Spatial Sorting of Heterogeneous Workers and Firms

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1 Introduction

I introduce a conceptual framework exploring the dynamics of location decisions among heterogeneous workers and firms. Workers s actively search for employment opportunities, considering that the productivity of workers in various locations is influenced by the spatial distribution of workforce, commonly known as the agglomeration effect.

A fundamental aspect of my research revolves around the exploration of workers' search behavior. Diverging from conventional models, workers can "partially" direct their search to specific locations by employing a mixed strategy. This strategy emerges as the optimal solution for the maximization problem of economic agents who weigh the costs associated with acquiring and processing information against their cognitive constraints, this assumption is commonly referred to as *rational inattention* (e.g. Christopher A. Sims (2003) and Christopher A. Sims (2006)). The optimal strategy assigns a positive probability to all available locations, with the probability increasing in proportion to the worker's anticipated surplus in each locale. The outcomes generated by this approach are consistent with those obtained using a discrete choice framework, as demonstrated in Lentz and Moen (n.d.).

Kaplan (n.d.) argues that better information - due to both information technology and falling travel costs - has made locations less of an experience good, thereby reducing the need for young people to experiment with living in different places. In my work, I explore the implications of this hypothesis by including the cost of information processing in the search process.

2 Model

2.1 Model Setup

Consider a total of N locations, each indexed by j. These locations are assumed to be ex-ante homogeneous, but their productivity, represented by Ω_j , is endogenously determined as a function of their size. The workforce is represented by a continuum of individuals, each characterized by their productivity denoted as $x \in [0, 1]$, along with their employment status. The distribution of productivity, x, follows an exogenous distribution $\ell(x)$. Within each location, there is an endogenous distribution of productivity denoted as $\ell^j(x)$. Unemployed workers receive a utility of $b_j(x)^1$, and

¹I consider instant utility nemployed workers may differ based on location, considering the varying cost of living, which tends to be higher in more productive locations.

individuals actively search for employment both when unemployed and employed. Movement across locations is possible but incurs a relocation cost denoted as $F^{j,i} \ge 0$.

A continuum of firms exists in the model, each characterized by their technology level denoted as $y \in [0, 1]$, following an exogenous distribution. The production function is defined as $f(x, y, \Omega_j)$, incorporating the location-specific productivity factor. Firms engage in recruitment by posting vacancies $v^{j}(y)$, incurring an increasing convex cost denoted as $c_{j}(v^{j}(y))$. Additionally, firms retain workers by responding to outside offers. Jobs are destroyed with probability δ .

Consider:

- $u^{j}(x)$ distribution of unemployed workers in location j,
- $h^{j}(x, y)$ distribution of matches in location j, represented by.

The distribution of skill in each location is obtained by: $u^{j}(x) + \int_{0}^{1} h^{j}(x, y) dy = \ell^{j}(x)$

Additionally, the total distribution of workers across all locations aggregates to the exogenous distribution of skill $\sum_{j=1}^{N} \ell^{j}(x) = \ell(x)$. The size of a location, denoted as μ^{j} , is obtained by integrating the location-specific worker distribution $\mu^{j} = \int_{0}^{1} \ell^{j}(x) dx$.

A crucial observation is the direct mapping between the distribution of unemployed and employed workers and the productivity of each location. This linkage highlights the interdependence between labor market dynamics and the productivity levels within a specific geographical area.

2.2 Job Search

Both unemployed and employed workers engage in job search activities. Workers are uncertain about the potential job market conditions in each location, and they must decide how to allocate their search efforts across different locations. The search process is characterized by a mixed strategy

$$\phi_u^j(x,i)$$
 and $\phi_s^j(x,y,i)$

that specifies the probability of searching in location i for an unemployed worker with productivity x in location j and a worker employed at a firm with productivity y in location j respectively. This probability (which is chosen by the worker) determines the rate at which the worker will receive an offer in location i, once an offer is received the worker will match randomly with vacancy taken from the distribution of firms in that location $\Gamma^i(\cdot)$ with pdf $\gamma^i(\cdot)$.

Note that searching in each location with equal probability is what the search literature calls random search while directing all searches to a particular location is called directed search, thus this model of *partially directed search* nests both random and directed search.

I assume that employed workers are less efficient at searching than unemployed workers, and this is captured by a parameter $s \in (0, 1)$. Total search intensity in location j is given by the aggregate search intensity of all workers in the economy, and their search strategies are defined as:

$$L^{j} = \sum_{j'=1}^{N} \left[\int \phi_{u}^{j'}(x,j) u_{+}^{j'}(x) dx + s \int \int \phi_{s}^{j'}(x,y,j) h_{+}^{j'}(x,y) dx dy \right]$$
(1)

Where $u_{+}^{j'}(x)$ and $h_{+}^{j'}(x, y)$ are the distribution of unemployed and employed workers in location j' after destruction shock. Let $v^{j}(y)$ represent the number of job opportunities posted by a firm y in location j and $V^{j} = \int v^{j}(y) dy$ represents the total number of job opportunities posted in location j.

As it is standard in the search literature, I assume that the total number of matches is characterized by a matching function $M^j = M(L^j, V^j)$ that depends on the total search intensity and the total number of vacancies posted, and the meeting rates of workers and vacancies are given by:

$$p^{j} = \frac{M\left(L^{j}, V^{j}\right)}{L^{j}}$$
 and $q^{j} = \frac{M\left(L^{j}, V^{j}\right)}{V^{j}}$

To summarize, an unemployed worker in location j with productivity x will receive an offer from a firm in location j' with productivity $y' \leq y$ with probability:

$$\phi_u^j(x,j')p^{j'}\int_0^y \Gamma^{j'}(y)dy$$

3 Dynamic Programming Problem

Let $U^{j}(x)$ be the value of unemployment for a type-x individual in location j and $J^{j}(x, y)$ denote the value of an (x, y) match in location j, including the continuation values if the worker and firm separate.

The key object in the model is

$$S^{j,i}(x,y) = J^{i}(x,y) - \left(U^{j}(x) + F^{j,i}\right)$$
(2)

the joint surplus created by a match of a type y firm at location i with a worker of type x in location j. I'm going to assume that matches only form if they generate nonnegative surplus; this is equivalent to immediate endogenous destructions if $S^{j,j}(x,y) < 0$ combined with an exogenous probability δ , I denote $\lambda^j(x,y) = (1-\delta) \mathbb{W}_{\{J^j(x,y) \geq U^{j,j}(x,y)\}}$ the probability that a match survives. If a match is destroyed, the worker gets a value of $U^j(x)$, and the firm gets 0.

The timing of the model is as follows:

- 1. Employed and unemployed workers in location j choose a location i to search according to their *optimal strategy*.
- 2. They receive an offer with probability p^i . If they don't receive an offer, they remain in their location at their current employment status.
- 3. If they receive an offer, they draw a firm from the local distribution of vacancies $\Gamma^{i}(y)$.
- 4. Workers and firms then bargain over the surplus that the new match will generate, to determine the value of the offer.
- 5. If the offer is *good enough*, the worker accepts, pays the cost of reallocation, and moves; if not, she stays at her location at her current employment status.

Upon Meeting the worker and the firm bargain over the surplus of the match. Contracts are determined as in Cahuc, Postel-Vinay, and Robin (2006) with the unemployed worker's outside option the value of unemployment in their current location $U^j(x)$ and the employed worker's outside option is full surplus extraction of current employer: $J^j(x, y)$. The results of this bargaining imply that a type x worker unemployed in location j that has a job opportunity with firm y' in location i is offered:

$$W^{j,i}(x,0,y) = U^j(x) + \xi S^{j,i}(x,y)$$
(3)

were ξ is the bargaining power of the worker.

A type x worker employed with firm y in location j that has a job opportunity with firm y' in location i is offered:

$$W^{j,i}(x,y',y) = J^j(x,y) + \xi(S^{j,i}(x,y') - S^{j,j}(x,y))$$
(4)

3.1 Unemployed Worker Value Function

Unemployed workers receive instant utility from living in location j, b(x, j), and anticipate the probability of getting an offer p^j in each location. They will choose the strategy that maximizes their future expected value knowing that in each location they will receive an offer which can be from any firm with a likelihood proportional to the share of total vacancies posted by each firm in each market. The worker will accept only the offers that promise her a higher value than unemployment²:

$$U^{j}(x) = b_{j}(x) + \beta U^{j}(x) + \beta \max_{\{\phi_{u}^{j}(x)\}} \left\{ \sum_{i=1}^{N} \phi_{u}^{j,i}(x) \left(p^{i} \xi \int S^{j,i}(x,y)^{+} d\Gamma^{i}(y) \right) - c(\phi_{u}^{j}(x)) \right\}$$
(5)

Next, I derive the search strategy adopted by workers, defining the concept and the criteria guiding their choices. The discussion includes the associated cost of the search. The optimal search strategy for each worker type and location is derived. Importantly, throughout this section, the type and employment status of the worker are abstracted, allowing me to focus on the exploration of the core elements of the search process.

3.1.1 Optimal Search Strategy

I assume that workers are rational inattentive in the style of Christopher A. Sims (2003), Christopher A. Sims (2006) and Matějka and McKay (2015). Workers at location j have prior knowledge of the value of moving to every location i; this value is dependent on the worker type, the meeting rates, and the vacancy distribution in location i. For simplicity, I will just denote $w^{j,i}$ the value of moving from j to i, and assume that prior knowledge is described by a joint distribution $G\left(\{w^{j,i}\}_{j=1}^N\right)$.

To refine this knowledge, workers can acquire information about the value of each location by searching. Workers are in essence acquiring information to reduce uncertainty (i.e. reduce entropy) associated with their prior knowledge. I make the simplifying assumption that ϕ_0^j , the unconditional

²Here $x^+ = \max\{0, x\}$

distribution of choosing each location is uniform. Workers in location j are faced with the following problem:

$$\max_{\phi^{j}} \left(\sum_{i=1}^{N} \phi^{j,i} w^{j,i} - c(\phi^{j} \mid \phi_{0}^{j}) \right) \quad \text{s.t.} \quad \sum_{i=1}^{N} \phi^{j,i} = 1 \quad \text{and} \quad \phi^{j,i} \ge 0 \tag{6}$$

With $c(\phi^j \mid \phi_0^j)$ being the cost of reducing the entropy of the prior. In this particular case of a noninformative prior, the cost is proportional to the Kullback–Leibler divergence (also called relative entropy and *I*-divergence) between the selected distribution and the prior (uniform):

$$c(\phi^{j}, \phi_{0}^{j}(x)) = c(\phi_{i}^{j}) = c \sum_{i=1}^{N} \phi^{j,i} \log\left(N\phi^{j,i}\right)$$
(7)

If processing information were costless i.e. c = 0 the worker would be able to perfectly direct their search towards the highest value location. With costly information processing (c > 0), the worker can still select to search randomly (i.e. uniform distribution) and pay the associated cost of 0, when the worker starts directing their search towards a particular location (the assumption is that this requires, acquiring more information of one of the potential values relative to the others) then the cost grows unboundedly large as the strategy gets closer to perfectly directing the search to a particular location.

• Besides (Matějka and McKay 2015) which formulates the problem in general terms, some works in the literature that use this cost structure are (Wu 2020) and (Cheremukhin, Restrepo-Echavarria, and Tutino 2020).

Solving the maximization problem we get the optimal strategy for each worker as:

$$\phi^{j}(i) = \frac{\exp\left(\left[w^{j,i}\right]/c\right)}{\sum\limits_{\tilde{j}=1}^{N} \exp\left(\left[w^{j,\tilde{j}}\right]/c\right)}$$
(8)

We already established that with costless information the limit behavior is that the worker perfectly directs their search to the highest value location. Next, note that when the cost of information is very high the limit behavior is that the worker searches randomly i.e. $\phi_i^j(x, j') = \frac{1}{N}$. Here the scaling parameter c plays a similar role to the noise parameter in the discrete choice model from (Lentz and Moen, n.d.).

Plugging the expected value of moving to each location into Equation (8) we get the following expression for the optimal strategy of unemployed workers:

$$\phi_u^{j,i}(x) = \frac{\exp\left(\left[p^i\xi\int S^{j,i}(x,y)^+d\Gamma^i(y)\right]/c\right)}{\sum\limits_{\tilde{i}=1}^N \exp\left(\left[p^{\tilde{i}}\xi\int S^{j,\tilde{i}}(x,y)^+d\Gamma^{\tilde{i}}(y)\right]/c\right)}$$
(9)

which we can into the value function in Equation (5) we get the Bellman equation for the value of unemployment:

$$(1-\beta)U^{j}(x) = b_{j}(x) + \beta c \underbrace{\log\left(\frac{1}{N}\sum_{j=1}^{N}\exp\left(p^{i}\frac{\xi}{c}\int S^{j,i}(x,y)^{+}d\Gamma^{i}(y)\right)\right)}_{\Lambda_{0}^{j}(x)}$$

3.2 Joint Value of a Match

To write the Bellman equation for the joint value of a match, we start by writing the expected value of searching in location i for a worker of type x employed at a firm of type y in location j. We know that the worker will receive an offer with probability p^i and that the firm will be drawn from the distribution of firms in each location, $\Gamma^i(y)$, and an offer from a firm y' is worth $W^{j,i}(x, y', y)$. The expected value of searching in location i is then:

$$(1-p^{i})J^{j}(x,y) + p^{i}\int \max\left\{J^{j}(x,y), W^{j,i}(x,y',y)\right\}d\Gamma^{i}(y')$$
(10)

pluggin the bargainign solution in Equation (4) we obtain:

$$(1 - p^{i})J^{j}(x, y) + p^{i} \int \max\left\{J^{j}(x, y), J^{j}(x, y) + \xi \left[S^{j, i}(x, y') - S^{j, j}(x, y)\right]\right\} d\Gamma^{i}(y')$$

$$\implies J^{j}(x, y) + p^{i} \int \xi \left[S^{j, i}(x, y') - S^{j, j}(x, y)\right]^{+} d\Gamma^{i}(y')$$
(11)

The employed worker will search in each of the N locations with some probability $\phi_e^{j,i}(x, y,)$ that they will choose optimally only if their job survies and will get their unemployment value if the job is destroyed. The Bellman equation for the value of a match is then:

$$J^{j}(x,y) = f(x,y,j) + \beta \left[(1 - \lambda^{j}(x,y))U^{j}(x) + \lambda^{j}(x,y)J^{j}(x,y) \right] \\ + \beta \lambda^{j}(x,y) \max_{\phi_{e}^{j}(x,y)} \left\{ \sum_{i=1}^{N} \phi^{j,i}(x,y)p^{i} \int \xi \left[S^{j,i}(x,y') - S^{j,j}(x,y) \right]^{+} d\Gamma^{i}(y') - c \left(\phi_{e}^{j}(x,y) \right) \right\}$$
(12)

From (8) we know that the optimal strategy for each employed worker is:

$$\phi_e^j(x,y,i) = \frac{\exp\left(\left[sp^i\xi\int\left[S^{j,i}(x,y') - S^{j,j}(x,y)\right]^+ d\Gamma^i(y')\right]/c\right)}{\sum\limits_{\tilde{i}=1}^N \exp\left(\left[sp^{\tilde{i}}\xi\int\left[S^{j,\tilde{i}}(x,y') - S^{j,j}(x,y)\right]^+ d\Gamma^{\tilde{i}}(y')\right]/c\right)}$$

Substituting the optimal strategy in the Bellman equation we get:

$$J^{j}(x,y) = f(x,y,j) + \beta \left[(1 - \lambda^{j}(x,y))U^{j}(x) + \lambda^{j}(x,y)J^{j}(x,y) \right] + \beta c \lambda^{j}(x,y) \underbrace{\log \left(\frac{1}{N} \sum_{i=1}^{N} \exp\left(\left[sp^{i} \frac{\xi}{c} \int \left[S^{j,i}(x,y') - S^{j,j}(x,y) \right]^{+} d\Gamma^{i}(y') \right] \right) \right)}_{\Lambda^{j}_{1}(x,y)}$$
(13)

3.3 Match Surplus

Next, I characterize the joint surplus of a match between a type x worker in location j and a type y firm in location i and show that the surplus encodes all the necessary information to characterize the dynamics of the model. First, define the instant surplus of a match between a type x worker in location j and a type y firm in location i as

$$s^{j,i}(x,y) = f(x,y,i) - b_j(x)$$

We can compute the surplus of matches in the same location as:

$$S^{j,j}(x,y) = s^{j,j}(x,y) + \beta \lambda^{j}(x,y) \underbrace{\left[J^{j}(x,y) - U^{j}(x)\right]}_{S^{j,j}(x,y)} + \beta c \left[\lambda^{j}(x,y)\Lambda_{1}^{j}(x,y) - \Lambda_{0}^{j}(x)\right]$$
(14)

Notice that $\lambda^j(x,y) > 0$ if and only if $S^{j \to j}(x,y) > 0$, thus $(1 - \delta) \max\{0, S^{j \to j}(x,y)\} = \lambda^j(x,y)S^{j \to j}(x,y)$. Finally, we can write the surplus of a match between a type x worker in location j and a type y firm in location j' as:

$$S^{j,i}(x,y) = J^{i}(x,y) - \left[U^{j}(x) + F^{j,i}\right]$$

= $\left[J^{i}(x,y) - U^{i}(x)\right] + U^{i}(x) - \left[U^{j}(x) + F^{j,i}\right]$
= $S^{i,i}(x,y) - \left[U^{j}(x) - U^{i}(x) + F^{j,i}\right]$ (15)

We can define the surplus equation implicitly as:

$$S^{j,i}(x,y) = \begin{cases} f(x,y,i) - b_j(x) + \beta \left[\lambda^j(x,y) S^{j,j}(x,y) + c \left(\lambda^j(x,y) \Lambda_1^j(x,y) - \Lambda_0^j(x) \right) \right] & \text{if } j = i \\ S^{i,i}(x,y) - [U^j(x) - U^i(x) + F^{j,i}] & \text{if } j \neq i \end{cases}$$
(16)

Notice that this way of writing the problem reduces the dimensionality from N^2 to N. From Equation (16) the following is evident:

• A worker in location j can be hired from a firm in location i by firm y if and only if the surplus of the match is positive:

$$S^{j,i}(x,y) \ge 0 \qquad \iff \quad J^i(x,y) - U^{j,i}(x) \ge 0$$

• A worker employed at location j by firm y can be poached by a firm y' in location i if and only if the surplus of the match is higher than the surplus of staying at the same firm in the same location:

$$S^{j,i}(x,y') > S^{j,i}(x,y) \quad \iff \quad J^i(x,y') > J^j(x,y) + F^{j,i}(x,y') = J^j(x,y) + F^{j,i}(x,y) + F^{j,i}(x,y) = J^j(x,y) + F^{j,i}(x,y) + F^{j,i}(x,y) = J^j(x,y) = J^j(x,y) + F^{j,i}(x,y) = J^j(x,y) + F^{j,i}(x,y) = J^j(x,y) = J^j(x$$

3.4 Vacancy Creation

Let $B^{j}(y)$ be the expected value of a type y vacancy making contact with a worker in location j. Vacancies meet unemployed and employed type-x workers at a rates

$$\frac{u^j_+(x)}{L^j}$$
 and $s\frac{h^j_+(x,y)}{L^j}$

The expected value of posting a vacancy is therefore, the surplus that the posting firm expects to add, potential matches with negative surplus are immediately destroyed therefore those add no surplus. In terms of the Bellman equation we can write:

$$B^{j}(y) = (1 - \xi) \sum_{i=1}^{N} \left(\int \phi_{u}^{i,j}(x) \frac{u_{+}^{i}(x)}{L^{j}} S^{i,j}(x,y)^{+} dx + \int \left(\int s \phi_{e}^{i,j}(x,y') \frac{h_{+}^{i}(x,y)}{L^{j}} \left[S^{j,i}(x,y) - S^{i,i}(x,y') \right]^{+} dx \right) dy' \right)$$

Firms will post vacancies until marginal cost equals expected benefit thus $c'_j(v^j(y)) = q^j B^j(y)$, a cost function that is increasing and concave guarantees non-degenerate distributions of vacancies. The vacancy creation

4 Labor Market Flows

Next, I delve into characterizing the distribution of unemployed workers and matches in alignment with individual decision rules, alongside distributions of vacancies posted in each location. To understand how agents move across locations, we introduce the probabilities $\hat{\phi}_{u}^{j,i}(x,y')$ and $\hat{\phi}_{s}^{j,i}(x,y,y')$, representing the likelihood that a searching worker in location j receives a suitable offer from firm y' at location i:

$$\hat{\phi}_{u}^{j,i}(x,y') = p^{i} \times \phi_{u}^{j}(x,i) \times \mathbb{H}_{\{S^{i,j}(x,y') \ge 0\}} \times \gamma^{i}(y')$$

$$\hat{\phi}_{s}^{j,i}(x,y,y') = sp^{i}\phi_{s}^{j}(x,y,i) \times \mathbb{H}_{\{S^{i,j}(x,y') > S^{j,j}(x,y') \ge 0\}} \times \gamma^{i}(y')$$

These probabilities capture the dynamics of worker movements and offer receptions, taking into account the conditions set by decision rules, location-specific factors, and the overall distribution of vacancies in each location.

The distribution of unemployed workers, denoted as $u^{j}(x)$, is determined by the probability that currently unemployed workers don't any suitable offers from firms in any location:

$$u^{j}(x) = u^{j}_{+}(x) \left(1 - \prod_{i=1}^{N} \int \hat{\phi}^{j,i}_{u}(x,y) dy'\right)$$

The law of motion for the distribution of matches, $h^{j}(x, y)$, is decomposed into three components:

$$h^{j}(x,y) = h_{U}^{j}(x,y) + h_{P}^{j}(x,y) + h_{R}^{j}(x,y)$$

1. The mass of workers hired from unemployment:

$$h_U^j(x,y) = \gamma^j(y) \sum_{i=1}^N u^i_+(x) \hat{\phi}_u^{i,j}(x,y)$$

2. The mass of workers successfully poached from other firms:

$$h_{P}^{j}(x,y) = \sum_{i=1}^{N} \left[\int h_{+}^{i}(x,y') \hat{\phi}_{s}^{i,j}(x,y',y) dy' \right]$$

3. The mass of workers that the firm can retain:

$$h_{R}^{j}(x,y) = h_{+}^{j}(x,y) \left(1 - \prod_{i=1}^{N} \left[\int \hat{\phi}_{s}^{j,i}(x,y,y') dy' \right] \right)$$

5 Computation of the Equilibrium

Note for any distribution of unemployed workers and matches across locations, we can aggregate and obtain the productivity of each location. This in turn implies a surplus function for each location and an endogenous distribution of vacancies and search effort across locations which in turn implies a new distribution of unemployed workers and matches across locations. The equilibrium is characterized by the fixed point of this process.

6 Preliminary Results

The matching function, denoted as M(L, V), is defined as $\omega_1 L^{\omega_2} V^{1-\omega_2}$, with the constraint that $\omega_1 > 0$. Vacancy costs are expressed by the function $c[v_t(y)] = \frac{c_0 v_t(y)^{1+c_1}}{1+c_1}$, where $c_0 > 0$ shand $c_1 > 0$. The production function f(x, y, j) involves parameters and terms influencing economic output, including agglomeration effects and cost of living factors represented by $\Omega(\mu_j)$ and $\chi(\mu_j)$, respectively. Here, $\Omega(\mu_j) = 1 + a_1 \mu_j^{b_1}$, and $\chi(\mu_j) = a_2 \mu_j^{b_2}$. Home production, denoted as b(x), is determined by \hat{b} multiplied by the production function $f(x, y^*(x), j)$, where $y^*(x) = \arg \max_y f(x, y, j)$. The distribution of worker types follows a Beta distribution, while firm types are uniformly distributed.

I compute the equilibrium of the model using two locations and find that for a set of parameters the model generated the following outcomes:

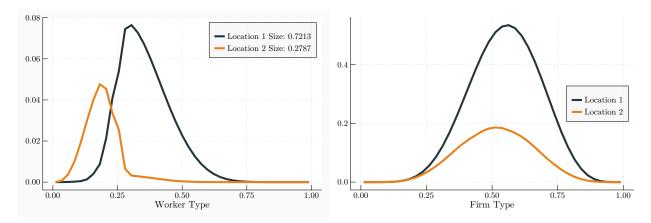


Figure 1: Distribution of Workers

Figure 2: Equilibirum Distribution of Vacncies

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