DEVELOPMENT OF THE GOODMAN DIAGRAM FOR THE DD5 COMPOSITE USING A MODULAR ANN

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Abstract: This work presents an analysis of how the learning rate in a modular Artificial Neural Network (ANN) influences the development of the Goodman Diagram for the DD5 composite material. Experimental data involving multiple S-N curves were collected, and a mathematical approach based on the Generalized Power Law was employed to determine the stress amplitude and the average number of cycles to failure. The Goodman Diagram was constructed by calculating the mean stress from the stress amplitude and fatigue ratio. Data normalization was applied to prevent neuron saturation and enhance the generalization capability of the ANN. For training, three fatigue ratio values (R = 0.1, 2, and 10) were chosen, while an additional value (R = 0.5) was reserved for validation purposes. The hidden layer neuron count varied from 5 to 25, with learning rates tested at 0.05, 0.1, and 0.5 over 3000 training epochs. Analysis of the mean squared error (MSE) demonstrated that learning rates of 0.05 and 0.1 yielded errors on the order of 10^{-4}, whereas the 0.5 rate produced higher errors around 10^{-3}. The best performance was achieved with the 0.1 learning rate, as its Goodman Diagram curves closely matched the experimental data, reflecting superior learning ability and robustness of the model. This methodology illustrates that modular ANNs provide an efficient, fast, and cost-effective alternative for fatigue life analysis of composite materials, supporting improvements in structural design with enhanced safety and reliability.

Keywords: Fatigue analysis, Goodman Diagram, Artificial Neural Network (ANN), Composite materials and Learning rate.

1. Introduction

The objective of this work is to analyze how the learning rate of a modular architecture Artificial Neural Network influences the results of the development of the Goodman diagram for the DD5 composite.

Each material has a tensile strength limit. However, when subjected to time-varying stresses, materials may fail at stress levels lower than this limit, characterizing fatigue failure (Souza, 1982,[1]; Budynas and Nisbett, 2016,[2]). Since machine elements are subjected to cyclic loads, their design must take into account the material's service life (Sutherland, 1999). An important relation for the study of fatigue is the fatigue ratio (R), which can be seen in Equation 1 (Souza, 1982,[1]; Freire Jr., 2005,[3]; Mott, 2015,[4]).

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{1}$$

1.1 Composite

The data used for training and validating the ANN refer to the DD5 material and were extracted from the work of Mandell and Samborsky (1997) [5]. The DD5 is a glass fiber structural composite widely used in applications requiring good mechanical performance under static and cyclic loading, such as wind turbine blades, nautical components, and automotive structures. Its performance strongly depends on the reinforcement layer configuration, fiber volume fraction, and polymer matrix characteristics. The DD5 consists of a laminate with a [0/±45/0] layer configuration, indicating that most fibers are oriented in the longitudinal direction (0°), while some are inclined at ±45° relative to the main loading direction. This hybrid architecture aims to provide strength both in tension and shear. The fiber distribution in the laminate comprises approximately 70% of fibers in the 0° direction, favoring uniaxial tensile strength, and 30% in the ±45° layers, which contribute to shear stiffness and material performance under complex loading conditions.

1.2 Goodman diagram

In the prevention of fatigue failures in composites, the Goodman Diagram has proven to be an effective tool, although it requires the development of a specific diagram for each type of composite analyzed. Obtaining the curve requires representative data of the material's behavior, which can be achieved by keeping the fatigue ratio constant and subjecting each specimen to a specific maximum stress value during testing. To determine the number of cycles to failure for different R values, new tests must be conducted, and specific S–N curves must be constructed for each condition (Freire Jr., 2005,[3]; Diniz, 2017,[6]). According to Bond (1999)[7], the construction of the Goodman Diagram requires, at a minimum, a mathematical model representing the S–N curve, obtained from experimental data from alternating stress tests, in addition to the tensile and compressive strength limits of the material. With this data, the diagram is developed using the mathematical model to determine the values of stress amplitude (σ_a) and mean stress (σ_{med}) for 10^3, 10^4, 10^5, 10^6, and 10^7 cycles. For composites, the main function of the Goodman Diagram is to define the safe regions of cyclic loading, indicating the number of cycles the material can withstand before failure. This is because, in most cases, these materials do not present a fatigue strength limit; that is, regardless of the applied stress, failure will occur after a finite number of cycles.

1.3 Mathematical equations

The specialized literature (Subramanian et al., 1995,[8]; Mandell et al., 1997,[5]; Sutherland, 1999,[9]) presents several equations aimed at modeling the fatigue behavior of composites. However, none of these mathematical expressions proves to be sufficiently comprehensive to satisfactorily represent the fatigue phenomenon in all types of composites. For a mathematical function to adequately represent fatigue behavior, it must consider at least three essential variables: the number of cycles to failure (N), the stress value (σ), and the fatigue ratio (R). Following this approach, Equation 2 expresses the stress amplitude (σ_a) as a function of the mean stress (σmed) and the number of cycles, without compromising the representation of the material's behavior:

$$\sigma_a = f(\sigma_{med}, N) \quad (2)$$

However, the previous equation does not satisfactorily represent the material's behavior, requiring a generalization. In this sense, Equation (3), proposed by Subramanian et al. (1995)[9], can be used to more adequately describe the relationship between maximum stress and the average number of cycles to failure ($\bar{N}$):

$$\sigma_{max} = A - B[\log(N)]^c \quad (3)$$

The addition of parameters A and C in Equation 3 aims to provide a better fit of the curve to the analyzed experimental results, allowing a representation closer to the fatigue behavior of composites. Another way to represent the S-N curve data is by using the Power Law (Equation 4). The main advantage of this equation is to provide smoothing of the curve for high numbers of cycles. However, it assumes that the material exhibits linear behavior on a log–log scale, which is not always experimentally observed. In such cases, it is possible to apply a generalization, presented in Equation 5 (Philippidis and Vassilopoulos, 2002,[10]; Wahl et al., 2002, [11]).

$$\sigma_{max} = a\bar{N}^{-B} \quad (4)$$

$$\log(\sigma_{max}) = A - B[\log(\bar{N})]^c \quad (5)$$

The parameters a, A, B, and C must be determined during the curve fitting process. With these values, it is possible to define the equation necessary for the development of the normalized Goodman Diagram.

$$\frac{\sigma_a}{UTS} = f\left(1 - \frac{\sigma_{med}}{UTS}\right)^u \left(\frac{UCS}{UTS} + \frac{\sigma_{med}}{UTS}\right)^v \quad (6)$$

In Equation 6, UTS and UCS represent, respectively, the tensile and compressive strength limits. The variables f, u, and v are parameters that vary as a function of the number of cycles to failure (N). According to Freire Jr. and Aquino (2009)[12], the main limitation of Equation 5 is the need for a large number of S-N curves to obtain satisfactory results. Another factor that disadvantages the model is the possible lack of correspondence between constant-life curves, which usually occurs when the number of available S-N curves is insufficient, compromising the faithful representation of the material's fatigue behavior. In their studies, Vassilopoulos et al. (2010)[13] proposed the development of a model capable of determining the fatigue behavior of a composite from a relatively reduced number of S-N curves. To this end, they mathematically demonstrated the existence of a relationship between mean stress (σ_{med}), fatigue ratio (R), and stress amplitude (σ_a), expressed in Equation 7. This model allows describing the material behavior using only two parameters, since the third can be obtained from the equation itself.

$$\sigma_a = \frac{1-R}{1+R}\sigma_{med} \quad (7)$$

Based on this proposal, equations were developed to represent each of the fatigue ratio regions, whose boundary conditions were defined from the experimental data of the S-N curves. The corresponding equations, as well as details about the construction of the constant-life Goodman

Diagram, can be found in Vassilopoulos et al. (2010)[13].

1.4 Artificial Neural Network

An artificial neural network (ANN) is a massively parallel processor composed of simple processing units, with a natural capacity to store experimentally obtained knowledge and make it available for use. Several characteristics make the field of ANNs attractive for solving complex problems. Among them, the ability to learn from examples and generalize information stands out; that is, the network can provide consistent responses for unknown situations or inputs even with a reduced amount of data. Therefore, it is essential to have a well-designed training set that is representative of real behavior, combined with an appropriate network architecture and an efficient training algorithm.

The activation function defines the neuron's output in terms of its internal activation level. Various types of activation functions can be applied, such as threshold function, piecewise threshold, ReLU, hyperbolic tangent (TanH), and sigmoid (Iyoda, 2000,[14]; Haykin, 2001,[15]; Glorot et al., 2011,[16]). Among these, the last is highlighted and was used in this work. The sigmoid function is considered the most common form of activation function in multilayer artificial neural networks trained by the back-propagation algorithm, as pointed out by Iyoda (2000)[14] and Silva (2009)[17]. It is defined as a strictly increasing function that offers an appropriate balance between linear and nonlinear behavior (Haykin, 2001,[15]). Equations 8 and 9 mathematically express the sigmoid function and its respective derivative.

$$\varphi(v) = \frac{1}{1+e^{-a_i v}} \quad (8)$$

$$\varphi'(v) = \frac{-(-a_i e^{-a_i v})}{(1+e^{-a_i v})^2} = a_i \varphi(v)[1 - \varphi(v)] \quad (9)$$

The main characteristic of the perceptron network is the use of neurons with nonlinear activation functions and a feedforward layered architecture. Generally, this type of network consists of a set of sensory units (input layer), one or more hidden layers of computational neurons, and an output layer. Its training is performed in a supervised manner through the error backpropagation algorithm.

Modular Networks are neural networks composed of two or more independent modules (subsystems), without direct connections between them. The outputs of these modules are mediated by a routing network, which does not provide feedback. This routing network combines the outputs of the modules to form the system's final output, as well as determines how input patterns are distributed among the modules (Haykin, 2001,[15]). According to Ishikawa (1995)[18], learning in modular networks occurs in two stages: the first involves training the synaptic weights in each expert module, and the second corresponds to adjusting the synaptic weights that connect the different modules.

The ability to learn through interaction with the environment or the information source is one of the main characteristics of neural networks. The fundamental concept of training a neural network consists of gradually modifying the synaptic weights, following a learning rule that determines how these weights will be adjusted (Haykin, 2001,[15]).

Supervised training is the most commonly used method for training ANNs. It is based on a predefined set of input and output data that guides the network's learning. Thus, the network is expected to respond approximately to the presented data. Furthermore, it is desirable that the network has the ability to generalize, i.e., provide consistent responses for both known and previously unseen data. According to Iyoda (2000)[14], a common way to implement supervised learning in neural networks is through iterative error correction procedures. Using a training algorithm structured to minimize error, combined with an adequate training set and a sufficient number of iterations, results in a neural network capable of performing tasks such as pattern classification and function approximation (Iyoda, 2000,[14]; Haykin, 2001,[15]).

The backpropagation algorithm is the most used method for training multilayer perceptron artificial neural networks (ANNs), based on the error correction rule. Its computational structure basically consists of two stages: feedforward processing, in which an input is applied to the network and its effect is propagated layer by layer, with weights kept fixed, until the output layer generates a set of actual network responses; and backward processing, in which an error signal is propagated in the opposite direction, layer by layer. At the end of this process, the network weights are adjusted according to the error correction rule (Braga et al., 2000,[19]; Iyoda, 2000,[14]; Haykin, 2001,[15]; Nied, 2007,[20]). Thus, the function of backpropagation training is based on minimizing the mean squared error as in Equation 10.

$$EMQ = \frac{1}{2Q}\sum_{1}^{Q}\sum_{k=1}^{K}(d_k - z_k)^2 \qquad (10)$$

Where EMQ is the mean squared error, Q is the total number of data points, K is the number of neurons in the output layer, and dk and zk are the desired response and the actual response of the k-th output neuron, respectively.

2. Methodology

The study development begins with the collection of experimental data, involving at least three S-N curves of the material under analysis. With these data, a mathematical approach based on the Generalized Power Law (Equation 5) was adopted, from which the values of stress amplitude (σ_a) and average number of cycles ($\overline{N}$) were determined. Finally, using the stress amplitude and the fatigue ratio, the mean stress (σ_{med}) was calculated according to Equation 7.

Normalization is a data preprocessing step aimed at preventing neuron saturation in the network, thus avoiding inadequate training. This method consists of restricting the input values of the S-N curves to a range between 0 and 1 for the stress amplitude, as shown in Equation 11.

$$\overline{N}_{nor} = \frac{\log(\overline{N})}{\log(N_{max})} \qquad (11)$$

Normalization of the other parameters also aids in the generalization of the ANN architecture usage. After normalizing the dataset during the preprocessing stage, the choice of network architecture constitutes the next step, being equally fundamental for successful training.

For network training, three fatigue ratio values were selected: $R = 0.1$, $R = 2$, and $R = 10$. To validate the synaptic weights obtained by the ANN, a dataset including all data plus an additional fatigue ratio, $R = 0.5$, was used. It is worth noting that the DD5 material employed has ultimate tensile strength (UTS) values of 1397 MPa and ultimate compressive strength (UCS) of -722 MPa.

The number of neurons in the hidden layer ranged from 5 to 25 in each module. For training, 3000 epochs were used, with learning rates varying between 0.05, 0.1, and 0.5.

The selection of synaptic weights for the representation of the Goodman Diagram considered the parameter set that showed the lowest mean squared error when evaluating all data.

The analysis of the dispersion of the Mean Squared Error (EMQ) values allows assessing two main characteristics of an artificial neural network (ANN): its generalization capability and the robustness (repeatability) of the trained algorithm. The errors obtained were on the order of 10^{-4} for learning rates 0.05 and 0.1, and on the order of 10^{-3} for the learning rate 0.5.

3. Results

The construction of the Goodman Diagram results from the combination of the experimental data used in training with the curves generated by the ANN. As illustrated in Figure 1 to Figure 3, the points on the lines, which represent each value of R, correspond to the experimental data used in training, except for $R = 0.5$, which was used only for network validation. The curves represent the set of points of stress amplitude versus mean stress for a given number of cycles.

Figure 1, Figure 2, and Figure 3 present the Goodman Diagrams obtained through the applied methodology, considering learning rates of 0.05, 0.1, and 0.5, respectively.

The curves located below the points presented in the graph indicate that the model's behavior remains within safe limits, ensuring structural integrity and the reliability of the obtained results. This relationship demonstrates that the model meets the established performance criteria, providing confidence for its practical application.

4. Conclusion

In the evaluation of the Mean Squared Error, which attests to the robustness (repeatability during training) of the trained algorithm, it is concluded that the adoption of only three S–N curves is sufficient to ensure consistent and representative results. This number of curves proves capable of adequately capturing the material's fatigue behavior, providing reliable information for both service life prediction and structural integrity assessment. Such an approach streamlines the modeling process, reducing experimental complexity without compromising estimation accuracy, thus enabling faster and more cost-effective fatigue life analyses

By analyzing the generated Goodman Diagrams, it is observed that the ANN trained with a learning rate of 0.1 delivered the best results, as the curves of this diagram were closer to the experimental data. Therefore, even compared to higher learning rates, the learning capability of the network with a 0.1 rate proved to be superior. This result indicates that increasing the processing rate, as well as extending the time allocated to performing the analyses, does not necessarily lead to an improvement in the quality or accuracy of the results obtained. In other words, even with greater computational capacity or longer processing time, the model's performance may remain unchanged, highlighting that efficiency is not solely associated with intensive resource usage, but rather with the adequacy of the parameters and methods employed.

Attachments

Figure 1. Goodman diagram for learning rates 0.05.

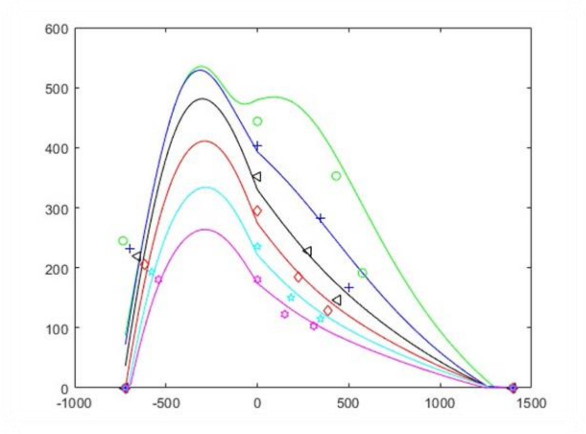


Figure 2. Goodman diagram for learning rates 0.1.

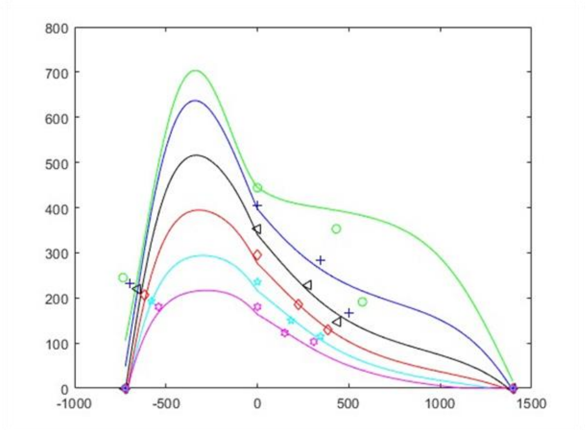
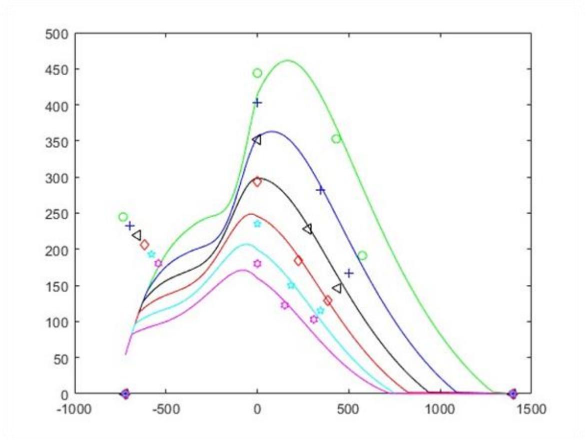


Figure 3. Goodman diagram for learning rates 0.5.



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