Social Security Reform, Retirement and Occupational Behavior*

Pedro Cavalcanti Ferreira† Rafael Parente‡ Bruno Delalibera§

August 5, 2022

Abstract

In most countries, the rules governing public and private pension systems are different, and so are hiring procedures, and job contracts. The tenures of government employees are longer and their wages, in general, higher. In this sense, social security reforms will affect not only the decision to leave the labor force, but also the choice of which sector to work. In this article, we study the impact of social security reforms on retirement and occupational behavior. We develop a life-cycle model with three sectors - private formal, private informal and public - and endogenous retirement to evaluate what are the macroeconomic and occupational impacts of social security reforms in an economy with multiple pension systems. In a model calibrated to Brazil, we simulate and quantitatively assess the long-run impact of reforms being discussed and/or implemented in different economies. Among them, the unification of pension systems and the increase of minimum retirement age. These reforms are found to affect the decision to apply to a public job, savings during the life cycle and skill composition across sectors. On the long run, they lead to higher output and capital, less informality and to average welfare gains. They also drastically reduce social security deficit.

Key Words: social security reform, public employment, public deficit, informality

JEL Codes: J26, H55, J45, J62

*We would like to thank the comments of Flavio Cunha, Carlos Eugênio Costa, Cezar Santos, Fernando Veloso, Marcelo Santos, Ana Luiza Dutra, Pedro Solti, Henrique Pires, João Lucas Thereze and seminar participants at EPGE FGV, the SED Meeting, Insper and SBE. Ferreira acknowledges the financial support of Faperj, INCT/CNPq and Capes. Parente acknowledges the financial support of Capes and the International Economics Section at Princeton University. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. All errors are our own.

†FGV/ EPGE Brazilian School of Economics and Finance and FGV Growth and Development. E-mail: pedro.ferreira@fgv.br

‡Princeton University. E-mail: rparente@princeton.edu

§Univesitat de Barcelona. E-mail: brunodelalibera@gmail.com

1 Introduction

This work explores quantitatively the effect of social security reforms on occupational and retirement behavior. This is a timely issue as several countries should face – or are already facing - problems in their social security systems due to ageing population and the financial fragility of their current systems. In particular, we study the effects of changes in social security systems with different rules for public and private retirees on individual and aggregate behavior. Among these reforms, we study the unification of the public and private systems and some reforms that are being discussed or implemented in different countries, such as raising the minimum retirement age.

Population ageing is the result of increasing life expectancy and falling fertility rates. According to the United Nations (2013), life expectancy around the world has been increasing steadily in the last 15 years. In 2000, life expectancy at birth was 65 years old. Nowadays, an individual is expected to live 70.5 years. Old age support ratio[1] is expected to decrease from 3.8 nowadays to 2.1 in 2050 and 1.6 in 2100 for the OECD countries. On top of that, Blöndal and Scarpetta (1999) and Gruber and Wise (2002) provide evidence that the workforce participation of the elderly population has declined in many OECD countries.

Pay-as-you-go (PAYG) social security systems exist in most of the countries in the world. Some of them have differentiated pension rules for public servants. Kings et al. (2007) document that over half of the OECD members have different pension rules for public and private workers. For instance, the US has segregated pension plans, depending on the jurisdiction. Beshears et al. (2011) report that the majority of public pension schemes are still defined-benefit, unlike the private sector retirement plans.

Holzmann and Hinz (2005) argue that most public pension systems are not financially sustainable. Of course, taking into account sustainability issues, government's budgetary deficit problems are even more problematic. An important example is the Brazilian crisis in the aftermath of the East Asian and Russian financial meltdowns, in 1998. It was documented that a fiscal deficit of more than 6% of the GDP triggered this crisis, and that two thirds of this deficit was due to the cost of pensions. Likewise, in Lebanon, public retirees' pensions is the third greatest expenditure item in the government's budget, even though they account for less than 3% of the population.

Given the upcoming demographic pressures and the financial situation of social security systems, it is important to evaluate the long run consequences of potential reforms[2]. To provide a tool for the evaluation of the consequences of these reforms, we build a life-cycle model with heterogeneous agents and endogenous occupational and retirement decisions. In particular, we construct an economy with three working sectors: private formal, private informal and the government, each with

[1] The ratio of individuals aged between 20 and 64 years old and those with more than 65 years old.
[2] See OECD (2015b) for a summary of what reforms are being implemented around the world.

its own pension system. We calibrate our economy to match key aspects of the Brazilian economy in 2013. With rapid aging population and over generosity of its pension system – specially that of public workers – this country can be seen as a typical, if not exaggerated, case of the social security problems countries are currently facing, or will face in the near future. The calibrated model reproduces closely data regarding sectorial and labor decisions, retirement claims characteristics, as well as social security deficit and public job application decision.

We run a group of exercises to evaluate the log-run effects of social security reforms that are being discussed or implemented in economies around the world. In particular, we evaluate the effects of a reform introduced in 2013, that ended the "integrality" provision for public retirees, imposing a cap in their benefit and approximating the public and private social security system. This is similar, for instance, to the recent project proposed by the Macron administration in France of a universal retirement system that eliminates special regimes for the public sector. We also evaluate current proposals of increasing minimum retirement age.

Following the approach developed by Huggett (1996) to model life-cycle economies with heterogeneous agents in general equilibrium, Huggett and Ventura (1999), Conesa and Krueger (1999), among many, studied potential social security reforms and their macroeconomic consequences. These articles treat retirement exogenously. This is not a plausible assumption to address early retirement provisions[3], for instance. Even though agents' earnings influence their retirement benefits, there is no trade-off between working one extra year to raise her future benefits and applying for retirement sooner to collect more benefits.

Imrohoroglu and Kitao (2012), Ferreira and dos Santos (2013), Jung and Tran (2012) and Gustman and Steinmeier (2005), among others, deal with endogenous retirement. Imrohoroglu and Kitao (2012) study the impact of two social security reforms on the US economy. They introduce health heterogeneity, as well as medical expenditures, which could act as determinants of social security benefit claims decision. None of these studies addresses retirement choices when agents face more than one working sector, however. This is important as social security reforms can induce workers to reallocate across different sectors, therefore having significant macroeconomic consequences such as fiscal deficit reduction.

If the unification of pension systems implies fewer benefits to public workers, high skill individuals may now opt for the private sector, increasing overall productivity of the economy. Segregated social security systems and the case of Brazil are dealt in Glomm et al. (2009), dos Santos and Pereira (2010), Dos Reis and Zilberman (2014) and dos Santos and Cavalcanti (2015). These articles emphasize how an overpaid and secure (in terms of job stability) public sector attracts the best human capital in the economy, and the macroeconomic consequences of such sector. What this literature have not done yet, as we do, is to develop a model to study occupational and retirement

[3]See Vestad (2013) for a list of countries that have early retirement possibilities.

choices of agents that face multiple working and retirement sectors.

The remaining of the paper proceeds as follows. Section 2 establishes key facts about social security systems around the world, as well as some peculiarities of the Brazilian system. Section 3 presents the key features of our model, including the problem of the agents and the stationary distribution. Section 4 defines the equilibrium concept we use in this work. Section 5 describes the calibration procedure. Section 6 validates our numerical solution, comparing non-targeted moments that our model generates to the data. In Section 7, we evaluate the steady state macroeconomic consequences of different social security reforms. Section 8 accesses the long-run welfare impacts of the recently adopted reforms. Finally, Section 9 concludes.

2 Motivation and Data

This section gathers data from different sources to motivate why quantifying the macroeconomic consequences of social security reforms is of first order for governments around the world. We show that population ageing and the current situation of public pension expenditures will tighten the governments' budget in the near future. We then detail the Brazilian situation, highlighting some aspects of the Brazilian economy that will guide our theoretical and quantitative analysis.

Population ageing is a widespread phenomenon that governments throughout the world will have to deal in the next decades. According to OECD data[4], the share of the elderly in the population is expected to increase significantly until 2100. For the OECD countries, the share of individuals over seventy years of age is expected to increase by almost 70% until 2050, from 12.2% to 20.6%. Even considering the world as a whole, which includes countries that are in a better demographic situation than most of the developed countries, this share is expected to more than double, going from 5% in 2015 to 11% in 2050.

At the same time that the elderly population will increase, there should also be a reduction in the proportion of people of working age in the future. Moreover, this is not a problem of high-income countries alone. Figure 1 shows that countries such as Brazil, Mexico and Chile are among those that can expect large decreases in the old age support ratio, the ratio between the working-age population (20-64 years old) over the retirement population (aged over 65 years old). In Mexico, for instance, old age support ratio is projected to move from 8.7 in 2013 to 2.8 in 2050, and further decrease to 1.4 by 2100. Old age support ratio is an important indicator when considering financial sustainability of social security systems, as most countries in the world adopt PAYG schemes, in which there are intergenerational transfers between the working population and the retired population. Therefore, this demographic shift would decrease revenues and tighten social

[4]See United Nations, Department of Economic and Social Affairs, Population Division (2017).

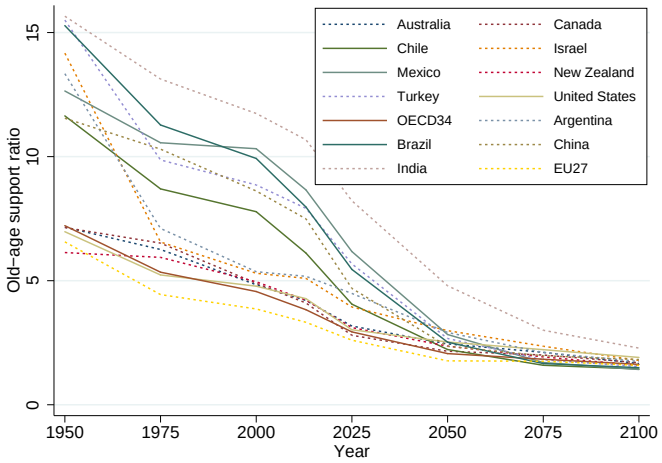

Figure 1: Old Age Support Ratio

security budget balance.

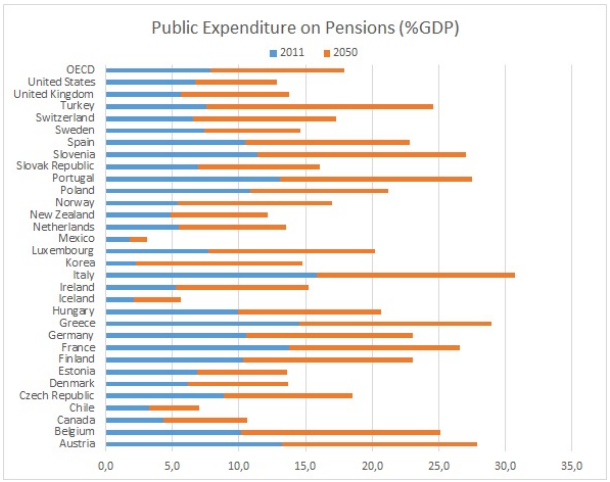

Figure 2: Social Security Expenditures

Public expenditure on pensions of the elderly population takes a large share of the GDP, and is projected to increase by 28% in the OECD countries until 2050, from 7.9% to 10%, as Figure 2 shows. The demographic change and higher expenditures with the elderly population suggest that the social security deficit will worsen in the future. Therefore, social security reforms ought to be discussed and implemented all over the world, as documented in OECD (2015b).

The Brazilian demographic prospects follow global trends. The average number of births per woman has been decreasing steadily since 1980. At the same time, life expectancy increased substantially. According to the World Bank, Brazil's life expectancy went from 54.7 years in 1960 to 73.6 in 2012.

The Brazilian Institute of Geography and Statistics' (henceforth IBGE) data projects that the share of individuals aged 65 and over will jump from 4% in 1980 to 22% around 2050. As a combination of these facts, the population age profile of the Brazilian economy has changed drastically over the last years, and is projected to worsen in the near future, with a fast increasing ratio of old to young individuals.

The Brazilian pension system ranks among the most generous in the world. Figure 3 shows that Brazil spends too much, compared to other countries in the world, on social security given its old age support ratio. According to the OECD, in 2011, the Brazilian economy spent 7.4% of its GDP with social security with an old age support ratio of nearly 8.4. Importantly, Brazil has a population as young as those of Mexico and Turkey, but spends as much as the USA, Sweden and the OECD average on social security, around 8% of GDP.

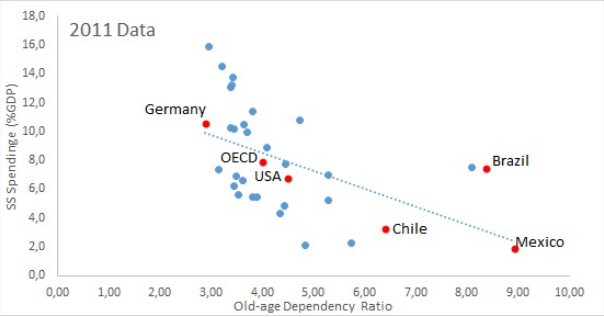

Figure 3: SS Expenditure vs. Old-age Support Ratio

However, Figure 3 is a static picture of the Brazilian social security situation. Tafner et al. (2015) calculate private retirement expenditure as a fraction of GDP through time in Brazil, and finds that it has steadily increased from 3.4% in 1991 to 5.8% in 2001 and 6.8% in 2011. The reason for these problems lay, essentially, in the Brazilian social security structure.

The Brazilian social security scheme has, like many other countries[5], different rules for public servants and private workers. On average, public pension schemes are characterized by lower contribution rates and higher replacement rates, when comparing to private retirement plans. For example, according to Kings et al. (2007): "Typically, public schemes are distinct from the national scheme in that the governance arrangements are relaxed (there is assumed to be less need for a fully funded arrangement) and they exceed the basic parameters as they are often significantly more generous". As for the US case, according to The Economist[6]: "Most public-sector workers can expect a pension linked to their final salary. Only 20% of private-sector workers benefit from such a promise.". Also, Beshears et al. (2011) report that there are significantly higher replacement

[5]See Kings et al. (2007) for OECD economies, Dickson et al. (2014) for an overview of European countries and Beshears et al. (2011) for the US, specifically.

[6] *Who Pays the Bill?* The Economist, July 27, 2013.

rates if an agent works in the public sector for the last part of her career, instead of working as a public servant first, and then retiring in the private sector.

In the Brazilian case, the constitutionally guaranteed provision of the so-called "integrality" ensured that public sector pensions would match the average of the 80% highest salaries before the retirement date. On the other hand, private benefits are defined as the arithmetic average of the 80% higher salaries over the life-cycle up to a limit, which does not reach 10 times the minimum wage. Therefore, the retirement benefits for private workers are potentially much lower than that of public workers.

Table 1: Social Security Deficit (Brazil, % GDP)

	Private	**Public**	**SS Deficit**
2013	0.94	1.18	2.12
2014	1.00	1.18	2.19
2015	1.44	1.22	2.66
2016	2.17	1.18	3.35

Table 1 shows that the public sector social security deficit accounts for nearly half of the total deficit. At the same time, in 2013, public retirees accounted for only 5.3% of total retirees in the economy. Hence, public retirement, relative to private retirement, is extremely generous, and a large part of such discrepancy is due to higher replacement rates. This is in line with the evidence for several other countries, mentioned above.

Table 2: Brazilian Early Retirement

	Retirement Age		**Expected Duration**	
Location	**Men**	**Women**	**Men**	**Women**
OECD	64	63	16	21
Latin America	62	60	17	21
World	62	60	16	21
Brazil:				
Private: Contribution	56	52	23	29
Private: Age Modality (Rural)	60	55	19	26
Private: Age Modality (Urban)	65	60	16	22

Early retirement among public and private workers is another concern. The retirement age is relatively low, as the country does not have a minimum retirement age for those in the private sector who have contributed to the social security system for 35 years or more. For the public servants, however, there is a minimum age of 60 years old. Table 2, taken from Tafner et al. (2015), shows that the average age in which individuals claimed for retirement benefits under the contribution modality[7] is 56 years old for men and 52 for women. The expected duration of

[7]This is a type of retirement Brazilians can opt to have, and it will be further detailed in Section 3.

retirement is 23 years for men and 29 for women.

There are large discrepancies between the numbers for Brazil and those for the rest of the world. OECD's average retirement age is 64, with an expected duration of 16 years for men. Retirement age in Brazil is also considerably lower than the Latin America average, which is 62 years of age.

The discrepancy between the retirement plans in the private and public sector is not the only reason why becoming a public servant is so attractive in Brazil. Figure 4 plots the compensation of general government employees[8] as a fraction of GDP versus the share of the labor force that is in the public sector. Relative to other countries and regions, the Brazilian government's size (11.4%), measured by the share of public employee, is only 60% of the OECD average government size, and is smaller than developed countries such as Poland (22.6%) and Italy (16%). In contrast, the Brazilian compensation as a share of GDP is higher than that of the OECD (11.6%), Poland (10.3%) and Italy (10.2%). Therefore, not only public workers retire in a relatively favorable situation with respect to private workers, but they are also better paid relative to other countries and regions in the world.

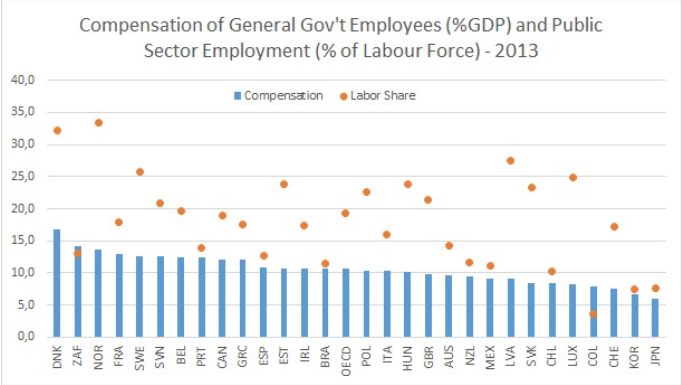

Figure 4: Public Employment Generosity

It is well documented for a large number of countries[9] that there is a significant wage differential between jobs in the public sector and private sector. As Table 3, in Section 5, shows, the same is true for Brazil. Using data from the 2013 Brazilian Household Survey (PNAD)[10], we use the Heckman (1979)-method[11] to regress log wages on several wage determinants, different controls and

[8] According to the OECD, compensation is defined as the sum of two components: 1) wages and salaries, and 2) employer's social contribution - such as retirement benefits. Data for Brazil is computed using the Primary Budget from National Accounts.

[9] See Belluzzo et al. (2005) for the Brazilian economy, Tansel (2004) for Turkey, Morikawa (2016) for the Japanese economy, Glinskaya and Lokshin (2007) for India and Depalo et al. (2013) for a study of the Euro area countries.

[10] "Pesquisa Nacional por Amostra de Domicílios", it is an annual cross-sectional household data survey published by IBGE

[11] Becoming a public sector worker may have a high selection bias. In order to become a public servant, individuals must score relatively well on an open exam. Therefore, it may be the case that only well-educated workers opt for

a public sector job dummy and estimate a public wage premium of around 0.37. That is, controlling for wage determinants such as industry, education, sex, and controlling for selection, working in the public sector increases average wages by 37%.

There is large evidence that public sector jobs are more secure than their counterpart in the private sector[12]. According to the Financial Times[13], after the 2008 crisis, Indian IT sector grooms shifted towards positions, such as civil servants and managers at state-owned enterprises, which have higher job security and payments. Munnell and Fraenkel (2013) state that, in the US: "Given the nature of their employment, state/local workers have historically been less vulnerable to layoffs than private sector workers. Moreover, despite the negative impact of the Great Recession on state/local employment, public workers still had a greater degree of job security than private workers during this period."

Figures 5 and 6 plot the density of tenure on the job for private and public sector workers in Brazil. The density of tenure in the public job is shifted to the right, compared to that of private workers. On average, a public servant has been on the job for 13.6 years, whereas a private worker has been in her current job for approximately 5.3 years. Since individuals that desire to smooth consumption would do so more efficiently in jobs that have longer duration, we can add the job stability to the list of features that make the public sector career relatively more attractive.

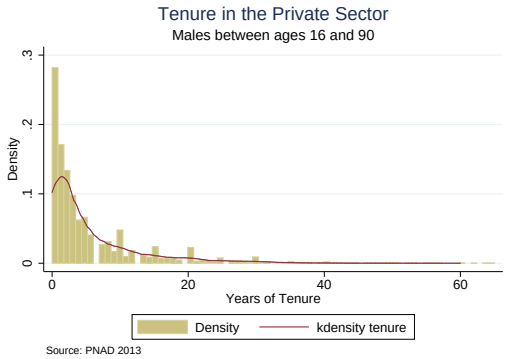

Figure 5

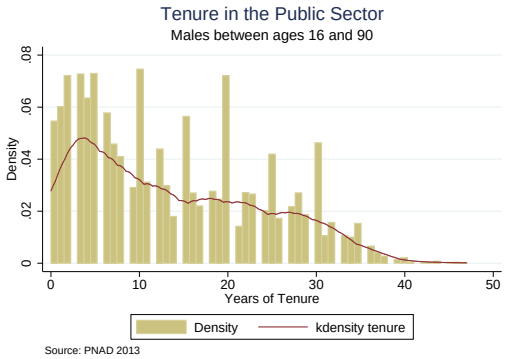

Figure 6

As for the distribution of the labor force across sectors, according to the 2013 PNAD data, 46.6% of males aged from 16 years old to 90 years old worked in the formal private sector, 18.2% in the informal private sector and 4.8% worked as public servants[14] The remaining 30.4% are not economically active.

trying to enter the public sector in the first place.

[12] See OECD (2015a) for OECD countries and Liu et al. (2014) for the Chinese economy.

[13] *Public servants give techies the push in India's marriage market.* Financial Times, April 25, 2009.

[14] We consider only statutory workers, those subject to the private social security plan described above.

Figure 7 plots the education distribution for each occupational choice considered in the model. It shows that the public sector distribution is shifted towards more educated individuals - 80% of public servants in Brazil have studied for more than 11 years[15]. In contrast, the distributions of informal workers or households that are economically inactive[16] are concentrated in low levels of education: 54% of individuals that are not economically active and 48% of informal workers have less than 7 years of schooling.

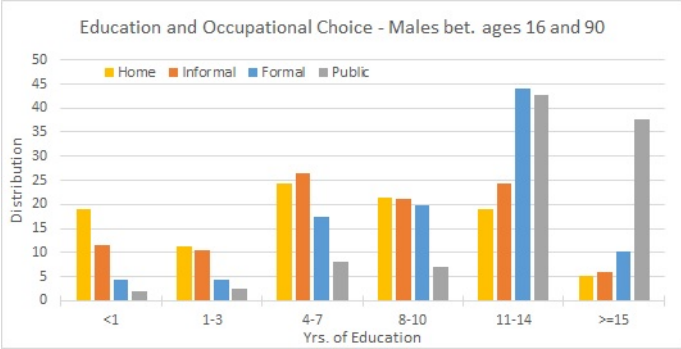

Figure 7: Distribution of Individuals

Interestingly, even though the public servants account for only 4.8% of the population, they represent nearly 20% of all individuals with more than 15 years of education. This number falls to 6.2% for individuals with 11-14 years of education, but it is still large comparing to the relative sector size. Clearly, the Brazilian public sector attracts a large share of high skill workers, way above its share in the total population. Moreover, the group of individuals that took the public exam (i.e., those who are trying to enter the public sector) represents 6% of the total population, but accounts for 23% of all individuals with more than 15 years of education.

As for informality, Bacchetta et al. (2009) study the informal sector in developing countries, and provide evidence that informality is negatively correlated with GDP and GDP growth. Moreover, the study shows that the share of informal employment in total employment in the 2000s was 52% for Latin America, 78% in Asia and 56% in Africa[17]. Informality also accounts for a large share of GDP. Bacchetta et al. (2009) state that the informal sector (excluding agriculture) accounted for 26% of GDP in Latin America in 2006. Finally, the study shows that the informal sector attracts less educated people: on average, around 65% of all informal workers in Latin America[18] are "low skill" workers. Less the 12% informal workers in the region are considered to be of "high skill".

[15] 43% have 11-14 years of education and 37% have more than 15 years.

[16] There is similar international evidence in Dickson et al. (2014).

[17] Country groupings: (i) Latin America: Argentina, Chile, Colombia, Costa Rica, Ecuador, Mexico, Panama, Uruguay, Venezuela, (ii) Asia: China, India, Indonesia, Pakistan, Sri Lanka, Thailand, (iii) Africa: Botswana, Cameroon, Egypt, Ethiopia, Ghana, Kenya, Malawi, South Africa, Tanzania, Zambia, Zimbabwe.

[18] This is the average in 2005 of Argentina, Brazil, Chile, Costa Rica, Equador, Mexico, Panama, Paraguay, Uruguay and Venezuela.

The Brazilian case is no different. Informality there also accounts for a large share of the working population and absorbs a disproportional share of less-qualified workers.

Brazil has a social assistance program in which individuals older than 60 with very low income can retire receiving as benefit the minimum wage. We estimated, using PNAD data, an income premium of 0.2 of formal workers relative to informal workers. Therefore, informal workers who earns during their lifetimes very low incomes could in principle retire receiving as benefit a minimum wage. Moreover, this does not require any contribution to the social security system. The data shows that, even though contribution is voluntary in Brazil, the share of informal sector workers that contributed in 2013 was around 17% only. Clearly, the incentives are not there for them to contribute.

In conclusion, a model studying the macroeconomic and occupational consequences of social security reforms should include the following features: (i) a well-paid and stable public sector which must have costly access (otherwise we would not see any discrepancy between education distribution across sectors nor high wage premiums), (ii) a public pension system that is attractive when compared to the private system and relatively expensive, and (iii) an informal sector absorbing low-skill individuals. Although the model will be calibrated to Brazil, we think that these features are common across a large share of poor to medium-income countries.

3 The Model

The economic environment in this paper consists of a life-cycle model of occupational choice and retirement behavior. Individuals can be either in the private sector, working for the government or retired from the labor force. All decisions are endogenous, in the sense that the individual will only apply for retirement or to the public sector job if it is worth it.

Aggregately, we have a three-sector economy with public and private production, the latter being either formal or informal. The government is responsible for paying (non-competitive) wages to its workers, in exchange for the production of a public good, and for managing a PAYG retirement system for both public and private sector retirees. In order to pay its bills, the government taxes consumption, capital and labor income. The formal private firms use a simple Cobb-Douglas technology with both capital and labor, whereas firms in the informal sector use a production function linear in labor. All firms act competitively.

The sources of exogenous uncertainty in the economy come from idiosyncratic shocks the private workers have in their labor efficiency, and the life span of the agents.

3.1 Demography, Preferences and Choices

The economy is populated by a continuum of mass one agents who may live at most T periods.

There is uncertainty regarding the time of death in every period so that everyone faces a probability ψ_{t+1} of surviving to the age $t+1$ conditional on being alive at age t (therefore we have $\psi_1 = 1$ and $\psi_{T+1} = 0$). This lifespan uncertainty entails that a fraction of the population leaves accidental bequests, which, for simplicity, are assumed to be distributed to all surviving individuals in a lump-sum basis (call it ζ_t).

The age profile of the population, denoted by $\{\varphi_t\}_{t=1}^{T}$ is modelled by assuming that the fraction of agents at age t in the population is given by the following law of motion $\varphi_t = \frac{\psi_t}{1+g_n}\varphi_{t-1}$ and satisfies $\sum_{t=1}^{T} \varphi_t = 1$, where g_n denotes the population growth rate. This normalization implies that $\varphi_1 = \left[1 + \sum_{t=2}^{T}(1+g_n)^{-(t-1)} \prod_{i=2}^{t} \psi_i\right]^{-1}$.

Agents enjoy utility over effective consumption, $\tilde{c}_t$, and leisure, l_t. They maximize their expected utility throughout life:

$$\mathbb{E}_0 \left[\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{k=1}^{t} \psi_k\right) u(\tilde{c}_t, l_t)\right]$$

Where β is the intertemporal discount factor and $\mathbb{E}_t$ is the expectation operator conditional on time t. The agents' period utility is assumed to take the form:

$$u(\tilde{c}_t, l_t) = \frac{[\tilde{c}_t^{\gamma}(1+l_t)^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

Where effective consumption is given by $\tilde{c} = c + \epsilon Y_G$, c_t being private consumption and Y_G a public good. If $\epsilon < 0$, the marginal utility of private consumption increases with an increase in Y_G and if $\epsilon > 0$, the opposite is true. Thus, our framework allows for substitutability or complementarity between public and private goods. The parameter σ determines the risk aversion and γ denotes the share of consumption in the utility.

In this economy, agents can be private workers, public servants or retirees. Denote such individual states as $m \in \{P, G, R\}$, respectively. Agents choose how much to consume, $c_t \geq 0$. They also make a labor decision, L_t. We will restrict the labor choice to be in $\{0, 1\}$. The remaining time is considered to be entirely leisure time. We assume that public servants are obligated to go to work. In contrast, private workers and retiree can choose between working (either formal or informal sector), or staying at home[19]. All agents in the economy can save and lend their savings to a

[19] We allow for retirees to endogenously choose whether to work or not. This is the case in Brazil, even though in some countries retirees are not allowed to work. We can easily shut down such market if we are interested in studying

private competitive firm, as usual.

Agents can choose to take an open exam and try their luck into the public sector. Taking this exam is costly, where the time cost is a function of their current age, $c_{ap}(t)$. We assume that retirees cannot enter the public sector, and that public workers cannot retake the exam.

As the workers become older, and conditional on meeting the eligibility requirements, they can apply for social security benefits and become retirees next period within their respective sector. The informal private workers retire as private retirees, the same retirement sector as the formal private workers (there is no informal retirement sector).

The public sector and the retirement sector are absorbing states. Once a worker applies and enters the public sector, there is no turning back. The same is true for the application for social security benefits.

3.2 Labor, Income and Efficiency

Conditional on their respective sectors, individuals make decisions on whether to work or not and on asset accumulation. Let w_f denote the competitive wage paid by formal private firms to their respective private workers, and w_i the wage paid by informal private firms. Thus, an individual aged t who decides to work $L_t \in \{0, 1\}$ produces a total of units of consumption before taxes given by:

$$y_t(m, w) = \begin{cases} we^{z_t+\eta_t} L_t & \text{if} \quad m \in \{P, R\} \\ \min\{(1+\theta) w_f e^{z_G+\eta_t}, \bar{y}_G\} & \text{if} \quad m = G \end{cases}$$

For $w \in \{w_f, w_i\}$[20].

In the model, z_t (the idiosyncratic productivity) is a random variable that evolves according to an AR(1) process given by: $z_t = \lambda_z z_{t-1} + \varepsilon_z$, with $\varepsilon_z \sim N(0, \sigma_z^2)$. There is no uncertainty regarding the public sector. z_G is the productivity that the private worker had when she decided to take the admission test for the public sector and succeeded. In our model, it will be constant over time. The function η_t is a deterministic age-specific component of labor efficiency. The parameter θ corresponds to the wage premium or economic rent that public sector workers receive relative to their counterparts in the formal private sector. The parameter $\bar{y}_G$ represents a wage ceiling that public workers are constitutionally subject to.

All agents in the economy pay capital income tax τ_k and consumption tax τ_c. Workers face labor income tax rate of $\tau_y(m)$, and those who are not retired must additionally contribute a fraction $\tau_{ss}(m)$ to the Social Security System. The revenue from $\tau_{ss}(m)$ is used to finance the social security

some other country's social security reforms.

[20] It is already imposed that the public workers must work, $L_t = 1$.

benefits of the retirees, and the revenue from $\tau_y(m)$ finances overall government expenditures not related to the social security system. Retirees pay a tax rate of $\tau_b(m)$ over their social security benefits. Informal workers do not pay labor income taxes, nor contribute to the SS system.

We assume that individuals save in a risk-free asset which pays an interest r. They cannot have negative assets at any age, so that the amount of assets carried over from age t to $t+1$ is such that $a_{t+1} \geq 0$. Furthermore, given that there is no altruistic bequest motive and death is certain at age $T+1$, agents at age T consume all their assets, that is, $a_{T+1} = 0$. We will normalize the continuation value after age T as zero.

The budget constraint for the non-retired individuals in the private sector is given by:

$$(1+\tau_c)c_t + a_{t+1} = [1+(1-\tau_k)r]a_t + (1-\tau_y(P))y_t(w,P) - \tau_{ss}(P)\min\{y_t(w,P), y_{\max}\} + \zeta_t$$

The budget constraint for the public sector workers is:

$$(1+\tau_c)c_t + a_{t+1} = [1+(1-\tau_k)r]a_t + (1-\tau_y(G)-\tau_{ss}(G))y_t(w_f,G) + \zeta_t$$

Lastly, for the retirees, the budget constraint is:

$$(1+\tau_c)c_t + a_{t+1} = [1+(1-\tau_k)r]a_t + (1-\tau_y(P))y_t(w,P) + (1-\tau_b(R))b(\cdot) + \zeta_t$$

Where $b(\cdot)$ stands for the social security benefits that a retiree (either public or private) receives[21].

3.3 Public Sector Recruitment

According to constitutional rules, the hiring process of civil servants is given by public competition. Thus, agents in the private sector who want to work in the public sector must take open exams and only those who obtain the best grades on these tests become eligible to fill a pre-determined number of job positions. Once a private worker takes the test and succeeds, she necessarily will become a public servant next period, and must work for the government until retirement.

The timing of the model is the following. First, an agent chooses to apply at t, paying the time cost $c_{ap}(t)$. Her score is revealed at $t+1$: $q_{t+1} \sim U[0,1]$. If $q_{t+1} \geq \bar{q}$ she will necessarily work for government from $t+1$ onwards. Otherwise, she will remain a private sector worker. The threshold score, $\bar{q}$, is chosen by the government in equilibrium to balance the demand and supply of public servants.

[21] Further on we will describe what are the arguments of the social security benefits function, as well as how the contribution to the social security for each type of agent works.

3.4 Social Security System

The social security in Brazil is a pay-as-you-go system, which transfers income from workers to retirees. The system is financed with payroll taxes, and has two very different regimes - the private sector regime and the public sector regime. The benchmark year of our calibration is 2013, hence the retirement benefits structure is modelled in order to mimic the retirement rules that used to prevail in Brazil at that year, before the implementation of a major social security reform.

3.4.1 Private Benefits

The private sector regime is organized under INSS, which stands for Instituto Nacional do Seguro Social (National Institute of Social Security), and establishes a contribution rate according to wage levels. Under the INSS retirement sector, we have two modalities of retirement - the age modality and the contribution modality.

If the worker is older than the normal retirement age, which is 65 years old, and have contributed more than 15 years in the formal private sector, she can apply for retirement under age modality. If the worker is older than 65 years old and has a labor income, y, less or equal to $\frac{1}{4}y_{min}$, where y_{min} is the minimum wage of the model economy, she can also apply for retirement. In this case, she will receive y_{min} as benefits, independently from her history of contributions. If the worker has not achieved the normal retirement age but have contributed for more than 35 years to the social security system, she can ask for retirement under the contribution modality. In both modalities, the value of the benefits will be calculated as a fraction of average past earnings, x:

$$b(t^r, x, m, t_C, y) = \begin{cases} \max\{\Psi(t^r, m, t_C)x, y_{min}\} & \text{if } (y \leq \frac{1}{4}y_{min} \ \& \ t^r \geq 65) \\ \Psi(t^r, m, t_C)x & \text{otherwise} \end{cases}$$

Where $m \in \{RP, RP^{age}\}$ stands respectively for the contribution modality and the age modality of retirement. Also, $\Psi(t^r, m, t_C)$ denotes the retirement replacement rate as a function of the age in which the worker applied for retirement, t^r, the retirement modality m, the number of years that the worker contributed formally to the social security system, t_C, and the current labor income, y.

The average lifetime earnings, x, is calculated by taking into account individual earnings up to the age of withdrawal from the labor force that are lower than the maximum taxable income, $y_{\max}$. Thus, the law of motion for x can be written as:

$$x_{t+1} = \frac{x_t(t-1) + min\{y_t(w_f, P), y_{\max}\}}{t}, \quad \text{for } t = 1, 2, ..., t^r \tag{1}$$

Only earnings from the formal sector are considered in the calculation of x. This is so because we

assume that individuals in the informal sector do not contribute to the social security system[22].

For those who apply for benefits under the contribution modality, the replacement rate is given by:

$$\Psi(t^r, RP, t_C) = f(t^r, t_C)$$

Where t^r and t_C are the same as before, and $f(t^r, t_C)$ is commonly known as the "fator previdenciário" which is not necessarily between [0,1]. Such discount was implemented by the Fernando Henrique Cardoso's presidency, in order to discourage the early retirement that occurred in Brazil. Its formula is given by:

$$f(t^r, t_C) = \frac{0.31 t_c}{E(t^r)} \left[1 + \frac{(t^r + 0.31 t_C)}{100}\right]$$

Where $E(t^r)$ is the life expectancy of the individual at the retirement age t^r, and t_C is the number of years contributed to the social security regime. Depending on the number of years that the private worker has contributed to the social security system and on the age of retirement, f can be bigger than 1. From now on, we will call f the social security factor [23].

Under the age modality, since the worker has already waited until the normal retirement age, the social security factor f is only applied if it increases the benefits - that is, if f is greater than one. Therefore, the replacement rate will be given by:

$$\Psi(t^r, RP^{age}, t_C) = \max\{f(t^r, t_C), 1\}\tilde{\Psi}(t_C)$$

Where $\tilde{\Psi}(t_C)$ is an additional discount, in which individuals aged 65 and over who have met the fifteen years minimum contribution requirement are entitled to 85% of their adjusted lifetime earnings: $\max\{f(t^r, t_C), 1\}x$. For each additional year worked beyond the lower limit, this fraction increases in one percentage point up to 100%. Such additional discount can be formally written as:

$$\tilde{\Psi}(t_C) = \begin{cases} 0.70 + \frac{t_C}{100} & \text{if} \quad t_C < 30 \\ 1 & \text{if} \quad t_P \geq 30 \end{cases}$$

[22] Even though informal workers can contribute so the SS system, only a small fraction do so. In 2013, nearly 84% of the informal workers have not contributed to the SS system. On average from 2002-2013, only 11% of the informal workers contributed.

[23] For instance, a 58 years old worker who contributed for 35 years receives only 86% of its past earnings when retiring.

3.4.2 Public Benefits

According to constitutional rules, the hiring process for civil servants is given by public competition. Once approved, selected and hired, civil servants have special rights, including a different pension system. In particular, retirement benefits for civil servants, until 2013, did not have an upper limit and corresponded to the average of the 80% highest wages received during the public career. Since we assume that there is no uncertainty regarding the labor income in the public sector, such average equals the last wage. Furthermore, retirement is mandatory at age 70 in the public sector and the individual must have at least 10 years working in the sector to be able to retire.

Civil servants older than 60 and that have contributed for at least 35 years can apply for benefits under the contribution modality (denote it by RG). The benefits given to the public servant can be expressed as:

$$b(t^r, z_G, RG, t_C, t_G) = y_{t^r}(w_f, G)$$

Where t^r is defined as before, t_C is the number of years that the public worker contributed to the Social Security System and t_G is the number of years that the individual has worked as a public servant.

Civil servants older than 65 can apply for benefits under the age modality (denoted by RG^{age}). In this case, individuals are entitled to a proportion $\frac{t_C}{35}$ of their last wage. It should be noticed that this formula entails low benefits for agents that reach 65 with a small number of contributions. If the public servant is older than 65 years old and has a labor income less or equal to $\frac{1}{4}y_{min}$, she can also apply for retirement under age modality. As in the private retirement case, her benefits will be equal to y_{min}, independently from her history of contributions. Formally, we have:

$$b(t^r, z_G, RG^{age}, t_C, t_G) = \begin{cases} \min\{\frac{t_C}{35}, 1\} y_{t^r}(w_f, G) & \text{if } t_G \geq 10 \text{ and } y > \frac{1}{4}y_{min} \\ y_{min} & \text{if } y \leq \frac{1}{4}y_{min} \end{cases}$$

3.5 Value Functions

For a given age, we will divide the individual states depending on what sector of the economy the individual is located. The state of an agent in the private sector is $s_P = (a, z, x, t_C) \in S_P \equiv \mathbb{R}_+ \times \mathcal{Z} \times \mathcal{X} \times \{0, 1, ..., T\}$, where a represent her asset holdings, z is the agent's idiosyncratic productivity in the private sector, x is her average past earnings in the private sector and t_C is the number of years contributed to the SS system in the formal sector. The relevant state for a public worker is $s_G = (a, z, t_C, t_G) \in S_G \equiv \mathbb{R}_+ \times \mathcal{Z} \times \{0, 1, ..., T\} \times \{1, ..., 10\}$, where t_G is the number of years the public worker has spent in the government sector. As for the retirees, their relevant state

is given by $s_R = (a, z, b) \in S_R \equiv \mathbb{R}_+ \times \mathcal{Z} \times \mathcal{B}$, where b stands for the benefits that these retirees are receiving.

The solution to the recursive problem of the agents yields the policy functions: leisure $d_t^l(s_m) \in \{0, 1\}$, asset holdings $d_t^a(s_m) \in \mathbb{R}_+$ and consumption $d_t^c(s_m) \in \mathbb{R}_{++}$ for all $m \in \{P, G, R\}$; retirement (for each modality) $d_t^{ss}(s_m) \in \{0, 1\}$ for $m \in \{P, G\}$; working for the informal sector $d_t^{inf}(s_m) \in \{0, 1\}$ for $m \in \{P, R\}$ and the public sector application $d_t^{ap}(s_P) \in \{0, 1\}$.

3.5.1 Retired Workers

At each age t, a retiree chooses between working (for either the formal or informal sectors) or staying at home. Her value function is given by:

$$V_t(s_R) = \max\{V_t^H(s_R), V_t^F(s_R), V_t^I(s_R)\}$$

The value of staying at home is given by:

$$V_t^H(s_R) = \max_{(c,a')\geq 0} u(\tilde{c}, 1) + \beta\psi_{t+1} \cdot \mathbb{E}\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + (1-\tau_b(R))b + \zeta$$

Where $s'_R = (a', z', b)$ and $\mathbb{E}\left[V_{t+1}(s'_R)\right] = \sum_{z'} \Pi(z, z')V_{t+1}(s'_R)$ is the standard expected value conditional on the current productivity, z.

Working for the formal private sector yields the value:

$$V_t^F(s_R) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1} \cdot \mathbb{E}\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + (1-\tau_y(P))y_t(w_f, P) + (1-\tau_b(R))b + \zeta$$

And the value of working in the informal sector is:

$$V_t^I(s_R) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1} \cdot \mathbb{E}\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + y_t(w_i, P) + (1-\tau_b(R))b + \zeta$$

3.5.2 Public Servants

At each age t, a public sector worker has three options: (i) asking for retirement under the contribution modality, if eligible; (ii) asking for retirement under age modality, if eligible and (iii) not asking for retirement and continue next period as a public worker.

In case she decides to retire, the social security benefits that she will earn from next period onwards are calculated according to the social security rules. Call them b'. Since the model is solved numerically, there will be a grid space for the benefits of the retirees, $\mathcal{B}$.[24]

Therefore, the value function of the public servant is:

$$V_t(s_G) = \max\{V_t^{NR}(s_G), V_t^R(s_G)\}$$

Where $V_t^{NR}(s_G)$ is the value for not retiring:

$$V_t^{NR}(s_G) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1}V_{t+1}(s'_G)$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + (1 - \tau_{ss}(G) - \tau_y(G))y_t(w_f, G) + \zeta$$

With $s'_G = (a', z, t_C + 1, t_G + 1)$. If eligible, the value for retiring from the labor force is given by:

$$V_t^R(s_G) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1}\mathbb{E}_b\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + (1 - \tau_{ss}(G) - \tau_y(G))y_t(w_f, G) + \zeta$$

Where $s'_R = (a', z', b' = b(t, z_G, Ret.Modality, t_C + 1, t_G + 1))$

3.5.3 Private Workers

At each age t, the formal private worker will consider: (i) asking for retirement in each modality, if eligible; (ii) going to work (either formally or informally) or staying at home and (iii) applying to the public sector job or not[25].

Figure 8 draws what are the possible combinations of decisions in (i)-(iii) that a private agent has. Each terminal node denotes a specific combination of choices. For example, the terminal

[24] It is not true that b' will necessarily be on $\mathcal{B}$, so we interpolate the retiree's next period expected value function: $\mathbb{E}_b\left[V_{t+1}(s'_{RG})\right] \equiv \alpha_b \cdot \mathbb{E}\left[V_{t+1}(a', z', b^1)\right] + (1-\alpha_b) \cdot \mathbb{E}\left[V_{t+1}(a', z', b^2)\right]$, where $b^1, b^2 \in \mathcal{B}$ are such that $b' = \alpha_b b^1 + (1-\alpha_b)b^2$.

[25] We assume that if the worker applies for retirement and takes the public exam, she will become a retiree next period for sure. Therefore, once the worker asks for retirement, she will never choose to take the public exam.

node denoted $V^{\mathrm{NR,T,I}}$, is the value that the private worker has when she decides to not apply for retirement, take the public exam and work for the informal sector, all within the same period.

To ease the understanding of the recursive problem, we order the agents' choices as we have drawn in the tree. First the agent will consider asking for retirement, contingent on that she will think about applying to the public sector, and contingent on both she will opt whether to work or not.

Therefore, value function of a private sector agent can be written as:

$$V_t(s_P) = \max\{V_t^{\mathrm{R}}(s_P), V_t^{\mathrm{NR}}(s_P)\}$$

Where, conditional on the retirement decision, $i \in \{\mathrm{R}, \mathrm{NR}\}$, there is a test-taking decision:

$$V_t^i(s_P) = \max\{V_t^{\mathrm{i,T}}(s_P), V_t^{\mathrm{i,NT}}(s_P)\}$$

And, given the previous considerations of retirement and test-taking decisions, $i \in \{\mathrm{R}, \mathrm{NR}\}$ and $j \in \{\mathrm{T}, \mathrm{NT}\}$, the private agent will choose whether to work (formally or informally) or to stay at home [26]:

$$V_t^{i,j}(s_P) = \max\{V_t^{\mathrm{i,j,H}}(s_P), V_t^{\mathrm{i,j,F}}(s_P), V_t^{\mathrm{i,j,I}}(s_P)\}$$

3.6 Agents' Stationary Distribution

The stationary distribution of agents is characterized by probability distribution functions $\mu_m : \{1, 2, ..., T\} \times S_m \to [0, 1]$, for all $m = \{P, G, R\}$, such that $\sum_{(m,t,s_m)} \mu_m(t, s_m) = 1$. That is, $\mu_m(t, s_m)$ is the measure of individuals at age t, in sector m and state s_m in the whole population.

For $t = 1$, the agent's just entered the economy, so we have no transition. Thus, their distribution will depend on initial conditions (hypothesis) of the model. Those hypothesis are: (i) every agent will start her life-cycle with zero initial assets, zero average past earnings and zero time of contribution; (ii) everybody will start as a worker in the formal private sector; (iii) the initial distribution of the idiosyncratic productivity will be the invariant distribution of the Markov process for z_t (call it $\bar{\Gamma}$).

[26] The details of each value function $V_t^{\mathrm{i,j,k}}(s_P)$, for $i \in \{\mathrm{R}, \mathrm{NR}\}$, $j \in \{\mathrm{T}, \mathrm{NT}\}$ and $k \in \{\mathrm{H}, \mathrm{F}, \mathrm{I}\}$ are in a separate appendix in ADDRESS.

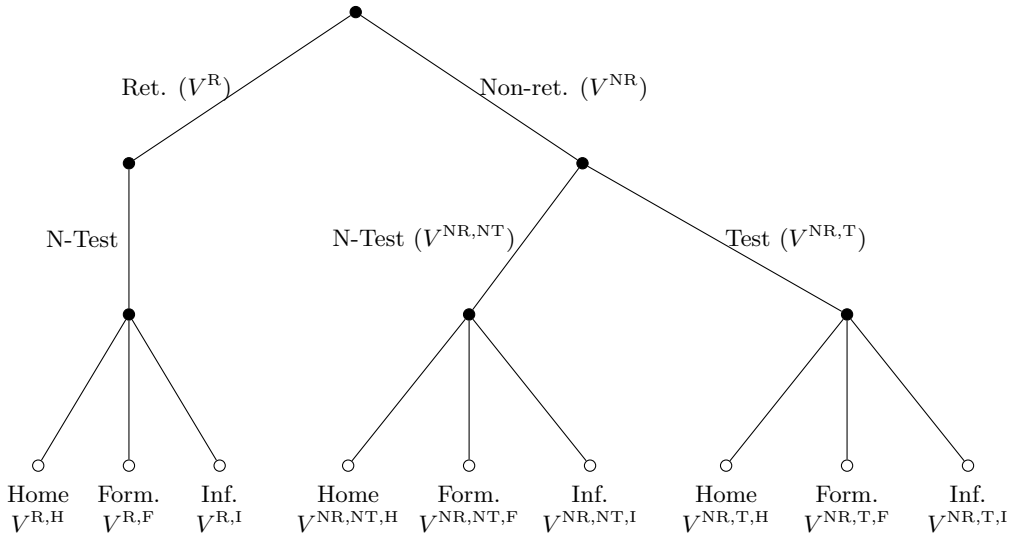

Figure 8: Private Agents' Decisions

Considering the above assumptions, for $t = 1$, let μ be given by:

$$\mu_P(1, s_P) = \begin{cases} \varphi_1 \bar{\Gamma}(z) & \text{if} \quad s_P = (0, z, 0, 0) \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_m(1, s_m) = 0 \quad \text{for } m \in \{G, R\}$$

For each of the remaining ages, the distributions will be derived using forward induction, considering the agents' policy functions, the transition matrix for the exogenous process z, survival probabilities and the probability of succeeding in the public exam and entering the public sector [27].

3.7 Technology

We assume that there are two representative firms in the model economy. One operating in the formal sector and one in the informal sector. The first one produces using both capital and labor, whereas the second one uses only labor. Both of them act competitively and maximize profits given input prices.

The production function of the formal sector is specified as a Cobb-Douglas function, given by $Y_f = F(K, N_f) = K^{\alpha} N_f^{1-\alpha}$, where K and N_f are the aggregate capital and private labor inputs and α is the capital's share in output. Capital is assumed to depreciate at a rate δ each period.

[27] For a formal deriviation of the function, see separate appendix at ADRESS .

We can write the problem of the firm as:

$$\max_{K,N_f} K^{\alpha} N_f^{1-\alpha} - w_f N_f - (r+\delta)K \tag{2}$$

Informal firms have linear technology in labor: $Y_i = A_i N_i$ and maximize profits[28] given by:

$$\Pi_i = \max_{N_i} A_i N_i - w_i N_i - \tau_i(N_i) \tag{3}$$

We consider $\tau_i(N_i)$ as an increasing and strictly convex expected cost of being caught by government authorities operating under informality. We follow the modelling and the discussion found in Ulyssea (2017), Meghir et al. (2015), Almeida and Carneiro (2012) and de Paula and Scheinkman (2011) in considering such costs, as the Brazilian law prohibits firms from operating informally.

3.8 The Government Sector

The government taxes capital, income, consumption, and social security benefits to finance social security coverage, government expenditures with the public servants and its own consumption. We assume that the government consumes a constant fraction of the formal GDP: $C_g = \alpha_g Y_f$.

The government hires a constant share $\bar{N}_G \in [0,1]$ of the population as public servants to produce a public good Y_G. We assume that the public good is a linear function of effective labor supply, L_G: $Y_G = L_G$.

In equilibrium, the government is responsible to choose $\bar{q}$ in order to balance the demand and supply of public workers.

4 Equilibrium

We define the *recursive competitive equilibrium* in this economy in a traditional way. A steady state competitive equilibrium consists of allocations of households and firms, prices (wages and interest rate), government taxes and threshold score, stationary distributions of agents, bequests and public goods such that: (i) households and firms optimize; (ii) individual and aggregate behaviors are consistent; (iii) the government sets threshold scores and consumption taxes to balance the size of the public sector as well as its budget constraint; (iv) the stationary distributions evolve with

[28] Even though the informal firms are acting competitively, it is easy to show that there are still positive profits for them. Since we focus on the households' behavior, we will not model firm entry and such in order for them to operate with zero profits.

optimal policy of the agents; (v) the amount of public good and the amount of bequests are consistent with individual behavior.

The only innovation in the equilibrium definition above, relative to the literature on life-cycle models, is that the government is responsible for setting the threshold score to have a constant share $\bar{N}_G$ of the population as public servants. In the online Appendix we formally state the equilibrium definition.

5 Data and Calibration

Tables 4 and 5, in the end of this section, summarize the parameters values of this exercise. We calibrated the economy to match features of the Brazilian economy in 2013. We used data from PNAD. An individual in our dataset is a men whose age is between 16 and 90, who is either: (i) economically inactive, (ii) economically active working in the formal[29] sector, which includes both the private sector and the public sector[30], or (iii) economically active, working in the informal sector.

5.1 Demography

The population age profile $\{\varphi_t\}_{t=1}^{T}$ depends on the population growth rate g_n, the survival probabilities ψ_{t+1} and the maximum age T that an agent can live. In this economy, a period corresponds to one year and an agent can live 75 years, so $T = 75$. Additionally, we assumed that an individual is born at age 16, so that the real maximum age is 90 years.

The data on survival probabilities are taken from IBGE's 2013 mortality tables. Figure 9 plots these probabilities as they are used in the model. The population growth rate is chosen to match the endogenous population age profile with the data. This yields a g_n equal to 0.01781. Figure 9 also plots the population age profile, comparing our model and the data.

5.2 Preferences and Technologies

The value of β is chosen so that the capital-to-output ratio is 2.5. This value is obtained from Morandi (2016), which applies the Perpetual Inventory Method to compute a historical series of the capital stock in Brazil. This number lies in the range of 2.5 and 3, values commonly used in the Macro literature for Brazil[31]. This resulted in a discount factor of 0.9697.

[29] In this paper, a formal worker is a worker that has his working form signed by his employer.
[30] We considered only statutory workers, who are subject to a segregated social security plan.
[31] See dos Santos and Cavalcanti (2015), Glomm et al. (2009) and Ferreira and do Nascimento (2005).

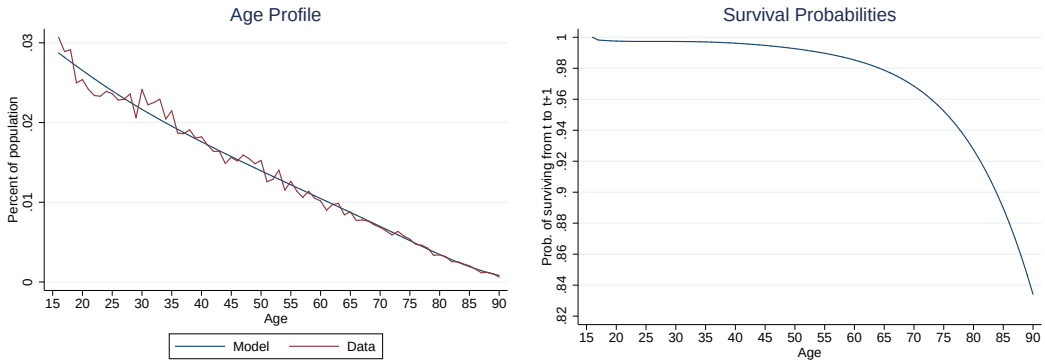

Figure 9: Calibrated Survival Probabilities and Population Age Profile.

The consumption share in the utility, γ, takes the value of 0.3814 to match the percentage of households that are economically active, taken from PNAD. According to the survey, in the year of 2013, the participation rate was 69.63%.

Since it is quite difficult to find a reasonable way to calibrate the marginal utility coefficient of the public good[32], for now, we will rely on Ferreira and do Nascimento (2005) and set $\epsilon = \frac{1}{2}$. Therefore, we proceed assuming that there is imperfect substitution between private and public consumption.

For the formal private sector technology parameters, we will set the capital share in output as $\alpha = 0.43$ and the depreciation rate at $\delta = 7\%$, as commonly used in the Macro literature.

As for the informal sector, we calibrate the informal firms' productivity of labor, A_i, to match the share of the informal workers in the population. From the 2013 PNAD we find that 18.2% of the work force is in the informal sector. This procedure results in $A_i = 1.593$.

The functional form of the expected cost of informality is: $\tau_i(N_i) = \xi_0 N_i^{\xi_1}$. Due to the highly similar functional form of the cost function and the profit maximization problem of the informal firms, the values used for ξ_0 and ξ_1 were taken from Meghir et al. (2015). This implies $\xi_0 = 0.955$ and $\xi_1 = 1.50$.

5.3 Income Processes

In the model economy, all the heterogeneity among agents that is not related to the age, asset accumulation and working sector is captured by the idiosyncratic productivity of work, z_t.

To calibrate the parameters of the AR(1) process, based on well established evidence for the US

[32]The empirical literature (e.g., Fiorito and Kollintzas (2004) and Ni (1995)) neither provides a plausible range of values for it nor a robust evidence of substitutability or complementarity between public and private consumption.

economy[33], we first set $\lambda_z = 0.96$. Then, we log-hourly wages on age and age squared[34] for the private sector workers, and assume that the MSE equals the unconditional variance of z: $MSE = Var(z) = \frac{\sigma_z^2}{1-\lambda_z^2}$. The regression estimates are detailed in Column (1) of Table 3. This procedure results in a variance of 0.0454. After determining the AR(1) parameters, we discretize it following Tauchen (1986), with 10 grid points.

To calibrate the efficiency profile, $\{\eta_t\}_{t=1}^{T}$, we follow Dos Reis and Zilberman (2014) and assumed the functional form: $\eta_t = \alpha_1^\eta t + \alpha_2^\eta t^2$. As mentioned before, for each choice of labor, L_t, the agent will receive a total wage of $y_t = we^{z_t+\eta_t}L_t$, regardless of the sector. Dividing by L_t, taking logs and substituting z_t, one gets the regression equation:

$$\log\left(\frac{y_t}{L_t}\right) = \log w + \alpha_1^\eta t + \alpha_2^\eta t^2 + \lambda_z z_{t-1} + \varepsilon_z \tag{4}$$

Two potential issues in regressing equation (4) are worth discussing. First, the regression suffers from an omitted variable problem, since z_{t-1} is not observed on the data. Therefore, using ordinary least squares on the above expression would result in biased and inconsistent estimates of α_1^η and α_2^η, since z_{t-1} is obviously correlated with t. To alleviate this problem, we control for individuals characteristics such as different races, whether she is head of the household, has a farm job, what is the occupational sector the individual is working and whether she lives in urban area.

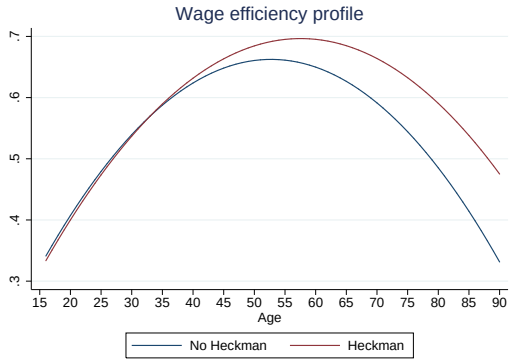

Figure 10: Efficiency Age Profile.

Selection bias is another problem that arises in regression (4). It could be the case that only high-z agents keep on working after 60 years old. Since we only use strictly positive wage data, the coefficients may be overestimating the impact of age on wages for people older than 60 years old.

[33]The literature on estimating the persistence coefficient typically finds values for λ_z close to 1. Flodén and Lindé (2001) estimate it to be 0.91, whereas French (2005) finds it to be around 0.98.

[34]That is, we want z to capture all variation in hourly wages that are not explained explicitly by the age of individuals.

We use the Heckman (1979) method to control for the potential selection bias in our regressions. To do so, we used dummies that may correlate with the probability of an individual going to work and are not correlated with her income. For instance, we controlled for whether the individual is single or not, has a living mother and lives with her. The resulting estimated values for α_1^η and α_2^η are 0.0242 and -0.000210, respectively. The coefficients are detailed[35] in Column (4), Table 3, and the age-efficiency profiles both with (Column (4)) and without (Column (3)) the Heckman's method are shown in Figure 10. The figure shows that there are indeed potential selection of older agents into the labor market. We take that into account and consider the red curve as our age-efficiency profile.

We estimate the wage premium of the public sector, $\theta = 0.3662$, using PNAD data. We include a civil servant dummy in the regression equation (4) to capture what is the average increase in the intercept when a worker goes from the private to the public sector. Table 3 summarizes the regression results.

Table 3: Income Process and Wage Efficiency Profile

	log(Income/Hour)			
	(1)	(2)	(3)	(4)
Variables	Private Sector	Private Sector	All Agents	All Agents
Age	0.0584***	0.0541***	0.0251***	0.0242057***
	(0.00006)	(0.00005)	(0.00006)	(0.00006)
Age^2	-0.000614***	-0.000467***	-0.000238***	-0.0002103***
	(0.0000008)	(0.0000007)	(0.0000008)	(0.0000008)
Yrs. of Schooling		0.0904***	0.0364***	0.0367912 ***
		(0.00003)	(0.00004)	(0.00004)
Civil Servant			0.3650586***	0.3661802***
			(0.0007)	(0.00007)
Other controls	No	No	Yes	Yes
Heckman	No	No	No	Yes
Observations	31,919,594	31,919,594	22,005,428	32,812,020
R-squared	0.0487	0.2578	0.4052	
Root MSE	0.76125	0.6724	0.5906	

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

[35]The complete table of our regression analysis is available upon request.

5.4 Government Sector

To calibrate the cost function of taking the public exam, we assumed a second-order polynomial form[36]: $c_{ap}(t) = \alpha_1^{ap} t^2 + \alpha_2^{ap} t + \alpha_3^{ap}$. We calibrated the parameters α_1^{ap} and α_2^{ap} to match the average age in which individuals take the public exam and the the number of agents who take it with less than 35 years old. The intercept, α_3^{ap} is chosen to match the total amount of test takers in the economy. This resulted in α_1^{ap}, α_2^{ap} and α_3^{ap} equal 0.000175, -0.0062 and 0.938 respectively.

Social security taxes of both public and private workers are taken directly from 2013 tax code. Public workers paid 11% of their income to the social security system. Private workers paid 8%, 9% or 11%, depending on their income, in the following manner:

$$\tau_{ss}(P) = \begin{cases} 8\% & \text{if } 0 \leq \min(y_t, y_{\max}) \leq \frac{1247.7}{4159} y_{\max} \\ 9\% & \text{if } \frac{1247.7}{4159} y_{\max} < \min(y_t, y_{\max}) \leq \frac{2079.5}{4159} y_{\max} \\ 11\% & \text{if } \frac{2079.5}{4159} y_{\max} < \min(y_t, y_{\max}) \end{cases} \tag{5}$$

Private retirees do not have any tax on their retirement benefits. Public retirees, on the other hand, must pay a 11% tax over the amount of their benefits that exceeds the maximum private benefit, $y_{\max}$. Formally, they must pay $11\% \max\{b - y_{\max}, 0\}$ to the social security system.

The private sector labor income tax and the capital tax rate are chosen following the Macro literature for Brazil[37]. We set them as $\tau_y(P) = 18\%$ and $\tau_k = 15.5\%$. As done in Immervoll et al. (2006), we set the labor income tax of the public servants as $\tau_y(G) = \frac{\tau_y(P)}{2} = 9\%$. The consumption tax rate is chosen to balance the government budget constraint in equilibrium.

The maximal value of a private pension, $y_{\max}$, is chosen to match the private sector social security deficit as a percentage of GDP, $SS_p = 0.94\%$, taken from de Oliveira (2016). The procedure resulted in $y_{\max} = 0.6398$. For the minimum wage, we set $y_{min} = \frac{678}{4159} y_{\max}$. These values were obtained from comparing the actual minimum wage and social security benefits in 2013. The ceiling on public sector earnings, $\bar{y}_G$ was chosen to match the ratio between the highest wage received by an individual in the public sector and the private sector, taken from PNAD.

The fraction of the working population in the public sector, $\bar{N}_G$, is set to be 4.8%, as it is calculated using data on sectorial occupation, also taken from PNAD. The share of GDP that is consumed by the government, α_g is set to be 0.1889, value taken directly from 2013 Brazilian National Accounts.

[36] Other convex cost functions would serve as well. For instance, $c_{ap}(t) = \alpha_1^{ap} e^{\alpha_2^{ap} t} + \alpha_3^{ap} t$. We chose a quadratic polynomial for simplicity. The choice of not considering a $c_{ap}(t, z)$ was also for the sake of simplicity.

[37] See, among many, Glomm et al. (2009), and Pereira and Ferreira (2010) .

5.5 Calibration Results

We summarize our calibrated parameters in two tables. Table 4 shows the parameters that were calibrated exogenously. Table 5 summarizes the main features of the internal calibration procedure.

Table 4: External Calibration Summary

Parameter	Description	Value	Source
$\{\psi_t\}_{t=1}^{T}$	Survival probabilities	-	IBGE
g_n	Population's growth rate	1.1781%	IBGE
σ	Risk aversion	4.8	Issler and Piqueira (2000)
ϵ	Public good ut. coef.	$\frac{1}{2}$	Ferreira and do Nascimento (2005)
α	Capital share in output	0.43	Standard value
δ	Depreciation	7%	Standard value
λ_z	Shock persistence	0.96	US economy
σ_z^2	Shock variance	0.0454	PNAD
θ	Public sector wage premium	0.3662	PNAD
α_1^{η}	Age eff. profile coef.	0.0242	PNAD
α_2^{η}	Age eff. profile coef.	-0.000210	PNAD
$\tau_{ss}(m)$, $\forall m$	SS income tax code	-	2013 tax rates
$\tau_b(m)$, $\forall m$	SS benefits tax code	-	2013 tax rates
$\tau_y(PF)$	Private sector's income tax	18%	Literature
$\tau_y(G)$	Public sector's income tax	9%	Immervoll et al. (2006)
τ_k	Capital tax	15.5%	Literature
y_{min}	Minimum wage	$\frac{678}{4159} y_{\max}$	2013's $\frac{\text{Min wage}}{\text{SS ceiling}}$
$\bar{N}_G$	Size of the govt. sector	4.8%	PNAD
ξ_0, ξ_1	Expected Cost of Informality	0.955, 1.50	Meghir et al. (2015)
α_g	$\frac{\text{Govt Consumpt.}}{\text{GDP}}$	18.89%	National Accounts

Table 5: Internal Calibration Results

Parameter	Target	Model	Data
$\beta = 0.9697$	$\frac{K}{Y}$	2.48	2.50
$\gamma = 0.3814$	Particip. Rate	69.13%	69.56%
$y_{\max} = 0.6398$	SS_p	0.94%	0.94%
$A_i = 1.593$	Informality's measure	19.0%	18.2%
$\bar{y}_G = 3.415$	Max. wage ratio (Public/Private)	0.664	0.667
$\alpha_1^{ap} = 0.000175$	Avg. age of test takers	27.6	29.5
$\alpha_2^{ap} = -0.0062$	% of test takers aged 16-35	0.854%	0.856%
$\alpha_3^{ap} = 0.938$	% of test takers	0.98%	1.05%

6 Equilibrium Features and External Validation

This section intends to externally validate our model. We compare the outcome the modelling economy to the data in different aspects, mainly related to working and retirement decisions and the social security deficit as a percentage of GDP.

First, for the calibrated economy, Table 6 summarizes the equilibrium variables' values. The tax structure used in the model yields a consumption tax of 14%, close to the Brazilian data[38]. The interest rate is high by international standards but not for Brazil.

Table 6: Equilibrium Variables

Variable	**Description**	**Value**
r	Interest Rate	7.17%
w_f	Formal Sector Wage	1.20
w_i	Informal Sector Wage	0.94
$\bar{q}$	Threshold Score	0.78
Y_G	Public Good	0.095
τ_C	Consumption Tax Rate	14.2%
ζ	Lump Sum Transfers	0.076

Figure 11 plots the average consumption and average savings. Average consumption is relatively smooth and has a hump-shape, just as documented by most of the literature on retirement in a life-cycle environment. The basic idea behind this shape and the relatively big decline after retirement is that once retired, agents would probably buy less "market" goods and more home-producing goods. Since what our model captures is the consumption of market goods, it is reasonable to have a decline in consumption after leaving the labor force. Average savings are in accordance with life-cycle aspects, individuals save the most when they are most productive, in order to smooth consumption when older.

Table 7: Average Idiosyncratic Productivity Across Sectors

Group	Formal	Informal	Home	Public
Avg. z	0.31	-0.43	-0.20	0.082

Table 7 shows the average idiosyncratic productivity for each sector in the economy. Following the evidence, low-z individuals endogenously choose to work in the informal sector, whereas the high-z agents go the the formal private sector, mainly, and the public sector. The discrepancy between the average z of the formal private sector and the public sector probably is due to the limit on the public sector earnings, $\bar{y}_G$.

Table 8 compares the aggregate distribution of the individuals generated by the model and in the

[38] For instance, see dos Santos and Pereira (2010).

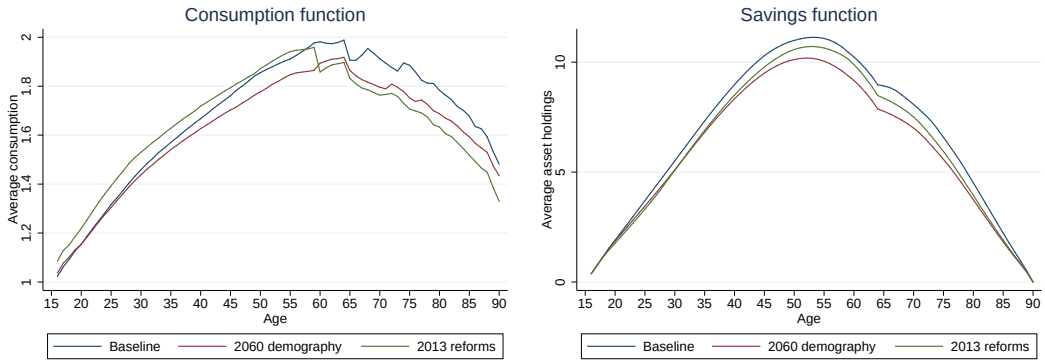

Figure 11: Consumption and Savings.

data.

Table 8: Distribution of Individuals (%)

	Retirement	**Public**	**Private**
Data	16.2	4.8	79.0
Model	12.4	4.7	83.0

Our model matches the aggregate distribution of individuals fairly well. [39] A more detailed analysis can be made by plotting how individuals are distributed across the three sectors of the economy, by age groups. Figure 12 represents the equilibrium distribution of individuals across sectors for the benchmark economy calibrated to the year of 2013.

Since the calibration procedure forces the match of the total economic participation rate, it is interesting to analyze whether agents are behaving as in the data along the life-cycle. The model is also able to reproduce the data quite well in this dimension. However, as shown in Figure 13, the model overestimates the number of young people in the labor market. Apart from that, our model captures the labor supply decision fairly well.

The intuition for the distance between the model and the data, is that young agents in the model economy start life with no initial assets and so they have to work early in life in order to compensate this lack of resources. Moreover, we are not modeling human capital accumulation nor schooling decisions. Most of the teenagers aged 16-18 in the data are probably ending their studies, preparing themselves to enter the market and still living with their parents. Therefore, the actual participation rate of young agents is low relative to the model.

Figure 14 plots, for each age group, the share of test takers in the population and compare it to the data. Even though the average age, the total amount of people and the total amount of test

[39] It is worth noticing that the Public column is internally calibrated, as the government chooses $\bar{q}$ to have $\bar{N}_G$ workers in equilibrium.

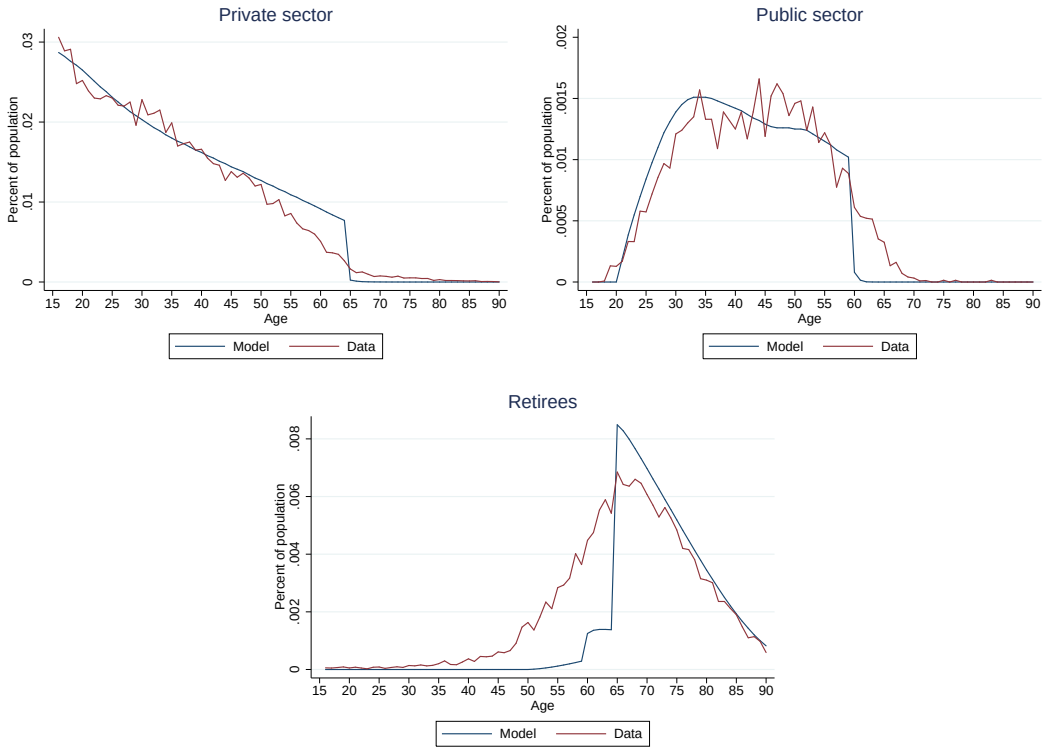

Figure 12: Agents' Equilibrium Distribution in Steady State.

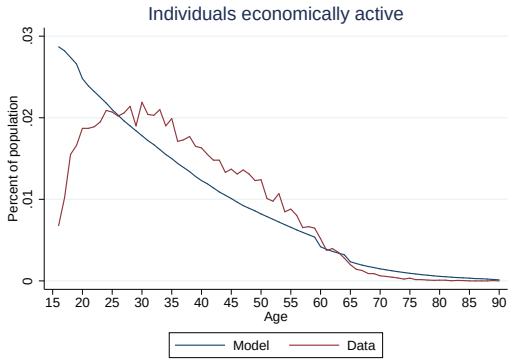

Figure 13: Participation Rate.

Figure 14: Public Test Takers.

takers between 16-35 years are internally calibrated, the share of test takers by age is close to the data. Apparently, the way we modeled all the process of applying to a public job is consistent with the actual process in the economy.

The model is also close to the social security deficit figures. In 2013, the social security deficit was of 2.12% of GDP. The private sector deficit accounted for 0.94% and the remaining 1.18% was due to the public sector. The model estimates that the public social security deficit is 1.27% of GDP,

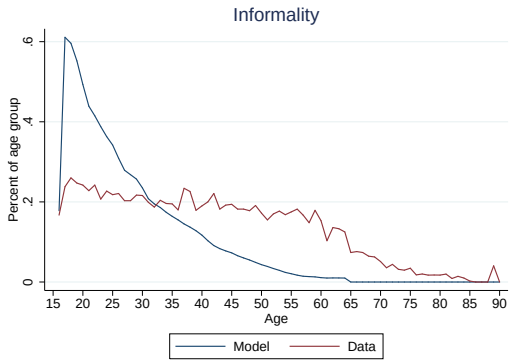

Figure 15: Informality by Age Groups.

close to the data, resulting a total deficit of 2.20% of GDP.

The model is also able to reproduce the early retirement decisions. Pereira (2013) shows that, in 2013, the average age in which Brazilian males apply for retirement under the contribution modality was around 55 years old. In our model economy, agents apply for social security benefits under the contribution modality, on average, when they are 58 years old . In addition, the average number of years contributed to the social security system, for those who claim for retirement under this modality, was around 35, precisely what is generated in our calibrated model.

Figure 15 plots the size of the informal sector by age. In the model, informality decreases significantly as individuals become older, a pattern that is also observed in the data, even though the intensity of the reduction is different. This is because, from the agent's point of view, it is too easy to alternate between formal and informal sector, whereas in reality this transition should be more sluggish.

7 Social Security Reforms

We evaluate the long term effects of the changes in the social security system. To do so, we first adjust the population growth rate to simulate the impact of demographic changes in the economy by 2060. We then evaluate the consequences of three different scenarios to study: (i) Demographic shift alone and no social security reform; (ii) Unification of the public and the private pension systems (and the demographic changes) and (iii) Increasing the minimum retirement age, on top of the reforms of scenario (ii).

7.1 No Social Security Reform

The 2060 demographic structure of the economy is constructed from IBGE data. The age profile used to calibrate our model was based on PNAD data, not on IBGE's Tables of Population Projections 2000-2060. This is so because we only considered individuals aged 16 to 90 years old who were either retirees, registered workers, unregistered workers or statutory public servants. Figure 16 shows that even though we restrict our sample, the demographic structure is not too far from that that considers the whole population.

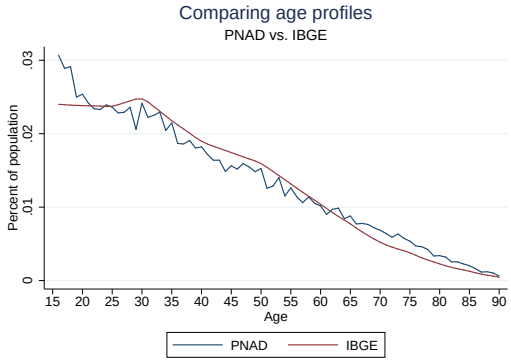

Figure 16: 2013 Comparison

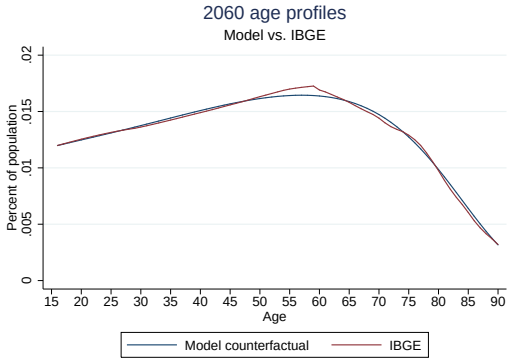

Figure 17: Population Projections

We then adjust the population growth rate to match the population age profile provided by IBGE for 2060. This procedure results in a population growth rate of $g_n = -0.01232$. Figure 17 plots the age profile for both the data and the model.

The demographic change implies a significant increase in the social security deficits of the public and private systems with respect to the present (i.e., the benchmark model). Public social security deficit goes from 1.26% to 4.12% of GDP and that of the private sector jumps from 0.94% to 4.1% of GDP. Those numbers tighten up the government's budget constraint, inducing an increase in the consumption tax rate from 14% to 24%. Many countries around the world are experiencing the same process of population aging. Several of them have overly generous dual social security systems, which in general favor public workers. The result above is an indication that these economies, and not only Brazil, will suffer from very negative fiscal conditions in the future if they do not change their retirement rules.

A second result, plotted in Figure 11, is that average consumption of young agents does not change much, decreasing a little for younger ages, while the consumption of the elderly falls significatively. Average savings decreases throughout the whole life-cycle. The increase in taxation affects people's income and hence savings and consumption. Consumption smoothing implies that savings absorb the largest share of the reduction in labor income. The capital-to-output ratio increases to 2.73,

being driven by the change in total output, which decreases 5%. We observe a large decrease in the participation rate. Unemployed households account for 42% of the population in this new environment, but informality decreases to , around 14% of total population. In summary, the demographic changes expected in the future will increase markedly social security deficit. The expansion in taxation required to finance it will lead in the future to less consumption and savings and higher unemployment and, of course, a decrease in the population welfare.

7.2 Social Security Reforms I: Toward a Unified System

In 2013, Law 12.618 imposed a ceiling on the social security benefits received by new entrants in the public sector. This cap is the same that limits private sector benefits, $y_{\max}$. The eligibility conditions did not change, neither did the benefits' formulas. The contribution to the social security system was limited to 11% of the minimum between the salary received by the public worker and $y_{\max}$. This reform approximated the two social security regimes, and intends to alleviate, at least in an *ex-ante* manner, the fiscal pressure of public retirement on public accounts.

We compare the steady state equilibrium of the economy with the 2060 demography and no reforms with that of the Brazilian economy with the new social security legislation (and the projected demography). A key result of the reforms is the reduction of the social security deficit. With the reforms, the social security system yields in (the steady state) equilibrium a deficit of 5% of GDP, as opposed to 8.2% without the reforms. The deficit of the private social security system remains similar to the previous exercise, at 4.2% of GDP. The difference is concentrated on the reduction of the public social security system. The deficit goes from 4.12% to 0.8% of GDP. The reduction of the SS deficit allows the equilibrium consumption tax rate to be lower, at 19.6%. This is expected, as the reform did not change the rules governing pensions of the private sector.

In the new equilibrium the capital-to-output ratio increases to 2.78. This represents an increase of 2% relative to the economy with demographic structure of 2060. Agents value the public sector for both its stability and its retirement advantages. Both of them acts as insurance against lifetime uncertainties. Once we shut down one of these key features, agents increase their precautionary savings to smooth consumption over the life-cycle. Figure 11 plots the average savings by age in the three economies. We highlight that the grey curve, corresponding to the economy with demographic change and social security reform, is above that for the economy without social security reform (but demographic change) for almost all ages in the life cycle. Consumption for the young agents increase relative to the other economies, even though it decreases at a faster rate for older agents: by the end of their life, as expected, individual save the same in the three economies. The 2013 social security reforms boosted aggregate capital from around 6.7 observed in the economy with no reforms but the 2060 demography, to 7.1, an expansion of 6%.

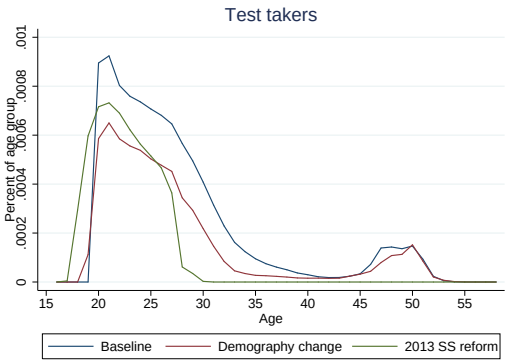

Figure 18: Test Takers.

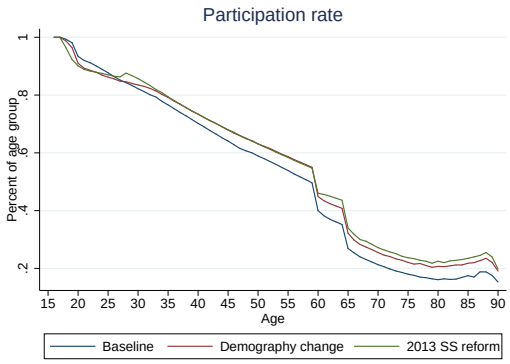

Figure 19: Participation Rate.

Social Security reform has a drastic effect on the profile of civil servants and of the distribution of test takers. Figure 18 shows that we no longer have "late-comer" test takers in the economy. That is, the reform eliminated people who worked in the private sector for most of their life, accumulated years of contribution to the social security system, but apply for a public job due to the better retirement conditions of the sector. There is now a concentration of test takers between 16 and 30 years old. Even though less people apply to public sector positions[40], the equilibrium threshold score is higher. The concentration of the test takers leads to this general equilibrium effect. Younger test takers implies younger public servants, which in turn means more time spent in the government. Therefore, we have a larger mass of public sector workers relative to the economy with old-aged test takers and no Social Security reform.

Regarding the distribution of agents in the economy, the key difference, due to a modification of the test takers curve, is the distribution of public servants. The average age in which agents apply for retirement under the contribution modality increases to 58.8. Figure 19 plots the economic participation rate. The economic participation rate increases for almost all age groups, except for individuals younger than 25. Part of the increase in the participation rate comes from the public sector workers who are staying longer on their job now. As retirement conditions are less generous, public employees postpone leaving the labor force to save for retirement.

The informality rate decreases to 13.8% of total population when compared to the current situation. Figure 20 shows that the informality rate increases for younger agents and that demographic changes are the main reason for this.

Finally, Table 9 presents sector average idiosyncratic productivity in the 3 scenarios. The 2013 Reform increases the average productivity in the Formal and Public sectors and decreases it in the Informal sector and that of non-working agents, relative to the 2060 economy with no reforms. Average productivity in the public sector increases by 10%, generating a new equilibrium with more

[40] 0.56% of the population.

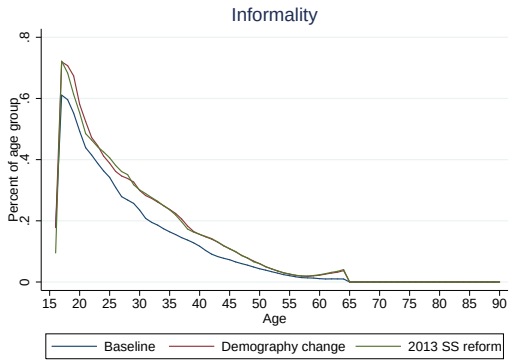

Figure 20: Informality Rate.

public goods produced. The social security cap trims out low-productivity agents that would enter the public sector exclusively because of high replacement ratios. Some, or most, of these workers join informality or unemployment.

Table 9: Average Idiosyncratic Productivity Across Sectors

Group	Formal	Informal	Home	Public
Benchmark	0.312	-0.426	-0.196	0.082
Dem. Changes	0.324	-0.372	-0.175	0.075
SS Reform: Unification	0.328	-0.375	-0.191	0.082

7.3 Social Security Reforms II: Increasing the Minimum Retirement Age

The Brazilian government is currently discussing with the Congress a social security reform with the following main aspects: (i) the end of the contribution modality, unifying the retirement modalities of private sector workers; (ii) imposing an overall minimum retirement age of 65 (62) years old for male (female) workers and (iii) changing in the replacement ratio's calculation, where now the worker will have to contribute for at least 25 years to retire [41]. Importantly, some of these points - e.g., increasing minimum retirement age - are components of most social security reforms being discussed or implemented around the world.

We run the model modifying the social security system according to these proposals. In order to do so we first impose the overall minimum retirement age of 65 years old, then analyze the change in the replacement rates. The key long-run impact of those reforms is to decrease the social security deficit in the future from 4.9% of GDP to 3.5% of GDP, with respect to the estimation of the previous case. Imposing the minimum retirement age alone takes us to a social security deficit of

[41] The proposed replacement ratio is the following: 70% of your past average wage, plus 1.5% for each year contributed over 25 but below 30, plus 2% for each year contributed over 30 but below 35 and plus 2.5% for each year contributed over 35 years of contribution. The replacement rate can never exceed 1.

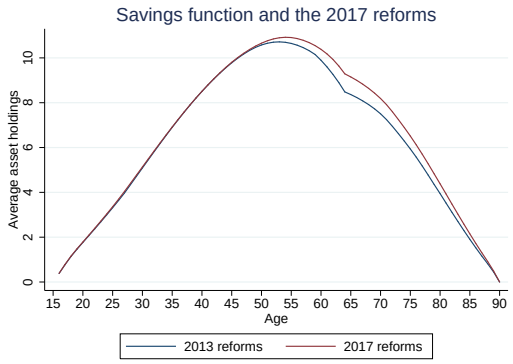

Figure 21: Average Life-cycle Savings.

4.5%. This accounts for 30% of the reduction in the deficit. In other words, the simulation using the parameters of the 2013 reform estimates that the social security deficit will decrease, in the long run, from 8.3% to 4.9% of GDP. By simply imposing the lower bound on the retirement age we observe that the deficit reduces further to 4.5%, and the replacement rate changes would push it down further to 3.5% of GDP.

As an equilibrium consequence, the consumption tax rate would further decrease to 16.8%. The minimum age alone would bring the consumption tax rate to 18%, explaining half of the reduction. Aggregate capital stock would increase to 7.05 as well as the capital-to-output ratio, which would go up to 2.7. Imposing the minimum retirement age would solely push the aggregate capital stock to 6.9. These responses are natural, as individuals use retirement as a mechanism to insure against income fluctuation and the system is now less generous, leading consequently to higher savings. Both the informal sector and the unemployment rate would drop in the long run to 14% and 38.2%, respectively, relative to the case with only the 2013 reforms (and 2060 demographic structure).

Figure 21 plots the average life-cycle savings and compares it to the case in which no additional reforms are implemented and only the 2013 reforms were in place. IT shows that the increase in savings will occur mainly for older-aged individuals. This is so because now individuals must wait until they turn 65 to retire from the labor force, with even lower replacement rates as before - now they must work for 25 years to retire, relative to the 15 years of contribution required before. However, at least in our model economy calibrated to Brazil, increasing the minimum retirement age and imposing close to universal systiem reduce but do not elimiate the large deficit expected in the future.

8 Welfare Analysis

In this section, we evaluate the long-run welfare effects of the social security reforms discussed above. The change in welfare, Δ_w, will be the proportional consumption variation of all agents in the economy that would equalize the average utility between the steady state equilibria. We will be comparing welfare changes between equilibria that have implemented different social security reforms relative to an equilibrium where the Brazilian demography has changed but the government did not adjust the social security rules. To ease notation, let variables with primes denote the new steady state, and variables without primes denote the old, baseline steady state. The change in welfare is given by:

$$\Delta_w \in \mathbb{R}: \quad \sum_{\tilde{s}} \mu(\tilde{s}) u(d_{c,t}(s), d_{l,t}(s), Y_G) = \sum_{\tilde{s}} \mu'(\tilde{s}) u((1-\Delta_w) d'_{c,t}(s), d'_{l,t}(s), Y_G)$$

Table 10 reports our welfare analysis under the above definition. A positive value for Δ_w means that agents are better in the new environment, with the given social security reform, relative to the benchmark case. That is, to achieve the benchmark's average utility, one must decrease consumption in the counterfactual environments. For the overall economy, the 2013 (e.g., unification of the pension systems) and future (i.e., minimum retirement age) reforms have similar impact on welfare, increasing it relative to a scenario with no reforms whatsoever. The reduction of the social security deficit allows the government to decrease taxes to balance its accounts, therefore individuals end up better off. Even though public agents must stay longer working for the government and public retirees have worse benefits, the gains generated by the deficit reduction compensates such losses for the whole economy. With the unification reforms, our model projects that, relative to the long run where no reform was implemented, agents are around 4.2% better off in terms of consumption equivalence. If in addition to this, we add minimum retirement age, agents would be 5.5% better off in terms of consumption equivalence, yielding an additional 1 percentage point of welfare gains.

Table 10: Welfare Gains (Δ_w)

Group of analysis	2013 reforms	Future reforms
All individ.	4.2%	5.5%
Private sector	4.7%	7.7%
Government	9.4%	12.3%
Retirees	-7.1%	2.3%

One can break down the analysis by different groups to find who are the winners and losers - that is, how much consumption should be given or taken to all agents within a sector in order to let them,

on average, indifferent between the two long-run equilibria[42].Table 10 shows that private agents will be 4.7% better off in the long run with the 2013 reforms and 7.7% with the 2017 reforms, as wages will be higher and consumption tax (significantly) lower. Public agents also benefit from both reforms. The changes in social security make higher-z agents move to the public sector, relative to the old steady state. Since z is highly influential to their labor income, they are much better off in the new steady state. Retirees are the ones who lose with the social security reform. The 2013 reforms induce losses of 7% relative to a steady state with the demographic change only. Interestingly, the 2017 reforms have a general equilibrium effect in the long run that reverses the welfare gains of the retirees in the economy, highlighting the large effects of a reduction in the consumption tax rate, induced by a smaller social security deficit.

9 Conclusion

Population ageing is putting a very strong strain on the solvency of social security systems around the world. The problem only tends to worsen, as this movement will accentuate in the future, so that the ratio between retired population and active workers will fall steadily. Add to it excessively generous retirement conditions, particularly in what concerns the public sector, and the end result is financially unsustainable pension systems that will tend to absorb a growing share of tax revenues.

Brazil is no different, if anything is an acute case of fiscal irresponsibility and can be used as an experiment for countries in similar situation. We modelled in detail the Brazilian economy and its social security system - not that far from many systems in the world - and simulated it plugging the demographic conditions projected to 2060. We found that, if nothing is changed, population ageing alone will increase social security deficit from around 2% today to more the 8% in less than 45 year from now. Given the necessary increase in taxation, the impact on the economy will be huge, as measure for instance by consumption reduction. Hence, the first lesson is that inertia and inaction will have a very high cost in the future.

We then introduce in the economy different reforms implemented in the recent past or currently in consideration. These reforms are not far from those discussed in many other countries. We find that the unification of social security systems, in a world were public employee face very generous retirement conditions, is able to decrease considerably - by half - social security deficit. Public worker stay longer in the job, increasing their saving, to compensate for the reduction in benefits. Moreover, the reforms will have in the long run positive effects on aggregate savings and output, at the same time that it changes considerably the decision to join the public sector. Now people apply earlier to a public job and, on average, they are more qualified.

[42]For each group, the measures used were rescaled in a way that the sum of all respective agents' weights equals one.

When the minimum retirement age is increased and higher (and longer) contributions are implemented, an additional 2 percentage points of GDP is subtracted from the deficit. Early retirement, hence, has a very high cost: a large fraction of workers retire today before they reach 60 years of age and the reform force they to work until 65 years of age. At the same time, simulations found that savings, capital and output increase in the long run. The welfare, on average, also increase. Hence, society as a whole has a lot to gain with sensible social security modifications.

Of course, given the political resistance of public workers against pension reforms - or of the whole population against any reform as it seems to be the case in France, for instance - the implementation of some of these changes will be very difficult if not impossible in the short run. That is so because many will lose during the transition to the new regime. However, the figures estimated in the present study show that the gains in the future are large and worth facing the opposition of interest groups.

References

Almeida, R. and Carneiro, P. (2012). Enforcement of labor regulation and informality. *American Economic Journal: Applied Economics*, 4(3):64–89.

Bacchetta, M., Ernst, E., Bustamante, J. P., et al. (2009). *Globalization and Informal Jobs in Developing Countries*. International Labour Organization Geneva.

Belluzzo, W., Anuatti-Neto, F., and Pazello, E. T. (2005). Distribuição de salários e o diferencial público-privado no Brasil. *Revista Brasileira de Economia*, 59:511 – 533.

Beshears, J., Choi, J. J., Laibson, D., and Madrian, B. C. (2011). Behavioral economics perspectives on public sector pension plans. Working Paper 16728, National Bureau of Economic Research.

Blöndal, S. and Scarpetta, S. (1999). The retirement decision in oecd countries. OECD Economics Department Working Papers 202, OECD Publishing.

Chen, K. (2010). A life-cycle analysis of social security with housing. *Review of Economic Dynamics*, 13(3):597 – 615.

Conesa, J. C. and Krueger, D. (1999). Social Security Reform with Heterogeneous Agents. *Review of Economic Dynamics*, 2(4):757–795.

de Oliveira, D. H. (2016). Balanço da seguridade social. *Ministério do Planejamento, Desenvolvimento e Gestão*.

de Paula, u. and Scheinkman, J. A. (2011). The informal sector: An equilibrium model and some empirical evidence from brazil. *Review of Income and Wealth*, 57:S8–S26.

Depalo, D., Giordano, R., and Papapetrou, E. (2013). Public-private wage differentials in euro area countries: evidence from quantile decomposition analysis. Temi di discussione (Economic working papers) 907, Bank of Italy, Economic Research and International Relations Area.

Dickson, M., Postel-Vinay, F., and Turon, H. (2014). The lifetime earnings premium in the public sector: The view from europe. *Labour Economics*, 31:141 – 161.

Dos Reis, A. C. and Zilberman, E. (2014). The insurance rrle of public employment. *Working Paper*.

dos Santos, M. and Cavalcanti, T. (2015). (Mis)Allocation Effects of an Overpaid Public Sector. 2015 Meeting Papers 1094, Society for Economic Dynamics.

dos Santos, M. and Pereira, T. N. (2010). Moving to a consumption-cased tax system: a quantitative assessment for brazil. *Revista Brasileira de Economia*, 64.

Ferreira, P. C. and do Nascimento, L. G. (2005). Welfare and growth effects of alternative fiscal rules for infrastructurei investment in brazil. *Manuscript*.

Ferreira, P. C. and dos Santos, M. R. (2013). The effect of social security, health, demography and technology on retirement. *Review of Economic Dynamics*, 16(2):350 – 370.

Fiorito, R. and Kollintzas, T. (2004). Public goods, merit goods, and the relation between private and government consumption. *European Economic Review*, 48(6):1367–1398.

Flodén, M. and Lindé, J. (2001). Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? *Review of Economic Dynamics*, 4(2):406–437.

French, E. (2005). The effects of health, wealth, and wages on labour supply and retirement behaviour. *The Review of Economic Studies*, 72(2):pp. 395–427.

Glinskaya, E. and Lokshin, M. (2007). Wage differentials between the public and private sectors in india. *Journal of International Development*, 19(3):333–355.

Glomm, G., Jung, J., and Tran, C. (2009). Macroeconomic implications of early retirement in the public sector: The case of brazil. *Journal of Economic Dynamics and Control*, 33(4):777 – 797.

Gruber, J. and Wise, D. A. (2002). Social security programs and retirement around the world: Micro estimation. Working Paper 9407, National Bureau of Economic Research.

Gustman, A. L. and Steinmeier, T. L. (2005). The social security early entitlement age in a structural model of retirement and wealth. *Journal of Public Economics*, 89(2):441 – 463.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica*, 47(1):153–161.

Holzmann, R. and Hinz, R. (2005). *Old-Age Income Support in the 21st Century*. The World Bank.

Huggett, M. (1996). Wealth distribution in life-cycle economies. *Journal of Monetary Economics*, 38(3):469–494.

Huggett, M. and Ventura, G. (1999). On the Distributional Effects of Social Security Reform. *Review of Economic Dynamics*, 2(3):498–531.

Immervoll, H., Levy, H., Nogueira, J. R., O'Donoghue, C., and Siqueira, R. B. d. (2006). The Impact of Brazil's Tax-Benefit System on Inequality and Poverty. IZA Discussion Papers 2114, Institute for the Study of Labor (IZA).

Imrohoroglu, S. and Kitao, S. (2012). Social security reforms: Benefit claiming, labor force participation, and long-run sustainability. *American Economic Journal: Macroeconomics*, 4(3):96–127.

Issler, J. V. and Piqueira, N. S. (2000). Estimating relative risk aversion, the discount rate, and the intertemporal elasticity of substitution in consumption for Brazil using three types of utility functions. *Brazilian Review of Econometrics*, 20:201–239.

Jung, J. and Tran, C. (2012). The extension of social security coverage in developing countries. *Journal of Development Economics*, 99(2):439 – 458.

Kings, J., Turkisch, E., and Manning, N. (2007). Public sector pensions and the challenge of an ageing public service. *OECD Working Papers on Public Governance*.

Liu, Z., Huang, F., Zhu, D., and He, H. (2014). Breaking the "Iron Rice Bowl" and Precautionary Savings: Evidence from Chinese State-Owned Enterprises Reform. 2014 Meeting Papers 764, Society for Economic Dynamics.

Meghir, C., Narita, R., and Robin, J.-M. (2015). Wages and informality in developing countries. *American Economic Review*, 105(4):1509–46.

Morandi, L. (2016). Novas estimativas do estoque de capital fixo brasileiro - 1950:2014. *Séries Históricas - FGV Instituto Brasileiro de Economia*.

Morikawa, M. (2016). A comparison of the wage structure between the public and private sectors in japan. *Journal of the Japanese and International Economies*, 39:73 – 90.

Munnell, A. H. and Fraenkel, R. C. (2013). Public sector workers and job security. *State and Local Pension Plans*, (31).

Ni, S. (1995). An empirical analysis on the substitutability between private consumption and government purchases. *Journal of Monetary Economics*, 36(3):593 – 605.

OECD (2015a). Government at a glance. *OECD Publishing*.

OECD (2015b). Pensions at a glance. *OECD Publishing*.

Pereira, E. d. S. (2013). Evolução das idades médias de concessão e dos tempos médios de contribuição das aposentadorias por tempo de contribuição concedidas entre 1996 e 2012. *Informe da Previdência Social*, 26/6.

Pereira, R. A. d. C. and Ferreira, P. C. (2010). Avaliacao dos impactos macro-economicos e de bem-estar da reforma tributaria no brasil. *Revista Brasileira de Economia*, 64:191 – 208.

Tafner, P., Botelho, C., and Erbisti, R. (2015). Reforma da previdência: a visita da velha senhora. *Editora Gestão Pública*.

Tansel, A. (2004). Public-Private Employment Choice, Wage Differentials and Gender in Turkey. IZA Discussion Papers 1262, Institute for the Study of Labor (IZA).

Tauchen, G. (1986). Finite state markov chain approximations to univariate vector autoregressions. *Economic Letters*, 20(2):177–181.

Ulyssea, G. (2017). Firms, informality and development: Theory and evidence from brazil. *Working Paper*.

United Nations, Department of Economic and Social Affairs, Population Division (2017). Probabilistic population projections based on the world population prospects: The 2017 revision. *Population Division, DESA*.

Vestad, O. L. (2013). Labour supply effects of early retirement provision. *Labour Economics*, 25:98 – 109. European Association of Labour Economists 24th Annual Conference, Bonn, Germany, 20-22 September 2012.

Appendix A Computing the Stationary Competitive Equilibrium

In this Appendix, we detail the computational methods used to quantitatively assess the macroeconomic consequences of social security reforms.

We numerically solved the model in Fortran 90. In order to do so, we discretized the asset space, the average past earnings space, the income process space and the social security benefits space. We did so in 200, 10, 10 and 20 points, respectively.

The grid on capital a is from 0 to 50, with its points concentrated over the lower bound[43]. The grid for x is equally spaced between 0 and $y_{\max}$. The grid for b is concentrated over 0 and limited by a $\bar{b}$, sufficiently high. The optimization procedure used was a simple grid-search method, exploring local monotonicity of the value functions. We do so because of several discrete-choices of the agents throughout the life cycle, making the value function problem highly non-concave, with potential multiple local maximums. We based our solution procedure on the following Proposition:

Proposition 1. *Conditional on an labor/occupational choice, the asset accumulation decision is increasing on small changes of today's assets.*

Proof. We will proceed by induction. The individuals' utility is[44]:

$$u(c_t, l_t) = \frac{[c_t^{\gamma}(1+l_t)^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

Individual states are $s_R = (a, z, b)$. In the last period, we have $a'_T(s_R) = 0$ for all s_R, once $V_{T+1}(s_R) = 0$. Therefore, the value function is:

$$V_T(s_R) = \max\{V_T^H(s_R), V_T^F(s_R), V_T^I(s_R)\}$$

The value of staying at home is given by:

$$V_t^H(s_R) = u(c_T, 1), \quad c_T = \frac{[1+(1-\tau_k)r]a + (1-\tau_b(R))b + \zeta}{1+\tau_c}$$

Working for the formal private sector yields the value:

$$V_t^F(s_R) = u(c_T, 0), \quad c_T = \frac{[1+(1-\tau_k)r]a + (1-\tau_y(P))y_t(w_f, P) + (1-\tau_b(R))b + \zeta}{1+\tau_c}$$

[43] We chose the upper bound on assets such that no agent (or rather in no state) with positive mass chooses optimally to hold the maximum amount of assets. That is, the upper bound restriction does not bind in equilibrium.

[44] For the sake of brevity and clarity, we will abstract from the public good Y_G in the proof, since it does not alter any of the results below.

And the value of working in the informal sector is:

$$V_t^I(s_R) = u(c_T, 0), \quad c_T = \frac{[1 + (1 - \tau_k)r]a + y_t(w_i, P) + (1 - \tau_b(R))b + \zeta}{1 + \tau_c}$$

The maximum is well defined in this case, since it is a discrete choice. Let $d_T^l(s_R)$ denote the optimal labor choice of the agent. Define $\tilde{y}_T(s_R)$ as the total income excluding the return on the assets. The optimal consumption at T can be written as:

$$c_T(s_R) = \frac{[1 + (1 - \tau_k)r]a_T + \tilde{y}_T(s_R)}{1 + \tau_c} \tag{6}$$

Now, let us show that the optimal asset accumulation given each value V^H, V^F and V^I in $T-1$ is monotone in a_{T-1}. For brevity, we will do so only for V^H. The value of an retiree that considers staying at home in $T-1$ is:

$$V_{T-1}^H(s_R) = \max_{(c,a')\geq 0} u(c, 1) + \beta\psi_T \cdot \mathbb{E}\left[u(c_T(s'_R), d_T^l(s'_R))\right]$$

$$\text{s.t.} \quad (1 + \tau_c)c + a' = [1 + (1 - \tau_k)r]a + \tilde{y}$$

The first order condition on an interior choice of a' is:

$$\frac{u_c(c, 1)}{1 + \tau_c} = \beta\psi_T \mathbb{E}\left[u_c(c_T(s'_R), d_T^l(s'_R))\frac{dc_T(s'_R)}{da'}\right]$$

But from (6), $\frac{dc_T(s'_R)}{da'} = \frac{1+(1-\tau_k)r}{1+\tau_c}$, so we have:

$$u_c\left(\frac{[1 + (1 - \tau_k)r]a_{T-1} + \tilde{y} - a'}{1 + \tau_c}, 1\right) = \beta\psi_T \mathbb{E}\left[[1 + (1 - \tau_k)r]u_c\left(\frac{[1 + (1 - \tau_k)r]a' + \tilde{y}_T(s'_R)}{1 + \tau_c}, d_T^l(s'_R)\right)\right]$$

Differentiating completely with respect to a_{T-1}, and assuming that changes in assets today are such that the intratemporal labor choice tomorrow stays unchanged, we have:

$$u_{cc}\left[1 + (1 - \tau_k)r - \frac{da'}{da}\right] = \beta\psi_T \mathbb{E}\left[[1 + (1 - \tau_k)r]^2 u'_{cc}\frac{da'}{da}\right]$$

Rearranging, one gets:

$$\frac{da'_T}{da_{T-1}} = \frac{u_{cc}[1 + (1 - \tau_k)r]}{u_{cc} + \beta\psi_T[1 + (1 - \tau_k)r]^2 \mathbb{E}\left[u'_{cc}\right]} \in (0, 1 + (1 - \tau_k)r]$$

Because $u_{cc} < 0$. Notice that this inequality holds for all situations, since the intertemporal tradeoff is not affected by intratemporal labor choices. Now, suppose that both inequalities hold for $t+1 \in \{2, ..., T-1\}$. That is, assume that Again, let us consider at the situation in which the agent considers staying at home:

$$V_t^H(s_R) = \max_{(c,a')\geq 0} u(c,1) + \beta\psi_{t+1} \cdot \mathbb{E}\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-\tau_b(R))b + \zeta$$

First order condition on an interior solution for a' is:

$$\frac{u_c(c,1)}{1+\tau_c} = \beta\psi_{t+1}\mathbb{E}\left[\frac{\partial V_{t+1}(s'_R)}{\partial a'}\right] \tag{7}$$

The envelope condition for the problem[45] is:

$$\frac{\partial V_t(s_R)}{\partial a} = \frac{1+(1-\tau_k)r}{1+\tau_c} u_c(c, d_t^l(s_R)), \quad c = \frac{[1+(1-\tau_k)r]a_t + \tilde{y}_t(s_R) - a_{t+1}(s_R)}{1+\tau_c}$$

Substituting into (7) yields:

$$u_c\left(\frac{[1+(1-\tau_k)r]a + \tilde{y} - a'}{1+\tau_c}, 1\right) = \beta\psi_{t+1}\mathbb{E}\left[[1+(1-\tau_k)r]u_c\left(\frac{[1+(1-\tau_k)r]a' + \tilde{y}' - a''}{1+\tau_c}, d_t^l(s_R)\right)\right]$$

Differentiating totally with respect to a small change in a, we get:

$$u_{cc}\left[1+(1-\tau_k)r - \frac{da'}{da}\right] = \beta\psi_T\mathbb{E}\left[[1+(1-\tau_k)r]u'_{cc}\left[1+(1-\tau_k)r - \frac{a''}{da'}\right]\right]\frac{a'}{da}$$

Rearranging, one gets:

$$\frac{da'_T}{da_{T-1}} = \frac{u_{cc}[1+(1-\tau_k)r]}{u_{cc} + \beta\psi_T[1+(1-\tau_k)r]\mathbb{E}\left[u'_{cc}\left[1+(1-\tau_k)r - \frac{a''}{da'}\right]\right]}$$

Which is in the interval $(0, 1+(1-\tau_k)r]$ due to our inductive hypothesis and the fact that $u(c,l)$ is strictly concave in consumption. □

We moreover make use of the following observation: All retirees, conditional upon working, will chose the sector with the highest net wage rate. That is, they will compare the wage rate in the

[45] For the complete description problem, see Section 3.5 in the text.

formal sector net of taxes and the wage rate in the informal sector, and will work on whichever sector has the highest rate.

The algorithm to find the general equilibrium used was an adaptation from the algorithm that is commonly used in the literature[46], including a fixed point over $\bar{q}$ to match $\bar{N}_G$.

The steps used to compute the stationary equilibrium are:

1. Guess initial values for $\Theta \equiv (r, w_i, Y_G, \zeta, \tau_c)$;
2. Take two boundaries for $\bar{q} \in [q_l, q_h]$;
2. Given such values, use firms FOCs to obtain w;
3. Solve the agents' problems backwards and find the respective policy functions;
4. Use the policy functions to compute the associated stationary distribution of households, by forward induction;
5. Aggregate the individual decisions and use the bissection method to find $\bar{q}$ such that $\bar{N}_G$ of the population is working as public servants;
6. Use individual decisions to calculate the implicit $\tilde{\Theta} \equiv (\tilde{r}, \tilde{w}_i, \tilde{Y}_G, \tilde{\zeta}, \tilde{\tau}_c)$;
7. Check whether $||\tilde{\Theta} - \Theta|| < \epsilon$. If not, update Θ, return to item 2 and iterate until convergence.

Appendix B Defining the Stationary Competitive Equilibrium

In this Appendix, we define the recursive, stationary equilibrium of the model.

A *recursive competitive equilibrium* consists of value functions $V : \{1, 2, ..., T\} \times S_m \to \mathbb{R}$ for all $m \in \{P, G, R\}$, policy functions for every age $t \in \{1, 2, ..., T\}$: (i) $d_t^l : S_m \to \{0, 1\}$, for the optimal decision of leisure, asset holdings $d_t^a : S_m \to \mathbb{R}_+$ and consumption $d_t^c : S_m \to \mathbb{R}_{++}$ for all $m \in \{P, G, R\}$; (ii) $d_t^{ss} : S_m \to \{0, 1\}$ retirement decisions for $m \in \{P, G\}$; (iii) $d_t^{inf} : S_m \to \{0, 1\}$, $m \in \{P, R\}$ for the optimal decision of working for the informal sector; (ii) $d_t^{ap} : S_P \to \{0, 1\}$ for the optimal decision of application to the public sector; competitive prices $\{r, w_f, w_i\}$, age dependent but time invariant measures of agents $\mu_m(t, s_m)$, government transfers ζ, taxes, an amount of public good, Y_G, and a threshold score $\bar{q}$ such that:

(1) $\{V_t, d_t^l(s), d_t^{ap}(s), d_t^{inf}(s), d_t^{ss}(s), d_t^a(s), d_t^c(s)\}_{t=1, s \in S}^T$ solve the problem in Section 3.5;

(2) Formal private firms solve problem (2) given $\{r, w_f\}$ and informal firms solve (3) given w_i and government monitoring costs; private firms;

[46] See Chen (2010), for an example.

(3) The individual and aggregate behavior are consistent:

$$K' = \sum_{m,\, \tilde{s}_m=(t,s_m)} \mu_m(\tilde{s}_m) d_{a,t}(s_m)$$

$$K = \frac{K'}{1+g_n}$$

$$N_f = \sum_{\tilde{s}_P=(t,s_P)} \mu_P(\tilde{s}_P) \mathbb{I}_{\{d_t^l(s_P)=0\}} \mathbb{I}_{\{d_t^{inf}(s_P)=0\}} e^{z_t+\eta_t} + \sum_{\tilde{s}_R=(t,s_R)} \mu_R(\tilde{s}_R) \mathbb{I}_{\{d_t^l(s_R)=0\}} \mathbb{I}_{\{d_t^{inf}(s_R)=0\}} e^{z_t+\eta_t}$$

$$N_i = \sum_{\tilde{s}_P=(t,s_P)} \mu_P(\tilde{s}_P) \mathbb{I}_{\{d_t^l(s_P)=0\}} \mathbb{I}_{\{d_t^{inf}(s_P)=1\}} e^{z_t+\eta_t} + \sum_{\tilde{s}_R=(t,s_R)} \mu_R(\tilde{s}_R) \mathbb{I}_{\{d_t^l(s_R)=0\}} \mathbb{I}_{\{d_t^{inf}(s_R)=1\}} e^{z_t+\eta_t}$$

(4) The government chooses $\bar{q}$ in order to balance people coming in and out:

$$\bar{N}_G = \sum_{\tilde{s}_G} \mu_G(\tilde{s}_G)$$

(5) Public goods' consistency:

$$Y_G = \sum_{\tilde{s}_G} \mu_G(\tilde{s}_G) e^{z_G+\eta_t}$$

(6) Final good market clears:

$$\sum_{m,\, \tilde{s}_m} \mu_m(\tilde{s}_m) d_{c,t}(s_m) + K' + C_g + \Pi_i = Y_f + Y_i + (1-\delta)K$$

(7) τ_C balances the government budget constraint:

$$\tau_C C + \tau_K r K + (\tau_y(P) + \tau_{ss}(P)) w N_{P,f} + \tau_y(P) w N_{R,f} + \tau_i(N_i) =$$
$$\sum_{\tilde{s}_G} (1 - \tau_y(G) - \tau_{ss}(G)) \mu_G(\tilde{s}_G) min\{(1+\theta) w e^{z_G+\eta_t}, \bar{y}_G\} + \sum_{\tilde{s}_R} (1 - \tau_b(R)) \mu_R(\tilde{s}_R) b(\tilde{s}_R) + C_g$$

(8) Bequests are rebated to the living ones:

$$\zeta = \frac{1+r}{1+g_n} \sum_{m,\, \tilde{s}_m=(t,s_m)} (1 - \psi_{t+1}) \mu_m(\tilde{s}_m) d_{a,t}(s_m)$$

Appendix C Private Agents' Value Function

- The value function of a Private agent is extensively given by[47]:

$$V_t(s_P) = \max\{V_t^{R,F}, V_t^{R,I}, V_t^{R,H}, V_t^{NR,T,F}, V_t^{NR,T,I}, V_t^{NR,T,H}, V_t^{NR,NT,F}, V_t^{NR,NT,I}, V_t^{NR,NT,H}\}$$

- $V_t^{R,F}$ is the value function of an individual who asks for retirement and works for the formal private firm:

$$V_t^{R,F}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1}\mathbb{E}_b\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-\tau_y(P))y_t(w_f, P) - \tau_{ss}(P)\min\{y_t(w_f, P), y_{\max}\} + \zeta$$

$$(x', t'_C) = \left(\frac{x(t-1) + min\{y_t(w_f, P), y_{\max}\}}{t}, t_C + 1\right)$$

Where the state tomorrow is $s'_R = (a', z', b' = b(t+1, x', Ret.Modality, t'_C, y_t(w_f, P)))$

- $V_t^{R,I}$ is the value function of an individual who asks for retirement and works for the informal private firm:

$$V_t^{R,I}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1}\mathbb{E}_b\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + y_t(w_i, P) + \zeta$$

$$(x', t'_C) = (x, t_C)$$

Being s'_R the same as before

- The individual who is considering to stay at home and ask for retirement has the following value function:

$$V_t^{R,H}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c}, 1) + \beta\psi_{t+1}\mathbb{E}_b\left[V_{t+1}(s'_R)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + \zeta$$

$$(x', t'_C) = (x, t_C)$$

[47] Note that we are using the fact that it is never optimal for an agent to take the public exam once she asks for retirement.

- The individuals who did not consider asking for retirement can think about applying to the public sector job

- The ones who consider to do so and still work for the private formal firms have the following value function:

$$V_t^{NR,T,F}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c},0) + \beta\psi_{t+1}\Big\{ P_r(q'\geq\bar{q})\cdot V_{t+1}(s'_G) + (1-P_r(q'\geq\bar{q}))\mathbb{E}\left[V_{t+1}(s'_P)\right] \Big\}$$

$$\begin{aligned} \text{s.t.} \quad (1+\tau_c)c + a' =& [1+(1-\tau_k)r]a + (1-\tau_y(P))(1-c_{ap}(t))y_t(w_f,P) \\ & - \tau_{ss}(P)\min\{(1-c_{ap}(t))y_t(w_f,P), y_{\max}\} + \zeta \end{aligned}$$

$$(x', t'_C) = \left(\frac{x(t-1) + min\{(1-c_{ap}(t))y_t(w_f,P), y_{\max}\}}{t}, t_C + 1 \right)$$

Where $s'_P = (a', z', x', t'_C)$ and $s'_G = (a', z, t'_C, 1)$

- The value function of an individual who is considering to take the test, and is also working as an informal worker is:

$$V_t^{NR,T,I}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c},0) + \beta\psi_{t+1}\Big\{ P_r(q'\geq\bar{q})\cdot V_{t+1}(s'_G) + (1-P_r(q'\geq\bar{q}))\mathbb{E}\left[V_{t+1}(s'_P)\right] \Big\}$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-c_{ap}(t))y_t(w_i,P) + \zeta$$

$$(x', t'_C) = (x, t_C)$$

- When an private agent considers taking the public exam and stay at home, she gets the following value function:

$$V_t^{NR,T,H}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c},1) + \beta\psi_{t+1}\Big\{ P_r(q'\geq\bar{q})\cdot V_{t+1}(s'_G) + (1-P_r(q'\geq\bar{q}))\mathbb{E}\left[V_{t+1}(s'_P)\right] \Big\}$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + \zeta$$

$$(x', t'_C) = (x, t_C)$$

- Once the private worker considers not asking for retirement and not taking the public exam, she can still opt between working for the formal firm, which gives her the value function of:

$$V_t^{NR,NT,F}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c},0) + \beta\psi_{t+1}\mathbb{E}\left[V_{t+1}(s'_P)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1+(1-\tau_k)r]a + (1-\tau_y(P))y_t(w_f,P) - \tau_{ss}(P)\min\{y_t(w_f,P), y_{\max}\} + \zeta$$

$$(x', t'_C) = \left(\frac{x(t-1) + min\{y_t(w_f, P), y_{\max}\}}{t}, t_C + 1 \right)$$

- Working for the informal firm and have the value function of:

$$V_t^{NR,NT,I}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c}, 0) + \beta\psi_{t+1}\mathbb{E}\left[V_{t+1}(s'_P)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + y_t(w_i, P) + \zeta$$

$$(x', t'_C) = (x, t_C)$$

- Staying at home with the value function of:

$$V_t^{NR,NT,H}(s_P) = \max_{(c,a')\geq 0} u(\tilde{c}, 1) + \beta\psi_{t+1}\mathbb{E}\left[V_{t+1}(s'_P)\right]$$

$$\text{s.t.} \quad (1+\tau_c)c + a' = [1 + (1-\tau_k)r]a + \zeta$$

$$(x', t'_C) = (x, t_C)$$

Appendix D Agents' Stationary Distribution

In this section, we formally derive the stationary distribution for the agents in the economy. We will only derive the distribution for agents older than 16[48].

Given the initial distribution above, the measure of each private agent in the economy (t, a', z', x', t'_C) can be written as the sum of four terms. The first one considers the mass of all agents that were in the private sector and have not applied to the public sector job, nor applied for retirement benefits. The second term takes into account the private workers that applied to the public sector and failed to get in. Both the first and second terms take into account the measure $\mu_P(t-1, a, z, x, t_C)$.

Within each term, we consider the transition probability of the idiosyncratic productivity, $\Pi(z, z')$, the optimal amount saved by the agents, $d^a_{t-1}(s)$, the updated average past earnings and updated time of contribution, conditional on optimal labor and informality choices, $\tilde{x}'$ and $\tilde{t}'$, retirement decisions $d^{ss}_{t-1}(s)$, the public sector application decision, $d^{ap}_{t-1}(s)$ and the probability of succeeding in the public sector exam, $P_r(\mathcal{U} \geq \bar{q})$.

Formally, the distribution for the formal private sector workers across ages $t = \{2, ..., T\}$ is recur-

[48] That is, for $t > 1$.

sively given by:

$$\begin{aligned}\mu_P(t,a',z',x',t'_C) =& \frac{\psi_t}{1+g_n}\cdot\Bigg\{\sum_{s_P=(a,z,x,t_C)}\Pi(z,z')\mathbb{I}_{\{d^a_{t-1}(s_P)=a'\}}\mathbb{I}_{\{\tilde{x}'=x'\}}\mathbb{I}_{\{\tilde{t}'_C=t'_C\}}(1-d^{ap}_{t-1}(s_P))*\\ &(1-d^{ss}_{t-1}(s))\cdot\mu_P(t-1,s_P)+\\ &\sum_{s_P=(a,z,x,t_C)}\Pi(z,z')\mathbb{I}_{\{d^a_{t-1}(s_P)=a'\}}\mathbb{I}_{\{\tilde{x}'=x'\}}\mathbb{I}_{\{\tilde{t}'_C=t'_C\}}d^{ap}_{t-1}(s_P)*\\ &(1-P_r(\mathcal{U}\geq\bar{q}))\cdot\mu_P(t-1,s_P)\Bigg\}\end{aligned}$$

Where the updated average past earnings and length of contribution to the social security system can be written as a function of the optimal leisure decision $d^l_{t-1}(s_P)$, informality decision $d^{inf}_{t-1}(s_P)$ and application to the public sector decision, $d^{ap}_{t-1}(s_P)$:

$$(\tilde{x}',\tilde{t}'_C)=\begin{cases}\left(\frac{x(t-2)+min\{(1-c_{ap}(t-1)d^{ap}_{t-1}(s_P))we(z,t-1),y_{\max}\}}{t-1}\,,\,t_C+1\right) & \text{if}\quad d^l_{t-1}(s_P)+d^{inf}_{t-1}(s_P)=0\\ (x,t_C) & \text{otherwise}\end{cases}$$

The distribution of the public servants has two components. The first one takes into account the private workers who took the test and succeeded. The second considers the decision of public workers who did not ask for retirement.

$$\begin{aligned}\mu_G(t,a',z',t'_C,t'_G) =& \frac{\psi_t}{1+g_n}\cdot\Bigg\{\sum_{s_P=(a,z,x,t_C)}\mathbb{I}_{\{z=z'\}}\mathbb{I}_{\{d^a_{t-1}(s_P)=a'\}}\mathbb{I}_{\{\tilde{x}'=x'\}}\mathbb{I}_{\{\tilde{t}'_C=t'_C\}}*\\ &d^{ap}_{t-1}(s_P)P_r(\mathcal{U}\geq\bar{q})\cdot\mu_P(t-1,s_P)+\\ &\sum_{s_G=(a,z,t_C,t_G)}\mathbb{I}_{\{z=z'\}}\mathbb{I}_{\{d^a_{t-1}(s_G)=a'\}}\mathbb{I}_{\{t_C+1=t'_C\}}\mathbb{I}_{\{t_G+1=t'_G\}}*\\ &(1-d^{ss}_{t-1}(s_G))\cdot\mu_G(t-1,s_G)\Bigg\}\end{aligned}$$

Where $\tilde{x}'$ and $\tilde{t}'_C$ have the same definition as before. The distribution of the retirees is also composed by two parts. First, we have agents who already were retired from the labor force. Second, we

account for the ones who recently asked for retirement[49].

$$\mu_{RP}(t,a',z',b') = \frac{\psi_t}{1+g_n} \cdot \Bigg\{ \sum_{s_R=(a,z,b)} \Pi(z,z') \mathbb{I}_{\{d^a_{t-1}(s_R)=a'\}} \mathbb{I}_{\{b'=b\}} \cdot \mu_{RP}(t-1,s_R) + \sum_{s_P=(a,z,x,t_C)} \alpha_b \cdot \Pi(z,z') \mathbb{I}_{\{d^a_{t-1}(s_P)=a'\}} d^{ss}_{t-1}(s_P) \cdot \mu_P(t-1,s_P) \Bigg\}$$

Where α_b is the interpolation coefficient of the function $b(\cdot)$ on the grid $\mathcal{B}$. A similar equation applies to compute the distribution of the retirees in the public sector, therefore we will omit it, for brevity.

[49] When solving the model, we divide the retirees between private (RP) and public (RG) retirees. The stationary distributions of the retirees are similar, therefore we will only derive for RP.