Term-Structure of Equity Risk and the Cross-Section of Currency Returns

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Abstract

We use the term-structure of equity risk to build novel currency risk factors and show that they are priced in the cross-section of currency returns. Currencies more exposed to the level and curvature risk-factors offer a lower risk premium, while those more exposed to the slope factor offer a higher risk-premium. A portfolio that buys the high risk factor exposure currencies and shorts the low risk factor exposure currencies captures risk-return dispersion, and the excess returns of these strategies can be understood as compensation for a globally traded shock.

Keywords: Equity risk, currencies, international finance, asset pricing.

JEL Classification: G12, G15, F31.

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1 Introduction

We build novel currency risk factors using information from the term-structure of equity risk and show that they are priced in a broad cross-section of currency excess returns, both in a cross-section of currency portfolios and in a cross-section of countries. The exposure of each currency to the term-structure of equity risk tells us about each country's sensibility to global economic shocks, and the risk factor as a high-minus-low exposure portfolio captures the return spread among different risk profile currencies. Using the first three principal components of the term-structure of equity risk, we are able to build currency risk factors from beta-sorted portfolios to capture this risk-return spread and price currency excess returns, exploring the interconnection between equity and currency markets.

Exchange rate dynamics are inherently linked to a country's risk characteristics, which makes a currency either a safer or a riskier investment (Hassan and Zhang, 2021). Identifying these risks is therefore relevant to understanding exchange rate movements. When turning to traditional asset pricing models, the literature has showed that they lead to the rise of various puzzles when it comes to pricing currency returns. As a solution, models that include rare disasters, such as Barro (2006) and Farhi and Gabaix (2016), have been used to try to explain these asset pricing puzzles. Farhi and Gabaix (2016) introduce a probability of world disaster where each country's exposure to these events is time-varying, helping explain FX puzzles. Another key observation comes from the fact that returns of various popular currency investment strategies have been linked to crash risk, as in Brunnermeier, Nagel, and Pedersen (2008), Farhi et al. (2015) and Chernov, Graveline, and Zviadadze (2018). Therefore, global risks and country exposure to these risks seems to be crucial for understanding currency dynamics.

Disaster and crash risks are challenging to measure directly. However, equity markets offer a viable proxy, taking advantage of the interconnections between equity and currency markets. Previous research has tried to explore this relationship. Looking at tail risk and the cross-section of currency returns, Fan, Londono, and Xiao (2022) build an equity tail risk factor using S&P500 options data and show that it is priced in the cross-section of currency returns. Others have also explored the predictive power of equity data to FX markets, where Londono and Zhou (2017) show that the US equity variance risk premium helps predict currency returns over time. And others have explored how equity downside risk can help explain the cross-section of currency returns, such as Lettau, Maggiori, and Weber (2014) and Dobrynskaya (2014). Despite these findings, the structure of how investors perceive equity risk over different horizons into the future and its effect on currency market dynamics remains underexplored.

We address this gap in literature by exploring how the term-structure of market-implied equity risk, measured by VIX futures and S&P 500 variance swaps, influences currency excess returns. These term-structures capture global risk-sentiment, and variations in the term-structure curve are indicative of changes in expected implied volatility and future realized variance, which in a broader scale are reflecting changes in risk-aversion. These movements can thus lead to portfolio rebalancing decisions and changes in capital flows away from riskier currencies and into safer currencies, affecting the excess returns of currency investments. Therefore, these term-structures carry valuable information about overall perceptions of risk at different horizons, as the shape of the curve itself indicates different risk expectations. So, the key question of this paper is to evaluate if the information contained in the equity risk term-structure is priced in the cross-section of currency returns, as the equity market can be used to gauge overall economic conditions and risk appetite.

We are able to extract common factors from these term-structures, which can be interpreted as a level, slope and curvature of equity risk. These factors are strongly linked to expectations of economic outlook, capturing information about global risk sentiment. Bad economic times are usually related to a rise in the level of the curve and a flattening, or even an inversion, of the slope of the curve. A currency's exposure to each of these term-structure common factors can serve as a proxy for the country's exposure to global risk sentiment, and tell us about its risk profile. With that, we are able to build currency risk factors for the level, slope and curvature of the term-structure by going long on the high exposure currencies and short on the low exposure currencies, capturing the risk-return spread among them. The evidence presented here suggests that these risk factors are able to provide additional explanatory power beyond established factors such as the Carry and Dollar factors proposed by Lustig, Roussanov, and Verdelhan (2011), while also being relatively good performers in terms of a currency investment strategy. This further highlights the interconnections between equity and currency markets.

This paper aims to make several contributions to the existing literature. First, it proposes novel global currency risk factors from equity derivatives markets that are priced in a broad cross-section of currency returns, contributing to the literature that tries to understand the cross-section of currency returns. Second, it demonstrates that country exposure to the term-structure of equity risk can identify global components in country-level risk, explaining the cross-section of currency returns and contributing to the literature on disaster and crash risk in foreign exchange markets. Third, it establishes a tradable currency portfolio built on exposures to equity derivatives factors, enhancing our understanding of the interconnection between equity and currency markets. By constructing novel currency risk factors based on equity derivatives, this paper provides a comprehensive analysis that not only enhances our understanding of currency market dynamics but also offers practical insights for academics and practitioners.

2 Stylized factor model of currency excess returns

We provide a simple factor model of currency excess returns for illustrative purposes, not going deep into the fundamental drivers behind the factors. We follow Lustig, Roussanov, and Verdelhan (2011), Verdelhan (2018), Fan, Londono, and Xiao (2022) and others in modelling the law of motion of the log nominal Stochastic Discount Factor (SDF) $m_{c,t+1}$ in each foreign country c. We assume a general specification, where the log nominal SDF of the foreign country c is driven by a country-specific factor u_c, a global factor u_g and a term-structure of risk factor TS_c, all independent from eachother:

$$-m_{c,t+1} = i_{c,t} + a_{c,t} + \delta_c u_{c,t+1} + \gamma_c u_{g,t+1} + \theta_c TS_{c,t+1}, \tag{1}$$

where $i_{c,t}$ is the risk-free interest rate of country c, $a_{c,t}$ is a constant such that $E_t[e^{m_{c,t+1}}] = e^{i_{c,t}}$, $u_{c,t+1}$ captures country-specific shocks to the SDF, $u_{g,t+1}$ captures global shocks to the SDF and $TS_{c,t+1}$ captures shocks related to the term-structure of risk. $u_{c,t+1}$ and $u_{g,t+1}$ can be any factors that drive currency returns.

In an interconnect world, we can assume that the term-structure of risk of each country c contains a component related to the global term-structure of risk, with a country loading φ_c, and a local country-specific component:

$$TS_{c,t+1} = \varphi_c TS_{t+1}^{global} + TS_{c,t+1}^{local} \tag{2}$$

Following the Asset Market View of exchange rates, as in Backus, Foresi, and Telmer (2001), the log change in the nominal exchange rate between the foreign and domestic currency is equal to the difference of the log nominal SDFs of the two countries, where the exchange rate is expressed in foreign currency units per one unit of domestic currency:

$$\Delta s_{c,t+1} = m_{t+1} - m_{c,t+1}, \tag{3}$$

where m_{t+1} is the log nominal SDF of the domestic country and $m_{c,t+1}$ is the log nominal SDF of the foreign country c. Note that we refer to the domestic country whenever we do not include the subscript c.

Then, the currency excess return rx_{t+1} for the domestic investor who invests in foreign currency c is given by:

$$\begin{aligned} rx_{c,t+1} &= -\Delta s_{c,t+1} + i_{c,t} - i_t \\ &= -(m_{t+1} - m_{c,t+1}) + i_{c,t} - i_t \\ &= -(-i_t - a_t - \delta u_{t+1} - \gamma u_{g,t+1} - \theta TS_{t+1} \\ &+ i_{c,t} + a_{c,t} + \delta_c u_{c,t+1} + \gamma_c u_{g,t+1} + \theta_c TS_{c,t+1}) + i_{c,t} - i_t \\ &= a_t - a_{c,t} + \delta u_{t+1} - \delta_c u_{c,t+1} + (\gamma - \gamma_c) u_{g,t+1} + \theta TS_{t+1} - \theta_c TS_{c,t+1}. \end{aligned} \tag{4}$$

Decomposing the term-structure of risk factor into the global and local component, as in equation (2), we have that:

$$\begin{aligned} rx_{c,t+1} = a_t - a_{c,t} - \overbrace{(\delta_c u_{c,t+1} + \theta_c TS^{local}_{c,t+1})}^{\text{foreign country shocks}} + \overbrace{(\delta u_{t+1} + \theta TS^{local}_{t+1})}^{\text{domestic country shocks}} \\ + \underbrace{TS^{global}_{t+1}(\theta\varphi + \theta_c\varphi_c) + (\gamma - \gamma_c)u_{g,t+1}}_{\text{global shocks}}. \end{aligned} \tag{5}$$

So, the currency excess return for a domestic investor who invests in foreign currency c is influenced by several factors. These include shocks related to both foreign and domestic SDFs, as well as local components of the term structure of risk. Additionally, it is impacted by shocks to the global component of the SDF and the global component of the term structure of risk.

TS^{global}_{t+1} is not observable, so we look into the conditional beta of the currency excess return with the domestic term-structure of risk factor, which itself contains a global component:

$$\begin{aligned}\beta_{TS,c,t} &= \frac{\mathbb{C}\text{ov}_t\left[rx_{c,t+1}, TS_{t+1}\right]}{\mathbb{V}\text{ar}_t\left(TS_{t+1}\right)} \\ &= \frac{\mathbb{C}\text{ov}_t\left[rx_{c,t+1}, \varphi TS^{global}_{t+1} + TS^{local}_{t+1}\right]}{\mathbb{V}\text{ar}_t\left(TS_{t+1}\right)} \\ &= \frac{\varphi(\theta\varphi - \theta_c\varphi_c)\mathbb{V}\text{ar}_t\left(TS^{global}_{t+1}\right) + \theta\mathbb{V}\text{ar}_t\left(TS^{local}_{t+1}\right)}{\mathbb{V}\text{ar}_t\left(TS_{t+1}\right)}. \end{aligned} \tag{6}$$

From that, we can see that if the term-structure of risk of the domestic country is not influenced by the global component (i.e. $\varphi = 0$), then $\beta_{TS,c,t}$ is equal to θ for all foreign countries. This is unlikely to be the case, as in reality there is a strong interconnection between countries and their economies, where shocks in one country's economy can have spillover effects into other countries. If the SDF of the foreign country does not have a component related to the term-structure of risk (i.e. $\theta_c = 0$) or the term-structure component of the foreign country is not exposed to the global term-structure of risk (i.e. $\varphi_c = 0$), then $\beta_{TS,c,t}$ would be the same across currencies. However, the most likely case is that the term-structure of risk of the domestic country is influenced by the global component (i.e. $\varphi \neq 0$), and both θ_c and φ_c varies across countries, resulting in a $\beta_{TS,c,t}$ that varies across currencies.

If instead we looked into the the conditional beta of excess currency returns with TS^{global}_{t+1}, we would see that

$$\begin{aligned}\beta^{global}_{TS,c,t} &= \frac{\mathbb{C}\text{ov}_t\left[rx_{c,t+1}, TS^{global}_{t+1}\right]}{\mathbb{V}\text{ar}_t\left(TS^{global}_{t+1}\right)} \\ &= \frac{(\theta\varphi - \theta_c\varphi_c)\mathbb{V}\text{ar}_t\left(TS^{global}_{t+1}\right)}{\mathbb{V}\text{ar}_t\left(TS^{global}_{t+1}\right)}, \end{aligned} \tag{7}$$

which tells us that sorting currencies on $\beta_{TS,c,t}$ would be equivalent to sorting on $\beta^{global}_{TS,c,t}$. So, as we do not observe TS^{global}_{t+1}, we can use the domestic term-structure of risk to proxy for $\beta^{global}_{TS,c,t}$, as long as $\varphi \neq 0$.

Then, as in Verdelhan (2018), we can use $\beta_{TS,c,t}$-sorted portfolios to extract the global component of any domestic term-structure of risk factor. So, the long-short portfolio of buying high term-structure of risk beta currencies and selling low term-structure of risk beta currencies will be a proxy for the global term-structure of risk factor:

$$fTS_{t+1} = \frac{1}{N_H}\sum_{i \in H} rx_i - \frac{1}{N_L}\sum_{i \in L} rx_i, \tag{8}$$

where N_H and N_L represent the number of currencies in the high and low beta portfolios, respectively. With a high enough number of currencies, each portfolio leg is diversified enough to make the portfolio dominated by the global term-structure of risk component and not by the local components. Therefore, this simple factor model suggests that we can use long-short beta-sorted currency portfolios to proxy for the global term-structure of risk factor. In the remainder of this paper, we use the US as the domestic country and take the perspective of the US investor, although this framework allows us to use any reference currency of choice.

3 Data

We obtain spot (s_t, in log) and 1-month forward exchange rates (f_t, in log) with respect to USD from Barclays Bank International (BBI), World Markets Company/Refinitiv (WMR) and Refinitiv, all via Datastream. We set the order of preference as WMR being the most preferable source and BBI as the least preferable, and we change the source of the data whenever it becomes available from a preferred source, as is commonly done in the literature. We also get data on real effective exchange rates from the IMF International Financial Statistics.

We use data for 48 countries[1]: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Croatia, Cyprus, Czech, Denmark, Egypt, Euro, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, Ukraine.

For the term-structure of equity risk, we use two different, but related, measures. First, we use monthly VIX futures contracts for 1 to 9-month expiration, which are futures of S&P500 implied volatility. We obtain this data from Datastream and for the time sample of January 2006 to March 2024. Second, we use monthly forward variance claims on

[1] We follow Kroencke, Schindler, and Schrimpf (2014) and Corte, Riddiough, and Sarno (2016), among others, and leave out the indicated countries due to large defiations of covered interest rate parity for the following periods: Egypt (01/01/2011 - 30/08/2013; 03/10/2016 - 28/02/2017); Indonesia (01/12/1997 - 31/07/1998; 01/02/2001 - 31/05/2005); Malaysia (01/05/1998 - 30/06/2005); Russia (01/12/2008 - 30/01/2009; 03/11/2014 - 27/02/2015); South Africa (01/08/1985 - 30/08/1985; 01/01/2002 - 31/05/2005); Turkey (01/11/2000 - 30/11/2001); and Ukraine (03/11/2014 - 12/04/2024).

S&P500 variance, obtained from quotes on variance swaps, for 1- to 12-month expiration[2]. Variance swaps are contracts that price future realized variance of the S&P500, and, when transforming into a variance forward[3], it gives us a measure of the market's risk neutral expectation of realized variance months into the future. This data starts in December 1995 and goes up to September 2013, and is obtained from Dew-Becker et al. (2017), whose primary source is a hedge fund. The use of both VIX futures and variance forwards enables us to have two samples to test the significance of our equity based currency risk factors, one from 1995 to 2013 and one from 2006 to 2024. Although these samples cover an overlapping window, the variance forwards sample has older data when euro area currencies still existed.

Figure 1 shows these equity risk term-structures. We can see that bad times are often related to upward shifts in the level of the curve and a flattening/inversion of the slope of the curve, as was the case in the years 2000, 2001, 2008 and 2020.

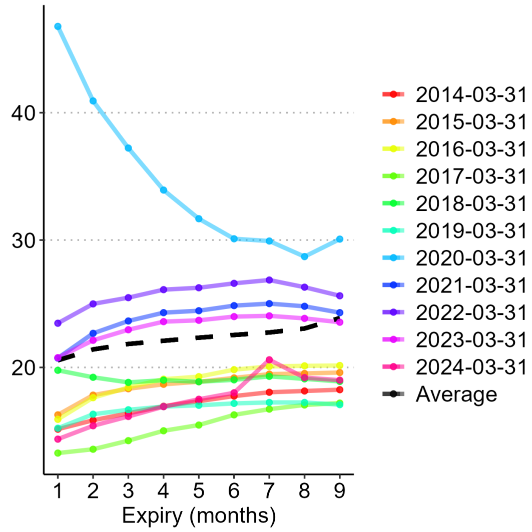
4 Empirical analysis

4.1 Equity-based currency risk factors

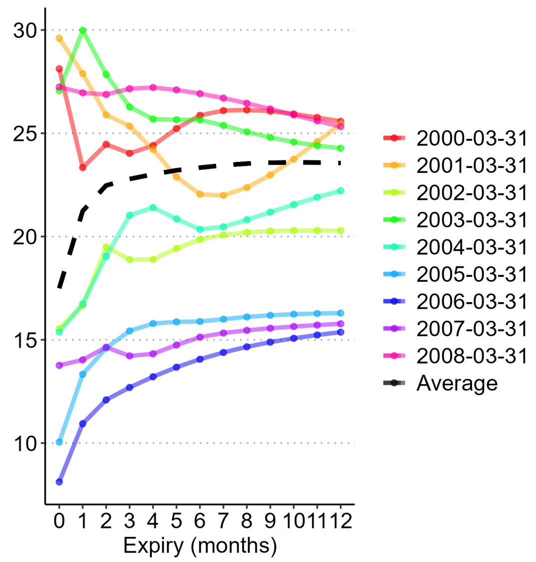
In order to extract the common components that drive variations in the term-structure of equity risk, we compute an expanding window Principal Component Analysis (PCA) on the standardized VIX and variance forwards term-structures, and use the first three principal components. In order to ensure enough data for the principal components, we start with a window of 50 months. Then at each following month we use all available past data, to ensure that for these following months after an initial startup period have

[2]Original expirations are of one, two, three, six, 12, and 24 months. Data obtained has a spline interpolation applied in order to obtain standardized maturities for all months between one month and 12 months.

[3]Given a risk-neutral (pricing) measure Q, the price of an n-month variance swap at the end of month t, VS_t^n, is$VS_t^n = E_t^Q\left[\sum_{j=1}^{n} RV_{t+j}\right]$, where RV_{t+m} is the sum of daily squared returns in month $t+m$, denoting the realized variance, and E_t^Q denotes the mathematical expectation under the risk-neutral measure conditional on information available at the end of month t. So VS_t^n is the expected sum of daily squared returns between months $t+1$ and $t+n$. Since an n-month variance swap is a claim to the sum of realized variance over months $t+1$ to $t+n$, an n-month variance forward is an asset with a payoff equal to realized variance in month $t+n$. No arbitrage implies that $F_t^n \equiv E_t^Q\left[RV_{t+n}\right] = VS_t^n - VS_t^{n-1}$, where F_t^n represents the market's risk-neutral expectation of realized variance n months in the future (at the end of month t). A one-month variance forward is exactly equivalent to a one-month variance swap, $F_t^1 = VS_t^1$.



(a) VIX futures selected term-structures



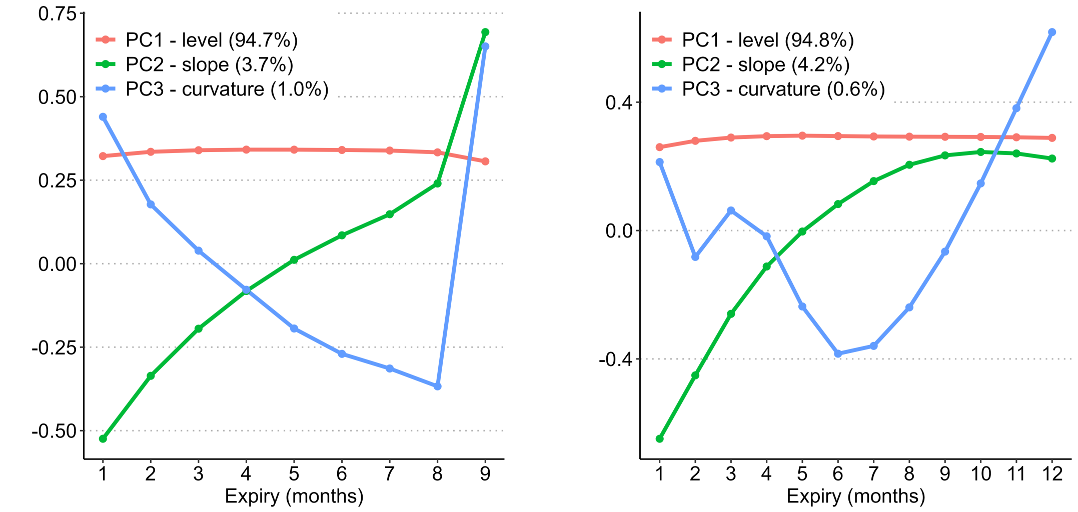
(b) Variance forward claims selected term-structures (in annualized % volatility units)

Figure 1: Term-structures of VIX futures and variance forward claims

no look-ahead bias. Figure 2 shows us the factor loadings of each contract on the first three PCAs of the full sample (i.e. the last data point) analysis, and we can see that together they explain about 99% of the variation of the curve, in both cases. We can see that all expirations load positively and about the same amount on PC1, indicating that it captures the level of the equity risk term-structure. On the other hand, we can see that the contracts have an increasing factor loading on PC2, indicating that shorter-term contracts move in the opposite direction of longer-term contracts, capturing the slope of the term-structure. Finally, PC3 can be interpreted as a curvature factor, since it loads positively on 1-month expiry contracts and longer expirations, while loading negatively on mid-term expirations, showing that the middle of the curve moves differently from both ends.

Figure 3 displays the time-series of the expanding window principal component analysis. The signals of the factor loadings are chosen such that increases in PC1 are related to upward shifts in the level of the curve, which are associated with bad times, while increases in PC2 are related to a steepening of the curve, which are related to good times. With respect to PC3, it is not as intuitive what happens to it in good/bad times.

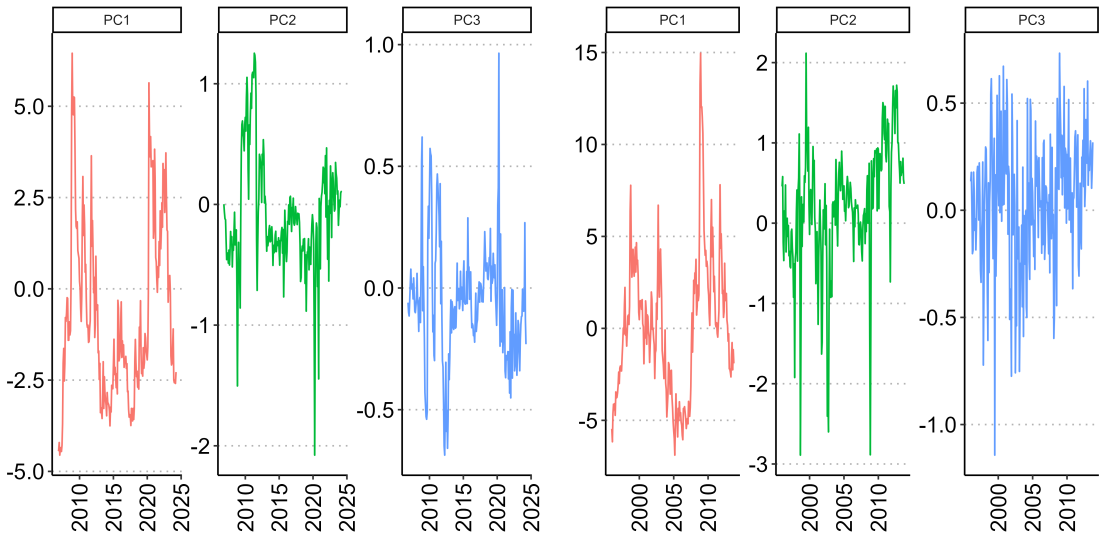
To evaluate whether these common equity risk term-structure factors are priced in the cross-section of currency returns, we sort currencies into five portfolios based on their



(a) VIX futures factor loadings

(b) Variance forward claims factor loadings

Figure 2: Factor loadings of VIX futures and variance forward claims PCA



(a) VIX futures PCAs time-series

(b) Variance forward claims PCAs time-series

Figure 3: Principal component time-series of expanding window PCA

lagged exposure to the term-structure principal components, $\beta_{TS^j,i}$. For that, we estimate the following rolling 24-month regression for each currency's monthly log excess return of buying a foreign currency in the forward market and then selling it in the spot market after one month, given by $rx_t = f_{t-1} - s_{t-1} - \Delta s_t$,

$$rx_{i,t} = \alpha_i + \beta_{DOL,i} DOL_t + \beta_{TS^j,i} \Delta TS_t^j + \varepsilon_{i,t}, \tag{9}$$

where DOL is the Dollar factor proposed by Lustig, Roussanov, and Verdelhan (2011), used to control for the overall variation in currency excess returns, and $TS = \{\text{PC1}, \text{PC2}, \text{PC3}\}$ represents the principal components extracted using VIX futures and variance forwards (separately), with one principal component added per regression.

Then, at the end of month t, we sort currencies based on $\beta_{TS^j,i}$ into five portfolios and compute the equal-weighted currency excess return of each portfolio in the following month. $\beta_{TS^j,i}$ proxies for the risk profile of each currency, therefore each beta-sorted portfolio is a diversified portfolio of similar risk profile countries. Our proposed currency risk factor for each equity risk term-structure common component is thus the high-minus-low (HML) beta-sorted portfolio, denoted as fTS^j, for each $j \in \{\text{PC1}, \text{PC2}, \text{PC3}\}$.

Table 1 displays summary statistics of the currency VIX futures beta-sorted portfolios, where we can see a decreasing pattern in the Sharpe Ratio in the beta level (PC1) and curvature (PC3) sorted portfolios as the quintile increases, and the opposite for the beta slope (PC2) sorted portfolios. This shows that sorting currencies based on their exposure to equity risk term-structure factors reveals a dispersion in risk-return relationships, indicating differing degrees of hedging effectiveness against global risk components. For variance forwards beta-sorted portfolios, we see a similar relationship, with exception of the beta curvature (PC3) sorted portfolios that exhibit an increasing pattern in the Sharpe Ratio, which could be due to the availability higher expiry contracts than in the VIX futures case. In both cases, the majority of the returns are being captured by the extreme portfolios, i.e. the highest quintile portfolio in the level (PC1) and curvature (PC3) case, and the lowest quintile portfolio in the slope (PC2) case.

The long-short beta-sorted portfolios also appear to be good performers in terms of currency investment strategies. As shown in Tables 1 and 2, for all cases, the Sharpe Ratios are fairly high in absolute terms and the Skewness is in line with most currency investment strategies in the literature, where they exhibit crash-risk. Additionally, Figure 4 displays the cumulative excess return of these risk factors over time. The level (PC1) risk factor displays an overwhelmingly negative return over time for both the VIX futures and variance forwards case, highlighting its role as a hedge. However, in practice, one could reverse the portfolio construction in order to obtain a positive return over time. The slope (PC2) risk factor, however, displays a large positive cumulative return over time, even during the financial crisis of 2008. This shows how these portfolios are able to adjust to the time-varying global risk conditions and switch into safer currencies in times of turmoil.

Table 1: VIX futures beta-sorted portfolios summary statistics

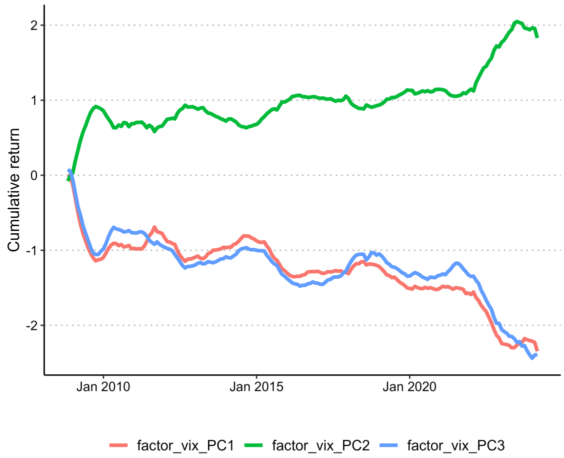
Portfolio	P1	P2	P3	P4	P5	HML
Panel A: sorts on β_{PC1}						
Avg Ann. Return	0.01	0.00	0.01	-0.02	-0.14	-0.15
t-stat	[0.27]	[0.03]	[0.48]	[-1.02]	[-3.80]	[-4.09]
Ann. Volatility	0.09	0.08	0.06	0.06	0.15	0.14
Sharpe Ratio	**0.11**	**0.00**	**0.17**	**-0.33**	**-0.93**	**-1.07**
Skewness	-0.38	-0.05	-0.41	-0.11	-1.06	-1.23
Panel B: sorts on β_{PC2}						
Avg Ann. Return	-0.13	-0.01	-0.00	-0.00	-0.01	0.12
t-stat	[-4.03]	[-0.44]	[-0.13]	[-0.19]	[-0.19]	[3.44]
Ann. Volatility	0.12	0.06	0.07	0.08	0.11	0.14
Sharpe Ratio	**-1.08**	**-0.17**	**-0.00**	**-0.00**	**-0.09**	**0.86**
Skewness	-0.74	-0.78	-0.38	-0.47	-0.57	0.62
Panel C: sorts on β_{PC3}						
Avg Ann. Return	0.00	-0.01	-0.01	-0.00	-0.15	-0.16
t-stat	[0.10]	[-0.37]	[-0.67]	[-0.04]	[-4.07]	[-4.21]
Ann. Volatility	0.09	0.08	0.06	0.07	0.15	0.15
Sharpe Ratio	**0.00**	**-0.12**	**-0.17**	**-0.00**	**-1.00**	**-1.07**
Skewness	-0.15	-0.78	-1.05	-0.26	-0.66	-0.85

Table 2: Variance forward claims beta-sorted portfolios summary statistics

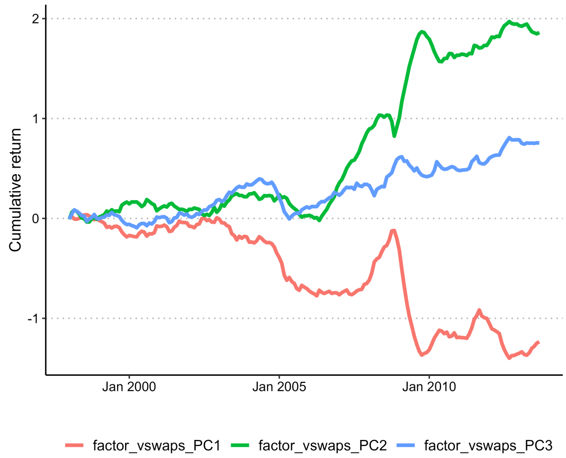
Portfolio	P1	P2	P3	P4	P5	HML
Panel A: sorts on β_{PC1}						
Avg Ann. Return	0.08	0.02	0.03	0.06	0.01	-0.08
t-stat	[3.07]	[1.02]	[1.82]	[3.20]	[0.18]	[-2.05]
Ann. Volatility	0.11	0.08	0.06	0.07	0.14	0.15
Sharpe Ratio	**0.73**	**0.25**	**0.50**	**0.86**	**0.07**	**-0.53**
Skewness	-0.75	-1.01	-0.35	-0.24	-1.32	-1.10
Panel B: sorts on β_{PC2}						
Avg Ann. Return	-0.00	0.02	0.03	0.06	0.11	0.11
t-stat	[-0.16]	[1.41]	[2.04]	[3.20]	[3.55]	[3.44]
Ann. Volatility	0.11	0.06	0.06	0.08	0.12	0.13
Sharpe Ratio	**-0.00**	**0.33**	**0.50**	**0.75**	**0.92**	**0.85**
Skewness	-1.13	-0.42	-0.10	-0.83	-0.94	0.36
Panel C: sorts on β_{PC3}						
Avg Ann. Return	0.04	0.01	0.04	0.04	0.08	0.04
t-stat	[1.45]	[0.51]	[2.48]	[2.37]	[2.89]	[1.70]
Ann. Volatility	0.11	0.08	0.06	0.07	0.11	0.10
Sharpe Ratio	**0.36**	**0.12**	**0.67**	**0.57**	**0.73**	**0.40**
Skewness	-0.96	-0.96	-0.07	0.24	-0.65	-0.05

Finally, the curvature (PC3) risk factor displays a cumulative return very similar to that of the level risk factor, in the case of VIX futures, but a moderately positive cumulative return over time for the variance forwards case.

One might argue that these beta-sorted portfolios are simply capturing the same relationship captured by the well-established Carry portfolios. Therefore, we explore how each



(a) VIX futures risk factors cumulative excess return



(b) Variance forward claims risk factors cumulative excess return

Figure 4: Risk factors cumulative excess return

beta-sorted portfolio is related to the Carry factor of Lustig, Roussanov, and Verdelhan (2011) in time-series regressions. The results in Tables 3 and 4 shows that the returns of the beta-sorted portfolios are not related to the performance of the Carry factor and are therefore capturing a different relationship among currencies.

Table 3: Time-series regression of VIX futures beta-sorted portfolios on the Carry factor

Portfolio	P1	P2	P3	P4	P5	HML
Panel A: sorts on β_{PC1}						
Intercept	-0.01	-0.02	-0.01	-0.01	0.02	0.03
	[-0.32]	[-0.72]	[-0.28]	[-0.27]	[0.25]	[0.34]
Carry Factor	0.04	0.05	0.03	-0.03	-0.38	-0.43
	[0.57]	[0.83]	[0.63]	[-0.67]	[-1.76]	[-1.23]
R^2	0.003	0.005	0.004	0.002	0.097	0.127
Panel B: sorts on β_{PC2}						
Intercept	0.01	-0.00	0.00	-0.01	-0.02	-0.03
	[0.16]	[-0.21]	[0.09]	[-0.58]	[-0.56]	[-0.46]
Carry Factor	-0.32	-0.01	-0.01	0.03	0.04	0.37
	[-2.21]	[-0.16]	[-0.18]	[0.50]	[0.43]	[1.32]
R^2	0.097	0.000	0.000	0.002	0.002	0.099
Panel C: sorts on β_{PC3}						
Intercept	-0.01	-0.01	-0.03	-0.00	0.03	0.04
	[-0.22]	[-0.33]	[-1.89]	[-0.10]	[0.50]	[0.53]
Carry Factor	0.02	0.01	0.05	0.00	-0.45	-0.47
	[0.31]	[0.08]	[1.28]	[0.08]	[-2.36]	[-1.67]
R^2	0.001	0.000	0.009	0.000	0.127	0.146

Note: Newey-West t-statistics between brackets.

Table 4: Time-series regression of variance forwards beta-sorted portfolios on the Carry factor

Portfolio	P1	P2	P3	P4	P5	HML
Panel A: sorts on β_{PC1}						
Intercept	-0.02	-0.01	0.01	0.05	0.06	0.08
	[-0.43]	[-0.27]	[0.57]	[1.49]	[0.97]	[1.06]
Carry Factor	0.20	0.06	0.03	0.01	-0.10	-0.30
	[2.90]	[1.29]	[0.73]	[0.22]	[-0.51]	[-1.20]
R^2	0.063	0.010	0.005	0.000	0.010	0.075
Panel B: sorts on β_{PC2}						
Intercept	0.04	0.03	-0.00	0.03	-0.03	-0.07
	[0.85]	[1.15]	[-0.08]	[0.97]	[-0.60]	[-1.22]
Carry Factor	-0.08	-0.01	0.07	0.06	0.28	0.36
	[-0.62]	[-0.35]	[1.59]	[1.48]	[3.48]	[2.20]
R^2	0.010	0.001	0.021	0.012	0.093	0.136
Panel C: sorts on β_{PC3}						
Intercept	0.45	0.36	-0.02	0.37	-0.38	-0.83
	[0.85]	[1.15]	[-0.08]	[0.97]	[-0.60]	[-1.22]
Carry Factor	-0.98	-0.16	0.79	0.74	3.30	4.28
	[-0.62]	[-0.35]	[1.59]	[1.48]	[3.48]	[2.20]
R^2	0.010	0.001	0.021	0.012	0.093	0.136

Note: Newey-West t-statistics between brackets.

4.2 Cross-sectional asset pricing tests

As suggested by the stylized factor model in Section 2, the return of the long-short beta-sorted portfolio can capture the global component in the term-structure of risk factor. Therefore, with the term-structure equity risk factors in hand, we can then test if they are priced in the cross-section of currency returns by estimating the price of risk associated with each fTS^j and its ability to explain the cross-section of currency excess returns. For that, we estimate two-stage asset pricing equations. We propose a 3-factor and a 5-factor model, where in the first stage we estimate a time-series regression of the excess return of each test asset on the risk factors,

3-factor model:

$$rx_{i,t+1} = \alpha_i + \beta_{i,DOL}DOL_t + \beta_{i,CAR}CAR_t + \beta_{fTS^j,i}fTS_t^j + \varepsilon_{i,t}$$

5-factor model: (10)

$$rx_{i,t+1} = \alpha_i + \beta_{i,DOL}DOL_t + \beta_{i,CAR}CAR_t + \beta_{fPC1,i}fPC1_t + \beta_{fPC2,i}fPC2_t + \\ + \beta_{fPC3,i}fPC3_t + \varepsilon_{i,t}$$

Then, in the second stage we estimate a cross-sectional regression using the $\hat{\beta}$'s from the first stage, where the λ's are the prices of risk that we are interested in,

$$\begin{aligned}
&\text{3-factor model:}\\
&\bar{rx}_i = \hat{\beta}_{DOL,i}\lambda_{DOL} + \hat{\beta}_{CAR,i}\lambda_{CAR} + \hat{\beta}_{fTS^j,i}\lambda_{fTS^j} + \eta_i\\
&\text{5-factor model:}\\
&\bar{rx}_i = \hat{\beta}_{DOL,i}\lambda_{DOL} + \hat{\beta}_{CAR,i}\lambda_{CAR} + \hat{\beta}_{fPC1,i}\lambda_{fPC1} + \hat{\beta}_{fPC2,i}\lambda_{fPC2} +\\
&\qquad + \hat{\beta}_{fPC3,i}\lambda_{fPC3} + \eta_i,
\end{aligned} \tag{11}$$

and compute the cross-sectional $R^2 = 1 - \frac{\frac{1}{N}\sum_{i=1}^{N}\hat{\eta}_i^2}{\text{var}\left(\bar{rx}_i\right)}$.

We use three different sets of currency test-assets, all of which we also include the factors themselves as test-assets. The first set includes the five Carry, five Momentum and five Value portfolios. The second set includes only the five Momentum and five Value portfolios, and the third set is the country level cross-section of excess returns, as recommended by Lo and MacKinlay (1990). The Carry portfolios are the five forward premium sorted portfolios, following Lustig, Roussanov, and Verdelhan (2011). The Momentum portfolios are built by sorting currencies on previous-month excess return, as in Menkhoff et al. (2012), and the Value portfolios are built by sorting currencies on the 5-year log-return of the real exchange rate, as in Menkhoff et al. (2017). The foreign exchange literature tends to focus more on explaining the Carry sorted portfolios, we go a step further and add additional portfolio cross-sections to test our hypothesis. We estimate the 3-factor model for all sets of test-assets, but the 5-factor model is only estimated for the country level cross-section, as in this case it is a larger cross-section and therefore has enough degrees of freedom in the regression.

The second stage results[4] are shown in Tables 5 and 6, where columns (1) to (4) are for the Carry, Momentum and Value test assets, columns (5) to (8) are for the Momentum and Value test assets, and columns (9) to (13) are the country level analysis. The first thing we can notice is that there is a big jump in the cross-sectional R^2 when we include any of the term-structure of equity risk currency factors, relative to model only with the Carry and Dollar factors, showing that these additional factors provide valuable information to pricing the cross-section of returns.

For the VIX futures based equity risk currency factors, we can see that level (PC1) and

[4]We use annualized monthly returns for the test assets and the risk factors.

curvature (PC3) have a negative price of risk, while slope (PC2) has a positive price of risk. A higher exposure to the level (PC1) and curvature (PC3) factors leads to a lower average return across assets, where an asset with a beta of one pays a risk premium of about -14% to -12% and -15% to -10% per year, respectively. Slope (PC2), on the other hand, pays a risk premium of about 13% for an asset with a beta of one, and goes to 21% in the 5-factor model. This highlights that these risk prices are economically significant across all three cross-sections, while also being statistically significant, with exception of the curvature (PC3) factor in the 5-factor model. It is also noticeable that in the 5-factor model the price of risk and the statistical significance of the Carry factor is slightly reduced.

For the variance forwards based equity risk currency factors, we observe a similar pattern, with some slight differences. Level (PC1) and slope (PC2) both continue to have a negative and positive price of risk, respectively, that is statistically and economically significant. However, curvature (PC3) has a positive price of risk, which could be due to the fact that we have longer expiry contracts in this case relative to VIX futures contracts. Additionally, curvature also remains statistically significant in the 5-factor model, together with the remaining factors.

These results tells us that the information captured by equity risk term-structures could be a proxy for changes in global risk-aversion and crash risk, and changes in it could lead to portfolio rebalancing decisions that affect currency returns. Therefore, currencies with a higher or lower exposure to these factors require a risk premium, showing that these novel currency factors are priced in the cross-section of currency returns. Previous models that include disaster risk seem to be successful in explaining asset pricing puzzles in the FX literature, these results show empirically that a measure related perceptions of global risk over different horizons into the future does indeed capture relevant information for explaining the cross-section of currency excess returns.

Table 5: Asset pricing tests of VIX futures term-structure factors - second stage

	Test assets												
	Carry + Momentum + Value				Momentum + Value				Country level				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
β_{fPC1}		-0.133^{***}				-0.138^{***}				-0.145^{***}			-0.123^{***}
		(0.027)				(0.023)				(0.030)			(0.035)
β_{fPC2}			0.124^{***}				0.128^{***}				0.139^{***}		0.215^{***}
			(0.026)				(0.022)				(0.034)		(0.067)
β_{fPC3}				-0.151^{***}				-0.154^{***}				-0.152^{***}	-0.100
				(0.030)				(0.023)				(0.032)	(0.084)
β_{CAR}	0.451^{***}	0.661^{***}	0.679^{***}	0.659^{***}	0.384^{***}	0.533^{***}	0.546^{***}	0.527^{***}	0.275^{***}	0.275^{***}	0.270^{***}	0.286^{***}	0.196^{**}
	(0.076)	(0.058)	(0.058)	(0.066)	(0.064)	(0.066)	(0.068)	(0.068)	(0.051)	(0.054)	(0.053)	(0.056)	(0.085)
β_{DOL}	0.033	0.064	0.074	0.111	0.047	0.041	0.045	0.065	-0.025	-0.024	-0.019	-0.007	-0.052
	(0.089)	(0.056)	(0.047)	(0.069)	(0.081)	(0.058)	(0.051)	(0.066)	(0.029)	(0.037)	(0.031)	(0.039)	(0.036)
R^2	0.792	0.940	0.945	0.929	0.868	0.940	0.945	0.940	0.825	0.827	0.832	0.829	0.837

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 6: Asset pricing tests of variance forward claims term-structure factors - second stage

	Test assets												
	Carry + Momentum + Value				Momentum + Value				Country level				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
β_{fPC1}		-0.094^{***}				-0.090^{***}				-0.436^{***}			-0.232^{**}
		(0.027)				(0.024)				(0.088)			(0.096)
β_{fPC2}			0.128^{***}				0.122^{***}				0.537^{***}		0.453^{***}
			(0.026)				(0.019)				(0.067)		(0.085)
β_{fPC3}				0.042^{*}				0.039^{**}				0.351^{***}	0.501^{***}
				(0.020)				(0.016)				(0.107)	(0.161)
β_{CAR}	0.532^{***}	0.718^{***}	0.726^{***}	0.745^{***}	0.411^{***}	0.619^{***}	0.624^{***}	0.672^{***}	0.782^{***}	0.792^{***}	0.505^{***}	0.591^{***}	0.481^{***}
	(0.074)	(0.046)	(0.043)	(0.034)	(0.073)	(0.069)	(0.052)	(0.045)	(0.111)	(0.120)	(0.113)	(0.105)	(0.108)
β_{DOL}	0.185^{**}	0.068	0.071^{*}	0.148^{***}	0.190^{**}	0.090	0.078^{*}	0.141^{***}	0.275^{***}	0.341^{***}	0.338^{***}	0.285^{***}	0.408^{***}
	(0.065)	(0.039)	(0.034)	(0.024)	(0.069)	(0.050)	(0.036)	(0.028)	(0.035)	(0.047)	(0.039)	(0.036)	(0.044)
R^2	0.816	0.958	0.961	0.976	0.789	0.929	0.952	0.965	0.693	0.708	0.748	0.688	0.786

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

5 Conclusion

This paper examines the asset pricing implications of the term-structure of equity risk on the cross-section of currency excess returns. In a stylized model of currency excess returns, we show that a currency's Stochastic Discount Factor is related to various local and global shocks, and sorting currencies on the beta of term-structure of risk enables us to proxy for global component in the term-structure of risk and build currency risk factors.

We build currency risk factors from long-short beta-sorted portfolios based on each currency exposure to principal components of the term-structure of VIX futures and S&P500 variance forwards. These term-structures are a measure of global risk-aversion and overall risk over different horizons into the future, and they have a factor structure that enables us to use the first three principal components and account for about 99% of their variation. Beta-sorting currencies based on the exposure to these principal components, as suggested by our stylized model, is able to differentiate low and high risk currencies, while being unrelated to the Carry factor.

Finally, we provide empirical evidence using three sets of test assets that our novel currency risk factors can help explain a large portion of the cross-section of currency excess returns. This explanatory power is independent of the well established Carry and Dollar factors of Lustig, Roussanov, and Verdelhan (2011) and suggests that both the VIX and variance forwards term-structures contain a global component that require risk premiums. This offers additional evidence to the risk-based explanation for anomalies in foreign exchange markets.

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