**Bargaining Process, the Risk of Breakdown, and some Contract Considerations in a Model about Negotiation:**
**A Theoretical Approach for Oil Industry**

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## Abstract

Oil and gas models are focused mainly in production, behavior, risk and forecast, so the present one constitutes a first attempt in trying to modellize oil sector behavior from the point of view of bargaining process. The purpose of this model, even if theoretically, is to present a theoretical approach of the bargaining process in oil and gas industry. I begin with a simple Nash bargaining solution including Edgeworth box for a better explanation of the behavior between two individuals that agree to participate in a bargaining process. Rubinstein's model where two players, government and oil companies, have a common interest to bargain over the partition of a "cake", but at the same time they have conflicting interests over the share of the cake and about the payoffs both of them could obtain from the bargaining situation. Even if this process can lead to an agreement, I take into account the risk of breakdown where one player or both may perceive that the negotiations might breakdown in a random manner for any reason that is not important for the outcome. The contract curve estimation shed light for analyzing the possibility to bargain about the selling and buying a commodity, in this first attempt, without taking into account the role of arbitrators and mediators, and any enforcement behavior from the part of each player.

Keyword: bargaining, contract theory, game theory
JEL: C78,

## I. Introduction

The theory of general equilibrium was considered one of the most impressive achievements in the history of model development. However, it quickly appeared that these models were not satisfactory when trying to explain strategic interactions between agents who can influence prices and have market power even though they interact through the price system. The existence of a continuum nonatomic agents, none of which can influence equilibrium prices and allocations, and the presence of many institutions that govern economic relationships impede the direct application of general equilibrium models in certain markets. Informational asymmetries are a source of another challenge because, unfortunately, they are pervasive in economic relationships; that is to say, firms know more about their costs than the government, and all agents take actions that are at least partly unobservable; these same behavior lowers the perfect knowledge assumed by rational expectations assumptions. While general equilibrium models offer interesting insights on the revelation of information by prices, their treatment of asymmetric information in not satisfactory, given that it can be expected that the agent who possesses private information, and even public ones, have a monopoly over the information and will try to manipulate it at the moment of bargain, for instance. In taking this into account, we must forsake general equilibrium models and turn the attention to other tools, it means, those belonging to game theory (Salanié, 2005.)

The theory of contracts thus evolves from the failures of general equilibrium theory, settled on a new way to study economic relationship. General equilibrium models are consistent but not realistic enough mainly in the area of strategic interactions between asymmetrically informed agents in well-defined institutional entities. The theory of contracts takes into account i) the partial equilibrium models when isolating the markets for one good, oil industry is a good example, sometimes two goods, from the rest of the economy; ii) the interactions of a small number of agents where one of them has access to private information; iii) the models sum up the constraints imposed by the prevailing institutional setting through a written agreement -preferable- or depending on a system of behavioral norms[1]; iv) the models are numerous mainly in the vein of noncooperative game theory with asymmetric information following a bargaining process ending on a bargain or a rupture, taking decisions at the same time or after some delays from the part of players interacting in the bargaining process.

Models about oil industry always have been focused on demand, supply, risk, prices and forecasts about the behavior of these variables, the shocks that hit the market of oil and natural gas and intents to explain the volatility of these same variables. Krichene (2005) presents a simultaneous equation model of the world crude oil and natural gas markets estimating price elasticities that are low whereas income elasticities are always significant and high. He compares elasticity estimates of the simultaneous equations model with some estimates of energy demand elasticities from the literature and confirms the hypothesis of extreme elasticity of demand and supply to price changes. The implications of low price

[1] Written agreements are explicit contracts that will be guaranteed by a court or a mediator or by "impartial" desired agents well established in the same contracts. An implicit contract is sustained by an equilibrium tacitly observed in the interactions between the agreeing agents intervening in a contract.

elasticities is that a small excess demand or supply may require large movements in prices to clear markets for crude oil and natural gas. Noting the importance of monetary policy in influencing aggregate demand and exchange rates, Krichene tries to establish a relation between oil prices, the U.S. dollar movements as measured by changes in the nominal effective exchange rate, the U.S. interest rates and to identify demand shocks arising from monetary policy. This point of view is very interesting vis-à-vis the crisis after an expansionary global monetary policy. Interest rates and nominal exchange rate influence inversely crude oil prices; rising interest rates and appreciating the exchange rate tend to depress crude prices, whereas declining interest rates and depreciating exchange rate have the opposite effect. As an implication of vulnerability to monetary shocks, coordination of monetary policy should explicitly include crude price volatility in the inflation target model.

From their part, Rossiaud and Locatelli (2008) envisage the slowdown in the growth of Russian oil output given the geological and techno-economical characteristics of the Russian oil sector that place structural constraints on future production trends. The paper does not propose any model but a mere description of the Russian oil industry behavior. Anyway, what is interesting from the point of view of the paper is that besides the cited characteristics in the Russian oil industry, it tries to analyze the sources of uncertainty that include institutional and organizational frameworks defined by the Russian state to control the activities of oil companies. This institutional framework determines the various incentives for operators affecting decisions on investing strategy and management of oil resources representing, thus, an essential factor in the probable profile of Russian production. Rossiaud and Locatelli found that almost years after the reorganization and privatization of the Russian oil industry undertaken as part of the transition process, it is clear that the industry's organizational model and institutional framework still have a long way to go before achieving stability. Obviously, the statements expressed in the paper risk to be taken as normative instead of being positive, so the conclusions could be interpreted as being political.

Burger et al. (2010) analyze oil-dependent economies, whether importers or exporters as being exposed to large and volatile shocks associated with oil price fluctuations. The macroeconomic impact of these supply shocks are pervasive, exposing the government's budget process and balance sheets as well as private-sector production and consumption to external fluctuations that are out of control mostly for developing countries. Insurance could be a source of study from the point of view of the bargaining process but this fact is not considered by the Burger's model. However, insuring against the impact of energy price shocks directly through futures or over-the-counter derivative contracts, is very difficult as the typical maturity of available instruments is either too short or the insurance is too costly for small, below-investment-grade economies. It will be useful to think about additional insurance and risk-sharing mechanism that can enhance the set of available (and tradable) financial instruments (1). A country's external capital structure (the composition of foreign assets and liabilities by instrument, currency, and maturity) can mitigate or exacerbate the impact of external shocks. For instance, foreign currency exposure may turn an otherwise benign real exchange response to an oil price shock into a negative financial shock with undesirable contractionary effects. Similarly, the maturity structure and instrument composition of foreign assets and liabilities (i.e., debt versus equity or particular sectors of world equity markets as opposed to others) may significantly affect the response to energy price and other external shocks.

Bems and de Carvalho Filho (2009) analyze the consequences over budget surplus of the oil exporting countries following the boom in commodity prices. the current account surpluses of the same oil-exporting countries have also widened significantly as oil prices soared. The size and the volatility of current account balances in oil-exporting countries bring to prominence questions about their role in the global imbalances and the appropriate macroeconomic policy response to fluctuations in oil prices and global economic activity. The paper is focused mainly in changes of the exchange rate as a consequence of capital flows to oil-exporting countries following supply side shocks and trying to take into account their particularities as oil-exporters, and proposing specific methodologies to specific oil-exporting countries following the price-based approach, the quantity-based approach and balance-sheet-base approach. Oil exporters can differ from other advanced and emerging market countries along multiple dimensions. For example: 1) the fiscal balance in oil-exporting countries is typically dominated by swings in fiscal revenues related to oil exports and is hence strongly correlated with the current account and more volatile than for non-oil-exporters; 2) because oil revenues accrue from the sale of a non-renewable resource, transfers from one generation to another play an important role in ensuring intergenerational equity (2); to avoid sharp decreases in absorption once oil exports decline, exhaustible-resource countries aim to accumulate foreign assets and use income from such assets to offset future decreases in the stream of oil income; such intergenerational transfers are more important for countries that expect to deplete their exhaustible resource endowment within a relatively short timeframe; as a result, oil exporters can be expected to exhibit large current account surpluses and higher net foreign asset positions; 3) oil-exporting countries are in general exposed to wider fluctuations in their external accounts, because their exports, by definition, are relatively undiversified and oil prices fluctuate more widely than the prices of other goods, such volatility is directly reflected in the higher volatility of their terms of trade, current accounts as a percent of GDP and income more generally; and finally 4) per capita output growth in oil exporting countries is systematically lower than in a sample of oil-importing advanced and emerging countries. At the same time, oil exporting countries exhibit lower dependency ratios and higher population growth rates.

Mun (2007) cites a Steve Hoye's work about his oil and gas experience using Monte Carlo simulation focused mainly on risk in oil and gas industry. Hoye's model is beset with multiple, significant risk factors that determine the resulting project's profitability, including: 1) d*ry-hole risk,* investing drilling dollars with no resulting revenue from oil or gas because none is found in the penetrated geologic formation;.2) *drilling risk,* high drilling costs can often ruin a project's profitability; although companies do their best to estimate them accurately, unforeseeable geological or mechanical difficulties can cause significant variability in actual costs; 3) *production risk,* even when oil or gas reservoirs are discovered by drilling, there is a high probability that point estimates of the size and recoverability of the hydrocarbon reserves over time are wrong; 4) *price risk,* along with the cyclical nature of the oil and gas industry, product prices can also vary unexpectedly during significant political events, overproduction and cheating, interruptions in supply such as large refinery fires, labor strikes, or political uprisings in large producing nations, and changes in world demand; 5) *political risk;* significant amounts of the world's hydrocarbon reserves are controlled by nations with unstable governments; companies that invest in projects in these countries take significant risks that the governments and leaders with whom they have signed contracts will no longer be in power when earned revenue streams should be shared contractually. In many well-documented cases, corporate investments in property, plants, and equipment (PPE) are simply nationalized by local governments, leaving companies without revenue or the equipment and facilities that they built to earn that revenue.

Baily (2007) considers equitable liens associated with the protection of an unpaid vendor in relation to the sale of land but applied to the unaccomplished duties of a company in relation to the one that provides oil exploitation areas in the home country. The established position is that, as soon as a binding contract for the sale of land has been made, the vendor has a lien on the property for the purchase money and a right to retain the property until payment is made. Equitable liens do not exist solely for the purpose of protecting the vendor of land. It also quickly became established that an equitable lien will protect a purchaser who has paid an instalment of the purchase price but not received a conveyance of the legal title to the land. In these circumstances, a purchaser will be protected by an equitable lien, which will operate to secure any payment that he has made in circumstances where the sale goes off through no fault of his own. It could be extracted from the body of the paper that the contract parties have established previously a third party in the form of a court in order to guaranteed the fulfillment of the contract.

Partnership and alliances among oil companies and contractors are considered by Kemp and Stephen (1999). The concepts have been adopted by some as devices which can reduce the adversarial element in the relationships between oil companies and their contractors. It is argued that the reduction of the adversarial element will encourage mutually beneficial co-operation which should stimulate a faster completion of certain identified tasks, innovations, and cost savings. Others fear that the employment of such contracts represents the transfer of more of the investment and operating risks to the contractors and a squeeze on their margins. The risk-sharing attributed to some typical types of development contracts, highlights the effects of bonus/penalties schemes over the behavior of private agents intervening in bargaining processes and contracts. An efficient schema should produce required incentives without introducing other unwanted incentives such as exaggeration of cost savings and understatement of certain costs in order to gain some advantages or elude some responsibilities. Kemp and Stephen (1999) use a Monte Carlo simulation as an appropriate method for measuring the risks and rewards of the two parts intervening in bargain and contract processes. A normal distribution of possible costs relating to an oil field development project is postulated. The width of the distribution (variance) indicates the extent of the risks faced by the investor. When bonus/penalty payments are introduced these are assumed to be paid on completion of the contract. The contract examined is a reimbursable one between the oil field investor and contractor.

This brief literature review shows us that there exists a lack of research concerning bargaining in oil industry between players denoted as countries and oil enterprises. Even if the revisited papers are very interesting, they do not consider a becoming important issue on bargaining modeling, even worst in the case of oil industry. The partition of a cake of size

$\pi$ must not be misleading in the sense of a distribution of a country between different interest, but only as a way to begin to analyze a certain bargain process, its implications for policy developers, the study of taxation, the importance of institutions when redacting contracts and a lot of issues which can improve the bargaining situations.

**II. The Attempt**

Any exchange situation in which a pair of individuals (or organizations excluding inanimate things) can engage in mutually beneficial trade but have conflicting interests over the terms of trade is a bargaining situation. There are two main reasons for studying bargaining situations. The first, the practical reason, is that many important and interesting human (economic, social and political) interactions are bargaining situations. Exchange situations (which characterize much of human economic interaction) are bargaining situations. The second reason, the theoretical one, for studying bargaining situations is that understanding such situations is fundamental to the development of the economic theory of markets.

Dixit et.al (2015) affirm that all bargaining situations have two things in common. First, the total payoff that the parties intervening in a negotiation are looking to enjoy, as a result of reaching an agreement, should be greater than the sum of the individual payoffs that each could achieve separately, i.e. the whole must be greater than the sum of the parts. Without the possibility of this excess value, or "surplus", the bargaining process would be pointless. The second important point, even general, follows from the first: it is not a zero-sum game. When it exists the possibility to obtain a surplus, the negotiation is about how to divide it up; each bargainer tries to get more for herself and leave less for the others. This may appear to be zero-sum but, in fact, if the agreement is not reached, no one will get any surplus at all. This alternative, is what creates the potential for the threats that make bargaining such a strategic matter.

Bargaining situations always begin considering the partition of a cake of size $\pi$ and the interaction of two individuals that have a common interest to trade, but at the same time, they have conflicting interests over the size of the cake each one wants to take, over the price at which to trade and over the share of the profit to each part following the bargaining process; for example, the seller would like to trade at a higher price, while the buyer would like to trade at a lower price. Any exchange situation is a bargaining situation. Stated in general and broad terms, a bargaining situation is a situation in which two players (individuals, firms, countries or a combination in between) have a common interest to cooperate but have conflicting interest over exactly how to cooperate. To put it differently, the players can mutually benefit from reaching agreement on an outcome from a set of possible outcomes (that contains two or more elements) but have conflicting interests over the set of outcomes.

In general, there are two main reasons for studying bargaining situations. The first reason is practical and includes many important and interesting human (economic, social and political) interactions that can be analyzed as bargaining situations; so, exchange situations (which characterize much of human economic interactions) are bargaining situations. The second reason is mostly theoretical in the sense that understanding such situations is fundamental to the development of the economic theory of markets.

The main issue that confronts the players in a bargaining situation is the need to reach agreement over exactly how to cooperate, before they actually cooperate (and obtain the fruits of that cooperation). Each player would like to reach some agreement rather than disagree and not to reach any agreement. But, at the same time, each player would like to reach an agreement that is favorable to him/her as possible. It is thus extremely possible that the players will strike an agreement only after some costly delay, or indeed fail to reach any agreement as it is witnessed by the history of disagreements and costly delayed agreements in many real-life bargaining situations.

Bargaining is any process through which the players on their own try to reach an agreement. This process is typically time consuming and involves the players making offers and counter-offers to each other. If the players get a third party to help them determine the agreement, this means that the agreement is not reached via bargaining but, for instance, via some arbitration process. The theory presented in this paper is a first attempt of studying bargaining situations in the oil industry and does not take into account an arbitration process even if it is briefly discussed.

A main focus of any theory of bargaining is on the efficiency and distribution properties of the outcome of bargaining. The former property relates to the possibility that the bargaining outcome is not Pareto efficient; this situation could arise, for example, if the players fail to reach an agreement or because they reach agreement after some costly delay. The delay is not only time consuming but mostly inefficient and money wasted time consuming. The distribution property, on the other hand, relates to the issue of exactly how the fruits of cooperation are divided between the players; i.e. how the gains from trade are divided.

In the oil industry it can be assumed that every bargaining process is not frictionless even if from the point of view of a country as a whole, this assumption conduces us to consider that players must incur on several costs during the bargaining process. The fact that a country is bargaining with several oil companies at the same time can be considered as a bargaining process pair by pair separately. However, Ambrus and En Lu (2008) present a dynamic model with multilateral bargaining situations where the players are involved in several bargaining processes. The model can be enriched by including the presence or more information in the bargaining process when a country is bargaining in several fronts at the same time. In the literature of bargaining this is known as the access of outside and/or inside options. If these outside options signify a gain for one of the players, the negotiations (almost) always could terminate forever in disagreement.

Another basic source of cost incurred by a player while bargaining comes from the possibility that the negotiations might randomly and exogenously breakdown in disagreement. Even if the probability of such an occurrence could be small, it nevertheless provides some friction in the bargaining process and as such may provide appropriate incentives to the players to compromise and reach an agreement. The Rubinstein's model considered later, takes into account this possibility of breakdown considering an offer and counter-offer process in a bargaining situation.

A bargaining situation is a game situation in the sense that the outcome of a bargaining process depends on both players' bargaining strategies whether or not an agreement is struck and the terms of agreement (if one is struck) depends on both players' actions during the bargaining process.

**III. Nash, Edgeworth and the partition of a cake (3)**

A bargaining solution may be interpreted as a formula that determines a unique outcome for each bargaining situation in some class of bargaining situations. Two players, $A$ and $B$, bargain over the partition of a cake of size $\pi$ , where $\pi > 0$. The set of possible agreements is

$$X = \{(x_A, x_B): 0 \leq x_i \leq \pi \text{ so } \pi = x_A + x_B\}$$

where $x_i$ is the share of the cake to player $i$. For each $x_i \in [0, \pi]$, $U_i(x_i)$ is player $i's$ utility from obtaining a share $x_i$ of the cake, where player $i's$ utility function $U_i : [0, \pi] \rightarrow \Re$ is strictly increasing and concave. If the players fail to reach an agreement, then player $i$ obtains a utility of $d_i$ where $d_i \geq U_i(0)$. There exists an agreement $x \in X$ such that $U_A(x) > d_A$ and $U_B(x) > d_B$ which ensure that there exists a mutually beneficial agreement. The utility pair $d = (d_A, d_B)$ is called the disagreement point. In order to define the Nash bargaining solution of this bargaining situation, it is useful to first define the set $\Omega$ of possible utility pairs obtainable through agreement; $\Omega$ denotes the two maps of indifference curves belonging to players A and B; i.e. the Edgeworth box. The set of bargaining situations can be expressed as

$$\Omega = \{(u_A, u_B): \text{there exists } x \in X \text{ such that } U_A(x_A) = u_A \text{ and } U_B(x_B) = u_B\}$$

Fix an arbitrary utility $u_A$ to player $A$ where $u_A \in [U_A(0), U_A(\pi)]$. From the strict monotonicity of $U_i$, there exists a unique share $x_A \in [0, \pi]$ such that $U_A(x_A) = u_A$; i.e. $x_A = U_A^{-1}(u_A)$ denotes the inverse of $U_A$. Hence

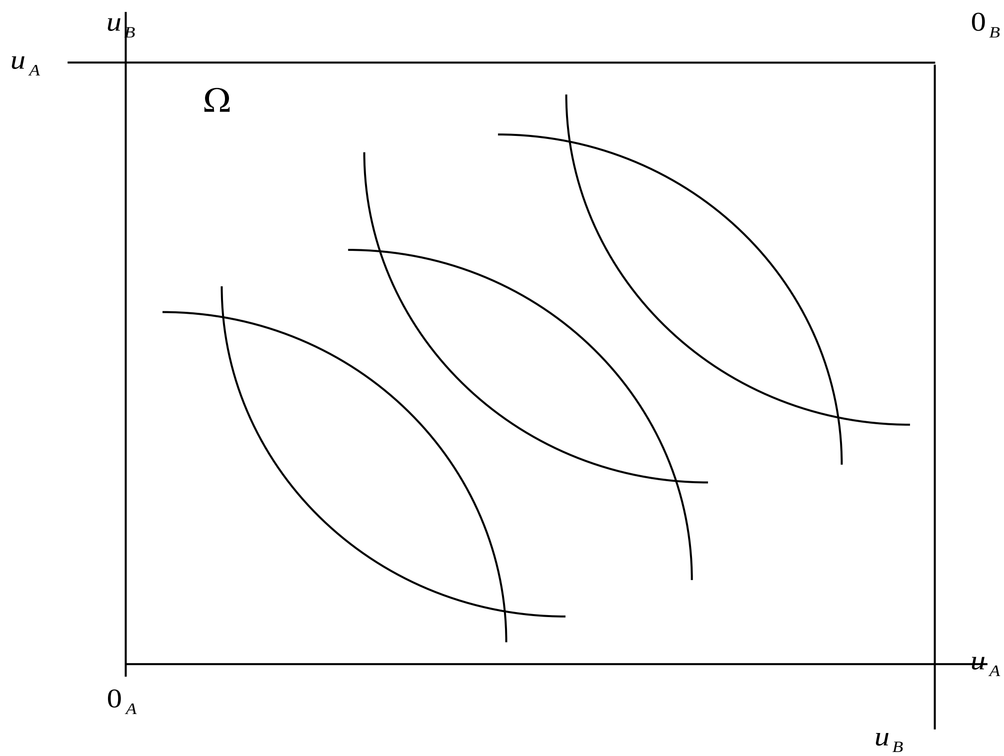
$$g(u_A) \equiv U_B\left(\pi - U_A^{-1}(u_A)\right)$$

is the utility player $B$ obtains when player $A$ obtains the utility $(u_A)$. This way of expressing the player $B$'s utility function does not mean that player $A$ is the first player to show his/her utility function followed by player $B$, it is only a manner to obtain player $B$'s utility in function of player $A$'s utility function; the same could be obtained beginning with player $B$'s utility function and after the player $A$'s utility function. Continuing the reasoning, it follows that

$$\Omega = \{(u_A, u_B): U_A(0) \leq u_A \leq U_A(\pi) \text{ and } u_B = g(u_A)\}$$

that is, $\Omega$ is the graph of the functions $U_A$ and $g:[U_A(0), U_A(\pi)] = U_B \rightarrow \Re$. Again, $\Omega$ represents the Edgeworth box for the utility functions of the two players as it is shown in Figure 1.

**Figure 1. The basic Edgeworth box.**




I assume now that the doting point expressed in the Edgeworth box is equal to the $d_i$ utilities when the players fail to reach an agreement and where, as it was early noted, $d_i \geq U_i(0)$.

The solution must be located over the $\pi$ curve showed in Figure 2. signaling the places where the indifference curves are tangent over the path in "climbing" the utility hill. Figure 2. shows the path of the utility functions for each player, adding the $d_i$ lines.

The Nash bargaining solution (NBS) of the bargaining situation described above is the unique pair of utilities denoted by $(u_A^N, u_B^N)$ that solves the following maximization problem

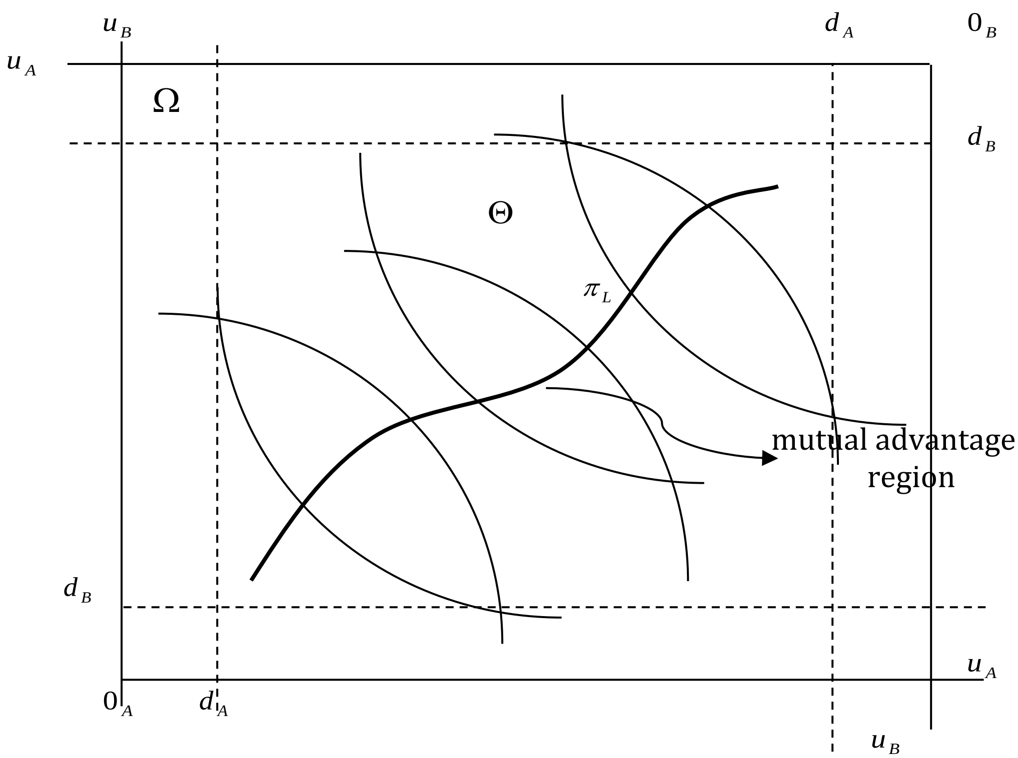
$$\max_{(u_A, u_B) \in \Theta} (u_A - d_A)(u_B - d_B)$$

where

$$\Theta \equiv \{(u_A, u_B) \in \Omega : u_A \geq d_A \text{ and } u_B \geq d_B\}$$
$$\Rightarrow \Theta \equiv \{(u_A, u_B) : U_A(0) \leq u_A \leq U_A(\pi);\ u_B = g(u_A); u_A \geq d_A \text{ and } u_B \geq d_B\}$$

This maximization problem is represented by the $\pi_L$ line in Figure 2.

**Figure 2. The Edgeworth box and the $\pi$ path**




In a frictionless situation, the solution could be found everywhere along the $\pi_L$ line (after we will see that $\pi_L$ becomes $\pi^N$ because it represents the optimal Nash bargaining solutions). Now I introduce the cost of bargaining in the form of a budget constraint, this constraint can express a cost, a price, a time consuming, another related variable or a mixed situation. So the restriction to the maximization problem can be stated as

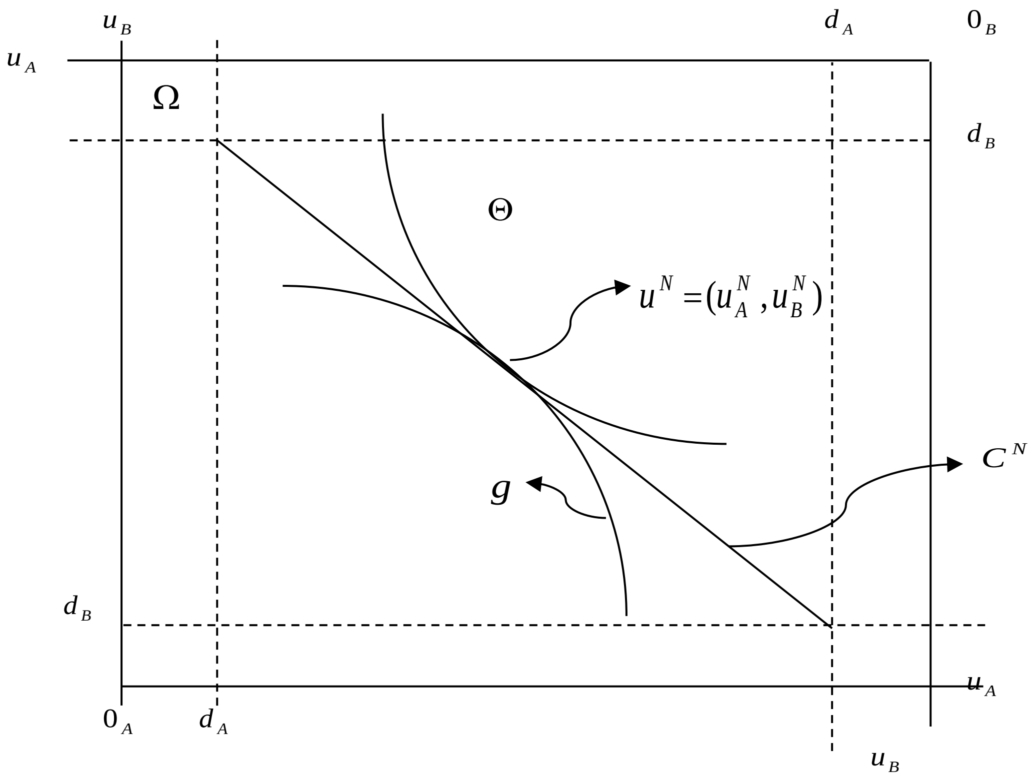
$$u_A d_A + u_B d_B = C^N$$

without the budget constraint there exists a continuum of utility pairs $(u_A, u_B) \in \Theta$, such that $u_A > d_A$ and $u_B > d_B$ over the $\pi_L$ line.

The maximization problem stated above including the budget constraint has a unique solution because the maximand $(u_A - d_A)(u_B - d_B)$ which is referred to as the Nash product is continuous and $g$ (the utility function of player *B*) is strictly decreasing and concave (for player *A*) and the set $\Theta$ is not empty.

Figure 3. illustrates the NBS; since $u_A^N > d_A$ and $u_B^N > d_B$, in the NBS the players reach agreement on $(x_A^N, x_B^N) = \left(U_A^{-1}(u_A^N), U_B^{-1}(u_B^N)\right)$.

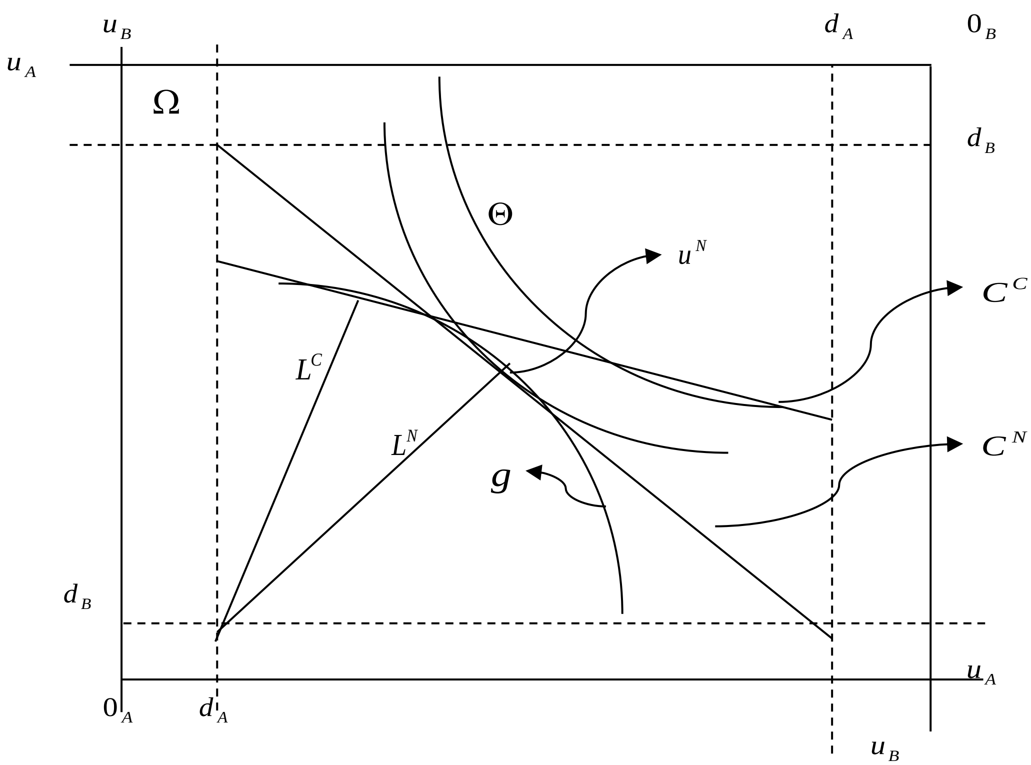
**Figure 3. The Edgeworth box and the Nash bargaining solution**




The Nash bargaining solution is the unique point $u^N$ on the graph of $g$ with the property that the slope of the line joining the points $u^N$ and $d$ is equal to the absolute value of the slope of the unique tangent to the graph of $g$ at $u^N$. Let us consider now any point $u$ on the Figure 4. to the left of $u^N$. The slope of the line $C^C$ is flatter than the $C^N$ one, whereas the slope of the line $L^C$ joining points $d$ and $u$ has increased relative to the slope of the line $L^N$, while the absolute value of the slope of the tangent $C^N$ to the graph of $g$ at $u$ has decreased relative to the absolute value of the slope on the tangent $C^N$. Therefore, the slope of $L^C$ is strictly greater than the absolute value of the slope of $C^C$. By a symmetric argument, it follows that the slope of the line joining the point $d$ with a point on the graph of $g$ to the right of the Nash bargaining solution represented by $L^N$ is strictly less than the absolute value of the slope of the tangent to the graph of $g$ at that point.

While Muthoo (2002) explains this changes only by movements over $g$, with the help of the Edgeworth box and the introduction of the budget constraint, I can explain not only these movements but the fact that even if it possible to attain points outside the $L^N$ path, these points will not be bargaining optimal solutions in the sense of Nash. Bargained solutions outside the $L^N$ path are possible when considering corner solutions and different types of indifference curves. Moreover, Muthoo considers function $g$ depending on $u_A$ that could be misleading, using the Edgeworth's box it is not necessary to consider this causal relation but every utility function separately.

**Figure 4. Nash, Edgeworth and other bargaining solutions**




## IV. The Rubinstein model

From now on and taking into consideration the Rubinstein's model, I include the possibility of a bargaining procedure: the players take turns to make offers and counter-offers to each other until the agreement is achieved. Rubinstein's model provides several insights about bargaining situations. One insight is that frictionless bargaining processes are indeterminate. A bargaining process may be considered "frictionless" if the players do not incur any costs by haggling (i.e. by making offers and counter-offers) in which case there is nothing to prevent them from haggling for as long as they wish. It seems intuitive that for the players to have some incentive to reach agreement, they should find it costly to haggle. Another insight is that a player's bargaining power depends on the relative magnitude of the player's respective cost of haggling with the absolute magnitudes of these costs being irrelevant to the bargaining outcome.

### *IV.1 The basic Rubinstein's model and the Edgeworth's box again*

Again, two players, *A* and *B*, bargain over the partition of a cake of size $\pi$ (where $\pi > 0$) according to the following, alternating offers procedure. At time 0 player *A* makes an offer to player *B*. An offer is a proposal of a partition of the cake. If player *B* accepts the offer, then agreement is struck and the players divide the cake according to the accepted offer. On the other hand, if player *B* rejects the offer, then he/she makes a counteroffer at time $t > 0$. If the counteroffer is accepted by player *A*, then agreement is struck. Otherwise, player *A* makes a counter-counteroffer at time $2t$. This process of making offers and counteroffers continues until a player accepts an offer.

Offers are made at discrete points in time: namely, at times $0, t, 2t, 3t, \ldots,$ where $\pi > 0$. An offer is a number greater than or equal to zero and less than or equal to $\pi$. It is useful to adopt the convention that an offer is the share of the cake to the proposer, and therefore $\pi$ minus the offer is the share to the responder. At time $nt$ when $n$ is even (i.e., *n* = 0, 2, 4, …) player *A* makes an offer to player *B*. If player *B*, accepts the offer, then the negotiation ends with agreement. On the other hand, if player *B* rejects the offer, then $n$ times units later, at time $(n+1)t$, player *B* makes an offer to player *A*. If player *A* accepts the offer, then the negotiations end with agreement. If the offer is not accepted, i.e. if player *A* rejects the offer, then $n$ time units later, at time $(n+2)t$, player *A* makes an offer to player *B*, and so on. The negotiations end if and only if a player accepts an offer.

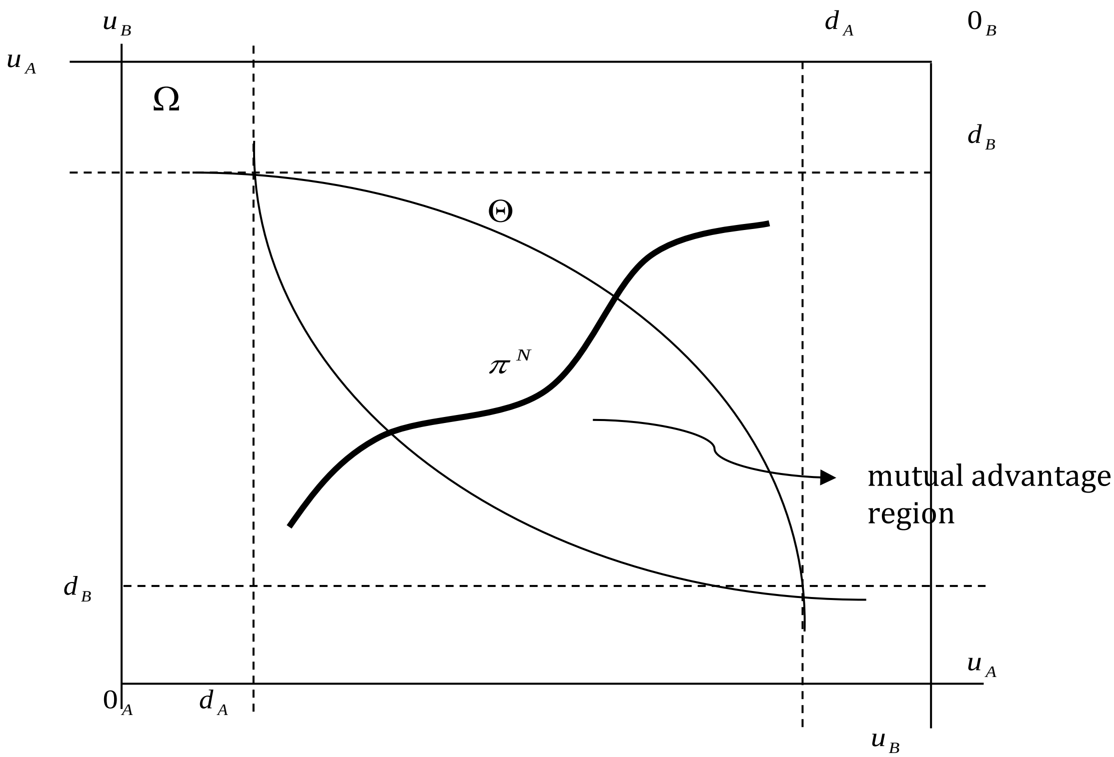
The payoffs are as follows. If the players reach agreement at time $nt$ (i.e., *n* = 0, 1, 2, …) on a partition that gives player $i$ $(i = A, B)$ a share $x_i \, (0 \le x_i \le \pi)$ of the cake, then player $i's$ payoff is

$$x_i \exp(-r_i \, nt)$$

where $r_i > 0$ is player $i's$ discount rate. On the other hand, if the players perpetually disagree (i.e., each player always rejects any offer made to him/her), then each player's payoff is zero, even if his/her utility can be positive (remember that $d_i > 0$). Figure 5. shows that the bargaining process following the Rubinstein's model over the alternating offers and counteroffers could take place over the path $\pi^N$ and inside the mutual advantage region. In this area the two players could climb their respective utility hills being gaining both of them additional utilities. However, is also clear that the two can decide for the point $(d_A, d_B)$ where there is some utility for the players but no payoff. Note, at the same time, that the $(d_A, d_B)$ point position becomes crucial when considering the offers and counteroffers of the two players. If the $(d_A, d_B)$ lines in Figure 5. are closer to the top of the respective utility hills, then the possibility of a breakdown becomes more probable. The Edgeworth's box, the Rubinstein's model and the bargaining equilibrium working together are more useful when considering the offer-counteroffer model. This result is not attainable considering only Muthoo's (2002) model that becomes too restrictive for the analysis.

This completes the description of the basic alternating-offers game. For notational convenience, define $\delta_i \equiv \exp(-r_i\, t)$ where $\delta_i$ is player $i's$ discount factor. Notice that $0 < \delta_i < 0)$

**Figure 5. Models mixing**




## V. Risk of breakdown

While bargaining the players may perceive that the negotiations might break down in a random manner for one reason or another. a potential cause for such a risk of breakdown is that the players may get fed up as negotiations become useless and/or protracted, and thus walk away from the negotiation meetings. This type of human behavior is random in the sense that the exact time at which a player walks away is random. Another possible cause for the existence of a risk of breakdown is that an outside intervention by a third party results in the disappearance of the gains from bargaining that exists in the first place between the two initial players. For example, while two firms bargain over how to divide the returns from the exploitation of a new crude oil or natural gas source, an outside firm may use its bargaining power, a certain type of bribery, or any other "available policy" to influence the decision about to continue or to stop the negotiation process. This kind of sources of breakdown are very difficult to model but certainly are at the same time very interesting to analyze.

### *V.1 A model with a risk of breakdown*

Two players (again and again), *A* and *B*, bargain over the partition of a cake of size $\pi$ (where $\pi > 0$) according to the alternating-offers procedure, but with the following modification:

immediately after any player rejects any offer at any time $nt$, with a probability $p$ (where $0 < p < 0)$, the negotiations break down in disagreement, and with probability $1-p$ the game proceeds to time $(n+1)t$ where the second player makes a counteroffer.

The payoffs are as follows. If the players reach agreement at time $nt$ ( $t$ = 0, 1, 2, 3, ... and $t > 0$ ) on a partition that gives player $i$ a share $x_i (0 \leq x_i \leq \pi)$ of the cake, then his/her payoff is $U_i(x_i)$, where $U_i : [0, \pi] \rightarrow \Re$ could be the von Neumann-Morgenstern procedure considering bayesian model behavior with utility functions and subjective probabilities. It is assumed that $U_i$ is strictly increasing and concave. If negotiations break down in disagreement at time $nt$, then player $i$ obtains a payoff of $b_i$, where $U_i(0) \leq b_i \; U_i(\pi)$. The payoff pair $(b_A, b_B)$ is called the breakdown point.

Since $b_i \in [U_i(0), U_i(\pi))$, there exists a number $z_i \in [0, \pi)$ such that $U_i(z_i) = b_i$; that is, $z_i = U_i^{-1}(b_i)$, where $U_i^{-1}(b_i)$ denotes the inverse of $U_i$ in the same way as it was developed in Section 3. Assume that $z_A + z_B < \pi$, which ensures that there exist mutually beneficial partitions of the cake.

If the players perpetually disagree, i.e., each player always rejects any offer made for the other, then player $i's$ payoff is

$$p_i \, b_i \sum_{n=0}^{\infty} (1-p)^n$$

which equals $b_i$. Thus, the point $(b_A, b_B)$ is a breakdown point.

At this point, it is useful to note that the model has become a very subjective one. The von Neumann-Morgenstern model cited above is very subjective because it needs subjective probabilities measures for being useful. Another way could take into account an estimation of a probit model, but it could be non practical for the probable lack of statistics. In this model uncertain and risk prospects are modeled as probability distributions over a given set of outcomes.

## VI. The model with asymmetric information

In some bargaining situations at least one of the players knows something of relevance that the other player does not. Bargaining models with asymmetric information fix well in situations when one of the player (or both of them) lacks skillfulness. A player may in general have private information about a variety of things that may be relevant for the bargaining outcome, such as preferences, outside and inside options. The payoff to each player (from trading) depends on the agreed price and on their own reservation value (4). A key assumption is that at least one player's reservation value is private information.

## VII. Contract theory

The theory here developed concerns bargaining and contract situations in which the outcome is determined entirely via bargaining process. In this first attempt, the role of arbitrators and mediators in helping the players reach agreement is not yet considered; at the same time, the benchmark is one between two parties who operate in a market economy with a well-functioning legal system where any contract the parties decide to write will be enforced perfectly by a court provided that it does not contravene any existing laws. It will be assumed that the contracting parties do not need to worry about whether the courts are able or willing to enforce the terms of the contract precisely. Judges are perfectly rational individuals whose only concern is to stick as closely as possible to the agreed terms of the contract. The penalties for breaching the contract will be assumed to be sufficiently severe that no contracting party will consider the possibility of not honoring the contract. Thus, only the economic aspects of the contract will be considered.

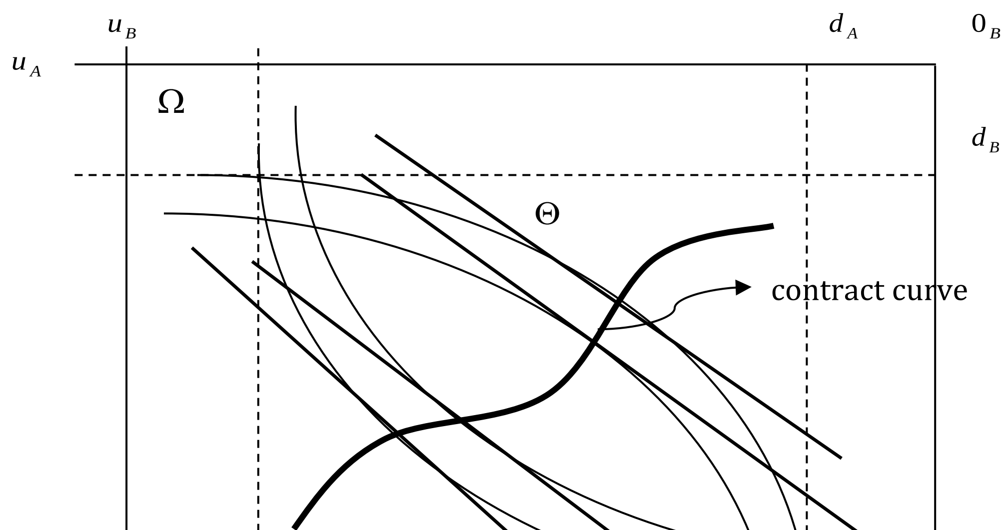
Let's begin considering the existence of only two types of fuels *1* and *2* for both players *A* and *B* in a bargaining situation. So, they face similar constraints given by:

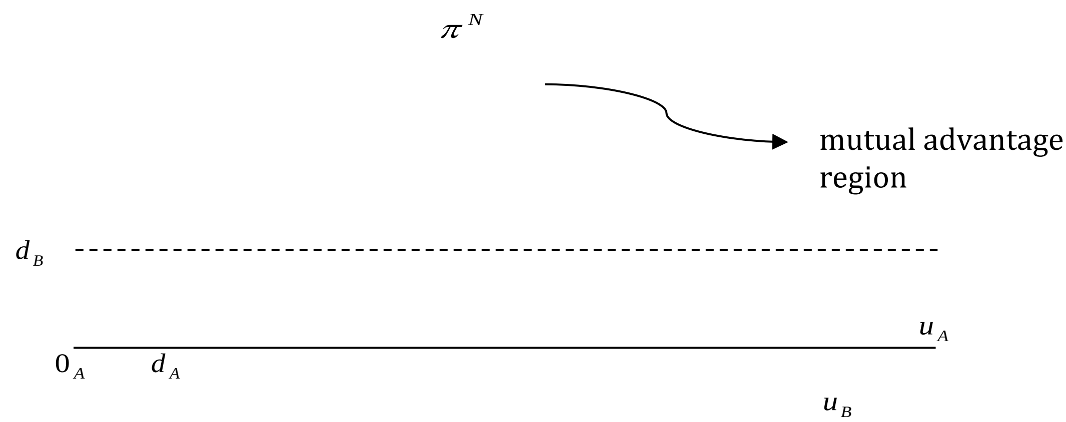
$$P_1\, Q_1^A + P_2\, Q_2^A \;=\; I_A$$

$$P_1\, Q_1^B + P_2\, Q_2^B \;=\; I_B$$

The difference between both players rests in their respective utility functions and in the pondered quantities each one will demand.

**Figure 5. The simple contract curve**






The contract curve contains all the Pareto-efficient allocations for which player's indifference curves are tangent, every point on the curve is efficient because one player cannot be made better off without making the other player worse off. Any bundle for which player *A's* indifference curve is tangent to player *B's* indifference curve lies on the contract curve, because no further trade is possible, so we can't reallocate goods to make one of them better off without harming the other. Starting at any endowment point, players A and B will trade to a bundle on the contract curve in the mutual advantage region.

Now, it is possible to derive the contract curve assuming that player *A's* utility function be $U_A(q_{A1}, q_{A2})$ where $q_{A1}$ is the amount of fuel 1 and $q_{A2}$ the amount of fuel 2. Similarly, player *B's* utility function is $U_B(q_{B1}, q_{B2})$. It is possible to determine the bundle that maximizes player *B's* wellbeing $U_B(q_{B1}, q_{B2})$ given that A's utility is constant at $\overline{U_A} = U_A(q_{A1}, q_{A2})$.

Using a Lagrangian multiplier, $\lambda$, we can write the Lagrangian corresponding to the maximum problem as:

$$\mathcal{L} = U_B(q_{B1}, q_{B2}) + \lambda \left[U_A\left(q_1 - q_{i1}, q_2 - q_{j2}\right) - \overline{U_d}\right] \qquad 6.1$$

where $q_1 = q_{A1} + q_{B1}$ is the pondered demanded amount of fuel 1 and $q_2$ is the pondered demanded amount of fuel 2. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q_{i1}} = \frac{\partial U_B}{\partial q_{i1}} = \lambda \frac{\partial U_A}{\partial q_{i1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial q_{i1}} = \frac{\partial U_B}{\partial q_{i1}} = \lambda \frac{\partial U_A}{\partial q_{i1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = U_A\left(q_1 - q_{i1}, q_2 - q_{i2}\right) - \overline{U_A} = 0$$

if we equate the right-hand sides of the two first equations, we have

$$MRS_B = \frac{\partial U_B / \partial q_{B1}}{\partial U_B / \partial q_{B2}} = \frac{\partial U_A / \partial q_{A1}}{\partial U_A / \partial q_{A2}} = MRS_A$$

That is, player *B's* marginal rate of substitution equals player *A's* marginal rate of substitution at an optimal bundle. In geometric terms, this condition says that B's indifference curve is tangent to *A's* indifference curve along the contract curve.

Actually, the determination of gas price for exporting is estimated by the following formulae:

$$GP = IP * \left(0{,}50 \frac{F1_n}{F1_0} + 0{,}25 \frac{F2_n}{F2_0} + 0{,}25 \frac{F3_n}{F3_0}\right)$$

where:
$GP$ is the gas price
$IP$ is the international price for every fuel in the basket, and
$Fi$ is the fuel multiplied by the estimated pondered weight

This gas price for exporting formulae is similar to the respective constraint faced by each player.

**Conclusion**
Even that bargaining and contract theories have had enormous theoretical developments, unfortunately, these attempts are not contrasted empirically. The model presented here is far from finished and it rests to intent a "real" application to the contracts design between the two players involved in the negotiation, i.e., a country and an enterprise. Asymmetric information rests, at the same time, an issue that must be taken into account in the following attempts.

The application of a Neumann-Morgenstern model when analyzing the risk of breakdown adds even more subjective estimation, this problem could be solved estimating a probit model but, this one at the same time, could possess another problem because of the probable lack of statistics or when they are available it rests to consider a probable contamination of these same statistics.

The Edgeworth box is very helpful when trying to explain the behavior of the two players and it rest to translate this model to the "real" behavior of the two players that bargain over the partition of a cake of size $\pi$ . When a certain model begins with the assumption of the existence of a function of utility, even if this assumption is very subjective, it does not mean that the model cannot explain the reality. However, it is clear that this issue must be taken into its real dimension when trying to carry the model to a more "real" level.

The contract curve can be estimated in an empirical way according to ARCH (Auto-Regressive Conditional Heteroscedastic) and GARCH models. It rests to apply the bargaining model results in an epistemological manner, a not simple task. The probability to include another considerations and behaviors such as risk, honesty, reputation and other issues, is enormous.

**Notes**

(1) The bargaining process could be enriched by taking into account these situations of bargaining over an insurance contract but, unfortunately, are not considered by Burger's model.

(2) This fact, the intergenerational negotiation could be very interesting even if too general, but, again, is not considered in the paper.

(3) This section is inspired in Muthoo (2002) Chapter 2. The Nash Bargaining Solution (pags 9-40), and Hirshleifer (1988) Chapter 13. The Benefices of Exchange, the Costs of Transaction and the Money Function (pags 442-491). The mix belongs to the author who is responsible for any misleading and misperception.

(4) The buyer's reservation value is the maximum price at which the buyer is willing to buy. By a same reasoning, the seller's reservation value is the minimum price at which the seller is willing to sell.

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