An alternative method for calculating concrete design compressive strength, f_{cd}

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Summary

The MC2020 standard introduces numeric models [$\beta_{cc}(t)$] that represent the development and reduction of concrete's compressive strength [$\beta_{c,sus}(t, t_0)$] due to cement hydration and long-term load effects. To promote sustainability, the guidelines for reference characteristic strength and control age have become more flexible, allowing ages greater than 28 days based on the specific structure and its construction timeline. For the calculation of the design strength, f_{cd}, two values for the coefficient α_{cc} (0.85 and 1.00) are proposed in MC2020 standard, which consider the combination of development and decrease in reduction. This article aims to conduct a numerical analysis of how the development and reduction of concrete strength [$\beta_{cc}(t) * \beta_{c,sus}(t, t_0)$] impacts the design compressive strength f_{cd}. It demonstrates that a continuous function can more accurately capture the interactions between these two phenomena, replacing the fixed values used for the current α_{cc}.

1 SAFETY VERIFICATION ON STRUCTURAL CONCRETE DESIGN

Subsection 12.4 "*Verification methods of structural performance*" from *fib* Model Code for Concrete Structures (2020) [1], MC2020, considers that the structural safety analysis of new buildings can be performed through the partial factor method, PFM. Among the 4 methods mentioned in MC2020, PFM is the method currently for designing new concrete structures in both Europe (EN 1992-1-1 [2]), and in Brazil (ABNT NBR 6118 [3]).

The PFM considers both mechanics of materials and actions on structures as random variables that can be represented by statistical distribution curves, from which only a characteristic value is assigned.

Characteristic values of actions (F_k), as well as the concrete (f_{ck}) and steel (f_{yk}) strengths, are modified by different partial factors. These are: the partial factor for the design value of actions (γ_f), and the partial factors for the design strength of concrete (γ_c) and steel (γ_s), which aim to account for statistical uncertainties. In addition to these, the design strength of concrete (f_{cd}) is further adjusted by two other factors, α_{cc} and η_{fc}.

In summary, the method involves increasing the design loads ($F_d = \gamma_f * F_k$) and decreasing the design material strengths, particularly for concrete ($f_{cd} = \alpha_{cc} * \eta_{fc} * f_{ck} / \gamma_c$). This process is simplified in Fig. 1.

[1] This article is part of the research from civil engineer Ricardo Boni for the master's program of Habilitation: Planning and Techonology in IPT in the state of São Paulo

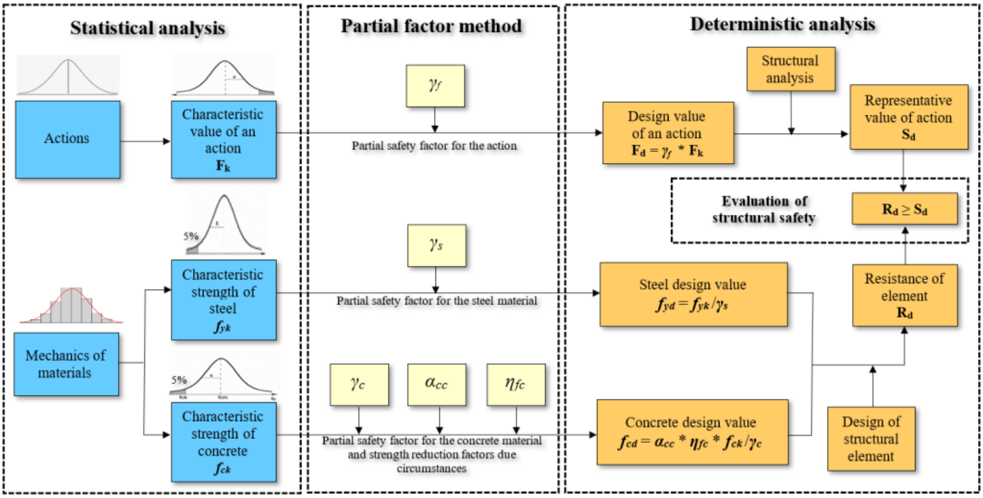

Fig. 1. Simplified scheme for concrete structure design through the partial factor method (PFM), according to MC2020. (adapted from Helene, 1993. [4])

2 DESIGN COMPRESSIVE STRENGTH (f_{cd})

The concrete compressive strength for design purposes (f_{cd}) is used in the design of compressed regions and is, in principle, the considered available, safe and “minimum” strength in the structure throughout its service life (> 50 years).

The MC2020, in subsection 14.19.1.4 “*Design compressive and tensile strengths*”, specifies the design compressive strength of concrete as follows:

$$f_{cd} = \alpha_{cc} * \eta_{fc} * f_{ck}/\gamma_c \quad (1)$$

where:

f_{cd} : design compressive strength;
f_{ck} : is the characteristic compressive strength;
γ_c : is the partial safety factor for concrete, being 1.5 for transient and persistent situations and 1.2 for incidental situations;
α_{cc} : is a coefficient taking account of long term effects on the compressive strength and of unfavourable effects from the way the load is applied. For normal design situations it may be assumed that the increase of the compressive strength after 28 days compensates the effect of sustained loading (α_{cc} = 1.0 for new structures);
η_{fc} : is a strength reduction factor accounting for the increased brittleness for higher strength concrete and may be calculed as:

$$\eta_{fc} = \left(\frac{40}{f_{ck}}\right)^{1/3} \leq 1.0 \quad (2)$$

3 PARTIAL SAFETY FACTOR FOR CONCRETE (γ_c)

It is known that the standard specimen used as a reference for design, control, and construction safety, indicated by f_{ck}, does not represent the effective compressive strength of the concrete in the structure, $f_{c,ais}$ (*actual in situ strength*), due several reasons, including:

a) The transportation, casting, vibration, curing and consolidation of fresh concrete at the construction site differ from the rigorous procedures applied to controlled standard specimens;

b) The standard specimen testing age is a fixed value (***t_{ref}***), whereas the required effective concrete strenght in the structure is needed for any age, meaning a minimum effective strenght is required from a specific age until the end of its service life;
c) the standard specimen does not receive any load until the test date, while the concrete in the structure is subjected to partial loads, receiving the “design load” at variable ages, above the reference test age, ***t_{ref}***;
d) the standard specimen is tested under controlled conditions in a rapid test (< 5 minutes), considered an “instantaneous” test, with a constant, increasing, and centrally applied load, while the structural concrete is subjected to long-term, eccentric, intermittent, and cyclic loads;
e) the standard specimen represents the concrete strength of a uniform batch and mixed in a single operation, such as a truck mixer, which can be cast on several structural elements that will have different degrees of vibration and curing, and consequently slightly different strengths from the standard specimen.

For all these reasons, it can be reasonable to infer that the concrete strength determined in the standard specimen represents the maximum potential of the concrete “at the mixer outlet”. No field operation or procedure can increase the concrete strength beyond that obtained in the standard test, as any testing error or non-standard operation will reduce the potential strength, never increase it.

Cremonini's doctoral research [5] found that the actual in-situ strength, as determined from extracted cores, is approximately 20% to 30% lower than the strength of standard specimens, when both are tested at 28 days, his finding is consistent with the concept of $\gamma_{m,c}$. According to the *fib* Model Code for Concrete Structures 2010 [6], MC2010, the partial safety factor for concrete, γ_c = 1.5 can be understood as the product of $\gamma_{m,c}$ * $\gamma_{Rd1,c}$ * $\gamma_{Rd2,c}$ where $\gamma_{m,c}$ is the partial safety factor for material properties = 1.39, $\gamma_{Rd1,c}$ is the partial safety factor for model uncertainty = 1.05 and $\gamma_{Rd2,c}$ is partial safety factor for geometrical uncertainty = 1.05.

Therefore, to calculate the concrete design strength with safety at any age throughout its service life, that is, to estimate f_{cd}, in addition to γ_c, it is still necessary to consider the effect of sustained loads ($\beta_{c,sus}$), the application age of these loads (t_0), and the effect of strength development due to cement hydration reaction over time (β_{cc}) after the reference age (t_{ref}).

In summary, the project specifies a concrete compressive strength f_{ck} at a certain reference age (t_{ref}) and its control through standard test. However, the safety verification must consider a design strength f_{cd}, which is slightly lower, to account for the differences between standard specimen and the actual structure. Additionally, the design must balance actions and phenomena that may either reduce or increase these actions over a service life of 50 years or more.

4 DEVELOPMENT OF STRENGTH WITH TIME (β_{cc})

The compressive strength of concrete develops from the contact of water with anhydrous cement, initiating chemical reactions of setting and hardening, with a gradual increase in compressive strength over time. To represent this increase in strength over time, the MC2020, in subsection 14.9.1 “*Development of strength with time*”, suggests using the following equation:

$$\beta_{cc}(t) = \frac{f_{cm}(t)}{f_{cm}} = e^{\left\{s_c\left[1-\left(\frac{t_{ref}}{t}\right)^{0.5}\right]\left(\frac{28}{t_{ref}}\right)^{0.5}\right\}} \tag{3}$$

where:

$\beta_{cc}(t)$: is a function to describe the strength development with time;
$f_{cm}(t)$: is the mean compressive strength in MPa at an age t in days;
f_{cm} : is the mean compressive strength in MPa at an specified reference age t_{ref} in days;
t : is the concrete age in days;
t_{ref}: age of concrete at which the concrete strength is determined in days
s_c: is a coefficient which depends on the strength development class of concrete, $0.1 \leq s_c \leq 0.6$.

Fig. 2 shows the strength development curves up to the age of 50 years, assuming t_{ref} = 28 days and the 6 main types of concrete characterized by the s_c values prescribed in MC2020. Note that the specimens are unloaded, under ideal conditions of constant temperature, pressure, and humidity, in a non-aggressive environment.

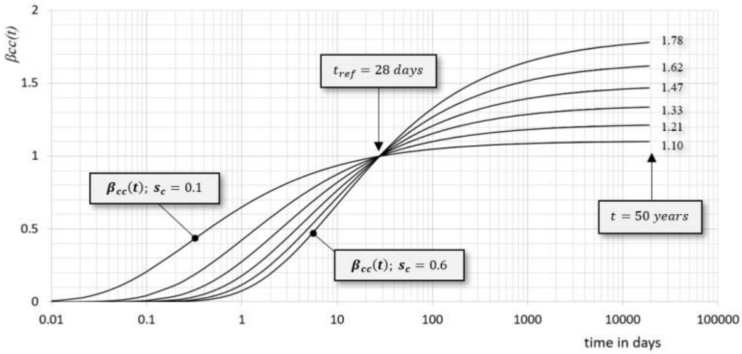

Fig. 2. Curves illustrating the relative development of concrete compressive strength over a 50-year period.

5 STRENGTH UNDER SUSTAINED LOADS ($\beta_{c,sus}$)

In 1960, Engineer Hubert Rüsch published the results of his experimental research entitled "*Researches Toward a General Flexural Theory for Structural Concrete*" [7]. The study revealed that the results of specimens subjected to long-duration loads (over 20 minutes) were significantly lower than those obtained from standard specimens subjected to rapid, monotonic loading tests typically conducted in control laboratories.

To represent the reduction in strength due sustained load effects, MC2020 prescribes in its subsection 14.9.3 "*Strength under sustained loads*" the use of the following equation:

$$\beta_{c,sus}(t,t_0) = \beta_{t0}(t_0) + [1 - \beta_{t0}(t_0)]\left[1 + 10^4\,\frac{(t-t_0)}{t_0}\right]^{-0.1} \tag{4}$$

where:

$\beta_{c,sus}(t, t_0)$: is a function which describes the decrease of strength with time under high sustained load[2]. It depends on the time under high sustained loads $(t - t_0)$ in days, as well as on age the concrete at loading in days. The function is defined for $(t - t_0)$ > 0.015 days (≈20 min);
$\beta_{t0}(t_0)$:is a parameter which considers the maturity of the concrete at loading. It can be calculated as:

$$\boldsymbol{\beta_{t0}}(\boldsymbol{t_0}) = 0.64 + 0.01 * \ln(\boldsymbol{t_0}) \tag{5}$$

t_0 : is the age of concrete at loading in days, is valid for $t_0 \geq 7$ days;
$t - t_0$: is the time under high sustained loads in days. For $t - t_0$ > 10 years, it shall be taken $t - t_0$ = 3650 days.

Fig. 3 graphically illustrates the function $\beta_{c,sus}(t, t_0)$ for high loads applied to concrete at various ages.

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[2] ***fib* MC2020** (2023) subitem 14.9.3.1 "*Sustained compressive strength*" comment: …a high sustained compressive stress can be defined as a stress of a magnitude of above 75% of the short time strength f_{cm}… …literature results show that this value may vary from 70% for lightweight aggregate concrete to 80% for high strenght concrete…

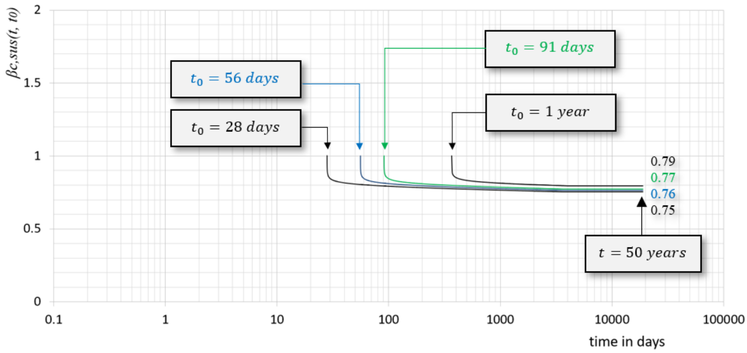

Fig. 3. Decrease in compressive strength up to 50 years of age as a function of the application date of sustained loads. Load applied at 28, 56, 91, and 365 days of age.

6 COEFFICIENT TAKING ACCOUNT OF LONG TERM EFFECTS AND OF UNFAVOURABLE EFFECTS FROM THE WAY THE HIGH LOAD IS APPLIED (α_{cc})

Based on the relative development and decrease expressions of concrete compressive strength as defined in MC2020, it is possible to analyze the development of concrete compressive strength considering the combined effect of these opposing phenomena, through the product of the functions $\boldsymbol{\beta_{cc}(t)}$ * $\boldsymbol{\beta_{c,sus}(t, t_0)}$.

The result of this product gives rise to numerous curves that develop as a function of age, concrete/cement characteristics, and the age at which the elevated load is applied and maintained for periods exceeding 20 minutes. Fig. 4 illustrates the envelope of curves for loads applied at 28 and 365 days of age, considering a control age ($\boldsymbol{t_{ref}}$) of 28 days and the different concretes provided in MC2020, characterized by $0.1 \leq s_c \leq 0.6$.

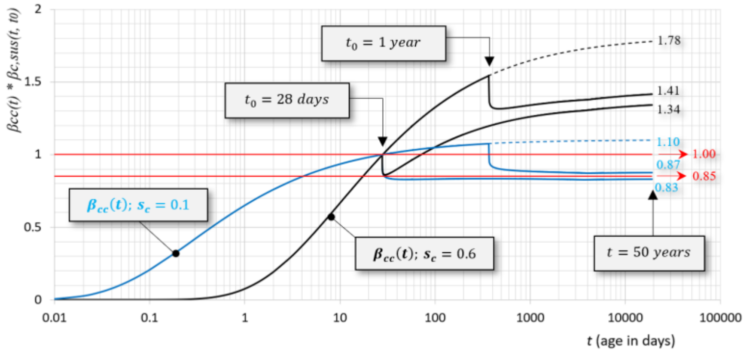

Fig. 4. Envelope of the relative evolution curves of compressive strength up to 50 years of age for high long-term loads applied at 28 or 365 days, with a reference age $\boldsymbol{t_{ref}}$ = 28 dias.

It is important to note that different choices for reference age ($\boldsymbol{t_{ref}}$) for concrete compressive strength result in distinct curves. In all cases, the product $\boldsymbol{\beta_{cc}(t)}$ * $\boldsymbol{\beta_{c,sus}(t, t_0)}$, which essentially represents the value of $\boldsymbol{\alpha_{cc}}$, assumes variable values, contrasting with the fixed values currently prescribed in MC2020 ($\boldsymbol{\alpha_{cc}}$ = 1.00 or $\boldsymbol{\alpha_{cc}}$ = 0.85).

7 PhD'S PROPOSAL FOR THE DETERMINATION OF α_{cc}

Generally, the first significant variable loads are applied to a structure months after the reference date, $\boldsymbol{t_{ref}}$, especially in the case of foundations and building columns. It occurs that, during this period, the concrete strength develops due to the continuous hydration of the cement. MC2020 and EN 1992-1-1 [2] consider that this development compensates for the adverse effect of applying a high sustained load and prescribe in these cases that a value of $\boldsymbol{\alpha_{cc}}$ = 1.00 should be adopted; otherwise, $\boldsymbol{\alpha_{cc}}$ = 0.85.

This paper suggests adopting a variable value for the coeficiente α_{cc}, which depends on the concrete strength class, type of cement used, and the time of load application to the structure. In other words, it is recommended to adopt:

$$\boldsymbol{\alpha_{cc}} = \min_{t}[\boldsymbol{\beta_{cc}}(\boldsymbol{t}) * \boldsymbol{\beta_{c,sus}}(\boldsymbol{t}, \boldsymbol{t_0})] \quad (6)$$

Table 1 presents the minimum values of the product $\beta_{cc}(t)$ * $\beta_{c,sus}(t, t_0)$ for t_{ref} = 28 days and different high long-term application ages t_0 (28, 56, 91 and 365 days).

Table 1 Minimum values of α_{cc} for t_{ref} = 28 days, t_0 varying from 28 days to 365 days, and for different concrete classes characterized by s_c.

t_0 [days]	s_c					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
28	0.83	0.84	0.85	0.85	0.86	0.86
56	0.84	0.89	0.92	0.96	0.99	1.02
91	0.85	0.91	0.96	1.01	1.06	1.12
365	0.87	0.95	1.04	1.13	1.22	1.31

Therefore, the concrete design strength would be calculated as follows:

$$\boldsymbol{f_{cd}} = \min_{t} \; [\boldsymbol{\beta_{cc}}(\boldsymbol{t}) * \boldsymbol{\beta_{c,sus}}(\boldsymbol{t}, \boldsymbol{t_0})] * \boldsymbol{\eta_{fc}} * \boldsymbol{f_{ck}}/\boldsymbol{\gamma_C} \; = \boldsymbol{\alpha_{cc}} * \boldsymbol{\eta_{fc}} * \boldsymbol{f_{ck}}/\boldsymbol{\gamma_C}$$

Note that the product of $\beta_{cc}(t)$ * $\beta_{c,sus}(t, t_0)$, depending on the concrete class considered and characterized by s_c, does not always have as its minimum value the results at 50 years of age. In most cases, the product of these functions can reach minimum values shortly after the application of the elevated load.

Table 2 presents the minimum relative values obtained over 50 years for the calculation of f_{cd} / f_{ck}, adopting a fixed γ_c equal to 1.5, with η_{fc} calculated according to MC2020 and α_{cc} calculated according to the minimum of the product of $\beta_{cc}(t)$ * $\beta_{c,sus}(t, t_0)$, considering as variables the reference age of the standard specimen (t_{ref}) and the date of application of the sustained high loads (t_0).

Table 2 Relative design compressive strength of concrete, f_{cd} / f_{ck} according to a certain reference age t_{ref} and a certain age of application of the loads t_0, being f_{cd} / f_{ck} = $\beta_{cc}(t)$ * $\beta_{c,sus}(t, t_0)$ * η_{fc} * 1 / γ_c.

high load	cement class	MC2020 vs PhD	t_{ref} = 28 days			t_{ref} = 56 days			t_{ref} = 91 days		
			C30	**C50**	**C70**	**C30**	**C50**	**C70**	**C30**	**C50**	**C70**
28 days	CS	MC2020	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
		PhD	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
	CN	MC2020	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
		PhD	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
	CR	MC2020	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
		PhD	0.57	0.52	0.46	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*	*nihil*
56 days	CS	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*
		PhD	0.68	0.61	0.53	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*
	CN	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*
		PhD	0.66	0.60	0.51	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*
	CR	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	*nihil*	*nihil*	*nihil*
		PhD	0.61	0.55	0.48	0.57	0.52	0.45	*nihil*	*nihil*	*nihil*
91 days	CS	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.75	0.66	0.56	0.63	0.57	0.50	0.57	0.53	0.47
	CN	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.71	0.63	0.53	0.61	0.56	0.49	0.57	0.53	0.46
	CR	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.64	0.56	0.47	0.59	0.53	0.45	0.56	0.52	0.45
365 days	CS	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.87	0.76	0.63	0.73	0.65	0.55	0.67	0.60	0.52
	CN	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.81	0.70	0.56	0.70	0.62	0.53	0.65	0.58	0.50
	CR	MC2020	0.67	0.62	0.55	0.57	0.53	0.47	0.57	0.53	0.47
		PhD	0.67	0.59	0.48	0.63	0.56	0.47	0.61	0.54	0.46

To facilitate visualization, Figs. 5, 6 and 7 present a graphical comparison between the values to be adopted for $\boldsymbol{f_{cd}}$ according to the defined $\boldsymbol{f_{ck}}$, $\boldsymbol{t_{ref}}$ and $\boldsymbol{t_0}$ and the values recommended by MC2020, considering concretes produced with CS, CN and CR cements.

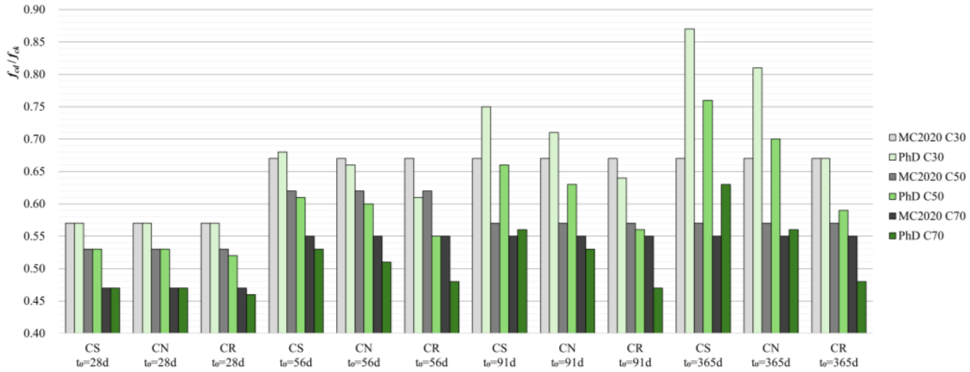

Fig. 5. Concrete compressive design strength $\boldsymbol{f_{cd}}$ / $\boldsymbol{f_{ck}}$ for $\boldsymbol{t_{ref}}$ = 28 days.

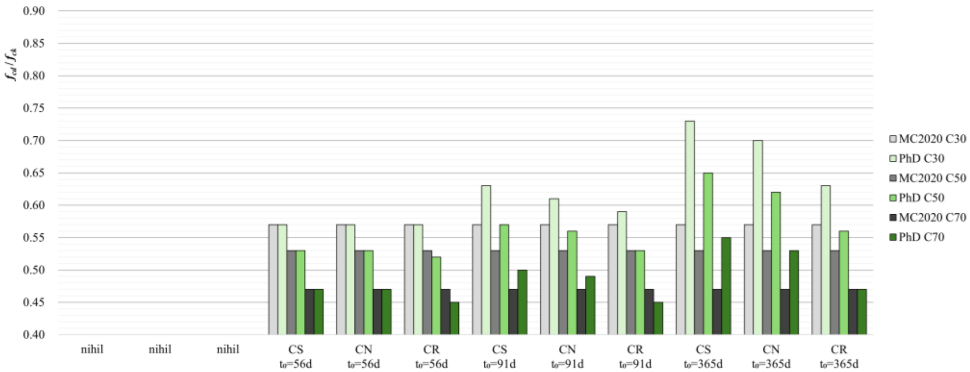

Fig. 6. Concrete compressive design strength $\boldsymbol{f_{cd}}$ / $\boldsymbol{f_{ck}}$ for $\boldsymbol{t_{ref}}$ = 56 days.

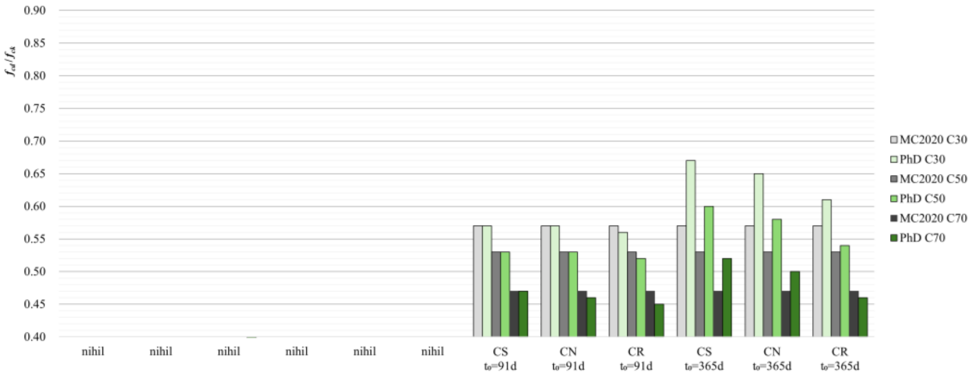

Fig. 7. Concrete compressive design strength $\boldsymbol{f_{cd}}$ / $\boldsymbol{f_{ck}}$ for $\boldsymbol{t_{ref}}$ = 91 days.

8 CONCLUSIONS

According to the *fib* Model Code 2020, MC2020, the values of the coefficient α_{cc} can be either 0.85 or 1.00, based on specific assumptions, however, they are both fixed coefficients. Consequently, the design compressive strength, f_{cd}, exhibits a range between 0.47*f_{ck} to 0.67*f_{ck}, with the specific value being influenced by the concrete class.

This article proposes a novel methodology for determining the design compressive strength f_{cd}. This methodology leverages established models that account for the increase in concrete strength over time due to cement hydration, as well as the reduction in strength caused by sustained high loads. Under these conditions, strengths ranging from 0.45*f_{ck} to 0.87*f_{ck} were obtained, significantly different from the fixed values prescribed in MC2020.

For the cases studied, addopting a fixed value for α_{cc} coefficient of 1.00 or 0.85, according MC2020, may lead to conservative design compressive strength, and consequently, uneconomical and unsustainable concrete structures, especially when slow-setting cements (CS) are used in up to 30%. On the other hand, when using concretes mixed with so-called rapid-setting cements (CR), the f_{cd} values and the resulting structures obtained using the fixed coefficients of MC2020 may be up to 15% non-conservative and daring.

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