Do Robust Predictors Improve the Accuracy of Inflation Forecasts in Moments of Structural Break?

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Abstract

The accurate prediction of inflation rates holds critical significance for both policymakers and economic agents. It is imperative to comprehend the limitations and strengths inherent in different models and information sets used to forecast inflation across varying time horizons. This study seeks to enhance the existing literature on Brazilian inflation forecasting by assessing the predictive efficacy of predictors robust to structural breaks, with a particular emphasis on the methodology introduced by Martinez et al. (2022). The findings of this study indicate that robust predictors exhibit notably superior performance during periods of instability and structural change. In the Brazilian context, these predictors outperform expert forecasts specifically during the COVID-19 pandemic period, as indicated by the Focus Survey. However, it is noteworthy that in the immediately preceding period, these models do not outperform the aforementioned survey.

Keywords: Inflation Expectations; Robust Estimators.
JEL CODES: C53, C22, E27

1 Introduction

The future behaviour of macroeconomic variables is an important input for the conduct of monetary policy. In an inflation target regime, the Central Bank has to react to the behaviour of inflation expectations. This is highlighted for the Brazilian case by Minella et al. (2002). Expectations also determine price adjustments, influence long-term investments and reflect the fiscal risks associated with an economy. However, in periods of extreme shocks and high uncertainty, the accuracy of these projections is significantly affected. Favero and Giavazzi (2004) shows, using past examples from emerging countries, how not anchoring expectations can lead to fiscal dominance risk and the difficult management of fiscal and monetary policy.

When we analyze the period of the Covid-19 pandemic, we observe that economic agents were exposed to strong uncertainty in assessing the economic situation, with inflation projections showing high dispersion and volatility. In May 2020, the median of the Focus survey projections [1] pointed to anchored inflation below 3.5% for the two following years, but with a high dispersion ranging from 2.4% to 5.0%. In May 2022, due to various shocks in the Brazilian economy, the expected inflation curve for the same period underwent a significant upward revision, reaching 8.2% and 4.2%, respectively. This erratic behaviour of inflation projections greatly affected the Central Bank's response. As a main consequence, the policy rate rose from 3.00% to 13.75% during this monetary cycle. In this environment of uncertainties and negative shocks, several recent works have sought to present the impacts of the Covid-19 pandemic on the basic tools for projecting economic variables; Bobeica and Hartwig (2023), Carriero et al. (2022), Lenza and Primiceri (2020), and Schorfheide and Song (2021). However, a large literature has already previously discussed alternative forecasting methods in moments of structural breaks. Pesaran and Timmermann (2007) discuss methods for selecting different estimation windows in the case of sequential breaks. Pesaran et al. (2013) and Giraitis et al. (2013) discuss tools for assigning different weights to estimates and obtaining an optimal projection in times of higher series volatility.

In the Brazilian case, to analyze this erratic behaviour of projections during periods of structural breaks, we use inflation expectations extracted from the Central Bank's Focus survey and reproduce the work of Martinez et al. (2022), who applied the robust predictor method to assist in projections of

[1]FOCUS is a survey among experts about their expectation on key variables of the Brazilian economy collected by Central Bank with financial institutions

economic variables in periods of structural breaks. The authors argue that alternative estimators are efficient in capturing sudden changes due to the high variance (and reduced bias) produced by projections. The relevance of the work, however, lies in using robust methods to estimate the long-term average of the desired variable, rather than the economic variable in question, since this long-term average is always susceptible to more relevant structural breaks over time. The authors find significant gains in forecasting the 10-year interest rate in the United States and the productivity rate of the United Kingdom.

The objective of this work, therefore, is to adjust Brazilian inflation expectations using the robust estimators proposed by the authors as a basis. We reproduce the alternative models presented by the author and compare them to the performance of the Focus survey between 2009 and 2022. It is observed that the projections produced by the alternative models indicate higher variance compared to the Focus survey. However, in periods of rapid inflation increases, the alternative models are more efficient and capable of producing significant gains compared to the estimates collected by the Central Bank's survey. This effectiveness is also tested through the analysis of the Root Mean Squared Errors (RMSE) and the test proposed by Diebold and Mariano (2002).

2 Literature Review

Deviation of inflation from the target leads to misguided and less efficient monetary policy decisions. For instance, excessive monetary tightening can lead to further economic downturns and potentially result in a scenario of extreme recession. However, if economic agents' expectations underestimate current inflation, monetary tightening will be less than required, leading to prolonged inflation persistence. Under this circumstance, obtaining good estimates is known to be challenging, as inflation is susceptible to tail shocks and significant structural breaks.

The outbreak of the COVID-19 pandemic proved to be a shock of intense magnitude to the global economic equilibrium, consequently leading to various errors in inflation projections and, as mentioned earlier, potentially guiding mistakes in monetary policy conduct during this period.

The literature on inflation forecasting methods in Brazil is extensive. Traditional models estimate the inflation rate through the Phillips Curve, which inversely relates unemployment to the inflation rate. There are various ways to estimate this relationship. Schwartzman (2006), for example, seeks to estimate a Phillips Curve with the disaggregation of the National Consumer Price Index (IPCA) between tradable, non-tradable, and monitored prices. The relevance and contribution of this work lie precisely in addressing this relationship with price disaggregation and replacing the output gap with capacity utilization calculated by the Brazilian Institute of Economics at the Getulio Vargas Foundation (FGV-IBRE). These various Phillips Curve estimations have proven to be the theoretical basis and one of the most traditional models for estimating the inflation rate. Other approaches, such as Mendonça et al. (2012), attempted to estimate a New Keynesian Phillips Curve for the Brazilian economy, finding robustness in the model and highlighting the importance of inflation expectations and past results for price dynamics in the economy.

To compare Phillips's curve-based methods to time series models, Arruda et al. (2011) compared these models with linear and non-linear models for Brazilian inflation. The authors concluded that the model, considering an expanded Phillips Curve that captures the exchange rate pass-through effect on the economy, had the lowest mean squared error among the other candidates, reinforcing the evidence that Phillips Curve-based and non-linear models can contribute to better predictive performance compared to linear models.

Other authors have focused on the relationship between the inflation rate and other macroeconomic variables. For example, Figueiredo et al. (2010) explored a large set of information and attempted to create models for forecasting future inflation. The work departs from more traditional models and presents the influence of other economic variables on the estimation. The author uses a large database with 368 monthly series from various sectors and characteristics of the economy, covering the period from 1995 to 2009. The forecasting method is based on factor models, which allow the reduction of the data dimension into various factors that hold valuable information for the estimations. Among the main conclusions, the author found that using a larger set of information does indeed improve forward estimations. Additionally, regarding the best-projected horizon in the work, the author highlights the performance of principal component models for projecting inflation six periods ahead.

Carlo and Marçal (2016) applied a set of inflation forecasting using aggregate and disaggregate from IPCA data. Furthermore, the authors used a different approach from the usual in comparing the efficacy of the models, through the Model Confidence Set (MCS), which can recognize the limitations of the database and assess the different model performances given the information in the chosen sample. The results suggest significant gains in treating data in disaggregated form for IPCA compared to considering aggregate data in Brazilian inflation. In addition, the authors also conclude that combining projections from aggregated models contributes to better short-term inflation projection performance.

In subsequent work, Garcia et al. (2017) used Machine Learning techniques and high-dimensional models to present pseudo-real-time IPCA projections estimated based on the available information up to that moment. The authors estimated daily projections for the five days before the official IPCA release and demonstrated a good approximation to the projections present in the Focus survey, indicating good alignment with financial institutions' estimates. Additionally, they used various high-dimensional models with different comparison criteria to obtain the best estimates. The study concluded that the use of more current techniques produces gains in both short-term and long-term forecasting horizons.

However, it is known that time series data are susceptible to unexpected shocks over time. The longer the forecasting horizon, the higher the level of uncertainty regarding the effects of these shocks. Several works address model selection methods that adapt to these structural breaks. Castle et al. (2015) used robust estimator methods in contrast to traditional models, which avoid the problem of sequential projection errors in the face of a more relevant structural break. The authors applied the alternative method to estimates of U.S. GDP growth and found gains in accuracy in times of significant shocks in the U.S. economy.

The contribution of this work is to apply the robust predictors with smoothing methods developed by Martinez et al. (2022) in an attempt to improve Brazilian inflation projections, especially aftershocks or structural breaks in the economy. This alternative method is used to improve labour productivity estimates in the UK, demonstrating that this variable has experienced many projection errors over time. According to the authors, the more traditional models in this forecasting literature are based on equilibrium correction theory, in which long-term parameters undergo few changes over time. However, in the presence of structural breaks, these parameters can vary and result in imprecise projections concerning economic variables. Therefore, the main objective of the authors is to demonstrate that the robust predictors with the smoothing method can help address persistent projection errors while maintaining the essential properties of other models.

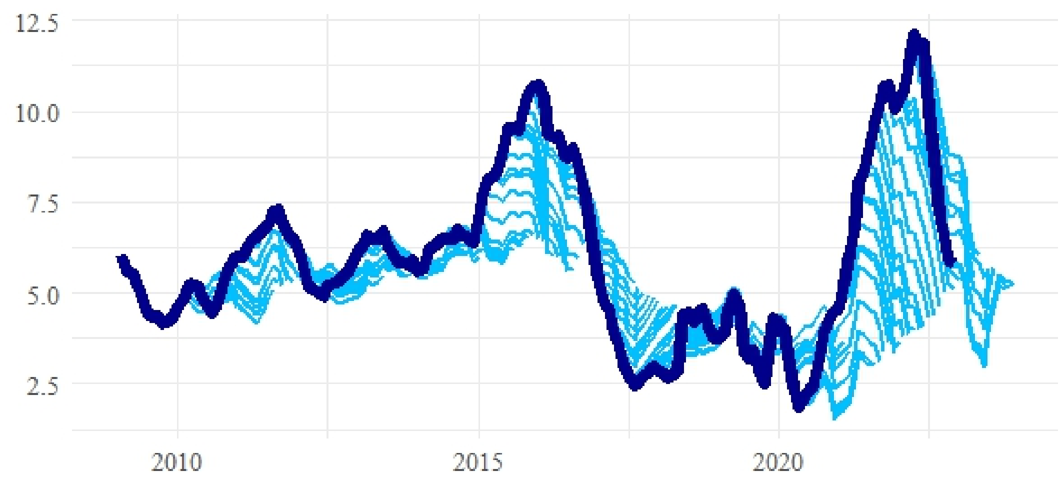
3 Data and Methodology

3.1 Inflation: Target series

This study focuses on monthly data of the Brazilian Consumer Price Index of Inflation, (IPCA), calculated by the Brazilian Institute of Geography and Statistics (IBGE). Additionally, we extracted expectations calculated by financial institutions in the Central Bank's Survey (FOCUS). In this case, the goal is to observe the expected inflation trajectory as forecasted by analysts from financial institutions over time and assess its accuracy. We opt to use the median forecasts.

In Figure 1, we can observe periods of persistent errors in comparison to the actual inflation results, especially during moments when structural breaks in the economy are observed. In mid-2015, for example, inflation experienced a strong shock due to economic policies adopted by the government at the time. In that year, electricity prices increased by 51.0% and gasoline had a 20.1% rise during the period, contributing significantly to the overall inflation increase. In 2017, the movement was in the opposite direction, showing rapid disinflation and ending the year at 3.0%, with deflation in food being one of the main downward highlights (-1.9%). Finally, during the 2020-21 period, the inflationary shock from the COVID-19 pandemic once again brought strong upward pressure on domestic prices. Food inflation at home reached an annual peak of 21.3%, marking the highest level since 2003; administered prices inflation reached 19.2% in November 2021, with significant pressure from gasoline and electricity. Lastly, the observed pressure on industrial prices and services illustrated the spread of this inflationary shock and its challenges for the monetary policy response during that period.

All these shocks observed over the past few years exhibited characteristics of unpredictability with significant effects on inflation projections. In the case of 2020-21, it can be observed that economic agents' expectations took months to adjust to this new equilibrium in the face of the uncertainties of the period, leading to a sequence of systematic errors in future projections of the variable.



Source: BCB and IBGE

Figure 1: Comparison IPCA (%YoY) vs. Focus

3.2 Basic Model

Autoregressive model is a method that uses past information to assist in forecasting future values of the desired variables. However, the authors propose an alternative way to present an autoregressive model of order 1. In the equation below, the AR(1) process considers a long-term mean variable (μ) and dynamics (ρ). It is assumed that $|\rho| < 1$.

$$x_t = \mu + \rho(x_{t-1} - \mu) + \varepsilon_t \tag{1}$$

Additionally, we can write that the iterated projection of multiple horizons ahead for this alternative AR(1) can be expressed as:

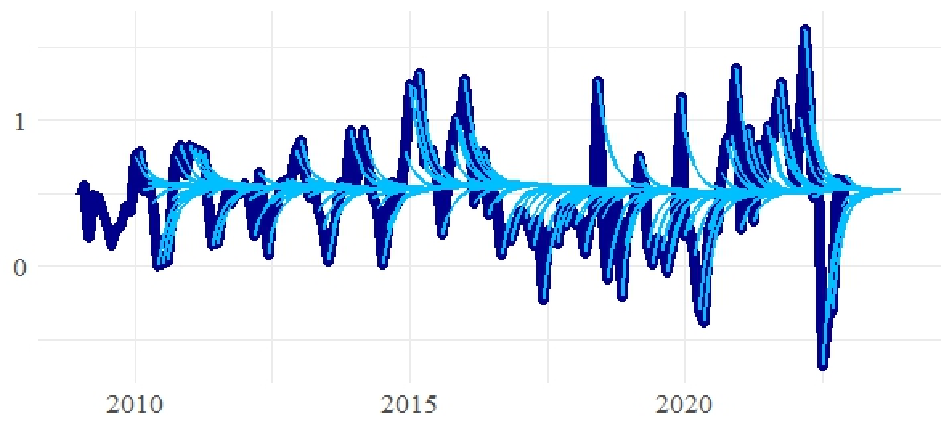
$$x^{AR}_{t+h|t} = \mu + \rho^h(x_t - \mu) \tag{2}$$

In the formula, h represents the desired projection horizon.

In the case of a structural break, however, autoregressive models have the characteristic of exhibiting a strong forecast bias at all horizons ahead. Therefore, they are not a good predictor for situations involving a structural break. Hence, another possibility within univariate regressions is prediction models based on a random walk. Due to being a random stochastic process, they have greater flexibility to adapt to new levels of economic parameters. In this case, it proves to be a more viable option compared to autoregressive models because it essentially relies on the most recent available information in the time series. The last observation contains important information on the structural change that has just happened. The random walk operates better the larger and more persistent a structural break is, as it is a model that quickly adjusts to the new equilibrium. It can be used as a benchmark given its simplicity. However, in the case of multiple sequential breaks, the random walk would exhibit higher variance in projections.

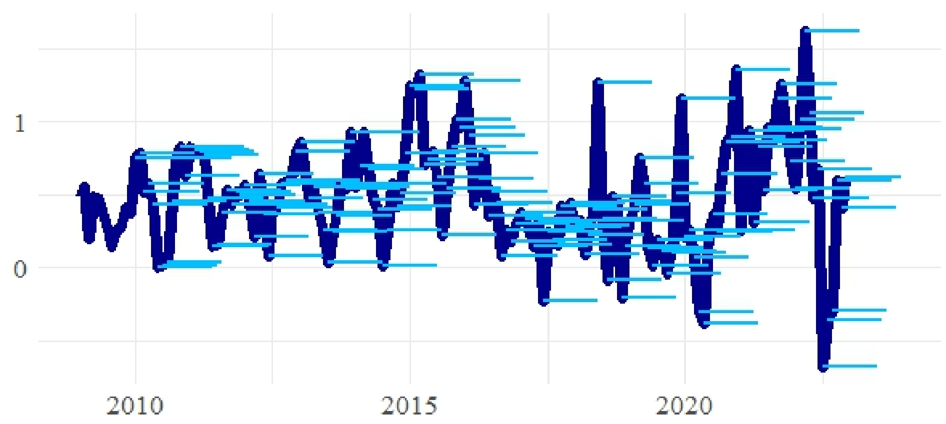
$$x^{rw}_{t+h|t} \;=\; x_t \tag{3}$$

In Figures 2 and 3, we can observe the differences between both forecast devices:



Dark blue - Actual Data
Light Blue - Forecasts
Source: Author's elaboration.

Figure 2: AR (1) Forecasts - Month over Previous Month



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Light Blue - Forecasts
Source: Author's elaboration.

Figure 3: Random Walk Forecasts - Month over Previous Month

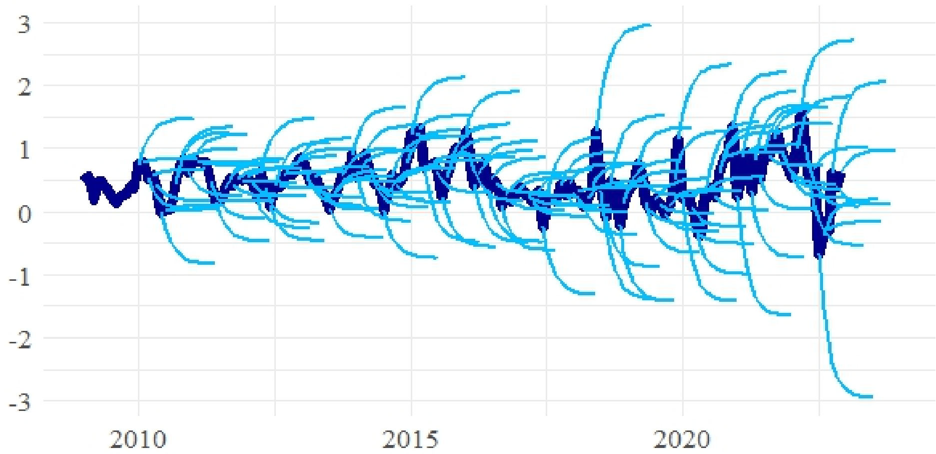
An alternative to the models presented is robust predictors, which, according to Hendry (2018), perform well for 1-period-ahead projections after a structural break occurs. In this scenario, by differencing the AR(1) model, we can eliminate the long-term mean (μ) and robustify the forecast. This way, the estimator can quickly adjust to a new equilibrium consistently estimated by the lagged value of the variable while maintaining a persistence parameter.

$$x_t^{rb} = x_{t-1} + \frac{\rho}{1-\rho}\Delta x_{t-1} \tag{4}$$

In this circumstance, the iterated projection of multiple horizons ahead will be:

$$x_{t+h|t}^{rb} = x_{t-1} + \left(1-\rho^h\right)\frac{\rho}{1-\rho}\Delta x_{t-1} \tag{5}$$

However, the problem with this alternative is the increase in the variance of the forecast error. In a moment of structural stability, the robust predictor is unbiased but very imprecise. In the event of a break, the low bias can compensate for the increase in the variance of the forecast. Figure 4 contains the plot of robust forecast for Brazilian CPI.



Dark blue - Actual Data
Light Blue - Forecasts
Source: Author's elaboration.

Figure 4: Robust Forecasts

3.3 Solution proposed by Martinez, Castle e Hendry (2021)

In this environment, Martinez et al. (2022) propose a reinterpretation using basic models as local estimators of the long-term mean (μ). In other words, they rely on "naive" methods to estimate the long-term mean of the desired variable rather than its actual value, assuming that it undergoes by significant structural changes over time and is subject to various shocks that alter the trajectory of the structural mean.

First, it is necessary to define more precisely the behaviour of the models during a structural break process. From now on, we will define h as the desired projection horizon after the occurrence of a break (b) that happened at time T. With this, after the occurrence of this break, we also assume that the parameters change and new values can be assigned. We refer to the post-break parameters as: μ^*, ρ^*. We begin with the authors' idea of seeking the expected value of the variable, x_{T+b}, b periods after a break. As we iterate x_{T+b} backwards, we will have:

$$\overline{\mu} = E\left[x_{T+b}\right] = \mu^* - {\rho^*}^b(\mu - \mu^*) \tag{6}$$

In which the new long-term mean ($\overline{\mu}$) will consist of a relationship among the post-shock parameters (μ^* and ρ^*) b periods after the occurrence of this shock. As the distance from the shock increases (with the growth of b), the long-term mean converges to μ^*, given that we assume $\rho < 1$. For this reason, we can conclude that the best estimator for the long-term mean is to find $\widehat{\mu} = x_{T+b}$. By substituting this relationship into the Equation 2 with pre-shock parameters, we will have:

$$x^{ar}_{T+b+h|T+b} = \mu + \rho^h(x_{T+b} - \mu) \tag{7}$$

$$x^{ar}_{T+b+h|T+b} = x_{T+b} + \rho^h(x_{T+b} - x_{T+b}) \tag{8}$$

$$x^{ar}_{T+b+h|T+b} = x_{T+b} \tag{9}$$

In other words, it is identical to the projection produced by a random walk model, illustrating that a random walk can be used to estimate the long-term mean.

This idea can be applied to robust methods, in which we will have:

$$\widetilde{\mu} = E\left[x_{T+b} + \frac{\rho}{1-\rho}\Delta x_{T+b}\right] \tag{10}$$

$$\widetilde{\mu} \;=\; \mu^{*} -\; \rho^{*^{b}} \;(\mu -\; \mu^{*}) + \left(\frac{\rho}{1-\;\rho}\right)\; \rho^{*^{b-1}}\,(1-\rho^{*})\;(\mu -\; \mu^{*}) \tag{11}$$

$$\widetilde{\mu} \;=\; \mu^{*} -\; \rho^{*^{b}} \;(\mu -\; \mu^{*}) \;\left(1 - \frac{\rho}{\rho^{*}}\left(\frac{1-\rho^{*}}{1-\rho}\right)\right) \tag{12}$$

The long-term mean converges to μ^{*} when $\rho = \rho^{*}$ or $b \to \infty$. With this, we can estimate the long-term mean as follows:

$$\widehat{\widetilde{\mu}} = \quad x_{T+b} + \frac{\rho}{1-\rho}\Delta x_{T+b} \tag{13}$$

$$\widehat{\widetilde{\mu}} = \;\frac{1}{1-\;\rho}\;\;(x_{T+b} -\;\; \rho x_{T+b-1}) \tag{14}$$

When we incorporate Equation 14 into the projection of the base model described in Equation 2, we arrive at a term identical to the robust predictor, showing how robust projections can also be reinterpreted as alternative estimators of the long-term mean (μ). Moreover, with this adaptation, we can reduce the bias of the projections while also reducing the variance of projection errors for longer horizons. The authors also emphasize that this interpretation can be extended to other models. However, for the development of this work, we will limit ourselves to robust and random walk models.

Martinez et al. (2022) also argue for the need to smooth these estimates to reduce the variance in these projections to make them competitive. For this purpose, the authors include the term $\frac{1}{n}$ for both applications described.

Thus, with this reinterpretation in mind, the main idea will be to use robust and random walk methods to estimate the long-term mean and incorporate this new mean into the base model described earlier. We will begin with the case of the random walk.

$$x^{RW}_{T+b+h|T+b} = \;\mu^{RW} +\; \rho^{h}(x_{T+b} -\; \mu^{RW}) \tag{15}$$

Thus, μ^{RW} will be given by:

$$\mu^{RW} \;=\; \frac{1}{n}\;\sum_{j=0}^{n-1} x_{T+b-j} \tag{16}$$

In other words, the long-term mean in the case of the smoothed random walk consists of a moving average of n periods behind the desired variable.

Additionally, the robust estimator model with smoothing, due to its lower variance in projection errors and better performance as it gets further from the break, can minimize the observed trade-off in robust estimators, making it the ideal method for estimating economic variables after a structural break, as emphasized by Martinez et al. (2022).

$$x^{RB}_{T+b+h|T+b} = \mu^{RB} + \rho^{h}(x_{T+b} - \mu^{RB}) \tag{17}$$

In this case, o μ^{RB} will be given by:

$$\mu^{RW} = \frac{1}{n}\sum_{j=0}^{n-1}(x_{T+b-j} + \frac{\rho}{1-\rho}\Delta x_{T+b-j}) \tag{18}$$

In other words, in the case of robust estimators, the authors include a dynamic factor that differs from previous results.

In conclusion, the authors suggest that the best estimate would be to use the average of the two models presented, resulting in the Smooth Average Robust Estimators, which will be the focus of the fourth section of this work.

4 Results

At this point, the goal is to apply the methodology presented in the previous chapter to Brazilian inflation. First, as done in the reference work, the models will be adapted to consider the expectations extracted from the Focus survey. The aim is to incorporate analyst information into the future trajectory of inflation and assess whether robust smoothing methods provide any improvement in accuracy compared to actual data. In the models, we will begin with an analysis of a base AR(1) model. Then, considering the Focus information, the robust and smoothed versions will be presented to examine the gains in the application of these alternative methods.

4.1 Base Model

The autoregressive models were initially estimated within the window from January 2000 to December 2009, which was expanded with each new known IPCA (Consumer Price Index for All Urban Consumers) data point. This way, a sequence of different inflation projections was obtained between 2000 and 2023.

The initial autoregressive model will be given by:

$$IPCA_t = \beta_0 + \beta_1 \left(IPCA_{t-1}\right) + \varepsilon_t \tag{19}$$

where $varepsilon_t$ is assumed to be an identically and independently distributed variable with finite variance.

In this model, we run a regression of the monthly variation of IPCA against a constant (β_0) and the lagged IPCA by one period. As previously explained, the projections of the autoregressive model are based on a portion of the most recent available information as the best estimate for the future trajectory of inflation. Therefore, in the presence of a more significant structural shock, autoregressive models will be biased in not accurately tracking the new behaviour of the analyzed variable.

Additionally, in Figure 5, we specify all the estimated coefficients β_1 in the AR(1) models, which will be important in the estimation of the robust smoothing models going forward. When analyzing Figure 5, we can observe an average coefficient of 0.65, indicating that, on average, about 65% of the current inflation can be explained by the information from the immediately preceding period, with some fluctuations over the analyzed period. However, it is possible to see a downward trajectory of this coefficient as we expand the window over time, suggesting that an instability or structural change took place after COVID.

Source: Author's elaboration.

Figure 5: AR(1) Coefficient

With the autoregressive coefficients in hand, we proceed to make a slight change to the base model by including two new parameters in the equation: μ_t, which represents a long-term average of the analyzed variable, and another coefficient that represents the median of expectations calculated by institutions, in this case, the Central Bank's Focus survey. With this set of information, the projection of IPCA h-periods ahead is given by:

$$IPCA_{t+h} = \mu_t + \beta_1^{h-1}(FOCUS_{t+1} - \mu_t) \tag{20}$$

The projection of IPCA takes into consideration the long-term inflation average (μ_t), autoregressive coefficients (β_1) and the estimates from the Focus survey for a period ahead ($FOCUS_{t+1}$). In the case of h=1, the IPCA projection will be entirely based on what was reported in the Focus survey on the last day of the preceding month, given the amount of information available for the short term. For longer horizons, the long-term average (μ_t) becomes more important for the variable's projections. This is why it's crucial to use the presented concept of smoothed robust predictors to find the best long-term average for the desired variable (μ_t).

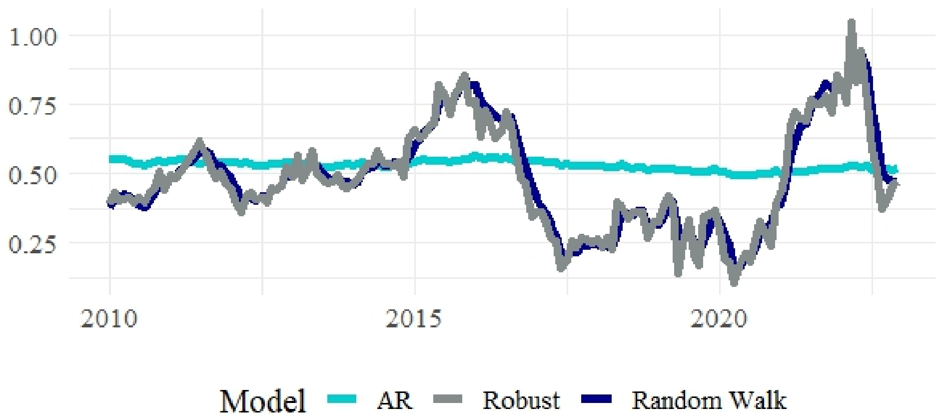
4.2 Estimating the Long-term Average

To estimate the long-term average using the methods of robust regressions with smoothing, as detailed in the previous chapter, we can summarize this process in Equation 21:

$$\mu_t(n,\gamma) = \frac{1}{n}\left[FOCUS_{t+1} + \sum_{j=0}^{n-2} IPCA + \gamma\frac{\beta_1}{1-\beta_1}(FOCUS_{t+1} - IPCA_{t-n+1})\right] \quad (21)$$

In which, the long-term average (μ_t) for period t will be equal to the FOCUS expected for a period ahead plus the sum of (n-2) past observations of the analyzed variable. In addition, if we have γ= 1 we will be a robust estimator with a smoothing environment. Otherwise, if, γ= 0, we will consider the version of the smoothed random walk. Finally, the choice of n is also relevant for the projections of the desired variable, translating into the smoothing process in estimating the long-term average. Thus, we choose $n = 12$ due to the authors' recommendations when dealing with higher-frequency data and given the seasonal characteristics of Brazilian inflation. If we adopt $n = 1$, we will be in an environment of robust estimators without smoothing.

With this, applying Equation 21, we can obtain the long-term average estimates which will be an important input to the projections. In Figure 6, we compare the difference between the three estimates of μ_t. The first method of estimation is the usual form of the AR(1) base model, where the long-term average of inflation approaches the historical average of monthly inflation. In this case, we observe that it is an estimate that undergoes few changes over time, indicating stability in this equilibrium of the variable. As an alternative, we applied the methods from Equation 21 and arrived at the other estimates in Figure 6. For the smoothed robust estimators, we observe greater volatility in the series, but it anticipates unexpected shocks in the variable more quickly. Finally, in the case of the smoothed random walk, as it is based on an average of the latest available information, it exhibits a smoother behaviour compared to the robust version.

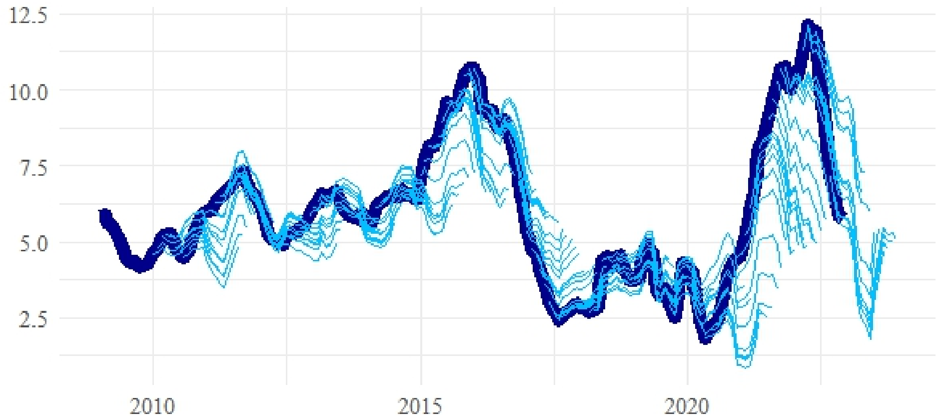


Source: Author's elaboration.

Figure 6: Long-term average comparison

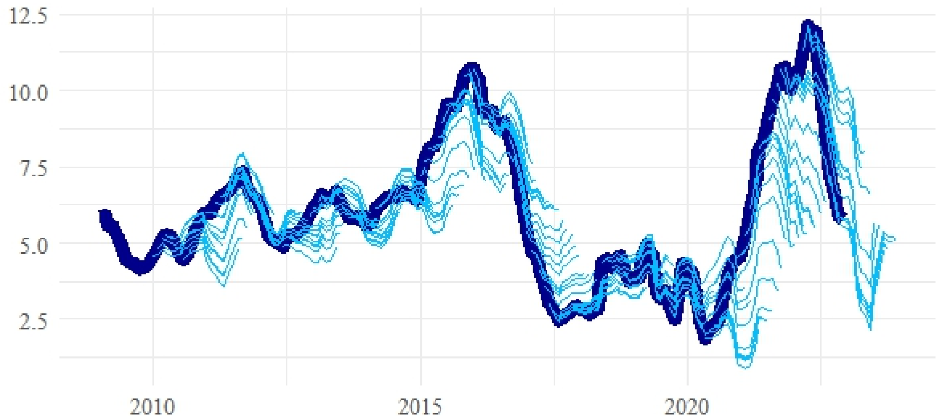
4.3 Forecasts

Based on the long-term estimates obtained, we can return to Equation 20 and calculate inflation estimates for 12 periods ahead. In Figures 7 and 8, we use the base model with the two different long-term mean estimates, i.e., estimated using robust methods (Figure 7) and random walk (Figure 8). We input projections for various windows set between 2009 and 2022 and compare them with the observed actual inflation curve during the same period. When analyzing the Figures, despite subtle differences between the estimates produced, it is still possible to see the characteristics of both alternatives. First, in the robust model, we observe that the projections exhibit less statistical memory about past results. For example, during periods of high inflation (2016 and 2022), we see a rapid reversal of projections, indicating a faster decline in inflation ahead. On the other hand, in the case of the random walk, we notice greater efficiency in situations of inflation persistence, as the model produces estimates with more significant statistical memory compared to the first case. In practice, the differences observed in the two models are justified by the profile of the μ_t estimates presented earlier.



Dark blue - Actual Data
Light Blue - Forecasts
Source: Author's elaboration.

Figure 7: Smooth Robust Forecasts



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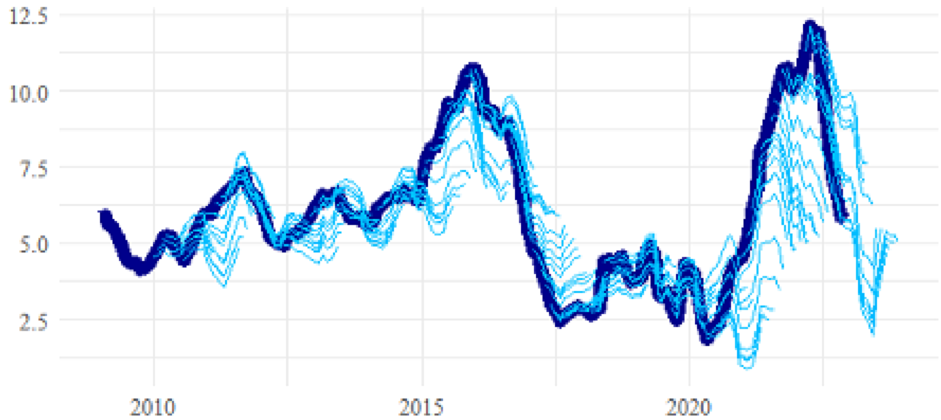
Light Blue - Forecasts
Source: Author's elaboration.

Figure 8: Smooth Random Walk Forecasts

Finally, Martinez et al. (2022) argues that to maintain the characteristics of both models, we can also extract the average of both previous projections, thereby indicating the best estimation for this situation.

4.4 Comparison with Focus Survey

In the previous sections, significant gains were observed in the use of robust smoothing methods compared to the performance of basic models, such as the autoregressive model. At this point, the goal is to compare the performance of the studied models against the median of projections from Brazilian institutions computed by the Central Bank (Focus Survey).



Dark blue - Actual Data
Light Blue - Forecasts
Source: Author's elaboration.

Figure 9: Average of Smooth Robust and Random Walk Forecasts

Visually, we compare in Figures 1 (Focus) and 9 (the average of smoothed robust and smoothed random walk the models) for IPCA versus the Focus median for the same analyzed periods. It can be identified that despite the Focus estimates having less volatility and appearing centered around the historical average of Brazilian inflation, the regression methods with smoothing robust devices better capture periods when inflation showed more significant increases, or periods that can be defined as more relevant structural breaks in the behaviour of domestic inflation, such as the years 2015 and 2021. Another way to verify the visual impression of the inflation trajectory is to observe the performance of the Mean Square Forecast Errors (MSFE) for all projected periods.

h	Focus	Smooth Robust	Smooth Random Walk	Average of Models
1	0.1870	0.1870	0.1870	0.1870
2	0.2316	0.2286	0.2293	0.2289
3	0.2560	0.2520	0.2528	0.2523
4	0.2718	0.2687	0.2693	0.2689
5	0.2826	0.2810	0.2813	0.2809
6	0.2906	0.2904	0.2900	0.2899
7	0.2977	0.2993	0.2984	0.2986
8	0.3040	0.3075	0.3063	0.3066
9	0.3095	0.3151	0.3137	0.3141
10	0.3140	0.3215	0.3198	0.3203
11	0.3174	0.3269	0.3248	0.3255
12	0.3204	0.3318	0.3293	0.3301

Source: Author's elaboration.

Table 1: MSFE 2009-2022

h	Focus	Smooth Robust	Smooth Random Walk	Average of Models
1	0.2050	0.2050	0.2050	0.2050
2	0.2565	0.2452	0.2468	0.2460
3	0.2735	0.2549	0.2578	0.2562
4	0.2788	0.2569	0.2618	0.2591
5	0.2826	0.2615	0.2670	0.2640
6	0.2855	0.2659	0.2714	0.2683
7	0.2886	0.2736	0.2788	0.2757
8	0.2913	0.2799	0.2848	0.2818
9	0.2938	0.2851	0.2894	0.2866
10	0.2956	0.2900	0.2937	0.2912
11	0.2958	0.2932	0.2964	0.2941
12	0.2948	0.2947	0.2972	0.2952

Source: Author's elaboration.

Table 2: MSFE 2015-2016

h	Focus	Smooth Robust	Smooth Random Walk	Average of Models
1	0.3208	0.3208	0.3208	0.3208
2	0.4007	0.3832	0.3831	0.3831
3	0.4450	0.4123	0.4105	0.4113
4	0.4875	0.4456	0.4401	0.4427
5	0.5144	0.4679	0.4591	0.4633
6	0.5306	0.4817	0.4700	0.4756
7	0.5431	0.4942	0.4798	0.4867
8	0.5576	0.5144	0.4988	0.5063
9	0.5704	0.5346	0.5186	0.5263
10	0.5815	0.5535	0.5374	0.5450
11	0.5895	0.5685	0.5518	0.5597
12	0.5946	0.5800	0.5628	0.5709

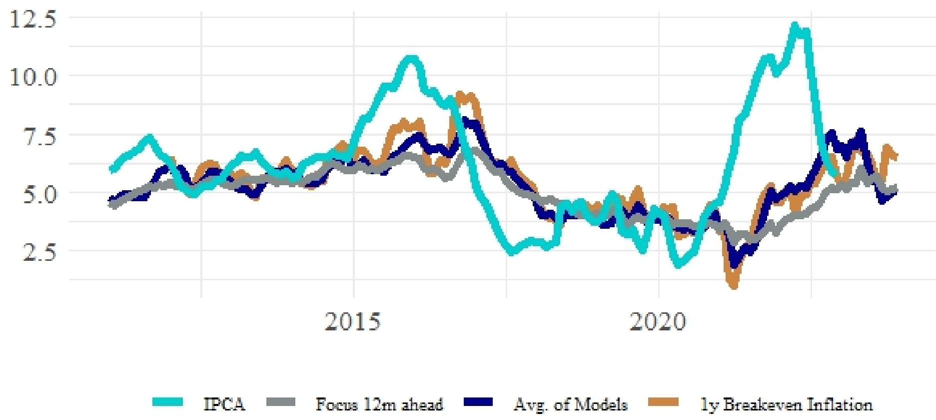
Source: Author's elaboration.

Table 3: MSFE 2020-2021

In Tables 1, 2 and 3, we can see that the robust methods performed well. When we analyze the entire sample (2009-2022), the robust methods are in line with Focus's projections. Therefore, the alternative models presented earlier do not show much differentiation from what is indicated by market expectations. It is important to note that for the period $h = 1$, the estimates from all models follow the Focus forecast because it contains the most available information. The deviations among the models start from $h = 2$.

When we examine the other tables, however, during two periods of higher inflation volatility, the robust methods demonstrate more accuracy compared to Focus, especially within periods two to eight ahead. For longer periods, between 9 and 12, all models accumulate high errors, and the robust methods do not stand out towards the end of the projected window. For the shorter range, among the three alternative models, there are no significant differences in MSFE analysis. In the 2015-16 period, the smoothed robust model seems to perform best compared to the others. Meanwhile, in the 2020-21 period, the random walk model with smoothing has the highest level of accuracy for the periods ahead.

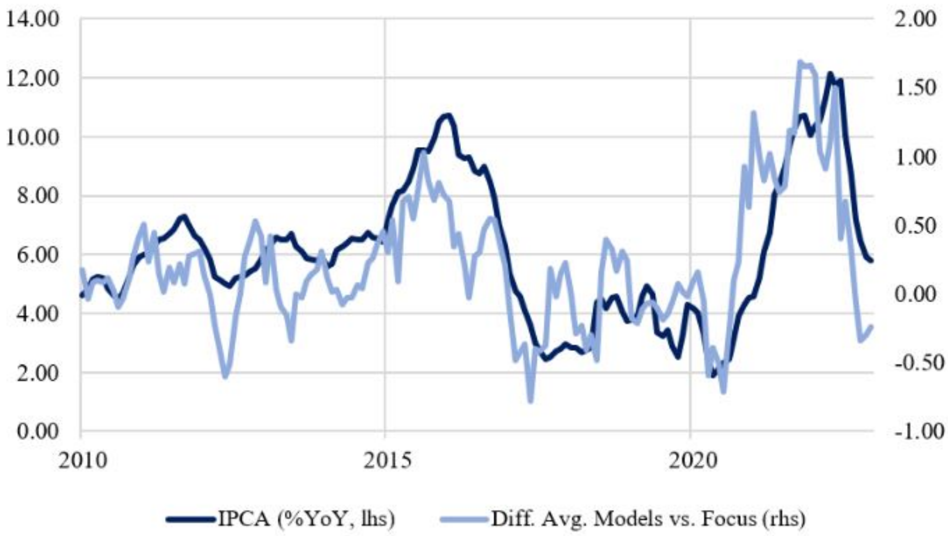
An additional comparison to be made is the performance of both models for the 12-month ahead inflation forecast. It is understood that both Focus and robust methods projections have relevant errors to the actual observed inflation. However, from Figure 10, we can see that robust methods respond more quickly to the increases observed in the most critical years for inflation, while the Focus median, especially in recent years, took more time to adapt to the new equilibrium. To complement this comparison, we also include the performance of implied inflation extracted from fixed-income market negotiations, showing that robust methods closely tracked the inflation indicated by market prices during the recent high inflation period.



Source: Author's elaboration.

Figure 10: Actual IPCA vs. Alternative Models

Additionally, looking at short-term inflation performance, when we take the simple difference between the robust estimates and the original expectations from the Focus Survey, we find a leading relationship with current inflation. (Figure 11) This suggests that in moments when robust estimates start to deviate from the original expectations, current inflation seems to react subsequently to this deterioration in long-term expectations.



Source: Author's elaboration.

Figure 11: Average Smoothed Models and Focus Deviation vs. Realized IPCA

The observations described align with the conclusion of Martinez et al. (2022) that because these models quickly adjust their long-term estimates (μ_t), they work better in periods of inflation shock. Traditional models take time to adapt to a new structural mean, while robust and random walk models with smoothing adjust more rapidly to these shocks.

4.5 Diebold and Mariano

Another way to compare the accuracy of different forecasts is through the methodology presented by Diebold and Mariano (2002), which involves comparing the accuracy between two models and determining whether the difference between the forecasts is significant or not. The test involves assessing the forecast difference through the following relationship:

$$d_t^{i,j} = g(e_t^i) - g(e_t^j) \tag{22}$$

That is, given the existence of two models (i, j), the variable d^{ij} captures the difference between a function $g(.)$ representing the error of both models under consideration. In this test, the null hypothesis is that the predictive powers of the models are equivalent to the alternative that there is a difference in predictive power. This test is more efficient for situations where you want to compare predictive power between the two models.

The test statistic is calculated as:

$$DM = \frac{\overline{d}}{\sqrt{Var(\overline{d})}} \tag{23}$$

In which $\overline{d}= T^{-1} \int_{j=1}^{T} d_{t+j}$, and T is the total number of predictions. According to the authors, the test statistic follows a normal distribution with a mean of zero and unit variance.

Applying the methodology of the authors, in the case of this work and opting by squared function for g, we have:

$$DM_t^{i,Focus} = e_{t,i}^2 - e_{t,Focus}^2 \tag{24}$$

In other words, we calculate the difference in squared errors between the two projections (alternative models vs. Focus) and test its significance by analyzing the p-values generated in the forward regressions.

By applying these concepts, we can reproduce the differences between the three alternative models and the performance of Focus for the 12 periods estimated ahead and over the years studied. We will focus on the comparison between the average of the models and the estimates of Focus. The other two comparisons are provided in the appendix of this work (Appendix A and B).

Period	Diebold-Mariano	Steps ahead - h											
2010-22		1	2	3	4	5	6	7	8	9	10	11	12
Smooth Robust	t-Statistic	NA	-0.90	-0.76	-0.56	-0.40	-0.43	-0.09	0.30	0.64	1.13	1.64	2.01
	p-value	NA	37.2%	45.1%	57.9%	69.3%	67.0%	93.0%	76.4%	52.1%	26.0%	10.3%	4.6%
Smooth RW	t-Statistic	NA	-0.78	-0.80	-0.66	-0.57	-0.67	-0.33	0.06	0.36	0.65	1.01	1.61
	p-value	NA	43.5%	42.4%	50.8%	57.0%	50.5%	74.0%	95.1%	72.0%	51.9%	28.3%	11.0%
Average	t-Statistic	NA	-0.85	-0.80	-0.63	-0.51	-0.58	-0.24	0.15	0.46	0.85	1.37	1.88
	p-value	NA	40%	43%	53%	61%	56%	81%	89%	65%	40%	17%	6%
2015-16		1	2	3	4	5	6	7	8	9	10	11	12
Smooth Robust	t-Statistic	NA	-0.38	-0.03	0.21	0.46	0.56	0.73	0.75	0.72	0.87	1.01	1.03
	p-value	NA	70.9%	97.7%	83.4%	66.2%	58.0%	47.4%	46.0%	47.6%	39.3%	32.3%	31.3%
Smooth RW	t-Statistic	NA	-0.28	0.03	0.22	0.40	0.45	0.57	0.57	0.46	0.60	0.80	0.82
	p-value	NA	78.4%	97.6%	82.7%	69.1%	66.6%	57.3%	57.7%	66.2%	55.5%	43.0%	42.1%
Average	t-Statistic	NA	-0.34	-0.01	0.21	0.42	0.49	0.64	0.65	0.58	0.73	0.91	0.92
	p-value	NA	74%	99%	84%	68%	63%	53%	52%	57%	47%	38%	37%
2020-21		1	2	3	4	5	6	7	8	9	10	11	12
Smooth Robust	t-Statistic	NA	-2.70	-3.12	-3.21	-3.13	-2.96	-1.91	-0.85	-238.00	0.10	0.50	1.06
	p-value	NA	1.1%	0.8%	0.6%	0.6%	0.9%	0.7%	40.8%	81.4%	91.9%	62.3%	30.1%
Smooth RW	t-Statistic	NA	-2.58	-3.01	-3.12	-3.08	-2.93	-1.73	-0.76	-0.21	-0.10	-0.02	0.30
	p-value	NA	2.3%	0.9%	0.7%	0.7%	0.9%	10.0%	45.6%	83.5%	92.6%	98.1%	76.9%
Average	t-Statistic	NA	-2.65	-3.08	-3.18	-3.13	-2.98	-1.84	-0.82	-0.24	-0.04	0.16	0.78
	p-value	NA	2%	1%	1%	1%	1%	8%	42%	81%	97%	88%	44%

Table 4: Diebold and Mariano Statistic and respective p-value

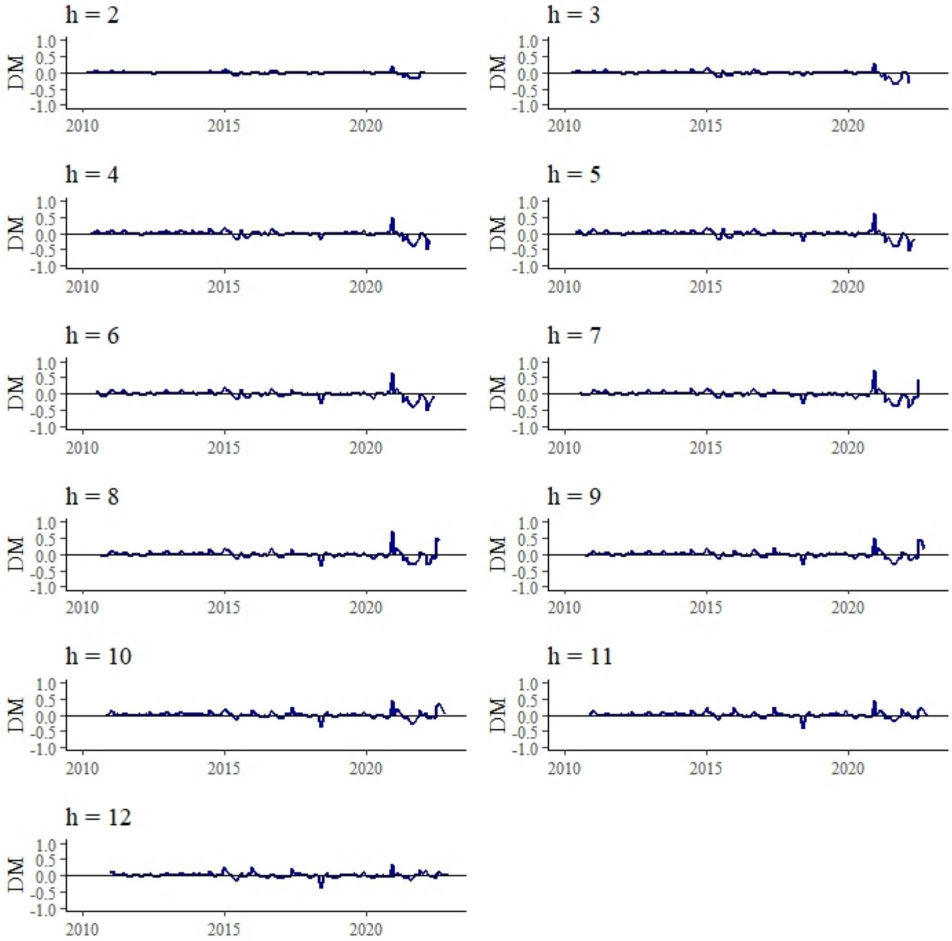


Figure 12: Plot of (22) using squared errors for Model Average versus Focus
Source: Author's elaboration.

In Figure 12, we can conclude that in comparison to the performance of Focus, the average of the robust models does not produce more accurate estimates than Focus during periods of inflation stability. Figures 13 and 14 contain the plots for other models. However, as observed in the analysis of RMSE, we see higher accuracy during periods of high inflation in the years 2015 and 2021. For estimation periods between 2 and 6 periods ahead, we identify a much better performance than the market median in the period 2021 and 2022. However, in Table 4, we measure the significance of these differences.

In these tables, non-significant p-values are observed for the entire sample. That is, when considering the period between 2009 and 2022, the differences between the alternative model and the Focus estimates are not statistically significant. However, the conclusion is quite different when we shorten the sample for periods of higher inflation during the COVID. In both situations, the differences between the two models are significant and indicate significant gains in using smooth robust forecasts during periods of high inflation volatility and uncertainty.

5 Limitations and Possible Extensions

The method implemented in this study has demonstrated its efficiency in forecasting macroeconomic variables, particularly during periods marked by significant structural breaks. As for potential extensions, the robustification process can be extended to various other types of forecasting models. Moreover, it holds applicability to other economic series characterized by substantial structural breaks, exemplified by the Brazilian Gross Domestic Product (GDP).

As for possible improvements, there are more sophisticated techniques for choosing the value of "n" used in the smoothing process of estimates. Works like Pesaran and Timmermann (2007) and Giraitis et al. (2013) discuss different ways to find the optimal smoothing window. Additionally, while we chose to follow the authors' recommendations by considering the average of the estimates (robust and random walk) as the main forecasting model for analysis, there are other alternative approaches, such as a Kalman filter, that might capture the characteristics of the two models in a more sophisticated way rather than a simple average. However, these are points to be addressed in future research.

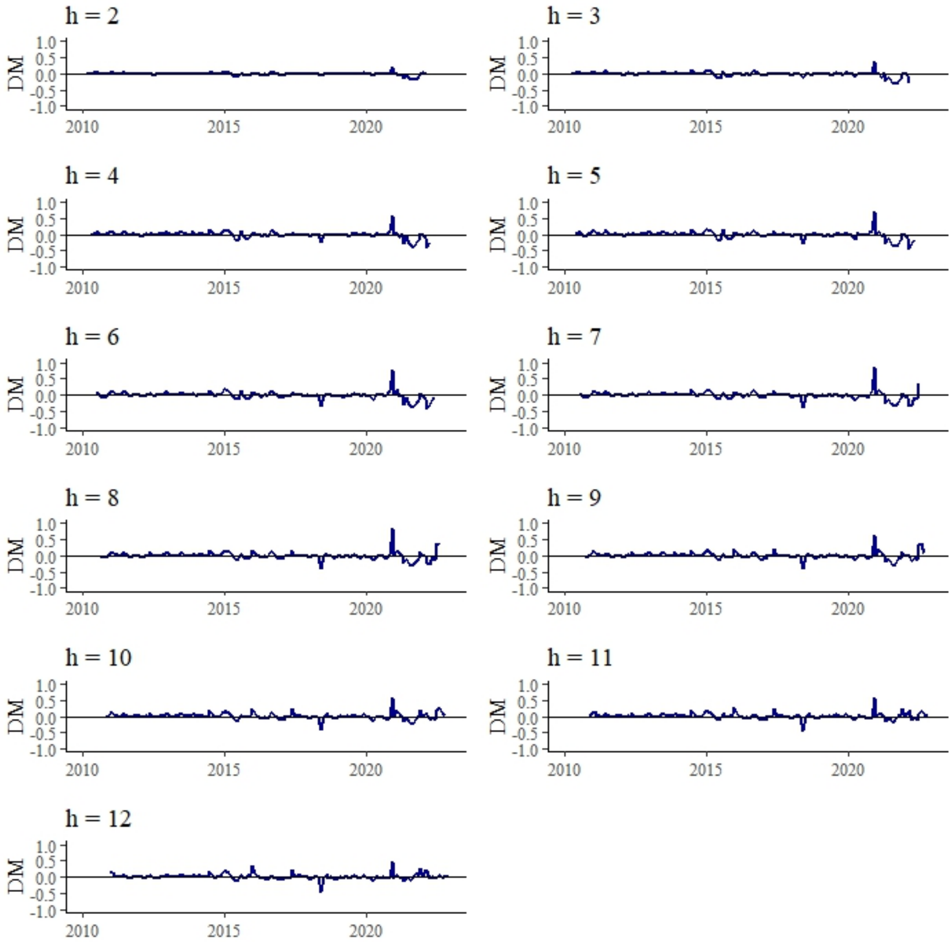


Figure 13: Plot of (22) using squared errors for Robust AR(1) versus Focus
Source: Author's elaboration.

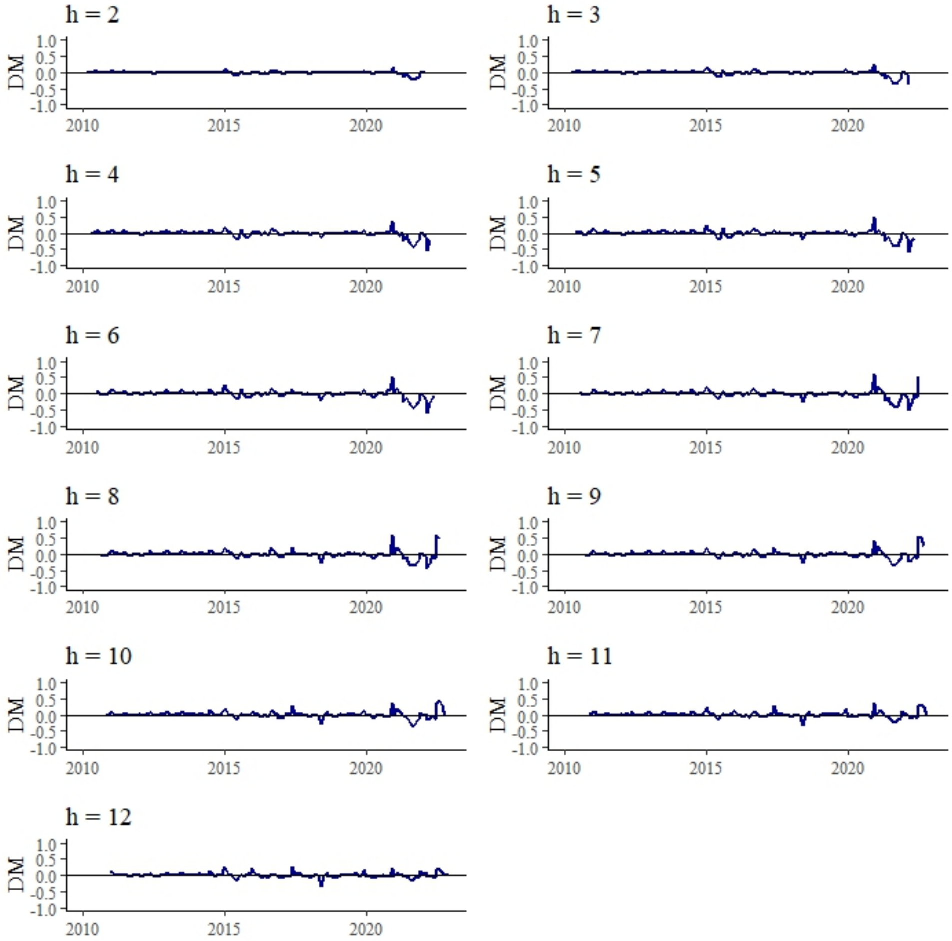


Figure 14: Plot of (22) using squared errors for Random walk versus Focus
Source: Author's elaboration.

6 Conclusions

This study aimed to test the effectiveness of the models proposed by Martinez et al. (2022) during periods of high inflation volatility in Brazil, which led to sequential errors in expectations compared to the observed variable. We replicated the alternative models presented by the authors and compared them with the performance of the Focus survey from 2009 to the end of 2022. It is observed that the projections produced by the alternative models indicate higher variance compared to Focus. However, during periods of rapid inflation spikes, the alternative models proved to be more efficient and capable of producing significant gains compared to the estimates produced by the Central Bank's expert survey (Focus). This effectiveness is also tested through the analysis of Root Mean Squared Errors (RMSE) and the test proposed by Diebold and Mariano (2002). When considering the entire sample (2009-22), the RMSE does not indicate a significant differentiation between the analyzed models. However, when selecting periods of high inflation (2015-16 and 2020-21), the robust methods significantly outperform the Focus estimates. The Diebold and Mariano (2002) analysis shows that this difference in performance is less relevant for the 2015-16 period but highly significant for the recent 2020-21 period.

Other analyses and comparisons presented in the study indicate that the forecasts made by robust estimators have a good approximation to the implied inflation calculated from market prices, replicating the characteristics of higher variance and a quicker response to inflation shocks in recent years. Additionally, we have also shown that the difference between robust estimates and the original Focus estimates precede, or at least accompany, the behaviour of current inflation. In other words, the erratic behaviour of inflation expectations and the stronger response of robust estimators seem to be closely linked to periods of higher inflation volatility.

Through these analyses, we have confirmed the significance of the proposed models and their effectiveness during periods of more significant structural breaks in the analyzed variable.

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