Real Options in Agricultural Commodities via Bayesian MCMC

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Abstract Classical real options valuation relies on point estimates of stochastic process parameters, ignoring the uncertainty inherent in parameter estimation. This paper proposes a Bayesian framework that propagates full posterior uncertainty from Markov Chain Monte Carlo (MCMC) estimation into real options valuation for agricultural commodities. We estimate three stochastic processes—Geometric Brownian Motion, Mean-Reverting Jump-Diffusion, and Mean-Reverting Stochastic Volatility—for five agricultural commodity futures (corn, soybean, wheat, live cattle, and soybean meal) using Hamiltonian Monte Carlo via Stan. For each posterior draw, we simulate price paths and compute option payoffs, yielding complete Bayesian distributions of real option values rather than scalar estimates. Results show that parameter uncertainty widens the distribution of timing and abandonment option values relative to classical approaches, with important implications for agricultural investment decisions under uncertainty.

Keywords: Real options; MCMC; Agricultural commodities; Stochastic processes; Bayesian inference.

JEL codes: Q14, C11, G13, G31.

1. Introduction

Agricultural commodity markets are characterized by high price volatility driven by weather shocks, policy changes, and global demand fluctuations (Deaton & Laroque, 1992; Geman, 2005). Investment decisions in this sector—such as expanding production capacity, switching between crops, or abandoning unprofitable operations—are inherently irreversible and must be made under considerable uncertainty (Dixit & Pindyck, 1994). Real options analysis provides a natural framework for evaluating such decisions, as it captures the value of managerial flexibility to defer, expand, contract, or abandon invest-

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ments in response to evolving market conditions (Myers, 1977; Trigeorgis, 1996).

However, standard real options valuation methods typically rely on point estimates of the underlying stochastic process parameters obtained via maximum likelihood estimation (MLE) or method of moments. This approach treats estimated parameters as known constants, thereby ignoring estimation uncertainty—a consequential omission when the underlying data exhibit regime changes, fat tails, or structural breaks common in commodity markets (Hamilton, 1989; Schwartz, 1997). As Tsekrekos & Yannacopoulos (2016) show in the context of optimal switching under stochastic volatility, the choice of stochastic specification can significantly affect real option values and the resulting investment decisions.

Three interconnected research gaps emerge from this literature. First, the *choice of stochastic process* is critical for real option valuation, yet most studies adopt a single specification without systematic comparison across competing models (Schwartz, 1997). Second, the *choice of method for parameter and volatility estimation*—typically MLE or method of moments—ignores the uncertainty inherent in the estimation itself, treating parameters as known constants rather than as distributions. Third, the *intersection of Bayesian methods and real options* remains largely unexplored: while Bayesian inference has become standard in financial econometrics (Johannes & Polson, 2010), its application to real options valuation has been limited to isolated cases that do not propagate full posterior uncertainty into option values (Slade, 2001).

This paper addresses these gaps by proposing a Bayesian framework that fully propagates parameter uncertainty into real options valuation. Our approach proceeds in three steps. First, we specify three stochastic processes of increasing complexity—Geometric Brownian Motion (GBM), Mean-Reverting Jump-Diffusion (MRJ), and Mean-Reverting Stochastic Volatility (MRS)—and estimate their parameters via Markov Chain Monte Carlo (MCMC) using Hamiltonian Monte Carlo (HMC) sampling (Hoffman & Gelman, 2014; Carpenter et al., 2017). Second, for each draw from the posterior distribution, we simulate forward price paths via Monte Carlo and compute the payoffs of two types of real options: timing (deferral) and abandonment. Third, we aggregate across posterior draws to obtain complete Bayesian distributions of option values, with credible intervals that capture both market uncertainty and parameter uncertainty.

We apply this framework to five major agricultural commodity futures traded on the Chicago Board of Trade (CBOT)—corn, soybean, wheat, live cattle, and soybean meal—over a twenty-year sample period (2005–2025).

Our empirical results show that ignoring parameter uncertainty leads to underestimation of the dispersion of real option values, potentially distorting investment decisions for agricultural producers and agribusiness firms.

This paper contributes to the literature by addressing these three gaps simultaneously. First, we compare real option values across three stochastic process specifications of increasing complexity (GBM, MRJ, MRS), enabling systematic assessment of how the *choice of stochastic process* affects investment decisions, with formal model comparison via leave-one-out cross-validation (LOO-CV) (Vehtari et al., 2017). Second, we replace point estimation with full Bayesian inference via MCMC, using Hamiltonian Monte Carlo in Stan (Carpenter et al., 2017), so that parameter and *volatility uncertainty* are captured by the posterior distribution rather than collapsed into a single estimate. Third, we bridge the *intersection of Bayesian methods and real options* by propagating complete posterior distributions into option valuation, yielding Bayesian distributions of option values—rather than scalar estimates—for agricultural commodities.

In addition to this introductory section, this paper is organized into five more sections. Section 2 reviews the relevant literature. Section 3 presents the methodology. Section 4 reports the empirical results. Section 5 discusses implications and limitations. Section 6 concludes.

2. Literature Review

2.1 Real options theory

The real options approach to investment valuation originated with Myers (1977), who recognized that corporate investment opportunities can be viewed as call options on real assets. Dixit & Pindyck (1994) formalized the theory, demonstrating that irreversibility and the ability to delay create option value beyond the traditional net present value (NPV). Trigeorgis (1996) extended the framework to accommodate multiple interacting options, while Copeland & Antikarov (2001) provided practical implementation guidance.

In the commodity sector, real options have been applied to mining investments (Slade, 2001), oil and gas exploration (Schwartz & Smith, 2000), and agricultural production decisions. Bastian-Pinto et al. (2009) valued switching options in the Brazilian ethanol-sugar complex, demonstrating the importance of flexibility in agricultural processing. Brandão et al. (2005) developed binomial decision tree methods for solving real options problems, which have been widely adopted in practice.

2.2 Stochastic processes for commodity prices

The choice of stochastic process is critical for real options valuation. Schwartz (1997) proposed three canonical models for commodity prices: a one-factor model with mean reversion, a two-factor model with stochastic convenience yield, and a three-factor model incorporating stochastic interest rates. Schwartz & Smith (2000) decomposed commodity price dynamics into short-term and long-term components. Lucia & Schwartz (2002) applied similar models to electricity markets.

The adequacy of Geometric Brownian Motion for commodity prices has been questioned by numerous studies documenting mean reversion (Pindyck, 2001), jumps (Eraker, 2001), and regime-switching behavior (Hamilton, 1989; Kim & Nelson, 1999). These empirical features motivate our consideration of multiple stochastic specifications.

2.3 Bayesian methods and MCMC in finance

Bayesian inference via MCMC has become a powerful tool in financial econometrics. Johannes & Polson (2010) survey MCMC methods for continuous-time financial models, while Jacquier et al. (2004) demonstrate the advantages of Bayesian estimation for stochastic volatility models. Eraker (2001) applied MCMC to diffusion models with applications to finance.

The development of Hamiltonian Monte Carlo (Hoffman & Gelman, 2014) and its implementation in probabilistic programming languages such as Stan (Carpenter et al., 2017) has improved the efficiency and reliability of Bayesian computation. Model comparison via the Widely Applicable Information Criterion (WAIC) and leave-one-out cross-validation (LOO-CV) (Vehtari et al., 2017) provides formal tools for selecting among competing stochastic specifications.

2.4 Bayesian approaches to real options

The intersection of Bayesian methods and real options remains relatively unexplored. Slade (2001) estimated real option values for mining investments but did not propagate full posterior uncertainty. Tsekrekos & Yannacopoulos (2016) showed that stochastic volatility specifications affect optimal switching decisions. To our knowledge, no study has simultaneously addressed the three interconnected gaps identified in this paper: (i) systematic comparison across multiple stochastic specifications for real options, (ii) Bayesian rather than frequentist parameter and volatility estimation, and (iii) full posterior uncertainty propagation into real option values. By combining Bayesian MCMC estimation of multiple stochastic processes with complete uncertainty propagation, this paper addresses all three gaps in a unified framework applied to agricultural commodities.

3. Methodology

3.1 Data and variables

We use weekly closing prices for five agricultural commodity futures contracts traded on the CBOT: corn (ZC=F), soybean (ZS=F), wheat (ZW=F), live cattle (LE=F), and soybean meal (ZM=F). The sample period spans January 2005 through December 2025, yielding approximately 1,040 weekly observations per commodity. We also include the S&P 500 index as a benchmark. Weekly log-returns are computed as $r_t = \ln(S_t/S_{t-1})$.

3.2 Stochastic process specifications

We consider three stochastic processes of increasing complexity for the commodity price dynamics.

3.2.1 Model 1: Geometric Brownian Motion (GBM)

The GBM, widely used for commodity price modeling since Black (1976), assumes that log-prices follow a random walk with drift:

$$\mathrm{d}S_t = \mu S_t \,\mathrm{d}t + \sigma S_t \,\mathrm{d}W_t \tag{1}$$

where S_t denotes the commodity price at time t, μ is the drift, $\sigma > 0$ is the volatility, and W_t is a standard Wiener process. The discretized log-return is:

$$r_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_t, \quad \varepsilon_t \sim \mathscr{N}(0,1) \tag{2}$$

3.2.2 Model 2: Mean-Reverting Jump-Diffusion (MRJ)

Following Schwartz (1997), we model log-prices with mean reversion and jumps:

$$\mathrm{d}\ln S_t = \kappa(\theta - \ln S_t)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t + J_t\,\mathrm{d}N_t \tag{3}$$

where $\kappa > 0$ is the mean-reversion speed, θ is the long-run equilibrium log-price, $J_t \sim \mathscr{N}(\mu_J, \sigma_J^2)$ is the jump size, and N_t is a Poisson process with intensity λ.

3.2.3 Model 3: Mean-Reverting Stochastic Volatility (MRS)

The MRS model extends the mean-reverting specification with time-varying volatility, following the stochastic volatility framework of Jacquier et al. (2004):

$$\mathrm{d}\ln S_t = \kappa(\theta - \ln S_t)\,\mathrm{d}t + \sigma_t\,\mathrm{d}W_t^{(1)} \tag{4}$$

$$\mathrm{d}\ln \sigma_t^2 = \alpha(\beta - \ln \sigma_t^2)\,\mathrm{d}t + \xi\,\mathrm{d}W_t^{(2)} \tag{5}$$

where $\alpha > 0$ is the volatility mean-reversion speed, β is the long-run log-variance, $\xi > 0$ is the volatility of volatility, and $\mathrm{Corr}(\mathrm{d}W_t^{(1)}, \mathrm{d}W_t^{(2)}) = \rho$.

3.3 Bayesian inference via MCMC

We adopt weakly informative priors (Gelman et al., 2013) for all model parameters, as summarized in Table 1. Prior distributions were calibrated following standard recommendations: location parameters are centered at data-driven values (e.g., $\theta \sim \mathcal{N}(\ln S_0, 1)$ anchored at the observed log-price), while scale parameters use half-normal priors that concentrate mass near zero but allow for larger values, consistent with the regularization approach advocated by Gelman et al. (2013). Posterior distributions are obtained via the No-U-Turn Sampler (NUTS), a variant of Hamiltonian Monte Carlo (Hoffman & Gelman, 2014), implemented in Stan (Carpenter et al., 2017). For each model and commodity, we run two parallel chains with 750 warmup iterations and 1,500 total iterations, yielding 1,500 posterior draws. The HMC/NUTS sampler achieves substantially higher effective samples per iteration than traditional Metropolis–Hastings, making two chains with 750 post-warmup draws each sufficient for reliable posterior inference (Carpenter et al., 2017). We then thin the posterior to 500 draws for the Monte Carlo option valuation stage.

Convergence is assessed via the potential scale reduction factor ($\hat{R}$) and effective sample size (ESS) for all parameters (Gelman et al., 2013). We target $\hat{R} < 1.05$ and ESS > 200 as minimum thresholds. Model comparison is performed using leave-one-out cross-validation (LOO-CV) (Vehtari et al., 2017).

3.4 Real options valuation

The key innovation of our framework is the propagation of full posterior uncertainty into option valuation. Algorithm 1 summarizes the procedure.

We evaluate two types of real options, approximated via European-style payoff functions evaluated at a one-year horizon. While a full real options

Table 1
Prior distributions for model parameters

Model	Parameter	Prior
GBM	μ (drift)	$\mathcal{N}(0, 0.5)$
	σ (volatility)	Half-$\mathcal{N}(0, 0.5)$
MRJ	κ (mean-reversion)	Half-$\mathcal{N}(0, 1)$
	θ (equilibrium)	$\mathcal{N}(\ln S_0, 1)$
	σ (diffusion)	Half-$\mathcal{N}(0, 0.5)$
	λ (jump intensity)	$\text{Beta}(2, 20)$
	μ_J (jump mean)	$\mathcal{N}(0, 0.3)$
	σ_J (jump std)	Half-$\mathcal{N}(0, 0.2)$
MRS	κ (price MR)	Half-$\mathcal{N}(0, 1)$
	θ (equilibrium)	$\mathcal{N}(\ln S_0, 1)$
	α (vol MR)	Half-$\mathcal{N}(0, 2)$
	β (long-run var)	$\mathcal{N}(\ln 0.04, 1)$
	ξ (vol-of-vol)	Half-$\mathcal{N}(0, 0.5)$
	ρ (correlation)	$\text{Uniform}(-1, 1)$

Algorithm 1 Bayesian Real Options Valuation

for each posterior draw $\boldsymbol{\theta}^{(s)}$, $s = 1, \ldots, S$ **do**
  for each simulated path $j = 1, \ldots, M$ **do**
    Simulate price path $\{S_t^{(s,j)}\}_{t=1}^T$ using $\boldsymbol{\theta}^{(s)}$
    Compute option payoff $\Pi^{(s,j)}$
  end for
  $V^{(s)} \leftarrow e^{-rT} \frac{1}{M} \sum_{j=1}^M \Pi^{(s,j)}$
end for
return $\{V^{(s)}\}_{s=1}^S$ ▷ Bayesian distribution of option value

analysis would involve optimal stopping (American-style exercise), the European approximation is standard in the literature for comparative frameworks (Copeland & Antikarov, 2001) and suffices to demonstrate the impact of parameter uncertainty on option values.

Timing option (deferral). The right to delay an investment and invest only when the commodity price exceeds a threshold:

$$\Pi_{\text{timing}} = \max(S_T - K, 0), \quad K = 1.10 \times S_0 \tag{6}$$

Abandonment option. The right to abandon an operation and recover a salvage value:

$$\Pi_{\text{abandon}} = \max(K_{\text{salvage}} - S_T, 0), \quad K_{\text{salvage}} = 0.80 \times S_0 \tag{7}$$

Note that simulation is conducted under the physical (historical) probability measure $\mathbb{P}$, using posterior draws of the estimated parameters. This contrasts with classical Black–Scholes pricing, which operates under the risk-neutral measure $\mathbb{Q}$. Consequently, the Bayesian and classical estimates are not strictly comparable in an arbitrage-pricing sense; rather, the comparison highlights the additional dispersion introduced by parameter uncertainty under real-world dynamics. For the Black–Scholes benchmark, we use MLE volatility estimates with the standard risk-neutral formula (Black, 1976).

4. Results

4.1 Exploratory analysis

Table 2 reports descriptive statistics for weekly log-returns of the five agricultural commodity futures. All series exhibit excess kurtosis and non-zero skewness, consistent with the well-documented leptokurtic behavior of commodity returns. The Jarque–Bera test rejects normality at the 1% level for all commodities, motivating our use of jump-diffusion and stochastic volatility specifications. Figure 2 confirms these departures from normality, showing heavier tails and asymmetry relative to the Gaussian benchmark. The Augmented Dickey–Fuller test confirms stationarity of log-returns at the 1% level across all series.

Live cattle displays the lowest weekly volatility (std. dev. 2.54%, approximately 18.3% annualized) and the most negative skewness (−0.78), while wheat exhibits the highest weekly volatility (4.59%, approximately 33.1% annualized) and the strongest positive skewness (0.91), reflecting distinct supply-side dynamics. Wheat also presents an extreme weekly return of 46.9%, likely associated with the supply shock following the Russian invasion of Ukraine in February 2022. These stylized facts suggest that a single stochastic process is unlikely to capture the heterogeneity across commodities.

Figure 1 displays the weekly price series over the 2005–2025 sample period. All commodities experienced substantial price spikes during the 2007–2008 food crisis and the post-COVID commodity boom (2020–2022), consistent with regime-switching behavior.

Table 2
Descriptive statistics of weekly log-returns (%)

Commodity	N	Mean	Std	Skew	Kurt	Min	Max	JB	ADF
Corn	1103	0.069	4.140	−0.558	7.246	−25.43	20.28	885.7***	−9.65**
LiveCattle	1099	0.085	2.538	−0.777	6.459	−16.00	9.84	658.5***	−9.93**
Soybean	1103	0.058	3.553	−0.697	9.038	−26.47	23.33	1764.6***	−8.81**
Soybean-Meal	1103	0.054	4.045	−0.510	5.651	−23.02	15.03	370.8***	−10.02**
Wheat	1103	0.045	4.585	0.914	13.424	−21.24	46.94	5147.6***	−12.10**

Weekly log-returns in percentage. JB: Jarque–Bera test statistic. ADF: Augmented Dickey–Fuller test statistic. ***, **, * denote significance at 1%, 5%, 10%.

4.2 MCMC estimation

We estimate each of the three stochastic process specifications for all five commodities via HMC/NUTS in Stan, yielding 15 separate MCMC runs. Table 3 reports posterior summaries for key parameters. All models achieved adequate convergence, with $\hat{R} < 1.02$ for all parameters. Effective sample sizes exceed 400 for the majority of parameters; a few parameters in the MRJ and MRS specifications exhibit lower ESS (minimum 223 for the MRJ equilibrium level of soybean meal), reflecting the greater complexity of these models. The MRS leverage correlation ρ remains largely unidentified by the data, with 95% credible intervals spanning nearly the full prior support $(-1, 1)$ across all commodities, indicating that the weekly frequency does not provide sufficient information to estimate this parameter.

Table 3
Posterior parameter estimates (mean [95% CI])

Commodity	Model	Param.	Mean	95% CI	$\hat{R}$	ESS
Corn	GBM	μ	0.0770	[-0.046, 0.200]	0.999	985
		σ	0.2987	[0.287, 0.311]	1.002	1152
	MRJ	κ	0.2439	[0.057, 0.518]	1.000	698
		θ	6.7479	[6.131, 7.879]	1.003	569
		σ	0.1929	[0.174, 0.213]	1.004	532
		λ	0.2274	[0.149, 0.309]	1.004	462
		μ_J	−0.0091	[-0.022, 0.002]	1.000	916
		σ_J	0.0661	[0.057, 0.079]	1.003	565

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Table 3 – *continued from previous page*

Commod-ity	Model	Param.	Mean	95% CI	$\hat{R}$	ESS
	MRS	κ	0.2566	[0.026, 0.574]	0.999	874
		θ	6.3019	[5.710, 7.342]	1.001	970
		α	2.9786	[1.629, 4.555]	1.000	940
		β	-2.8228	[-3.141, -2.504]	1.000	1843
		ξ	2.1056	[1.686, 2.585]	0.999	899
		ρ	-0.0154	[-0.942, 0.947]	0.999	3943
Soybean	GBM	μ	0.0625	[-0.050, 0.176]	1.000	1275
		σ	0.2565	[0.246, 0.267]	1.000	1153
	MRJ	κ	0.2975	[0.084, 0.573]	1.001	495
		θ	7.6410	[7.167, 8.719]	1.002	320
		σ	0.1950	[0.182, 0.208]	1.000	689
		λ	0.1006	[0.057, 0.159]	1.000	436
		μ_J	-0.0274	[-0.051, -0.011]	1.002	727
		σ_J	0.0695	[0.056, 0.088]	1.000	651
	MRS	κ	0.2886	[0.024, 0.657]	1.000	842
		θ	7.2323	[6.836, 8.230]	1.002	515
		α	2.3270	[1.077, 3.759]	1.001	884
		β	-3.1313	[-3.470, -2.804]	1.000	1536
		ξ	1.6410	[1.258, 2.046]	1.002	816
		ρ	-0.0080	[-0.948, 0.951]	0.999	3707
Wheat	GBM	μ	0.0768	[-0.061, 0.214]	1.000	3554
		σ	0.3307	[0.317, 0.345]	1.000	3578
	MRJ	κ	0.7421	[0.209, 1.261]	1.001	1987
		θ	6.3256	[6.090, 6.568]	0.999	1857
		σ	0.2907	[0.272, 0.311]	1.001	2598
		λ	0.0336	[0.009, 0.078]	1.001	2341
		μ_J	0.0245	[-0.041, 0.106]	1.001	2027
		σ_J	0.1350	[0.085, 0.237]	1.002	2011
	MRS	κ	0.7549	[0.228, 1.265]	1.000	2501
		θ	6.2662	[6.026, 6.465]	1.005	836
		α	2.0308	[0.679, 3.683]	1.001	2241
		β	-2.5019	[-2.840, -2.178]	1.000	2843
		ξ	1.2597	[0.862, 1.696]	1.002	2397
		ρ	0.0099	[-0.953, 0.954]	1.000	2835
LiveCattle	GBM	μ	0.0604	[-0.023, 0.138]	1.002	915
		σ	0.1832	[0.176, 0.191]	1.001	1145
	MRJ	κ	0.2053	[0.078, 0.396]	1.010	828
		θ	5.7971	[5.184, 6.876]	1.016	578
		σ	0.1347	[0.124, 0.146]	1.001	786
		λ	0.1616	[0.097, 0.240]	1.000	722
		μ_J	-0.0170	[-0.028, -0.008]	1.000	927
		σ_J	0.0404	[0.033, 0.049]	1.000	895
	MRS	κ	0.1235	[0.024, 0.282]	1.001	747
		θ	5.6858	[4.978, 6.885]	1.011	586

Continued on next page

Table 3 – *continued from previous page*

Commodity	Model	Param.	Mean	95% CI	$\hat{R}$	ESS
		α	2.6877	[1.144, 4.482]	1.002	365
		β	−3.6158	[-3.883, -3.312]	1.002	654
		ξ	1.5021	[1.066, 1.989]	1.000	418
		ρ	0.0085	[-0.952, 0.953]	0.999	1909
Soybean-Meal	GBM	μ	0.0660	[-0.063, 0.200]	1.001	996
		σ	0.2920	[0.280, 0.305]	0.999	1072
	MRJ	κ	0.3912	[0.078, 0.809]	1.003	395
		θ	6.2144	[5.786, 7.289]	1.017	223
		σ	0.2260	[0.201, 0.248]	1.002	403
		λ	0.1485	[0.072, 0.251]	1.004	400
		μ_J	−0.0150	[-0.039, 0.001]	1.003	405
		σ_J	0.0664	[0.053, 0.084]	1.004	605
	MRS	κ	0.4345	[0.034, 0.888]	1.001	2238
		θ	5.7729	[5.335, 6.353]	1.001	1458
		α	1.9531	[0.798, 3.365]	1.001	2586
		β	−2.7686	[-3.134, -2.411]	1.000	2997
		ξ	1.4204	[1.054, 1.824]	1.000	2298
		ρ	0.0092	[-0.947, 0.948]	1.000	3103

Posterior means with 95% Bayesian credible intervals from HMC/NUTS estimation. $\hat{R}$: potential scale reduction factor. ESS: effective sample size.

For model comparison, Table 4 presents the LOO-IC values for each commodity–model combination. Lower LOO-IC indicates better out-of-sample predictive performance.

Table 4
Model comparison via LOO-IC

Commodity	GBM	MRJ	MRS
Corn	−3889.7 (83.8)	−4051.3 (67.6)	−4200.6 (62.9)
Soybean	−4226.3 (95.4)	−4394.4 (65.3)	−4564.0 (55.3)
Wheat	−3661.3 (121.7)	−3767.9 (63.5)	−3860.5 (61.5)
LiveCattle	−4951.7 (78.0)	−5075.0 (63.1)	−5113.6 (63.2)
Soybean-Meal	−3941.2 (72.0)	−4033.4 (61.7)	−4166.2 (56.7)

LOO-IC: Leave-One-Out Information Criterion (lower is better). Standard errors in parentheses.

4.3 Real option values

The central contribution of this paper is the Bayesian distribution of real option values obtained by propagating posterior uncertainty through Monte Carlo simulation. For each of the 500 thinned posterior draws, we simulate 5,000 price paths over a 52-week horizon and compute discounted option payoffs.

Table 5 compares the Bayesian option values (posterior mean and 95% credible interval) with classical Black–Scholes point estimates using MLE parameters. Although the two approaches operate under different probability measures (physical vs. risk-neutral), the comparison shows that the Bayesian credible intervals are considerably wider than the classical point estimates, reflecting parameter uncertainty that the classical approach ignores.

Table 5
Bayesian real option values vs. classical Black–Scholes

Commodity	Model	Timing Option		Abandon Option		BS Timing
		Mean	95% CI	Mean	95% CI	(point)
Corn	GBM	53.08	[27.9, 93.5]	9.91	[4.0, 18.6]	43.01
	MRJ	41.99	[18.5, 74.9]	8.18	[2.8, 15.5]	43.01
	MRS	38.81	[18.7, 66.1]	6.25	[1.3, 12.5]	43.01
LiveCattle	GBM	13.80	[6.2, 24.0]	1.13	[0.4, 2.6]	11.97
	MRJ	3.58	[0.4, 9.2]	3.50	[1.1, 8.1]	11.97
	MRS	9.27	[2.5, 16.9]	1.24	[0.1, 3.0]	11.97
Soybean	GBM	96.53	[47.5, 174.7]	16.93	[6.2, 32.9]	84.49
	MRJ	77.88	[39.4, 132.3]	12.47	[3.8, 28.6]	84.49
	MRS	75.79	[34.9, 125.2]	7.72	[0.3, 17.7]	84.49
Soybean-Meal	GBM	32.93	[15.1, 57.7]	6.81	[2.6, 13.7]	28.16
	MRJ	28.35	[14.2, 47.7]	3.76	[1.1, 9.0]	28.16
	MRS	21.65	[9.5, 36.7]	3.49	[0.5, 8.6]	28.16
Wheat	GBM	68.22	[32.7, 121.1]	15.49	[6.7, 29.0]	56.40
	MRJ	55.61	[30.2, 95.5]	5.58	[1.3, 13.8]	56.40
	MRS	35.77	[16.2, 61.5]	6.11	[0.8, 15.6]	56.40

Option values in USD. 95% CI denotes Bayesian credible interval from posterior uncertainty propagation. BS: Black–Scholes with MLE point estimates.

Figure 3 displays the posterior distributions of timing option values across models and commodities, and Figure 4 shows the corresponding abandonment option distributions. Figure 5 further illustrates this comparison, showing that the Bayesian posterior means diverge from the classical Black–Scholes point estimates across models and commodities.

5. Discussion

Parameter uncertainty matters. The Bayesian credible intervals for option values are considerably wider than the point estimates obtained from classical Black–Scholes valuation, even accounting for the fact that the two approaches operate under different probability measures. This finding aligns with Tsekrekos & Yannacopoulos (2016), who showed that the choice of volatility specification materially affects option values, and provides empirical evidence that ignoring parameter uncertainty can lead to overconfident investment decisions. For agricultural producers and agribusiness firms, this means that classical NPV and Black–Scholes analyses may understate the risk associated with investment timing and abandonment decisions.

Model choice affects option values. The three stochastic process specifications (GBM, MRJ, and MRS) yield materially different distributions of option values. Models incorporating mean reversion (MRJ and MRS) tend to produce lower timing option values compared to GBM, reflecting the tendency of mean-reverting prices to return to equilibrium rather than trend indefinitely. This finding is consistent with Schwartz (1997), who documented the importance of mean reversion for commodity derivative pricing. Notably, while the MRS model achieves the best LOO-IC across all commodities (Table 4), the leverage correlation parameter ρ remains unidentified, suggesting that the model's superior fit derives primarily from its flexible volatility dynamics rather than from the price-volatility correlation channel. This raises a caveat: LOO-IC favors the MRS partly because it has $T-1$ latent volatility states that absorb heteroskedasticity, and users should weigh this flexibility against the model's greater complexity.

Commodity heterogeneity. The stylized facts documented in Section 4.1—the variation in skewness, kurtosis, and volatility across commodities—translate into material differences in option value distributions. Live cattle, with its lower volatility and stronger mean reversion, exhibits narrower option value distributions than wheat or corn. This suggests that a uniform approach to real options valuation across agricultural commodities is inappropriate.

Practical implications. Our framework provides decision-makers with a complete probability distribution over option values rather than a single number. This enables risk-aware investment analysis: rather than asking "is the option worth exercising?" based on a point estimate, managers can assess the probability that the option value exceeds a given threshold, incorporating both market and parameter uncertainty. For agricultural cooperatives and processing firms, the timing and abandonment option distributions offer valuable information for capacity planning and exit decisions.

Limitations. Several limitations warrant discussion. First, our Monte Carlo simulation uses physical measure parameters estimated from historical data, whereas rigorous option pricing requires risk-neutral dynamics. The comparison with Black–Scholes—which operates under the risk-neutral measure—is therefore approximate; the observed differences reflect both parameter uncertainty and the discrepancy between pricing measures. Future work should incorporate market prices of risk to enable consistent risk-neutral valuation. Second, the weekly frequency may smooth out intra-week dynamics. Third, we model each commodity independently, whereas a portfolio-level analysis might capture diversification benefits. Fourth, our option payoffs are evaluated at a fixed terminal date (European-style); American options with early exercise features would require optimal stopping methods such as Longstaff–Schwartz simulation. Fifth, the leverage correlation parameter ρ in the MRS model is unidentified at weekly frequency, suggesting that the stochastic volatility component may require higher-frequency data or informative priors to be fully estimated.

6. Conclusion

This paper proposes and implements a Bayesian framework for real options valuation in agricultural commodities that propagates parameter uncertainty from MCMC estimation into option value distributions. Applying three stochastic process specifications—GBM, MRJ, and MRS—to five CBOT commodity futures over 2005–2025, we show that classical point-estimate approaches understate the uncertainty in timing and abandonment option values.

Our main finding is that the Bayesian credible intervals for option values are considerably wider than classical Black–Scholes point estimates, with important implications for agricultural investment decisions. The framework provides practitioners with a distribution-based approach for evaluating irreversible investments under uncertainty, bridging a gap between Bayesian econometrics and real options theory that has remained largely unexplored in the agricultural commodity domain.

Future research could extend this framework in several directions: American-style options with optimal stopping via Bayesian Longstaff–Schwartz methods (Longstaff & Schwartz, 2001), risk-neutral pricing with estimated market prices of risk, switching options across commodity pairs, regime-switching dynamics, and multi-commodity portfolio optimization under posterior uncertainty.

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Conflict of interest The authors declare no conflict of interest.

Artificial Intelligence This research utilized AI tools to assist in data analysis, manuscript drafting, and figure generation. All AI-generated content was critically reviewed and validated by the authors to ensure accuracy and alignment with the scientific integrity of the study.

Data availability All data used in this study are publicly available from Yahoo Finance. Replication code is available from the corresponding author upon reasonable request.

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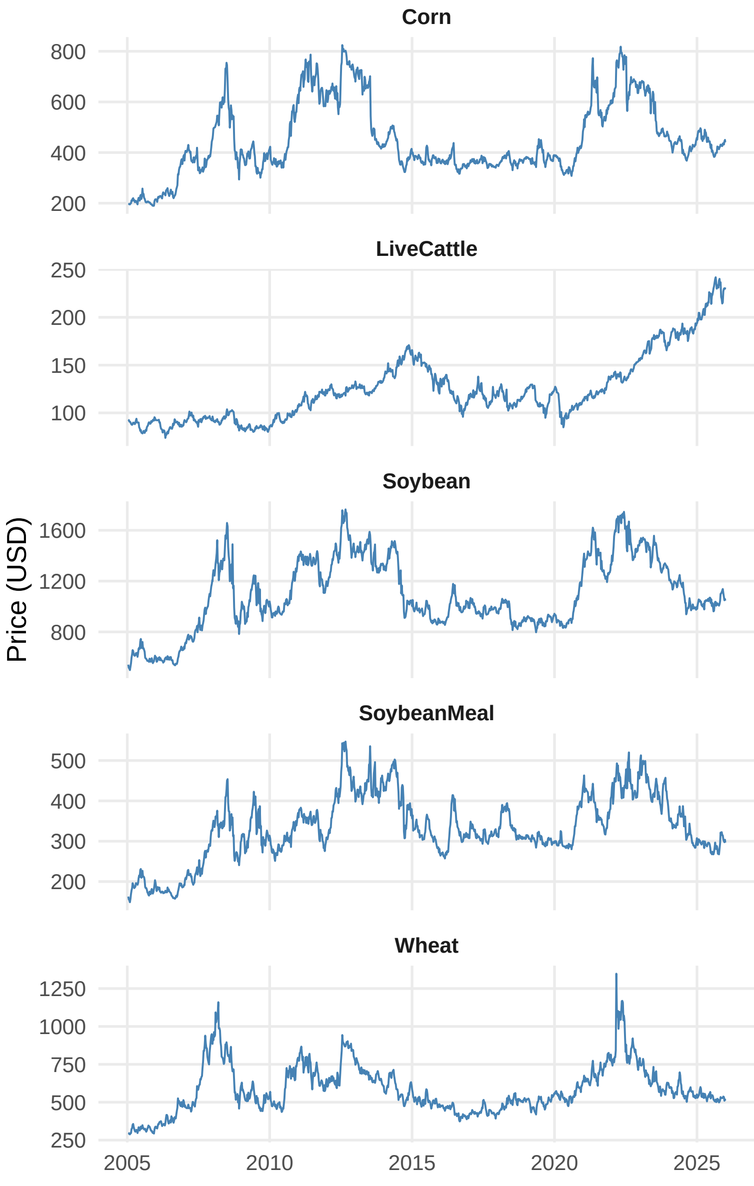

Figure 1
Weekly closing prices for five agricultural commodity futures (2005–2025).

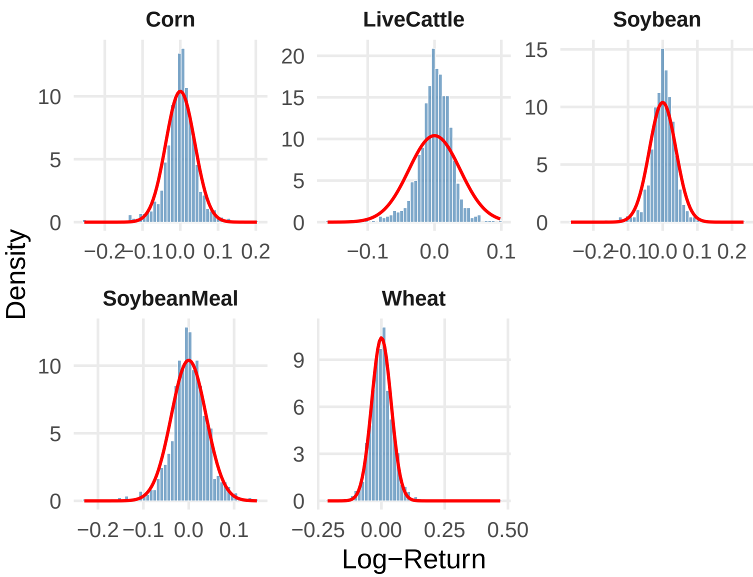

Figure 2

Empirical distribution of weekly log-returns with normal density overlay. The heavier tails relative to the Gaussian benchmark motivate jump and stochastic volatility models.

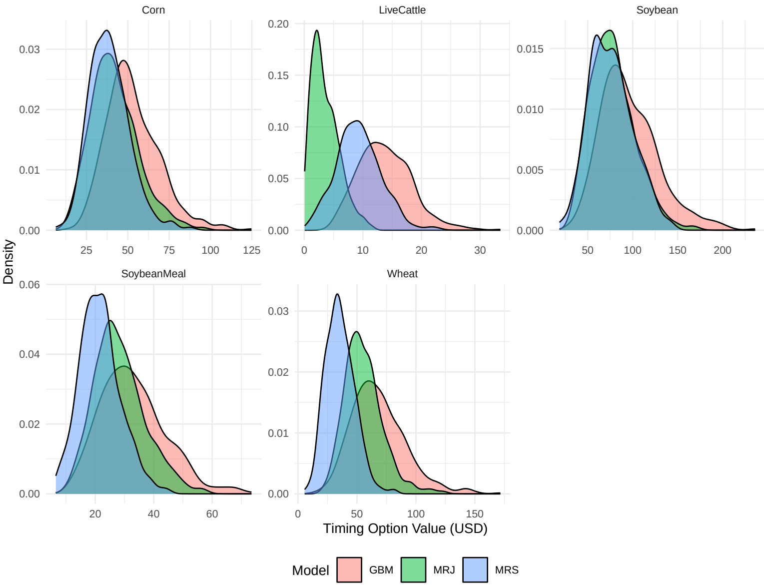

Figure 3
Posterior distributions of timing option values by commodity and model.

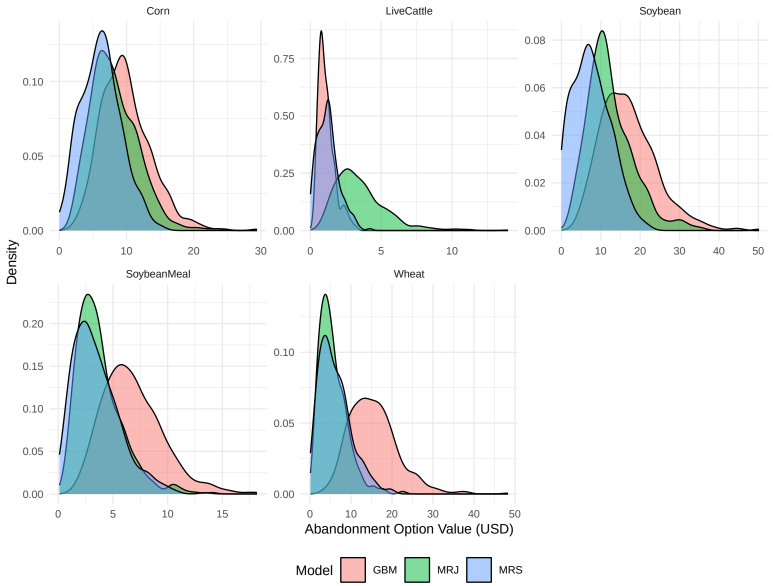

Figure 4
Posterior distributions of abandonment option values by commodity and model.

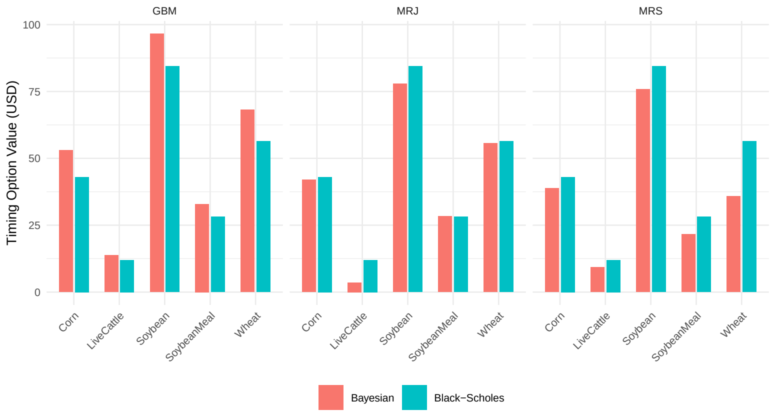

Figure 5
Comparison of Bayesian posterior mean vs. classical Black–Scholes timing option values by commodity and model.