Exploring time-varying efficient price between prices of Ethereum and Gold*

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Abstract

We investigate whether Ethereum price has a cointegration relationship with the gold price. We analyze that the possible cointegration is time-varying due to the development that has taken place in the cryptocurrency market over time and evaluate the effects of this possible change in behavior between cryptocurrency price and gold price. Our procedure is based on vector error-correction model with time-varying parameters and volatility stochastic from Koop et al. (2011). We include the gold price variable as this is a hedging asset under periods of market turmoil. Our sample is daily between 08/06/2015 and 12/30/2022. In the period of covid-19, we find that there was an increase in uncertainty, but Ethereum do not play the role of a hedge asset. Price of Ethereum follows the price of gold more in the long-term relationship between 2015 and 2018.

Keywords: Ethereum price, Efficient price, Gold price, Time-varying cointegrating relationship.

1 Introduction

Cryptocurrencies have been gaining the attention of investors around the world in a context of decentralisation and low transaction costs. Despite the security of a blockchain network, the value of these coins is not tied to any tangible asset in general, but to an algorithm capable of consolidating trades. Thus, a market price is defined based on the relationship between supply and demand.

A field in the literature seeks to explain the mechanisms that influence the pricing of cryptocurrencies which is the efficient price. The efficient price can be understood as the long-term behaviour of the asset price. This long-term behaviour subtracts from short-term swings, the presence of outliers, or some transitory disruptions in the market (Kapar and Olmo, 2021).

The reason for finding the efficient price of Bitcoin is that investors can use it in strategic trading decisions. For example, consider a situation where there is a long-term relationship, which would be the efficient price of Bitcoin. A deviation from long-term equilibrium can trigger investor decisions to buy or sell in the belief that the price of Bitcoin will return to long-term equilibrium. Also cryptocurrencies have the potential to affect sectors on the real side of the economy, which generates economic and social concern with digital asset markets (Liu and Tsyvinski, 2021).

Our aim is to investigate how the criptocurrency price behaves in the long term. The emphasis is on the relationship that may exist between the price of cryptocurrency and the price of gold. We analyze

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the efficient price of the Ethereum from the cointegrating relationship, if any. We consider the case that cointegration is time-varying due to the development that has taken place in the cryptocurrency market over time and evaluate the effects of this possible change in behavior. We estimate the time-varying version of vector error-correction model (VECM) with stochastic volatility from Koop et al. (2011). We study the second largest cryptocurrency by market capitalization: Ethereum. We use daily data between 08/06/2015 and 12/30/2022.

Our research proposal is directly related to Adebola et al. (2019). Adebola et al. (2019) analyze the relationship between cryptocurrencies and gold prices through cointegration, if any. They use fractional cointegration as a methodology. We have a difference in relation to Adebola et al. (2019) because we analyze the presence of cointegration with time-varying parameters (TVP) based on Koop et al. (2011). Our motivation for the time-varying behavior for cointegration comes from the presence of pronounced rises and falls in cryptocurrency prices. For example, Yaya et al. (2019) analyze the behavior of cryptocurrency prices before and after the 2017 and 2018 crash in the cryptocurrency market. Also cointegration refers to variables moving toward a long-term equilibrium. The time-varying cointegration relation can be understood as a slowly changing equilibrium, in which variables are attracted at any point in time, but not necessarily at all points in time (Saikkonen and Choi, 2004; Bierens and Martins, 2010; Koop et al., 2011). Thus, moments of pronounced rises in the price of the cryptocurrency could be associated with periods in which there is no attraction towards equilibrium.

Furthermore, we explain why we use the price of gold as a variable for the relationship. Gold is considered a hedge asset under periods of market turmoil. At the same time, Bitcoin has similar characteristics to gold for hedging for example, which was corroborated by the positive reaction for both assets during stock price shocks (Kapar and Olmo, 2021). Because of this, we study Ethereum price.

The result indicates that the price of Ethereum follows the price of gold more in the long-term relationship between 2015 and 2018. However, the coefficients of the speed of adjustment are not statistically significant at 10%, indicating that cointegration ceases to exist from 2018 mostly. In the period where cointegration is present, the Ethereum price responds to deviations in the long-term relationship with gold price, as expected.

This paper is organised in five sections besides this introduction. The second section provides an overview of the literature on the efficient price of cryptocurrencies and the gold price. The third section covers the methodology we apply. We describe its source in the fourth section. In the fifth section, we report the results of the estimation. Finally, we present our concluding remarks.

2 Literature Review

The growth of cryptocurrencies led to studies that were concerned with explaining their behaviour, especially Bitcoin. In addition to the factors that determine the price of this asset, Kristoufek (2013), Bouoiyour et al. (2014), and Kapar and Olmo (2021) analyse its efficient price, for example.

Kapar and Olmo (2021) seek to explain the cryptocurrency price dynamics by adding other variables such as gold price, which is a hedge asset for turbulent moments in the market. Based on Corbet et al. (2019), the demand for gold increased significantly during the Bitcoin crash in January 2018. Klein et al. (2018) study the two assets together and conclude that their behaviors are exactly the opposite in times of distress. The price of gold may have similar characteristics to Bitcoin according to Kapar and Olmo (2021).

Adebola et al. (2019) study the relationship between cryptocurrency and gold prices. One of the methodologies is fractional cointegration. The authors considered the 12 cryptocurrencies with the highest market capitalizations. They analyze whether the price of each cryptocurrency cointegrates (fractionally) with the price of gold, that is, they consider cointegration only between two variables. We rely on Adebola et al. (2019) to study the cointegration of the cryptocurrency price with the gold price considering only two variables in the cointegration test and in the model estimation. They find cointegration between cryptocurrency and gold prices only for Bytecoin and Bitcoin and partially for Dash and Stellar.

Yaya et al. (2019) discuss the crash in the cryptocurrency market between 2017 and 2018. For example, Cheung et al. (2015), Cheah and Fry (2015), Corbet et al. (2018), Su et al. (2018), and Chaim and Laurini (2019) find evidence of explosive behavior in the price of Bitcoin, that is a temporary pattern. For example, Bitcoin price reached 19,000 USD before the crash of December 2017 (Yaya et al., 2019). But, at 25 November 2018, Bitcoin price was 4,141 USD. That is, cryptocurrency prices had skyrocketed and had pronounced drops as well. This motivates the study of cointegrating relationships that vary over time. In addition to being a market that has been evolving over time. Therefore, we choose to use time-varying cointegration based on Koop et al. (2011).

Another variable that will be analyzed is the fear index that is the VIX. Based on trading in S&P 500 index options, VIX serves as an indicator of expected volatility in the index over the coming 30-day period. The fear index captures market sentiments by the Fed's financial stress index. Kapar and Olmo (2021) use the Federal Reserve (Fed) financial stress index to capture market sentiment and analyze whether there is a cointegration relationship with the price of Bitcoin. That is, whether market sentiment affects the price of Bitcoin. Unlike Kapar and Olmo (2021), we will use the VIX in the future because it is a measure that comes directly from market negotiations.

Our contribution is to use the VEC model with time-varying parameters and stochastic volatility following Koop et al. (2011). We consider the possibility of time-varying cointegration, which the literature do not study. In addition, we seek to cover a cryptocurrency Ethereum that has been less studied than Bitcoin and therefore we can bring reflections on it.

According to Liu and Tsyvinski (2021), cryptocurrency returns have low exposure to equity, currency and commodity markets, which could be a sign of influence of specific factors in digital currency markets. But Ciaian et al. (2018) study the interdependence between the price of Bitcoin and the price of other cryptocurrencies. Their result is that the interdependence between other cryptocurrency and Bitcoin prices is more present in the short term than in the long run. They obtain the presence of Bitcoin price cointegration with four other cryptocurrencies out of the 16 considered. This indicates that although Bitcoin is the virtual dominant currency, there is no price interdependence between Bitcoin and most cryptocurrencies in the long run, indicating that the factors that affect cryptocurrency prices might not be the same in the long run with respect to the price of Bitcoin. That is, along these lines, we can expect that a possible cointegration relationship (if any) between Bitcoin and gold prices may not be representative of the cryptocurrency market. We will take whether this occurs in our study comparing with Ciaian et al. (2018). In this version of the article, we estimate the relationship between the price of gold and Ethereum.

3 Methodology

Basically, we will analyze whether there is cointegration between Ethereum ($Peth_t$) and gold prices ($Pgold_t$). Our procedure will be to specify the VECM model if there is presence of cointegration. Furthermore, we allow the short-term dynamics, speed of adjustment term and cointegrating vectors to be time-varying or time-invariant. Our procedure is based on Koop et al. (2011). We choose the model specification based on the information criterion to consider whether there is a cointegration and whether the parameters are time-varying. Consider vector y_t for $t = 1, ..., T$, we write TVP-VECM as

$$\Delta y_t = \Phi_t + \alpha_t \beta_t' y_{t-1} + \sum_{h=1}^{l} \Gamma_{h,t} \Delta y_{t-h} + \varepsilon_t \tag{1}$$

where $y_t = (Peth_t, Pgold_t)'$, ε_t are independent $N(0, \Omega_t)$, $n \times r$ matrices α_t and β_t are full rank, r is the number of cointegrating relationships. We follow Koop et al. (2011) that only the cointegrating space is identified, and not the cointegrating vectors. The notation for the space spanned by β is $p_t = sp(\beta_t)$. We specify β_t to be semi-orthogonal to achieve identification according to Strachan and Inder (2004).[1] So, the cointegrating space at time t has a distribution centered over the cointegrating space at time $t-1$. Also the evolution of cointegrating space p_t is gradual similar to TVP-VAR models. Also we can express prior beliefs about the marginal distribution of the cointegrating space at time t.

The parameters $\gamma_t = (\alpha_t, \Gamma_{1,t}, ..., \Gamma_{l,t}, \Phi_t)$ follow a standard state equation such as

$$\gamma_t = \rho \gamma_{t-1} + \vartheta_t \tag{2}$$

where ϑ_t is a vector of states. Equation (2) is commonly used as a state equation in TVP-VAR models. We use the same specification of multivariate stochastic volatility model for Ω_t as Primiceri (2005).

We restrict β_t to achieve identification and we consider β_t^* the unrestricted matrix of cointegrating vectors without identification imposed. β_t^* is related to the restricted β_t as

$$\beta_t = \beta_t^* (\kappa_t)^{-1} \tag{3}$$

where $\kappa_t = (\beta_t^{*'} \beta_t^*)^{1/2}$. Our state equation for the time variation in the cointegrating space is based on $b_t^* = vec(\beta_t^*)$ for $t = 2, ..., T$ as

$$b_t^* = \overline{\rho} b_{t-1}^* + \eta_t \tag{4}$$

where $\eta_t \sim N(0, I_{nr})$ for $t = 2, ..., T$, and $b_1^* \sim N\left(0, I_{nr} \frac{1}{1-\overline{\rho}^2}\right)$, $\overline{\rho}$ is a scalar and $|\overline{\rho}| < 1$ assures the cointegrating space today has a distribution that is centered over last period's cointegrating space. Also we normalize the error covariance matrix in the state equation (4) to the identity. We consider a single vector $Tnr \times 1$ $b^* = (b_1^{*'}, ..., b_T^{*'},)$. From the conditional distribution in Equation (4), we can infer that the joint distribution of b^* is normal with zero mean and the standard covariance matrix of a stationary AR(1) process.

We allow for multivariate stochastic volatility and all the parameters in a_t evolve according to random walks, where $A_t = (\alpha_t^*, \Gamma_{1,t}, ..., \Gamma_{l,t}, \Phi_t)$ and $a_t = vec(A_t)$. As we mention before, we also allow variants of this model that restrict the parameters of this model to be time-invariant. If $r = 0$, we have TVP-VAR model.

We have a four sets of state equations: two associated with the measurement error covariance matrix, one for the cointegrating space given in Equation (4), and one for the conditional mean coefficients. The prior for the initial condition for the cointegration space implies a uniform for p_1. The prior for $\overline{\rho}$ is uniform over the interval (0.999, 1).[2] After a burn-in of 1000 iterations, we use 20,000 consecutive iterations using MCMC algorithm.

[1] I.e., $\beta_t' \beta_t = I_r$. This identification constraint does not restrict the estimable cointegration space. If we assume a uniform prior for β_t, the implication is a uniform prior on p_t according to Strachan and Inder (2004).

[2] $\overline{\rho} = 0.99$ leads to unrealistically huge changes in the cointegrating space according to Koop et al. (2011).

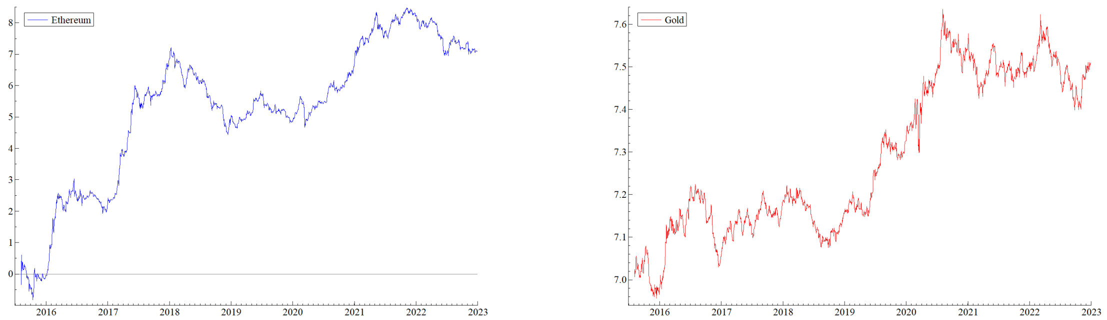
4 Data

The sample starts from the year 2015, when data is available for Ethereum price. The data frequency is daily. Our sample covers between 08/06/2015 and 12/30/2022. Our study analyzes the long-term relationship, but a limitation is that we have less than ten years of sample period. Although there are Ethereum transactions on Saturdays, we prefer to have the synchronization with the gold price.

We use the Ethereum price in dollars from Coincodex. We extract data on the price of gold in dollars from Bloomberg. We use the variables Gold and Ethereum price in logarithm.

Figure 1 presents the logarithm of the price of the Ethereum price (on the right) and gold price (on the right). The price of gold has a lower level over the time considered, with an increase from mid-2019, mainly from December, to the highest level in the series in the sample with the increase in uncertainty due to the covid-19 pandemic. Ethereum price mainly rises from the second half of 2020. The historic peak of Ethereum price occurs in 2021. In the period of covid-19, both prices rose in a period of uncertainty. Visually, the analyses series appear to be non-stationary which is a condition for us to use the VEC model.

Figure 1: Visual analysis of the logarithm of the price of the two series: Ethereum (on the left) and Gold (on the right)



5 Results

We analyze in this section whether there is a long-term relationship between Ethereum and gold prices, whether such a relationship is time-varying, if any. The first step is for us to choose the number of lags of the VAR model in level by the information criterion. Table 1 reports the information criteria for the VAR in level with the different lags. We have Akaike information criterion (AIC), Bayesian information criterion (BIC), and Hannan-Quinn information criterion (HQ). The three criteria indicate two lags for the VAR model in level, which leads that the VEC model has a lag $l = 1$.

Next we choose the VEC model specification. Table 2 presents the information criteria for the different specifications of the VEC model with $l = 1$. We compare three points: (i) unrestricted and restricted intercept for the cointegrating relationship, (ii) whether there is the presence of cointegration, and (iii) whether the model parameters are time-varying (TVP) or invariant (TIP). The information criteria indicate that the best model has one cointegrating vector, with restricted intercept and TVP.

The Figure 2 shows the estimate of the cointegrating coefficient associated with the price of gold and the 90% confidence interval. The price of Ethereum starts to have a stronger relationship with the price of gold over time due to the increase in the coefficient. For example, the 1% rise in the price of gold was accompanied by a rise of 0.3% in the price of Ethereum in 2016 and moved into the range between 0.8% and 0.9% in 2022. The price of Ethereum began to follow the price of gold proportionally over time, although this coefficient reduced throughout 2022.

Table 1: Information criteria to select the number of lags of the VAR model in level

Lags	AIC	BIC	HQ
2	**-16599.3**	**-16571.7**	**-16589.1**
3	-16595.1	-16556.4	-16580.8
4	-16592.6	-16542.9	-16574.3
5	-16589.7	-16529.0	-16567.3
6	-16583.4	-16511.5	-16556.9
7	-16583.9	-16501.0	-16553.3
8	-16585.1	-16491.2	-16550.5
9	-16579.4	-16474.5	-16540.7
10	-16575.2	-16459.1	-16532.4
11	-16570.0	-16442.9	-16523.1
12	-16566.5	-16428.4	-16515.6
13	-16567.0	-16417.8	-16512.0
14	-16562.8	-16402.6	-16503.7
15	-16560.0	-16388.7	-16496.8

Table 2: Information criteria for selecting among VEC model specifications

Intercept	Parameters	r	AIC	BIC	HQ
Unrestricted	TIP	0	-15747.4	-15730.8	-15741.3
Unrestricted	TIP	1	-17279.8	-17246.7	-17267.6
Restricted	TIP	0	-17272.0	-17261.0	-17267.9
Restricted	TIP	1	-17180.7	-17147.5	-17168.5
Unrestricted	TVP	0	-17087.5	-17070.9	-17081.4
Unrestricted	TVP	1	-17361.9	-17331.4	-17350.7
Restricted	TVP	0	-17275.8	-17264.8	-17271.8
Restricted	TVP	1	**-17660.8**	**-17630.3**	**-17649.6**

The Figure 3 shows the estimate of the coefficient $\hat{\alpha}_t$ of Ethereum price (on the left) and Gold price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval. The Ethereum price responds to deviations from the long-term relationship in part of the analyzed time - in moments like the years 2015 to early 2018 -, although this is not the case in the entire period. The gold price does not respond to deviations in the long-term relationship, with the exception of specific moments.

Figure 2: Estimate of the coefficient $\hat{\beta}_t$ associated with the price of gold and the 90% confidence interval

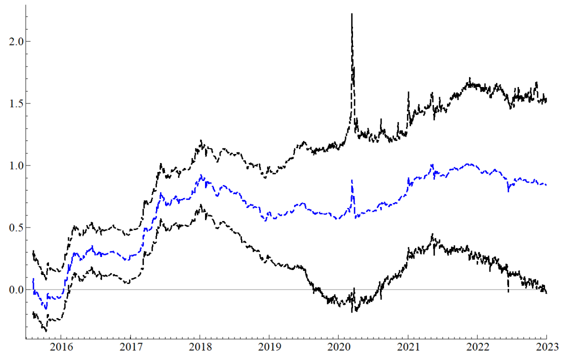
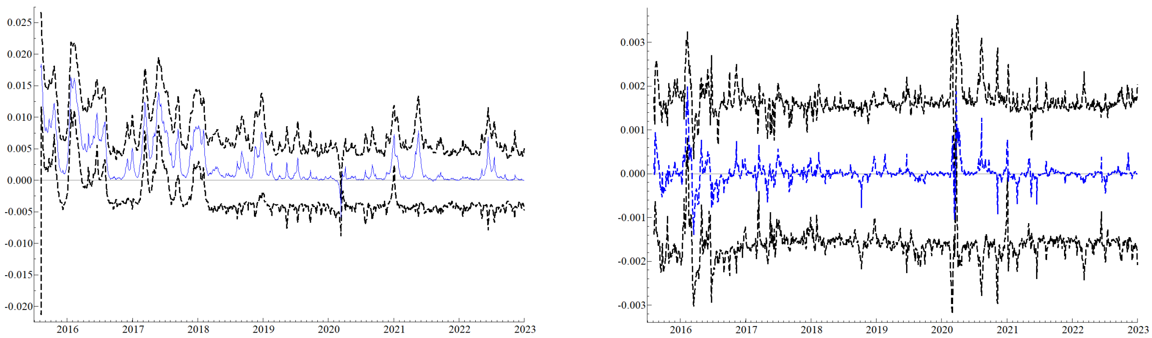


Figure 3: Estimate of the coefficient $\hat{\alpha}_t$ of Ethereum Price (on the left) and Gold Price (on the right) response to deviations from the long-term relationship, and the 90% confidence interval



6 Conclusion

This article seeks to model the efficient price or long-term behavior of Ethereum by a VEC model explained by the gold price. We consider the case of the presence of time-varying cointegration (for the long-term relation vector and for the loadings vector) with stochastic volatility based on Koop et al. (2011). The data are daily between 08/06/2015 and 12/30/2022.

The result indicates that the price of Ethereum follows the price of gold more in the long-term relationship between 2015 and 2018. However, the coefficients of the speed of adjustment are not statistically significant at 10%, indicating that cointegration ceases to exist from 2018 mostly. In the period where cointegration is present, the Ethereum price responds to deviations in the long-term relationship with gold price, as expected. In the period of covid-19 there was an increase in uncertainty and the possibility of analyzing whether Ethereum would be a hedge asset. According to our result, Ethereum did not play this role.

This is a work in progress where we will be adding other variables like the VIX. Furthermore, we will consider the relation of Bitcoin price to these variables. Still we will increase the number of consecutive iterations after burn-in to 36,000 as Koop et al. (2011) because the process is computationally intensive.

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