The Quest for Alpha in Equity Gamma

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Abstract

This paper examines the relative value in and opportunities of gamma trades in equity options markets. It has been well documented in the literature that shorting index options straddles, with or without delta hedging, tends to produce positive expected returns, albeit at the expense of very high risks. Buying all the single name straddles against the short index straddle, a popular strategy known as "dispersion trade", has been shown to significantly improve the risk return profile in other applied work. This strategy is well known to be related to the fact that implied correlation is often priced excessively high. Most econometric work highlighting these options strategies were conducted in the late 90´s and early 2000´s, before the 2008 financial crises and 20220 Covid crisis, and since then their attractiveness have been substantially mitigated. Dispersion trades usually assign market index weight to each single name straddle, without considering the relative value opportunities among them, ignoring the fact that some single name implied volatilities might be overpriced, and others underpriced. We suggest a measure for the relative price of gamma trades and hedge ratios to optimally allocate among single name straddles. We use this relative pricing to build a dynamic long/short gamma strategy which favors the "cheaper" straddles and avoid the "expensive" ones at any point in time. This significantly improves the risk return profile when compared to dispersion trades in the S&P500 market. Finally, we conduct a broad screening of all the worldwide equities indices and subindices where historical implied volatility is available to indicate where opportunities for our dynamic long/short gamma strategy appear very promising.

Key words: Straddles; Delta Hedged; Dynamic Gamma Trading; Alpha Harvesting, Risk Return

JEL Classification: G11, G12, G13, G17

Introduction

Since the seminal works by Black and Scholes (1973) and Merton (1973), investment strategies focused on volatility and gamma have received a lot of scrutiny from both scholars and practitioners. A common instrument used in these strategies is called straddles, which is a combination of a Call and a Put option over the same underlying, with the same maturity and same strike price, usually set to be at-the-money at the onset of the trade.

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Buying (or selling) straddles tends to be market neutral, not exposed to directional risks of the underlying asset, particularly when the exposure is delta hedged until the expiry of the options. The purchase of straddles benefits from sudden increase in volatility and from a large change in the underlying price; but suffers from the time decay of the options.

In Artur Sepp (2013), the expected profit-and-loss (P&L) of a delta hedged long straddle, from the onset of the trade until maturity, is shown to be approximately proportional to the difference between the squares of the realized and implied volatilities, σ_R and σ_I. The constant of proportionality is related to the gamma of the options.

$$\mathbb{E}[P\&L] \cong \Gamma[(\sigma_R^2 - \sigma_I^2)]T$$

Where $\Gamma(\sigma) = \frac{1}{2}\frac{\partial^2 Straddle_{Price}}{\partial S^2}S^2 = \frac{K}{2\sqrt{2T}\pi\sigma^2}exp\left\{-\frac{1}{8}\sigma^2 T\right\}$, S being the underlying price.

This result holds under discrete hedging and is independent of the hedging frequency.

Hence, if options are delta-hedged to maturity, the holder (seller) of a straddle exposure is expected to make a profit when the implied volatility is smaller (higher) than the subsequent realized volatility. A similar result easily extends and will be key to our dynamic long/short gamma strategy below.

The authors also develop an analytical approximation for the variance of the delta-hedged P&L, which is significantly more complicated and challenging to compute. In this version of our dynamic long/short gamma strategy, we will only make use of the simpler expected P&L formula above. However, in futures versions we plan to use their P&L variance (and covariances) approximation to build a portfolio of gamma trades closer to the optimal frontier.

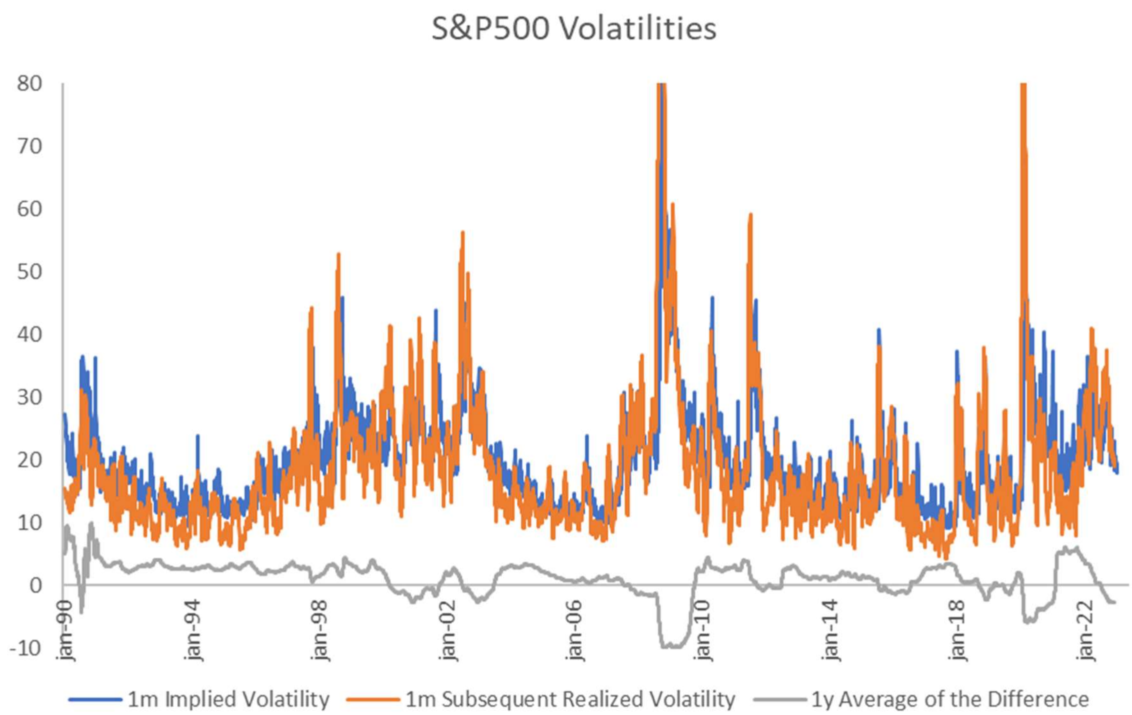
Nonetheless, it is worth mentioning that the author shows that there is an irreducible component of P&L variance that cannot be eliminated by increasing the hedging frequency. It does decrease, but never vanishes, with the hedging frequency. Therefore, there is P&L risk even if the realized volatility is higher than the implied volatility, and even ignoring transaction costs. This is because in the real world (as opposed to the risk neutral Black-Scholes-Merton world) there is misspecification of model parameters: realized and implied volatilities are usually different.

When we consider transaction costs, there is a trade-off between the hedging frequency, the expected P&L and its variance. The author elegantly tackles this issue with his analytic approximations for the first two moments of the delta-hedged P&L by optimizing the Sharpe Ratio of the strategy.

In our dynamic long/short gamma strategy, transaction costs are important and may be very high in less liquid markets. However, the optimal hedging frequency for each market was calibrated through simulations, instead of using these more complicated analytical approximations.

Systematically Selling Equity Index Gamma

Systematically selling straddles on an equity index, such as the S&P 500, with or without dynamic delta hedge, had been a popular and significantly profitable strategy after the crash of 1987 and throughout the 90´s and early 2000´s.



Source: Bloomberg, Constância Investimentos

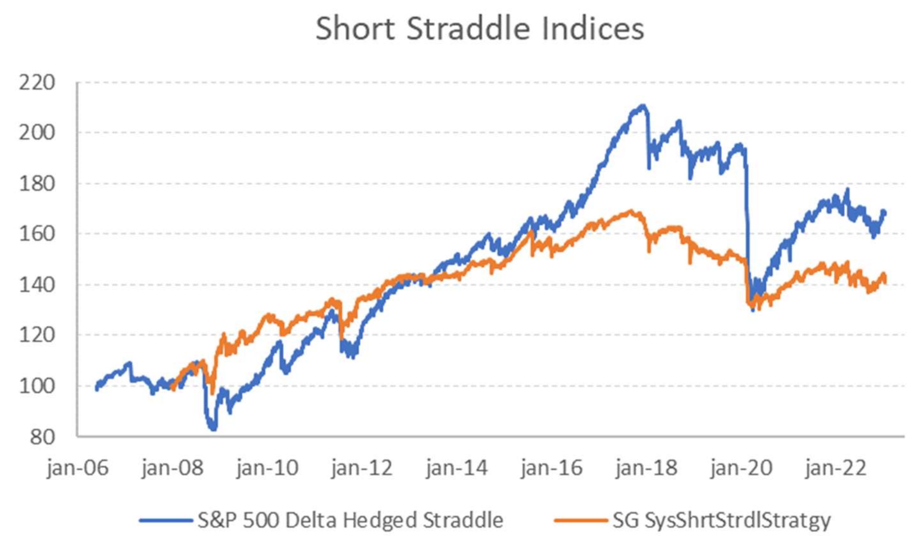
Intimately related to the success of this naïve strategy is the spread between implied and realized volatilities of the S&P 500, which has been persistently positive for most of the historical period, and only occasionally flip sign into deep negative territory, generally during market sell offs.

This apparent asset price puzzle has been extensively studied and well documented in the finance literature with various studies consistently reporting Sharpe ratios above 1.0 for this simple strategy, even after transactions were accounted for. See for example Han (2008) Coval & Shumway (2001) and Broadie et al. (2009).

Finance theory indeed suggests it is adequate that a strategy earns a significant positive excess return if it is exposed to the "peso" problem of rare but large losses, especially if these losses coincide equity market drawdowns and economic bad times. Therefore, it is noteworthy to mention that all the studies mentioned above used datasets that did not include the 2008 financial crisis and 2020 Covid pandemic, the only two other periods in the last 40 years as tumultuous as and comparable to the 1987 crash in terms of extreme volatility.

Although selling at-the-money equity index straddles had offered historically very attractive payoffs with apparent high empirical Sharpe ratio around the Goldilocks economy of the 90's, even then the favorable risk return balance would have been somewhat mitigated once adjusted by the high negative skew and excess kurtosis risks embedded into the strategy.

Because of the strategy´s popularity, indices have been created to facilitate the tracking and evolution of its performance. Below we plot 2 of these indices since their onset: the *S&P 500 Delta Hedged Straddle* (available since 2006) and the *SG Systematic Short Straddle Index* (available since 2008).



Source: Bloomberg, Constância Investimentos

These two straddle indices are constructed with slightly different methodologies:

-The *S&P 500 Delta Hedged Straddle* uses 3 months straddles rolled at the maturity dates, the delta hedged is rebalanced whenever the absolute value of the delta is above 10% delta, and the position is sized at the rolling date for the following 3-month period to have 1% Vega.

- The *SG Systematic Short Straddle Index* uses 1 month Straddles, there is no delta hedging or vega sizing but straddle positions are smoothly entered every trading day for the rolling date to the expiry date.

Despite the differences in methodologies, both constructions deliver modest Sharpe ratios of about 0.3 since inception. This lackluster performance for the last 15 years should be further adjusted downwards by transaction costs, which these indices ignore, and the very negative skew and high excessive kurtosis of the plots above.

Dispersion Trades

A close cousin of our dynamic long/short gamma strategy explained below is the so-called *dispersion trades*, which is also based on the exploitation of often overpriced index options relative to individual stock options.

However, popular dispersion trades usually purchase straddles on *all* single name options available, irrespective of their relative costs and how cheap or expensive each individual implied volatility appears.

When there is a selection process of the single name straddles that the dispersion strategies should be exposed to, it is based on some PCA analysis to reduce the number of trades for liquidity reason, or some correlation filter that is uniform across all stocks and either activates or deactivates all the trades. See for example Schneider and Stübinger (2020).

To the best of our knowledge, there is no consideration for possible relative attractiveness among the different single name straddles in the literature. And this is the main contribution of our dynamic long/short gamma strategy. We explicit a selection mechanism that only allows for exposure on single name options considered "cheap" by showing that the expected return of each long/short component in the dispersion trade is proportional to the difference between (i) the (projection of) the realized volatilities $\frac{\sigma_{R,j}}{\sigma_{R,Index}}$ and (ii) the relative implied volatilities $\frac{\sigma_{I,j}}{\sigma_{I,Index}}$.

It is strikingly obvious that a component of the dispersion trade with extremely high implied volatility should be excluded from the basket because it is excessively expensive. So, it surprising that this hasn´t been incorporated in any dispersion strategy. The math developed later in this article supports this intuition and points a way to account for it.

Also, dispersion trades usually assign weights to each individual option equal to the index weight of the stock. This would map into a hedge ratio $\lambda = 1$, instead of the hedge ratio $\lambda = \frac{\sigma_{Implied,j}}{\sigma_{Implied,Index}}$ that our analysis below suggests.

As occurred with the simpler strategy of outright selling equity index gamma, there is well documented evidence that dispersion trade strategies have had excellent risk adjusted performance, see for example Marshall (2009), Schneider and Stübinger (2020) and Ferrari, Poy and Abate (2019).

However, again the period used to conduct these simulations exclude both the 2008 global financial crisis and the 2020 Covid stress. Once these are accounted for, the attractiveness of these strategies also diminishes considerably and deliver Sharpe ratio below 0.5 in the last 15 years. Our dynamic long/short gamma strategy navigates this stress periods better and manage to deliver a superior risk adjusted return.

The Dataset

Implied volatility data and risk-free rates used in this manuscript is from Bloomberg. The stock price data, including outstanding shares, dividends, and stock split information, were retrieved from the S&P Capita lQ.

Implied volatility data for the single name equities and indices is available in Bloomberg starting January 2005 for the S&P500 stocks and for some of the more liquid markets, but somewhat shorter for some emerging markets and subindices.

Hence, the simulations we carry out for our dynamic long/short gamma strategy, which we explain below, start in early 2005. This encompasses 2008 and 2020 which, as we have mentioned, were particularly challenging periods from any gamma strategy.

We used the adjusted historical data on equity prices and returns since January 2000 for the regressions we´ll need.

Some of the authors cited above have used datasets for implied volatility from Markit or OptionMetrics Ivy which extends further back into the early 90´s for some single names. However, implied volatility data from Bloomberg is very convenient because the volatility surface is already delivered interpolated by tenor e delta. Also, the availability of data for several options market beyond S&P500 allows us to conduct a broad comparative analysis of our strategy across geographies and subindices.

For the computations of the simulated options prices we use the second front option contract as the implied volatility data shortest tenor is one month. At the onset of the simulated trades we choose the strikes (we assume they exist) of the straddles with 50% delta, both for the single names and for the index. Then we relied on the Black-Scholes formula and used the simple interpolation between the 50% delta 1 month and 2 months.

Data on implied volatility for 25% delta and 10% delta is also available in Bloomberg. Hence, in a future version of this work we could attempt more elaborate interpolation schemes across strikes and tenors, using for example Dupire´s equation, which uses incorporates a richer set of information of the implied volatility surface. See Dupire (1994).

The period considered in the simulations was from the beginning of January 2005 to end of February 2023.

Relative Value in Gamma Exposures

As mentioned, previous academia work provides historical evidence that strongly supports some level of expected profitability for shorting equity index straddles, albeit accompanied by high levels of risk. On the other hand, the conclusion that there is a sizeable empirical premium of implied volatilities over realized volatilities is dubious at best. See for example Bakshi, G., & Kapadia, N. (2003), Bollen & Whaley (2004), Dennis et al. (2006), and Driessen et al. (2009).

In fact, our internal research at Constancia Investimentos suggests that buying straddles on individual equities tend to accrue a very small positive excess return in many important equity markets.

A possible explanation for this pricing discrepancy between single names and index straddles is a potential overdemand for hedging using index options as the main instrument, and an oversupply of single names implied volatilities through covered Call writing strategies.

Hence, a strategy that buys a straddle over a single name equity *j* and hedges it with a short equity index straddle has the potential to enjoy the alpha on both ends, while significantly reducing volatility and higher order risks.

Although the long/short gamma trade is profitable on average, its performance can be substantially improved by choosing an appropriate hedge ratio, which we denote by λ, and entering it only when the relative price is attractive.

Extending the algebra from Artur Sepp´s (2013) article, the expected P&L of the long/short gamma trade is given by

$$\mathbb{E}[P\&L] \cong \left[\Gamma_j\left(\sigma_{R,\,j}^2 - \sigma_{I,\mathrm{j}}^2\right) - \lambda.\,\Gamma_{index}\left(\sigma_{R,\,index}^2 - \sigma_{I,index}^2\right)\right]T$$

$$= \left[\left(\Gamma_j.\,\sigma_{R,\,j}^2 - \lambda.\,\Gamma_{index}.\,\sigma_{R,\,inde}^2\ \right) - \left(\Gamma_j.\,\sigma_{I,\mathrm{j}}^2 - \lambda.\,\Gamma_{ind}\quad.\,\sigma_{I,\mathrm{index}}^2\right)\right]T$$

And we set λ such that the content of the second parentheses is equal to zero, $\lambda = \frac{\Gamma_j.\sigma_{\mathrm{implied,j}}^2}{\Gamma_{Index}.\sigma_{\mathrm{implied,Index}}^2}$.

Note that simple algebra gives us $\frac{\Gamma_j.\sigma_I,j}{\Gamma_{index}.\sigma_I,index} = exp\left\{-\frac{1}{8}\left[\sigma_{I,j}^2 - \sigma_{I,index}^2\right]T\right\}$, which is close to 1 for T = 1/12 (1 month straddles) even for very high levels of implied volatilities.

Hence the hedge ratio λ is approximately equal to the ratio of implied volatilities $\lambda \cong \frac{\sigma_{I,\mathrm{j}}}{\sigma_{I,\mathrm{index}}}$

Substituting this choice of λ into the expected P&L equation above we get

$$\mathbb{E}[P\&L] \cong \sigma^2_{R,\,index}\Gamma_j\left[\left(\frac{\sigma^2_{R,\,j}}{\sigma^2_{R,\,index}} - \frac{\sigma^2_{I,j}}{\sigma^2_{I,index}}\right)\right]T \text{ , so whenever } \frac{\sigma^2_{R,\,j}}{\sigma^2_{R,\,index}} \gg \frac{\sigma^2_{I,j}}{\sigma^2_{I,index}} \text{ , we have } \mathbb{E}[P\&L] \gg 0.$$

We mentioned in the beginning of this article the well-known result that the holder of a delta-hedged straddle profits when the implied volatility is smaller than the subsequent realized volatility. The equation allow us to extend this assertion in a very intuitive way, and conclude that the holder of a long/short gamma trade (with hedge ratio λ) makes money when the ratio of the implied volatilities is smaller than the subsequent ratio of the realized volatilities.

Obviously, we don´t know what the subsequent realized volatilities for any stock *j* or equity index are. However, these ratios of the volatilities have well behaved, stationary, mean-reverting time series, and we can resort to well stablished econometric technics to project it.

Projecting the Ratio of Realized Volatilities

To obtain $Projected\left(\frac{\sigma^2_{R,\,j}}{\sigma^2_{R,\,index}}\right)$ we set the notation for 1 month lag $\lambda^j_{t+1m} = \frac{\sigma_{R,\mathrm{j,t+1m}}}{\sigma_{R,\mathrm{index,t+1m}}}$ and run rolling log-AR(1) regressions adding a component for market dependence:

$$Ln\left(\lambda^j_{t+1m}\right) = \beta^j_0 + \beta^j_1 Ln\left(\lambda^j_t\right) + \beta^j_2 Ln\left(\sigma_{R,\mathrm{inde\ \ ,t}}\right) + \epsilon^j_t$$

For all the 50 stocks of the S&P500 that we ran regressions for, we obtained β^j_1 different than zero at the 5% significance level, indicating mean reverting behavior for the volatility ratios, as would have been expected.

We have added the term $Ln\left(\sigma_{R,\mathrm{index,t}}\right)$ to the regression above to allow for a single common factor in general volatility levels. As argued in the recent work of Ding, Engle, Li and Zheng (2022) and Kapadia, Linn, and Paye (2020), there is strong empirical evidence that a single common factor in market volatility is sufficient and, indeed, in our context the S&P500 sample we used supports this result.

To show this, we compute the correlation matrix of the *N x T* residual errors $\zeta = \left(\epsilon^j_t\right)_{t\le T, n\le N}$ from the regressions above and compare its eigenvalue's structure $(\lambda_n)_{n\le N}$ with the Marchenko–Pastur distribution from random matrix theory.

Initial results for random matrix theory were obtained by Marchenko and Pastur (1967) and later significantly extended by other authors, see for example Mehta (1995). For our purposes, the main result is the fact that, when shocks are i.i.d., the asymptotic limit (when $T \longrightarrow \infty$, $N \longrightarrow \infty$ and $Q = \frac{T}{N}$ is fixed) of eigenvalues structure of the correlations converge to the density $\rho(\lambda)$ given by

$$\rho(\lambda) = \frac{Q}{2\pi}\frac{\sqrt{(\lambda_{max}-\lambda)(\lambda_{min}-\lambda)}}{\lambda}, \qquad \lambda^{max}_{min} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}.$$

In our case, T = 276 months, N = 50 stocks, which give us Q = 5.52.

After computing the maximum and minimum eigenvalues of the sample $Correl(\zeta)$ of the residual above we confirm they are adherent to the support of the Marchenko–Pastur distribution. Hence, we cannot outright reject the null hypothesis that the residuals are purely random, suggesting that the single common factor added to the regression equation above was enough to explain the variability in the data.

The table below reports the regression coefficients for the 15 most liquid single names. Blank spaces indicate that the coefficient was insignificant at the 5% level.

Singla name	Intercept	Beta 1	Beta 2
APPLE INC	0.099	0.227	-0.206
MICROSOFT CORP	0.125	0.047	-0.170
AMAZON.COM INC	0.360	0.046	-0.204
BANK OF AMERICA CORP	0.517	0.623	0.125
JPMORGAN CHASE & CO	0.493	0.424	0.081
ALPHABET INC-CL A	0.085	0.132	-0.199
WELLS FARGO & CO	0.491	0.580	0.122
CITIGROUP INC	0.638	0.531	0.149
EXXON MOBIL CORP	0.206	0.463	
JOHNSON & JOHNSON	-0.294	0.417	-0.143
CHEVRON CORP	0.217	0.423	-0.020
FORD MOTOR CO	0.558	0.343	
PFIZER INC		0.122	-0.136
INTEL CORP	0.175	0.100	-0.189
BOEING CO/THE	0.376	0.405	

Source: S&P CapitalIQ, Constância Investimentos

It is interesting to note that most, but not all, β_2^j coefficients are significant, and the majority have negative sign, but a relevant number, about 20% of them, have positive sign.

This sign of β_2^j regression coefficient is particularly relevant to our dynamic long/short gamma strategy because drawdowns tend to occur when the general level of volatility and correlation increases. And in these stressed market environments, the $Projected\left(\frac{\sigma_{R,j}^2}{\sigma_{R,Index}^2}\right)$ from the regression will naturally lead the strategy to reduce exposures of single names with $\beta_2^j < 0$ and focus on the ones with positive loads to the common factor. As most single names have negative loads, the total exposure of the strategy will naturally diminish in high volatility times and will focus on the single names that are more volatile in these periods.

Dynamic Long/Short Gamma Strategies

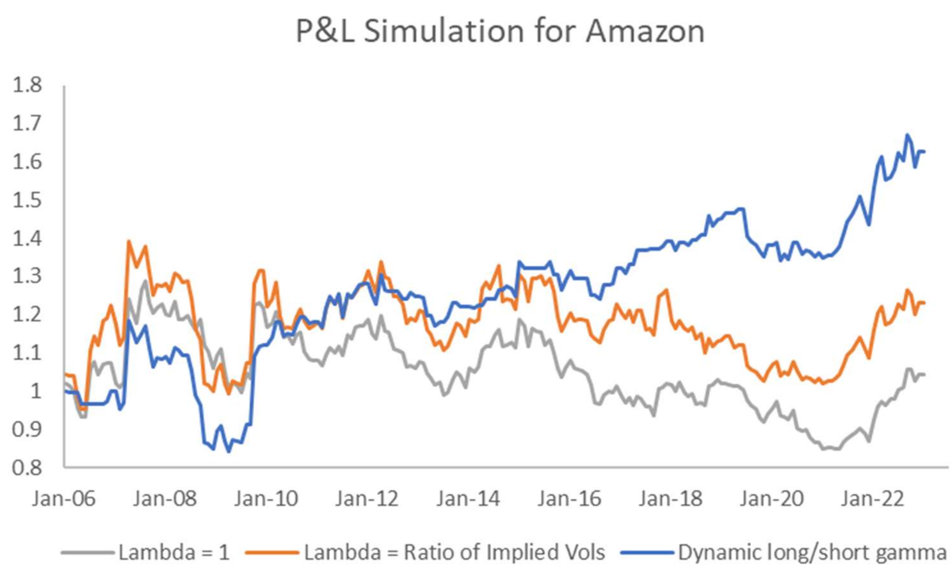
Intuition always suggests that any trade should be entered only when it is cheap and avoided when expensive. After the heavy lifting done by the algebra from the previous sections, it is straightforward to translate this commonsense statement for our dynamic long/short gamma strategy into the mathematical condition:

- when $\frac{\sigma^2_{I,j}}{\sigma^2_{I,index}} < Projected\left(\frac{\sigma^2_{R,\,j}}{\sigma^2_{R,\,index}}\right)$, go long \$1 notional of the single name straddle j and short \$$\lambda$ notional of the index straddle.

This innovative selection process allows the strategy to concentrate only on the single name straddles that are expected to be profitable when delta-hedged to maturity.

For single name equities *j* where the long-term average of $\sigma^2_{I,j}/\sigma^2_{I,index}$ is approximately equal to the long term average of $\sigma^2_{R,\,j}/\sigma^2_{R,\,inde}$, the long/short gamma trade would be active approximately 50% of the time. On the other hand, for equities where the gap between the latter and the former is significantly positive, the alpha to be extracted will be higher and the mathematical condition above will respond by keeping the trade predominantly active.

The plot below displays the P&L simulation for the long/short gamma strategy for Amazon stock.



Source: Bloomberg, Constância Investimentos

The gray line is the P&L evolution of a 1 month 50% delta Amazon Straddle versus short 1 month 50% S&P500 Straddle strategy which is active all the time and have hedge ratio $\lambda = 1$. The straddles are rolled at maturity to the next expiry date and daily delta-hedged. This P&L would be one of the components of a dispersion trade strategy. Note that, within this simulation horizon, which includes 2008 and 2020 market stress periods, it has negligible average return and is almost indistinguishable from a random walk.

The orange plot maintains the strategy active all the time but sets the more efficient hedge ratio $\lambda = \frac{\sigma_{\mathrm{I,Amazon}}}{\sigma_{I,\mathrm{S\&P500}}}$ and the blue line is our dynamic long/short gamma strategy, it uses the same efficient hedged ratio and deactivates the trade when the expected P&L is negative. It delivers a Sharpe ratio of 0.35 before transaction cost and just over 0.20 after transaction costs, which we assumed to be 0.5 implied vols for each straddle and 2.5 bps for each delta hedged rebalance.

The attractiveness of this strategy for any individual US equity is modest, but the correlation among them is negligible, particularly if the delta-hedge is done frequently. Therefore, when we consider our broader set of S&P500 stocks the dynamic long/short gamma strategy achieves a decent risk return with Sharpe ratio of about 0.83 after transaction costs.

Furthermore, in other equity markets and indices less efficient than S&P500, such as Ibovespa, STXE 50 (EUR), US KBW bank index, among others (see table in the next section), simulations of our dynamic long/short gamma strategy indicate potential of very solid risk adjusted gains.

Beyond the S&P 500

Given its popularity and the reasonable level of efficiency of the aggregate US equity markets, it doesn´t surprise us that most of the alpha in these gamma strategies have been exploited way in the last several years. But there are some indications that the opportunities to extract alpha with our dynamic long/short equity gamma strategy may still be available in other markets. Natural candidates are the indices where the simpler short index straddle performed even when the more challenging periods of 2008 global financial crises and 2020 Covid outbreak are considered.

For example, if we cherry pick and construct the equivalent delta-hedge short straddle indices for US Banks and STXE 50 EUR we would get Sharpe ratios of around 0.6, and for Ibovespa this risk return measure is close to 1.0. Transaction costs diminish these Sharpe Ratios by 30% to 40% if delta-hedge is executed daily, and a bit less if the frequency of the rebalance is optimized by trial and error to once or twice per week.

At the risk of incurring data mining, we conducted a broad exploratory analysis beyond the S&P 500 by scrutinizing all the equity indices where data on implied volatility is available from Bloomberg, which is only about 10% of the almost 300 equity indices worldwide. Preliminary results give us cautious enthusiasm about extending this strategy to subsets of the US market and selective foreign equity indices.

The table below reports the historical average of the ratio of 1 month realized and implied volatility. After highlighting the caveats (i) all the analysis of the table below were conduct in-sample and (ii) for many indices the size of the implied volatility dataset is far from optimal, it is plausible to assert that the markets/indices with smaller historical average of this ratio suggests higher potential for harvesting alpha with our dynamic long/short gamma strategy.

	Country	Index	Historical Average of the Ratio Realized / Implied Volatility	Data Available Since
Americas	US	DOW JONES INDUS. AVG	0.96	2005-01-03
	US	KBW BANK INDEX	0.93	2005-01-03
	US	NASDAQ 100 STOCK INDX	1.02	2005-01-03
	US	PHILA GOLD & SILVER INDX	1.03	2005-01-03
	US	PHILA SEMICONDUCTOR INDX	1.02	2005-01-03
	US	OIL SERVICE SECTOR INDEX	0.97	2005-01-03
	US	PHILA UTILITY INDEX	1.02	2005-01-03
	US	RUSSELL 1000 INDEX	1.00	2003-01-25
	US	RUSSELL 2000 INDEX	0.98	2005-01-03
	US	S&P 100 INDEX	0.98	2005-01-03
	US	S&P 500 INDEX	0.97	2005-01-03
	BZ	BRAZIL IBOVESPA INDEX	0.93	2015-12-04
Europe	EC	Euro Stoxx 50 Pr	0.97	2008-10-23
	EC	STXE 50 (EUR) Pr	0.90	2009-07-17
	EC	STXE 600 (EUR) Pr	0.97	2009-07-17
	GB	FTSE 100 INDEX	0.97	2009-07-17
	GE	DAX INDEX	0.98	2009-07-20
	GE	MDAX PERF INDEX	0.93	2009-07-20
	FR	CAC 40 INDEX	0.96	2009-07-20
	SZ	SWISS MARKET INDEX	0.96	2009-07-17
	IT	FTSE MIB INDEX	0.98	2009-07-20
	NE	AEX-Index	0.96	2009-07-20
	DE	OMX Copenhagen 25 Index	1.01	2017-11-17
	NO	OBX STOCK INDEX	0.86	2009-07-17
	NO	OMX Oslo 20 GI Index	0.91	2013-11-01
	SW	OMX STOCKHOLM 30 INDEX	0.95	2009-06-11
	AS	AUSTRIAN TRADED ATX INDX	0.90	2009-07-20
	GR	FTSE/ASE Large Cap	0.92	2009-07-20
	TU	BIST 30 Index	0.97	2013-07-31
Asia	JN	NIKKEI 225	1.00	2006-05-12
	HK	HANG SENG INDEX	0.99	2009-07-16
	HK	HANG SENG CHINA ENT INDX	0.98	2009-07-16
	TA	TAIWAN TAIEX INDEX	0.92	2009-01-10
	AU	S&P/ASX 200 INDEX	0.93	2007-06-20
	IN	Nifty 50	0.91	2007-06-26
	SK	KOSPI 200 INDEX	0.99	2009-06-10
	TH	THAI SET 50 INDEX	1.00	2003-01-25

Source: Bloomberg, Constância Investimentos

Connection with Implied Correlation Trades

As highlighted by Ferrari, Poy and Abate (2019), dispersion trades are closely related to correlation strategies and their performance can be improved making use of this duality.

Let´s recall that the volatility of the index is given by

$$\sigma_{Index}^2 = \sum_{i=1}^{i=n} \sum_{k=1}^{k=n} w_i w_k \rho_{ik} \sigma_i \sigma_k$$

A simple way to compute the average implied correlation $\bar{\rho}$ is to force the equation above to hold for implied volatilities and assuming that all $\rho_{ik} = \bar{\rho}$, $i \neq k$.

A more elaborate way that preserves the structure of the historical correlation matrix is developed in Kawee and Nattachai Numpacharoen (2013), the Implied, where the equation above is forced to hold for implied volatilities and all historical correlations are bumped or bumped down by the same amount.

To improve the overall performance of dispersion trades, Ferrari, Poy and Abate (2019) carry out similar computations of implied correlations as described above to time in-and-out of all the straddle exposures. In summary, they only activate the dispersion strategy when implied correlation is significantly above the realized correlation.

Similar interpretation and duality can be carried to our dynamic long/short gamma strategy. To develop the intuition, assume a simplified world where:

(i) all assets have the same index weight; $w_i = w_k = \frac{1}{N}$;
(ii) 2-by-2 correlations are the same $\rho_{ik} = \bar{\rho}$, $i \neq k$ and
(iii) Volatilities are the same (realized and implied) $\sigma_i = \sigma_k = \bar{\sigma}$

Then the math simplifies to

$$\sigma_{Index}^2 = \frac{N}{N^2}\bar{\sigma}^2 + \frac{N(N-1)}{N^2}\bar{\rho}\bar{\sigma}^2 \cong 0 + 1.\bar{\rho}\bar{\sigma}^2 \quad \Longrightarrow \quad \frac{\sigma_j^2}{\sigma_{Index}^2} \cong \frac{\bar{\sigma}^2}{\bar{\rho}\bar{\sigma}^2} = \frac{1}{\bar{\rho}}$$

Remember that the mathematical condition for our dynamic long/short gamma strategy to selectively enter the single name equity straddle *j* versus the straddles of the index is

$$\frac{\sigma_{I,j}^2}{\sigma_{I,Index}^2} < Projected\left(\frac{\sigma_{R,j}^2}{\sigma_{R,Index}^2}\right),$$

And the simplification above indicates that this is equivalent to $\frac{1}{\bar{\rho}_{projected}} \gg \frac{1}{\bar{\rho}_{implied}}$.

Again, the trade is only triggered when implied correlation is higher enough. The dynamic long/short gamma strategy is profitable partially because implied correlation is often overpriced, and the trade across many individual equities resembles a short exposure on implied correlations.

However, differently than some dispersion trades, where either all single name straddles are active or inactive (Ferrari, Poy and Abate (2019)), in our dynamic long/short gamma strategy the trigger related to implied correlation is single name specific, and only the attractive single equity straddles will be purchased at any point in time.

Conclusion

We have developed a relative value measure for gamma trades between single name stocks and their equity index by comparing the ratio of implied volatilities with the projected ratio of realized volatilities.

This allowed us to build a dynamic long/short gamma strategy that outperforms the simpler and more popular gamma trades of outright selling index straddles and dispersion trades. The decent simulated risk rewards for the efficient S&P500 market in the last 15 years, which were particularly challenging for gamma trades due to the 2008 and 2020 crisis, is very encouraging. Preliminary estimations show very promising potential in other geographies and subindices, where wider margins and opportunities seem to exist.

Further improvements to our strategy may be obtained in future research endeavors by developing a non-trivial extension of Artur Sepp´s (2013) analytical approximation of the second moment of the delta-hedged P&L for our hedged straddles (long single name vs short index) set up. This would allow us to build more balanced portfolios of hedged straddles that might be closer to the Markovitz optimal frontier.

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