Does price efficiency improve portfolio allocation? An empirical evidence for cryptocurrencies

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Abstract

This paper proposes a new investment strategy in the cryptocurrency market based on a two-step procedure. The first step is the computation of the asset's levels of efficiency in a universe of cryptocurrencies. Price returns' efficiency degrees are measured by their corresponding levels of multifractality, obtained by the multifractal detrended fluctuation analysis method. The higher the multifractality, the higher the inefficiency in terms of the weak-form of market efficiency. Cryptocurrencies are then ranked in terms of efficiency. The second step is the construction of portfolios under the Markowitz framework composed of the most efficient digital coins. Minimum variance, maximum Sharpe ratio, equally weighted, and efficient-based portfolios were considered. The former strategy is also proposed, where the weights are computed proportionally to the asset's levels of efficiency. The main findings are that cryptocurrency price returns are multifractal, and their levels of efficiency change over time. In periods of high volatility and high price depreciation (bear market), a better performance is associated with portfolios composed of the most efficient cryptocurrencies.

Keywords: Portfolio Allocation, Cryptocurrencies, Price efficiency, MF-DFA, Multifractality.

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1. Introduction

The modern portfolio selection theory, proposed by Markowitz (1952), or also known as the mean-variance approach, is a classic portfolio management tool, widely considered by academics and market participants. The theory provides a framework to find optimal weights for portfolios associated with the lowest level of risk for a given return. Weights computation is based on the estimation of the means and covariances of portfolio assets returns (Xing et al., 2014).

The empirical literature has indicated a poor out-of-sample portfolios performance when means and covariances are inaccurate under the methodology of Markowitz (Merton, 1980). Jagannathan & Ma (2003), Jiang et al. (2019), Bodnar et al. (2018), Bodnar et al. (2017), Frahm & Memmel (2010) are examples of different approaches to compose minimum variance portfolios in equity markets in order to avoid the need of portfolio mean returns estimation (Jagannathan & Ma, 2003). Other studies suggested different methodologies to improve the accuracy of portfolio risk and return estimation in the mean-variance framework, such as fuzzy logic (Mashayekhi & Omrani, 2016; Yoshida, 2009), data envelopment analysis (Essid et al., 2018; Lim et al., 2014), autoregressive and moving average models (Pinto et al., 2011), extreme value volatility models (Karmakar, 2017; Dimitrakopoulos et al., 2010), Bayesian techniques (Bodnar et al., 2017), cointegration and correlation methods for index tracking (Sant'Anna et al., 2017), artificial neural networks (Yu et al., 2008; Fernández & Gómez, 2007), support vector machines (Paiva et al., 2019), particle swarm optimization (Silva et al., 2019), realized volatility estimators (Caldeira et al., 2017), and jump-diffusion processes (Lian & Chen, 2019). They found that risk-return estimation methods are directly associated with weights estimation error and, therefore, the out-of-sample portfolios performance.

Besides these developments, a little attention has been devoted to the problem of selecting the assets before computing portfolio weights (optimization step) under the mean-variance framework. Generally, it is supposed to be an universe of assets, then, based on a defined investor objective, weights are obtained from the solution of an optimization problem. Hence, the aim of this work is to propose a new investment strategy, where the assets level of (in)efficiency is used to select the assets before the computation of portfolio weights under the Markowitz framework.

The Efficient Market Hypothesis (EMH), proposed by Fama (1965), is one of the most important theoretical frameworks in finance, used as the basis for the development of several pricing models and decision-supporting systems. The EMH essentially assumes that asset prices reflect all relevant information, which are available to all market participants. Many studies in the empirical finance literature have investigated the validity of the EMH, especially its weak-form, which states that prices changes follow a random walk dynamics, thus implying the unpredictability of security returns based on historical data[1].

Empirically, asset returns do not follow the weak-form of the EMH, as the random walk dynamic is considered very restrictive. Lack of liquidity, higher transaction costs, insider trading and errors in investors' judgments are the main sources of inefficiency, particularly in periods of higher levels of uncertainty, which deviates the actual behavior of markets from the efficiency. As a consequence, predictable patterns emerge from price irregularities that might be persistent for short periods in some cases (Diniz-Maganini et al., 2021). Besides this aspect, which is important for investors, market efficiency also plays a crucial role for the economies in general by promoting an effective resources allocation (Rizvi & Arshad, 2016).

A recent and novel framework for measuring asset returns efficiency is the multifractal detrended fluctuation analysis (MF-DFA). From econophysics, the method provides a tool for examining financial time series stylized facts such as multifractality, asymmetry, persistence and long-memory dependencies. These are especially essential for portfolio management, as they are related to future prices predictability, and thus market inefficiency (Al-Yahyaee et al., 2020). In

[1] Markiel & Fama (1970) and Titan (2015) are surveys regarding the empirical analysis of the weak-form of market efficiency.

addition, a particular advantage related to finance is that MF-DFA is able to provide the construction of a measure to rank the markets based on their efficiency degrees by evaluating the spectrum of generalized Hurst exponents[2,3,4].

In this context, this paper aims to investigate how assets efficiency influences portfolio performance under the mean-variance framework. For instance, our main research question is: do portfolios composed of more adherent assets to the weak-form of market efficiency provide a better risk-return relation? To address this issue, MF-DFA is used to select the most efficient assets before optimizing the portfolios (computing weights). Then, portfolios out-of-sample performances are evaluated in terms of risk and return under different investors objectives and market dynamics (bull, bear and volatile markets). Maciel (2021) particularly evaluates how efficiency influences equity portfolios performance in the Brazilian market. The author showed that most efficient equities resulted in portfolios with lower levels of systematic risk (betas), indicating that the lack of efficiency is related to higher sensitivity to macroeconomic and conjuncture changes. However, only a particular out-of-sample period is considered, lacking robustness for different market dynamics.

Particularly, the empirical analysis of this paper concerns the evaluation of

[2]The Hurst exponent, referred to as the "index of dependence" or "index of long-range dependence", is used as a measure of long-term memory of time series. Originally developed in hydrology and commonly studied in fractal geometry, it relates to the autocorrelations of the time series, and the rate at which these decrease as the lag between pairs of values increases (Hurst, 1951).

[3]Traditional nonlinear variance ratio tests or autocorrelation functions are not able to identify multifractal structures. Fractal properties are associate to time series that present heavy tails and long-memory. As these features are commonly observed in financial asset price returns (stylized facts), the use of MF-DFA appears as a suitable technique to evaluate random walk properties in such series, as stated by the econophysics literature (Arshad et al., 2016; Ali et al., 2018; Tiwari et al., 2019b).

[4]The works of Mensi et al. (2018), Sukpitak & Hengpunya (2016), Dewandaru et al. (2015), Tiwari et al. (2019a), Shahzad et al. (2017), Zhu & Zhang (2018) and Rizvi & Arshad (2017) are examples of using MF-DFA to evaluate the weak-form of market efficiency in financial markets, mostly stock markets.

cryptocurrency portfolios. The reasons for selecting this sort of asset are threefold: i) cryptocurrencies have risen rapidly in market capitalization over the past years; as liquidity is a drive of efficiency, strategies involving digital coins based on their levels of efficiency may provide benefits to compose portfolios; ii) due to the high average returns and low correlations of crypto-assets, investors are interested in their potential for portfolio allocation strategies and/or diversification (Petukhina et al., 2021), thus profitable approaches to perform this task is of practical interest; iii) the literature has verified the multifractal behavior of cryptocurrencies (Mnif et al., 2022; Al-Yahyaee et al., 2020), as discussed in the following.

Considering Bitcoin, Ethereum, Monero, Dashcoin, Litecoin and Ripple, Al-Yahyaee et al. (2020) indicated that the corresponding digital coin returns are inefficient, and that the level of efficiency is time-varying. Among the coins, Dashcoin was considered the least inefficient, whereas Litecoin was the cryptocurrency associated with the highest level of inefficiency. Similarly, Cheng et al. (2019) also found evidence on the multifractality of cryptocurrency returns, stating that the investor behavior under different time scales also exhibits a nonlinear state in such markets. Moreover, using data from 2010 to 2017, Al-Yahyaee et al. (2018) have showed an increase on Bitcoin multifractality, and that this digital coin is more inefficient than the gold, stock and currency markets[5]. Thus, the literature has found that cryptocurrencies (and the financial markets in general) are multifractal and, especially, significant impacts on market level of efficiency were verified after the COVID-19 outbreak. However, the evaluation on how the level of efficiency impacts portfolio allocation strategies is still not not verified by the literature, particularly for cryptocurrencies.

The methodology of this work comprises the use of the most negotiated

[5]Ozkan (2021), Diniz-Maganini et al. (2021), Mnif et al. (2020), Naeem et al. (2021a), Naeem et al. (2021b), Mensi et al. (2020), Choi (2021) and Mensi et al. (2021) also found evident on the impacts of the COVID-19 pandemic on the level of efficiency in different markets.

cryptocurrencies to compose long-only minimum variance and maximum Sharpe ratio portfolios using data of nineteen digital coins for the period from January 2018 to October 2022. The main objective is to propose and evaluate a novel strategy for portfolio management based on a two-step procedure by considering cryptocurrency levels of efficiency before computing portfolio weights. The first step comprises the ranking of cryptocurrencies based on their multifractality levels (computed by MF-DFA), and the second step is the computation of portfolios weighted considering the most efficient digital coins. Out-of-sample performance is evaluated using several return and risk measures and including comparisons against an equally weighted portfolio as a benchmark. An alternative heuristic portfolio selection is also proposed, where the weights are computed according to each cryptocurrency level of efficiency. This allocation strategy is a new strategy to compose portfolios based on (in)efficiency, named, (in)efficiency-based cryptocurrency portfolios. In addition, empirical experiments also include the performance evaluation of the proposed strategies during recent systemic extreme events: the Corona Virus Disease (COVID-19) pandemic and the Ukraine-Russia war.

The contributions of this paper to the literature and market participants can be described as follows. First, the use of MF-DFA to select the most efficient cryptocurrencies to compose portfolios was not considered by the literature to our best knowledge. Second, it is suggested a new trading strategy that accounts for cryptocurrencies' level of multifractality to compute portfolio weights as a heuristic portfolio composition. Third, it considers an extensive empirical experiment focusing on different digital coins instead of solely on Bitcoin and Ethereum as most of the studies. Fourth, the suggested approach provides an alternative and simple trading strategy which may be useful for market participants to improve their decision-making processes for cryptocurrency portfolio construction. Finally, this study gives the evaluation of efficiency-based investment strategies during bull, bear and more volatile market dynamics, and also in periods of extreme events such the COVID-19 pandemic and the Ukraine-Russia war.

After this introduction, Section 2 describes the methodology, the MF-DFA method, the strategies for portfolio selection, as well as the out-of-sample performance measurements. Empirical experiments are detailed in Section 3. Finally, Section 4 concludes the paper and suggests topics for future research.

2. Methodology

This paper proposes a new strategy to compose cryptocurrency portfolios. The main idea is first to measure the adherence of the digital coins to the weak-form of market efficiency using MF-DFA. Precisely, it is evaluated how cryptocurrency returns dynamics approaches a random walk process considering their corresponding levels of multifractality. The level of multifractality, as a proxy for the degree of (in)efficiency, is used to select the most efficient cryptocurrencies to compose portfolios, and then weights are computed under the mean-variance framework. The role of asset efficiency is evaluated in portfolio performance. Cryptocurrency portfolios are optimized using minimum variance and maximum Sharpe ratio strategies, as investors objectives. Finally, comparisons are made considering benchmarks such as the equally weighted portfolio. Further an heuristic strategy is also proposed in this work: an (in)efficient-based allocation strategy. It simply computes portfolio weights proportional to the levels of efficiency of each digital coin. Results are evaluated in terms of risk and return metrics for different out-of-sample periods mimicking moments of bull, bear and volatile markets, and also considering contexts of extreme events such as the COVID-19 pandemic and the Ukraine-Russia war. MF-DFA method, the portfolios construction frameworks, and the performance measures are detailed in the following.

2.1. Multifractal detrended fluctuation analysis

Multifractal detrended fluctuation analysis, proposed by Kantelhardt et al. (2002), uses generalized Hurst exponents and is a powerful tool for detecting multifractality in a time series. Properties like persistence, anti-persistence and random walk behavior can be measured through MF-DFA. Particularly for asset returns series, according to the values of the q-th order Hurst exponents, the adherence of the weak-form of market efficiency can be evaluated, as well as the measuring of the corresponding level of (in)efficiency.

Let $\{r(t)\}$, for $t = 1, 2, \dots, T$, be a non-stationary time series of length T. In this work, $r(t) = \ln[P(t)] - \ln[P(t-1)]$ are the log-returns, and P_t stands for the equity price at t. To apply the MF-DFA technique, a new sequence, denoted by $y(t)$, called the profile function, is constructed as:

$$y(t) = \sum_{k=1}^{i} [r(k) - \bar{r}], \quad i = 1, 2, \dots, T, \tag{1}$$

where $\bar{r} = (1/T) \sum_{t=1}^{T} r_t$.

The time series $y(t)$ is divided into $T_s \equiv \text{int}(T/s)$ windows of equal length s, where s is the scale parameter. These segments must be non-overlapping. As the length T may not be a multiple of the scale parameter s, the constructed intervals may disregard a short part of the profile function. Thus, the subdivision is performed from the opposite end and a total $2T_s$ sub-intervals is constructed (Tiwari et al., 2019b; Bai & Zhu, 2010). This mechanism avoids any information lost[6].

For each window, $\{\nu = 1, \dots, 2T_s\}$, the next step comprises the fitting of a polynomial of order m (usually, $m = 1$) using least squares to compute the local tendency. Then the variance is calculated for $\nu = 1, \dots, T_s$ and $\nu = T_s + 1, \dots, 2T_s$, respectively:

$$F^2(s, \nu) = \frac{1}{s} \sum_{t=1}^{s} \{y[(\nu - 1)s + t] - y_\nu^m(t)\}^2, \tag{2}$$

$$F^2(s, \nu) = \frac{1}{s} \sum_{t=1}^{s} \{y[T - (\nu - T_s)s + t] - y_\nu^m(t)\}^2, \tag{3}$$

[6] As suggested by Rizvi & Arshad (2014), the scaling range assumed the values of $s_{min} = 10$ and $s_{max} = (T/4)$, where T is the series' number of observations.

where y_ν^m corresponds to the fitting polynomial with order m in the ν-th segment. To avoid overfitting and facilitate calculation a linear polynomial ($m = 1$) is considered, as suggested by Choi (2021).

The q-th order fluctuation function, $F_q(s)$, is obtained by averaging over all segments:

$$F_q(s) = \left\{ \frac{1}{2T_s} \sum_{\nu=1}^{2T_s} [F^2(s,\nu)]^{q/2} \right\}^{1/q}. \tag{4}$$

The q-order Hurst exponent is defined as the slopes $h(q)$ of regression lines for each q-order root mean square $F_q(s)$. The order q, $q \in \Re, q \neq 0$, encompasses the effect of varying degrees of fluctuation on $F_q(s)$, thus it is related to small (larger) fluctuations when $q < 0$ ($q > 0$). Notice that the standard DFA is obtained when $q = 2$ (Tiwari et al., 2019b).

From the fluctuation function, the final step of MF-DFA comprises the computing of the scale index, based on the log-log plots of $F_q(s)$ against s for each value of q. A linear pattern in the log-log scale is obtained if $F_q(s)$ is in accordance with the called power-law: $F_q(s) \sim s^{h(q)}$.

A fluctuation function value $F_q(s)$ is computed for each segment s. The q-order generalized Hurst index, $h(q)$, corresponds to the slope of $ln\,(F_q(s)) \sim ln(s)$. The dependence of $h(q)$ on q provides relevant information concerning the pattern of a time series (Ali et al., 2018). A time series is monofractal if $h(q)$ does not depend on q. Otherwise, a series is multifractal when $h(q)$ depends on q and monotonically decreases as q increases.

If the Hurst exponent is equal to 0.5, $h(q) = 0.5$, the time series is a random walk independent process. In such a case, the stochastic process is adherent to the weak-form of market efficiency. Finally, when $0 < h(q) < 0.5$ ($0.5 < h(q) < 1$) the time series correlations are anti-persistence (persistence), indicating the rejection of a random walk dynamics.

The exponent $h(q)$ relates to the multifractal scaling exponents $\tau(q)$ as:

$$\tau(q) = qh(q) - 1. \tag{5}$$

To estimate multifractality, q and $\tau(q)$ are transformed to α and $f(\alpha)$ using a Legendre transform (Choi, 2021):

$$\alpha = \frac{d}{dq}\tau(q), f(\alpha) = \alpha(q)q - \tau(q), \tag{6}$$

where $f(\alpha)$ is the multifractal spectrum or singularity spectrum and α is the singularity strength.

Hence, the level of multifractality can be calculated as follows (Choi, 2021; Tiwari et al., 2019b; Ali et al., 2018):

$$\Delta h = max(h(q)) - min(h(q)). \tag{7}$$

The higher Δh a stronger degree of multifractality is observed. This measure of multifractality provides a mechanism to rank the series according to their levels of multifractality or efficiency. If a series follows the random walk hypothesis (weak-form of market efficiency), $h(q) = 0.5$ for distinct values of q. Hence, a market is said to be weak-form efficient when Δh is zero. Otherwise, the market is inefficient and the higher the Δh value the higher its inefficiency. This property provides a mechanism to rank different markets in terms of (in)efficiency, and also allows the measure of efficiency over time. Thus, based on the Δh values, cryptocurrencies levels of efficiency can be evaluated in order to compose portfolios with the most efficient digital coins, which is the main purpose of this paper.

An alternative way to compute the level of multifractality is the width of the multifractal spectrum $\Delta\alpha$:

$$\Delta\alpha = max(\alpha) - min(\alpha). \tag{8}$$

Likewise, a wider multifractal spectrum implies a stronger degree of multifractality. Finally, we can compute the the asymmetry parameter (Θ) that estimates the asymmetry of the multifractal spectrum (Choi, 2021):

$$\Theta = \frac{\Delta\alpha_L - \Delta\alpha_R}{\Delta\alpha_L + \Delta\alpha_R}, \tag{9}$$

where $\Delta\alpha_L = \alpha_0 - \alpha_{min}$, $\Delta\alpha_R = \alpha_{max} - \alpha_0$, α_0 is the α value at the maximum of $f(\alpha)$. According to Choi (2021), $\Theta > 0$ is associated with left-sided asymmetry, where subsets of large fluctuations contribute substantially to the multifractal spectrum. Otherwise, $\Theta < 0$ indicates right-sided asymmetry in the spectrum, which means that smaller fluctuations constitute a dominant multifractality source.

2.2. Strategies for portfolio selection

Using the level of multifractality (Δh), calculated from MF-DFA, the cryptocurrencies are ranked in terms of efficiency, the lower the level of multifractality the higher the efficiency (i.e. more adherent to the random walk, a process that describes the weak-form of market efficiency). Then, (in)efficient-based portfolios are constructed under the mean-variance framework of Markowitz (1952) for two different investing strategies: the Minimum Variance Portfolio (MVP) and the Maximum Sharpe Ratio Portfolio (MSR).

Under a universe of N assets and the corresponding portfolio represented by the vector of weights $\mathbf{w} = (w_1, w_2, \ldots, w_N)^T$, the MVP is obtained as the solution of the following optimization problem:

$$\min_{\mathbf{w}} \left[\sigma_p^2\right] = \min_{\mathbf{w}} \left[\mathbf{w}^T \Phi \mathbf{w}\right], \text{ subject to } \sum_{i=1}^{N} w_i = 1, \tag{10}$$

where $\sigma_p^2 = \mathbf{w}^T \Phi \mathbf{w}$ is the portfolio variance, Φ a $N \times N$ matrix of covariances, and the constraint $\sum_{i=1}^{N} w_i = 1$ stands for a full-invested portfolio.

We considered long-only portfolios, hence $w_i \geq 0 \; \forall i$ constraints must be included in (10). The solution of MVP problem depends only on the covariance matrix and does not require returns means estimation, as estimation errors on this later statistic have considerable impact on the portfolio weights (Merton, 1980).

Otherwise, the Maximum Sharpe Ratio Portfolio (MSR) is the portfolio in the efficient frontier of Markowitz associated with the maximum return-risk relation:

$$\max_{\mathbf{w}} \left[\mathrm{SR}_p\right] = \max_{\mathbf{w}} \frac{\mathbf{w}^T \mu_p}{\sqrt{\mathbf{w}^T \Phi \mathbf{w}}}, \text{ subject to } \sum_{i=1}^{N} w_i = 1, \tag{11}$$

$\mu_p = (\mu_1, \mu_2, \ldots, \mu_N)^T$ is the vector of assets mean returns.

The Equally Weighted Portfolio (EWP) strategy is also considered in this work as a benchmark. For a universe of N stocks, the weights in the EWP are calculated as:

$$w_i = \frac{1}{N},\ \forall i. \tag{12}$$

Finally, an heuristic portfolio allocation strategy is also proposed in this work. The basic idea is that the participation of each asset in the portfolio (measured by its corresponding weight) is proportional to its level of efficiency (measured by the Δh metric - degree of multifractality). Thus, the weights in the efficiency-based portfolio (EBP), are computed as:

$$w_i = \frac{\Delta h_i^{-1}}{\sum_{j=1}^{N} \Delta h_i^{-1}}, i = 1, 2, \ldots, N. \tag{13}$$

It is considered the inverse value of Δh, Δh^{-1}, as efficiency is higher when Δh is the lowest. Hence, weights computation using Eq. (13) gives higher participation to the more efficient digital coins relative to the other cryptocurrencies in the portfolio. The aim of this approach is to verify how efficiency can drive an alternative way to compute cryptocurrency portfolio weights.

Hence, MVP, MSR, EWP and EBP portfolios will be constructed by the more efficient and considering all digital currencies considered in this work. This method aims to verify whether or not cryptocurrencies efficiency can influence portfolio out-of-sample performance.

Additionally, this work does not consider the possibility of short positions, as the possibility of short-selling is affected by market conditions associated with the costs of cryptocurrency loans. Moreover, since the aim of this research is the empirical evaluation of a strategy based on selecting assets based on their efficiency levels, allowing for short positions may cause a distortion of the results in favor of some method as portfolios performances can be significantly affected by the level of leverage exposure setting, an additional decision parameter that requires a sensitivity analysis for proper comparisons.

Covariance matrix is estimated using the most simple methodology, i.e. the sample covariance matrix[7]. It is computed based on the time series of asset returns over a pre-specified period. Thus, $\Phi = [\phi_{i,j}]$, where $\phi_{i,j}$ is the covariance between assets i and j, is estimated as:

$$\hat{\phi}_{i,j} = \frac{1}{T}\sum_{t=1}^{T}(r_{i,t}-\mu_i)(r_{j,t}-\mu_j), \tag{14}$$

where $r_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$ is the log-return of asset i at t, $P_{i,t}$ the asset price at t, and $\mu_i = \frac{1}{T}\sum_{t=1}^{T} r_{i,t}$ the mean return of asset i.

2.3. Performance assessment

Performance is evaluated with a backtesting approach. Based on an in-sample set, cryptocurrency levels of efficiency are measured. Portfolios are composed by the most efficient cryptos, as well as considering all assets, for comparisons. Portfolios weights are computed under different strategies, i.e. MVP, MSR, EWP, and EBP. Performance is then evaluated in an out-of-sample set with non-rebalancing[8] using different risk and return measures, such as: annualized returns, cumulative returns, annualized volatility, average one-day Value-at-Risk (VaR) and Shape ratio.

The annualized returns of a portfolio, r_p^A, is used for comparing competing portfolios in terms of profitability, and is calculated as:

$$r_p^A = \left[\prod_{t=1}^{T}(1 + r_{p,t})\right]^{360/T} - 1, \tag{15}$$

where 360 stands for the average number of days over a year.

The cumulative return, r_p^C, calculates the geometric return over a period of

[7] More sophisticated methods for covariance matrix estimation may be used, such as EWMA and multivariate GARCH-family models. However, testing different methodologies for covariances in portfolio selection is beyond the main objective of this work.

[8] Rebalancing schemes can be considered, however, the identification of the time of rebalancing, as well as the consideration of transaction costs, are complex tasks, being considered as future work due to length limitations.

time:

$$r_p^C = \left[\prod_{t=1}^{T} (1 + r_{p,t}) \right] - 1. \tag{16}$$

To measure the portfolio risk, the annualized volatility, σ_p^A, is calculated as:

$$\sigma_p^A = \sqrt{360} \cdot \sigma_p = \sqrt{360} \cdot \sqrt{\frac{1}{T} \sum_{t=1}^{T} (r_{p,t} - \mu_p)^2}, \tag{17}$$

where μ_p is the mean portfolio return.

Another measure of risk, usually used by market participants, is the Value-at-Risk (VaR). The VaR is defined such that the probability of a loss greater than VaR is (at most) γ%. Formally, the VaR is defined as:

$$\text{VaR}_\gamma = \inf\{x \in \Re : \text{Prob}(r_p < x) \leq \gamma\}. \tag{18}$$

The one-day VaR is computed for the portfolios using its nonparametric approach based on historical data (Historical VaR) at a $\gamma = 5\%$ confidence level (quantile). Hence, the average daily $\text{VaR}_{5\%}$ is calculated as an alternative metric for portfolio risk.

The portfolio Sharpe ratio measures the returns of the portfolio, adjusted to the risk: $SR_p = \frac{r_p}{\sigma_p}$.

3. Empirical analysis

The main objective of this paper is to evaluate how efficiency influences the performance of cryptocurrency portfolios. Our methodology can be summarized by two simple steps:

STEP 1: Compute the levels of multifractality (Δh), as a proxy of efficiency, for all the cryptocurrencies considered. Digital coins are then ranked in terms of efficiency to compose portfolios with the most efficiency cryptocurrencies;

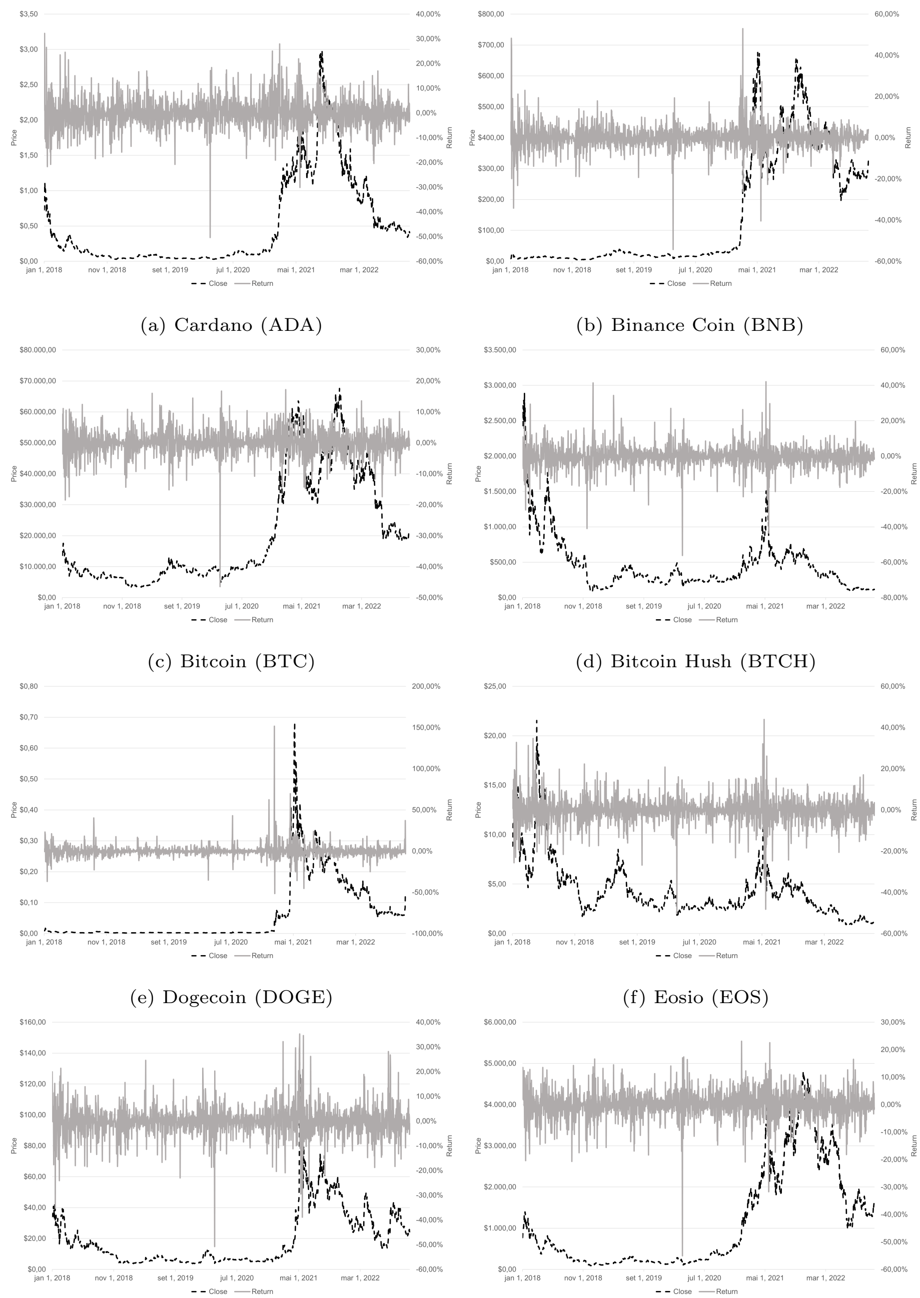
STEP 2: Optimize portfolio weights for minimum variance and maximum Sharpe ratio strategies, and calculate weights of equally weight and (in)efficiency-based portfolios, for this former approach, weights are proportional to the assets level of efficiency - Eq. (13).

Portfolios considering all the digital coins, without taking into account the efficiency, are also evaluated for comparisons, in order to measure the role of multifractality to drive portfolio composition. The following subsections describe the data, the analysis of (in)efficiency using MF-DFA, and the performance of cryptocurrency portfolios when multifractality is used to select the corresponding digital coins to invest in.

3.1. Data

Database comprises daily closing USD prices of nineteen cryptocurrencies for the period from Jan. 1, 2018 to Oct. 31, 2022 within a total of 1,765 observations[9]. Our sample starts in 2018 due to the low number of cryptocurrencies at this period with relevant liquidity and historical information availability, as the analysis in this study aims to consider the higher number of cryptocurrencies, different from the works that focused on Bitcoin and Ethereum solely. The most traded digital coins were selected considering the ones that provide daily prices with a positive traded volume for all the days in the period evaluated. The sample ends in Oct. 31, 2022 as this was the last available data information when the research was performed. Selected cryptocurrencies are: Cardano (ADA), Binance Coin (BNB), Bitcoin (BTC), Bitcoin Hush (BTCH), Dogecoin (DOGE), Eosio (EOS), Ethereum Classic (ETC), Ethereum (ETH), Filecoin (FIL), KuCoin Token (KCS), Chainlink (LINK), Litecoin (LTC), Decentraland (MANA), Tron (TRX), Waves (WAVES), Stellar (XLM), Aerum (XRM), Ripple (XRP), and Zcash (ZEC). Figure 1 shows the temporal evolution of cryptocurrencies prices and log-returns for the period considered. It is clear the high volatility dynamic of the cryptocurrency market, especially from the beginning of 2020 where price changes are significant.

[9]Data were collected at `https://coinmarketcap.com/`.



(a) Cardano (ADA)

(b) Binance Coin (BNB)

(c) Bitcoin (BTC)

(d) Bitcoin Hush (BTCH)

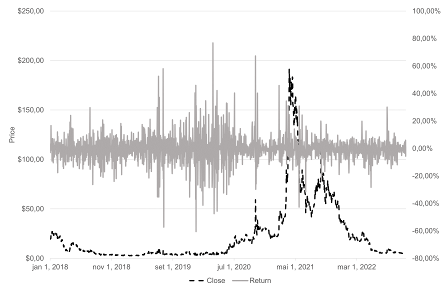
(e) Dogecoin (DOGE)

(f) Eosio (EOS)

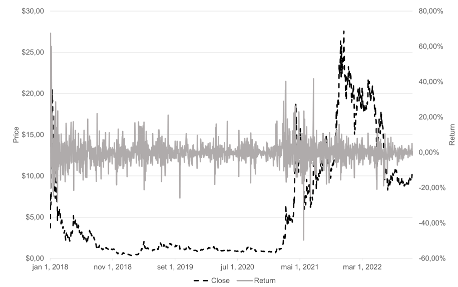
(g) Ethereum Classic (ETC)

(h) Ethereum (ETH)

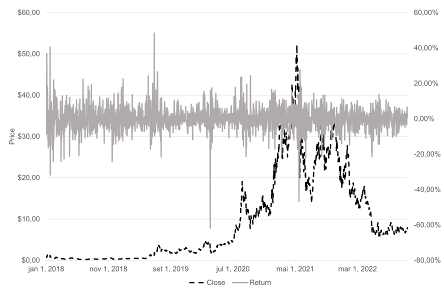
Fig. 1: Temporal evolution of prices and returns of the cryptocurrencies.



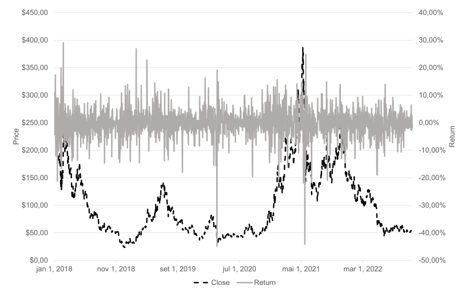
(i) Filecoin (FIL)



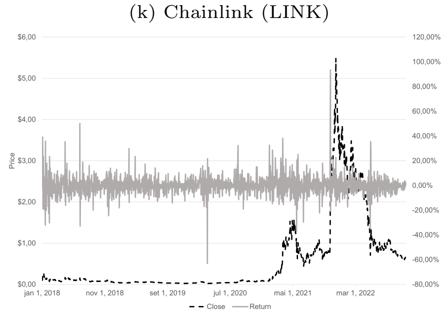
(j) KuCoin Token (KCS)



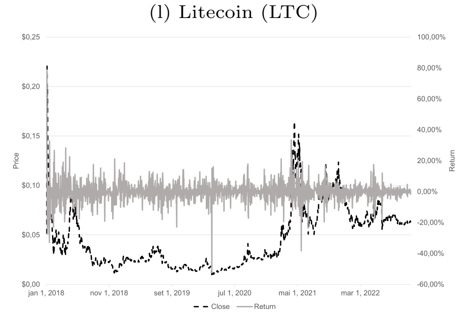
(k) Chainlink (LINK)



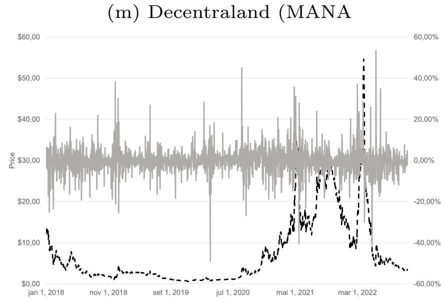
(l) Litecoin (LTC)



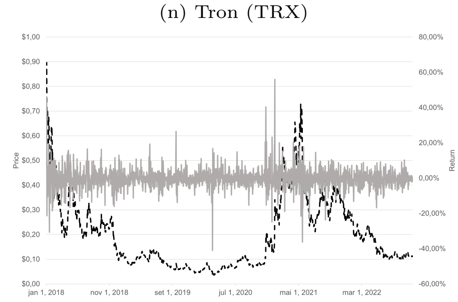
(m) Decentraland (MANA



(n) Tron (TRX)

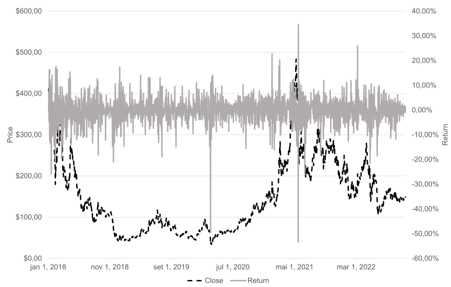


(o) Waves (WAVES)

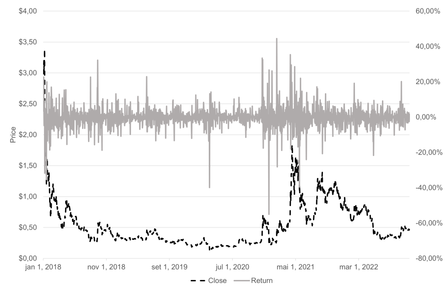


(p) Stellar (XLM)

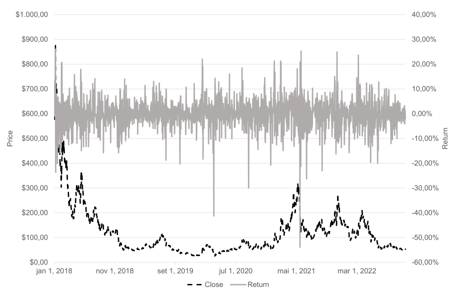
Fig. 1: Temporal evolution of prices and returns of the cryptocurrencies (continued).



(q) Aerum (XRM)



(r) Ripple (XRP)



(s) Zcash (ZEC)

Fig. 1: Temporal evolution of prices and returns of the cryptocurrencies (continued).

Table 1 provides summary statistics for the log-returns of all cryptocurrencies considered in this work. Generally, log-returns are zero mean with similar and high values of standard deviation (high volatility). Cryptocurrencies returns are right- (9 cases) and left-skewed (10 cases), as the skewness values are positive and negative, respectively. In terms of kurtosis, high values are observed for all currencies, revealing heavy tail returns distributions. Finally, it is notable that relevant positive and negative extreme returns are verified (maximum and minimum returns, respectively), revealing that the range of returns variation is significant, which is a feature of a high volatile market as the cryptocurrency (see Table 1).

3.2. Cross-validation strategy

To measure the adherence of cryptocurrency returns to the weak-form of the efficiency market hypothesis, the corresponding degrees of multifractality are measured by the Δh values calculated from MF-DFA, as defined in Eq.

(7). If the process follows a random walk process, $\Delta h = 0$ (Choi, 2021; Tiwari et al., 2019b). The higher the multifractality, the lowest the efficiency of the digital coin price returns. Multifractality is used as a tool for ranking the cryptocurrencies in terms of efficiency to compose portfolios that consider the most efficient assets.

Table 1: Summary statistics for the cryptocurrencies log-returns for the period from Jan. 1, 2018 to Oct. 31, 2022.

Crypto	mean	median	std. dev.	max	min	skewness	kurtosis
ADA	-0.03%	0.00%	5.94%	32.21%	-50.37%	-0.0573	8.3162
BNB	0.21%	0.10%	5.74%	52.92%	-54.28%	0.2783	19.5442
BTC	0.02%	0.11%	3.92%	17.18%	-46.47%	-1.0605	16.2055
BTCH	-0.18%	-0.11%	6.12%	42.08%	-56.14%	-0.3337	14.6259
DOGE	0.15%	-0.12%	7.73%	151.62%	-51.49%	5.1377	96.7217
EOS	-0.12%	0.00%	6.37%	43.94%	-50.32%	-0.2270	11.5944
ETC	-0.01%	0.00%	6.20%	35.24%	-50.78%	-0.0817	11.0905
ETH	0.04%	0.08%	5.10%	23.07%	-55.07%	-0.9990	12.9982
FIL	-0.07%	-0.25%	9.52%	76.82%	-60.46%	0.5185	12.8250
KCS	0.06%	0.00%	6.63%	67.41%	-49.58%	1.2319	18.1515
LINK	0.15%	0.08%	6.99%	48.42%	-61.75%	-0.1765	10.6099
LTC	-0.08%	0.00%	5.32%	29.06%	-44.90%	-0.6057	10.7313
MANA	0.11%	0.05%	7.73%	93.33%	-62.98%	1.2050	21.3737
TRX	0.02%	0.10%	6.34%	78.68%	-52.35%	0.8463	23.0337
WAVES	-0.07%	0.00%	6.85%	53.52%	-48.92%	0.4059	12.2479
XLM	-0.07%	-0.10%	5.87%	55.93%	-41.00%	0.6865	14.9447
XRM	-0.05%	0.17%	5.41%	34.49%	-53.42%	-1.0623	14.6456
XRP	-0.09%	-0.09%	5.97%	44.46%	-55.04%	-0.0858	15.8544
ZEC	-0.13%	-0.14%	6.00%	26.07%	-53.94%	-0.5919	9.7118

For cross-validation and robustness purposes, the whole sample was divided into three in-sample/out-of sample sets as detailed in Table 2. To justify the name of each sample, Figure 2 illustrates the evolution of Bitcoin (BTC) prices over each in-sample/out-of-sample sets. Portfolios are evaluated for three different out-of-sample sets that are associated with distinct market behavior. From Figure 2, notably the year of 2020 verifies a price appreciation of BTC price, being considered a bull market. BTC prices are more volatile during the year of 2021, and a bear market is verified in the year of 2022, where BTC price decreases significantly. As BTC is a leading cryptocurrency, the remaining ones generally follow a similar market behavior for the years of 2020, 2021 and 2022.

Table 2: In-sample and out-of-sample sets for the evaluation of efficiency-based cryptocurrencies portfolios.

	in-sample set			out-of-sample set		
Sample	start date	end date	# obs.	start date	end date	# obs.
Bull	1/01/2018	12/31/2019	730	1/01/2020	12/31/2020	366
Volatile	1/01/2019	12/31/2020	731	1/01/2021	12/31/2021	365
Bear	1/01/2020	12/31/2021	731	1/01/2022	10/31/2022	304

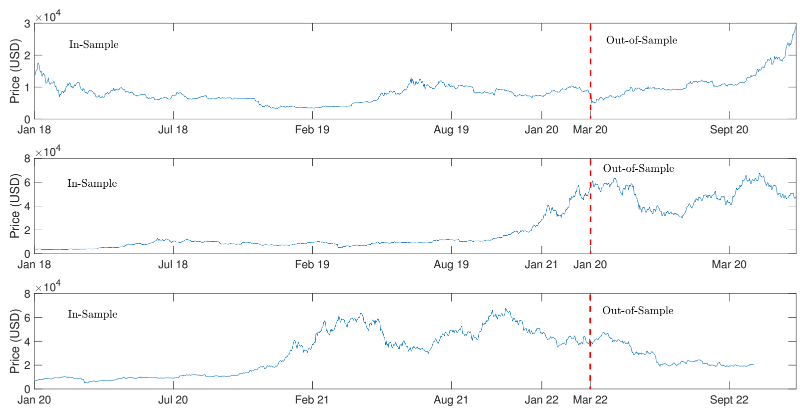


Fig. 2: Bitcoin price evolution for the three in-sample/out-of-sample sets considered in this work.

To further characterize the out-of-sample sets dynamics, Table 3 provides some statistics of log-returns calculated for each set (2020, 2021, and 2022). In terms of return means, positive returns are verified for the years of 2020 and 2021, and a negative average is observed in 2022, due to the "crypto winter" year, where most digital coins suffered a significant price drop. Concerning the volatility of returns, standard deviation values are higher in 2021 (see Table 3), as shown in the price variation dynamics of BTC in Figure 2. Finally, returns 345 are generally left-skewed in 2020 and 2022, and right-skewed in the year of 2022, whereas returns kurtosis decreases over the samples (Table 3).

Table 3: Summary average statistics for all the cryptocurrencies log-returns for the three out-of-sample sets, i.e. 2020, 2021 and 2022. Q1, Q2 and Q3 correspond to the first, second and third quartiles, respectively.

Statistic	Average	Min.	Q1	Q2	Q3	Max.
Panel A: 2020						
mean	0.0027	-0.0007	0.0017	0.0027	0.0042	0.0054
std dev	0.0593	0.0382	0.0520	0.0548	0.0609	0.1289
skewness	-1.6368	-4.0733	-2.4057	-1.7484	-0.7192	0.8132
kurtosis	24.6266	7.9739	17.5035	24.3189	29.5039	53.8678
Panel B: 2021						
mean	0.0037	0.0004	0.0012	0.0023	0.0052	0.0102
std dev	0.0748	0.0420	0.0661	0.0734	0.0775	0.1327
skewness	0.3123	-1.4252	-0.3475	0.0433	0.6319	4.6076
kurtosis	13.1441	4.4856	8.5052	10.8375	13.2864	51.3770
Panel C: 2022						
mean	-0.0029	-0.0061	-0.0036	-0.0029	-0.0017	-0.0006
std dev	0.0493	0.0339	0.0419	0.0494	0.0538	0.0826
skewness	-0.1306	-0.9558	-0.4179	-0.2788	0.0098	1.2273
kurtosis	7.0725	3.9633	4.9235	6.5125	7.8775	13.4677

Finally, it is worth noticing that the year of 2020 is marked by the COVID-19 outbreak, and the year of 2022 by the ongoing full-scale Ukraine-Russia war that started in February, 2022. Hence, the out-of-sample sets, besides representing different market dynamics, also incorporate systemic events which allow a more robust analysis on the effects of efficiency in cryptocurrency portfolio management.

3.3. Efficiency analysis of cryptocurrencies

Concerning the in-sample sets, the multifractality of all cryptocurrencies returns is measured using MF-DFA[10]. Table 4 shows the degree of multifractality (Δh) and the width of the multifractal spectrum ($\Delta\alpha$) for the cryptocurrencies during the three in-sample periods: bull, volatile and bear markets. The degree of multifractality is calculated from the generalized Hurst exponents, $H(q)$, for different values of the fluctuation order, q. A series is monofractal if $H(q)$ is constant for all q, hence $\Delta h = 0$. On the other hand, series is multifractal, fractal structure varies according to the measurement conditions, the higher Δh the higher the multifractality and the less efficient is the market - higher deviation of a random walk dynamics[11].

The results in Table 4 indicate that both the degree of multifractality (Δh) and the width of the multifractal spectrum ($\Delta\alpha$) are generally higher than zero for all digital coins considered in this paper. Larger values of Δh and/or $\Delta\alpha$ are associated with stronger multifractality; this in turn implies less efficiency for the corresponding cryptocurrency, since the multifractal property correlates negatively with market efficiency (Choi, 2021). Thus, in general, all cryptocurrencies present a multifractal behavior, indicating that the weak-form of market efficiency is rejected in all in-samples and for all digital coins.

To evaluate how (in)efficiency changes over time, Table 4 also presents the variation of Δh, $\Delta(\Delta h) = [(\Delta h_t/\Delta h_{t-1}) - 1]$, considering the values from the current in-sample set Δh_t in relation to the value calculated from the previous period Δh_{t-1} - a simple variation ratio. $\Delta(\Delta h) > 0$ ($\Delta(\Delta h) < 0$) indicates a increase (decrease) on the corresponding cryptocurrency inefficiency. For all the 37 calculated values of $\Delta(\Delta h)$, 23 of them are positive, indicating that for most of the cryptos evaluated, an increase in the inefficiency is verified. In some cases, this variation is quite high, as for example for Filecoin, where Δh

[10] All experiments in this work were performed using R software.

[11] The Hurst exponents, $H(q)$, from $q = -4$ to $q = 4$ are not presented here due to length limitations but are available upon request.

increased 488% for the years of 2019-2020 compared to the period of 2018-2019. When the sample of 2019-2020 is considered in relation to 2019-2020, approximately half of the Δh variations (10 of 19) are negative, which means that the year of 2020, marked by the COVID-19 pandemic, caused asymmetrical effects on the cryptocurrencies levels of multifractality, as some observed an improve on the level of efficiency and the remaining are characterized by a decrease of the adherence to the weak-form of market efficiency. However, concerning the years of 2020-2021, compared to 2019-2020, $\Delta(\Delta h)$ values are mostly positive (14 of 19), which means that the cryptocurrencies returns are less weak-form efficient (see Table 4). This result indicates that the systemic event as the Ukraine-Russia war has negatively impacted the level of efficiency in the digital coin markets[12]. Finally, the estimated value of the asymmetry parameter Θ is generally positive for all cryptocurrency return series during the periods considered (see Table 4). Thus, cryptocurrency returns exhibit left-sided asymmetry, which implies that subsets of large fluctuations contribute substantially to the multifractal spectrum.

To illustrate the fractal behavior of the cryptocurrency returns, Figure 3 shows the MF-DFA findings for Ripple (XRP) price returns for the three in-sample periods considered in this work, as an example. Figure 3-a illustrates the fluctuation function $\log_2(F_q(s))$ versus $\log_2(s)$ plot - $q = -4$ (black), $q = 0$ (red), and $q = 4$ (green). It can be observed that the local slope of the plots changes with crossover time scales, which is evidence of multifractality. Multifractality is also confirmed by the dependence of the Hurst exponent on the values of q, as verified in Figure 3-b: as q increases, $H(q)$ shows a downward trend. When a time series exhibit mono fractality, the generalized Hurst exponent should not vary with q.

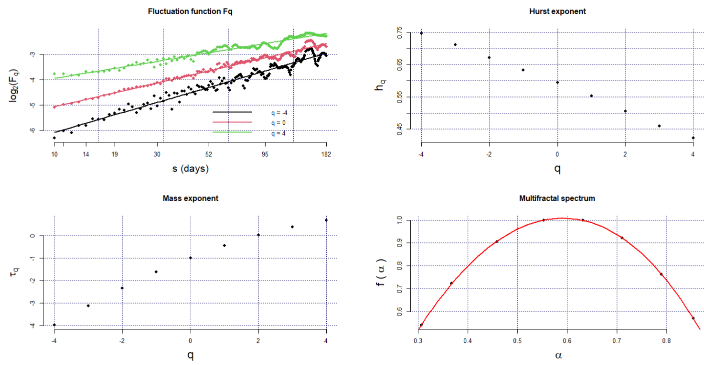
[12]It is important to highlight that the year of 2021 is associated with a bear market and this behavior might be associated with the decrease of the level of efficiency of the corresponding cryptocurrencies. However, the analysis of the temporal dynamics of multifractality if out of the scope of this paper.

Table 4: Multifractality parameters for the cryptocurrency return series during periods of bull, stable and bear markets. Δh is the degree of multifractality, $\Delta\alpha$ is the width of the multifractal spectrum, Θ is the asymmetry parameter, and $\Delta(\Delta h) = [(\Delta h_t/\Delta h_{t-1}) - 1]$ is the variation of Δh considering the values from the current in-sample set Δh_t in relation to the value calculated from the previous period Δh_{t-1}. $\Delta(\Delta h) > 0$ ($\Delta(\Delta h) < 0$) indicates an increase (decrease) on the corresponding cryptocurrency inefficiency.

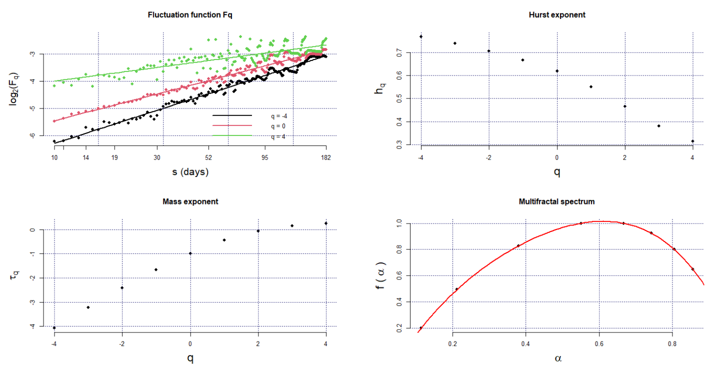
Crypto	Period	α_{max}	α_{min}	α_0	$\Delta\alpha$	Δh	$\Delta(\Delta h)$	Θ
ADA	Bull (2018-2019)	0.7098	0.5492	0.5978	0.1606	0.0844	-	-0.3948
	Stable (2019-2020)	0.6776	0.4587	0.6281	0.2189	0.1190	41%	0.5477
	Bear (2020-2021)	0.9577	0.4572	0.7432	0.5005	0.2992	151%	0.1429
BNB	Bull (2018-2019)	0.7900	0.0452	0.5562	0.7448	0.3998	-	0.3722
	Stable (2019-2020)	0.6596	0.2623	0.5498	0.3973	0.2149	-46%	0.4473
	Bear (2020-2021)	0.6449	0.4067	0.6387	0.2382	0.1233	-43%	0.9479
BTC	Bull (2018-2019)	1.0177	0.3504	0.6478	0.6673	0.4207	-	-0.1086
	Stable (2019-2020)	0.9027	0.2373	0.6771	0.6654	0.3894	-7%	0.3219
	Bear (2020-2021)	0.9314	0.3164	0.6754	0.6150	0.3435	-12%	0.1675
BTCH	Bull (2018-2019)	0.7394	0.3319	0.6109	0.4075	0.2179	-	0.3693
	Stable (2019-2020)	0.7030	0.3584	0.6148	0.3446	0.1865	-14%	0.4881
	Bear (2020-2021)	0.7030	0.3110	0.6016	0.3920	0.2309	24%	0.4827
DOGE	Bull (2018-2019)	0.8894	0.2994	0.7201	0.5900	0.3680	-	0.4261
	Stable (2019-2020)	0.9431	0.0520	0.7172	0.8911	0.5857	59%	0.4930
	Bear (2020-2021)	1.2026	0.1677	0.8178	1.0349	0.6941	19%	0.2564
EOS	Bull (2018-2019)	0.8032	0.4965	0.5793	0.3067	0.1702	-	-0.4601
	Stable (2019-2020)	0.7570	0.3851	0.5714	0.3719	0.2180	28%	0.0019
	Bear (2020-2021)	0.7843	0.2559	0.6006	0.5284	0.3184	46%	0.3047
ETC	Bull (2018-2019)	0.7619	0.2751	0.5830	0.4868	0.2930	-	0.2650
	Stable (2019-2020)	0.7467	0.6006	0.6271	0.1461	0.0644	-78%	-0.6372
	Bear (2020-2021)	0.7074	0.3177	0.6314	0.3897	0.2034	216%	0.6100
ETH	Bull (2018-2019)	0.7863	0.4170	0.5889	0.3693	0.2154	-	-0.0690
	Stable (2019-2020)	0.7556	0.3189	0.6284	0.4367	0.2387	11%	0.4174
	Bear (2020-2021)	0.8150	0.3288	0.6551	0.4862	0.2837	19%	0.3422
FIL	Bull (2018-2019)	0.4856	0.3161	0.4320	0.1695	0.0918	-	0.3676
	Stable (2019-2020)	0.9210	0.0711	0.4779	0.8499	0.5322	480%	-0.0427
	Bear (2020-2021)	0.9953	0.3451	0.6455	0.6502	0.4138	-22%	-0.0760
KCS	Bull (2018-2019)	0.8675	0.0965	0.6866	0.7710	0.4380	-	0.5307
	Stable (2019-2020)	1.0066	0.5496	0.7403	0.4570	0.2827	-35%	-0.1654
	Bear (2020-2021)	0.9879	0.3581	0.7354	0.6298	0.4165	47%	0.1982
LINK	Bull (2018-2019)	0.8233	0.3241	0.6422	0.4992	0.3015	-	0.2744
	Stable (2019-2020)	0.8839	0.3250	0.7112	0.5589	0.3336	11%	0.3820
	Bear (2020-2021)	0.8300	0.3281	0.6517	0.5019	0.2904	-13%	0.2895

Table 4: Multifractality parameters for the cryptocurrency return series during periods of bull, stable and bear markets. Δh is the degree of multifractality, $\Delta\alpha$ is the width of the multifractal spectrum, Θ is the asymmetry parameter, and $\Delta(\Delta h) = [(\Delta h_t/\Delta h_{t-1}) - 1]$ is the variation of Δh considering the values from the current in-sample set Δh_t in relation to the value calculated from the previous period Δh_{t-1}. $\Delta(\Delta h) > 0$ ($\Delta(\Delta h) < 0$) indicates an increase (decrease) on the corresponding cryptocurrency inefficiency (continued).

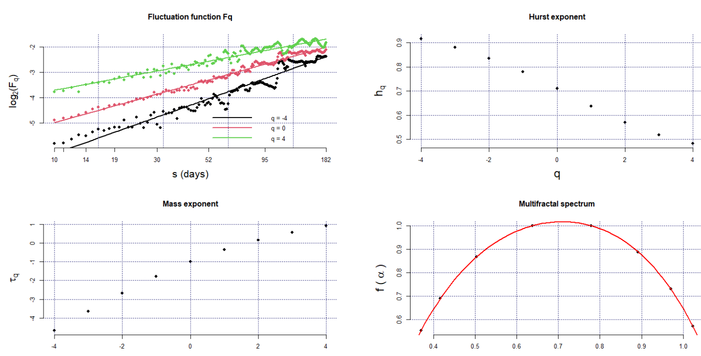
Crypto	Period	α_{max}	α_{min}	α_0	$\Delta\alpha$	Δh	$\Delta(\Delta h)$	Θ
LTC	Bull (2018-2019)	0.7049	0.3514	0.5236	0.3535	0.2032	-	-0.0257
	Stable (2019-2020)	0.7679	0.4051	0.5668	0.3628	0.2104	4%	-0.1086
	Bear (2020-2021)	0.8456	0.3507	0.6169	0.4949	0.2963	41%	0.0758
MANA	Bull (2018-2019)	0.5957	0.1717	0.4865	0.4240	0.2359	-	0.4849
	Stable (2019-2020)	0.6014	0.2785	0.5502	0.3229	0.1621	-31%	0.6829
	Bear (2020-2021)	0.9619	0.1690	0.7243	0.7929	0.4911	203%	0.4007
TRX	Bull (2018-2019)	0.8813	0.1218	0.6219	0.7595	0.4283	-	0.3169
	Stable (2019-2020)	0.7624	0.3245	0.6097	0.4379	0.2687	-37%	0.3026
	Bear (2020-2021)	0.8437	0.4101	0.6755	0.4336	0.2776	3%	0.2242
WAVES	Bull (2018-2019)	0.7179	0.3281	0.5741	0.3898	0.2215	-	0.2622
	Stable (2019-2020)	0.6565	0.2800	0.5316	0.3765	0.2094	-5%	0.3365
	Bear (2020-2021)	1.0134	0.2947	0.7106	0.7187	0.4508	115%	0.1574
XLM	Bull (2018-2019)	0.7896	0.3721	0.6148	0.4175	0.2528	-	0.1626
	Stable (2019-2020)	0.7810	0.2211	0.6522	0.5599	0.3253	29%	0.5399
	Bear (2020-2021)	0.8065	0.1705	0.6825	0.6360	0.3906	20%	0.6101
XRM	Bull (2018-2019)	0.7281	0.3472	0.5375	0.3809	0.2291	-	-0.0008
	Stable (2019-2020)	0.6860	0.3418	0.5350	0.3442	0.1744	-24%	0.1226
	Bear (2020-2021)	0.6369	0.3039	0.5740	0.3330	0.1773	2%	0.6222
XRP	Bull (2018-2019)	0.8531	0.3068	0.6320	0.5463	0.3246	-	0.1906
	Stable (2019-2020)	0.8556	0.1141	0.6667	0.7415	0.4532	40%	0.4905
	Bear (2020-2021)	1.0232	0.3704	0.7793	0.6528	0.4341	-4%	0.2528
ZEC	Bull (2018-2019)	0.7682	0.3923	0.5929	0.3759	0.2277	-	0.0673
	Stable (2019-2020)	0.7316	0.4913	0.6212	0.2403	0.1218	-47%	0.0811
	Bear (2020-2021)	0.7433	0.3890	0.5900	0.3543	0.2052	68%	0.1346



(a) In-sample set from 2018 to 2019



(b) In-sample set from 2019 to 2020



(c) In-sample set from 2020 to 2021

Fig. 3: The MF-DFA results of Ripple (XRP) price returns using data for the three different periods for the in-sample data. a) Fluctuation functions for $q = -4$, $q = 0$, $q = 4$. b) Generalized Hurst exponent for each q. c) Mass exponent, $\tau(q)$. d) Multifractal spectrum.

Figure 3-c provides the Renyi exponent $\tau(q)$ over different values of q. The monofractal and white-noise time series has a mass exponent $\tau(q)$ with a linear q-dependency. The linear q-dependency of $\tau(q)$ leads to a constant h_q of these time series because h_q is the tangent slope of $\tau(q)$. In contrast, the multifractal time series has mass exponents $\tau(q)$ with a curved q-dependency, which is the case of XRP returns and, consequently, a decreasing singularity exponent h_q (see Figure 3-a). Finally, Figure 3-d shows the multifractal of the Hölder spectrum f_α versus the Hölder exponent, α. If a time series is monofractal, then f_α would reduce to the Hurst exponent, such that $\alpha = H$, and $f_\alpha = 1$. Concerning XRP price returns, the multifractal spectrum shows an inverted parabola shape, which validates our previous results of multifractality (see Figure 3-d).

The previous results (see Table 4) have indicated that cryptocurrency price returns present multifractality, emerging the aim of this work which is to verify how the level of multifractality, as a measure of (in)efficiency, influences the performance of crypto-portfolios when constructed considering the most efficient coins. To perform this task, the cryptocurrencies must be ranked in terms of multifractality. Table 5 presents the efficiency rankings based on the corresponding levels of multifractality, measured by Δh, for the three periods considered in this work. It is important to notice that the rankings change considerably according to the in-sample set considered, confirming that efficiency changes over time, which is an aspect worth considering in portfolio management.

Based on the results from Table 5, portfolios with the most efficient cryptocurrencies are composed based on the rankings of each in-sample set. Finally, without taking into account assets levels of efficiency, portfolios composed of all the 19 digital coins are considered for comparisons. Therefore, the next step is the definition on how many digital coins to consider in the portfolios. As our sample is composed of 19 assets, approximately 20% most efficient cryptos were selected, i.e. the total of 4 cryptocurrencies, which are marked in Table 5. A higher rate for this selection could be also adopted, however, as our sample is composed of only 19 assets, the resulting portfolios will contain a few digital coins, which narrows the diversification mechanism, the main feature of the

Markowitz framework.

Table 5: Efficiency rankings during three in-sample periods for all cryptocurrencies. Efficiency is measured by the multifractality degree (Δh). The lower the multifractality, the more efficient is the cryptocurrency, in terms of the weak-form of market efficiency. (*) indicates the 20% most efficient cyptocurrencies in each in-sample set.

Periods	2018-2019		2019-2020		2020-2021	
Rank	Crypto	Δh	Crypto	Δh	Crypto	Δh
1	ADA*	0.0844	ETC*	0.0644	BNB*	0.1233
2	FIL*	0.0918	ADA*	0.1190	XRM*	0.1773
3	EOS*	0.1702	ZEC*	0.1218	ETC*	0.2034
4	LTC*	0.2032	MANA*	0.1621	ZEC*	0.2052
5	ETH	0.2154	XRM	0.1744	BTCH	0.2309
6	BTCH	0.2179	BTCH	0.1865	TRX	0.2776
7	WAVES	0.2215	WAVES	0.2094	ETH	0.2837
8	ZEC	0.2277	LTC	0.2104	LINK	0.2904
9	XRM	0.2291	BNB	0.2149	LTC	0.2963
10	MANA	0.2359	EOS	0.2180	ADA	0.2992
11	XLM	0.2528	ETH	0.2387	EOS	0.3184
12	ETC	0.2930	TRX	0.2687	BTC	0.3435
13	LINK	0.3015	KCS	0.2827	XLM	0.3906
14	XRP	0.3246	XLM	0.3253	FIL	0.4138
15	DOGE	0.3680	LINK	0.3336	KCS	0.4165
16	BNB	0.3998	BTC	0.3894	XRP	0.4341
17	BTC	0.4207	XRP	0.4532	WAVES	0.4508
18	TRX	0.4283	FIL	0.5322	MANA	0.4911
19	KCS	0.4380	DOGE	0.5857	DOGE	0.6941

Finally, portfolios have their weights calculated with different strategies: minimum variance portfolio (MVP), maximum Sharpe-ratio portfolio (MSR), and efficient-based portfolio (EBP). Weights in EBP are not computed by solving an optimization problem but calculated as an heuristic strategy, as described in Eq. (13). An equally weighted portfolio is also considered as a benchmark. Portfolios performances are presented and discussed in the following.

3.4. Portfolios performance

Portfolios performance metrics are reported in Table 6. Concerning the year of 2020, a period of bull market for most of the cryptocurrencies, MVP and MSR portfolios performed significantly better than the remaining strategies, EW, and EBP, in terms of annualized returns (r_p^A) and cumulative returns (r_p^C) - returns are more than twice higher for MVP and MSR in contrast to the competitive portfolios. This confirms the benefits of using the Markowitz framework over heuristic approaches to compute portfolios weights. It is worth to note that, besides the higher level of returns of MVP and MSR, these strategies also presented generally lower annualized volatility and VaR values than EW and EBP. Hence, as a result, it is observed to have a relevant higher risk-return relations of MVP and MSR, when measured by the corresponding Sharpe ratios (SR) - see Table 6.

Moving on for the analysis of the role of cryptocurrencies levels of efficiency, for the year of 2020, all strategies, MVP, MSR, EWP, and EBP provided significant higher returns, lesser risks, and higher Sharpe ratios when they are composed by all the 19 digital coins - Table 6. Portfolios composed by the more efficient assets showed the worst performance. This is a consequence of a bull market, the portfolio with more cryptos took advantage of the generally price appreciation.

It is also worth mentioning that the historical data used for MVP and MSR optimization processes (in-sample set) provide information of a different dynamics compared to the one that the performance is tested (out-of-sample set). The COVID-19 pandemic affected both mean returns and risk (correlations/ standard deviations), as well as the efficiency levels across different kinds of markets; hence, market dynamics changes have additional impacts on MVP and MSR portfolios.

Table 6: Portfolios performance metrics. The results are calculated for the three out-of-sample sets, representing periods of bull (2020), volatile (2021) and bear (2022) markets. MVP, MSR, EWP, and EBP stand for the minimum variance, maximum Sharpe ratio, equally weighted, efficient-based portfolios, respectively. Subscripts *most* represents the portfolios optimized considering the most efficient cryptocurrencies, ranked using MF-DFA for the returns data from the corresponding in-sample sets. Subscript *all* corresponds to the portfolio with all nineteen cryptocurrencies. Results in bold indicate the best Sharpe ratio relation within an investor strategy (MVP, MSR, EWP, and EBP). (*) stands for the best Sharpe ratio relation in an out-of-sample set (2020, 2021 and 2022), regardless of the investor strategy.

Portfolios	r_p^A	r_p^C	σ_p^A.	VaR	SR
Panel A: bull market (2020)					
MVP_{most}	0.4504	0.7161	0.8007	-0.0767	0.5625
MVP_{all}	0.8606	1.4640	0.6402	-0.0519	**1.3443***
MSR_{most}	0.3952	0.6220	0.7977	-0.0766	0.4954
MSR_{all}	0.8599	1.4626	0.6444	-0.0517	**1.3344**
EWP_{most}	0.2246	0.3421	0.8216	-0.0791	0.2733
EWP_{all}	0.3270	0.5083	0.7317	-0.1163	**0.4470**
EBP_{most}	0.2356	0.3596	0.8011	-0.0790	0.2941
EBP_{all}	0.3423	0.5335	0.7538	-0.1189	**0.4541**
Panel B: volatile market (2021)					
MVP_{most}	1.0372	1.8030	1.0745	-0.0935	0.9653
MVP_{all}	1.1968	2.1265	1.0546	-0.0871	**1.1349**
MSR_{most}	1.1175	1.9644	1.0452	-0.0911	**1.0692**
MSR_{all}	0.8061	1.3544	0.9963	-0.0845	0.8091
EWP_{most}	1.2777	2.2946	1.0593	-0.0931	**1.2062**
EWP_{all}	0.6796	1.1193	0.9249	-0.0809	0.7348
EBP_{most}	1.5153	2.8039	1.1289	-0.0974	**1.3423***
EBP_{all}	0.7281	1.2084	0.9409	-0.0824	0.7738
Panel C: bear market (2022)					
MVP_{most}	-0,4783	-0,5439	0,6834	-0,0765	**-0,6999***
MVP_{all}	-0,5765	-0,6453	0,5481	-0,0631	-1,0519
MSR_{most}	-0,4742	-0,5395	0,6688	-0,0762	**-0,7090**
MSR_{all}	-0,5730	-0,6417	0,5469	-0,0628	-1,0476
EWP_{most}	-0,5330	-0,6009	0,7001	-0,1108	**-0,7613**
EWP_{all}	-0,6130	-0,6818	0,6263	-0,1034	-0,9787
EBP_{most}	-0,5126	-0,5798	0,7028	-0,1110	**-0,7293**
EBP_{all}	-0,5912	-0,6601	0,6218	-0,1030	-0,9508

During the year of 2021, the cryptocurrency market verified more volatile (risky) dynamics. In this scenario, the results changed considerably compared to the 2020 out-of-sample period. From Panel B of Table 6, we can verify that portfolios composed by the most efficient cryptocurrencies are associated with the best risk-return relation (higher Sharpe ratio values within a strategy are highlighted in bold). The only exception is the MVP portfolio, which provides a better Sharpe ratio when all cryptocurrencies are considered, i.e. without taking into account the efficiency levels of the assets.

The year of 2021 displays a riskier crypto dynamics, thus, a more uncertain environment to trade digital coins. The question that arises is: why do portfolios composed with more efficient assets perform better in that scenario? The theoretical explanation for this performance would follow two possible arguments. The first is the one discussed by Hong et al. (2007). The authors indicated that due to investors' limited processing capabilities it can take time for investors to digest and act upon news. The second argument is that investor sentiment can drive prices away from their fundamental value, as stated by DeLong & Magin (2009). Hence, due to the higher uncertain dynamic in 2021, portfolios associated with the more efficient stocks performed better as the least efficient assets suffer from a more noisy dynamic due to investors sentiments and slower news processing. Particularly, the best approach of all investors strategies during this period is the efficiency-based portfolio (EBP) composed by the more efficient assets, with a Sharpe ratio of 1.3423 (see Panel B of Table 6). In this strategy, weights are computed heuristically in accordance to the asset's level of efficiency (the higher the efficiency, the higher the weight).

Finally, Panel C of Table 6 presents the performance of the portfolios in a cryptocurrency bear market, i.e. for the year of 2022, where most of the digital coin prices fall significantly. During this period, all portfolios showed losses in terms of annualized and cumulative returns, following the pattern of the so-called "winter crypto", an extended period of depressed cryptocurrency asset prices compared with prior peaks. Nonetheless, some portfolios verified higher losses in relation to the other strategies. Generally, portfolios composed by the more efficient crypto-assets result in lower levels of losses. Higher losses are associated with the portfolios that ignored asset levels of efficiency (composed by all 19 cryptocurrencies). These approaches are associated with lower risk (lower annualized volatility) - see Panel C of Table 6. In both periods of 2021 and 2022, investing in the most efficient did not produce portfolios with the lowest volatilities. The portfolios constructed with the 19 cryptos were the ones that showed lower levels of risk. This fact may have occurred because those strategies contained more assets and, thereby, they contained less idiosyncratic risk, due to diversification. Concerning both metrics, risk and return, Sharpe ratio values from the portfolios with the most efficient digital coins are higher (less negative), indicating a lower level of losses per unit of standard deviation.

4. Conclusion

This paper empirically assessed whether or not selecting cryptocurrencies by their degree of efficiency to compose portfolios provide a better performance. Efficiency of each digital coin was measured in terms of returns adherence to the weak-form of market efficiency (adherence to a random walk dynamics). This was done using MF-DFA to compute the corresponding degrees of multifractality, as a proxy for (in)efficiency. Classic allocation policies, such as minimum variance, maximum Sharpe ratio and equally weighted portfolios were evaluated. In addition, new heuristic strategies were proposed, where the weights are proportional to the asset's degrees of (in)efficiency. In a sample of 19 cryptocurrencies, portfolios were constructed considering the 20% most efficient digital coins (within a total of 4 cryptos). Out-of-sample performance considered different market dynamics, such as bear, bull and volatile cryptocurrency markets.

The findings indicated that, in a bull market like the year of 2020, portfolios composed by all the digital coins provided a significantly better risk-return relatin. On the other hand, for the out-of-sample sets of 2021 and 2022, representing a more volatile and a bear market, respectively, when the most efficient cryptocurrencies are selected to compose portfolios, better results, in terms of Sharpe ratio, are achieved. These portfolios, however, besides producing a higher risk-return relation, are associated with a generally higher level of volatility in comparison to the alternative approaches. Summing up, the empirical findings indicate that taking into account the degree of efficiency when composing cryptocurrency portfolios influenced the corresponding performances in terms of risk and return.

The findings of this study provide important implications. First, cryptocurrency prices display inefficient behavior during the evaluated period, which brings out the possibility to forecast future pricing movements based on historical information. Thus, financial decisions that assume the random walk hypothesis must be thoughtfully revised, especially when important theoretical financial models are based on this assumption. This is quite relevant during the COVID-19 phase, where the digital became less efficient. Second, it was found that the level of efficiency plays a significant role when this feature is taken into account to compose crypto-portfolios. Hence, accessing the assets level of efficiency may help investors to perform better trading strategies.

Future work shall consider the inclusion of constraints in portfolio allocation, like allowing short positions, enforcing targets of return, volatility or diversification degree, consideration of transaction costs, etc. The inclusion of rebalancing policies, such as calendar or threshold-based, and the evaluation of different covariance estimation methods would also enrich future studies. Finally, as (in)efficiency was considered as a driver for selecting crypto-portfolios, it would also be interesting to evaluate portfolios optimized towards that feature, i.e. minimizing or maximizing portfolio's degree of (in)efficiency.

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