Imperfect Competition and Leverage in the Banking Sector

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Abstract

This paper evaluates the role of financial frictions and imperfect banking competition in the Brazilian business cycle. I estimated a dynamic stochastic general equilibrium (DSGE) model that incorporates a Cournot banking sector where banks accumulate capital subject to a capital adequacy requirement. The findings show that the spread is more significant in scenarios with imperfect banking competition and bank capital adequacy requirements. The amplified countercyclical spread, which arises from the interaction of the imperfect banking competition and bank stress channels, tends to amplify the response of output, consumption, and other macroeconomic variables to adverse shocks. The results show that most of the spread increase in Brazil is due to financial shocks, especially after 2008.

1 Introduction

Following the 2008 financial crisis, there was an increased focus on incorporating financial frictions into Dynamic Stochastic General Equilibrium (DSGE) models. Most existing models in the literature explore financial frictions in a perfectly competitive banking sector. However, one can observe that the banking sector tends to be imperfectly competitive. In recent decades, the world banking system has been characterized by high asset concentration. Averaging across countries, the share of assets held by the five largest banks as a share of total commercial banking assets is about 80%.[1] In Brazil, the situation is no different. There was a substantial increase from 50% to over 85% in the share of the assets of the five largest banks from 2000 to 2019.

In an imperfectly competitive banking environment, banks can set a loan rate markup above their marginal cost to maximize earnings by controlling the economy's credit supply. This move by banks can directly affect the spread (the difference between the rates charged on loans and the rates paid on deposits) and generate effects that are not just restricted

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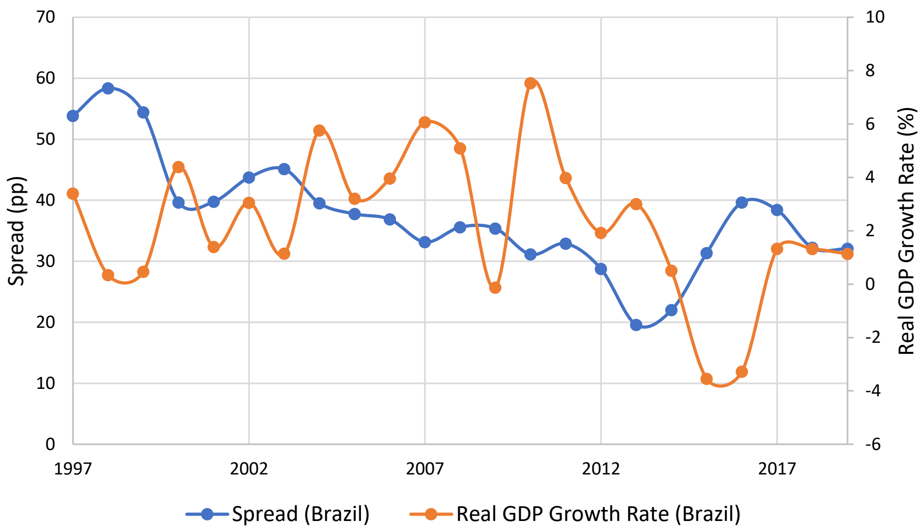
[1] Five banks' asset concentration data can be found on the Global Financial Development database on the website www.worldbank.org/en/publication/gfdr/data.

to the credit market but also expand to the real economy, causing a drop in employment and output. Firms that are financially constrained cannot raise their investments and hire employees because of the high-interest rates on loans taken out in banks, making it difficult to develop business activities.[2]

This study evaluates the role of financial frictions and banking intermediation on the real business cycle in Brazil, specifically on the spread charged by Brazilian banks. To achieve this goal, we estimate a DSGE model that incorporates a Cournot banking sector, in which banks accumulate capital subject to capital adequacy requirement. Figure 1 shows the countercyclicality of the spread, using country-level data for Brazil from 1997 to 2019. The model shows that both imperfect banking competition and banks' capitalization costs can lead to a countercyclical spread that amplifies the aggregate fluctuations in output, investment, consumption, and other macroeconomic variables by raising the cost of credit in the presence of adverse shocks. In this context, we set an additional question in the study: Which channel is most important to explain the spread increase in the recession?

Figure 1: Spread and real GDP growth in Brazil from 1997 to 2019



Note: The annual spread (in percentage points) from the World Bank is the difference between the lending rate (charged by banks on loans to the private sector) and deposit rate (offered by commercial banks on three-month deposits). The graph shows the unweighted average spread for Brazil (blue line) over time. The orange line corresponds to Brazil's annual real GDP growth rate.

Our model incorporates two main channels that affect the spread: imperfect banking competition and bank balance-sheet stress. How the spread changes through an imperfect banking competition channel in response to adverse shocks depends on the entrepreneur's loan demand elasticity and the number of banks in the economy. The demand for loans becomes more inelastic when the expected future capital prices (entrepreneurs' collateral) decrease and the expected marginal product of capital increases. In the presence of

[2] Joaquim et al. (2019) showed that an increase in local banking concentration in Brazil, going from 4 to 3 symmetrical banks, leads to a rise in the spread by 5.88 p.p. and a reduction of 17.1% in the volume of new loans made by private banks to firms.

entrepreneurs' binding collateral constraints, low expected capital prices after adverse shocks indicate a reduced borrowing capacity of entrepreneurs, and a high expected margin of the product means that borrowers operate below the optimal scale due to the tighter binding borrowing constraints. As a result, entrepreneurs are more financially constrained, leading to a higher inelastic demand for loans. The lower elasticity of loan demand gives banks in imperfect competition an incentive to charge a high loan rate, thus generating a high spread.

Regarding the bank stress channel, the spread is also affected by banks' balance sheet conditions and, in turn, depends on banks' capital accumulation and their cost of capitalization. Banks receive capital from retained earnings while keeping their capital-to-loans ratio close to an exogenous optimal leverage target to avoid punitive costs due to deviations from this optimal target. The optimal leverage target can be derived from mandatory capital requirements for banking activities (e.g., those explicitly established in the Basel Accords) or the country's Central Bank. Through changes in the banks' leverage ratio and the identity bank's balance sheet, the bank's capital influences the amount of credit offered, the setting of loan rates, and the spread.

The model considers four main adverse shocks to determine which channel best explains the countercyclical character of the spread, in the context of the Brazilian market: productivity shocks, collateral shocks, investment shocks, and financial shocks. Productivity shocks affect entrepreneurs' production function. Collateral shocks reduce the fraction of guarantees that entrepreneurs provide to obtain bank' loans. Investment shock hit capital producers by increasing the cost of raising new capital. A financial shock is an unexpected shock that deteriorates bank balance sheet conditions and introduces essential feedback loops between the financial and real sides of the economy. Our results show that the estimated structural parameters for Brazil amplify the adverse shock effects of the parameters seen in the European Union, and that financial shocks mainly explain spread fluctuations after the 2008 financial crisis.

The remainder of this paper is organized as follows. Sections 2 describe how our model contributes to the existing literature on imperfect banking competition in which banks accumulate capital subject to capital adequacy requirements. Section 3 describes the study's model. In Section 4, we estimate the structural parameters of the model using Brazilian data for 2000-2019 using a Bayesian approach. Section 5 shows that perfect banking competition can reduce banks' market power and improve aggregate production in the economy. Section 6 shows a dynamic analysis that verifies the effects of adverse shocks (productivity, collateral, investment, and financial) on the real economy and the banking sector, highlighting possible feedback loops between both sectors. In Section 7, we vary the essential parameters of the model to verify how they affect the spread and fluctuations of the main macroeconomic variables using sensitivity analysis. Section 8 shows the historical decomposition of the accumulated output and spread in Brazil. Section 9 presents the conclusions.

2 Theoretical Reference

Our study is related to recent efforts in the literature to incorporate imperfect banking competition, where banks accumulate capital subject to a capital adequacy requirement into DSGE models with financial frictions. The starting point for incorporating financial frictions into DSGE models to study the relationships between financial markets and the real economy dates back to the 2008 crisis. Financial friction can be understood as the difficulty faced by agents in conducting transactions due to information asymmetry, agency costs, or collateral constraints. These market failures can act as financial accelerators and amplify output, inflation, and interest rate fluctuations, as the spread fluctuates. Specifically, the financial market can significantly impact the real economy. There are two prevalent explanations in the literature to explain spread fluctuations: an imperfect banking competition channel, where banks have market power and can manage the spread, and a bank stress channel, in which the spread depends on bank balance sheet conditions.

Imperfect bank competition is often modeled using the Dixit and Stiglitz (1977) framework, in which the banking sector is monopolistically competitive and comprises small banks (Gerali et al. (2010), Andrés and Arce (2012), Hafstead and Smith (2012), Airaudo and Olivero (2019)). In this monopolistic banking competition approach, the spread (loan rate markup) is constant over business cycles without additional assumptions. In Gerali et al. (2010), changes in the spread depend on exogenous shocks to the interest rate elasticities of loan and deposit demand and the degree of interest rate stickiness. Andrés and Arce (2012) introduced an endogenous spread change, modeling imperfect competition using the Salop (1979) spatial model of horizontal product differentiation, where a finite number of banks compete on price dimensions. Banks determine the optimal loan rate according to the effects of their pricing policies and the volume of loans required by each borrower. Hafstead and Smith (2012) expand Bernanke et al. (1999) standard financial accelerator model by including a heterogeneous and monopolistically competitive banking sector in which the spread depends on the bank's marginal cost of producing new loans. In Airaudo and Olivero (2019), monopolistically competitive banks fix lending rates prospectively, as current interest rates also affect future demand for loans by financially constrained firms given the existence of borrowers' specific banking habits.

Another segment of the literature models an endogenous change in spread by introducing large banks into the Dixit and Stiglitz (1977) framework (Cuciniello and Signoretti (2014)) or modeling large banks in an oligopolistic competition framework (Li (2019)). Cuciniello and Signoretti (2014) show that the spread level is positively connected to the level of entrepreneurs' leverage, and monopolistic banking competition can amplify aggregate fluctuations after monetary policy contractions. However, this result is supported by the strategic interaction between banks with market power and the Central Bank's inflation target. Li (2019) highlights evidence that the banking sector tends to be dominated by only a few large banks (OECD and EU countries) and uses the Cournot banking sector to model oligopolistic banking competition. Li (2019) shows that the spread decreases in the number of banks and the loan demand elasticity to the loan rate, and reveals a new shock propagation mechanism using imperfect banking competition that operates through the dynamics of the expected marginal product of physical capital.

In contrast, the literature models the bank stress channel through bank balance sheets, which focuses on how bank capital accumulation affects the loan supply and the spread (Markovic (2006), Gerali et al. (2010), Meh and Moran (2010)). Markovic (2006) shows that the bank stress channel can be an essential part of the monetary policy transmission mechanism that affects interest rates, mainly when there are large direct shocks to banks' balance sheets that lead to the deterioration of the health of the banking sector. These shocks can occur due to structural reforms in the banking sector or regulatory changes in capital accumulation. As a result, banks may find it costlier to raise the new capital needed to complete bank capital regulatory requirements. The higher cost of accumulating bank capital is transferred to the cost of borrowing for firms by increasing loan interest rates.

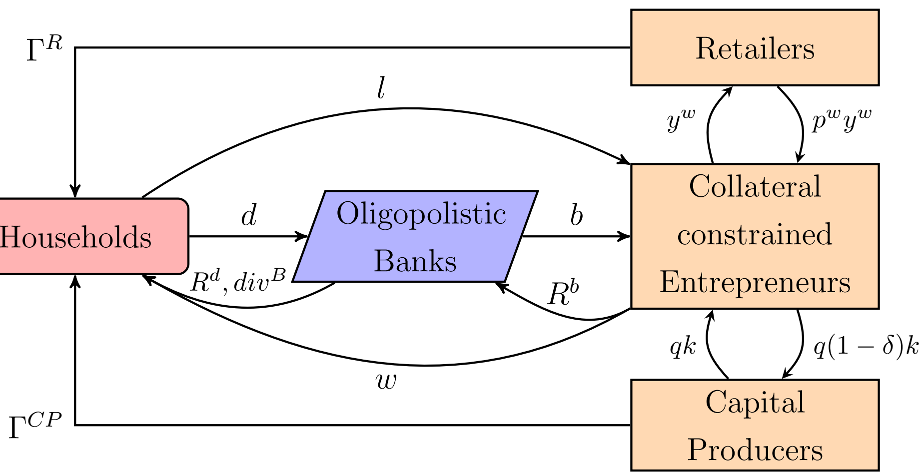
Gerali et al. (2010) show that bank balance-sheet constraints establish a link between the financial and real sides of the economy. The authors model the bank accumulation of capital from retained earnings, and banks pay a cost when their leverage ratio deviates from the optimal target level. Through this leverage ratio (banks' capital-to-assets ratio) and the bank balance sheet conditions, bank capital influences the number of loans issued and the spread. The more significant the deviation from the optimal target or the higher the deviation cost (bank capitalization cost), the higher the impact on the spread. In addition, bank balance sheet conditions can amplify shocks from the real side of the economy, if the banking sector is not at the optimal leverage ratio when a shock occurs. Finnaly, Meh and Moran (2010) build a model in which the balance sheet of banks affects the propagation of shocks (technology, monetary, and financial) and macroeconomic performance. The authors showed that bank's net worth (bank capital) increases the capacity of an economy to absorb adverse shocks, that is, banking sectors that are well-capitalized demonstrate smaller declines in bank lending in periods of negative technology shocks, mitigating the decline in economic activity. However, economies in which banks display low capital accumulated during adverse technology shocks significantly reduce lending, causing a decrease in economic activity. Unlike technology shocks, financial shocks that originate in the banking sector and produce a sudden shortage of bank capital lead to reductions in bank lending and economic activity due to reduced investments.

Our model is based on Li (2019) and Gerali et al. (2010). However, it has several modifications. There is Cournot banking competition in Li (2019), but no bank capital accumulation. Gerali et al. (2010) model monopolistic banking competition, where banks accumulate all profits (zero-dividend policy). In our study, there is Cournot banking competition, in which banks accumulate a portion of their profits and pay dividends to households. Moreover, banks pay a cost when they deviate from the optimum capital adequacy requirement as in Gerali et al. (2010). This cost is imposed on banks to prevent them from maintaining a high level of leverage, thereby putting the banking system at risk of financial collapse. This study's main contribution is that it reveals that the spread is directly affected by Cournot banking competition and bank stress channels, with possible feedback loops between the two channels. The imperfect competition channel increases the spread through a drop in the elasticity of loans to the loan rate, when the number of banks in the economy is low. In contrast, the spread can be amplified through banks' high capitalization costs when their leverage level deviates from the established optimal target. Furthermore, the bank stress channel may amplify macroeconomic shocks arising from the real sector of the economy in the presence of a high cost of capitalization or if these shocks significantly affect the level of bank leverage (capital-to-loans ratio).

3 Model

The model has six main agents: households, collateral constrained entrepreneurs, capital producers, retailers, oligopolistic banks, and the Central Bank. Figure 2 shows the relationship between the agents in the model. Households consume and work, while entrepreneurs produce wholesale goods y^w using physical capital k bought from capital producers and labor l supplied by households. Entrepreneurs pay price q for the physical capital purchased and w for household work. Two types of financial instruments provided by oligopolistic banks are available to economic agents, deposits and loans. Households can save resources by depositing d in banks and receiving an interest rate R^d whose value is controlled by the Central Bank through a Taylor rule. Entrepreneurs borrow b from banks to finance their investment activities. Entrepreneurs face a borrowing constraint when taking out a bank loan tied to tomorrow's collateral value (value of their physical capital). If entrepreneurs do not honor their loan rate payments, banks can confiscate a fraction of the entrepreneurs' collateral.

Figure 2: Dynamics of the model



The banking sector operates under Cournot competition, banks choose the level of loans that maximize the dividends div^B paid to households. Bank loans are financed by the amount of household deposits and bank capital accumulated. On the production side, retailers buy the wholesale good y^w at a nominal price p^w from entrepreneurs in a competitive market and use it as the only input to produce differentiated retail goods at no extra cost. Capital producers buy the non-depreciated physical capital of entrepreneurs and use it to create new capital for wholesale goods production. Capital producers derive a market price for physical capital, which determines the value of entrepreneurs' collateral. Both the retailer and capital producer sectors are owned by the households and return their respective profits (Γ^R, Γ^{CP}) to the same at the end of the period.

3.1 Households

A continuum of identical infinitely-lived households of unit mass maximizes the following expected utility function:

$$\max_{\{c_t, l_t, d_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] \tag{1}$$

which depends on consumption c_{t+s} and labor supply l_{t+s}, with $\beta \in (0,1)$ being the subjective discount factor for households. In each period, households consume c_t, save d_t (in terms of real final consumption), and offer l_t hours of labor. Time is normalized to 1, and $(1 - l_t)$ can be defined as the amount of leisure of households in period-t, and ϕ_l is the relative utility weight of leisure time.

Assume that households own the capital production sector, retail firms, and bank shareholders in this economy. In addition, assume that there are no risk-free bonds, therefore, in equilibrium, households keep only bank deposits d_t. Nominal deposits d_{t-1} saved in period $t-1$ which yields a gross nominal interest rate R^d_{t-1} at the beginning of period t. In addition to deposit gains $R^d_{t-1} d_{t-1}$, households have income from work $w_t l_t$, profits from the capital formation sector Γ^{CP}_t, retail firms Γ^R_t, and dividends div^B_t paid by banks. Thus, the representative household has the following budget constraint:

$$c_t + d_t = \frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma^{CP}_t + \Gamma^R_t + div^B_t \tag{2}$$

where $\pi_t \equiv \frac{p_t}{p_{t-1}}$ denotes the gross inflation rate and p_t is the unit price of the final consumption good. We denote the Lagrange multiplier associated with the representative household budget constraint by λ_t and the first-order conditions with respect to consumption c_t (3), labor supply l_t (4), and bank deposits d_t (5) can be written as follows:[3]

$$\lambda_t = \frac{1}{c_t} \tag{3}$$

$$\lambda_t w_t = \frac{\phi_l}{(1 - l_t)} \tag{4}$$

$$\lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R_t^d}{\pi_{t+1}} \right] \tag{5}$$

where equation (5) is the intertemporal Euler equation, which can also be written as:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{R_t^d}{\pi_{t+1}} \right] \tag{6}$$

where $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \frac{c_t}{c_{t+1}}$ is the stochastic discount factor in period t for real payoffs in period $t+1$, with $u(c_t) = \ln(c_t)$.

3.2 Entrepreneurs

There is a continuum of perfectly competitive entrepreneurs of unit mass that have some capital endowment in the initial period. In period $t-1$, entrepreneurs acquire physical capital k_{t-1} from capital producers at the real price q_{t-1} and in period t hire labor l_t from households that will be used as inputs to produce a wholesale good y_t^w through constant-returns-to-scale Cobb-Douglas production technology:

$$y_t^w = z_t k_{t-1}^{\alpha} l_t^{1-\alpha} \tag{7}$$

where $\alpha \in (0,1)$ is the output elasticity of the physical capital. The wholesale good y_t^w produced in period t is then sold to retailers at a nominal price p_t^w, who then produce the final consumption good y_t sold at a nominal price p_t. The total factor productivity z_t follows an autoregressive process AR(1):

$$\ln(z_t) = \psi_z \ln(z_{t-1}) + \varepsilon_t^z \tag{8}$$

where $\psi_z \in (0,1)$ reflects the persistence of z_t and ε_t^z is a productivity shock with variance σ_z^2. Let β^E denote the subjective discount factor of the entrepreneurs. It is assumed that $\beta^E < \beta$ to ensure that entrepreneurs are net borrowers and households are net savers in the steady-state and its neighborhood, following Iacoviello (2005). The objective of entrepreneurs is to maximize their expected lifetime utility:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta^E)^s \ln\left(c_{t+s}^E\right) \tag{9}$$

[3] The households' optimization problem is described in Appendix A.

subject to a budget constraint:

$$c_t^E + q_t k_t + w_t l_t + \frac{R_{t-1}^b b_{t-1}}{\pi_t} = \frac{y_t^w}{x_t} + q_t(1-\delta)k_{t-1} + b_t \tag{10}$$

where $x_t \equiv \frac{p_t}{p_t^w}$ denotes the markup of the price of the final consumption good y_t over the price of the wholesale good y_t^w. The loans taken out in the banking sector in period t are represented by b_t and R_t^b denotes the interest rate that entrepreneurs pay for these loans. At the end of period t, entrepreneurs can sell non-depreciated capital $(1-\delta)k_{t-1}$ to capital producers at price q_t, where δ is the depreciation rate of physical capital. The wholesale good y_t^w produced in period t is then sold to the retailers at the price p_t^w. On the expenditure side of entrepreneurs, the outflow of funds is given by consumption c_t^E, cost of capital investment $q_t k_t$, wage payments $w_t l_t$ and gross loans interest payments $\frac{R_{t-1}^b b_{t-1}}{\pi_t}$.

An agency problem is introduced, following Kiyotaki and Moore (1997), assuming costly debt enforcement. If entrepreneurs fail to honor their debts, banks may confiscate part of their assets. Assuming that physical capital k_t can be used as collateral assets, let $m_t^k \in (0,1)$ denote the fraction of physical capital collateral that banks can confiscate if entrepreneurs fail to repay their loans. Consequently, the maximum amount that entrepreneurs can borrow is such that the gross nominal debt interest payment $R_t^b b_t$ is equal to the expected value of their assets that banks can recover $m_t^k \mathbb{E}_t[q_{t+1}(1-\delta)k_t]$ if they do not make their payments. Thus, entrepreneurs are subject to the following borrowing constraint:

$$b_t \leq m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)k_t \pi_{t+1}}{R_t^b} \right] \tag{11}$$

The pledgeability ratio m_t^k is subject to collateral shocks and follows an autoregressive process AR(1):

$$\ln\left(m_t^k\right) = \psi_{m^k} \ln\left(m_{t-1}^k\right) + \varepsilon_t^{m^k} \tag{12}$$

where $\psi_{m^k} \in (0,1)$ indicates the persistence of the m_t^k and $\varepsilon_t^{m^k}$ is the collateral shock with variance $\sigma_{m^k}^2$. Let $\lambda_{1,t}^E$ and $\lambda_{2,t}^E$ denote the Lagrangian multipliers associated with the budget constraint (10) and borrowing constraint (11), respectively. Then, the first-order conditions of entrepreneurs' optimization problem in relation to entrepreneurs' consumption c_t^E (13), labor demand l_t (14), loan demand b_t (15), and capital demand k_t (16) are:[4]

$$\lambda_{1,t}^E = \frac{1}{c_t^E} \tag{13}$$

$$w_t = (1-\alpha)\frac{y_t^w}{x_t l_t} \tag{14}$$

$$\lambda_{2,t}^E = \lambda_{1,t}^E - \beta^E \mathbb{E}_t \left[\lambda_{1,t+1}^E \frac{R_t^b}{\pi_{t+1}} \right] \tag{15}$$

[4]The entrepreneurs' optimization problem is described in Appendix B.

$$\lambda^E_{1,t} q_t = \beta^E \mathbb{E}_t \left[\lambda^E_{1,t+1} \left(\alpha \frac{y^w_{t+1}}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + \lambda^E_{2,t} \mathbb{E}_t \left[\frac{m^k_t (1-\delta) q_{t+1} \pi_{t+1}}{R^b_t} \right] \tag{16}$$

Combining equations (13) and (15), we obtain the following expression in the steady-state:

$$\lambda^E_2 = \frac{1}{c^E} \left(1 - \beta^E \frac{R^b}{\pi} \right) \tag{17}$$

The value of the steady-state of the gross real interest rate $\frac{R^d}{\pi}$ is determined by the households' subjective discount factor such that $\frac{R^d}{\pi} = \frac{1}{\beta}$, according to the Euler equation (6). To ensure that the borrowing constraint is always binding in the steady-state, λ^E_2 must be positive, implying $\beta^E < \beta$. The heterogeneity in β and β^E guarantees that entrepreneurs are net borrowers in the steady-state.[5] Based on the budget constraint (10), the entrepreneur's net worth n_t in period t, after the productivity shock z has been realized and the wholesale good y^w_t produced, is defined by:

$$n_t = \frac{y^w_t}{x_t} - w_t l_t + q_t (1-\delta) k_{t-1} - \frac{R^b_{t-1} b_{t-1}}{\pi_t} \tag{18}$$

where $q_t(1-\delta)k_{t-1}$ is the total value of capital stock and $\frac{R^b_{t-1} b_{t-1}}{\pi_t}$ is the loan interest payment at the beginning of period t. Then, budget constraint (10) can be written in terms of n_t:

$$c^E_t + q_t k_t = n_t + b_t \tag{19}$$

which implies that the entrepreneur finances consumption c^E_t and purchases new capital k_t through bank loans b_t and retained earnings n_t. Under the assumption of log utility, c^E_t is a fixed proportion of the accumulated profits n_t:

$$c^E_t = (1 - \beta^E) n_t \tag{20}$$

The real loan demand b_t can also be written as the total purchasing cost of new capital over internal financing or savings $\beta^E n_t$:[6]

$$b_t = q_t k_t - \beta^E n_t \tag{21}$$

where $\beta^E n_t$ is the portion of retained earnings that are not consumed and that can be used to purchase new capital. Note that the binding borrowing constraint (11) determines market loan demand, which implies an inverse relationship between the equilibrium loan rate R^b_t and loan quantity b_t. Under an imperfectly competitive banking scenario, each individual bank determines the amount of b_t and consequently affects R^b_t. In particular, for given asset prices q_{t+1} and π_{t+1}, a higher loan rate R^b_t corresponds to a lower loan

[5] In the literature, the standard approach assumes $\beta^E < \beta$ to ensure that the borrowing constraint permanently binds in the steady-state and its neighborhood, as long as the size of shocks are sufficiently small (Iacoviello (2005), Andrés and Arce (2012), Gerali et al. (2010)).

[6] In the presence of binding budget constraints (19).

quantity b_t and this affects the demand for physical capital.

3.3 Capital Producers

A continuum of perfectly competitive capital producers of unit mass is introduced to obtain an explicit expression for the capital price q_t (Gambacorta and Signoretti (2014)). Capital producers buy non-depreciated capital $(1-\delta)k_{t-1}$ from entrepreneurs and buy final consumption goods i_t from retailers to produce new capital k_t at the end of period t:

$$k_t = (1-\delta)k_{t-1} + i_t \tag{22}$$

where i_t is gross investment, and k_t is the newly produced capital that will be sold back to the entrepreneurs at the real price q_t. The capital k_t will be used in the production of the wholesale good in period $t+1$. Following Christiano et al. (2005), we assume that old capital can be converted into new capital at a one-to-one rate subject to a quadratic investment adjustment cost $f\left(\frac{i_t}{i_{t-1}}\right) = \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2$ with $f(1) = f'(1) = 0$, $f''(1) > 0$. The adjustment cost specification shows that few units of new capital can be produced for one investment unit whenever $\frac{i_t}{i_{t-1}}$ deviates from the unitary value in the steady-state. In addition, $\chi > 0$ reflects the magnitude of the adjustment cost and s_t^{qk} is the total factor productivity of the investment i_t that follows an autoregressive AR(1):

$$\ln\left(s_t^{qk}\right) = \psi_{s^{qk}} \ln\left(s_{t-1}^{qk}\right) + \varepsilon_t^{qk} \tag{23}$$

where $\psi_{s^{qk}}$ measures the degree of persistence of s_t^{qk} and ε_t^{qk} is an investment productivity shock with variance $\sigma^2_{s^{qk}}$.

The capital producer chooses the level of gross investment i_t that maximizes the sum of the expected discounted future profits from the sale of new capital k_t at price q_t minus the payment of input costs $(q_t(1-\delta)k_{t-1} + i_t)$ and investment adjustment cost $f\left(\frac{i_t}{i_{t-1}}\right) i_t$:

$$\max_{\{i_t, k_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[q_t k_t - q_t(1-\delta)k_{t-1} - i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t \right] \tag{24}$$

where $\Lambda_{t,t+s} = \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor because households own capital producers. Replacing (22) in (24), the objective function can be simplified as follows:

$$\max_{\{i_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ (q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t \right\} \tag{25}$$

The capital producer's problem returns the relation (26) to the capital price q_t taking the first-order condition with respect to i_t:[7]

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 + \chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\left(\frac{i_t}{i_{t-1}}\right) s_t^{qk} - \chi \mathbb{E}_t \left\{\Lambda_{t,t+1}\left(\frac{i_{t+1} s_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 s_{t+1}^{qk}\right\} \tag{26}$$

In the steady-state, the real capital price q_t is equal to one since $i_{t-1} = i_t = i_{t+1}$. All profits Γ_t^{CP} made outside the steady-state ($q \neq 1$) by the capital producers sector returns to households, where $\Gamma_t^{CP} = (q_t - 1)i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t$.

3.4 Retailers

A continuum of retailers of unit mass, indexed by i, buy the wholesale good $y_t^w(i)$ at a nominal price $p_t^w(i)$ from entrepreneurs and use it as the only input to produce differentiated retail goods costlessly.[8] Each retailer i produces a different variety $y_t(i)$ and charges a nominal price $p_t(i)$ for the differentiated product. The output of the final consumption good y_t is a constant elasticity of substitution (CES) composite of all the different varieties produced by retailers (using the Dixit and Stiglitz (1977) framework):

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{27}$$

where $\epsilon > 1$ is the elasticity of intertemporal substitution between different varieties.

Each retailer i then sells his unique variety, applying a markup over the wholesale price, taking into account the demand that he faces, characterized by a stochastic price-elasticity ϵ_t^y. Retailers' prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by ι_p. Whenever retailers want to change their price beyond what indexation allows, they face a quadratic adjustment cost parameterized by a coefficient κ_π. Then, retailers must choose $\{p_t(i)\}_{t=0}^{\infty}$ to maximize profits given by:[9]

$$\Gamma^R = \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{t,t+s}\left[p_t(i)y_t(i) - p_t^w(i)y_t(i) - \frac{\kappa_\pi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)^2 p_t y_t\right] \tag{28}$$

subject to downward-sloping demand from consumers' maximization of a consumption aggregator:

$$y_t(i) = y_t\left(\frac{p_t(i)}{p_t}\right)^{-\epsilon_t^y} \tag{29}$$

[7]The capital producers' optimization problem is described in Appendix C.
[8]The retailers are monopolistically competitive.
[9]The retailers' optimization problem is described in Appendix D.

The price-elasticity ϵ_t^y follows an autoregressive AR(1):

$$\ln(\epsilon_t^y) = \psi_y \ln\left(\epsilon_{t-1}^y\right) + \varepsilon_t^y \tag{30}$$

where ψ_y measures the degree of persistence of ϵ_t^y, and ε_t^y is a price-elasticity shock with variance σ_y^2. In symmetrical equilibrium, $p_t(i) = p_t$, the first-order conditions imply at Phillips curve, given by:

$$\frac{\epsilon_t^y}{x_t} - \kappa_\pi \left(\pi_t - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_t + \beta\mathbb{E}_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\pi_{t+1} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right] = \epsilon_t^y - 1 \tag{31}$$

where $\Lambda_{t,t+s} = \beta^s\frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor since households own retail firms, and $x_t = \frac{p_t(i)}{p_t^w(i)} = mc_t(i)$ is the markup of the final good price.

3.5 Central Bank

Assume a Taylor rule implements a monetary policy, which responds to both the deviation of the gross inflation rate from inflation target $\overline{\pi}$, and the divergence of output from its steady-state $\overline{y}$. The Central Bank controls the gross nominal interest rate R_t^d on bank deposits, following the Taylor rule (32):

$$R_t^d = \rho_r R_{t-1}^d + (1-\rho_r)\left[\overline{R}^d + \phi_\pi(\pi_t - \overline{\pi}) + \phi_y(y_t - \overline{y})\right] + \varepsilon_t^R \tag{32}$$

where $\overline{R}^d, \overline{\pi} = \pi^{ss}$ and $\overline{y} = y^{ss}$ represent steady-state values, and ε_t^R is the monetary policy shock with variance is σ_R^2. The coefficient $\rho_r \in [0,1]$ is the interest rate persistence parameter, and $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are feedback parameters that reflect the sensitivity of the interest rate R_t^d to inflation and output deviations to the steady-state.

3.6 Imperfect Banking Competition (Cournot)

The Cournot banking sector is used to characterize oligopolistic competition and capture banks' market power once the banking sector is dominated by a few large players. Banks' quantity-setting decisions affect the market loan rate in Cournot equilibrium. Assume that there are N banks in the economy indexed by j, that operate under Cournot competition. Each bank considers the effect of its choice $b_t(j)$ on entrepreneurs' capital and loan demand through the equilibrium lending rate but ignores the general equilibrium effects and takes other aggregate prices and quantities as indicated.

The capital accumulated by banks can be used along with deposits collected to finance new loans for entrepreneurs. Then, banks have the following balance-sheet identity:

$$b_t(j) = d_t(j) + k_t^B(j) \tag{33}$$

where $k_t^B(j)$ is the bank's capital, $d_t(j)$ is the deposit received from households, and $b_t(j)$

is the loan made to entrepreneurs in period t by bank j. Both sources of finance are perfect substitutes from the standpoint of the bank's balance sheet. The choice of banks is defined by an additional assumption of an optimal capital-to-loans ratio (i.e., the inverse of leverage) exogenously given by τ^B, from which it is costly to deviate. This cost can be considered a trade-off for the bank that arises when deciding how many own resources (bank capital) to keep, or as an approach to study the implications and costs of regulatory capital requirements. Given this assumption, bank capital plays a fundamental role in determining the conditions for credit supply in terms of both quantity and loan prices. Banks' capital accumulates from retained earnings:

$$k_t^B(j) = (1-\delta^b)k_{t-1}^B(j) + \Gamma_t^B(j) - div_t^B(j) \tag{34}$$

where $\Gamma_t^B(j)$ is the overall real profit made by bank j, δ^B measures the resources used in managing capital, and div_t^B is the dividend paid to households in period t. Deviations from the optimal target τ^B imply a quadratic cost given by $\Omega_t^B(j)$:

$$\Omega_t^B(j) = \frac{\kappa_{k^B}}{2}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)^2 k_t^B(j) \tag{35}$$

where bank j pays a quadratic cost parameterized by a coefficient κ_{k^B} whenever the capital-to-loans ratio $\frac{k_t^b(j)}{b_t(j)}$ deviates from the optimal target value τ^B. As we assume that bank capital is accumulated from retained earnings, the models' banks are at the center of a feedback loop between the real and financial aspects of the economy. Banks' profits and capital may be negatively affected when macroeconomic conditions deteriorate. Depending on the nature of the shock that hits the economy, banks may respond to the subsequent weakening of their financial position (i.e., decreasing leverage) by reducing the number of loans b they are willing to extend to the private sector, thus amplifying the original contraction of the real variables.

Modeling the leverage position of banks and the setting of loan rates subject to bank capital requirements allows us to introduce a series of shocks that originate on the credit supply side, and thus study their effects on the spread and the real economy. We can examine the impact of a drastic weakening in the balance sheet position of the banking sector. Then, the bank j profit in the banking system organized under Cournot competition in period t is:

$$\Gamma_t^B(j) = \frac{1}{\pi_t}\left[R_{t-1}^b\left(b_{t-1}(j) + \sum_{m\neq j} b_{t-1}(m)\right) b_{t-1}(j) - R_{t-1}^d d_{t-1}(j) - \Omega_{t-1}^B(j)\right] \tag{36}$$

where R_t^b is the nominal interest rate paid by entrepreneurs for loans $b_t(j)$ taken from bank j, R_t^d is the nominal interest rate determined by the Central Bank paid on household' deposits, and $b_t(m)$ are loans granted by banks $m \neq j$ in the banking system. In an imperfect competition environment, R_t^b represents the inverse of the loan demand function, which depends on b_t and, therefore, $b_t(j)$. The dependence of R_t^b on $b_t(j)$ means that each bank j exercises a certain degree of control over the equilibrium gross loan interest rate R_t^b by changing its own quantity of loans $b_t(j)$ given the other quantity of loans $b_t(m)$ granted by banks $m \neq j$ in the banking system with a Cournot structure. Thus, replacing the balance-sheet identity (33) in equation (36) yields:

$$\Gamma_t^B(j) = \frac{1}{\pi_t}\left[R_{t-1}^b\left(b_{t-1}(j) + \sum_{m\neq j} b_{t-1}(m)\right)b_{t-1}(j) - R_{t-1}^d(b_{t-1}(j) - k_{t-1}^B(j)) - \Omega_{t-1}^B(j)\right] \tag{37}$$

It is possible to define the capital accumulation $k_t^B(j)$ as follows (replacing (37) in (34)):

$$k_t^B(j) = \left(1 + \frac{R_{t-1}^d}{\pi_t} - \delta^b\right)\frac{k_{t-1}^B(j)}{s_t^{k^B}} + \left(\frac{R_{t-1}^b - R_{t-1}^d}{\pi_t}\right)b_{t-1}(j) - div_t^B(j) - \Omega_{t-1}^B(j) \tag{38}$$

Bank capital accumulation (38) is subject to an unexpected financial shock $s_t^{k^B}$ that follows an autoregressive AR(1):

$$\ln\left(s_t^{k^B}\right) = \psi_{s^{k^B}} \ln\left(s_{t-1}^{k^B}\right) + \varepsilon_t^{k^B} \tag{39}$$

where $\psi_{s^{k^B}}$ measures the degree of persistence of $s_t^{k^B}$ and $\varepsilon_t^{k^B}$ is a financial shock that destroys the capital accumulated by banks with variance $\sigma_{k^B}^2$. Therefore, each bank j maximizes the sum of the present discounted value of future dividends subject to the bank capital accumulation law (38):[10]

$$\max_{\{b_t(j), k_t^B(j), div_t^B(j)\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}[\ln\left(div_{t+s}^B(j)\right)] \tag{40}$$

where $\Lambda_{t,t+s} = \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor, since households own the banks. Solving the banks' optimization problem with respect to $b_t(j)$ yields the following first-order condition:

$$\mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{\lambda_{t+1}^B(j)}{\pi_{t+1}}\left[\frac{\partial\Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)}b_t(j) + R_t^b - R_t^d\right)\right]\right\} = 0 \tag{41}$$

In Cournot equilibrium, the total optimal loan quantity is $b_t = b_t(j) + \sum_{m\neq j} b_t(m)$ and each bank produces a share of the total quantity of loans b_t. In addition, the total optimal bank capital is $k_t^B = k_t^B(j) + \sum_{m\neq j} k_t^B(m)$, and each bank accumulates a share of total bank capital in the banking system k_t^B. Assuming that the banks are identical, $b(j) = \frac{b_t}{N}$ and $k_t^B(j) = \frac{k_t^B}{N}$ in equilibrium. Since $\frac{\partial R_t^b}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}\frac{\partial b_t}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}$ in Cournot equilibrium, the

[10]The banks' optimization problem is described in Appendix E.

first-order condition (41) can be written as:

$$\mathbb{E}_t \Lambda_{t,t+1} \left\{ \frac{\lambda^B_{t+1}}{\pi_{t+1}} \left[\frac{\partial \Omega^B_t}{\partial b_t} - \left(\frac{\partial R^b_t}{\partial b_t} \frac{b_t}{N} + R^b_t - R^d_t \right) \right] \right\} = 0 \tag{42}$$

where market loan demand is given by entrepreneurs' binding borrowing constraints (11). The loan rate R^b_t has a direct negative effect on market loan demand b_t because an increase in R^b_t reduces the entrepreneurs' borrowing capacity. In addition, R^b_t also has an indirect effect on b_t by influencing entrepreneurs' demand for physical capital k_t.[11] When bank j chooses $b_t(j)$ to maximize dividends, we need to consider how entrepreneurs would respond by changing their demand for physical capital $\frac{\partial k_t}{\partial R^b_t}$, which affects the level of investments in the economy.

The entrepreneur's demand for physical capital k_t decreases in loan rate R^b_t because $\frac{\partial k_t}{\partial R^b_t} < 0$, and the interest rate elasticity of the capital demand $PEK_t \equiv -\frac{\partial k_t}{\partial R^b_t} \frac{R^b_t}{k_t}$ monotonically decreases in the expected marginal product of capital:

$$PEK_t = \frac{1}{1-\alpha} \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R^b_t} \right]}{\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1} \right]} \right) \tag{43}$$

where $MPK_{t+1} \equiv \frac{\alpha z_{t+1}(k_t)^{\alpha-1}(l_{t+1})^{1-\alpha}}{x_{t+1}}$ is the marginal product of capital in real terms. The market loan demand elasticity, PED_t, captures their dependency on capital demand elasticity, PEK_t. The elasticity of entrepreneurs' loan demand concerning the equilibrium gross loan rate R^b_t under Cournot competition is:[12]

$$PED_t \equiv -\frac{\partial b_t}{\partial R^b_t} \frac{R^b_t}{b_t} = 1 + PEK > 0 \tag{44}$$

Solving the first-order condition (42), we find the following expression for the loan interest rate R^b_t, with $\Lambda_{t,t+1} > 0$ and $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t} > 0$:

$$R^b_t = \frac{R^d_t - \kappa_{k^B} \left(\frac{k^B_t}{b_t} - \tau^B \right) \left(\frac{k^B_t}{b_t} \right)^2}{\left(1 - PED_t^{-1} \frac{1}{N} \right)} \tag{45}$$

where N is the number of banks, and κ_{k^B} are the banks capitalization costs. From equation (45), the loan interest rate R^b_t decreases in the number of banks N (more banking competition), and in the loan demand elasticity PED_t. Entrepreneurs respond quickly to the increased loan interest rate R^b_t and reduce the number of loans b_t demanded, forcing banks to charge lower loan interest rates. The capital-to-loans rate, $\frac{k^B_t}{b_t}$, is below the optimum target τ^B, so the condition $R^b \geq R^d$ is always valid.

[11]It can be seen by the equation (16).
[12]See Appendix F.

3.7 Equilibrium Conditions

Equilibrium was imposed on deposit and loan markets. According to Walras' law, if $n-1$ markets are in equilibrium, then the n^{th} market is also in equilibrium. Thus, in equilibrium, the aggregate resource constraint is:

$$c_t + c_t^E + i_t + div_t^B + \frac{\kappa_\pi}{2}(\pi_t - \overline{\pi})^2 y_t + \frac{\chi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t + \frac{\kappa_{k^B}}{2}\left(\frac{k_t^B}{b_t} - \tau^B\right)^2 k_t^B = y_t \quad (46)$$

This is also a good market-clearing condition. In equilibrium, the new capital supplied by capital producers equals entrepreneurs' capital demand, and the labor supplied by households equals entrepreneurs' labor demand. In addition, the Cournot equilibrium of banking sector can be written as $b_t^B = \sum_{j=1}^N b_t(j)$, $d_t^B = \sum_{j=1}^N d_t(j)$ and $k_t^B = \sum_{j=1}^N k_t^B(j)$, where the supply of loans from the banking sector b_t^B equals market loan demand b_t, demand for deposits from the banking sector d_t^B equals the supply of deposits from households d_t, and the banking system capital k_t^B is equal to the sum of the N banks' capital. From equation (33), the total loan supply b_t^B equals the total deposit holding in the banking system plus the total capital accumulated k_t^B in the banking sector, $b_t^B = d_t^B + k_t^B$.

4 Calibration and Bayesian Estimation

4.1 Data

Our model uses a quarterly time series of six Brazilian variables from 2000-Q3 to 2019-Q4 (77 observations). We chose the following variables: GDP, investment, IPCA inflation (official inflation index adopted in Brazil), nominal short-term interest rate (Selic), loans to firms (working capital), and deposits.[13] The GDP, investment, and IPCA inflation data are sourced from the Brazilian Institute of Geography and Statistics (IBGE).[14] The other data that reference the short-term interest rate (Selic), loans to firms, and deposits were extracted from the Central Bank of Brazil (BCB).[15] The short-term interest rate (Selic) is the benchmark Brazilian interest rate used as a basis for setting other rates in the financial system. The data used for estimation that show a trend are stationary using a one-sided HP filter with a smoothing parameter set at 1,600 (while all rates are demeaned). To avoid stochastic singularity, we consider the number of observable variables equal to the number of shocks in the model. Table 1 summarizes the variables used in the estimation:

[13] In Appendix G there is a more detailed description of the data used in the estimation.
[14] IBGE data can be found on the website www.ibge.gov.br.
[15] BCB data can be found on the website www.bcb.gov.br.

Table 1: Description of the observable variables used in the estimation

Variables	Series	Source
y_t^{obs}	GDP - seasonally adjusted	IBGE
i_t^{obs}	Investment - seasonally adjusted	IBGE
π_t^{obs}	IPCA index - CPI inflation	IBGE
R_t^{obs}	Short-term nominal interest rate - Selic annualized	BCB
b_t^{obs}	Credit operations with nonearmarked funds (end of period)	BCB
d_t^{obs}	Deposit money banks - Time deposits, savings and others	BCB

4.2 Calibration

Table 2 reports the values of the calibrated parameters. The idea is to use calibration parameters that can represent the Brazilian economy more closely. The parameter calibration not found in Brazil is calibrated according to the strategy of Li (2019) and Gerali et al. (2010). Household subjective discount factor β and entrepreneurs subjective discount factor β^E follows De Castro et al. (2015) and Gerali et al. (2010), respectively, and are equal to $\beta = 0.989$ and $\beta^E = 0.97$. The capital share α and depreciation rate of physical capital δ are calibrated with values of 0.44 and 0.015 respectively, according to De Castro et al. (2015).

Table 2: Calibrated parameters

Parameter	Value	Description
Households		
β	0.989	Subjective discount factor
ϕ_l	1.8	Relative utility weight on leisure time
Entrepreneurs		
β^E	0.97	Subjective discount factor
α	0.44	Physical capital share
δ	0.015	Depreciation rate for physical capital
Retailers		
ϵ	6	Elasticity of substitution between retail goods
Banks		
N	5	Number of banks
δ^B	0.09	Cost of managing the bank capital's position
τ^B	0.16	Target for the capital-to-loans ratio

The calibration for ϕ_l and ϵ is in line with Li (2019) and Gerali et al. (2010). The relative utility weight for leisure time ϕ_l in household' utility was 1.8. In the retailers' market, the calibration for the elasticity of substitution among retail goods ϵ was set to 6 to generate a 20% final good price markup $\left(x = \frac{\epsilon}{\epsilon-1}\right)$ in the steady-state, a value that is frequently used in the literature. Regarding the banking sector, we set the number of banks as $N = 5$ because the five largest banks dominate most of the share of assets in the Brazilian banking system. Finally, the target for the capital-to-loans ratio τ^B is set at

0.16 (16%) and the cost for bank capital management position δ^B is set at 0.09, following Ferreira et al. (2015).

4.3 Estimation Methodology

In this section, we present the techniques used to estimate the model parameters. Our model is estimated using full-information likelihood-based Bayesian methods, following An and Schorfheide (2007), and Smets and Wouters (2007). The choice of this Bayesian estimation technique is based on the analysis of the best estimation techniques by several authors. Rabanal and Rubio-Ramírez (2005) argued that the Bayesian approach can estimate the entire DSGE model, unlike the GMM method, which is based on a particular equilibrium relationship. Another argument that the authors use is that the Bayesian technique is more efficient for small samples than the GMM. In addition, the Bayesian approach allows for the use of prior distributions that function as weights in the posterior distribution process. Herbst and Schorfheide (2015) emphasized that the use of a priori distributions facilitates the process of identifying the model's parameters and minimizes the problem of extreme values, that is, coefficient values that do not reflect the reality of the economy represented in the model.[16]

We linearized the equations that describe the model around the steady-state. The solution takes the form of a state-space model used to compute the likelihood function using the Kalman filter. The Bayesian approach allowed us to choose prior distributions for the model parameters added to the likelihood function. The posterior distributions of the parameters are estimated using the Metropolis-Hastings algorithm. The Bayesian method uses the priors of the parameters $p(\theta)$ combined with the likelihood of the DSGE model $L(Y|\theta)$ to produce the posterior joint distribution of the parameter vector $p(\theta|Y)$, where $Y = \{y_1, \cdots, y_n\}$. The likelihood function is used to update the prior beliefs regarding the parameters conditioned on the sample information. We also adopted a Monte Carlo Markov chain (MCMC) sampling algorithm to simulate the parameter vector θ distribution because the posterior distributions are difficult to characterize. The random-walk Metropolis-Hastings algorithm, which belongs to the class of MCMC algorithms, is used to generate a sample that approximates (draws) the posterior distributions. In the MCMC method, the objective is to obtain a sample of the posterior distribution and calculate sample estimates of the distribution characteristics through iterative simulation techniques based on Markov chains. The idea is to simulate a random walk in parameter space θ that converges to a stationary distribution of interest for estimation.

4.4 Prior Distribution

The prior distributions chosen to estimate the parameters can be seen in Table 3. The prior information mainly follows Gerali et al. (2010). The prior investment adjustment

[16]The model sample period runs from 2000:Q3 to 2019:Q4. Our estimation is done with Dynare 4.6.1.

costs χ follow a gamma distribution with a mean of 2.5 and a standard deviation of 1.0. The gamma distribution is also used as a prior for the banks' capitalization cost κ_{k^B} and the price adjustment cost of retailers κ_π. We defined κ_{k^B} with a mean of 10.0 and standard deviation of 5.0 and set κ_π with a mean of 50.0 and standard deviation of 20.0. The price indexation parameter ι_p was set to a mean of 0.65 and a standard deviation of 0.20. The normal and gamma distributions are used as priors for the monetary policy rule parameters. The exception is the parameter that determines the degree of interest rate persistence ρ_r, which the literature uses as a beta distribution. The prior mean for ρ_r was 0.75, and the standard deviation was 0.10. The coefficient for the output gap ϕ_y follows a normal distribution with a mean of 0.10 and a standard deviation of 0.15. The coefficient of the response to inflation ϕ_π follows a gamma distribution with a mean of 2.0 and a standard deviation of 0.50. The prior means for all autoregressive coefficients $(\psi_z, \psi_k, \psi_{qk}, \psi_\pi, \psi_{k^B})$ were set to 0.80, with a standard deviation of 0.10. For these coefficients, we use the beta distribution as the prior distribution. The priors' means of the shocks $(\sigma_z, \sigma_k, \sigma_{qk}, \sigma_\pi, \sigma_{k^B}, \sigma_R)$ are assumed to follow inverse-gamma distribution with a value of 0.01 and standard deviation of 0.05.

Table 3: Prior and posterior distribution of structural parameters

Parameter description			Prior			Posterior		
			Distribution	Mean	Std. dev	Mean	90%HP	
κ_π	Adjustment Cost	prices	gamma	50.0	20.0	86.35	35.28	98.29
χ		investment	gamma	2.5	1.0	5.020	2.987	6.906
κ_{k^B}		bank capital	gamma	10.0	5.0	22.19	17.15	27.04
ρ_R	Taylor Rule	persistence	beta	0.75	0.10	0.620	0.501	0.736
ϕ_π		inflation	gamma	2.0	0.5	1.565	0.759	2.303
ϕ_y		output	normal	0.10	0.15	0.336	0.220	0.451
ι_p	Indexation	prices	beta	0.65	0.20	0.735	0.509	0.985
ψ_z	AR process	productivity	beta	0.80	0.10	0.919	0.875	0.965
ψ_k		collateral	beta	0.80	0.10	0.391	0.265	0.519
ψ_{qk}		investment	beta	0.80	0.10	0.404	0.268	0.547
ψ_y		output	beta	0.80	0.10	0.795	0.648	0.960
ψ_{k^B}		financial	beta	0.80	0.10	0.903	0.873	0.932
σ_z	Shocks	productivity	inv. gamma	0.01	0.05	0.035	0.029	0.040
σ_k		collateral	inv. gamma	0.01	0.05	0.050	0.043	0.058
σ_{qk}		investment	inv. gamma	0.01	0.05	0.061	0.050	0.072
σ_y		output	inv. gamma	0.01	0.05	0.082	0.024	0.146
σ_R		policy rate	inv. gamma	0.01	0.05	0.064	0.046	0.081
σ_{k^B}		financial	inv. gamma	0.01	0.05	0.097	0.085	0.114

4.5 Posterior Results

Table 3 also reports the posterior mean and 90 percent probability intervals for the structural parameters.[17] Posterior distribution draws of the model parameters are obtained

[17] Appendix H provides the graphs of the priors and posteriors of the structural parameters.

using the Metropolis-Hastings algorithm. We run ten parallel chains, each chain with a length of 100,000.[18] The scale factor was set to deliver acceptance rates of 30 percent. The posterior mean estimated from Brazil data for the adjustment cost of prices $\kappa_\pi = 86.35$ and investment adjustment cost $\chi = 5.02$ are higher than their prior, and both are high compared to the values for the European Union, see Gerali et al. (2010); $\kappa_\pi = 30.57$; and Li (2019), $\chi = 2.50$. Regarding the cost of capitalization of banks $\kappa_{k^B} = 22.19$, we found a higher value for Brazil than that found for Europe $\kappa_{k^B} = 11.49$, in Gerali et al. (2010). This high value of Brazilian banks' capitalization cost means that any shock that affects the capital-to-loans ratio can amplify the spread variation. Regarding the estimated parameters of the Taylor rule, the interest rate persistence is estimated as $\rho_r = 0.62$, the response to the deviation of inflation from the target has a posterior mean equal to $\phi_\pi = 1.56$, and the response to the output gap is equal to $\phi_y = 0.33$. The estimate of the price indexation parameter found a value equal to $\iota_p = 0.73$. Estimates of autoregressive coefficients show that some shocks, such as productivity and financial shocks, have high persistence. However, collateral and investment shocks have lower persistence.

5 Comparative Static with Number of Banks N

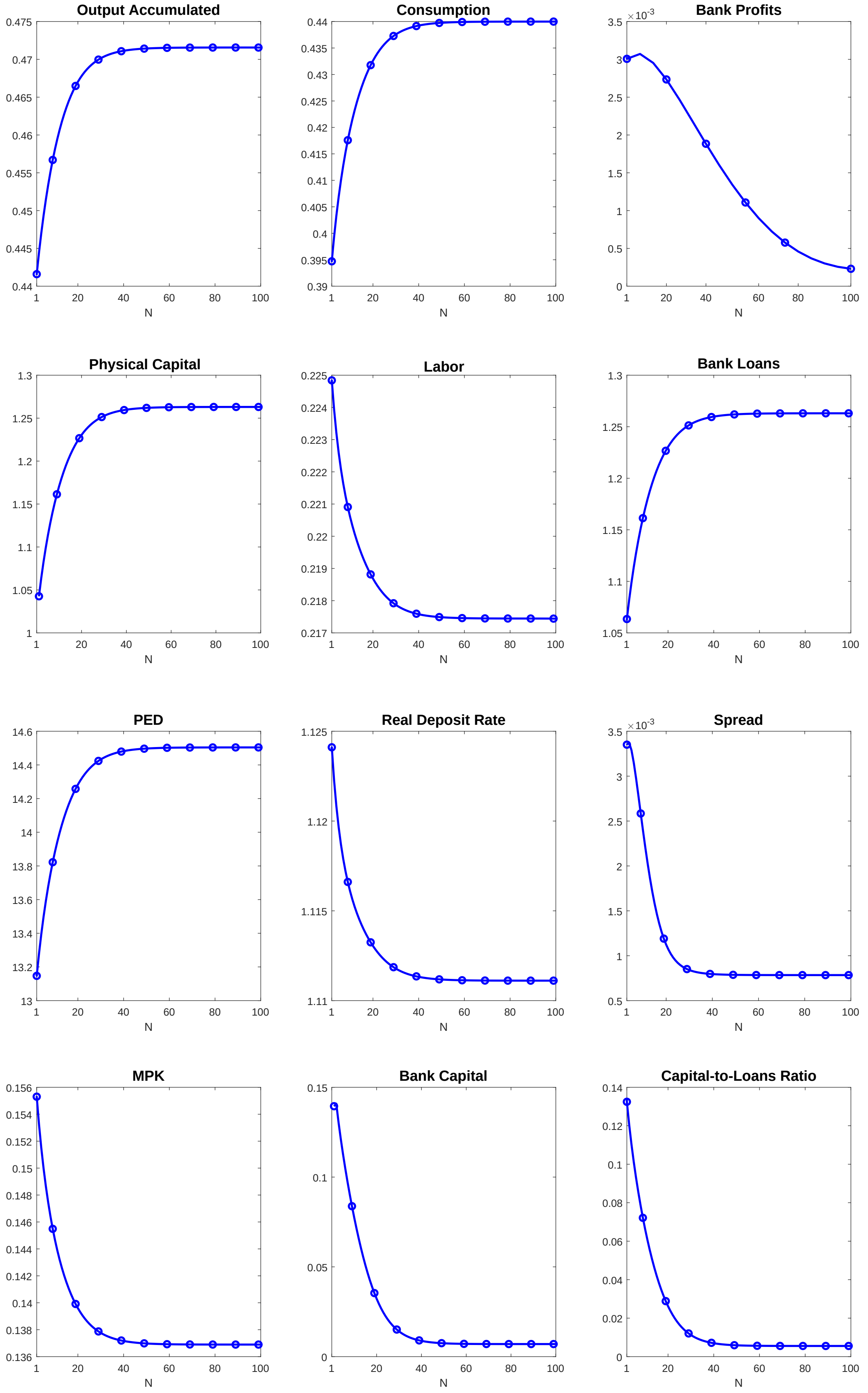
Figure 3 shows how the steady-state values of the main variables change when the number of banks N increases from 1 to 100 in the absence of any shock. A higher N implies more banking competition and, consequently, banks lose the ability to readjust the loan rate via the imperfect competition channel, causing a drop in the spread. The lower cost of borrowing causes collateral-constrained entrepreneurs to increase their demand for loans to finance physical capital purchases, thus increasing the economy's accumulated output and aggregate consumption.

A lower loan rate makes physical capital k cheaper relative to labor l for entrepreneurs, increasing the $\frac{k}{l}$ ratio and thus reducing the marginal product of capital $\alpha z \left(\frac{k}{l}\right)^{\alpha-1}$.[19] Capital demand elasticity PEK (43) increases when the marginal product of capital (MPK) is lower and, consequently, increases the loan demand elasticity (PED is more elastic). This model is different from perfect banking competition models as the spread does not become zero when banks are organized under Cournot competition and accumulate capital k^B subject to capital adequacy requirements. This occurs because the

[18]The fraction of parameter vectors initially generated to be dropped as a burning before posterior simulations was set at 0.5.

[19]Since $\alpha < 1$ under the assumption of a production function of constant returns to scale, the MPK decreases in the capital-labor ratio.

Figure 3: Steady-state values for $N \in [1, 100]$



Note: The spread is expressed in percentage points, and MPK is calculated as $\alpha z k^{\alpha-1} l^{1-\alpha}$.

loan rate R^b (45) will not be equal to the interest rate R^d paid on deposit d due to the existence of the banks' capitalization cost ($\kappa_{k^B} \neq 0$) in the bank stress channel. When N increases, banks can optimize by setting their capital-to-loans ratio $\frac{k^B}{b}$ to less than their optimal target value τ^B such that $R^b > R^d$ remains valid. From (36), a lower loan rate decreases the banks' profits, which also reduces accumulated banking capital (34). With a higher bank loan amount in the economy and less capital accumulation by banks, the capital-to-loans ratio $\frac{k^B}{b}$ decreases.

6 Dynamic Analysis

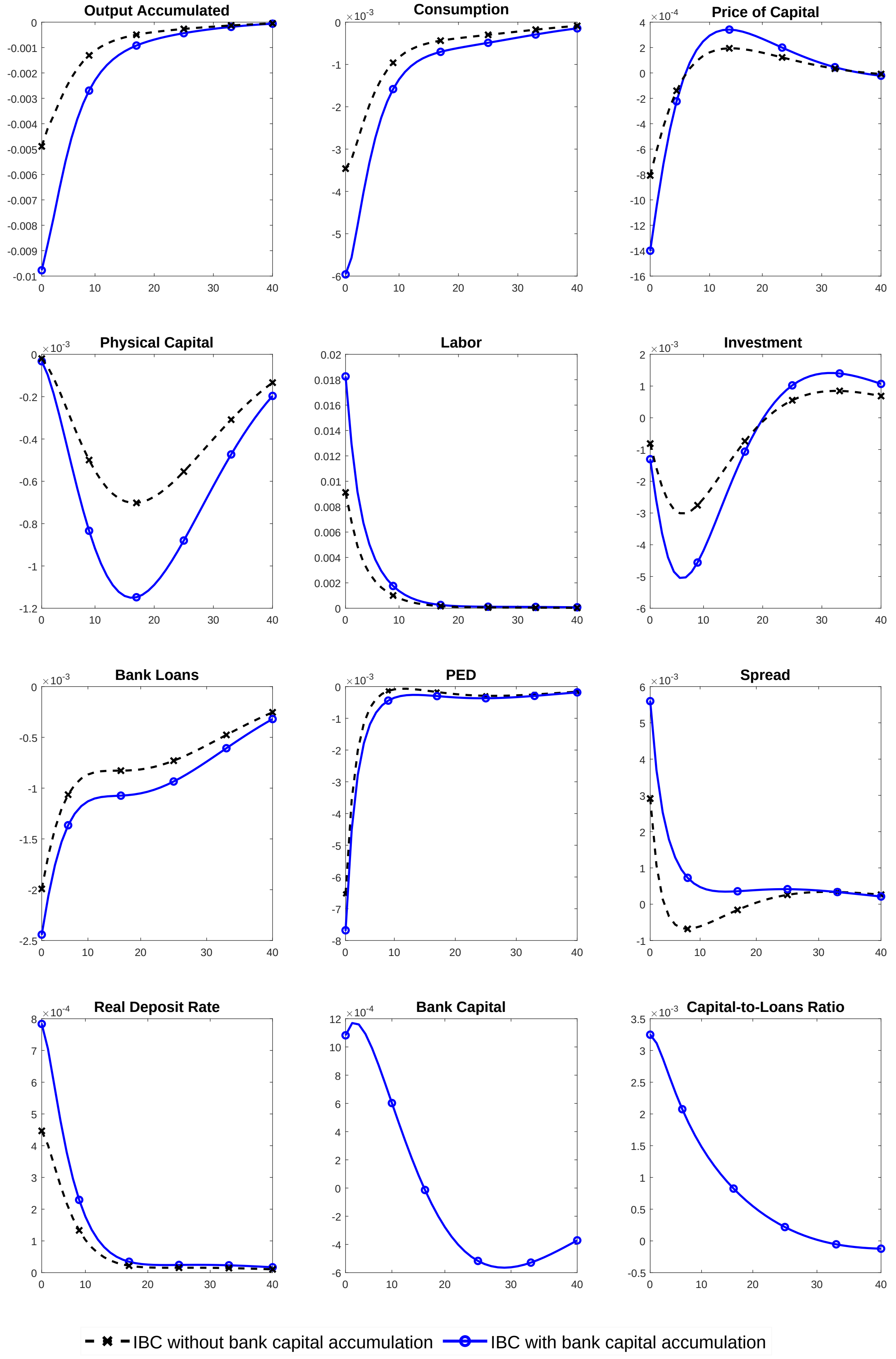
This section investigates how the adverse shocks propagate through the imperfect banking competition (IBC) and bank stress channels. We consider four adverse shocks: (i) productivity, (ii) collateral, (iii) financial, and (iv) investment. There are two scenarios for this exercise. The first is the scenario with IBC without capital accumulation by banks ($k^B = 0$). The second scenario is an IBC with banks' capital accumulation subject to the capital adequacy requirement ($k^B \neq 0$).

6.1 Productivity Shock

Figure 4 shows the impulse responses after a persistent negative productivity shock z. The rise in the spread after the negative productivity shock that affects the entrepreneurs' production function is due to the bank's market power (N low) and the fall in loan demand elasticity (PED). The negative productivity shock reduces the marginal productivity of capital (MPK) and, consequently, the entrepreneurs' demand for physical capital, causing a drop in their price q. A fall in the price of physical capital (entrepreneurs' collateral) reduces entrepreneurs' borrowing capacity (11) and makes them more financially constrained. The result is a more inelastic PED, as seen in Figure 4. With IBC, banks take advantage of the lower loan demand elasticity by reducing their loan quantity b to achieve a higher equilibrium loan rate R^b, which increases the spread.

The productivity shock has a secondary effect on the spread through the bank stress channel: the banks' movement to reduce the number of loans b increases the capital-to-loans ratio from two actions. The first is the drop in loans b, and the second is the increase in bank capital accumulated k^B due to the higher loan rate charged to financially constrained entrepreneurs. Banks always keep their capital-to-loans ratio $\frac{k^B}{b}$ below the optimal τ^B.

Figure 4: Impulse responses to a negative productivity shock



Note: The horizontal axis shows the quarters after a negative productivity shock z at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

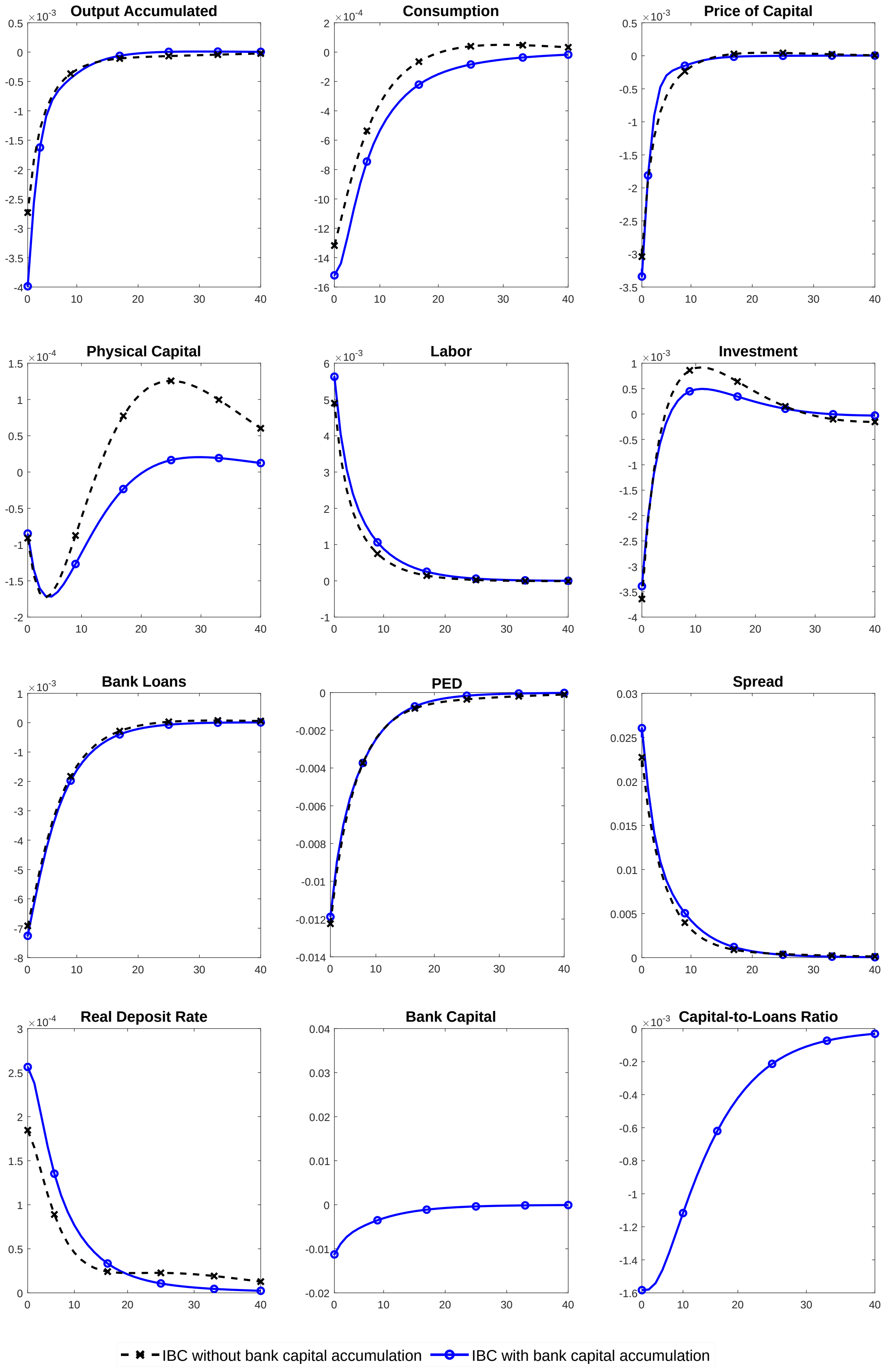
Whenever adverse shocks affect the capital-to-loans ratio, the effects of these shocks are transmitted to the loan rate and, consequently, the spread due to the existence of capitalization cost κ_{k^B} (45). The amplifying effect on the countercyclical spread in an IBC scenario with a bank stress channel occurs because of banks' capitalization cost κ_{k^B} that interferes in the equilibrium loan rate R^b charged to entrepreneurs. Under IBC with the bank stress channel, the responses of output, investment, physical capital, and households' consumption are amplified about the scenario that banks do not accumulate capital. The amplification effect of real variables can be explained by the existence of the bank stress channel, which amplifies the countercyclical spread due to the banking sector's cost of capitalization. Figure 4 demonstrates a slower recovery of physical capital when the bank stress channel is present, because of the higher cost of credit generated by this channel, which directly amplifies the drop in accumulated output and consumption.

6.2 Collateral Shock

This section investigates the negative shock in the pledgeability ratio m^k, which reduces the guarantees entrepreneurs offer to banks to borrow (Figure 5). The adverse collateral shock is a supply-side shock since it directly affects the credit supply to entrepreneurs and, therefore, the output accumulated. The exogenous reduction in the pledgeability ratio m^k directly reduces the fraction of physical capital used as collateral by entrepreneurs to obtain loans from banks and, therefore, lowers the entrepreneur's borrowing capacity through the binding collateral constraint (11). The decrease in the fraction of entrepreneurs' collateral makes them financially constrained and reduces the loan demand elasticity, PED. Banks in imperfect competition take advantage of the fragility of entrepreneurs and increase their loan rates and spread. With the higher cost of credit, entrepreneurs reduce their level of investments, generating a drop in accumulated output.

The bank stress channel also has the power to amplify the spread when an adverse collateral shock occurs. Figure 5 shows that a lower pledgeability ratio m^k directly reduces entrepreneurs' borrowing capacity and banks' capital accumulation, even if banks set a higher loan rate R^b. Unlike the production shock that affects the production function of entrepreneurs, the negative collateral shock reduces the demand for loans from entrepreneurs, affecting banks' ability to profit with a high-interest rate on loans. This reduction in bank capital accumulation causes a drop in the capital-to-loans ratio and generates a second spread increase when bank stress channel is active. The responses of accumulated output,

Figure 5: Impulse responses to a negative collateral shock



Note: The horizontal axis shows the quarters after a negative collateral shock m^k at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

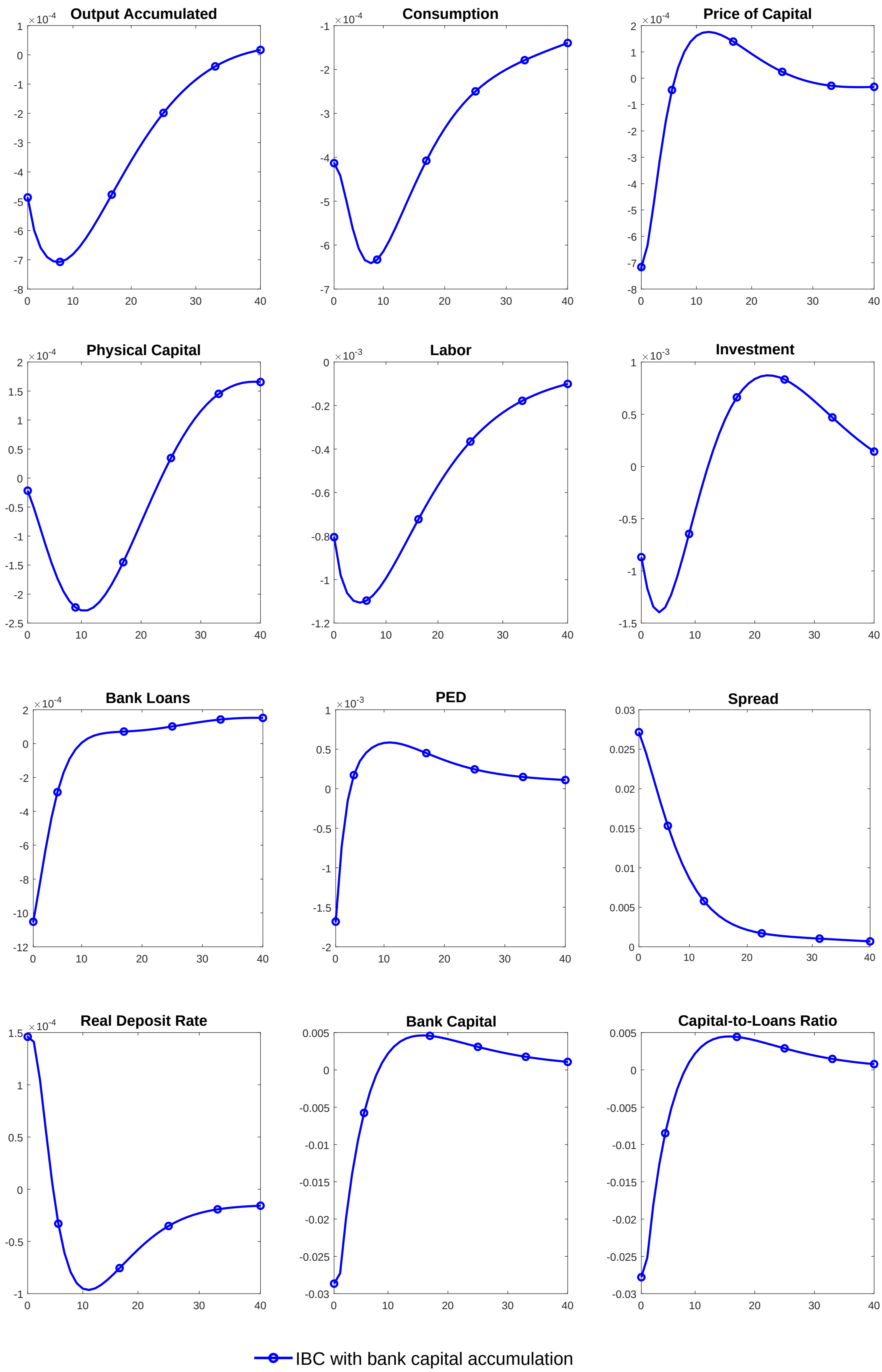
physical capital, and consumption also have an amplifying effect compared to a scenario with only an IBC channel.

6.3 Financial Shock

To assess the importance of bank stress channel for the countercyclical movement of the spread, it is also essential to recognize how the effects of shocks on banks' balance sheets affect the availability and cost of credit. This section examines what happens when a bank's capital experiences a strong negative shock. To run the simulation, we introduce the possibility of an unexpected and persistent contraction of bank capital k^B. The negative financial shock is persistent because the estimated parameter for financial shock persistence indicates a value of $\psi_{k^B} = 0.903$. Figure 6 shows that the adverse financial shock reduces the capital-to-loans ratio $\frac{k^B}{b}$ away from the optimal target τ^B, requiring a fast recovery of banks. The decline in bank capital leaves banks too leveraged and has a burden of costs of their deviation from the optimal capital requirement. Banks then rebalance assets and liabilities by reducing borrowing and, consequently, increasing the loan rate. The reduction in loans is a significant concern for banks, as they need to approximate their $\frac{k^B}{b}$ to the optimal target τ^B quickly.

A rapid increase in the spread contributes to rebuilding the bank's capital stock after an adverse financial shock. The spread increase will be more significant if the bank's capitalization cost is high. The financial shock in the bank balance sheet has a secondary effect on the spread increase through an IBC channel. Entrepreneurs reduce investments due to the increase in credit costs, which reduces their borrowing constraint. A high loan rate makes them more financially constrained, as can be seen from the drop in PED in Figure 6. In turn, banks increase the spread more when entrepreneurs have an inelastic PED, amplifying the fluctuations of variables on the real side of the economy. Moreover, the fall in entrepreneurs' investment affects labor, which negatively deviates from the initial steady-state. A lower demand for k reduces its price q. The accumulated output y also decreases with a lower investment level, as reflected in the consumption drop. Figure 6 shows that the economy starts a recovery process ten quarters after the negative financial shock. However, despite this change in the downward trajectory, the output takes about 40 quarters to return to the initial equilibrium. In contrast, consumption has a recovery time of more than 40 quarters. The PED returns to its initial value 20 quarters after the initial shock. Physical capital starts its recovery before the output recovery. This increase

Figure 6: Impulse responses to a negative financial shock



Note: The horizontal axis shows the quarters after a negative financial shock sk^B at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

in demand for k raises the prices of capital q, which stabilize after 30 quarters. The labor presents a slower recovery, taking more than 40 quarters for its complete recovery.

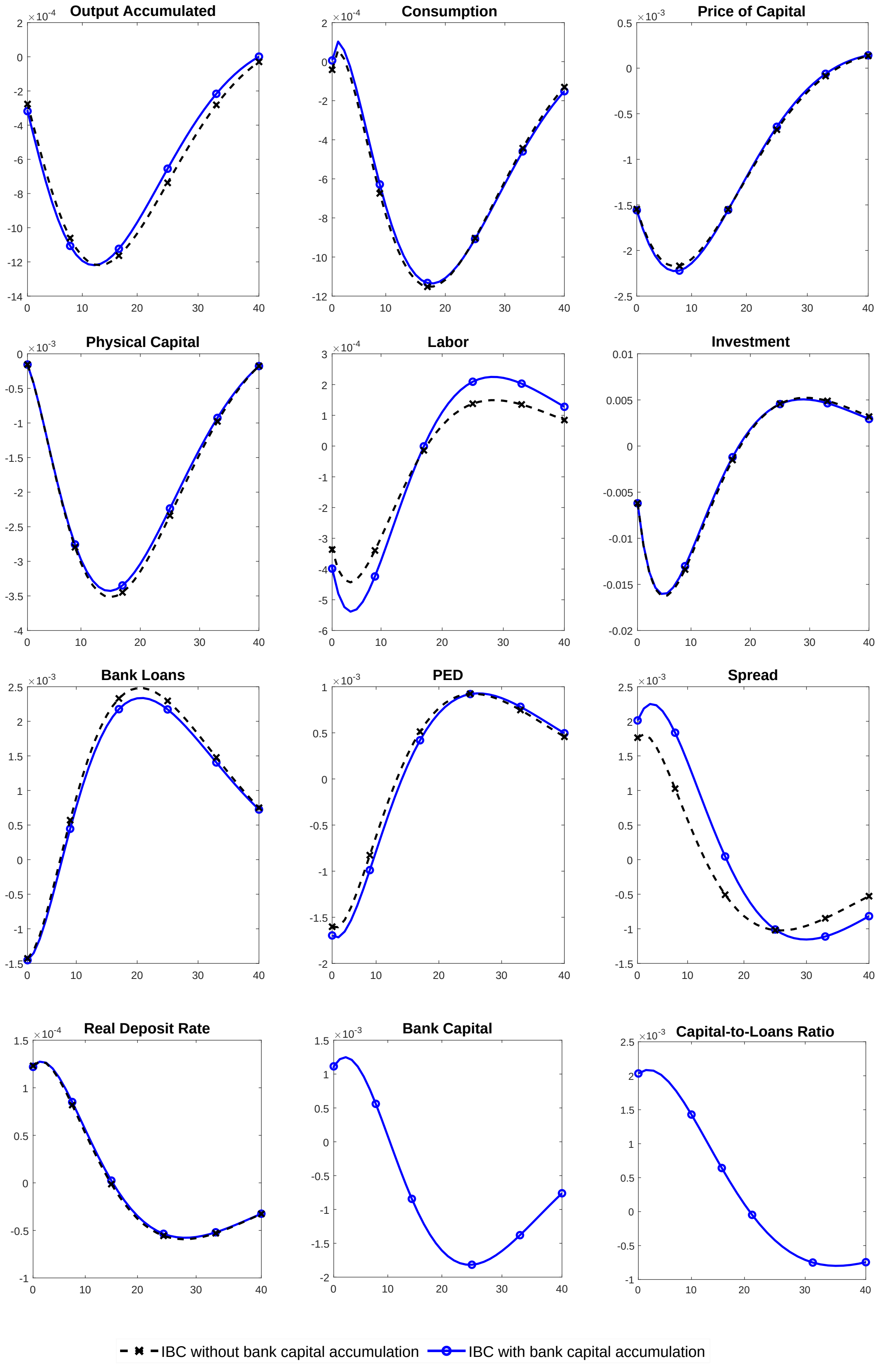
6.4 Investment Shock

This section investigates the effects of an adverse investment shock that affects capital producers and increases the cost of producing new capital that is sold to entrepreneurs. The lower use of physical capital by entrepreneurs causes a decrease in the price of capital q. Negative investment shock does not have a high persistence. The parameter indicating persistence was estimated with a value equal to $\psi_{qk} = 0.404$, as shown in Table 3. The price of capital fall directly affects entrepreneurs' PED and, consequently, their borrowing constraint by reducing the value of their collateral. Entrepreneurs with inelastic loan demand elasticity provide the necessary incentive to increase the spread in an imperfect competitive banking environment.

Figure 7 shows that the drop in physical capital lasts for about 20 quarters after the negative investment shock and the total recovery took more than 40 quarters. The drop in entrepreneurs' activities immediately affects labor. The sharp decline in labor reversed 20 quarters after the shock. The accumulated output shows a persistent decline 15 quarters after the negative investment shock and requires more than 40 quarters to recover fully. The drop in consumption, in turn, is motivated by a reduction in output levels, and it takes more than 40 quarters to return to the initial equilibrium.

The investment shock propagates mainly through IBC channel and has a secondary effect on the bank stress channel. The increase in the spread allows banks to increase the accumulation of capital k^B, which, together with the fall in loans b, increases the capital-to-loans ratio. Figure 7 shows that although there is an increase in the capital-to-loans ratio, the difference in the target $(\frac{k^B}{b} - \tau^B)$ is negative, which amplifies the spread. The simulation shows that negative shocks that do not affect the entrepreneurs' collateral and the banks' balance sheet, such as productivity and investment shocks, allow banks organized under imperfect competition to accumulate more capital as the spread increases. The spread amplifying effect dissipated after 20 quarters. The low persistence estimated for the investment shock from the Brazilian data can explain this rapid return to equilibrium.

Figure 7: Impulse responses to a negative investment shock



Note: The horizontal axis shows the quarters after a negative investment shock sq^k at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables, interest rates and the spread in percentage points.

7 Sensitive Analysis

This section checks the robustness of the baseline results in Section 6 by changing the investment adjustment cost parameter χ and banks' capitalization cost κ_{k^B} in the bank stress channel. We compared the estimated values of these parameters in Brazil with values used by the European Union by Li (2019) and Gerali et al. (2010), whereas all other parameters were calibrated and estimated as the baseline analysis. Third, we test the robustness of the model by increasing the number of banks N and verifying the effects on the model's macroeconomic variables.

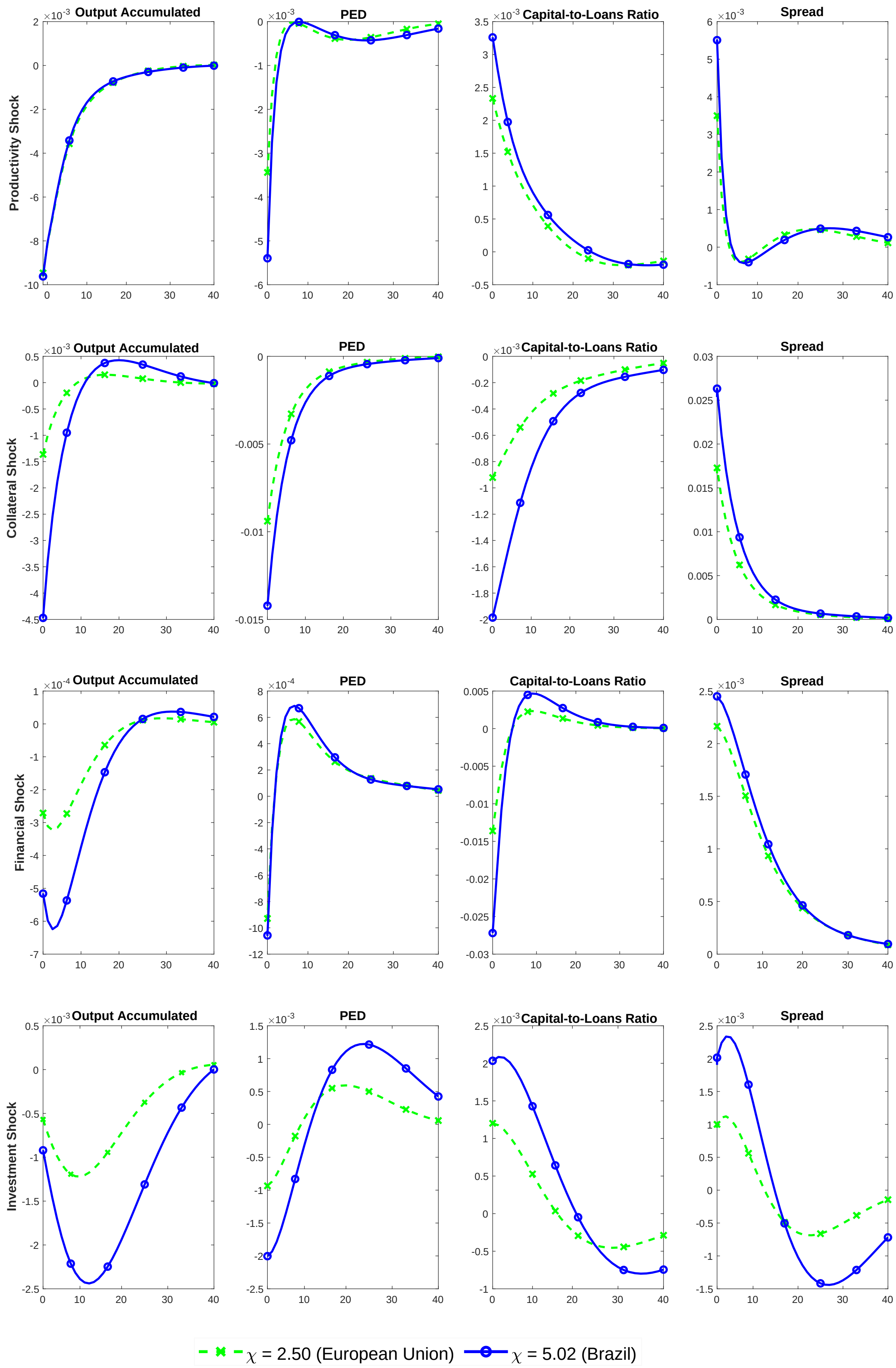
7.1 The Investment Adjustment Cost χ

After the estimations described in Section 4, we find a value for the investment adjustment cost equal to $\chi = 5.02$ higher than $\chi = 2.5$, the value used by Li (2019) for the EU. Figure 8 shows that the increase in investment cost χ does not significantly affect the spread when we consider the productivity, collateral, and financial shocks. The most significant impact on the spread is seen in the negative investment shock that affects the capital producers responsible for χ.

The decrease in the PED is also accentuated by the higher χ. The higher the reduction in the price of capital q due to the negative investment shock and the increase in χ causes a higher fall in the PED and a consequent higher increase in the spread by IBC channel. A higher investment adjustment cost χ significantly slows entrepreneurs' utilization of physical capital. As a result, the drop in output is persistent for investment shocks, taking approximately 40 quarters to reach the initial equilibrium. The productivity shock does not have an amplifying effect on the accumulated output when there is an increase in the adjustment cost χ.

Productivity and investment shocks increase the capital–to-loans ratio. Banks in imperfect competition can immediately increase capital accumulation in response to these adverse shocks. For a negative financial shock, a higher χ cost amplifies the capital-to-loans ratio fluctuation only initially, dissipating ten quarters after the shock. However, a large effect is seen on the capital-to-loans ratio in the presence of an adverse collateral shock when χ is high, as collateral-constrained entrepreneurs use physical capital to borrow. A high χ reduces the availability of entrepreneurs' collateral and demand for loans, which causes a significant loss of bank capital accumulation in the presence of collateral shock.

Figure 8: Impulse responses to negative shock with different χ



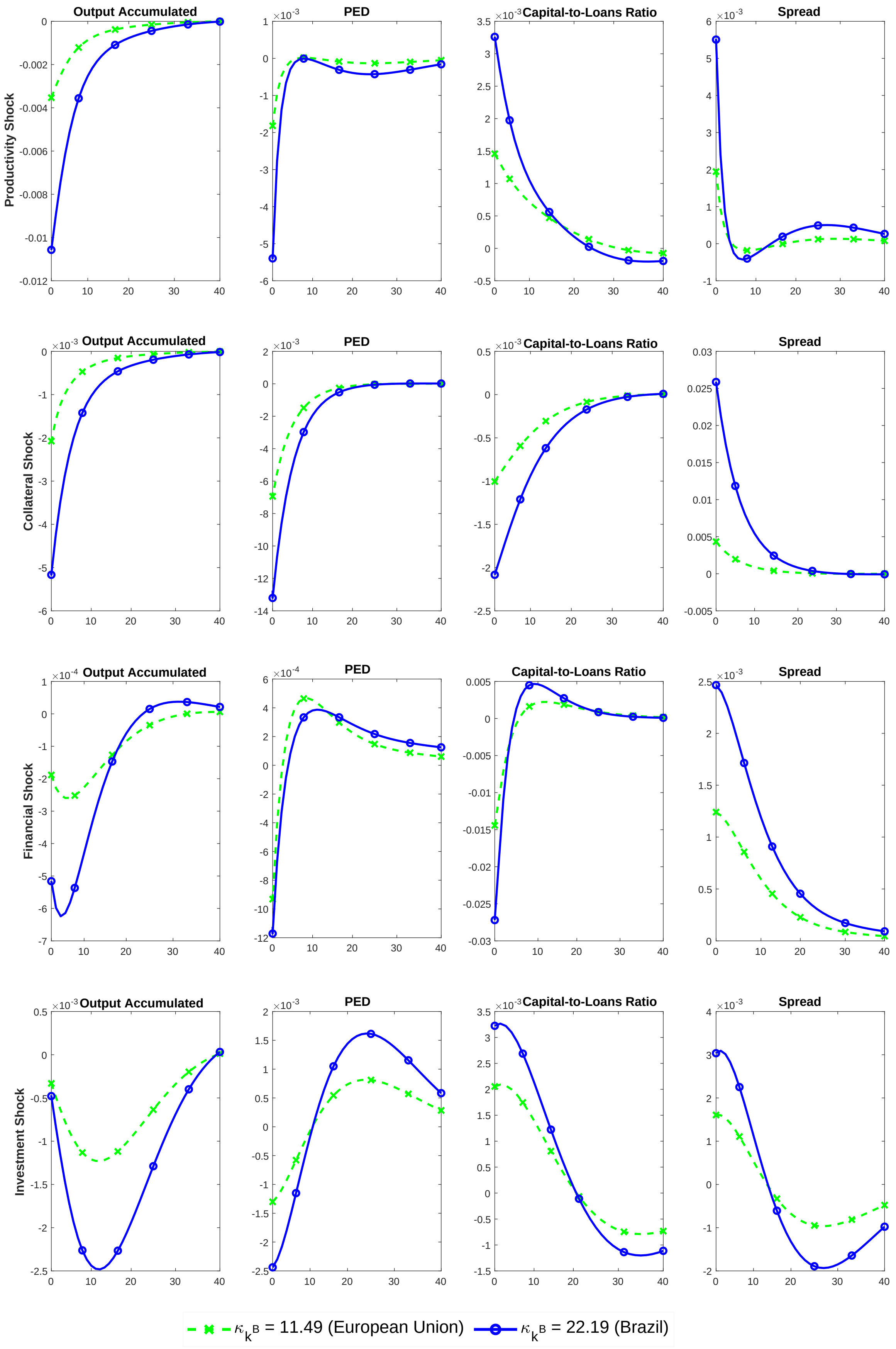
Note: The horizontal axis shows quarters after a negative shock at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread.

7.2 The Banks' Capitalization Cost κ_{k^B}

Figure 9 shows the effects of different shocks that hit the economy when we vary banks' capitalization cost κ_{k^B}. The estimated banks' capitalization cost for Brazil is $\kappa_{k^B} = 22.19$, which is higher than the cost $\kappa_{k^B} = 11.49$ for EU banks (Gerali et al. (2010)). The cost κ_{k^B} has the power to amplify the spread variation given shocks that pass through the capital-to-loans ratio $\frac{k^B}{b}$. The unexpected adverse financial shock operates in the bank stress channel and destroys the bank capital accumulated k^B, causing an immediate drop in the capital-to-loans ratio. This capital-to-loans ratio movement has increased the spread. The increase in the spread will be larger for high values of the banks' capitalization cost, which amplifies the fluctuations of the real variables that are affected by the spread. A higher loan rate and lower borrowing capacity make entrepreneurs more financially constrained (inelastic PED), increasing the spread from a secondary effect via the IBC channel.

The fall in accumulated output resulting from an adverse financial shock that slows down entrepreneurial activity is amplified by higher κ_{k^B} value. About the negative productivity shock, the increase in κ_{k^B} does not significantly affect the spread variation. The spread trajectory returns to the initial equilibrium after ten quarters. An increase in bank capital and low loan supply increases the capital-to-loans ratio. This $\frac{k^B}{b}$ movement contributes to an increase in the spread to a lesser magnitude via the bank stress channel. A similar capital-to-loans ratio movement is observed for investment shock. For productivity, collateral, and investment shocks, the economy's accumulated output recovers at the initial equilibrium 40 quarters after the adverse shocks and in the presence of higher κ_{k^B}. Adverse collateral shock also produces an amplifying effect on the spread. With less physical capital to use as collateral, entrepreneurs have an inelastic PED, which encourages banks in imperfect competition to charge a higher spread. This reduction in entrepreneurs' loan demands prevents banks from increasing their capital. Bank capital accumulation decreases and reduces the capital-to-loans ratio. The drop in the capital-to-loans ratio combined with the high κ_{k^B} cost significantly increased the spread.

Figure 9: Impulse responses to negative shock with different κ_{k^B}



Note: The horizontal axis shows quarters after a negative shock at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread.

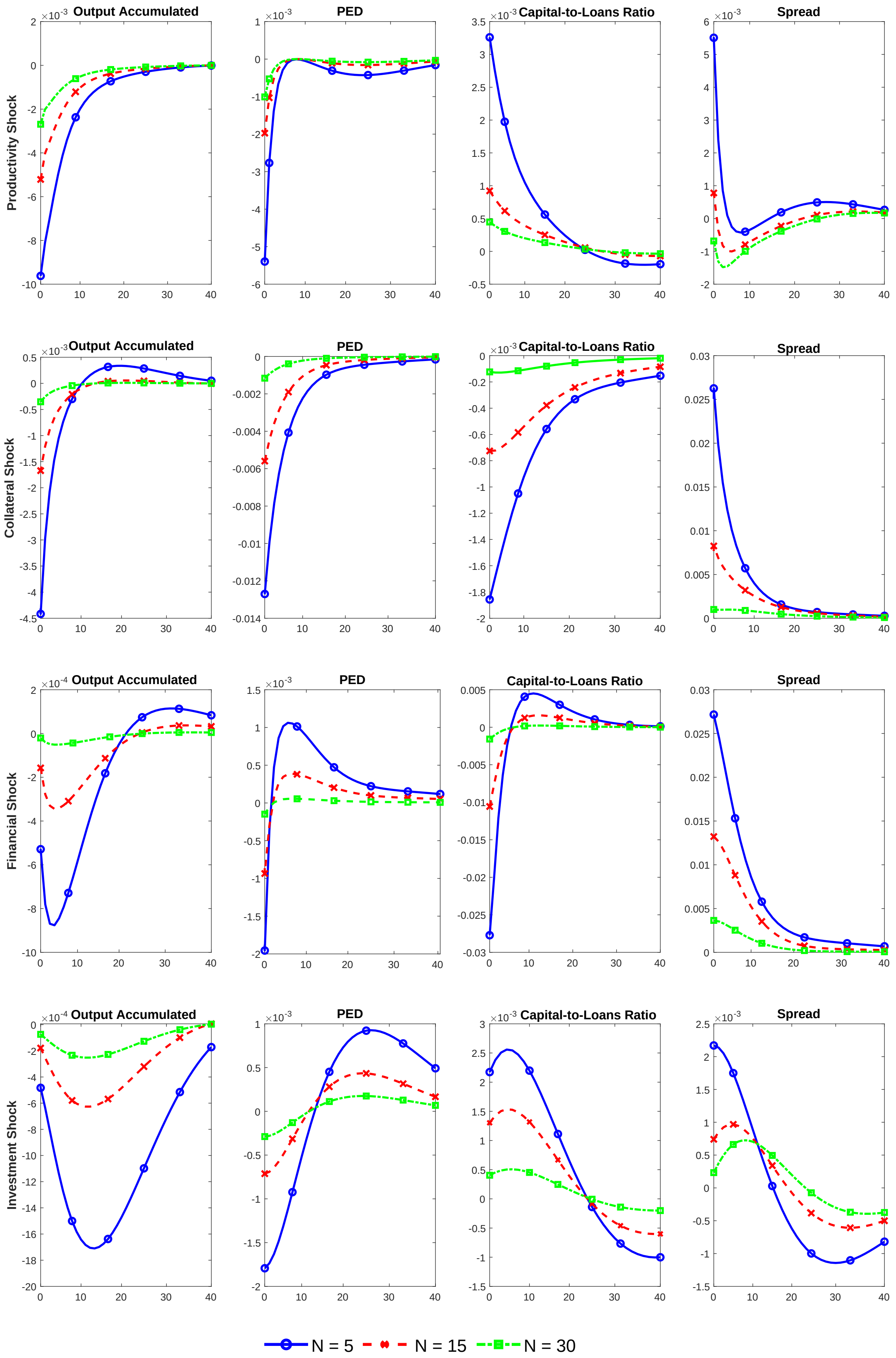
7.3 Effects of Banking Competition

Figure 10 shows the impulse responses of accumulated output, loan demand elasticity, capital-to-loans ratio, and the spread after adverse shocks when we vary the number of banks N (five, 15, and 30, that is, closer to perfect competition). For $N = 5$, the banks have a higher power to increase the loan rate via IBC channel after adverse shocks. Consequently, the spread increase effects are transmitted to the real variables in the model. A high spread reduces the activity conducted by entrepreneurs because of the higher borrowing cost, causing a sharp drop in the accumulated output of the economy.

If we allow new banks to enter the banking system until $N = 15$, the spread amplifying effect decreases considerably. The loan rate falls with increased banking competition. Consequently, the spread decreased. In this scenario, entrepreneurs can leverage more, and PED increases. Increased borrowing for entrepreneurial activity affects the accumulated output of the economy, attenuating its decline. In addition, we reduce the barriers to entry for new banks, and bank competition increases until $N = 30$. In this case, the spread amplification effect practically disappears after adverse productivity, collateral, and investment shocks, as shown in Figure 10. Many banks reduce the impact of shocks that originate in IBC channel on the spread.

However, more banking competition cannot completely cancel out the effect of financial shock on the spread, because this shock emerges in the bank stress channel. The increase in the number of banks to $N = 30$ reduces the spread variation about the scenario with $N = 5$ but does not make it equal to zero. An unexpected reduction in k^B causes the fall in the capital-to-loans ratio and generates a spread increase, regardless of the number of banks. Perfect banking competition can reduce the effects of a financial shock on the spread because when the shock passes to the IBC channel, the high number of banks makes it impossible for them to readjust the loan rate, given the drop in entrepreneurs' loan demand elasticity. Consequently, the financial shock that passes through the IBC channel has little impact on the spread.

Figure 10: Impulse responses to negative shock with different N

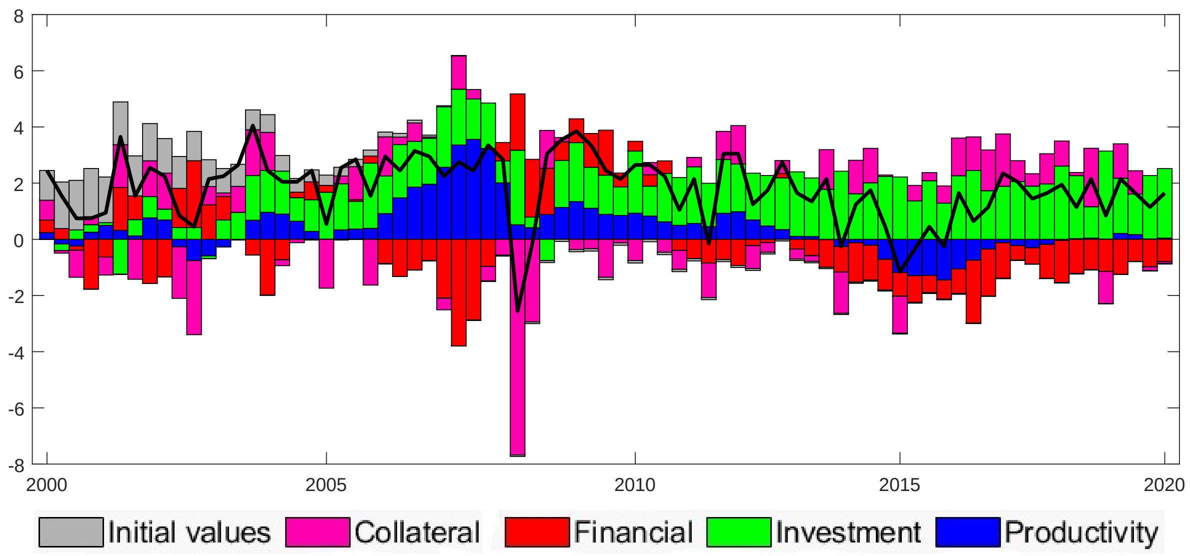


Note: The horizontal axis shows quarters after a negative shock at the beginning of period one. The vertical axis shows the percentage deviation from the steady-state for real variables and the spread.

8 Historical Decomposition

This section shows how productivity, collateral, investment, and financial shocks explain the output and spread fluctuations. This historical shock decomposition was obtained by fixing the model's parameters at the posterior mean, and using the Kalman smoother to obtain the values of the innovations for each shock.[20] The shocks mentioned are essential for explaining the output dynamics in the selected period. Figure 11 shows the historical decomposition of the accumulated output.

Figure 11: Historical decomposition of the accumulated output



Note: The decomposition is computed using the posterior mean distribution of the model. Macro shocks include productivity, investment, and collateral shocks. Financial shocks include bank balance sheet shocks.

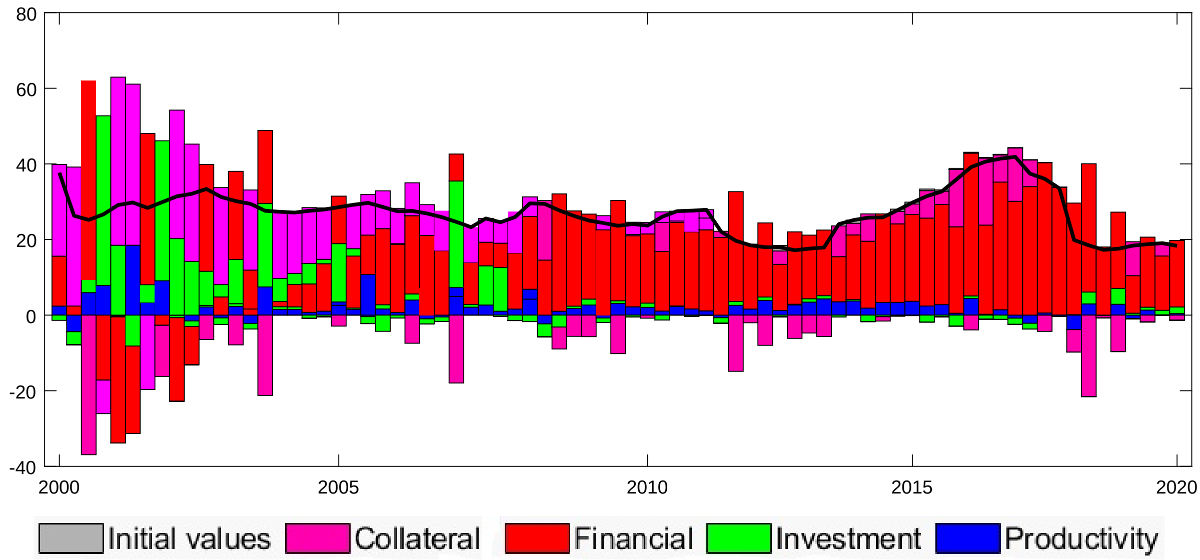
The investment shock acts as a component that drives the output increase, especially after the 2008 financial crisis. Productivity shocks also played an important role in the output increase until 2014. However, with the onset of the Brazilian recession in 2015 productivity shocks contributed to a drop in output. The collateral shock that affects entrepreneurs' assets contributes to increases and falls in output at different times over the selected period. The collateral shock was the main driver of the drop in output during the 2008 financial crisis. In Brazil, countercyclical measures stood out in the fight against the crisis. The domestic market was generally encouraged by increased credit, lower interest rates, and tax cuts. In 2009, it was already possible to notice a recovery in the Brazilian economy, both in growth and the return of financial flows to the country. Finally, the financial shock contributed more to the drop in output than to its growth, except during the credit expansion policy during the 2008 crisis. This negative contribution is due to the high capitalization cost of Brazilian banks, which leads to the effects of financial shocks

[20]The historical decomposition shows the historical deviations of the endogenous variables from their respective steady-state values into the contribution coming from the different types of shocks.

on borrowers through a high loan rate. The period with the most significant negative contribution was from 2014 to 2019.

Figure 12 shows the spread decomposition in the productivity, collateral, investment, and financial shocks. It is possible to notice a more significant influence of financial shocks on spread fluctuations. From 2008 onwards, financial shocks generated most of the spread increase. Unexpected financial shocks that hit banks' balance sheets and reduce the bank capital, provide the necessary incentive for banks to readjust the rate charged on loans. Consequently, this increased the spread. The collateral shock also played a more important role in the spread fluctuation, which declined over time. In contrast, the productivity shocks maintained a practically constant influence on the spread over time. Finally, the investment shock, which had more significant participation in the spread increase during 2000-2008, reduced its power after 2008.

Figure 12: Historical decomposition of the spread



Note: The decomposition is computed using the posterior mean distribution of the model. Macro shocks include productivity, investment, and collateral shocks. Financial shocks include bank balance sheet shocks.

9 Conclusions

Our study evaluates the role of financial frictions and imperfect banking competition in Brazil's business cycle. We estimate a DSGE model for Brazil that incorporates a Cournot banking sector, in which banks accumulate capital subject to capital adequacy requirements. We identified two channels, IBC and bank stress, and studied how these channels amplify the four different shocks. The imperfect competition channel amplifies shocks when the number of banks is low, and bank stress channel amplifies the shocks when banks' capitalization is high. The estimated parameters for the 2000-2019 interval show that the amplification of shocks is larger in Brazil than in the European Union.

Our findings show that the spread is more significant in scenarios with imperfect banking competition and bank capital adequacy requirements. The amplified countercyclical spread, which arises from the interaction of the two channels, tends to amplify the response of output, consumption, and other macroeconomic variables to adverse shocks. We also show that most of the spread increase in Brazil was due to financial shocks, mainly after 2008. Collateral shocks, which once had a more significant influence on spread fluctuations, lost influence after 2008. The financial shocks that increase the spread contribute the most to the decrease in accumulated output in Brazil. With the onset of the Brazilian recession in 2015, the productivity shock also contributed to the drop in output. Conversely, investment shocks played a different role, mainly contributing to output growth from 2008 to 2019. Finally, we show that new banks' access to the banking system reduces the spread amplifying effect and the effects of adverse shocks on the real economy.

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Appendices

A Households' Optimization Problem

The representative household maximizes their utility subject to the budget constraint (47):

$$\begin{aligned} \max_{\{c_t, l_t, d_t\}} \quad & \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] \\ \text{s.t.} \quad & c_t + d_t = \frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma^{CP}_t + \Gamma^R_t + div^B_t \end{aligned} \tag{47}$$

The lagrangian for this problem can be written as:

$$L = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(c_{t+s}) + \phi_l \ln(1 - l_{t+s})\right] + \lambda_t \left[\frac{R^d_{t-1} d_{t-1}}{\pi_t} + w_t l_t + \Gamma^{CP}_t + \Gamma^R_t + div_t - c_t - d_t\right]$$

and the first-order conditions are:

$$\begin{aligned} [c_t]: \quad \frac{1}{c_t} - \lambda_t &= 0 \\ \lambda_t &= \frac{1}{c_t} \end{aligned} \tag{48}$$

$$\begin{aligned} [l_t]: \quad \lambda_t w_t - \phi_l \frac{1}{(1 - l_t)} &= 0 \\ \lambda_t w_t &= \frac{\phi_l}{(1 - l_t)} \end{aligned} \tag{49}$$

$$\begin{aligned} [d_t]: \quad -\lambda_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R^d_t}{\pi_{t+1}}\right] &= 0 \\ \lambda_t &= \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{R^d_t}{\pi_{t+1}}\right] \end{aligned} \tag{50}$$

where we can do the following simplification in the Euler's equation:

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{R^d_t}{\pi_{t+1}}\right] \tag{51}$$

using $\Lambda_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$.

B Entrepreneurs' Optimization Problem

The entrepreneur's objective is to maximize the expected lifetime utility:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta^E)^s \ln\left(c^E_{t+s}\right) \tag{52}$$

subject to a budget constraint:

$$c^E_t + q_t k_t + w_t l_t + \frac{R^b_{t-1} b_{t-1}}{\pi_t} = \frac{y^w_t}{x_t} + (1-\delta) q_t k_{t-1} + b_t \tag{53}$$

and subject to a borrowing constraint:

$$b_t \leq m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta) k_t \pi_{t+1}}{R^b_t} \right] \tag{54}$$

which $m^k_t \in (0,1)$ denote the fractions of physical capital collateral that can be confiscated by banks when the entrepreneurs fail to repay their debt. The entrepreneurs' lagrangian can be written as:

$$L = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta^E\right)^s \ln\left(c^E_{t+s}\right) - \lambda^E_{1,t} \left\{ c^E_t + q_t k_t + w_t l_t + \frac{R^b_{t-1} b_{t-1}}{\pi_t} - \frac{y^w_t}{x_t} - (1-\delta) q_t k_{t-1} - b_t \right\}$$
$$- \lambda^E_{2,t} \left\{ b_t - m^k_t \mathbb{E}_t \left[\frac{q_{t+1} k_t (1-\delta) \pi_{t+1}}{R^b_t} \right] \right\}$$

The first-order conditions are:

$$(c^E_t): \quad \frac{1}{c^E_t} - \lambda^E_{1,t} = 0$$
$$\lambda^E_{1,t} = \frac{1}{c^E_t} \tag{55}$$

$$(b_t): -\lambda^E_{2,t} - \beta^E \mathbb{E}_t \left[\lambda^E_{1,t+1} \frac{R^b_t}{\pi_{t+1}} \right] + \lambda^E_{1,t} = 0$$
$$\lambda^E_{2,t} = \lambda^E_{1,t} - \beta^E \mathbb{E}_t \left[\lambda^E_{1,t+1} \frac{R^b_t}{\pi_{t+1}} \right] \tag{56}$$

$$(l_t): -\lambda^E_{1,t} w_t + \lambda^E_{1,t} (1-\alpha) \frac{z_t (k_{t-1})^{\alpha} (l_t)^{-\alpha}}{x_t} \frac{l_t}{l_t} = 0$$
$$w_t = (1-\alpha) \frac{y^w_t}{x_t l_t} \tag{57}$$

$$(k_t): -\lambda^E_{1,t} q_t + \beta^E \mathbb{E}_t \left[\lambda^E_{1,t+1} \frac{\alpha z_{t+1} (k_t)^{\alpha-1} (l_{t+1})^{1-\alpha}}{x_{t+1}} \frac{k_t}{k_t} \right] + \beta^E \mathbb{E}_t [\lambda^E_{1,t+1} (1-\delta) q_{t+1}]$$
$$+\lambda^E_{2,t} \mathbb{E}_t \left[\frac{m^k_t (1-\delta) q_{t+1} \pi_{t+1}}{R^b_t} \right] = 0$$

$$\lambda^E_{1,t} q_t = \beta^E \mathbb{E}_t \left[\lambda^E_{1,t+1} \left(\alpha \frac{y^w_{t+1}}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + \lambda^E_{2,t} \mathbb{E}_t \left[\frac{m^k_t (1-\delta) q_{t+1} \pi_{t+1}}{R^b_t} \right] \tag{58}$$

It is possible to find an expression for k_t replacing (54), (55) and (56) in (58):

$$\frac{q_t}{c^E_t} = \beta^E \mathbb{E}_t \left[\frac{1}{c^E_{t+1}} \left(\alpha \frac{y^w_{t+1}}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + \left[\frac{1}{c^E_t} - \beta^E \mathbb{E}_t \left(\frac{R^b_t}{c^E_{t+1}} \right) \right] \mathbb{E}_t \left[\frac{m^k_t (1-\delta) q_{t+1} \pi_{t+1}}{R^b_t} \right]$$
$$q_t = \beta^E \mathbb{E}_t \left[\frac{c^E_t}{c^E_{t+1}} \left(\alpha \frac{y^w_{t+1}}{x_{t+1} k_t} + (1-\delta) q_{t+1} \right) \right] + c^E_t \left[\frac{1}{c^E_t} - \beta^E \mathbb{E}_t \left(\frac{R^b_t}{c^E_{t+1}} \right) \right] \frac{b_t}{k_t} \tag{59}$$

C Capital Producers' Optimization Problem

Capital producers buy the non-depreciated capital from entrepreneurs and the final good from retailers to produce new capital sold to entrepreneurs. The capital producers' optimization problem can be written as:

$$\max_{\{i_t, k_t\}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[q_t k_t - q_t (1-\delta) k_{t-1} - i_t - \frac{\chi}{2} \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right)^2 i_t \right]$$
$$\text{s.t.} \quad k_t = (1-\delta) k_{t-1} + i_t \tag{60}$$

where $\Lambda_{t,t+s} \equiv \beta^s \frac{u'(c_{t+s})}{u'(c_t)}$ is the stochastic discount factor since the households are themselves the capital producers. The objective function (60) can be simplified to:

$$\max_{\{i_t, k_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ q_t [k_t - (1-\delta) k_{t-1}] - i_t - \frac{\chi}{2} \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right)^2 i_t \right\}$$
$$\max_{\{i_t, k_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ q_t i_t - i_t - \frac{\chi}{2} \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right)^2 i_t \right\}$$
$$\max_{\{i_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left\{ (q_t - 1) i_t - \frac{\chi}{2} \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right)^2 i_t \right\} \tag{61}$$

The first order condition in relation to i_t is:

$$(q_t - 1) - \frac{\chi}{2} \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right)^2 - i_t \left\{ \chi \left(\frac{i_t s^{qk}_t}{i_{t-1}} - 1 \right) \frac{s^{qk}_t}{i_{t-1}} \right\}$$
$$-\mathbb{E}_t \left\{ -\Lambda_{t,t+1} i_{t+1} \chi \left(\frac{i_{t+1} s^{qk}_{t+1}}{i_t} - 1 \right) \frac{i_{t+1} s^{qk}_{t+1}}{i^2_t} \right\} = 0$$

Thus, the price of capital q_t is:

$$1 = q_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 - i_t \left\{\chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\frac{s_t^{qk}}{i_{t-1}}\right\}$$
$$+\mathbb{E}_t \left\{\Lambda_{t,t+1} i_{t+1} \chi \left(\frac{i_{t+1} s_{t+1}^{qk}}{i_t} - 1\right)\frac{i_{t+1} s_{t+1}^{qk}}{i_t^2}\right\} \quad (62)$$

Rearranging the terms:

$$q_t = 1 + \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 + \chi\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)\left(\frac{i_t}{i_{t-1}}\right) s_t^{qk}$$
$$-\chi \mathbb{E}_t \left\{\Lambda_{t,t+1}\left(\frac{i_{t+1} s_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 s_{t+1}^{qk}\right\} \quad (63)$$

And, the profits of capital producers can be written as:

$$\Gamma_t^{CP} = (q_t - 1) i_t - \frac{\chi}{2}\left(\frac{i_t s_t^{qk}}{i_{t-1}} - 1\right)^2 i_t \quad (64)$$

D Retailers' Optimization Problem

We know that the problem of the representative retail firm producing the consumption final good y_t can be written as:

$$\Gamma^R = \mathbb{E}_t \sum_{t=0}^{\infty} \Lambda_{t,t+s}\left[\frac{p_t(i)^{1-\epsilon_t^y} y_t}{p_t} - \frac{p_t^w(i) p_t(i)^{-\epsilon_t^y} y_t}{p_t} - \frac{\kappa_\pi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)^2 p_t y_t\right] \quad (65)$$

The first-order condition in relation to $p_t(i)$ is:

$$(1-\epsilon_t^y)\frac{p_t(i)^{-\epsilon_t^y} y_t}{p_t} + \epsilon_t^y \frac{p_t^w(i) p_t(i)^{-\epsilon_t^y - 1} y_t}{p_t} - \kappa_\pi \left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\frac{p_t y_t}{p_{t-1}(i)}$$
$$-\beta \mathbb{E}_t \left[\Lambda_{t,t+1}\kappa_\pi \left(\frac{p_{t+1}(i)}{p_t(i)} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right) p_{t+1} y_{t+1}\left(-\frac{p_{t+1}(i)}{p_t(i)^2}\right)\right] = 0 \quad (66)$$

Dividing the expression above by y_t:

$$(1-\epsilon_t^y)\frac{p_t(i)^{-\epsilon_t^y}}{p_t} + \epsilon_t^y \frac{p_t^w(i) p_t(i)^{-\epsilon_t^y - 1}}{p_t} - \kappa_\pi \left(\frac{p_t(i)}{p_{t-1}(i)} - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\frac{p_t}{p_{t-1}(i)}$$
$$-\beta \mathbb{E}_t \left[\Lambda_{t,t+1}\kappa_\pi \left(\frac{p_{t+1}(i)}{p_t(i)} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right) p_{t+1}\frac{y_{t+1}}{y_t}\left(-\frac{p_{t+1}(i)}{p_t(i)^2}\right)\right] = 0 \quad (67)$$

In symmetrical equilibrium, or $p_t(i) = p_t$, first order conditions imply the Phillips curve nonlinear, given by:

$$\begin{aligned}(1-\epsilon_t^y) + \frac{\epsilon_t^y}{x_t} - \kappa_\pi \left(\pi_t - \pi_{t-1}^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_t \\ +\beta\mathbb{E}_t\left[\Lambda_{t,t+1}\kappa_\pi\left(\pi_{t+1} - \pi_t^{\iota_p}\overline{\pi}^{1-\iota_p}\right)\pi_{t+1}^2\frac{y_{t+1}}{y_t}\right] = 0 \end{aligned} \tag{68}$$

where $x_t = \frac{p_t(i)}{p_t^w(i)} = mc_t(i)$ is the markup of the final good price.

E Banks' Optimization Problem with Capital Accumulation

Banks maximize the dividends payable to shareholders (households). The bank j's optimization problem is:

$$\begin{aligned} \max_{\{b_t(j),k_t^B(j),div_t^B(j)\}} \quad & \mathbb{E}_t\sum_{s=0}^{\infty}\Lambda_{t,t+s}[\ln\big(div_{t+s}^B(j)\big)] \\ \text{s.t.} \quad & k_t^B(j) = (1-\delta^B)k_{t-1}^B(j) + \Gamma_t^B(j) - div_t^B(j) \\ & div_t^B(j) \geq 0 \\ & k_t^B(j) \geq 0 \\ & b_t(j) \geq 0 \end{aligned} \tag{69}$$

The lagrangian is:

$$L = \sum_{s=0}^{\infty}\Lambda_{t,t+1}\mathbb{E}_t\left\{\ln\big(div_t^B(j)\big) + \lambda_t^B(j)\left[k_t^B(j) + div_t^B(j) - (1-\delta^B)k_{t-1}^B(j) - \Gamma_t^B(j)\right]\right\}$$

Thus, the first-order conditions are:

$$\begin{aligned} [div_t^B(j)] : \frac{1}{div_t^B(j)} + \lambda_t^B(j) = 0 \\ \lambda_t^B(j) = -\frac{1}{div_t^B(j)} \end{aligned} \tag{70}$$

$$[b_t(j)] : \mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{\lambda_{t+1}^B(j)}{\pi_{t+1}}\left[\frac{\partial\Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)}b_t(j) + R_t^b - R_t^d\right)\right]\right\} = 0 \tag{71}$$

$$[k_t^B(j)] : \lambda_t^B(j) + \mathbb{E}_t\lambda_{t+1}^B(j)\left\{\frac{1}{\pi_{t+1}}\left[\frac{\partial\Omega_t^B(j)}{\partial k_t^B(j)} - R_t^d\right] - (1-\delta^B)\right\} = 0 \tag{72}$$

Then using $\frac{\partial R_t^b}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}\frac{\partial b_t}{\partial b_t(j)} = \frac{\partial R_t^b}{\partial b_t}$ in (71), we get the following expression:

$$\begin{aligned}
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t(j)} b_t(j) + R_t^b - R_t^d\right) &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t}\frac{b_t}{N} + R_t^b - R_t^d\right) &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(\frac{\partial R_t^b}{\partial b_t}\frac{b_t}{R_t^b}\frac{1}{N} + 1\right) R_t^b + R_t^d &= 0 \\
\frac{\partial \Omega_t^B(j)}{\partial b_t(j)} - \left(1 - PED_t^{-1}\frac{1}{N}\right) R_t^b + R_t^d &= 0
\end{aligned}$$

Thus, isolating R_t^b and using $b_t = \frac{b_t(j)}{N}$ and $k_t^B = \frac{k_t^B(j)}{N}$, we get the loan interest rate R_t^b:

$$\begin{aligned}
R_t^b &= \frac{R_t^d + \frac{\partial \Omega_t^B(j)}{\partial b_t(j)}}{\left(1 - PED_t^{-1}\frac{1}{N}\right)} \\
R_t^b &= \frac{R_t^d - \kappa_{k^B}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)\left(\frac{k_t^B(j)}{b_t(j)}\right)^2}{\left(1 - PED_t^{-1}\frac{1}{N}\right)} \\
R_t^b &= \frac{R_t^d - \kappa_{k^B}\left(\frac{k_t^B}{b_t} - \tau^B\right)\left(\frac{k_t^B}{b_t}\right)^2}{\left(1 - PED_t^{-1}\frac{1}{N}\right)} \qquad (73)
\end{aligned}$$

and, come backing to (72):

$$\begin{aligned}
\lambda_t^B(j) + \mathbb{E}_t\lambda_{t+1}^B(j)\left\{\frac{1}{\pi_{t+1}}\left[\frac{\partial \Omega_t^B(j)}{\partial k_t^B(j)} - R_t^d\right] - (1-\delta^B)\right\} &= 0 \\
\mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B(j)}\right\}\left\{(1-\delta^B) + \frac{1}{\pi_{t+1}}\left[R_t^d - \frac{\partial \Omega_t^B(j)}{\partial k_t^B(j)}\right]\right\} &= \frac{1}{div_t^B(j)}
\end{aligned}$$

Then:

$$\begin{aligned}
\mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B(j)}\right\}\left\{\frac{1}{\pi_{t+1}}\left[R_t^d - \kappa_{k^B}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)\left(\frac{k_t^B(j)}{b_t(j)}\right) - \frac{\kappa_{k^B}}{2}\left(\frac{k_t^B(j)}{b_t(j)} - \tau^B\right)^2\right]\right\} \\
= \frac{1}{div_t^B(j)} - \mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B(j)}\right\}(1-\delta^B)
\end{aligned}$$

Rearranging the above terms and using again $b_t = \frac{b_t(j)}{N}$ and $k_t^B = \frac{k_t^B(j)}{N}$ with $div_t^B = \frac{div_t^B(j)}{N}$, we get:

$$\mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B}\right\}\left\{(1-\delta^B)+\frac{1}{\pi_{t+1}}\left[R_t^d-\kappa_{k^B}\left(\frac{k_t^B}{b_t}-\tau^B\right)\left(\frac{3}{2}\left(\frac{k_t^B}{b_t}\right)-\frac{\tau^B}{2}\right)\right]\right\}=\frac{1}{div_t^B}$$

$$div_t^B=\left(\mathbb{E}_t\Lambda_{t,t+1}\left\{\frac{1}{div_{t+1}^B}\right\}\left\{(1-\delta^B)+\frac{1}{\pi_{t+1}}\left[R_t^d-\kappa_{k^B}\left(\frac{k_t^B}{b_t}-\tau^B\right)\left(\frac{3}{2}\left(\frac{k_t^B}{b_t}\right)-\frac{\tau^B}{2}\right)\right]\right\}\right)^{-1} \quad (74)$$

where $\Lambda_{t,t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}$.

F Calculating the Elasticity of Loan Demand to the Loan Rate (PED)

The PED can be calculated replacing (55) and (56) in (58):

$$\frac{1}{c_t^E}q_t=\beta^E\mathbb{E}_t\left[\frac{1}{c_{t+1}^E}\left(\alpha\frac{y_{t+1}^w}{x_{t+1}k_t}+(1-\delta)q_{t+1}\right)\right]$$
$$+\left(\frac{1}{c_t^E}-\beta^E\mathbb{E}_t\left[\frac{1}{c_{t+1}^E}\frac{R_t^b}{\pi_{t+1}}\right]\right)m_t^k\mathbb{E}_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]$$

Rearranging the terms:

$$\frac{1}{c_t^E}q_t=\beta^E\mathbb{E}_t\left[\frac{1}{c_{t+1}^E}\left(\alpha\frac{y_{t+1}^w}{x_{t+1}k_t}+(1-\delta)q_{t+1}\right)\right]+\frac{1}{c_t^E}m_t^k\mathbb{E}_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]$$
$$-\beta^E\mathbb{E}_t\left[\frac{1}{c_{t+1}^E}\frac{R_t^b}{\pi_{t+1}}\right]m_t^k\mathbb{E}_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]$$

Thus,

$$\frac{1}{c_t^E}\left(q_t-m_t^k\mathbb{E}_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right]\right)=$$
$$\beta^E\mathbb{E}_t\left[\frac{1}{c_{t+1}^E}\left(\alpha\frac{y_{t+1}^w}{x_{t+1}k_t}+(1-\delta)q_{t+1}-m_t^k\mathbb{E}_t[q_{t+1}(1-\delta)]\right)\right] \quad (75)$$

Using (7) in (75):

$$q_t-m_t^k\mathbb{E}_t\left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b}\right] = \beta^E\mathbb{E}_t\left[\frac{c_t^E}{c_{t+1}^E}\frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}}(k_t)^{\alpha-1}\right]$$
$$+ \beta^E\mathbb{E}_t\left[(1-\delta)q_{t+1}-m_t^k\mathbb{E}_t\left[q_{t+1}(1-\delta)\right]\right] \quad (76)$$

Define now:

$$A_{k,t} \equiv q_t - m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \quad (77)$$

$$B_{k,t} \equiv \beta^E \mathbb{E}_t \left[\frac{c_t^E}{c_{t+1}^E} \frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}} \right] > 0 \quad (78)$$

$$C_{k,t} \equiv \beta^E \mathbb{E}_t \left[(1-\delta) q_{t+1} - m_t^k \mathbb{E}_t \left[q_{t+1}(1-\delta) \right] \right] \quad (79)$$

Then, we can write (76) as:

$$A_{k,t} = B_{k,t}(k_t)^{\alpha-1} + C_{k,t} \quad (80)$$

Isolating capital in the above equation:

$$\frac{A_{k,t} - C_{k,t}}{B_{k,t}} = (k_t)^{\alpha-1}$$

$$\left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{1}{\alpha-1}} = k_t \quad (81)$$

We know that R_t^b is present in $A_{k,t}$, then deriving (81) in relation to R_t^b:

$$\frac{\partial k_t}{\partial R_t^b} = \frac{1}{\alpha-1} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{\frac{1}{\alpha-1}-1} \left(\frac{\partial A_{t,k}}{\partial R_t^b} \frac{1}{B_{k,t}} \right) \quad (82)$$

The derivative $\frac{\partial A_{t,k}}{\partial R_t^b}$ is equal to:

$$\frac{\partial A_{t,k}}{\partial R_t^b} = m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R_t^b)^2} \right] \equiv D_{k,t} > 0 \quad (83)$$

Thus, returning in (82) with $u_1 = \frac{1}{\alpha-1}$:

$$\begin{aligned} \frac{\partial k_t}{\partial R_t^b} &= u_1 \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{u_1-1} \left(\frac{D_{k,t}}{B_{k,t}} \right) \\ &= u_1 \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{u_1} \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{-1} \left(\frac{D_{k,t}}{B_{k,t}} \right) \\ &= u_1 \left(\frac{A_{k,t} - C_{k,t}}{B_{k,t}} \right)^{u_1} \left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}} \right) \\ &= u_1 \left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}} \right) k_t < 0 \end{aligned} \quad (84)$$

We will have the negative derivative since $A_{k,t} - C_{k,t} > 0$ (it has already been shown that $u_1 < 0$ and $D_{k,t} > 0$). This condition can be guaranteed by (80):

$$A_{k,t} - C_{k,t} = B_{k,t}(k_t)^{\alpha-1} > 0 \quad (85)$$

Then, taking the derivate of b_t in binding borrowing constraint with respect to R_t^b, we get:

$$\frac{\partial b_t}{\partial R_t^b} = m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_{b,t}} \right] \frac{\partial k_t}{\partial R_t^b} < 0 \tag{86}$$

The market loan demand b_t is downward-slopping in R_t^b. To find an expression for PED_t, elasticity of loan market b_t to the loan rate R_t^b, we do:

$$PED_t \equiv -\frac{\partial b_t}{\partial R_t^b} \frac{R_t^b}{b_t} \equiv -\frac{R_t^b}{b_t} \frac{\partial b_t}{\partial R_t^b} \tag{87}$$

We know that:

$$b_t = m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)k_t\pi_{t+1}}{R_t^b} \right] = \left(\frac{1}{R_t^b} \right) m_t^k \mathbb{E}_t [q_{t+1}(1-\delta)k_t\pi_{t+1}]$$

Thus,

$$\begin{aligned}
\frac{\partial b_t}{\partial R_t^b} &= -\left(\frac{1}{(R_t^b)^2} \right) m_t^k \mathbb{E}_t [q_{t+1}(1-\delta)k_t\pi_{t+1}] + \left(\frac{1}{R_t^b} \right) m_t^k \mathbb{E}_t [q_{t+1}(1-\delta)\pi_{t+1}] \left(\frac{\partial k_t}{\partial R_t^b} \right) \\
&= -\left(\frac{1}{R_t^b} \right) m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)k_t\pi_{t+1}}{R_t^b} \right] + m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \left(\frac{\partial k_t}{\partial R_t^b} \right) \\
&= -\frac{b_t}{R_{b,t}} + m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \left(\frac{\partial k_t}{\partial R_t^b} \right)
\end{aligned} \tag{88}$$

Replacing in (87):

$$\begin{aligned}
PED_t &= -\frac{R_t^b}{b_t} \left[-\frac{b_t}{R_t^b} + m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \left(\frac{\partial k_t}{\partial R_t^b} \right) \right] \\
&= 1 - \frac{R_t^b}{b_t} \left[m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \left(\frac{\partial k_t}{\partial R_t^b} \right) \right] \\
&= 1 - \frac{R_t^b}{b_t} \left[m_t^k \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R_t^b} \right] \frac{k_t}{k_t} \left(\frac{\partial k_t}{\partial R_t^b} \right) \right] \\
&= 1 - \frac{R_t^b}{b_t} \frac{b_t}{k_t} \left(\frac{\partial k_t}{\partial R_t^b} \right) \\
&= 1 + PEK_t > 0
\end{aligned} \tag{89}$$

where $PEK_t \equiv -\frac{\partial k_t}{\partial R_t^b} \frac{R_t^b}{k_t}$ denote the elasticity of entrepreneurs' capital demand to the loan rate. The $PED_t \equiv -\frac{R_t^b}{b_t} \frac{\partial b_t}{\partial R_t^b} > 0$ because $\frac{\partial b_t}{\partial R_t^b} < 0$ and the entrepreneurs' demand for capital decreases with increases in loan rate, $\frac{\partial k_t}{\partial R_t^b} < 0$. Before define:

$$MPK_t \equiv \frac{\alpha z_t (k_{t-1})^{\alpha-1} (l_t)^{1-\alpha}}{x_t} \tag{90}$$

as the marginal product of capital in terms of the final good. And:

$$\Lambda^E_{t,t+1} \equiv \beta^E \frac{u'(c^E_{t+1})}{u'(c^E_t)} = \beta^E \frac{c^E_t}{c^E_{t+1}} \tag{91}$$

as a stochastic discount factor for entrepreneurs. Thus,

$$PEK_t \equiv -\frac{\partial k_t}{\partial R^b_t}\frac{R^b_t}{k_t} = u_1 \left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}}\right) k_t \frac{R^b_t}{k_t} = u_1 \left(\frac{D_{k,t}}{A_{k,t} - C_{k,t}}\right) R^b_t \tag{92}$$

which $u_1 = \frac{1}{\alpha-1} < 0$. Thus, we can written PEK as:

$$PEK_t = -u_1 \left(\frac{D_{k,t}}{B_{k,t}(k_t)^{\alpha-1}}\right) R^b_t = -u_1 \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R^b_t)^2}\right]}{\beta^E \mathbb{E}_t \left[\frac{c^E_t}{c^E_{t+1}} \frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}}\right](k_t)^{\alpha-1}} \right) R^b_t$$

And,

$$PEK - u_1 \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{(R^b_t)^2}\right]}{\mathbb{E}_t \left[\Lambda^E_{t,t+1} \frac{\alpha z_{t+1}(l_{t+1})^{1-\alpha}}{x_{t+1}}\right](k_t)^{\alpha-1}} \right) R^b_t = -u_1 \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R^b_t}\right]}{\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1}\right]} \right)$$

Thus,

$$PEK = \frac{1}{1-\alpha} \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R^b_t}\right]}{\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1}\right]} \right) > 0 \tag{93}$$

It can be seen that PEK_t depend positively on the m^k_t and the expected discounted values of the future prices of capital, $\mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)}{R^b_t}\right]$. Furthermore, PEK_t depends negatively on the expected discounted values of the marginal product of capital, $\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1}\right]$, in terms of the final good. Replacing (93) in (89), we get that:

$$PED_t = 1 + \frac{1}{1-\alpha} \left(\frac{m^k_t \mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R^b_t}\right]}{\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1}\right]} \right) > 0 \tag{94}$$

It can be seen that higher values of $\mathbb{E}_t \left[\Lambda^E_{t,t+1} MPK_{t+1}\right]$ and also lower values of the $\mathbb{E}_t \left[\frac{q_{t+1}(1-\delta)\pi_{t+1}}{R^b_t}\right]$ reduce the elasticity of the loan demand PED_t. In addition, an decrease in m^k_t (after a negative collateral shock) directly reduces PED_t and indirectly by increasing in expected MPK_{t+1} (with the reduction of m^k_t, entrepreneurs obtain less loans and decrease their production, which increases MPK_{t+1}).

G Data and Sources

We use six quarterly macroeconomic variables of the Brazilian economy. Data comprises the period between 2000-Q3 to 2019-Q4. Below, we present the chosen variables with their respective sources:

1. Gross domestic product (GDP) - quarter versus an immediately previous quarter (%) seasonally adjusted. Source: SCNT from the IBGE;

2. Gross Fixed Capital Formation - quarter versus an immediately previous quarter (%) seasonally adjusted. Source: SCNT from the IBGE;

3. Consumer Price Index (IPCA) as a proxy of price inflation. Source: National System of Consumer Price Index (SNIPC) of the IBGE;

4. Interest rate policy (Selic) quarterly. Source: BCB;

5. Loans to entrepreneurs: Credit operations with non-earmarked funds - Consolidate balance (end of period) - Working capital - quarter versus an immediately previous quarter (%). Source: BCB;

6. Deposits: Extended payment methods - Deposit money banks - Time deposits, savings, and others - quarter versus an immediately previous quarter (%). Source: BCB.

H Prior and Posterior Distributions

Figure 13: Prior and posterior distributions