRED	5	0
160	Industrial Production: Durable Goods: Primary Metal	IPG331S	FRED	5	0
161	Industrial Production: Durable Goods: Machinery	IPG333S	FRED	5	0
162	Industrial Production: Durable Goods: Aerospace and Misc. Transportation Equipment	IPG3364T9S	FRED	5	0
163	Industrial Production: Non-Durable Goods: Food	IPG311S	FRED	5	0
164	Industrial Production: Non-Durable Goods: Petroleum and Coal Products	IPG324S	FRED	5	0
165	Industrial Production: Non-Durable Goods: Chemical	IPG325S	FRED	5	0
166	Industrial Production: Non-Durable Goods: Plastics and Rubber Products	IPG326S	FRED	5	0
167	Industrial Production: Non-Durable Goods: Petroleum Refineries	IPG32411S	FRED	5	0
168	Industrial Production: Non-Durable Goods: Pharmaceutical and Medicine	IPG3254S	FRED	5	0
169	Industrial Production: Non-Durable Goods: Plastics Material and Resin	IPN325211S	FRED	5	2
170	Industrial Production: Construction Supplies	IPB54100S	FRED	5	0
171	Industrial Production: Non-Energy, Total	IPX5001ES	FRED	5	0
172	Industrial Production: Energy, Total	IPB50089S	FRED	5	0

Table 5 – Leading series of economic activity (continued)

	DESCRIPTION	NAME	SOURCE	TCODE	RLAGS
173	Industrial Production: Electric and Gas Utilities	IPUTIL	FRED	5	0
174	Industrial Production: Electric Power Generation, Transmission, and Distribution	IPG2211S	FRED	5	0
175	Industrial Production: Mining: Oil and Gas Extraction	IPG211S	FRED	5	0
176	Industrial Production: Mining: Copper, Nickel, Lead, and Zinc Mining	IPG21223S	FRED	5	0
177	Industrial Production: Mining: Coal Mining	IPN2121S	FRED	5	0
178	Industrial Production: Mining: Iron Ore Mining	IPN21221S	FRED	5	3
179	Industrial Production: Mining: Drilling Oil and Gas Wells	IPN213111S	FRED	5	0
	Economic uncertainty				
180	Policy-related economic uncertainty index for Australia	EPU_Australia	Economic Policy Uncertainty	1	1
181	Policy-related economic uncertainty index for Brazil	EPU_Brazil	Economic Policy Uncertainty	1	1
182	Policy-related economic uncertainty index for Canada	EPU_Canada	Economic Policy Uncertainty	1	1
183	Policy-related economic uncertainty index for Chile	EPU_Chile	Economic Policy Uncertainty	1	1
184	Policy-related economic uncertainty index for China	EPU_China	Economic Policy Uncertainty	1	1
185	Policy-related economic uncertainty index for Colombia	EPU_Colombia	Economic Policy Uncertainty	1	1
186	Policy-related economic uncertainty index for France	EPU_France	Economic Policy Uncertainty	1	1
187	Policy-related economic uncertainty index for Germany	EPU_Germany	Economic Policy Uncertainty	1	1
188	Policy-related economic uncertainty index for Greece	EPU_Greece	Economic Policy Uncertainty	1	1
189	Policy-related economic uncertainty index for India	EPU_India	Economic Policy Uncertainty	1	1
190	Policy-related economic uncertainty index for Ireland	EPU_Ireland	Economic Policy Uncertainty	1	1
191	Policy-related economic uncertainty index for Italy	EPU_Italy	Economic Policy Uncertainty	1	1
192	Policy-related economic uncertainty index for Japan	EPU_Japan	Economic Policy Uncertainty	1	1
193	Policy-related economic uncertainty index for Korea	EPU_Korea	Economic Policy Uncertainty	1	1
194	Policy-related economic uncertainty index for Netherlands	EPU_Netherlands	Economic Policy Uncertainty	1	1
195	Policy-related economic uncertainty index for Russia	EPU_Russia	Economic Policy Uncertainty	1	1
196	Policy-related economic uncertainty index for Spain	EPU_Spain	Economic Policy Uncertainty	1	1
197	Policy-related economic uncertainty index for Singapore	EPU_Singapore	Economic Policy Uncertainty	1	1
198	Policy-related economic uncertainty index for UK	EPU_UK	Economic Policy Uncertainty	1	1
199	Policy-related economic uncertainty index for US	EPU_US	Economic Policy Uncertainty	1	1
200	Geopolitical Risk Index of Caldara and Iacoviello: Argentina	GPRC_ARG	Geopolitical Risk	1	0
201	Geopolitical Risk Index of Caldara and Iacoviello: Brazil	GPRC_BRA	Geopolitical Risk	1	0
202	Geopolitical Risk Index of Caldara and Iacoviello: China	GPRC_CHN	Geopolitical Risk	1	0
203	Geopolitical Risk Index of Caldara and Iacoviello: Colombia	GPRC_COL	Geopolitical Risk	1	0
204	Geopolitical Risk Index of Caldara and Iacoviello: Hong Kong	GPRC_HKG	Geopolitical Risk	1	0

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
205	Geopolitical Risk Index of Caldara and Iacoviello: India	GPRC_IND	Geopolitical Risk	1	0
206	Geopolitical Risk Index of Caldara and Iacoviello: Indonesia	GPRC_IDN	Geopolitical Risk	1	0
207	Geopolitical Risk Index of Caldara and Iacoviello: Israel	GPRC_ISR	Geopolitical Risk	1	0
208	Geopolitical Risk Index of Caldara and Iacoviello: Korea	GPRC_KOR	Geopolitical Risk	1	0
209	Geopolitical Risk Index of Caldara and Iacoviello: Malaysia	GPRC_MYS	Geopolitical Risk	1	0
210	Geopolitical Risk Index of Caldara and Iacoviello: Mexico	GPRC_MEX	Geopolitical Risk	1	0
211	Geopolitical Risk Index of Caldara and Iacoviello: Philippines	GPRC_PHL	Geopolitical Risk	1	0
212	Geopolitical Risk Index of Caldara and Iacoviello: Russia	GPRC_RUS	Geopolitical Risk	1	0
213	Geopolitical Risk Index of Caldara and Iacoviello: Saudi Arabia	GPRC_SAU	Geopolitical Risk	1	0
214	Geopolitical Risk Index of Caldara and Iacoviello: South Africa	GPRC_ZAF	Geopolitical Risk	1	0
215	Geopolitical Risk Index of Caldara and Iacoviello: Thailand	GPRC_THA	Geopolitical Risk	1	0
216	Geopolitical Risk Index of Caldara and Iacoviello: Turkey	GPRC_TUR	Geopolitical Risk	1	0
217	Geopolitical Risk Index of Caldara and Iacoviello: Ukraine	GPRC_UKR	Geopolitical Risk	1	0
218	Geopolitical Risk Index of Caldara and Iacoviello: Venezuela	GPRC_VEN	Geopolitical Risk	1	0
219	Geopolitical Risk Index of Caldara and Iacoviello: Recent GPR	GPR	Geopolitical Risk	1	0
220	Geopolitical Risk Index of Caldara and Iacoviello: Recent GPR Threats	GPRT	Geopolitical Risk	1	0
221	Geopolitical Risk Index of Caldara and Iacoviello: Recent GPR Acts	GPRA	Geopolitical Risk	1	0
222	Geopolitical Risk Index of Caldara and Iacoviello: Historical GPR	GPRH	Geopolitical Risk	1	0
223	Geopolitical Risk Index of Caldara and Iacoviello: Historical GPR Threats	GPRHT	Geopolitical Risk	1	0
224	Geopolitical Risk Index of Caldara and Iacoviello: Historical GPR Acts	GPRHA	Geopolitical Risk	1	0
	Leading indicator				
225	OECD Composite Leading Indicator (CLI) for Australia	CLI_Australia	OECD	2	0
226	OECD Composite Leading Indicator (CLI) for Brazil	CLI_Brazil	OECD	2	0
227	OECD Composite Leading Indicator (CLI) for Canada	CLI_Canada	OECD	2	0
228	OECD Composite Leading Indicator (CLI) for China	CLI_China	OECD	2	0
229	OECD Composite Leading Indicator (CLI) for France	CLI_France	OECD	2	0
230	OECD Composite Leading Indicator (CLI) for Germany	CLI_Germany	OECD	2	0
231	OECD Composite Leading Indicator (CLI) for India	CLI_India	OECD	2	0
232	OECD Composite Leading Indicator (CLI) for Indonesia	CLI_Indonesia	OECD	2	0
233	OECD Composite Leading Indicator (CLI) for Italy	CLI_Italy	OECD	2	0
234	OECD Composite Leading Indicator (CLI) for Japan	CLI_Japan	OECD	2	0

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
235	OECD Composite Leading Indicator (CLI) for Korea	CLI_Korea	OECD	2	0
236	OECD Composite Leading Indicator (CLI) for Mexico	CLI_Mexico	OECD	2	0
237	OECD Composite Leading Indicator (CLI) for South Africa	CLI_South_Africa	OECD	2	0
238	OECD Composite Leading Indicator (CLI) for Spain	CLI_Spain	OECD	2	0
239	OECD Composite Leading Indicator (CLI) for Turkey	CLI_Turkey	OECD	2	0
240	OECD Composite Leading Indicator (CLI) for United Kingdom	CLI_UK	OECD	2	0
241	OECD Composite Leading Indicator (CLI) for United States of America	CLI_USA	OECD	2	0
242	OECD Composite Leading Indicator (CLI) for Big four European	CLI_Big4_European	OECD	2	0
243	OECD Composite Leading Indicator (CLI) for G7	CLI_G7	OECD	2	0
244	OECD Composite Leading Indicator (CLI) for NAFTA	CLI_NAFTA	OECD	2	0
245	OECD Composite Leading Indicator (CLI) for Major five Asia	CLI_Major5_Asia	OECD	2	0
	Real business conditions in the U.S.				
246	Aruoba-Diebold-Scotti Business Conditions Index	ADS_INDEX	Federal Reserve Bank of Philadelphia	1	0
	Quantitative Easing				
247	Total Assets (US$ trillions), Federal Reserve	QE_FED	Federal Reserve Bank of St. Louis	5	0
248	Total Assets (US$ trillions), Federal Reserve + European Central Bank + Bank of Japan	QE_FED_ECB_BOJ	Federal Reserve Bank of St. Louis	5	0
	Energy Outlook				
249	Liquid Fuels Consumption, World (million barrels per day)	STEO.PATC_WORLD.M	Short-Term Energy Outlook, U.S. EIA	5	0
250	Liquid Fuels Consumption, OECD (million barrels per day)	STEO.PATC_OECD.M	Short-Term Energy Outlook, U.S. EIA	5	0
251	Liquid Fuels Consumption, non-OECD (million barrels per day)	STEO.PATC_NON_OECD.M	Short-Term Energy Outlook, U.S. EIA	5	0
252	Crude Oil Production Capacity, OPEC (million barrels per day)	STEO.COPC_OPEC.M	Short-Term Energy Outlook, U.S. EIA	5	0
253	Petroleum Product Supply, Total (million barrels per day)	STEO.PASUPPLY.M	Short-Term Energy Outlook, U.S. EIA	5	0
254	Crude Oil Production, U.S. (million barrels per day)	STEO.COPRPUS.M	Short-Term Energy Outlook, U.S. EIA	5	0
255	Crude Oil and Other Liquids Inventory, U.S. (million barrels)	STEO.PASC_US.M	Short-Term Energy Outlook, U.S. EIA	5	0
256	Petroleum Net Imports, U.S. (million barrels per day)	STEO.PAIMPORT.M	Short-Term Energy Outlook, U.S. EIA	5	0
257	Net Inventory Withdrawals, Crude Oil and Other Liquids, U.S. (million barrels per day)	STEO.T3_STCHANGE_US.M	Short-Term Energy Outlook, U.S. EIA	5	0
258	Natural Gas Henry Hub Spot Price, U.S. (dollars per thousand cubic feet)	STEO.NGHHMCF.M	Short-Term Energy Outlook, U.S. EIA	5	0
259	Cost of Coal Delivered to Electric Generating Plants, U.S. (dollars per million Btu)	STEO.CLEUDUS.M	Short-Term Energy Outlook, U.S. EIA	5	0
260	Coal Production, U.S. (million short tons)	STEO.CLPRPUS_TON.M	Short-Term Energy Outlook, U.S. EIA	5	0
261	Coal Consumption, U.S. (million short tons)	STEO.CLTCPUS_TON.M	Short-Term Energy Outlook, U.S. EIA	5	0
262	Consumption of Electricity, U.S. (billion kilowatthours)	STEO.ELCOTWH.M	Short-Term Energy Outlook, U.S. EIA	5	0
263	Raw Steel Production, U.S. (million short tons per day)	STEO.RSPRPUS.M	Short-Term Energy Outlook, U.S. EIA	5	0

Table 5 – Leading series of economic activity (continued)

	Description	Name	Source	tcode	rlags
264	Aircraft Utilization, U.S. (revenue ton-miles/day thousands)	STEO.RMZZPUS.M	Short-Term Energy Outlook, U.S. EIA	5	0
265	Vehicle Miles Traveled, U.S. (million miles/day)	STEO.MVVMPUS.M	Short-Term Energy Outlook, U.S. EIA	5	0
	Financial markets				
266	Europe Brent Spot FOB U$/BBL Daily	EUROPE_BRENT_SPOT_FOB_U$	Thomson Reuters	5	0
267	Baltic Exchange Dry Index (BDI)	BALTIC_DRY	Thomson Reuters	5	0
268	CBOE SPX VOLATILITY VIX	VIX	Thomson Reuters	1	0
269	US Dollar index DXY	US_DOLLAR_INDEX	Thomson Reuters	5	0
270	MSCI Emerging Markets U$	MSCI_EM	Thomson Reuters	5	0
271	MSCI World U$	MSCI_WORLD	Thomson Reuters	5	0
272	EURO STOXX 50	EURO_STOXX50	Thomson Reuters	5	0
273	S& P500 ES ENERGY	SP500_ENERGY	Thomson Reuters	5	0
274	S& P GSCI Energy Total Return - RETURN IND. (OFCL)	SP_GSCI_ENERGY	Thomson Reuters	5	0
275	CRB BLS Spot Index (1967=100)	CRB	Thomson Reuters	5	0
276	CRB BLS Spot Index Metals	CRB_METALS	Thomson Reuters	5	0
277	CRB BLS Spot Index Foodstuffs	CRB_FOOD	Thomson Reuters	5	0
278	Futures Brent crude oil, Intercontinental Exchange (ICE), 1 month	FUTURE_BRENT_M1	Thomson Reuters	5	0
279	Futures Brent crude oil, Intercontinental Exchange (ICE), 2 months	FUTURE_BRENT_M2	Thomson Reuters	5	0
280	Futures Brent crude oil, Intercontinental Exchange (ICE), 3 months	FUTURE_BRENT_M3	Thomson Reuters	5	0
281	Futures Brent crude oil, Intercontinental Exchange (ICE), 4 months	FUTURE_BRENT_M4	Thomson Reuters	5	0
282	Futures Brent crude oil, Intercontinental Exchange (ICE), 5 months	FUTURE_BRENT_M5	Thomson Reuters	5	0
283	Futures Brent crude oil, Intercontinental Exchange (ICE), 6 months	FUTURE_BRENT_M6	Thomson Reuters	5	0
284	Futures Brent crude oil, Intercontinental Exchange (ICE), 7 months	FUTURE_BRENT_M7	Thomson Reuters	5	0
285	Futures Brent crude oil, Intercontinental Exchange (ICE), 8 months	FUTURE_BRENT_M8	Thomson Reuters	5	0
286	Futures Brent crude oil, Intercontinental Exchange (ICE), 9 months	FUTURE_BRENT_M9	Thomson Reuters	5	0
287	Futures Brent crude oil, Intercontinental Exchange (ICE), 10 months	FUTURE_BRENT_M10	Thomson Reuters	5	0
288	Futures Brent crude oil, Intercontinental Exchange (ICE), 11 months	FUTURE_BRENT_M11	Thomson Reuters	5	0
289	Futures Brent crude oil, Intercontinental Exchange (ICE), 12 months	FUTURE_BRENT_M12	Thomson Reuters	5	0
290	Futures Brent crude oil, Intercontinental Exchange (ICE), 24 months	FUTURE_BRENT_M24	Thomson Reuters	5	0
291	Futures Brent crude oil, Intercontinental Exchange (ICE), 36 months	FUTURE_BRENT_M36	Thomson Reuters	5	0
292	Futures Brent crude oil, Intercontinental Exchange (ICE), 48 months	FUTURE_BRENT_M48	Thomson Reuters	5	0
293	Futures Brent crude oil, Intercontinental Exchange (ICE), 60 months	FUTURE_BRENT_M60	Thomson Reuters	5	0
294	Futures Brent crude oil, Intercontinental Exchange (ICE), 72 months	FUTURE_BRENT_M72	Thomson Reuters	5	0

Note: "Name" means the series' internal name usually provided in the release data files. "tcode" means the code of the transformation applied to the series. "rlags" means the approximated lag of the last observation available in relation to t. We consider the lag to be zero if the observation of $t-1$ is normally available by the 15th of month t. Transformation codes: (1) no transformation, (2) Δx_t, (3) $\Delta^2 x_t$, (4) $\log(x_t)$, (5) $\Delta \log(x_t)$, (6) $\Delta^2 \log(x_t)$, (7) $\Delta\left(x_t/x_{t-1}-1\right)$.

Appendix B. Main techniques for dating recessions and computing their probability of occurrence

Stock and Watson (1989, 2002, 2014)

On different papers, Stock and Watson have had a large contribution to (and influence on) the literature of dating recessions and computing their probability of occurrence. In this section, we will focus mainly in three of their papers for the sake of space.

Stock and Watson (1989) lay down the foundations of what most of this literature would become in the future. They ask: (i) "is it possible to develop a formal probability model that gives rise to the indexes of leading and coincident variables?"; (ii) "what are the best variables to use as components of the leading index?"; (iii) "given these variables, what is the best way to combine them to produce useful and reliable indexes?"

Instead of focusing on an aggregate measure of economic activity, they follow Burns and Mitchell's (1946, p. 3) definition that a business cycle "consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals..." So, Stock and Watson (1989) proposed the statistical foundations for implementing the main ideas proposed by Burns and Mitchell. By looking at a large number of economic-activity series at the same time, they opted not to follow a single aggregate series as GDP, which led further down the road to the use of techniques that could eventually deal with big data.

To model coincident and leading indices of economic activity, they constructed a state-space model where X_t denotes an $n \times 1$ vector of the logarithms of macroeconomic variables that are hypothesized to move contemporaneously with overall economic conditions. X_t consists of two components: the common unobserved scalar variable, or "index," C_t, and an n-dimensional component, u_t, standing for the idiosyncratic component. The state-space model reads as:

$$\begin{aligned}
\underset{n\times 1}{\Delta X_t} &= \underset{n\times 1}{\beta} + \underset{n\times 1}{\gamma(L)} \underset{1\times 1}{\Delta C_t} + \underset{n\times 1}{u_t}, \\
\underset{n\times n}{D(L)} u_t &= \underset{n\times 1}{\varepsilon_t}, \\
\underset{1\times 1}{\phi(L)} \Delta C_t &= \delta + \eta_t,
\end{aligned}$$

where L denotes the lag operator, and $\phi(L)$, $\gamma(L)$ and $D(L)$ are respectively scalar, vector, and matrix lag polynomials.

They describe their model as follows: "The main identifying assumption expresses the core notion of the dynamic factor model that the co-movements of the multiple time series arise from the single source ΔC_t." So, based on the estimates of the state-space model above, their coincident index is $C_{t|t}$ – the minimum mean square error linear estimate of this single common factor, produced by applying the Kalman filter to the estimated system. The leading index, on the other hand, is the estimate of the growth of this unobserved factor over the next six months, computed using a set of leading variables, i.e., $C_{t+6|t} - C_{t|t}$.

Regarding the issue of prediction of recessions and expansions, they link the leading index of economic activity with a recession index – an estimate of the probability that the economy will be in a recession six months hence.

Stock and Watson (2002) delves into forecasting using principal components when employing a multitude of predictors. So, their approach can be viewed as a big-data approach, where one employs a dimension-reduction mechanism for estimating common dynamic factors across a large set of variables. It addresses the challenge of making accurate predictions amidst high-dimensional datasets, typical in economic analyses. The authors distill essential information from a vast array of economic variables by using principal component analysis (PCA), which reduces data dimensionality and helps identifying key patterns and statistical relationships in the database.

To fix ideas, consider the joint model for $(X_t', y_t)'$, where the objective is to predict y_{t+h} employing the large dataset of predictors X_t, i.e., N is large:

$$\begin{aligned} \underset{N\times 1}{X_t} &= \underset{N\times r}{\Lambda}\underset{r\times 1}{F_t} + \underset{N\times 1}{e_t}, \\ \underset{1\times 1}{y_{t+h}} &= \underset{1\times r}{\beta_F'}\underset{r\times 1}{F_t} + \underset{1\times m}{\beta_w'}\underset{m\times 1}{w_t} + \underset{1\times 1}{\varepsilon_{t+h}}, \end{aligned}$$

where the vector F_t embeds r latent factors, w_t includes m additional predictors for y_{t+h}, e.g., its own lags, Λ, β_F', and β_w' are appropriate loadings, and errors e_t are cross-sectional diversifiable.

The classical way of estimating factors F_t is by using PCA, with loadings being later estimated by least squares. One of the key points discussed is PCA's role in mitigating multicollinearity, a common issue in regression analysis with numerous predictors. By transforming original predictors into a smaller set of orthogonal components, PCA alleviates multicollinearity, thus enhancing the stability and interpretability of the regression model.

The article also explores practical implementation of PCA-based forecasting methods, offering insights into model selection, estimation techniques, and evaluation metrics. Empirical results underscore the approach's effectiveness in generating accurate forecasts for various economic indicators. Notably, in some cases, a small number of factors can capture close to or more than 50% of the variation in a large dataset.

According to Stock and Watson (2014), there are two main approaches in the business cycle dating literature. The first approach, initiated by Burns and Mitchell (1946), involves identifying turning points individually in a large number of time series, and then searching for a common date called an aggregate turning point. Stock and Watson refer to this approach as *"date-then-average"*. By incorporating information from diverse economic indicators, this approach provides a robust and comprehensive assessment of turning point probabilities, thereby enhancing the accuracy and reliability of turning point forecasts. This approach leverages the collective wisdom embedded within multiple economic indicators to capture the nuanced dynamics of the business cycle.

The second, more recent approach, seeks turning points in only a few, or just one, aggregate time series (e.g., GDP). Stock and Watson call this approach *"average-then-date"*. Such method begins by aggregating information from various economic indicators, typically by computing the average value of each predictor over a specified period, such as a quarter. Subsequently, specific dates are assigned to each observation based on its corresponding quarter, thus aligning the aggregated data with a time series framework. The idea is to enhance the accuracy and reliability of turning point estimates by synthesizing information from multiple sources into a unified and interpretable format.

The authors examine both approaches and propose a non-parametric definition of turning points, allowing the construction of estimators, sampling distributions, and standard errors of turning points using a sample of a set of 270 time series. In addition, Stock and Watson highlight the importance of real-time data in turning point estimation, enabling policymakers and analysts to react promptly to changes in economic conditions.

This last point raises some controversy. Indeed, their approach has generalized the "dating" approach in Harding and Pagan (2006). As is well-known, real-time use of these techniques is not straightforward, since it requires dealing with endpoints in the "dating" process. For dating protocols that require a minimum 6-month period to date a recession, one has to predict the series being dated 6-months ahead, which brings forecast-uncertainty into the "dating," once interest lies with "date-than-average." But, forecast uncertainty six months hence can be substantial, which may be a problem for the final "average" estimated in real time.

Despite this potential shortcoming, historical results for dating recessions are impressive and come really close to those obtained by the NBER committee.

Chauvet and Hamilton (2006) and Hamilton (2011)

Hamilton's (2011) article aims to answer whether there is any effective technique for dating recessions in real time. Part of the literature uses the techniques proposed by the author himself, Hamilton (1989), in his seminal article on models of two (or more) regimes, with a latent state variable representing the regimes of the economy, driven by a Markov Chain. Part of this literature uses common unobservable factors, which can be identified using principal component analysis, e.g., Stock and Watson (1989, 1991, 2002).

Hamilton (2011) asks the following question: given that we have NBER dates for the U.S., what is the advantage of trying to implement an automatic mechanism (algorithm) for forecasting U.S. recessions? The author answers this question as follows:

1. *Timeliness.* The Business Cycle Dating Committee has issued its announcements of the beginning and end of recessions usually much after the event. For example, the NBER dated the recession to 1990-91, starting in August 1990 and ending in March 1991. It did not make the announcement that the recession had started until April 1991 – a month after which the NBER itself later decided to be the end of the recession. The

end of the 2001 recession was announced in July 2003 – 28 months after the recession was considered over.

2. *Apolitical Mechanism.* A purely objective algorithm for determining recession dates in real time ensures that the process is completely apolitical. While no one has accused the NBER of altering its announcements based on political considerations, there is undeniably pressure to delay the announcement that a recession has begun or speed up the announcement that a recovery has begun if the goal is to help the incumbent.

3. *Structural Mechanism.* Creating a mechanical way of recognizing the inflection points of business cycles allows us to elucidate what we really mean when we say "the economy is in recession". If the dates assigned by the NBER represent the answer, what is the question? The whole process seems to assume that there are some very different factors operating in the economy at different times, and that these changes have observable implications. Mechanization of the dating procedure can help clarify exactly how and why we assign the dates we do.

The next question asked by Hamilton is *why is it so difficult to implement a recession-prediction algorithm for the U.S.*? Again, the answer is given in three different ways:

1. *Predictability.* If people could predict recessions, they probably wouldn't happen. Firms wouldn't be stuck with inventories, labor and capital they wouldn't need, and the Fed would likely ease its monetary policy stance sooner. Economists know that stock prices are difficult (or impossible) to predict if the market is working properly. It may be that economic recessions, by their very nature, imply similar limitations to their forecasting.

2. *Data revision.* The data available in real time may send different signals than the same data later reviewed. For example, the real GDP growth rates for each quarter of 2001, as reported in the late 2002 *vintage*, show 3 successive quarters of declining real GDP, which sounds unmistakably like a recession. However, data from the same quarters by the January 30, 2002 *vintage* show that the recession was already over. This shows the difficulty of using GDP data to date recessions, as these are subject to many revisions that, in some cases, may change the perception of the state of the economy depending on the *vintage*.

3. *Changes over time.* One factor that makes it difficult to recognize real-time business cycle turning points is the fact that key economic relationships continually change over time and with available information.

In a slightly less optimistic tone, Hamilton then proposes to aim for something more modest, trying to recognize a turning point soon after it has occurred using robust algorithms against the revised data and structural changes out-of-sample, which seems to have a reasonable track record (at least until now). The algorithm that Hamilton (2011) proposes is based on the approach in Chauvet and Hamilton (2006).

Chauvet and Hamilton consider the following question: what is different about the behavior of GDP during the quarters that the NBER classifies as recessions compared to those characterized as expansions? They answered this question by collecting the growth rate of U.S. GDP between 1947Q2 and 2004Q2, in which the NBER ended up describing 45 of these quarters as part of an economic recession. This subsample of 45 observations has an average growth rate of -1.2% (expressed as annual growth) and a standard deviation of 3.5. The remaining 180 expansion quarters had a mean of 4.5 and a standard deviation of 3.2. The observed sample of 225 observations can be seen as a mixture of these two distributions, with 20% coming from the distribution of recession periods and the remaining 80% from the distribution of booming periods.

Let $S_t = 1$ if the quarter t is eventually declared by the NBER as part of a recession, and $S_t = 2$ if quarter t is eventually declared part of an expansion. Denote the GDP growth rate by y_t. We can define the joint probability that the quarter t is declared a recession and has a GDP growth rate of y_t as follows:

$$P(S_t = 1, y_t) = f(y_t \,|S_t = 1) \, P(S_t = 1),$$

where $f(y_t \,|S_t = 1)$ is the conditional density of y_t given $S_t = 1$, in which $P(S_t = 1)$ is the probability of occurrence of a recession, previously calculated as 0.2. Analogously:

$$P(S_t = 2, y_t) = f(y_t \,|S_t = 2) \, P(S_t = 2),$$

where $P(S_t = 2) = 0.8$. The optimal inference that interests us is to know what is the probability of a recession being dated by the NBER when we have a certain value of GDP growth as a condition:

$$P(S_t = 1 \,|y_t) = \frac{P(S_t = 1, y_t)}{f(y_t)} = \frac{P(S_t = 1, y_t)}{P(S_t = 1, y_t) + P(S_t = 2, y_t)},$$

so we can simplify:

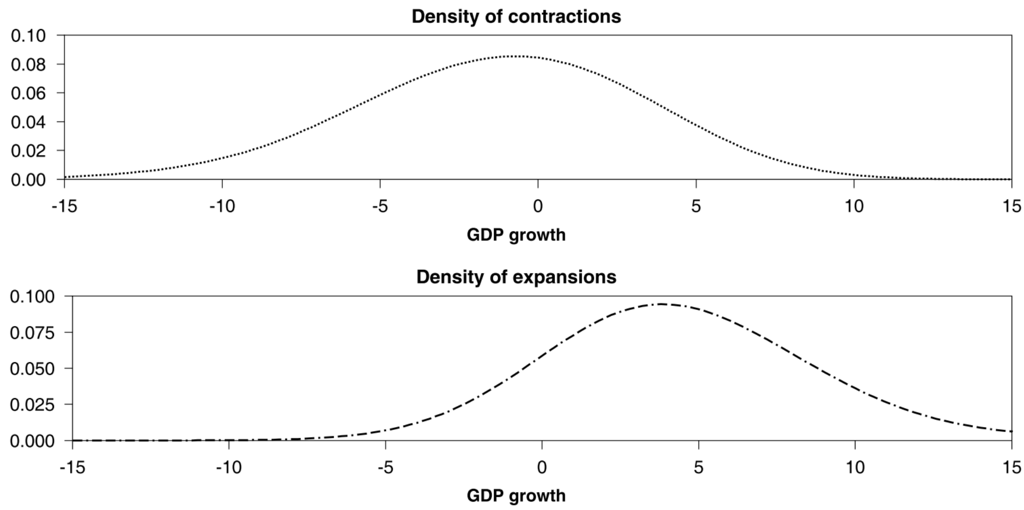
$$\begin{aligned} P(S_t = 1, y_t) &= f(y_t \,|S_t = 1) \times 0.2, \\ P(S_t = 2, y_t) &= f(y_t \,|S_t = 2) \times 0.8. \end{aligned}$$

When we separate the observations into contractions and expansion periods, we obtain the following densities in figure B.1.

This shows that these densities have very different means, although their respective variances are not very different. The average of recessive periods is a GDP growth (annualized) of -1.2%, whereas the average of expansion periods is of 4.5%. Therefore, if we observe a GDP growth of 6%, for example, the probability of a recession being dated is quite low. On the other hand, if we observe a GDP growth of -6%, this probability will be quite high.

This rule (or algorithm) seems to satisfy the requirements of being simple and robust. Unfortunately, this is not very useful, since the vast majority of observations will fall into a

Figure B.1 – Conditional probability density function (pdf)



Source: Hamilton (2011)

range where they will not provide clear signals. However, there is a second feature of NBER dates that can be quite useful - the value of S_t it is quite likely to be the same as S_{t-1}. For 95% of the observations for which $S_{t-1} = 2$, S_t was also equal to 2, whereas 78% of the observations for which $S_{t-1} = 1$ were also followed by $S_t = 1$. So, even if y_t alone does not give us a very useful idea of the signal, the value of y_{t-1} can help us refine it. Obviously, there is no reason to stop conditioning in the period $t-1$, when we can condition the model using information up to the beginning of the sample, i.e., calculating:

$$P\left(S_t = 1 \left| y_t, y_{t-1}, \cdots, y_1 \right.\right),$$

which is a filtered version of the probability of occurrence of a recession dated by the NBER.

Chauvet and Hamilton (2006) also considered a smoothed version of it, which uses the entire sample, $y_1, y_2, \cdots, y_T$, as follows:

$$P\left(S_t = 1 \left| y_T, y_{T-1}, \cdots, y_1 \right.\right).$$

The model estimation entails the following steps. It is assumed that:

$$\begin{aligned} y_t \left| S_t = 1 \right. &\sim \mathcal{N}\left(\mu_1, \sigma^2\right), \\ y_t \left| S_t = 2 \right. &\sim \mathcal{N}\left(\mu_2, \sigma^2\right). \end{aligned}$$

The density of the first observation is given by:

$$f\left(y_1; \mu_1, \mu_2, \sigma^2, \pi\right) = \pi \phi\left(y_1; \mu_1, \sigma^2\right) + (1-\pi)\, \phi\left(y_1; \mu_2, \sigma^2\right),$$

where π is the unconditional probability of occurrence of a recession dated by the NBER and $\phi(\cdot)$ is the probability density function (pdf) of a Normal random variable. Next, Chauvet and Hamilton compute recursively the conditional densities $P\left(S_t = 1 \left| y_t, y_{t-1}, \cdots, y_1 \right.\right)$ and then characterize the log-likelihood function as:

$$\mathcal{L}(\theta; y_T, y_{T-1}, \cdots, y_1) = \sum_{t=1}^{T} \log \left[f\left(y_t \left| y_{t-1}, y_{t-2}, \cdots, y_1 ; \theta\right.\right)\right].$$

The results of the estimation of the model of Chauvet and Hamilton (2006), were compared with those from Hamilton for the GDP data, classified according to the NBER:

Table B.1 – Parameter estimates

Table 1
Parameter estimates based on (1) the characteristics of expansions and recessions as classified by the NBER, and (2) the values that maximize the observed sample log likelihood of postwar GDP growth rates over the period 1947:Q2–2004:Q2.

Parameter	Interpretation	Value from NBER classifications	Value from GDP alone
μ_1	Average growth in expansion	4.5	4.62
μ_2	Average growth in recession	−1.2	−0.48
σ	Standard deviation of growth	3.4	3.34
p_{11}	Prob. expansion continues	0.95	0.92
p_{22}	Prob. recession continues	0.78	0.74

Note: Reproduced from Chauvet and Hamilton (2006).

Source: Hamilton (2011)

The results are apparently excellent, with the exception of average GDP growth in NBER recessions. But, we must keep in mind that they were obtained in an *in-sample* analysis. As Hamilton (in fact, Niels Bohr) reminded us, predictions are very difficult, especially about the future, i.e., *out-of-sample*.

Hamilton (2011) also discusses the results of two experiments. In the first one, the filtered probability is calculated as $P(S_t = 1 \left| y_t, y_{t-1}, \cdots, y_1 \right.)$ using the latest GDP vintage (available in 2004). In the second experiment, to evaluate the usefulness of this algorithm in real time (on date t), he discusses the use of GDP data that would be available in t according to its vintage; see the database maintained by the Federal Reserve Bank of Philadelphia - Croushore and Stark (2003). These data are used both to estimate the parameter vector θ and to make inference about $P(S_t = 1 \left| \cdot \right.)$. However, the use of such data resulted in a considerable deterioration of results. Therefore, Hamilton recommends waiting an extra quarter to use revised data, which allows for additional precision, before dating recession in t.

Thus, the proposed mechanism is to make inferences about S_t only after the next quarter's GDP growth rate, y_{t+1}, is available, calculating $P\left(S_t = 1 \left| y_{t+1}, y_t, \cdots, y_1 ; \widehat{\theta}_{t+1}\right.\right)$, where $\widehat{\theta}_{t+1}$ represents the parameters estimated by maximum likelihood using as data $y_1, y_2, \cdots, y_t, y_{t+1}$.

In order to implement the real-time algorithm, based on $P\left(S_t = 1 \left| y_{t+1}, y_t, \cdots, y_1 ; \widehat{\theta}_{t+1}\right.\right)$, Chauvet and Hamilton (2006) recommended the following decision rule. When $P\left(S_t = 1 \left| y_{t+1}, y_t, \cdots, y_1 ; \widehat{\theta}_{t+1}\right.\right)$ first exceeds 0.65, a recession is declared to be underway. At this point, the probable start of the recession is attributed to the beginning of the most recent set of observations for which $P\left(S_{t-j} = 1 \left| y_{t+1}, y_t, \cdots, y_1 ; \widehat{\theta}_{t+1}\right.\right)$ exceeds 0.5. The recession call remains in effect until $P\left(S_t = 1 \left| y_{t+1}, y_t, \cdots, y_1 ; \widehat{\theta}_{t+1}\right.\right)$ falls below 0.35, time at which the probable end point for the recession is assigned as the beginning of the most recent set of observations for which $P\left(S_{t-j}=1\,|y_{t+1},y_t,\cdots,y_1\,;\widehat{\theta}_{t+1}\right)$ is smaller than 0.5. The results of this implementation of the real-time algorithm are shown below:

Table B.2 – Turning points of business cycle in the U.S.

Table 2
Business cycle turning points and dates on which announcements were issued by NBER and the GDP-based algorithm.

Start of recessions

Peak as determined by NBER	Date NBER made declaration	Recession start as determined by algorithm	Date algorithm made declaration	Algorithm announcement lead (−) or lag (+) in months
1969:Q4	N.A.	1969:Q2	May 1970*	N.A.
1973:Q4	N.A.	1973:Q4	May 1974*	N.A.
1980:Q1	Jun 1980	1979:Q2	Nov 1979*	−7
1981:Q3	Jan 1982	1981:Q2	Feb 1982*	+1
1990:Q3	Apr 1991	1989:Q4	Feb 1991*	−2
2001:Q1	Nov 2001	2001:Q1	Feb 2002*	+3
2007:Q4	Dec 2008	2007:Q4	Jan 2009	+1

Start of expansions

Trough as determined by NBER	Date NBER made declaration	Recession end as determined by algorithm	Date algorithm made declaration	Algorithm announcement lead (−) or lag (+) in months
1970:Q4	N.A.	1970:Q4	Aug 1971*	N.A.
1975:Q1	N.A.	1975:Q1	Feb 1976*	N.A.
1980:Q3	Jul 1981	1980:Q2	May 1981*	−2
1982:Q4	Jul 1983	1982:Q4	Aug 1983*	+1
1991:Q1	Dec 1992	1991:Q4	Feb 1993*	+2
2001:Q4	Jul 2003	2001:Q3	Aug 2002*	−12
2009:Q2	Sep 2010	2009:Q2	Apr 2010	−5

Notes: Starred entries denote simulated real-time declarations, unstarred are actual real-time declarations. N.A. indicates that the information is not available.

Source: Hamilton (2011)

The results in the table above show that the peak and valley dates of the algorithm and the NBER are either identical or very close. As for the timeliness of the algorithm's dates vis-à-vis the NBER, there is no clear advantage. In fact, half the time NBER dates peaks and valleys before the algorithm, and half the time NBER dates peaks and valleys after the algorithm.

Issler and Vahid (2006)

Here we present a summary of the techniques proposed in Issler and Vahid (2006) to model the U.S. business cycles and to predict recessions, relying heavily on their own text. For a complete account, readers are referred to their paper.

The canonical-correlation approach of Issler and Vahid follows the common features literature, initiated by Engle and Kozicki (1993), Vahid and Engle (1993), and Engle and Issler (1995). There, economic series share common components that can be removed by linear combination. Examples are trends, cycles, seasonality, volatility, etc. The best known examples are common trends (cointegration, after Engle and Granger, 1987), and common cycles (common serial correlation, after Engle and Kozicki, 1993).

Issler and Vahid (2006) employ canonical-correlation analysis to model U.S. business cycles and to predict U.S. recessions relying on the NBER-committee dating of those episodes. Hotelling (1935, 1936) introduced canonical-correlation analysis as a multivariate-statistic tool. Akaike (1976) properly referred to canonical variables as "the information interface channels between past and present" and referred to canonical correlations as the "strength" of these channels.

In the context of Issler and Vahid, consider a set of traditional *coincident* series regarding the U.S. business cycles (income, output, employment and sales), denoted by $x_t = (x_{1t}, x_{2t}, x_{3t}, x_{4t})'$, and the set of m ($m \geq 4$) of their predictors collected in a vector z_t (this includes lags of x_t and the lags of the *leading* series). Canonical-correlation analysis transforms x_t in four independent linear combinations $A(x_t) = (\alpha_1' x_t, \alpha_2' x_t, \alpha_3' x_t, \alpha_4' x_t)$, with the property that $\alpha_1' x_t$ is the linear combination of x_t which is more (linearly) predictable using z_t, i.e., using $\gamma_1' z_t$; $\alpha_2' x_t$ is the second most linearly predictable using z_t, i.e., using $\gamma_2' z_t$, after controlling for $\alpha_1' x_t$; and so on.

The linear combinations $\alpha_i' x_t$ are not correlated with each other, and the four linear combinations of z_t, $\Gamma(z_t) = (\gamma_1' z_t, \gamma_2' z_t, \gamma_3' z_t, \gamma_4' z_t)$ have the property that $\gamma_i' z_t$ is the linear combination of z_t which has the highest quadratic correlation with $\alpha_i' x_t$, for $i = 1, 2, 3, 4$, and are not correlated with each other as well. The R^2s between $\alpha_i' x_t$ and $\gamma_i' z_t$ for $i = 1, 2, 3, 4$, denoted by $(\lambda_1^2, \lambda_2^2, \lambda_3^2, \lambda_4^2)$, are the squares of the canonical correlations between x_t and z_t. Finally, we can test recursively whether or not these canonical correlations are statistically zero – a null hypothesis which implies no prediction from the z_ts to the x_ts. This allows separating *cycles* from *noise*.

The use of the NBER-committee dating is done through a Probit model:

$$\text{NBER}_t = \begin{cases} 1 & \text{if } \mathbb{E}(y_t^* \mid I_{t+h}) < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where y_t^* is a variable representing the unobserved state of the economy, i.e., the latent U.S. business cycle, NBER_t is a dummy signaling U.S. recessions, where we must stress that future information ($\mathbb{E}(y_t^* \mid I_{t+h})$) is used by the NBER-committee in determining the state of the economy in period t. The key identifying assumption in Issler and Vahid is the following:

Hypothesis 1 (Issler and Vahid, 2006): *There is a linear index (of the cyclical parts) of the coincident series that has exactly the same pattern of correlation with previous information as the unobserved state of the economy.*

Advancing to their empirical results, they only found three significant cycles, $c_{it} = \alpha_i' x_t$, $i = 1, 2, 3$, that are predictable from the past, so *Hypothesis 1* boils down to:

$$\mathbb{E}(y_t^* - \beta_0 - \beta_1 c_{1t} - \beta_2 c_{2t} - \beta_3 c_{3t} \mid I_{t-1}) = 0. \tag{9}$$

Using equation (9), it follows that:

$$\begin{aligned}\mathbb{E}\left(y_t^* \mid I_{t-1}\right) &= \beta_0+\beta_1\mathbb{E}\left(c_{1t} \mid I_{t-1}\right)+\beta_2\mathbb{E}\left(c_{2t} \mid I_{t-1}\right)+\beta_3\mathbb{E}\left(c_{3t} \mid I_{t-1}\right) \\ &= \beta_0+\beta_1 c_{1t}+\beta_2 c_{2t}+\beta_3 c_{3t}+\omega_t,\end{aligned} \tag{10}$$

where c_{it} is decomposed as $c_{it}=\mathbb{E}\left(c_{it} \mid I_{t-1}\right)-\omega_{it}$, $i=1,2,3$, $\omega_t=\beta_1\omega_{1t}+\beta_2\omega_{2t}+\beta_3\omega_{3t}$, and $\mathbb{E}\left(\omega_t \mid I_{t-1}\right)=0$. It becomes clear that ω_t is correlated with c_{it}, $i=1,2,3$. Moreover, since we can always write:

$$\mathbb{E}\left(y_t^* \mid I_{t+h}\right)=\mathbb{E}\left(y_t^* \mid I_{t-1}\right)+\xi_t+\xi_{t+1}\cdots+\xi_{t+h}, \tag{11}$$

where ξ_{t+j} is the "surprise" associated with the new information that arrives in period $t+j$, $j=0,1,2,...,h$, we arrive at:

$$\begin{aligned}\mathbb{E}\left(y_t^* \mid I_{t+h}\right) &= \beta_0+\beta_1 c_{1t}+\beta_2 c_{2t}+\beta_3 c_{3t}+u_t, \\ u_t &= \omega_t+\xi_t+\xi_{t+1}\cdots+\xi_{t+h},\end{aligned} \tag{12}$$

where u_t is unpredictable given information in period $t-1$, that is, $\mathbb{E}\left(u_t \mid I_{t-1}\right)=0$, but it has a $MA(h)$ structure, being correlated with c_{it}, $i=1,2,3$, mainly due to the term ω_t.

To consistently estimate β_0,β_1,β_2 and β_3, Issler and Vahid employ the two-stage conditional maximum likelihood estimator (2SCML) proposed by Rivers and Vuong (1988) because of its relative simplicity. This method uses instrumental variables. The natural instruments are the variables z_t, since the canonical-correlation analysis yields estimates of $\gamma_1' z_t$, $\gamma_2' z_t$, $\gamma_3' z_t$, which are the best linear predictors c_{1t},c_{2t},c_{3t}, respectively.

Issler and Vahid also propose a coincident index – "instrumental variable coincident index" (IVCI) – relying on estimates of betas and cycles:

$$\begin{aligned}\Delta IVCI_t &= \widehat{\beta_1}\widehat{c_{1t}}+\widehat{\beta_2}\widehat{c_{2t}}+\widehat{\beta_3}\widehat{c_{3t}} \\ &= \widehat{\beta_1}\widehat{\alpha_1'}x_t+\widehat{\beta_2}\widehat{\alpha_2'}x_t+\widehat{\beta_3}\widehat{\alpha_3'}x_t \\ &= \left(\widehat{\beta_1}\widehat{\alpha_1'}+\widehat{\beta_2}\widehat{\alpha_2'}+\widehat{\beta_3}\widehat{\alpha_3'}\right)x_t,\end{aligned} \tag{13}$$

a simple linear combination of the coincident series x_t. A leading index can also be obtained using (13) by replacing c_{1t},c_{2t},c_{3t} with their linear optimal predictors $\lambda_1\gamma_1' z_t$, $\lambda_2\gamma_2' z_t$, $\lambda_3\gamma_3' z_t$, respectively. The "instrumental variable leading index" (IVLI) is a linear combination of z_t:

$$\Delta IVLI_t=\mathbb{E}_{t-1}\left(\widehat{\beta_1}\widehat{c_{1t}}+\widehat{\beta_2}\widehat{c_{2t}}+\widehat{\beta_3}\widehat{c_{3t}}\right)=\left(\widehat{\beta_1}\widehat{\lambda_1}\widehat{\gamma_1'}+\widehat{\beta_2}\widehat{\lambda_2}\widehat{\gamma_2'}+\widehat{\beta_3}\widehat{\lambda_3}\widehat{\gamma_3'}\right)z_t, \tag{14}$$

where the operator $\mathbb{E}_{t-1}(\cdot)$ represents the conditional expectation using information up to $t-1$.