Specialization, Skill Mismatch, and Labor Market Risk

Fernando Lopes[*] Cache Ellsworth[†]

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Abstract

In this paper, we build a model of the labor market with search frictions in which workers choose how much to invest in a multidimensional set of skills before entering the labor force. Specialization can generate a larger match value between workers and firms conditional on being matched. We show that search frictions have important consequences for skill allocation, distorting skill choices and affecting the quality of matches in the labor market. Workers become less specialized as a way to insure against labor market risk, reducing the match value between workers and firms. This induces a novel type of cost of search frictions, which we call skill distortion. We propose a method to separately identify the output cost of skill distortion in our model and show that it is quantitatively relevant.

1. Introduction

Mismatch between workers' skill sets and jobs' skill requirements is prevalent and can generate sizable output and wage losses (Guvenen et al., 2020; Lise and Postel-Vinay, 2020). Most occupations usually require a combination of cognitive, language, and social skills, but there is large variation from job to job in the intensity with which workers are required to use these skills to produce output. When considering multidimensional skills, then, worker specialization is a key component in how to determine how productive a particular match really is.

[*]University of Wisconsin-Madison. fllopes@wisc.edu.
[†]University of Wisconsin-Madison. crellsworth@wisc.edu.

Workers who are more specialized in a narrow set of tasks can be more productive conditional on finding the right job. In the presence of search frictions, however, specialized workers face the risk of finding an occupation that doesn't match their skill set – or worse, not finding a job at all. When employment is uncertain and workers are risk averse, the decision to focus a great deal of time and money on building a single skill might then seem less attractive. Additionally, even if the worker managed to find a job that is a good match for their skills, in the event that they lose that job, they will have a harder time finding another one, so unemployment presents a larger income risk for specialized workers. This trade-off is at the core of how workers decide to invest in their skills prior to entering the labor market, with diversification in skills acting then as insurance for workers when there is risk in the labor market. This diversification may result in lower output, as workers invest in skills they might not end up using in their occupation.

This paper documents the costs and benefits of specialization for workers and how workers choose their skill portfolio to insure against labor market risk. We develop a structural model of the labor market with search frictions and endogenous investment in multidimensional skills to capture the response of workers to labor market conditions. Workers choose a skill mix before they enter the labor market according to their their idiosyncratic preferences and abilities, as well as market conditions. A model with these features is capable of capturing the trade-off between a higher wage conditional on being matched and the insurance skill diversification provides. We discipline this model using data on worker skills and wages from the National Longitudinal Survey of Youth 1979 (NLSY79), along with data on skill requirements from the Occupational Information Network (O*NET). With these two data sources, we can infer the supply and demand of skills in a certain market to calibrate our model. The model allows us to quantify the effects of search frictions on mismatch and, in turn, how much output is lost due to incomplete specialization for insurance motives.

This particular dimension of mismatch is relevant because, as we find in this paper, a significant part of mismatch between workers and firms comes from workers' investment in skills that happen before they enter the labor market. Thus, any policy aiming to improve match quality that only affects the decisions of workers once they enter the labor force will be muted and policy effectiveness will be diminished. One such example of a policy that could be impacted by this is unemployment insurance. One of the main objectives of unemployment insurance is to improve match quality in the labor market, giving workers additional incentive to wait until the "right job" comes along. However, if part of what determines what is the "right job" for a worker is determined by labor market conditions before the worker entered the labor market (say, when they were in college), then there is a degree of mismatch that cannot be remedied by just increasing the value of waiting for the worker.

We understand mismatch between workers and firms as coming from two sources. The first, more widely discussed in the literature, is misallocation – search frictions create matches that shouldn't exist by pairing workers and firms that are not a good fit for each other but, due to the cost of waiting for a better match, end up having a positive match value.

The second, a novel mechanism, is what we dub skill distortion: workers choose to invest in skills they wouldn't have invested in the absence of search frictions as insurance. This generates mismatch because workers have lower relevant skills for their particular occupation because they had to spend time and resources learning other skills to insure against the uncertainty of the matching process. Another effect of skill distortion is that it changes the optimal allocation of workers in the labor market due to distorted skill choices (so that even if workers were allocated efficiently to jobs conditional on their skill distribution, we would still have lower output due to distorted skill choices).

We define specialization precisely by how dispersed the skills of a worker are. A specialized worker is someone who is very skilled in a narrow range of competencies (say, mathematics and logical thinking), but not very proficient in all the others (public speaking and manual labor, for example). In contrast, a diversified worker is someone who has similar capabilities in many or all skills. We capture this with a measure of the variance of the match value (or productivity) of a worker across all the occupations in the labor market, which we call skill remoteness, in line with Macaluso (2017). This measure can be understood also as the expected mismatch that worker would experience when matched with a random occupation. Armed with this definition, we find a distribution of worker specialization that is skewed right with a large tail towards being very specialized.

Empirical analysis of wage data suggests that specialization has important consequences for workers. It is associated with higher mismatch on average, suggesting specialized workers have a harder time finding an occupation that's a good fit for them. Since mismatch has direct consequences on productivity, this translates to a wider wage distribution for specialized workers: they do well if they can find the right job, but the worst-case scenario is much worse for them since their productivity varies so much between occupations.

Our key result is quantifying how much output is lost due to skill distortion. In the absence of search frictions, workers know exactly for which occupation they will be hired in equilibrium and thus will tailor their skill set to that specific occupation or their ideal occupation. In this case, there is no place for diversification of skills as insurance. Diversification only matters for output and wages insofar as search frictions are present in the labor market. We leverage this knowledge to obtain counterfactual matches and skill choices when there is no risk in the labor market. With this, we are able to perform an accounting exercise in our model that allows us to disentangle how much mismatch observed in the data comes from each source, misallocation and distortion. We find that as much as 18% of the additional mismatch generated by search frictions comes from before workers enter the labor market, that is, from distorted skill choices.

Related Literature. Some papers have studied the relationship between specialization and productivity, such as Becker and Murphy (1992), Bolton and Dewatripont (1994), Yang and Borland (1991), and more recently Bassi et al. (2023). The closest paper to our research question, focusing on the trade-offs faced by specialized workers, is Martellini and Menzio (2021). They find that specialized workers have experienced large wage gains relative to non-specialized workers as a result of search frictions declining since 1980. Their main mechanism for explaining this phenomenon is similar to ours: secular reductions in search frictions allowed these workers to find jobs that are a better match for their skills and thus earn more. Our main contribution relative to their paper is to endogenize skill choice as a function of search frictions, which allows search frictions to distort the choices of workers even before they go into the job market.

One paper that investigates the cost of specialization through skill remoteness is Macaluso (2017). This paper is similar to our own by investigating the costs of displacement for workers who have a skill profile that doesn't match the jobs in their local labor market. We build on her work by endogenizing the skill choice of workers as a function of the supply and demand of skills in the labor market. The skill mismatch between workers and firms is an equilibrium object, so any policy that doesn't take into consideration the dynamic incentives for insurance in the face of labor market risk misses a crucial mechanism through which the policy might operate.

Similarly, our paper builds off the larger multidimensional skill and skill mismatch literatures (such as Guvenen et al., 2020, Lise and Postel-Vinay, 2020, Lindenlaub, 2017, and Heckman et al., 2006) which finds skill mismatch has a large impact on worker's wages[1]. Guvenen et al. (2020), in particular, measures multidimensional skill mismatch from a worker-specific

[1]For a complete survey on the effects of heterogeneous human capital on workers, see Sanders and Taber (2012)

measure of cognitive and non-cognitive skills to cognitive and non-cognitive skill requirements that are occupation-specific. We follow in their footsteps by modeling firms with an exogenous profile of skill requirements and workers choosing their skills to respond to the distribution of firms in the economy. Then, like in their model, inefficiencies arise from a mismatch between workers' skills and a firm's skill requirements. We do allow, however, for there to be a mismatch in absolute skills as well, since the random search protocol might pair up a productive worker with an unproductive firm (and vice-versa). Having these two types of inefficiency at once is key for our quantification exercise, which aims to show that distortion from the insurance motive is a relevant force.

2. The Costs of Specialization

This section details our data sources, sample construction and exhibits some evidence of the costs of becoming more specialized for workers. We mainly use two data sources: NLSY79 for workers characteristics and O*NET for occupation characteristics. In NLSY79, we observe a worker's occupation, wage, demographic characteristics as well as test scores immediately they enter the labor market. To construct skill measures for workers, we follow closely the procedure outlined in Guvenen et al. (2020).

For math and verbal skills, we use scores in distinct sections of the Armed Forces Vocational Aptitude Battery (ASVAB), a vocational test taken by all respondents of the NLSY79 when they were 18 years old. For math skills, we take the average of the Arithmetic Reasoning and Mathematics Knowledge sections; for verbal skills, we take the average of the Word Knowledge and Paragraph Comprehension sections. For social skills, on the other hand, we take an average of two tests available also in NLSY79, the Rotter Locus of Control Scale (RLCS) and Rosenberg Self-Esteem Scale (RSES). RLCS measures respondents' feelings about oneself, their self-worth, and satisfaction. RSES, on the other hand, measures respondents' feelings about their autonomy in the world and the primacy of their self-determination rather than chance.

To measure skill requirements specific to each occupation, we obtain data from O*NET with the importance of particular competences for the execution of that job. For math skill requirements, we average thirteen variables measuring the importance of mathematical reasoning; for verbal skill requirements, we average thirteen variables measuring the importance of verbal understanding; finally, for social skill requirements, we average six variables measuring the importance of social interactions. To connect O*NET to NLSY79, we use the occupation code crosswalk provided by the Defense Manpower Data Center (DMDC). To make measures comparable then, we rank workers and firms along each dimensions according to their percentile rank.

One simple way to measure the concentration of worker skills is to take the vector of skills for each worker and compute the distance from that vector to any of the axes. The idea is that a vector of skills that lies, say, on the math axis is fully specialized in math skills, but a vector that lies right in the middle is fully diversified. A simple function that captures the ditance between a vector and the closest axis is $1-\sin(2\theta)$, where θ is the angle between the vector and the closest axis. This function is well behaved, symmetric, and bounded between 0 and 1, with 0 being a vector on the 45-degree line and 1 being a vector that lies on an axis.

We compute this distance meaure for every worker on our sample, as well as occupations and find the graph in Figure 1. In blue we have the histogram of this measure of specialization for worker skills and in orange we have the distribution for occupation skill requirements. We can see that the distribution of worker skills is heavily skewed towards zero, that is, most workers in our sample are generalists. Still, there is a small number of workers that are very specialized. Looking at the distribution of skill requirements, we see that most occupations require a generalist set of skills as well, but there is a significant number of occupations that require very specific skill sets, with some bunching very close to one.

Comparing this to the distribution of worker skills, we can see that there is a significant number of jobs that would benefit from having more specialized workers in the market, but workers do not accumulate their skills for those specific jobs. This is inconsistent with a frictionsless economy (or even a directed search process), since these firms would be willing to pay high wages for workers with those skills, so if worker knew for sure that they would be able to find those jobs, they would accumulate skills to match their requirements. Our model offers a way to interpret the right tail of these distributions: because matching is random in the labor market, workers cannot focus on those jobs, because the downside of not finding that specific job is too dire, so the distribution of worker skills is more skewed towards zero.

Now we are ready to introduce our measure of mismatch between a worker and a firm. For individual i in occupation j, mismatch is measured as

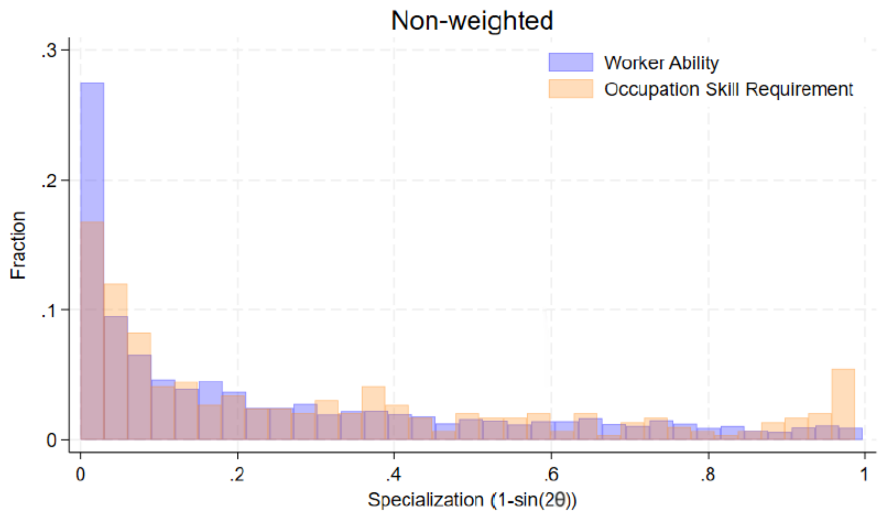

Figure 1: Distance between skill/skill requirement vectors and the closest axis. The measure of distance is $1 - \sin(2\theta)$, where θ is the angle between the skill/skill requirement vector and the closest axis. In blue, we have the measure for all worker skills in NLSY79. In orange, we have the measure for all occupation skill requirements in O*NET.

$$m_{i,j} = \frac{1}{S}\sum_{s=1}^{S}\Big(q(x_{i,s}) - q(y_{j,s})\Big)^2 \tag{1}$$

where $x_{i,s}$ represents the ability of individual i for skill s; $y_{j,s}$ represents the requirements of occupation j for skill s; and $q(\cdot)$ represents the quantile rank function. This measure captures the gap between a worker i's skill and the level of skill required to perform occupation j. It is a quadratic measure, so larger gaps penalize productivity more than small gaps. It is also a symmetric measure, so workers that are too qualified for that job also generate output losses. This captures the idea that an overqualified worker is usually demotivated or must be compensated through amenities to compensate for the disutility of working, which reduces match value. A similar argument to this is employed in Rosen (1986); Lise and Postel-Vinay (2020); Boerma et al. (2023).

Our first piece of evidence comes from regressing log wages on the mismatch between a worker's skills and the skill requirements at the occupation in which they are employed. Formally, we denote by $j(i,t)$ the occupation at which worker i is employed at time t. We run

$$\log(wage)_{i,t} = \beta_0 + \beta_1 m_{i,j(i,t)} + \eta Z_{i,t} + \varepsilon_{i,t}$$

Dependent variable: Log wages		
	(1)	(2)
Mismatch (z-score)	-0.046	-0.019
	(0.01)	(0.01)
Constant	7.283	6.003
	(0.01)	(0.14)
Controls	-	X
N	44,651	44,651
R^2	0.00	0.29

Table 1: Regression of log wages on the measure of mismatch of a worker with the occupation that worker was employed in at the moment of the wage observation. First column includes no controls and second column includes demographic controls.

where $Z_{i,t}$ denotes a set of worker-specific controls, like ethnicity, sex, educational level, and job tenure. We convert the values of mismatch to a z-score to make interpretation more intuitive. As it can be seen in Table 1, we find that workers that have higher mismatch with their occupation earn on average 0.02 log points lower wages. In monetary terms, this translates to the average worker earning \$22.5/week less due to mismatch.

The key measure for our purposes, however, is a measure of specialization. The decision to specialize in certain skills for workers is risky insofar they cannot perform equally in a wide number of jobs. Our stance on how to measure specialization (and we will further justify it within our quantitative model) is by measuring the risk of mismatch a worker faces in the job market. Then, we introduce the following measure of skill remoteness for individual i in year t

$$R_{i,t} = \sum_{j=1}^{J} \omega_{j,t} m_{i,j} \tag{2}$$

where $\omega_{j,t}$ is the share of occupation j at time t in NLSY79. Two key ideas inform this measure. First is that specialization translates to risk, that is, $R_{i,t}$ can be understood as the average mismatch of worker i is they were randomly matched to a job, but also as the variance in their productivity across all the skill requirements in the labor market. That means that if, for example, there are three types of jobs in an economy, each requiring a high level of a certain skill, then a worker that is fully specialized in math will have a higher variance in their productivity than someone with a balanced combination of all three. This is a very similar notion of specialization to that in Martellini and Menzio (2021). The second is that the risk of specialization depends on conditions of the market. An accountant that is fully specialized in math faces high risk in a market where most occupations value social skills, but faces no risk at all in a market where 90% of occupations value math skills, even if they are not accounting jobs per se.

For our second piece of evidence, we plot in Figure 2 moments of the wage distribution conditional on the level of specialization. On the x-axis, we have the rank of workers in the skill remoteness measures, divided into 50 bins, while on the y-axis we have log wages. We plot the 90th and 10th percentiles of the wage distribution for each bin. The gap between the top and the bottom of the wage distribution widens as workers become more specialized. This provides insight into the nature of the risk more specialized workers face. Wage dispersion increases for more specialized workers, but mostly due to the left tail of the distribution falling out. This means that workers that are more mismatched to occupations available in the economy are exposed to approximately 0.2 lower wages when comparing the most specialized to the least specialized workers. This is potentially a reason why very few workers want to be at the right tail of the distribution in Figure 1 even though there is a significant number of occupations there: the downside of failing to meet a firm willing to hire them for a very specialized occupation is much worse.

Finally, we intend to show that specialization does indeed imply a higher probability of the worker being mismatched with their occupation. To this end, we regress the worker's mismatch with their current occupation on their measure of skill remoteness. To eliminate concerns of endogeneity, we compute remoteness by removing their current occupation. That is, we define alternative measure

$$\tilde{R}_{i,j} = \sum_{j' \neq j} \omega_{j',t} m_{i,j'}$$

That is, the measure of skill remoteness when we leave out occupation j. Then, we run the following regression

$$m_{i,j(i,t)} = \beta_0 + \beta_1 \mathbb{1}\{\tilde{R}_{i,j(i,t)} \text{ top } 50\%\} + \eta Z_{i,t} + \gamma_t + \zeta_{j(i,t)} + \varepsilon_{i,j(i,t)}$$

where $\mathbb{1}\{\tilde{R}_{i,j(i,t)} \text{ top } 50\%\}$ is a binary variable indicating when the worker's skill remoteness is above the median when computed without their current occupation. We also include fixed

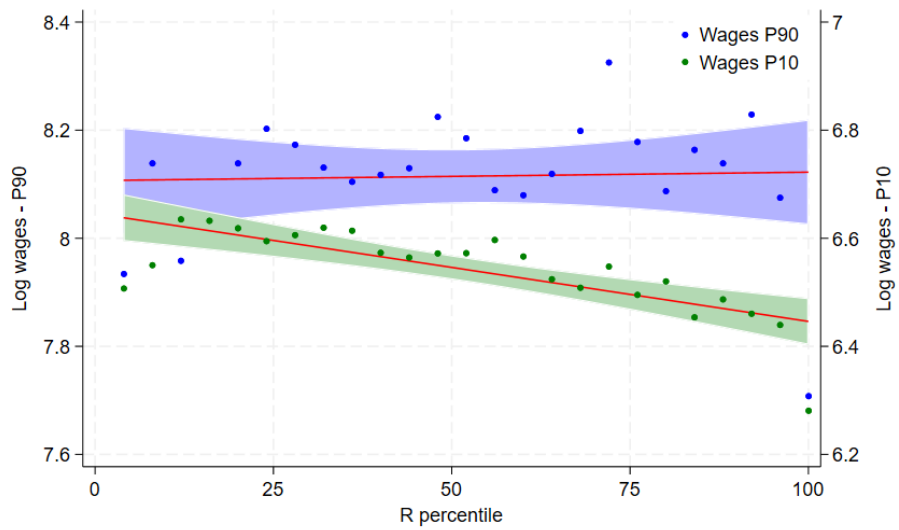

Figure 2: Wage distribution conditional on the level of specialization, as measured by skill remoteness. On the x-axis, we have workers grouped into 50 bins of skill remoteness R and then ranked. On the y-axis, we have log wages. The blue dots represent the 90th percentile of the wage distribution for each bin and green represents the 10th percentile. Blue and green confidence intervals for the nonparametric regression are shown for 90th and 10th percentile, respectively.

effects γ_t for time and $\zeta_{j(i,t)}$ for occupation, as well as the usual demographic controls.

The goal of this regression is to show that specialization (skill remoteness) is correlated with more mismatched at their current occupation. It seems tautological, but note that this need not be the case. Suppose there is a worker who is fully specialized in verbal skills and there is only one job who requires that level of specialization, making the match between that worker and every other occupation very poor. If the labor market were frictionless, even if that worker has a high level of remoteness, they would be able to find exactly the job where their mismatch is zero. Specialization, then, does not necessarily imply a bad match. It is only associated with worse matches if labor market risk, in the form of search frictions, plays a significant role in mediating workers and firms. As we can see in Table 2, that is indeed the case, with worker who are more specialized experiencing more mismatch with their current occupation on average.

Taken together, the evidence suggests two key features of the interaction between specialization and risk. First, Workers are exposed to different mismatch risks depending on their skill profile. More specifically, more specialized workers are more exposed to mismatch risk in the form of a lower left tail of the wage distribution. Second, workers want to avoid mismatch, as it

Dependent variable: Mismatch

	(1)	(2)
$\tilde{R}_{i,j(i,t)}$ top 50%	0.016	0.021
	(0.003)	(0.003)
Constant	0.097	0.099
	(0.002)	(0.042)
N	44,655	44,655
r^2	0.00	0.17
$m_{i,j(i,t)}$ mean for $\tilde{R}_{i,j(i,t)}$ bottom 50%	0.096	0.096
Controls	-	X

Table 2: Regression of mismatch on the alternate measure of remoteness computed by removing the current occupation of the worker.

can be very costly in terms of wages. Then, labor market risk provides an incentive for workers to diversify their skill sets. To capture this, we need a model with specific ingredients, which we highlight in the next section. These ingredients are: endogenous skill choice; multidimensional skills and skill requirement, to have a meaningful notion of specialization; labor market risk, in the form of search frictions; and risk-averse workers, so that they have an insurance motive for diversification.

3. Toy Model

In this section, we will propose a simple model that allows us to illustrate how labor market uncertainty leads to underinvestment in specialization and how such a model might be useful to inform our empirical strategy. This is merely an exercise to build intuition before we move on to the full quantitative model. First, we solve for the equilibrium without any frictions, then we introduce search frictions and, finally, we show how it might be used to quantify the wage and output losses that come from the risk effect of search frictions, disentangling it from misallocation effects.

This is a static one-to-one assignment model of workers and firms. There is a mass of workers equal to one, each with total skill endowment x, distributed between $[\underline{x}, \overline{x}]$ according to cdf $F(x)$. Workers have preferences over wages, w, given by $u(w)$, where u is increasing and strictly concave. Before going to the labor market, workers decide how to divide their total skill endowment between cognitive and non-cognitive skills, which we denote by x_1 and x_2 respectively. That is, they choose x_1 and x_2 to maximize their expected utility under the constraint that $x_1 + x_2 = x$. The wage is determined by Kalai bargaining over total match output, with $\eta \in (0,1)$ being the bargaining power of workers.

The other side of the market is populated by a mass one of firms that come in two types, which we call 1 and 2. Firms type 1 have productivity y_1 and firms type 2 have productivity y_2, where we assume $y_1 > y_2$. Additionally, we assume that the share of firms type 1 is given by λ. To simplify, each type of firm has a specific type of technology. Firms type 1 can only use the cognitive skills of the worker as an input, which we denote by x_1, to produce $x_1 y_1$ units of output. Firms type 2 use only the non-cognitive skills of the worker as an input, which we denote by x_2, to produce $x_2 y_2$ units of output.

The timing of the model is as follows: (i) workers decide how to allocate their total skill endowment between cognitive and non-cognitive skills; (ii) workers go to the job market and matches are realized; (iii) firms and workers bargain over wages; (iv) production happens, workers are paid and consume their wages.

Our main equilibrium object will be an assignment function $\mu : [\underline{x}, \overline{x}] \times [\underline{x}, \overline{x}] \times \{y_1, y_2\} \to \{0, 1\}$ that takes value one if worker (x_1, x_2) is matched with firm y and zero otherwise. Then, an equilibrium is given by an assignment function μ and a distribution of skills $H(x_1, x_2)$ such that μ maximizes the profit of the firm, the choice of (x_1, x_2) maximizes the expected utility of the worker given μ and H, and the allocation is feasible, that is,

$$\lambda \int_{\underline{x}}^{x} \mu(x_1, x_2, y_1) dH(x_1, x_2) + (1-\lambda) \int_{\underline{x}}^{x} \mu(x_1, x_2, y_2) dH(x_1, x_2) = F(x), \forall x$$

3.1. Equilibrium without search frictions

First, we solve for the equilibrium without any search frictions present in the labor market. In this case, we can solve the social planner's problem of choosing the assignment function to maximize total output[2] subject to feasibility. This problem can be written as

$$\max_{\mu} \quad \lambda \int \mu(x_1,x_2,y_1)x_1y_1 dH(x_1,x_2) + (1-\lambda)\int \mu(x_1,x_2,y_2)x_2y_2 dH(x_1,x_2)$$
$$\text{s.t.} \quad \mu(x_1,x_2,y_1) = 1 - \mu(x_1,x_2,y_2)$$

This is equivalent to choosing an indicator function $\alpha(x_1,x_2)$ that takes value 1 when worker (x_1,x_2) is matched with a firm type 1 and 0 when this worker is matched with a firm type 2. Then, the planner's problem becomes

$$\max_{\alpha} \quad \int [\lambda\alpha(x_1,x_2)x_1y_1 + (1-\lambda)(1-\alpha(x_1,x_2))x_2y_2]\, dH(x_1,x_2)$$

From the first order condition of the problem above, we have that

$$\frac{x_1}{x_2} \geq \frac{(1-\lambda)y_2}{\lambda y_1} \Leftrightarrow \text{worker is matched with firm 1}$$

That is, workers will be allocated according to their comparative advantage, as well as the relative abundance and productivity of each type of firm. Given this result, we can show the following simple Lemma.

Lemma: *There exists a skill threshold x^* such that*

$$(x_1,x_2) = \begin{cases} (0,x), & \text{if } x < x^* \\ (x,0), & \text{if } x \geq x^* \end{cases}$$

Moreover, x^* is such that $F(x^*) = 1-\lambda$.

Proof. First, we argue that workers must fully specialize. Define $x_1^*(x_2) \equiv \frac{(1-\lambda)y_2}{\lambda y_1}x_2$. Suppose that a worker satisfying $x_1 \geq x_1^*(x_2)$ is choosing $x_1 < x$. Planner optimality requires that this worker is matched with a firm type 1 and thus earns wage $\eta x_1 y_1$. Clearly, this worker would earn more by choosing $x_1 = x$. Similarly, if some worker with skills satisfying $x_1 < x_1^*(x_2)$ were not choosing $x_2 = x$, they could reduce x_1 and increase x_2 in the first stage to earn a higher wage for sure. Then, we can only have two types of worker, $(x,0)$ and $(0,x)$. Now we argue that workers

[2] This is not formally the social planner's problem, since workers have concave utility. However, given that we are working a transferable utility environment and the total consumption of both agents must add up to output, the planner's preferred allocation is the one that maximizes output, so the problem defined above is equivalent to the planner's problem.

with higher absolute skills will specialize in the cognitive skill. Suppose that wasn't the case. That is, there is some worker with $x \geq x^*$ that chooses $(x_1, x_2) = (0, x)$. This worker would be matched with firm 2 and earn $\eta x y_2$, which means at least one worker with absolute skill $x' < x$ is matched with a firm type 1. If worker x chose instead $(x_1, x_2) = (x, 0)$, they could be allocated to a firm type 1, which would strictly prefer that worker, since $(1-\eta)xy_1 > (1-\eta)x'y_1$. This would also be advantageous for the worker, since $\eta x y_1 > \eta x y_2$. Then, there exists a threshold on absolute skill x^* that separates workers. The value of x^* is pinned down by feasibility. □

This Lemma gives us two interesting properties of the frictionless model. First, all workers will fully specialize in one type of task. Because workers are certain about the type of employment they will find, they invest all their time in learning the skills necessary to perform that job and none in skills that are irrelevant to the firm that will employ them. Second, workers are sorted across types of jobs according not only to their comparative advantage but their absolute advantage as well. Since the production function is supermodular in skills and firm productivity and we assumed that firms that use cognitive skills are more productive, workers that are capable of investing more in the cognitive skill will be matched with those firms. Then, specialization induces sorting also on the extensive margin, allocating better workers to better jobs by allowing them to take full advantage of their higher skill endowment.

3.2. Equilibrium with search frictions

Suppose now that there is a frictional job market, where workers meet and bargain with firms according to their share in the population. In our static environment, this is the same as saying there is random matching in this economy, that is, a worker meets a firm type 1 with probability λ and a firm type 2 with probability $1-\lambda$. If a worker rejects an offer, they become unemployed and receive $w = 0$. Then, a worker type (x_1, x_2) has probability λ of earning $w = \eta x_1 y_1$ and probability $1-\lambda$ of earning $w = \eta x_2 y_2$. For simplicity, we assume $u(w) = \ln w$ for this section. Then, the problem of a worker with skill x becomes

$$\begin{aligned} \max_{x_1, x_2} \quad & \lambda \ln(\eta x_1 y_1) + (1-\lambda) \ln(\eta x_2 y_2) \\ \text{s.t.} \quad & x_1 + x_2 = x \end{aligned}$$

Taking the FOC yields

$$\frac{x_1}{x_2} = \frac{\lambda}{1-\lambda}$$

for every worker. That is, the choice of skill investment is now independent of the worker type and depends only on the relative abundance of each type of job. Then, for every worker, we have $(x_1, x_2) = (\lambda x, (1-\lambda)x)$.

Clearly, output will be lower in this case. This is because of two reasons. First, random matching makes it so we are no longer matching the best workers with the best firms, which is optimal since the production function is supermodular. Second, workers are no longer fully specialized, so they will not produce as much output when they are matched. We can interpret this as risk creating an externality in the labor market. Workers only internalize the cost of risk to themselves and diversify their skills to hedge against random matching. But they don't internalize the cost they impose on the firm (in the form of lower profits) when they reduce their supply of skills in which they would have a comparative advantage absent frictions.

Having these two solutions on hand is the basis for our counterfactual analysis. We can compute total mismatch in the economy using data from NLSY79 and O*NET. By calibrating the model to fit the data, we can perform the counterfactual of setting search frictions to zero and solving the model again. By doing this, we can find the counterfactual matches and skill choice of workers. With this, we can decompose mismatch in two parts: misallocation, by looking only at mismatch generated by differences in matches; and skill distortion, by looking only at mismatch generated by differences in skill choice. We will detail the procedure in Section 5.

4. Quantitative Model

We now present the full quantitative model we use to capture the effect of search frictions on skill choice and mismatch. The economy is populated by a measure one of workers that live for T periods. When workers die, they are replaced by an identical worker. However, before entering the labor market, newborn workers allocate total skill endowment into two types of skills $\vec{x} = (x_1, x_2)$, with $x_1 + x_2 = x$. Now, workers also have a preference type z which impacts the cost of accumulating one type of skill versus the other. Thus, worker types are distributed according to joint distribution $F(x, z)$.

The way preferences for certain types of skill affect skill choice is through the relative cost of investing in one skill versus the other. Every worker pays a disutility cost of allocating skills

(x_1, x_2) given by

$$z\frac{x_1^{1+\varepsilon}}{1+\varepsilon}+\frac{x_2^{1+\varepsilon}}{1+\varepsilon}$$

with $\varepsilon > 0$. Note that z is then the cost of investing in skill 1 relative to skill 2. The purpose this serves in the model is to match the range specialization we observe in the data. With absolute skill endowment x we can match the fact that some workers have strictly higher scores in all dimensions that other workers. Preference shock z, then, allows us to match the fact that both high skill and low skill workers vary in terms of specialization. Another way z could be interpreted is that some workers find it easier to convert raw skill into math or social skills than others, effectively turning it into a parameter characterizing the skill production frontier.

Workers derive linear utility of consumption[3], discount future utility at rate $\beta \in (0, 1)$ and have two possible states, employed or unemployed. When employed, they consume their wages. In addition, there is unemployment benefit (or costless home production) b given to all workers while they are unemployed.

There is also a mass of firms (or occupations) with skill requirements $\vec{y} = (y_1, y_2)$ distributed according to $G(\vec{y})$. A firm of type $\vec{y}$ employing with a worker of type $\vec{x}$ produces output $f(\vec{x}, \vec{y}) = \vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2$. This particular form of the production function is chosen because the allocation that maximizes it is also the same allocation that minimizes our measure of mismatch in the economy[4], so that mismatch has a direct translation into output cost.

The labor market is fricctional, with matches between workers and firms happening randomly and uniformly accross the population. Both the employed and the unemployed conduct job search each period, with meeting rate λ_0 for unemployed workers and λ_1 for employed workers. Wage is determined by Kalai bargaining, with workers having bargaining power η (meaning that workers appropriate a fixed share η of total surplus). Jobs are subject to an exogenous destruction shock, which happens with probability δ each period.

The timing of the model is as follows. Newborn workers, before entering the labor market choose skill allocation $\vec{x}(x, z)$ to maximize the value of unemployment net of disutility costs.

[3]The concavity in preferences that will give us the insurance motive for diversification now comes from the convex disutility of accumulating skills.

[4]Computing mismatch between $\vec{x}$ and $\vec{y}$, we have $m(\vec{x}, \vec{y}) = \frac{1}{2}(x_1 - y_1)^2 + \frac{1}{2}(x_2 - y_2)^2 = A - B(x_1 y_1 + x_2 y_2)$, where A and B are constants that disappear when we take cross derivatives (so that they are irrelevant to determining the optimal allocation, according to Becker, 1973). Then, it is easy to see that $f(\vec{x}, \vec{y})$, since it is a strictly decreasing linear transformation of $m(\vec{x}, \vec{y})$, will be maximized whenever m is minimized.

They enter the labor market next period as an unemployed worker ate age 1. For all other periods, workers search in the labor market and matches are realized. Then, production occurs and workers consume wages or home production. Finally, exogenous destruction shocks are realized. Value functions are evaluated after search but before production occurs.

Then, the problem of a newborn worker is given by:

$$\max_{\vec{x}} \quad \underbrace{U_1(\vec{x})}_{\substack{\text{Value of} \\ \text{unemployment with } \vec{x}}} \quad - \quad \underbrace{\left(z\frac{x_1^{1+\varepsilon}}{1+\varepsilon} + \frac{x_2^{1+\varepsilon}}{1+\varepsilon} \right)}_{\substack{\text{Disutility of} \\ \text{skill accumulation}}} \quad \text{s.t.} \quad x = x_1 + x_2 \tag{3}$$

where $U_1(\vec{x})$ is the value of unemployed of a worker at age 1.

The value of an unemployed worker of age $t < T$ is given by

$$U_t(\vec{x}) = b + \beta \Bigg(\underbrace{\lambda_0 \int_Y \max\{V_{t+1}(\vec{x},\vec{y}), U_{t+1}(\vec{x})\} dG(\vec{y})}_{\text{Probability of matching with firm } \vec{y}} + \underbrace{(1-\lambda_0) U_{t+1}(\vec{x})}_{\substack{\text{Probability of} \\ \text{not matching}}} \Bigg) \tag{4}$$

with $U_T(\vec{x}) = 0$, $V_t(\vec{x},\vec{y})$ being the value of worker $\vec{x}$ being employed at occupation $\vec{y}$, and Y being the set of all occupations.

We can then write the value of an employed worker of type $\vec{x}$ working in occupation with skill requirements $\vec{y}$ as

$$\begin{aligned} V_t(\vec{x},\vec{y}) = w(\vec{x},\vec{y}) + \beta \Bigg(& \underbrace{\lambda_1 (1-\delta) \int_Y \max\{V_{t+1}(\vec{x},\vec{y}), V_{t+1}(\vec{x},\vec{y}')\} dG(\vec{y}')}_{\text{Probability of meeting with firm } \vec{y} \text{ and no destruction}} \\ & + \underbrace{\delta U_{t+1}(\vec{x})}_{\substack{\text{Probability of} \\ \text{destruction}}} + \underbrace{(1-\delta)(1-\lambda_1) V_{t+1}(\vec{x},\vec{y})}_{\substack{\text{Probability of no destruction} \\ \text{and no meeting}}} \Bigg) \end{aligned} \tag{5}$$

where $w(\vec{x},\vec{y})$ is the solution to the Kalai bargaining problem and $V_T(\vec{x},\vec{y}) = 0$.

Finally, the value of a firm of type $\vec{y}$ employing a worker $\vec{x}$ of age t is

$$J_t(\vec{x},\vec{y}) = f(\vec{x},\vec{y}) - w(\vec{x},\vec{y}) + \beta\Big(\underbrace{\lambda_1(1-\delta)J_{t+1}(\vec{x},\vec{y})\int_Y \mathbb{1}\{V_{t+1}(\vec{x},\vec{y}) \geq V_{t+1}(\vec{x},\vec{y}')\}dG(\vec{y}')}_{\text{Probability of worker meeting other firm and not leaving}} + \underbrace{(1-\delta)(1-\lambda_1)J_{t+1}(\vec{x},\vec{y})}_{\substack{\text{Probability of no destruction}\\ \text{and worker doesn't meet}}}\Big) \tag{6}$$

With these value functions in hand, we can solve the model and calibrate parameters to match moments in NLSY79 and O*NET, which we detail in the next chapter.

4.1. Calibration

For calibration, we focus on two types of skills: math (which will be identified as skill 1 in the model) and social (skill 2). We do this because math and verbal scores have very high correlation in the data, due to both being collected from the same source, the ASVAB test.

The key parameters to identify pertain to the distribution $F(x,z)$, which will give us the mapping between underlying types and skill choices in our counterfactual. We assume x and z are independent, with marginal cdfs $F_x(\cdot)$ and $F_z(\cdot)$, respectively. We set F_x to a normal distribution and match the mean and standard deviation of this distribution to the mean and variance of the total score (summing up x_1 and x_2 for each worker) in our sample, which provides a direct mapping between model and data.

As for F_z, we assume z follows a lognormal distribution. The reason for this is the specification of the relative cost in the disutility. Values of z close to zero mean that the worker has much lower relative disutility to accumulating skill 1. We can see this by looking at the ratio of the marginal disutility of 1 relative to 2, which is given by $z(x_1/x_2)^{\varepsilon}$. With low values of z then, we would be able to match all workers with high scores in math and low social skill scores. To match workers with the opposite skill composition, then, we need to have that the relative cost of accumulating social skills is very low, which translates to allowing for very high values of z. The choice of a lognormal distribution then is warranted, since the support is unbounded above and the distribution has a heavy tail. In order to calibrate this distribution, we target the second moments of the distribution of (x_1, x_2) in NLSY79, since z is the parameter

Assigned Parameters

Parameter	Description	Value	Source
Environment:			
β	Discount factor	0.96	-
η	Share of Income	0.5	-
ε	Convexity of disutility	0.2	-
b	Value of Leisure	0.05	-
T	Work Lifespan	40	Avg retirement age
δ	Match Survival Prob.	0.235	Yearly EU Rate - CPS
λ_0	Meeting rate for Unemployed	0.18	Yearly UE Rate - CPS
λ_1	Meeting rate for Employed	0.88	Yearly EE Rate - CPS
Ability Endowment:			
$F_x(x)$	Distribution of total ability	$N(\mu_x, \sigma_x)$	NLSY79
μ_x	Mean of total ability	1.51	NLSY79
σ_x	SD of total ability	0.69	NLSY79
$F_z(z)$	Distribution of cost to skill 1	Lognormal(μ_z, σ_z)	NLSY79
Skill Requirements:			
$G(y_1, y_2)$	Distribution of occ skill requirements	–	O*NET

Estimated Parameters

Parameter	Description	Moment (NLSY79)	Value	Target	Model
μ_z	Mean of relative skill cost	Variance of ability 1	0.00	0.082	0.082
σ_z	SD of relative skill cost	Covariance of ability 1 and 2	0.11	0.023	0.026

Table 3: Calibrated parameters from NLSY79 and O*NET

that governs the dispersion of skills. We hit the targeted moments reasonably well with this this distribution.

The identification of the distribution of skill requirements is conducted nonparametrically from O*NET. We work with 292 occupations and match the distribution of skill requirements to that of the occupations in O*NET. To solve the model, we discretize the support of F_x and F_z into 20 bins each, allowing for up to 400 types of workers.

5. Counterfactual

In this section, we intend to disentangle how much of observed mismatch comes from the misallocation effect of search frictions and how is due to skill distortion. To accomplish this, then, we need to shut down search frictions in the labor market, allowing workers and firms to match instantaneously to their desired counterparts. This makes the labor market function essentially as a centralized market, so we need only to solve the assignment problem to find what matches would be realized in equilibrium in the absence of search frictions. Since this is a frictionless economy, it is much easier to solve the planner's problem to find the allocation and then implement the split of the surplus following the Kalai bargaining procedure.

The social planner, then, solves the following assignment problem[5], given a distribution of skill composition $H(\vec{x})$:

$$
\begin{aligned}
\max_{\mu(\cdot,\cdot)} \quad & \int_Y \int_X \mu(\vec{x},\vec{y}) f(\vec{x},\vec{y}) dH(\vec{x})\, dG(\vec{y}) \\
\text{s.t.} \quad & \int_Y \mu(\vec{x},\vec{y}) dG(\vec{y}) = h(\vec{x}), \forall \vec{x} \\
& \int_X \mu(\vec{x},\vec{y}) dH(\vec{x}) = g(\vec{y}), \forall \vec{y} \\
& \mu(\vec{x},\vec{y}) \geq 0, \forall \vec{x},\vec{y}
\end{aligned}
$$

where, remember, μ is the assignment function that tells us what share of workers type $\vec{x}$ work in occupations type $\vec{y}$, $h(\cdot)$ is the density function of H, and $g(\cdot)$ is the density function of G. Note that the planner maximizes total surplus, since this is a transferable utility environment. The feasibility constraints guarantee that the mass of workers in each occupation matches the mass of that particular occupation and vice-versa. Given μ, we can solve for workers' skill choice problem as before.

As we argued before, this linear programming problem is equivalent to minimizing total mismatch, so this counterfactual yields the minimum possible mismatch in this economy given the technology available (distribution of skill requirements), skill endowments and preferences. Notice that the minimum mismatch need not be zero, since workers might still be

[5]Unlike in our toy model, this is actually the social planner's problem, since workers and firm have linear utility. All disutility of skill allocation (which is responsible for the concavity in worker's utility) is paid before entering the labor market, which the planner takes as given here.

willing to endure some mismatch with their occupation if the disutility of accumulating skills to perfectly match the occupation's requirements is too high.

The planning problem above gives us the efficient allocation in the labor market given a distribution of skills $H(\vec{x})$, which is an endogenous object that itself depends on the matches in the labor market. We must solve, then, for H and μ jointly. To achieve this, we employ a recursive algorithm. We start by guessing some distribution, say H^0 and solve the linear programming problem above, yielding μ^0. We then feed μ^0 into the skill choice problem and find the distribution of $\vec{x}$ that workers would if they were to encounter μ^0 in the labor market, which we call H^1. We iterate in this manner until $\sup_{\vec{x}} |H^{i+1}(\vec{x}) - H^i(\vec{x})|$ and then we set $H(\vec{x}) = H^{i+1}(\vec{x}), \forall \vec{x}$ and $\mu(\vec{x}, \vec{y}) = \mu^i(\vec{x}, \vec{y}), \forall \vec{x}, \vec{y}$. It is straightforward to show that the operator described above is a contraction and thus a unique fixed point exists.

Armed with the counterfactual matching function and distribution of skills, we can decompose what part of frictions come from matches that should not have been realized, but did so due to the cost of waiting (misallocation), and what part comes from choices that predate the happenings in the labor market (skill distortion). To do so, we employ the following accounting exercise. We start with the formula for mismatch for a certain worker. By adding and subtracting the skills and occupation requirements of the counterfactual match for that worker, manipulating and then summing up, we can decompose mismatch into misallocation, distortion, and an interaction term, as below.

$$m_i = \frac{1}{S} \sum_s (q(x_{i,s}) - q(y_{j(i),s}))^2 \tag{7}$$

$$= \frac{1}{S} \sum_s (\underbrace{q(x^*_{i,s}) - q(y_{j(i),s})}_{=\alpha_{i,s}} + \underbrace{q(x_{i,s}) - q(y^*_{j(i),s})}_{=\delta_{i,s}} - \underbrace{(q(x^*_{i,s}) - q(y^*_{j(i),s})}_{=m^*_{i,s}}))^2 \tag{8}$$

$$\Rightarrow \quad \underbrace{\frac{1}{N} \sum_i m_i}_{\text{Average Mismatch}} \approx \underbrace{\frac{1}{S} \frac{1}{N} \sum_i \sum_s \alpha^2_{i,s}}_{\text{Average Misallocation}} + \underbrace{\frac{1}{S} \frac{1}{N} \sum_i \sum_s \delta^2_{i,s}}_{\text{Average Distortion}} + \underbrace{2 \frac{1}{S} \frac{1}{N} \sum_i \sum_s \delta_{i,s} \alpha_{i,s}}_{\text{Interaction term}} \tag{9}$$

In Equation 8 above, $x^*_{i,s}$ is the accumulated skill a worker i chooses for skill s in the absence of frictions; $y^*_{j(i),s}$ is the requirement of occupation $j(i)$ of skill s, where $j(i)$ is the occupation worker i is matched with in the absence of frictions (obtained from μ); finally, $m^*_{i,s}$ is the mismatch that occurs between worker i and their occupation for skill s even in the absence of

	Value	% of Total
Total mismatch	0.0490	100%
Additional mismatch ($\alpha + \delta +$ Interaction)	0.0408	83.2%
α	*0.0335*	*68.4%*
δ	*0.0073*	*14.8%*
Interaction	*0.0000*	*0%*
Mismatch without frictions	0.0082	16.8%

Table 4: Mismatch decomposition results using frictionless counterfactual

frictions, which is we assume to be small.

Then, in Equation 9 below, we collect terms and aggregate. The term $\alpha_{i,s}$ captures the difference between the worker's efficient skill choice and the skill requirement of the occupation that they actually perform in the data. We interpret this as isolating the effect of only misallocation: even if workers did not diversify their skills in reaction to labor market risk, we would still see some mismatch solely due to random matching in the economy. The term $\delta_{i,s}$ captures the difference between the worker's actual skill level and the skill requirement of the occupation to which that worker would be efficiently assigned. We interpret this as isolating the effect of skill distortion: even if the worker had found the job that is preferred by the planner, they still wouldn't be able to perform that job as well due to mismatch that comes from their insurance decision. The term $\delta_{i,s}$ then, reflects the distortion that comes from decisions taken before the labor market is the novel cost of search frictions we identify. Finally, there is an interaction term between the two.

Notice that the left hand side of Equation 9 can be computed directly from the data. Each term in the right hand side can be computed by combining the results from our counterfactual with data. We can then compute each term, decomposing mismatch as in Table 4. First of all, we can see that the mismatch we observe without frictions is around 0.0082, while the mismatch we observe in the data is around 0.049. This suggests that search frictions are responsible for increasing mismatch in the economy (and, thus, increasing output loss due to mismatch) sixfold, a sizable loss.

Second, by looking at the decomposition of the mismatch that is brought by search frictions, we see that misallocation is the most relevant one, accounting for almost 70% of all mismatch we observe in the data. Skill distortion, however, has a significant impact too: it represents

14.8% of all mismatch observed in NLSY79, meaning that almost 15% of the output loss that comes from skill mismatch comes from before workers enter the labor market and thus cannot be remedied with labor market policies, such as unemployment insurance.

Looking at how much skill distortion represents as a share of the additional mismatch induced by search frictions, we can compute

$$\frac{\delta}{\alpha+\delta+\text{Interaction}} = 17.9\%$$

to see that skill distortion accounts for almost 18% of the total cost of search frictions. Given our choice of production function, this translates to an additional output loss of 22% induced by search frictions. This suggests that the welfare costs of search frictions might be understated if we ignore how they affect the decisions of agents before they enter the labor market.

6. Concluding Remarks

We developed a model of the labor market with risk in the form of search frictions and random matching where workers must invest in a multidimensional set of skills. Workers differ in their total skill level and preferences for accumulating each type of skill. Job differ in the way they combine those skills to produce output. Search frictions cause workers to be mismatched with their jobs, not just due to random matching, but also because workers internalize the risk of not finding their ideal job, so they diversify their skills as insurance. We have shown in our counterfactuals that skill diversification in response to labor market risk has a quantitatively relevant impact on mismatch and output.

A number of other aspects of skill distortion merit further analysis. Future avenues of research into this phenomenon could include the characterization of optimal policies and investigation into other sources of risk. For the latter, we believe that search frictions are a relevant source of skill distortion, but not the only one. Other extensions that could have interesting interactions with labor market risk include cycles and skill-biased technological progress. Aggregate productivity cycles (and other types of aggregate shocks) could create cycles of skill distortion, with agents hedging against recessions by diversifying their skill sets even more. That way, we would have predictions on how skill investment and labor demand react to cycles when different types of labor are imperfect substitutes. This could cause slow recoveries, as agents that enter the labor market during recessions are usually less productive in any single job due to diversification. It could also explain part of the scarring effect of recessions on wages. Skill-biased technological change, on the other hand, is a force towards more specialization into cognitive and math skills, which could counteract the insurance motive of diversification.

As for policy, we believe this model offers a strong and tractable framework for policy analysis. The first direct consequence of our counterfactual is that, as we have argued above, optimal labor market policy (as in Chetty, 2006 and Baily, 1978) may not be as effective in improving match quality if agents can react to labor market conditions by accumulating skills before they enter the labor market. A full characterization of optimal unemployment insurance that accounts for match quality and the response of skill choice would be an interesting extension and could deliver a new characterization of the optimal policy. Two other policies that can be evaluated and characterized within this model are on-the-job training programs (allowing workers to accumulate skills on the job and reduce mismatch with tenure) and education subsidies (reducing the cost of acquiring certain skills and creating incentives for specialization if certain jobs demand it).

Finally, a key assumption of this paper is that agents react to the composition of occupations in the economy by accumulating different sets of skills. Some direct evidence of skill choice being influenced by labor market conditions, preferably using micro-level data, would be an important step to advance our understanding of this phenomenon.

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