Jumps and Jolts: A Continuous-Time Model for Electricity Forward Contract Pricing

,

,

This paper addresses the limitations of traditional power derivative pricing models, which inadequately account for spikes on forward contracts dynamics. We propose a continuous-time model that incorporates both jump processes and a time-varying drift to capture shifts in hedging supply and demand. The model also includes correlated Brownian motions to reflect common shocks affecting contracts with different delivery periods. Using the generalized method of moments (GMM), we estimate model parameters using daily settlement prices of Norwegian forward electricity contracts. Simulation exercises validate the reliability of the parameter estimators. Furthermore, we demonstrate the model's improved forecasting accuracy compared to an ARMA-GARCH model, highlighting its potential value for traders and risk managers in portfolio management.

Keywords: Power derivatives; Jumps; GMM.

JEL codes: C58, C22, G13.

1. Introduction

Since Merton's seminal contribution, empirical research in asset pricing has consistently documented the presence of jumps across a wide range of asset classes, including commodities ((Chevallier and Ielpo, 2014)), exchange rates ((Erdemlioglu et al., 2015)), and equities ((Bollerslev et al., 2008)), among others.

Moreover, (Alexeev et al., 2019) provide evidence that neglecting jumps in portfolio management can lead to under-diversification and increased exposure to extreme events. However, existing models for pricing power derivatives often fail to adequately capture such rare events. This study addresses this limitation by proposing and estimating a model that more effectively captures jumps in forward electricity returns.

Forward electricity contracts are vital tools for energy companies to hedge against market volatility (see (Frestad, 2012); (Sanda et al., 2013)). Unlike

 https://doi.org/10.xxxx/xxxxxxxx

most other commodities, electricity is non-storable and offers limited opportunities for energy conversion ((Bessembinder and Lemmon, 2002)). Nevertheless, the two primary modeling approaches employed by practitioners and researchers (see (Benth et al., 2008); (Deschatre et al., 2021)) often overlook the fact that contracts with different maturities may exhibit distinct statistical properties or ignore the occurrence of rare events.

The traditional approach to modeling power-based derivatives assumes that spot electricity prices follow a stochastic differential equation (SDE), with forward prices derived using no-arbitrage arguments (see (Deng, 2000); (Lucia and Schwartz, 2002); (Benth et al., 2014); (Gudkov and Ignatieva, 2021)). However, (Geman and Roncoroni, 2006) and (Nomikos and Soldatos, 2008) emphasize that spot and forward electricity prices differ in their statistical characteristics—such as jump frequency, volatility behavior, and the possibility of negative prices ((Keles et al., 2012))—casting doubt on the effectiveness of SDE-based spot models in capturing forward price dynamics.

In contrast, (Clewlow and Strickland, 1999) and (Rasmussen and Stensland, 2000) advocate for modeling forward prices directly. (Benth and Koekebakker, 2008) further formalizes the necessary conditions such models must satisfy to ensure arbitrage-free pricing. Nonetheless, much of the empirical work based on this framework focuses primarily on volatility and disregards the presence of jumps (see (Rasmussen and Stensland, 2000); (Benth and Koekebakker, 2008); (Blanco et al., 2018); (Algieri et al., 2021)).

A notable exception is (Callegaro et al., 2022), who employ a Hawkes process (see (Hawkes, 2018)) to model jump behavior in forward prices. However, their model does not account for other important features of power forward contracts, such as a time-to-maturity risk premia, as considered in (Diko et al., 2006) and (Benth and Paraschiv, 2018).

To address these gaps, we propose a continuous-time model that incorporates both jumps and a time-varying drift term to reflect fluctuations in the supply and demand for hedging, in line with (Benth and Paraschiv, 2018). Furthermore, the model includes correlated Brownian motions to capture the influence of common shocks across contracts with different delivery dates.

We estimate the model using the generalized method of moments (GMM) introduced by (Hansen, 1982), chosen for its flexibility and computational efficiency, as it avoids strong distributional assumptions and the complexity of simulation-based estimation. Through simulation exercises, we demonstrate the robustness of our parameter estimates. As a secondary contribution, we provide a novel set of moment conditions that may be valuable for continuous-time models incorporating rare events.

Our empirical analysis uses daily settlement prices for Norwegian quarterly and annual forward electricity contracts traded on NASDAQ OMX. This dataset is chosen deliberately: (Sanda et al., 2013) identify these contracts as primary hedging instruments for Norwegian energy firms, and (Paulsen and Bergsholm, 2020) highlights the Nordic electricity derivatives market as one of the most liquid globally.

We also compare the forecasting performance of our model against a benchmark ARMA-GARCH model, showing that our approach can offer improved predictive accuracy and practical value for traders and risk managers.

2. Literature review

The wave of privatization and deregulation between 1980 and 2000 fundamentally transformed electricity markets, exposing participants to significant price volatility and prompting the emergence of power-based derivatives as essential hedging instruments (Bacon, 1995; Knittel and Roberts, 2005). Traditional methods for valuing commodity derivatives—such as those developed by (Gibson and Schwartz, 1990) and (Miltersen and Schwartz, 1998)—are not directly applicable to electricity, primarily due to its non-storability (Bessembinder and Lemmon, 2002).

To overcome this limitation, several studies have modeled the dynamics of spot electricity prices using stochastic differential equations (SDEs) (Deng, 2000; Lucia and Schwartz, 2002; Cartea and Figueroa, 2005; Pirrong, 2011; Füss et al., 2015). While empirical evidence suggests that spot-based models can generate reliable estimates of time-varying forward premia and accurately reflect spot price evolution (Wei and Lunde, 2023; Laudagé et al., 2024), critics argue that this approach has limitations. Notably, (Geman and Roncoroni, 2006) and (Nomikos and Soldatos, 2008) point out that spot and forward electricity prices often exhibit different statistical characteristics, making it unlikely that solutions derived from SDEs under no-arbitrage conditions accurately capture the true distribution of market-traded forward prices.

An alternative approach focuses on modeling forward prices directly under a risk neutral measure rather than deriving them from spot prices. This methodology, grounded in the framework introduced by (Heath et al., 1992), has been adopted by researchers such as (Clewlow and Strickland, 1999), (Rasmussen and Stensland, 2000), and (Benth and Koekebakker, 2008) and according to (Deschatre et al., 2021) it became the most popular approach among market practicioners. Moreover, (Hinz et al., 2005) shows that this framework allows for an equilibrium between forward and capacity markets.

Initial implementations of this framework typically relied on one-factor models to describe the forward curve. However, these models have been criticized for their inability to capture the complex volatility structures observed in power derivatives ((Fanelli et al., 2016)). In response, more recent work has introduced multi-factor models that incorporate calendar effects, seasonality, and the Samuelson effect (Kiesel et al., 2009; Fanelli et al., 2016). Although such models offer improvements in option pricing—especially in addressing volatility smiles—they remain empirically under-tested in contexts involving forward electricity trading or practical risk management ((Kiesel et al., 2009; Fanelli et al., 2016; Fanelli and Schmeck, 2019)).

To further enhance model realism, some studies have proposed more sophisticated drift structures that reflect economic factors such as hedging pressure[1] . For instance, (Di Poto and Fanone, 2011) introduce a multifactor model where drift exhibits seasonal dynamics and the risk factors follow Lévy processes. Similarly, (Benth and Paraschiv, 2018) suggest modeling both drift and volatility as functions of time-to-maturity. Another variation, presented by (Blanco et al., 2018), defines the equilibrium price of specific market segments as the central tendency in an SDE, with stochastic fluctuations driven by Lévy processes.

Despite their theoretical soundness, these models face two main limitations. First, their multi-step estimation procedures can introduce inefficiencies in parameter inference. Second, while many attempt to capture price spikes through heavy-tailed distributions, multiple risk factors, or time-varying volatility, such complexity increases the risk of misspecification. Moreover, the effect of these features often depends on the forecast horizon.

A more recent contribution by (Callegaro et al., 2022) addresses some of these challenges using Hawkes processes to capture jump behavior in a multifactor model. Their approach shows promise in identifying price spikes, supported by Kolmogorov-Smirnov tests. However, it lacks rigorous validation of its jump identification method and relies on separate estimation procedures, potentially leading to inefficiency. Additionally, it does not integrate the time-to-maturity dependence of drift highlighted in (Benth and Paraschiv, 2018).

Augmenting these modeling efforts, (Algieri et al., 2021) study electricity futures risk premia by extracting real-world and risk-neutral densities from options traded on the EEX between 2010 and 2017. Assuming a Heston model under the risk-neutral measure, they achieve closed-form expressions for densities and replicate smile effects. While their statistical validation is

[1] See (Hirshleifer, 1991) for a description of this phenomenon and (Basu and Miffre, 2013) and (Stødal et al., 2023) for empirical evidence of the presence of this phenomenon on power derivatives.

comprehensive, the practical implications for trading—such as applications in value-at-risk or expected shortfall—are not explored. Furthermore, they do not benchmark their model against alternative frameworks, including those of (Kiesel et al., 2009), which also yield closed-form solutions for option valuation.

3. Model

The models discussed in Section (2) suggest that, with the exception of (Callegaro et al., 2022), the spikes observed in electricity forward returns are typically addressed through one of three approaches: (i) assuming that model residuals follow a heavy-tailed distribution, (ii) presuming that jumps in the spot market are perfectly transmitted to forward markets, or (iii) employing stochastic volatility models. However, the first approach is prone to specification errors, the second lacks empirical support (see Geman and Roncoroni, 2006; Nomikos and Soldatos, 2008), and the third, as noted by (Eraker et al., 2003), often fails to adequately capture sudden and short-lived market disruptions.

To address these limitations, we propose a model defined by equations (1)–(4), where parameters are estimated using the generalized method of moments (GMM) developed by (Hansen, 1982).

Let $P_{i,t}$ denote the forward electricity price of contract i at time t. This price is modeled as the sum of a deterministic component, $\Lambda_{i,t}$, and a stochastic component, $X_{i,t}$:

$$\ln P_{i,t} = \Lambda_{i,t} + X_{i,t}, \tag{1}$$

The deterministic component, $\Lambda_{i,t}$, reflects the long-term equilibrium price relevant for hedging purposes—potentially corresponding to the equilibrium forward price described by (Bessembinder and Lemmon, 2002). In contrast, the stochastic component, $X_{i,t}$, captures all sources of uncertainty, including those related to hedging demands and forward risk premia.

The stochastic component $X_{i,t}$ evolves according to:

$$dX_{i,t} = (\alpha_i \left(\frac{V_t}{252}\right) + \beta_i X_{i,t})dt + \sigma_i dW_{i,t} + G_{i,t} dN_{i,t} \tag{2}$$

V_t is a deterministic, monotonically decreasing variable representing the number of business days remaining until the rollover of the contract, bounded between 1 and 252; $dW_{i,t}$ denotes the Brownian motion associated with forward contract i at time t, $G_{i,t}$ is an independent Gaussian variable representing the jump size of forward contract i at time t, while $dN_{i,t}$ captures the jump process of forward contract i at time t:

$$P(dN_{i,t} = 1) = \lambda_i dt, \tag{3}$$

$$P(dN_{i,t} > 1) = o(dt) \tag{4}$$

The jump dynamics follows a Poisson process with intensity λ_i. Moreover,

$$E(dW_{i,t}dW_{j,t}) = \rho_{i,j}dt, \; i \neq j \tag{5}$$

where $\rho_{i,j}$ is the correlation between forward contracts i and j and α_i, β_i, σ_i are constants in the model. Finally $dW_{i,t}$ and $dN_{i,t}$ are independent stochastic process.

In our model, the drift term consists of two components: one that is inversely proportional to the time-to-maturity, and another that is proportional to the stochastic component. The rationale behind this specification is twofold. First, it allows the model to capture the relationship between time-to-maturity and the risk premium, as documented by (Benth and Paraschiv, 2018). Second, it reflects the idea that economic agents demand a return proportional to the level of uncertainty—captured by the stochastic component—when taking positions in forward electricity contracts, whether long or short.

Additionally, the jump component on our model allows us to capture the presence of skewness and heavy tails on the data, which according to (Arismendi Zambrano, 2019) are important features to describe the behavior of asset returns. Moreover, our model could potentially be used to price other derivatives such as options using methods described in (Glasserman, 2004).

One potential criticism of our model is that it does not explicitly incorporate the Samuelson effect. However, as we demonstrate empirically in Section 6, this phenomenon does not appear to be present in our data. A similar conclusion is reached by (Blanco et al., 2018), who adopt a constant volatility model to describe the dynamics of German forward electricity prices. Their findings indicate that, despite the widespread presence of heteroskedasticity in many commodity markets, assuming constant volatility can still yield a reasonable approximation in certain market environments—particularly when volatility remains relatively stable over time or when other components of the model, such as the drift, account for a substantial share of the observed variation. This provides additional support for our modeling approach, indicating that excluding the Samuelson effect is empirically justified given the characteristics of the data under analysis.

4. Empirical strategy

The first step we take in this work to achieve its objective is to establish that the forward curve is composed by the non-overlaping quarterly and one year ahead forward electricity contracts. We decide working with those delivery periods because according to Table 3 of (Sanda et al., 2013), quarterly and one year ahead forward electricity contracts are the main contracts that Norweigian energy companies use to hedge their positions.

In the second step of this work, we address the issue that our sample data are incompatible with the model proposed in section 6 due to being sampled at discrete time points. Additionally, we need to account for rolling over effects that may arise when we replace a contract at the end of the year.

To address these issues, we follow (Aït-Sahalia et al., 2015) and (Chen and Qi, 2024) and build a discrete version of it[2]. In addition, we use dummy variables to represent the long-term equilibrium prices, which will reflect the average price of hedging over a delivery period.

$$\ln P_{i,t} = \Lambda_{i,t} + X_{i,t}, \tag{6}$$

$$\Lambda_{i,t} = C_i + \sum_{h}^{h+\delta} \gamma_{i,h} D_{h,t} + \pi_i R_t \tag{7}$$

$$\Delta X_{i,t} = \alpha_i \left(\frac{V_t}{252}\right) \Delta t + \beta_i X_{i,t} \Delta t + \sigma_i \varepsilon_{i,t} + J_{i,t} \Delta N_{i,t}, \tag{8}$$

$$E(\Delta X_{i,t} \Delta X_{j,t}) = \sqrt{\sigma_i^2 \sigma_j^2} \rho_{i,j} \Delta t, i \neq j \tag{9}$$

[2]Check (Phillips and Yu, 2009) and (Sauer, 2011) for an analysis of different discretization schemes on the estimations of continuous time series models.

$$P(\Delta N_{i,t} = 0) = 1 - \lambda_i \Delta t, \tag{10}$$

$$P(\Delta N_{i,t} = 1) = \lambda_i \Delta t, \tag{11}$$

$$P(\Delta N_{i,t} > 1) = o(\Delta t) \tag{12}$$

$D_{h,t}$ is a dummy variable that takes the value 1 when h is equal to the same year as the time period t and 0 otherwise. R_t is another dummy variable that takes the value 1 when we roll over the contracts[3] and 0 otherwise. $\Delta t = \frac{1}{252}$, and $\varepsilon_{i,t}$ is a random Gaussian variable with mean 0 and standard deviation $\sqrt{\Delta t}$ for contract i at time t. $J_{i,t}$ is an iid normal random variable with mean μ_i and standard deviation σ_i^J, representing the jump size of contract i at time t. C_i, $\gamma_{i,h}$, and π_i are constants in the model.

Equations (6)–(12) indicate that there are 23 parameters to be estimated. Following the methodology of Benth and Paraschiv (2018) and Blanco et al. (2018), we divide the estimation procedure into two phases. In the first phase, we estimate the coefficients $\gamma_{i,j}$ by performing time series regressions of $\ln P_{i,t}$ on $D_{j,t}$.

In the second phase, we collect the residuals from the previous phase and use them to estimate the parameters α_i, β_i, σ_i, μ_i, σ_i^J, λ_i, and $\rho_{i,j}$ using the package gmm, which is implemented in R by (Chausse, 2010). According to (Tankov, 2003) and (Hall, 2015), the main advantage of GMM over maximum likelihood estimation (MLE) is that it relies solely on moment conditions to generate consistent estimators. Thus, we do not need to specify the complete probability distribution of the data.

(Hall, 2003), (Newey and Smith, 2004), and (Hall, 2015) show that the bias and consistency of GMM parameters depend on the choice of moment conditions and the weighting matrix. In the GMM literature, several econometric procedures are developed to address these questions (see (Andrews and Lu, 2001), (Liao, 2013), (Cheng and Liao, 2015), and (Hirukawa, 2023)), but there is no consensus in the literature on which approach is the best. Thus, we conduct several simulation exercises to provide evidence that our decisions can produce reliable estimators and that are reported on Section 5.

[3] We roll over all contracts when the yearly contract stops trading, which according to (Nasdaq, 2024) is after the third day before the delivery period.

4.1 Moment Conditions

In this work, we use the conditional moment conditions represented by equations (13)-(24) and estimate two sets of parameters simultaneously. This approach allows us to estimate the correlation between the prices, represented by the parameter $\rho_{i,j}$. Therefore, we use 23 equations to estimate 14 parameters of our model, satisfying the overidentifying condition of (Hansen, 1982).

$$E(\Delta X_{i,t}|X_{i,t},V_t) - (\alpha_i \frac{V_t}{252}\Delta t + \beta_i X_{i,t}\Delta t + \lambda_i \mu_i \Delta t) = 0 \tag{13}$$

$$E(\Delta X_{i,t}^2|X_{i,t},V_t) - E^2(\Delta X_{i,t}|X_{i,t},V_t) - (\sigma_i^2\Delta t + \lambda_i \Delta t(\mu_i^2 + (\sigma_i^J)^2) = 0 \tag{14}$$

$$E((\Delta X_{i,t} - E(\Delta X_{i,t}|X_{i,t},V_t))^4|X_{i,t},V_t) - (\lambda_i \Delta t(\mu_i^4 + 6\mu_i^2(\sigma_i^J)^2 + 3(\sigma_i^J)^4)) = 0 \tag{15}$$

$$E(\hat{\varepsilon}_{i,t,1}X_{i,t}|X_{i,t},V_t) = 0 \tag{16}$$

$$E(\hat{\varepsilon}_{i,t,1}(1 + X_{i,t}^2)|X_{i,t},V_t) = 0 \tag{17}$$

$$E(\hat{\varepsilon}_{i,t,2}X_{i,t}|X_{i,t},V_t) = 0 \tag{18}$$

$$E(\hat{\varepsilon}_{i,t,2}(1 + X_{i,t}^2)|X_{i,t},V_t) = 0 \tag{19}$$

$$E(\hat{\varepsilon}_{i,t,3}(1 + X_{i,t}^2)|X_{i,t},V_t) = 0 \tag{20}$$

$$E(I_{|\Delta X_{i,t}|>\tau_i}|X_{i,t},V_t) - (\lambda_i \Delta t + \theta_i) = 0 \tag{21}$$

$$E(\Delta X_{i,t} I_{|\Delta X_{i,t}|>\tau_i} | X_{i,t}, V_t) - (\mu_i \lambda_i \Delta t) = 0 \tag{22}$$

$$E(\hat{\varepsilon}^2_{i,t,4}) - (\lambda_i \Delta t(\mu_i^2 + (\sigma_j^J)^2)) = 0, \tag{23}$$

$$E(\Delta X_{i,t} \Delta X_{j,t} I_{|\Delta X_{i,t}|\leqslant\tau_i} | X_{i,t}, X_{j,t}, V_t) - \sqrt{\sigma_i^2 \sigma_j^2} \rho_{i,j} \Delta t = 0, \text{ where} \tag{24}$$

$$\hat{\varepsilon}_{i,t,1} = \Delta X_{i,t} - (\alpha_i \frac{V_t}{252} \Delta t + \beta_i X_{i,t} \Delta t + \lambda_i \mu_i \Delta t) \tag{25}$$

$$\hat{\varepsilon}_{i,t,2} = \Delta X^2_{i,t} - \Delta\bar{X}^2_{i,t} - (\sigma_i^2 \Delta t + \lambda_i \Delta t(\mu_i^2 + (\sigma_i^J)^2) \tag{26}$$

$$\hat{\varepsilon}_{i,t,3} = (\Delta X_{i,t} - \Delta\bar{X}_{i,t})^4 - (\lambda_i \Delta t(\mu_i^4 + 6\mu_i^2(\sigma_i^J)^2 + 3(\sigma_i^J)^4)) \tag{27}$$

$$\hat{\varepsilon}_{i,t,4} = \Delta X_{i,t} I_{|\Delta X_{i,t}|>\tau_i} - \lambda_i \Delta t \mu_i, \text{ where} \tag{28}$$

τ_i is the threshold for forward contract i, T is the length of our time series and θ_i is a nuisance parameter whose role is going to be explained below. Next, we will explain the motivating fundamental for each moment condition.

The equations (13), (14) and (15) represent, respectively, the first moment, variance and fourth centered moment. It is worth mentioning that (Aït-Sahalia et al., 2015) also uses similar condition to estimate parameters of a continuous time model with stochastic volatility and jumps.

(Aït-Sahalia et al., 2015) also propose to include a moment condition based on the covariance of Brownian motion to estimate correlation between their variables but in this work we also include an indicator function as shown in Equation (24). We implemented the indicator function to ensure $\rho_{i,j}$ 1 and 1, as our simulations showed more frequent violations without this safeguard.

The moment conditions represented by equations (17)-(20) are inspired by the work of (Chan et al., 1992), where the author uses residuals from the moment conditions to estimate a continuous time model that describes the behavior of the short-term interest rates in the USA. These conditions may represent the fact that past information cannot be used for predicting changes in the forward contract prices.

We build moment conditions described by equations (21)-(23) using non-parametric estimators for jump intensity and jump size shown[4] in (Mancini, 2004), (Mancini, 2009), and (Figueroa-López and Mancini, 2019). The key insight behind the construction of those estimators is that when the time lag between observations is small, the contribution of the continuous components becomes asymptotically negligible, so it becomes possible to disentangle jumps from diffusions[5].

We rely on proposition 3 from (Figueroa-López and Mancini, 2019) and the theorem 4.9 from (Çinlar, 2011) to establish moment condition given by Equation (21) because the former shows that when the threshold respects the inequality given by 29, then the convergence result represented by 30 is true and the latter guarantee that the first moment of $I_{|\Delta X_{i,t}|>\tau_i}$ will also converge to the first moment of $I_{\Delta N_{i,t}>0}$.

Proposition 1 ((Figueroa-López and Mancini, 2019, Prop. 3))**.**

Let $dX_t = a_t dt + \sigma_t dW_t + dJ_t$ be a jump-diffusion process with $J_t = \sum_{i=1}^{N_t} \gamma_i$ for a non-explosive counting process N and real-valued random variables γ_j, a, σ are càdlag and a.s. $\underline{\sigma}^2 := inf_{s\in[0,T]} \sigma_s^2 > 0$.

Assuming that $M_i(\omega) \in [\inf_{s\in[t_{i-1},t_i]} \sigma_s(\omega)^2, \overline{\sigma}^2]$, where $\overline{\sigma}^2 := sup_{s\in[0,T]} \sigma_s^2$, if

$$\tau_i > \sqrt{2M_i \Delta t \ln(\frac{1}{\Delta t})}, \tag{29}$$

holds, then

$$I_{|\Delta X_{i,t}|>\tau_i} \xrightarrow{\text{a.s.}} I_{\Delta N_{i,t}>0} \tag{30}$$

The moment conditions described by Equations (22) and (23) represent, respectively, the first and second moments of jump size. In order to establish

[4] It is worth mentioning that (Deschatre et al., 2020) also use results from (Mancini, 2004) to build estimators for the intensity parameter and mean reversion of jump process of spot electricity prices.

[5] (Aıt-Sahalia, 2004) uses a similar insight to explain why maximum likelihood estimation is capable of disentangling jumps from diffusions.

them we rely on results of (Mancini, 2004) and (Mancini, 2009), who show that jump size estimator given by Equation (31) is consistent.

$$\hat{J}_{i,t} = \Delta X_{i,t} I_{|\Delta X_{i,t}| > \tau_i} \tag{31}$$

It is straightforward noticing that a crucial point in establishing moment conditions (21)-(23) is choosing the correct threshold and (Figueroa-López and Nisen, 2013) show that a wrong choice may lead to biased and inefficient estimators. In this work, we decide[6] working with the threshold represented by Equation (32) because it respects the necessary condition given by proposition 1.

$$\tau(\sigma_i) = \sqrt{2\sigma_i^2 (\Delta t)^{\phi} \ln(\frac{1}{\Delta t})}, \, \phi \in (0,1) \tag{32}$$

A point of concern in our estimation strategy is the problem of the speed of convergence of the estimator of the threshold towards 0 discussed in (Figueroa-López and Nisen, 2013), that is, the choice of ϕ may introduce some bias in our parameter estimates, as it can result in our threshold being too large or too small, leading to misclassification of jumps. Moreover, we do not know the true value of the diffusion component (σ_i), so errors in its estimates may also introduce some bias on $\hat{\tau}$ and consequently on the other parameters.

To address potential problems arising from measurement errors in the threshold we use the same strategy as (Bollerslev and Zhou, 2002), that is, we include a nuisance parameter (θ_i) on the moment condition given by Equation (21). A possible interpretation of θ_i is that it is the average misclassification of jumps of $\hat{\tau}$.

4.2 Weighting matrix

(Hall, 2003) emphasizes that a crucial step in GMM estimation is the selection of the weighting matrix. In this study, we adopt the inverse of the covariance matrix, following (Hansen, 1982), who demonstrates that this choice yields efficient estimators. To estimate the covariance matrix, we employ nonparametric kernel-based methods as described in (Den Haan and Levin, 1997) and (Hirukawa, 2023), resulting in a heteroskedasticity- and autocorrelation-consistent (HAC) estimator.

[6] We also report simulation results the threshold proposed by (Figueroa-López and Nisen, 2013) in http://dx.doi.org/10.13140/RG.2.2.34687.11682 and show evidence that it does not generate estimators reliable as the one generated using Equation (32).

According to (Den Haan and Levin, 1997), the implementation of HAC covariance estimation requires several methodological choices, including the selection of the kernel function, the determination of bandwidth and truncation parameters, and the decision on whether to prewhiten the residuals. However, as noted in the survey articles by (Den Haan and Levin, 1997), (Wang and Wu, 2012), and (Hirukawa, 2023), there is no universally optimal approach to address these choices. Therefore, we rely on the outcomes of our simulation exercises to guide these decisions in the context of our estimation.

4.3 Initial parameters

Finally, the last point of concern that we need to address is that GMM estimation is sensitive to initial parameters because as (Hall, 2003) points out, the J-statistic is usually nonlinear, so we need to use optimization methods that are often highly sensitive to starting parameters, that is, if our initial guess are too far way from the true values, then the estimates produced by the optimization algorithm may be severly biased. Moreover, wrong starting values may lead to estimates that make the criterion function diverge to infinity.

As a result of this, we develop different sets of starting values to estimate parameters model. To build the first set of starting values that we use to estimate interest parameters, we employ the same procedure as (Aït-Sahalia et al., 2015), that is, we get initial values for the continuous part (α_i, β_i and σ_i) and jump components separetely (λ_i,μ_i and σ_i^J).

First, we select one of the thresholds given by Equation (32) and estimate it using the quadratic variation as an estimator[7] of σ_i. Then, we remove from our sample observations whose absolute value are greater than the threshold estimated in the previous step and use the remaining data to estimate a two-step GMM (see (Hall, 2003)) to obtain estimates for the continuous part using moment conditions given by equations (13) - (19).

In the third stage, we estimate jump components using the estimators represented by equations (33)-(36). Following this, we use iterated GMM proposed by (Hansen et al., 1996) with moment conditions given by equations (13)-(23) for each contract i individually.

$$\hat{\sigma}_i^{QV} = \sqrt{\frac{\sum_{t=1}^{T} \Delta X_{i,t}^2}{\frac{T}{252}}} \tag{33}$$

[7](Figueroa-López and Mancini, 2019) use the same estimator to initiate its estimation algorithm.

$$\hat{\lambda}_i = \frac{\sum_{t=1}^{T} I_{|\Delta X_{i,t}| > \hat{\tau}(\hat{\sigma}_i^{QV})}}{\frac{T}{252}} \tag{34}$$

$$\hat{\mu}_i = \frac{\sum_{t=1}^{T} \Delta X_{i,t} I_{|\Delta X_{i,t}| > \hat{\tau}(\hat{\sigma}_i^{QV})}}{T} \tag{35}$$

$$\hat{\sigma}_i^J = \sqrt{\frac{\sum_{t=1}^{T} (\Delta X_{i,t} I_{|\Delta X_{i,t}| > \hat{\tau}^1(\hat{\sigma}_i^{QV})} - \hat{\mu}_i)^2}{T-1}}, \text{ where} \tag{36}$$

In the fourth stage, we use $\tau(\hat{\sigma}^{QV})$ to exclude from our sample all values that jump, that is, we create a database where $\hat{\Delta X}_{i,t}^{C}$ is different from 0, which is given by Equation 37, and use it to create a starting value for the estimator of ρ given by Equation 38. We then combine with parameters estimated in previous step to create the first set of starting values and estimate parameters of contracts i and j jointly using moment conditions represented by equations (13)-(24).

$$\hat{\Delta X}_{i,t}^{C} = \Delta X_{i,t} I_{|\Delta X_{i,t}| \leqslant \hat{\tau}(\hat{\sigma}_i^{QV})} \tag{37}$$

$$\hat{\rho}_{i,j} = \frac{\hat{Cov}(\hat{\Delta X}_{i,t}^{C}, \hat{\Delta X}_{j,t}^{C})}{\sqrt{\hat{Var}(\hat{\Delta X}_{i,t}^{C}) \hat{Var}(\hat{\Delta X}_{j,t}^{C})}}, i \neq j, \tag{38}$$

If this set of parameter make the criterion function to diverge, then we try estimating parameters by using 0 as starting values of jump components. And even if it still generates convergence problems, we use 0 as starting value for all components of the continuous time model[8]

5. Simulation

The Generalized Method of Moments (GMM) provides a semiparametric estimation strategy that remains operative in settings where the full likelihood

[8] In order to save space we put the other simulation exercises results and estimated parameters of Equation (6) in `http://dx.doi.org/10.13140/RG.2.2.34687.11682`.

is either analytically intracTable or theoretically unspecified. Its robustness to misspecification of the underlying stochastic process, combined with its ability to accommodate endogeneity through moment orthogonality, renders it particularly attractive for structural inference in dynamic models (see (Hall, 2003), (Genaro and Astorino, 2022)). However, the method's practical implementation hinges critically on the specification of the moment condition set, for which no canonical selection procedure exists. To date, the literature has offered procedures that are primarily comparative or selection-on-efficiency—such as those developed by (Andrews, 1999), (Doran and Schmidt, 2006), (Cheng and Liao, 2015), (Caner et al., 2016), and (Frazier and Renault, 2019)—none of which resolve the ex ante indeterminacy in choosing among admissible moment restrictions. That is, they enable post-estimation evaluation of estimator efficiency conditional on a candidate set of moments but offer no principled method for identifying a subset that is both admissible and information-maximizing.

To probe this deficiency, we undertake a simulation-based exploration of moment selection under a continuous-time jump-diffusion framework. The data-generating process corresponds to the system of stochastic differential equations defined in (8)–(12), which incorporates mean-reverting diffusions with correlated Brownian components and a pure-jump process. We simulate 5,000 trajectories and estimate the structural parameters in each repetition via GMM using the moment conditions enumerated in (13)–(24), implemented through the gmm package (see (Chausse, 2010)). To address concerns regarding the influence of initial transients on finite-sample moments, we discard the first 1,008 observations from each simulated path—each of which initially consists of 4,789 points—in accordance with the burn-in procedure of (Duffie and Singleton, 1993).

Our stochastic specification for jump magnitudes departs from the canonical treatment in (Deng, 2000), who assumes two independent Poisson processes governing positive and negative jumps with exponential size distributions. In contrast, we posit a single compound Poisson process with normally distributed jumps centered at zero, thereby imposing symmetry in jump directionality. To maintain consistency with the overall jump intensity and second-moment structure of Deng's specification, we scale the jump intensity and standard deviation parameters by a factor of two. This rescaling ensures comparability in the contribution of jump risk to the process's higher-order dynamics.

Crucially, our identification strategy hinges on the moment structure's ability to disentangle the latent components driving the observed process. Since the GMM framework does not leverage the full conditional density, parameter identification relies entirely on the informativeness and non-redundancy of the moment conditions. In this context, the presence of nuisance parameters—those not directly of inferential interest—complicates identification by inflating the dimensionality of the system without necessarily augmenting identification strength. Nonetheless, as evidenced in Table 1, our simulation results demonstrate that the true structural parameters fall within the 95% simulation-based confidence intervals (defined by the 2.5% and 97.5% quantiles), despite observable bias. This suggests that the chosen moment structure, though not globally optimal, retains sufficient rank and informational content to yield estimators that are regular, root-n consistent, and practically viable in finite samples.

Table 1
Simulation exercise results

Parameter	True value	Mean	Median	0.05%	2.5%	5%	95%	97.5%	99.5%
λ_1	12.160	11.912	11.844	7.507	8.747	9.276	14.729	15.445	16.976
μ_1	0.000	0.000	-0.001	-0.074	-0.054	-0.045	0.046	0.056	0.076
$(\sigma_1^J)^2$	0.144	0.139	0.136	0.082	0.095	0.100	0.183	0.196	0.225
σ_1^2	0.548	0.555	0.557	0.515	0.528	0.532	0.581	0.586	0.598
β_1	-1.700	-1.687	-1.677	-2.611	-2.304	-2.191	-1.235	-1.166	-0.977
α_1	3.400	3.392	3.356	1.566	2.088	2.289	4.657	4.951	5.637
θ_1	-	-0.013	-0.015	-0.030	-0.026	-0.024	-0.006	-0.004	0.003
λ_2	14.000	13.755	13.779	0.000	9.775	10.672	17.015	18.002	21.898
μ_2	0.000	0.004	0.000	-0.047	-0.029	-0.025	0.026	0.034	0.382
$(\sigma_2^J)^2$	0.048	0.369	0.046	0.000	0.030	0.033	0.061	0.065	0.942
σ_2^2	0.116	0.117	0.117	0.108	0.110	0.112	0.123	0.124	0.126
β_2	-1.800	-1.799	-1.766	-2.949	-2.582	-2.402	-1.312	-1.228	-0.989
α_2	0.870	0.860	0.846	0.267	0.426	0.496	1.266	1.376	1.573
θ_2	-	-0.012	-0.014	-0.040	-0.028	-0.025	-0.003	0.001	0.023
$\rho_{1,2}$	0.200	0.193	0.188	0.092	0.123	0.134	0.241	0.254	0.279

This Table reports results from simulation exercises in which we use the empirical strategy that we describe in section 4 using the method proposed by (Andrews, 1991). We simulate 5,000 trajectories of 2 time series with each 3,781 observations and the threshold we use to generate these results is $\sqrt{2\sigma_i^2(\Delta t)^{0.999}log(\frac{1}{\Delta t})}$.

6. Data

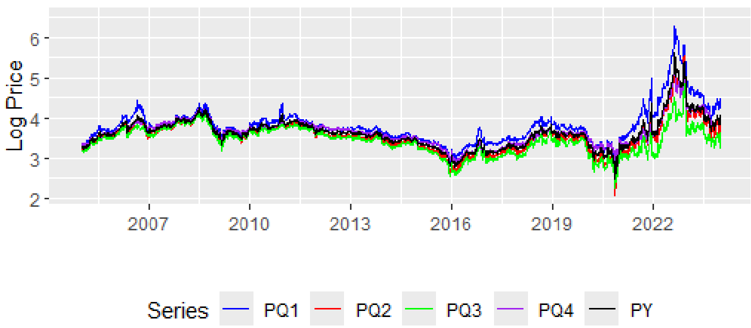
Our empirical analysis relies on daily settlement prices for quarterly and yearly electricity futures contracts traded on Nasdaq OMX, covering the Nordic power market. The sample spans the period from January 3, 2005, to December 29, 2023, yielding a total of 4,773 observations. The data were retrieved from the Eikon database and correspond to the contracts specified on pages 77–80 of (Nasdaq, 2024). Given that Nasdaq OMX is widely recognized as a central venue for electricity derivatives trading in Europe (see (Bouveret et al., 2023)), the dataset is well suited for the objectives of this study.

A salient feature of these futures contracts is that they are cash-settled against the daily Elspot System Price published by Nord Pool for the Nordic region. While both the quarterly and yearly contracts share this settlement mechanism, they differ in terms of their trading calendars. Yearly futures commence trading on the first banking day of the tenth calendar year preceding the delivery period and expire on the third-last banking day prior to delivery. Quarterly futures, in contrast, initiate trading on the first banking day of the penultimate year before delivery and cease trading on the final banking day preceding the contract period.

The dataset presents missing observations on five specific dates—January 6, 2005; February 15, 2005; May 18, 2005; June 24, 2005; and November 4, 2010. To mitigate potential biases introduced by these gaps, we employed a combination of imputation techniques, including last-observation-carried-forward and Kalman filter-based interpolation.

Figure 1 depicts the evolution of futures logarithmic prices over the sample period. Between 2005 and 2010, both quarterly and yearly contracts exhibited substantial price appreciation followed by sharp corrections. These dynamics likely reflect shifts in market expectations concerning forward supply-demand conditions, potentially influenced by climatic variability and hydrological reservoir levels. In the subsequent period (2011–2016), a pronounced decline in futures prices is observed, mirroring the broader downtrend in European spot electricity prices (see (Pepermans, 2019)). This co-movement suggests that the structural drivers depressing short-term prices—such as market liberalization ((Pepermans, 2019)), increased penetration of renewables ((Auer and Haas, 2016)), declining coal and natural gas prices ((Kougias and Szabó, 2017)), and volatility in emissions allowance markets ((Salmela et al., 2020))—also exerted downward pressure on longer-dated market expectations.

Figure 1
Price Dynamics: The graph depicts the time series from 02/01/2005 to 29/12/2023 of forward electricity contract logarithmic prices. P_{Qi} is the price of the forward electricity contract with energy delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year



The dynamics of logarithmic prices illustrated in Figure 1 suggest that contracts with different delivery periods typically move in the same direction. This pattern is expected, as divergent movements would create arbitrage opportunities in the market. This observation is further corroborated by the values in Table 2, which reveal high correlation coefficients between the prices of these contracts.

Table 2
Full Sample - logarithmic prices correlation matrix

	$\log(P_{Q1})$	$\log(P_{Q2})$	$\log(P_{Q3})$	$\log(P_{Q4})$	$\log(P_Y)$
$\log(P_{Q1})$	1.000				
$\log(P_{Q2})$	0.943	1.000			
$\log(P_{Q3})$	0.776	0.934	1.000		
$\log(P_{Q4})$	0.926	0.973	0.913	1.000	
$\log(P_Y)$	0.968	0.993	0.905	0.983	1.000

This Table reports unconditional correlation of daily quarterly and one year-ahead forward electricity logarithmic prices from 02/01/2005 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year.

Following 2016, electricity futures prices exhibited a renewed upward trajectory, a trend plausibly linked to the sustained rise in European Union Emissions Trading Scheme (EU ETS) allowance prices (see (Salmela et al., 2020)). The escalation intensified markedly in 2022, coinciding with the geopolitical and energy market disruptions triggered by Russia's invasion of Ukraine.These structural breaks and regime shifts suggest that the underlying futures price processes may exhibit nonstationary behavior.

To formally assess this, we conducted unit root diagnostics using both the Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt–Shin (KPSS) tests on the full sample and on the pre-COVID subsample in order to mitigate potential biases in the statistical characterization of futures price behavior that may be attributable to pandemic-induced market dislocations. Both unit root tests indicate that at a 5% level of significance our time series are not stationary, as expected.

Tables 3 and 4 report summary statistics for the level of forward logarithm prices across the full sample and the pre-2020 subsample, respectively. The results reveal that both mean and median values are highest for contracts expiring in the first and fourth quarters, indicating elevated price levels during winter delivery periods. This seasonal pricing pattern is consistent with findings in (Fleten and Lemming, 2003), who attribute such dynamics to increased electricity demand for heating during colder months.

Table 3
Full Sample - logarithmic prices: descriptive statistics

Stat	$\ln(P_{Q1})$	$\ln(P_{Q2})$	$\ln(P_{Q3})$	$\ln(P_{Q4})$	$\ln(P_Y)$
Aver.	3.814	3.549	3.441	3.663	3.631
Med.	3.753	3.542	3.475	3.647	3.616
Std-Dev.	0.467	0.394	0.340	0.347	0.389
Skew.	1.798	0.773	-0.048	0.976	1.111
Kurt.	8.349	5.735	3.299	5.566	6.296
Min.	2.464	2.067	2.239	2.863	2.472
Max.	6.310	5.561	4.925	5.438	5.631
ADF. Stat	-2.345	-2.977	-3.389*	-2.330	-2.535
KPSS. Stat	4.881***	4.271***	10.107***	4.614***	4.145***

This table reports descriptive statistics of daily quarterly and one year-ahead forward electricity prices logarithm from 02/01/2005 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 4
Subsample - logarithmic prices: descriptive statistics

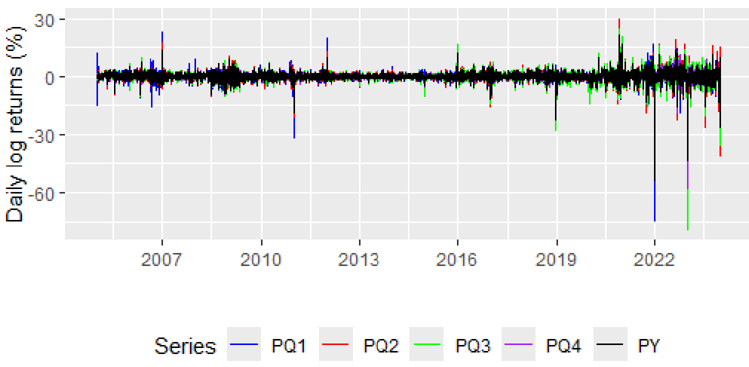
Stat	$\ln(P_{Q1})$	$\ln(P_{Q2})$	$\ln(P_{Q3})$	$\ln(P_{Q4})$	$\ln(P_Y)$
Aver.	3.715	3.513	3.457	3.618	3.581
Med.	3.734	3.540	3.490	3.645	3.610
Std-Dev.	0.259	0.283	0.297	0.258	0.269
Skew.	-0.134	-0.426	-0.439	-0.433	-0.377
Kurt.	2.929	2.951	2.939	2.822	2.887
Min.	2.939	2.660	2.536	2.863	2.791
Max.	4.439	4.190	4.164	4.257	4.245
ADF. Stat	-3.208*	-2.887	-2.836	-2.487	-2.815
KPSS. Stat	11.709***	15.425***	18.635***	16.837***	15.778***

This table reports descriptive statistics of daily quarterly and one year-ahead forward electricity prices from 02/01/2005 until 30/12/2019. P_{Qi} is the price of the forward electricity contract with energy delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

The descriptive statistics reported in Tables 3 and 4 further underscore the structural impact of the 2020–2023 period on the statistical properties of the futures series. In particular, estimated volatilities are markedly lower when observations from 2020 onward are excluded, suggesting that post-2020 market dynamics introduced heightened variability. Moreover, prior to mid-2019, the distributional characteristics of third-quarter and yearly futures contracts exhibit neither significant skewness nor excess kurtosis. This absence of heavy tails contrasts with the behavior of spot electricity prices, which have historically displayed pronounced non-normality (see (Lucia and Schwartz, 2002)).

Figure 2 plots daily log returns for both yearly and quarterly contracts and offers a complementary perspective. Unlike price levels, the return series exhibit no discernible deterministic trend, consistent with weak-form market efficiency. However, the presence of sharp discontinuities—manifested as return spikes—points to potential jump behavior in forward electricity prices, aligning with the stylized facts observed in high-frequency energy markets.

Figure 2
Daily returns: The graph depicts the time series from 02/01/2005 **to** 29/12/2023 **of daily log returns of forward electricity contract.** P_{Qi} **is the price of the forward electricity contract with energy delivery in** i**th quarter of the following year and** P_Y **is the price of the forward electricity contract with delivery in the following year**



The descriptive statistics reported in tables 5 and 5 support this observation; even when constrained to samples up until 2019, heavy tails persist. Notably, volatility increased following the inclusion of periods affected by COVID-19 and the Ukrainian conflict into our sample. This suggests that both events contributed to heightened uncertainty surrounding power derivatives. Until 2019, daily returns for yearly and quarterly contracts exhibited moderate negative skewness, indicating a consistent incentive for market participants to engage in hedging activities.

However, despite these disruptive events results from unit root tests, Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt–Shin (KPSS), indicate that daily log returns of our sample are stationary at 5% level significance. Therefore, we can potentially use the parameter estimates in the subsample to conduct our forecast exercises on section 8.

Table 5
Full sample - Returns: Descriptive statistics

Stat	$\Delta\ln(P_{Q1})$	$\Delta\ln(P_{Q2})$	$\Delta\ln(P_{Q3})$	$\Delta\ln(P_{Q4})$	$\Delta\ln(P_{Y})$
Aver.	0.000	0.000	0.000	0.000	0.000
Med.	0.000	0.001	0.001	0.000	0.000
Std-Dev.	0.028	0.030	0.029	0.020	0.025
Skew.	-4.004	-2.809	-5.195	-5.331	-3.474
Kurt.	112.469	55.610	130.482	140.109	71.374
Min.	-0.743	-0.523	-0.791	-0.574	-0.537
Max.	0.267	0.299	0.246	0.131	0.216
ADF. Stat	-17.249***	-17.723***	-17.990***	-16.735***	-17.114***
KPSS. Stat	0.049	0.033	0.034	0.059	0.046

This table reports descriptive statistics of daily log price returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 6
Subsample - Returns: Descriptive statistics:

Stat	$\Delta\ln(P_{Q1})$	$\Delta\ln(P_{Q2})$	$\Delta\ln(P_{Q3})$	$\Delta\ln(P_{Q4})$	$\Delta\ln(P_{Y})$
Aver.	0.000	0.000	0.000	0.000	0.000
Med.	0.000	0.000	0.000	0.000	0.000
Std-Dev.	0.020	0.019	0.019	0.014	0.017
Skew.	-0.905	-1.113	-1.210	-0.736	-0.958
Kurt.	35.877	22.838	21.762	10.871	19.155
Min.	-0.320	-0.274	-0.275	-0.140	-0.225
Max.	0.230	0.175	0.169	0.081	0.138
ADF. Stat	-15.448***	-16.834***	-17.282***	-16.839***	-16.487***
KPSS. Stat	0.095	0.100	0.106	0.157	0.119

This table reports descriptive statistic of daily log price returns of quarterly and one year-ahead forward electrcity from 03/01/2005 until 30/12/2019. PQ_i is the price of the forward electricity contract with delivery in *ith* quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Finally, we address the issue whether Samuelson effect is present on our data or not. In order to this, we use a similar empirical setup[9] as (Jaeck and Lautier, 2016), where they estimate a regression between the absolute log returns and the time-to-maturity of the forward contracts and if the slope parameter time-to-maturity, then it would be an evidence that Samuelson effect is present on the data.

On our empirical framework, on the other hand, we run a regression between the estimated absolute difference of the stochastic component from equation 6 that did not overcome the threshold given by equation 32 with $\phi = 0.999$ and the time to maturity from 03/01/2005 to 30/12/2019 as represented in equation 39. The idea behind our procedure is that we do not want our results being influenced by Ukraine war, by jumps or by specific year specific shocks.

$$100|Z_{i,t}| = a_i + b_i TTM_{i,t} + u_{i,t}, \; where \tag{39}$$

$Z_{i,t}$ is the value of the data sample we create on the fourth stage of section 4 of forward contract i at time period t, $TTM_{i,t}$ is the amount of day at time t of forward contract i before rolling-over the contract, $u_{i,t}$ is an error term, a_i and b_i are model parameters for forward contract i.

The results of tables 7 - 11 indicate that at level of significance of 5% there is no Samuelson effect on our sample. A possible explanation for this fact is that as in (Anderson and Danthine, 1983) the resolution of uncertainty may happen in some other time but those are questions that we leave for future research.

Table 7
Samuelson effect test results for P_Y

Parameter	Estimate	Std. Error	Test. Stat	p-value
$\hat{a}_{P_Y}$	1.168***	0.034	34.365	< 0.01
$\hat{b}_{P_Y}$	0.000	0.000	-0.942	0.346

This table reports estimated coefficients from equation 39 for forward electricity contract with delivery period for the next year (P_Y) from 03/01/2005 until 30/12/2019. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

[9]Check subsection 3.1 and equation 2 from (Jaeck and Lautier, 2016) for further details.

Table 8
Samuelson effect test results for P_{Q1}

Parameter	Estimate	Std. Error	Test. Stat	p-value
$\hat{a}_{P_{Q1}}$	1.155***	0.036	31.939	< 0.01
$\hat{b}_{P_{Q1}}$	0.000	0.000	-0.007	0.994

This table reports estimated coefficients from equation 39 for forward electricity contract with delivery period for the next year (P_{Q1}) from 03/01/2005 until 30/12/2019. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 9
Samuelson effect test results for P_{Q2}

Parameter	Estimate	Std. Errors	Test. Stat	p-value
$\hat{a}_{P_{Q2}}$	1.162***	0.034	34.397	< 0.01
$\hat{b}_{P_{Q2}}$	0.000	0.000	-0.866	0.387

This table reports estimated coefficients from equation 39 for forward electricity contract with delivery period for the next year (P_{Q2}) from 03/01/2005 until 30/12/2019. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 10
Samuelson effect test results for P_{Q3}

Parameter	Estimates	Std. Error	Test. Stat	p-value
$\hat{a}_{P_{Q3}}$	1.162***	0.034	34.589	< 0.01
$\hat{b}_{P_{Q3}}$	0.000	0.000	-1.066	0.286

This table reports estimated coefficients from equation 39 for forward electricity contract with delivery period for the next year (P_{Q3}) from 03/01/2005 until 30/12/2019. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 11
Samuelson effect test results for P_{Q4}

Parameter	Estimates	Std. Error	Test. Stat	p-value
$\hat{a}_{P_{Q4}}$	1.164***	0.036	32.730	< 0.01
$\hat{b}_{P_{Q4}}$	0.000	0.000	-0.661	0.509

This table reports estimated coefficients from equation 39 for forward electricity contract with delivery period for the next year (P_{Q4}) from 03/01/2005 until 30/12/2019. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

7. Results

In this section we report results of estimating parameters of the model represented by equations (1)-(4) using data from 03/01/2005 to 30/12/2019, so our results are not contaminated by events such as the Ukranian war. Moreover, by leaving out the daily forward prices from the years of 2020-2023 we gather enough data to conduct our forecasting exercises that we report in section 8.

The results for the J-test reported on Table 12 show that at a level of significance of 5% the moment constraints are satisfied, which indicate that our model is well specified. The estimates of Tables 13-16 show that $\hat{\alpha}_i$ is positive but it is only statistically different from 0 at a 5% level of significance on third and fourth quarter contracts, which indicates that the relation between the drift and time-to-maturity that is reported in (Benth and Paraschiv, 2018) is not ubiquituos.

On the other hand, $\hat{\beta}_i$ is negative and statistically different from 0 considering the same level of significance for all forward contracts. Consequently, our results indicate that on average when prices deviate from their long term equilibrium, economic agents must receive an incentive to correct unbalance on the suppy and demand for hedging. Those results are alinged with the empirical evidence from (Diko et al., 2006), (Benth and Paraschiv, 2018), (Blanco et al., 2018) and (Algieri et al., 2021) that forward contracts present time varying risk premiums.

Additionaly, our estimates indicate that the unitary cost by disequilibrium ($\hat{\beta}_i$) is not equal on all contracts and that the more expensive contract are the forward contracts for the thirds and fourth quarters. In other words, economic agents require a greater compensation to provide hedge during the winter season.

Our estimates also indicate that rare events are pervasive in all forward contracts and that on on average their impact is not statistically significant at a 5% level. However,they present different levels of intensity ($\hat{\lambda}_i$) and standard deviations (σ_i^J), which indicates that information processes

The results of Table 13 also show evidence that quarterly and yearly forward prices jump but on average their impact ($\hat{\mu}_i$) at a level of significance of 5% is not statistically different from 0. Moreover, the difference between jump intensity ($\hat{\lambda}_i$) and their standard deviation ($\hat{\sigma}_i^J$) indicate that there are differences in the information flow for specific quarters and one year-ahead contracts and how sensitive they are to new information.

Table 12
Results of the J-test

Model	Test. Stat	p-value
P_{Q1}-P_Y	3.790	0.705
P_{Q2}-P_Y	4.325	0.633
P_{Q3}-P_Y	6.620	0.357
P_{Q4}-P_Y	4.124	0.660

This table reports results from J-test from (Hansen, 1982) using daily log returns of quarterly and yearly forward electricity contracts from 03/01/2005 until 29/12/2023. The first column indicates which contracts are used to estimate parameters of our model; for example, a model P_{Qi}-P_Y indicates that the daily log returns from the forward contract with the delivery period of the ith quarter of the following year and daily log returns from the yearly contract are the data used to estimate parameters. $***$ indicates that test statistic is significant at 1% level, $**$ indicates that test statistic is significant at 5% level and $*$ indicates that test statistic is significant at 10% level.

Table 13
Parameter estimates for $P_{Q1} - P_Y$

Parameter	Estimate	Std. Error	Test-Stat	p-value
$\hat{\lambda}_{P_{Q1}}$	3.625***	0.775	4.677	< 0.01
$\hat{\mu}_{P_{Q1}}$	-0.013	0.017	-0.817	0.414
$\hat{\sigma}^J_{P_{Q1}}$	0.128***	0.019	6.888	< 0.01
$\hat{\sigma}_{P_{Q1}}$	0.242***	0.004	66.798	< 0.01
$\hat{\beta}_{P_{Q1}}$	-3.498***	1.093	-3.200	< 0.01
$\hat{\alpha}_{P_{Q1}}$	0.105	0.127	0.826	0.409
$\hat{\theta}_{P_{Q1}}$	0.006**	0.003	2.469	0.014
$\hat{\lambda}_{P_Y}$	3.620***	0.750	4.825	< 0.01
$\hat{\mu}_{P_Y}$	-0.018	0.015	-1.177	0.239
$\hat{\sigma}^J_{P_Y}$	0.111***	0.014	7.865	< 0.01
$\hat{\sigma}_{P_Y}$	0.223***	0.003	66.054	< 0.01
$\hat{\beta}_{P_Y}$	-3.666***	1.060	-3.457	< 0.01
$\hat{\alpha}_{P_Y}$	0.142	0.118	1.209	0.227
$\hat{\theta}_{P_Y}$	0.005**	0.002	1.981	0.048
$\hat{\rho}_{P_{Q1},P_Y}$	0.946***	0.014	67.048	< 0.01

This table reports GMM estimates of our model parameters using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019. Prior to estimation, forward electricity price data were interpolated using the previous available value. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 14
Parameter estimates for P_{Q2}-P_Y

Parameter	Estimate	Std. Error	Test-Stat	p-value
$\hat{\lambda}_{P_{Q2}}$	4.124***	0.804	5.132	< 0.01
$\hat{\mu}_{P_{Q2}}$	-0.021	0.015	-1.436	0.151
$\hat{\sigma}^J_{P_{Q2}}$	0.115***	0.014	8.386	< 0.01
$\hat{\sigma}_{P_{Q2}}$	0.243***	0.004	65.097	< 0.01
$\hat{\beta}_{P_{Q2}}$	-3.438***	1.024	-3.357	< 0.01
$\hat{\alpha}_{P_{Q2}}$	0.182	0.130	1.397	0.162
$\hat{\theta}_{P_{Q2}}$	0.005**	0.003	1.972	0.049
$\hat{\lambda}_{P_Y}$	3.463***	0.662	5.234	< 0.01
$\hat{\mu}_{P_Y}$	-0.020	0.016	-1.304	0.192
$\hat{\sigma}^J_{P_Y}$	0.113***	0.013	8.688	< 0.01
$\hat{\sigma}_{P_Y}$	0.222***	0.003	66.144	< 0.01
$\hat{\beta}_{P_Y}$	-3.534***	1.029	-3.434	< 0.01
$\hat{\alpha}_{P_Y}$	0.149	0.118	1.265	0.206
$\hat{\theta}_{P_Y}$	0.005**	0.002	2.521	0.012
$\hat{\rho}_{P_{Q2},P_Y}$	0.966***	0.009	110.910	< 0.01

This table reports GMM estimates of our model parameters using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019. Prior to estimation, forward electricity price data were interpolated using the previous available value. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 15
Parameter estimates for P_{Q3}-P_Y

Parameter	Estimate	Std. Error	Test-Stat	p-value
$\hat{\lambda}_{P_{Q3}}$	6.056***	1.228	4.932	< 0.01
$\hat{\mu}_{P_{Q3}}$	-0.012	0.010	-1.210	0.226
$\hat{\sigma}^J_{P_{Q3}}$	0.092***	0.012	7.345	< 0.01
$\hat{\sigma}_{P_{Q3}}$	0.240***	0.004	64.816	< 0.01
$\hat{\beta}_{P_{Q3}}$	-3.307***	0.963	-3.433	< 0.01
$\hat{\alpha}_{P_{Q3}}$	0.259**	0.125	2.067	0.039
$\hat{\theta}_{P_{Q3}}$	-0.001	0.004	-0.260	0.795
$\hat{\lambda}_{P_Y}$	4.261***	0.976	4.367	< 0.01
$\hat{\mu}_{P_Y}$	-0.008	0.013	-0.590	0.555
$\hat{\sigma}^J_{P_Y}$	0.104***	0.017	6.113	< 0.01
$\hat{\sigma}_{P_Y}$	0.223***	0.003	66.329	< 0.01
$\hat{\beta}_{P_Y}$	-3.491***	0.960	-3.638	< 0.01
$\hat{\alpha}_{P_Y}$	0.184	0.118	1.565	0.118
$\hat{\theta}_{P_Y}$	0.003	0.003	0.848	0.397
$\hat{\rho}_{P_{Q3},P_Y}$	0.959***	0.011	83.916	< 0.01

This table reports GMM estimates of our model parameters using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019. Prior to estimation, forward electricity price data were interpolated using the previous available value. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 16
Parameter estimates for P_{Q4}-P_Y

Parameter	Estimate	Std. Error	Test-Stat	p-value
$\hat{\lambda}_{P_{Q4}}$	3.593***	0.684	5.254	< 0.01
$\hat{\mu}_{P_{Q4}}$	-0.022	0.013	-1.619	0.105
$\hat{\sigma}^J_{P_{Q4}}$	0.095***	0.011	8.544	< 0.01
$\hat{\sigma}_{P_{Q4}}$	-0.187***	0.003	-63.078	< 0.01
$\hat{\beta}_{P_{Q4}}$	-3.950***	1.005	-3.929	< 0.01
$\hat{\alpha}_{P_{Q4}}$	0.195**	0.099	1.982	0.047
$\hat{\theta}_{P_{Q4}}$	0.006**	0.002	2.641	0.008
$\hat{\lambda}_{P_Y}$	3.567***	0.654	5.452	< 0.01
$\hat{\mu}_{P_Y}$	-0.014	0.014	-0.978	0.328
$\hat{\sigma}^J_{P_Y}$	0.109***	0.013	8.619	< 0.01
$\hat{\sigma}_{P_Y}$	0.224***	0.003	64.425	< 0.01
$\hat{\beta}_{P_Y}$	-3.573***	0.995	-3.592	< 0.01
$\hat{\alpha}_{P_Y}$	0.144	0.119	1.213	0.225
$\hat{\theta}_{P_Y}$	0.004**	0.002	1.968	0.049
$\hat{\rho}_{P_{Q4},P_Y}$	0.921***	0.012	76.933	< 0.01

This table reports GMM estimates of our model parameters using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019. Prior to estimation, forward electricity price data were interpolated using the previous available value. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Finally, the estimate of the correlation matrix shown on Table 17 align with the results from (Blanco et al., 2018), which shows that that yearly and quarterly future contracts returns are strongly positively correlated. Therefore, shocks in different in specific quarters may spread to the whole forward curve.

Table 17
Correlation matrix estimate

	P_{Q1}	P_{Q2}	P_{Q3}	P_{Q4}	P_Y
P_{Q1}	1.000				
P_{Q2}	0.915	1.000			
P_{Q3}	0.874	0.932	1.000		
P_{Q4}	0.838	0.886	0.894	1.000	
P_Y	0.946	0.966	0.959	0.921	1.000

This matrix reports parameter $\hat{\rho}_{P_{Qi},P_Y}$ and $\hat{\rho}_{P_{Qi},P_{Qj}}$, $i \neq j$.

8. Forecast performance

On section 7 we provide evidence that our model is able to characterize the dynamics of yearly and quarterly power-based derivatives, since at a level of 5% of siginificance the results of J-test on table 12 indicate that the overindentifying conditions are satisfied. Now in this section, we show that our model may be useful for risk managers and traders to hedge their companies risks and manage their portfolio by comparing its forecasting accuracy with an alternative model[10].

We choose as a competing model the ARMA-Garch(p,q,1,1) representation of the stochastic component of daily log returns of quarterly and yearly electricity forward contracts because according to (Liu and Shi, 2013) it is an useful model to describe electricty price dynamics. Moreover, we use Akaike information criteria (AIC) and Bayesian information criteria (BIC) (see (Cavanaugh and Neath, 2019)) to select the p and q orders of this model[11].

[10]We report the comparison with ARMA-Garch models whose specification was chosen using AIC criteria because it generate the lowest mean absolute error (MAE) in comparison with the models that use the BIC. However, the results are reported on the appendix.

[11]The values of AIC and BIC for different ARMA-Garch specifications can be seen in a supplementary material that can be found in `http://dx.doi.org/10.13140/RG.2.2.34687.11682` . Moreover, we use R package rugarch from (Galanos, 2024) to estimate ARMA-Garch parameters.

We compare models accuracy by calcluating the the absolute error for the 1, 5 and 10 days ahead forecasts for both models and then applying model confidence set test proposed by (Hansen et al., 2011) and that was implemented on R package MCS by (Bernardi and Catania, 2018). We estimate daily absolute errors for quarterly and yearly contracts and we also estimate for each forecasting day the mean average absolute error for both models in order to have an estimate of the forecasting error of the forward curve as a whole.

On our analysis, we consider two forecasting periods, that is, from 02/01/2020 to 29/12/2023 and from 02/01/2020 to 30/12/2021, so we can have evidence that test results is not being driven the effects of the Ukranian war. In tables 18-23 we report the results of the confidence set for parameters estimated until 2019. By comparing the mean absolute errors for the two forecasting time horizons we can notice that both models present larger errors when the forecasting periods include the year of 2022 and that at a 5% level of significance across all forecasting periods and time horizons in our analysis, the results of the confidence set test indicate that forecasts of the forward curve using our model is more accurate than the ARMA-Garch[12] forecasts for the forward curve.

[12]The ARMA-Garch orders selected according to AIC criterion for P_Y, P_{Q1}, P_{Q2}, P_{Q3} and P_{Q4} were respectively (7,7,1,1), (5,7,1,1), (9,5,1,1), (8,7,1,1) and (10,5,1,1).

Table 18
Results of the confidence set test results for 1 day-ahead forecasts from 02/01/2020 to 29/12/2023

1D	Cont.	ARMA-Garch	Test. Stat.	p-value
P_Y	1.975	3.514	-3.878***	< 0.01
P_{Q_1}	3.570	5.520	-3.860***	< 0.01
P_{Q_2}	2.010	2.470	-3.472***	< 0.01
P_{Q_3}	1.179	1.183	-0.250	0.803
P_{Q_4}	1.554	1.652	-1.267	0.205
Curv	2.058	2.868	-4.081***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 1 day-ahead forecasts of forward electricity contracts prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 02/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year, P_Y is the price of the forward electricity contract with delivery in the following year and Curv is the average of absolute errors in each day. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 19
Results of the confidence set test results for 5 days-ahead forecasts from 08/01/2020 to 29/12/2023

5D	Cont.	ARMA-Garch	Test. Stat.	p-value
P_Y	5.895	13.451	-6.707***	< 0.01
P_{Q_1}	11.014	22.563	-4.254***	< 0.01
P_{Q_2}	6.060	8.111	-4.485***	< 0.01
P_{Q_3}	3.193	3.425	-2.719***	< 0.01
P_{Q_4}	4.877	4.296	1.898*	0.052
Curv	6.208	10.369	-4.397***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 5 days-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 08/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year, P_Y is the price of the forward electricity contract with delivery in the following year and Curv is the average of absolute errors in each day.$***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 20
Results of the confidence set test results for 10 days-ahead forecasts from 15/01/2020 to 29/12/2023

10D	Cont.	ARMA-Garch	Test. Stat.	p-value
P_Y	9.462	21.332	-5.109***	< 0.01
P_{Q_1}	18.043	36.342	-6.053***	< 0.01
P_{Q_2}	9.565	12.997	-3.946***	< 0.01
P_{Q_3}	4.916	5.446	-2.226**	0.024
P_{Q_4}	7.730	6.387	-2.459**	0.014
Curv	9.943	16.501	-5.133***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 10 days-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 15/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year, P_Y is the price of the forward electricity contract with delivery in the following year and Curv is the average of absolute errors in each day. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively.

Table 21
Results of the confidence set test results for 1 day-ahead forecasts from 02/01/2020 to 30/12/2021

1D	Cont.	ARMA-GARCH	Test. Stat.	p-value
P_Y	0.858	0.963	-5.897***	< 0.01
P_{Q_1}	1.444	1.532	-2.658***	< 0.01
P_{Q2}	0.874	0.954	-4.100***	< 0.01
P_{Q3}	0.658	0.681	-2.510**	0.012
P_{Q_4}	0.542	0.540	0.232	0.819
Curv	0.875	0.934	-4.912***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 1 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by hansen2011. Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 02/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. *** indicates that test statistic is significant at the 1% level; ** indicates that test statistic is significant at the 5% level; * indicates that test statistic is significant at the 10% level.

Table 22
Results of the confidence set test results for 5 days-ahead forecasts from 08/01/2020 to 30/12/2021

5D	Cont.	ARMA-GARCH	Test. Stat.	p-value
P_Y	2.189	3.089	-7.477***	< 0.01
P_{Q1}	3.799	4.613	-4.237***	< 0.01
P_{Q_2}	2.282	2.717	-3.677***	< 0.01
P_{Q_3}	1.737	2.132	-5.404***	< 0.01
P_{Q_4}	1.454	1.452	0.051	0.959
Curv	2.292	2.801	-6.815***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 5 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by hansen2011. Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 08/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. *** indicates that test statistic is significant at the 1% level; ** indicates that test statistic is significant at the 5% level; and * indicates that test statistic is significant at the 10% level.

Table 23
Results of the confidence set test results for 10 days-ahead forecasts from 15/01/2020 to 30/12/2021

10D	Cont.	ARMA-GARCH	Test. Stat.	p-value
P_Y	3.194	4.966	-8.503***	< 0.01
P_{Q_1}	5.511	6.992	-5.318***	< 0.01
P_{Q_2}	3.312	4.278	-5.946***	< 0.01
P_{Q_3}	2.623	3.602	-6.634***	< 0.01
P_{Q_4}	2.071	2.097	-0.280	0.779
Curv	3.342	4.387	-9.637***	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 10 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by hansen2011. Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 15/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. *** indicates that test statistic is significant at the 1% level; ** indicates that test statistic is significant at the 5% level; and * indicates that test statistic is significant at the 10% level.

9. Concluding remarks

On this work we propose a model to describe the dynamics of forward electricity contracts that not only takes into account the time-to maturity risk premium but also incorporate the presence of common shocks and rare events without relying on analytical solutions to SDEs. Moreover, we provide evidence that its parameters can be viably estimated without making any assumptions about the probability distribution of the data.

On the empirical section of our paper we find evidence that the forward risk premium depends on deviations from long term equilibrium prices but the time-to maturity premium is not statistically significant at level of 5% in every contract. Moreover, we find evidence that the cost per unit of providing hedging for winter quarters are the most expensive, which goes in the same direction of the results found by (Fleten and Lemming, 2003).

Additionally, our estimates also align with the results of (Blanco et al., 2018) in the sense that they also find that yearly and quarterly contracts are highly positivelly correlated, which indicate that shocks from specific delivery periods spread to the whole forward curve. Moreover, our empirical results point out that jumps are pervasive in all forward contracts but happen at different intensities, so specific information regarding delivery periods do not arrive at the same frequency.

Finally, we show evidence that our model outperformed ARMA-Garch in our forecast exercises, thus show it is potentially useful for market practicioners and risk managers to describe the behavior of the forward electricity curve.

Acknowledgments

Conflict of interest The authors declare no conflict of interest.

Artificial Intelligence This research utilized AI tools to assist in data analysis, manuscript drafting, and figure generation. All AI-generated content was critically reviewed and validated by the authors to ensure accuracy and alignment with the scientific integrity of the study. The use of AI adhered to ethical guidelines, ensuring transparency and compliance with academic standards. Any biases or limitations inherent to the AI tools were carefully considered in the interpretation of results. The authors affirm that the AI tools did not compromise the originality or integrity of the work.

References

Aıt-Sahalia, Y. (2004). Disentangling diffusion from jumps, *Journal of Financial Economics* **74**(3): 487–528.

Alexeev, V., Urga, G. and Yao, W. (2019). Asymmetric jump beta estimation with implications for portfolio risk management, *International Review of Economics & Finance* **62**: 20–40.

Algieri, B., Leccadito, A. and Tunaru, D. (2021). Risk premia in electricity derivatives markets, *Energy Economics* **100**: 105300.

Anderson, R. W. and Danthine, J.-P. (1983). The time pattern of hedging and the volatility of futures prices, *The Review of Economic Studies* **50**: 249–266.

Andrews, D. W. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation, *Econometrica: Journal of the Econometric Society* **59**: 817–858.

Andrews, D. W. (1999). Consistent moment selection procedures for generalized method of moments estimation, *Econometrica* **67**(3): 543–563.

Andrews, D. W. and Lu, B. (2001). Consistent model and moment selection procedures for gmm estimation with application to dynamic panel data models, *Journal of econometrics* **101**(1): 123–164.

Arismendi Zambrano, J. C. (2019). Higher-order tail moments in asset-pricing theory, *HANDBOOK OF GLOBAL FINANCIAL MARKETS: Transformations, Dependence, and Risk Spillovers*, World Scientific, pp. 689–741.

Auer, H. and Haas, R. (2016). On integrating large shares of variable renewables into the electricity system, *Energy* **115**: 1592–1601.

Aït-Sahalia, Y., Cacho-Diaz, J. and Laeven, R. J. (2015). Modeling financial contagion using mutually exciting jump processes, *Journal of Financial Economics* **117**(3): 585–606.

Bacon, R. W. (1995). Privatization and reform in the global electricity supply industry, *Annual review of energy and the environment* **20**(1): 119–143.

Basu, D. and Miffre, J. (2013). Capturing the risk premium of commodity futures: The role of hedging pressure, *Journal of Banking & Finance* **37**(7): 2652–2664.

Benth, F. E., Benth, J. S. and Koekebakker, S. (2008). *Stochastic modelling of electricity and related markets*, Vol. 11, World Scientific.

Benth, F. E. and Koekebakker, S. (2008). Stochastic modeling of financial electricity contracts, *Energy Economics* **30**(3): 1116–1157.

Benth, F. E. and Paraschiv, F. (2018). A space-time random field model for electricity forward prices, *Journal of Banking & Finance* **95**: 203–216.

Benth, F. E., Klüppelberg, C., Müller, G. and Vos, L. (2014). Futures pricing in electricity markets based on stable carma spot models, *Energy Economics* **44**: 392–406.

Bernardi, M. and Catania, L. (2018). The model confidence set package for r, *International Journal of Computational Economics and Econometrics* **8**(2): 144–158.

Bessembinder, H. and Lemmon, M. L. (2002). Equilibrium pricing and optimal hedging in electricity forward markets, *the Journal of Finance* **57**(3): 1347–1382.

Blanco, I., Peña, J. I. and Rodríguez, R. (2018). Modelling electricity swaps with stochastic forward premium models, *The Energy Journal* **39**(2): 1–34.

Bollerslev, T. and Zhou, H. (2002). Estimating stochastic volatility diffusion using conditional moments of integrated volatility, *Journal of Econometrics* **109**(1): 33–65.

Bollerslev, T., Law, T. H. and Tauchen, G. (2008). Risk, jumps, and diversification, *Journal of Econometrics* **144**(1): 234–256.

Bouveret, A., Di Nello, D., Gutierrez, J. and Haferkorn, M. (2023). Eu energy derivatives markets: Structure and risks, *Revista de Estabilidad Financiera/Banco de España, 44 (primavera 2023), p. 39-70* .

Callegaro, G., Mazzoran, A. and Sgarra, C. (2022). A self-exciting modeling framework for forward prices in power markets, *Applied stochastic models in business and industry* **38**(1): 27–48.

Caner, M., Maasoumi, E. and Riquelme, J. A. (2016). Moment and iv selection approaches: A comparative simulation study, *Econometric Reviews* **35**(8-10): 1562–1581.

Cartea, A. and Figueroa, M. G. (2005). Pricing in electricity markets: a mean reverting jump diffusion model with seasonality, *Applied Mathematical Finance* **12**(4): 313–335.

Cavanaugh, J. E. and Neath, A. A. (2019). The akaike information criterion: Background, derivation, properties, application, interpretation, and refinements, *Wiley Interdisciplinary Reviews: Computational Statistics* **11**(3): e1460.

Chan, K. C., Karolyi, G. A., Longstaff, F. A. and Sanders, A. B. (1992). An empirical comparison of alternative models of the short-term interest rate, *The journal of finance* **47**(3): 1209–1227.

Chausse, P. (2010). Computing generalized method of moments and generalized empirical likelihood with R, *Journal of Statistical Software* **34**(11): 1–35.

Chen, J. and Qi, S. (2024). Limit-hitting exciting effects: Modeling jump dependencies in stock markets adhering to daily price-limit rules, *Journal of Banking & Finance* **163**: 107184.

Cheng, X. and Liao, Z. (2015). Select the valid and relevant moments: An information-based lasso for gmm with many moments, *Journal of Econometrics* **186**(2): 443–464.

Chevallier, J. and Ielpo, F. (2014). Twenty years of jumps in commodity markets, *International Review of Applied Economics* **28**(1): 64–82.

Çinlar, E. (2011). *Probability and stochastics*, Springer.

Clewlow, L. and Strickland, C. (1999). Valuing energy options in a one factor model fitted to forward prices, *Available at SSRN 160608* .

Den Haan, W. J. and Levin, A. T. (1997). 12 a practitioner's guide to robust covariance matrix estimation, *Handbook of statistics* **15**: 299–342.

Deng, S. (2000). *Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes*, University of California Energy Institute Berkeley.

Deschatre, T., Féron, O. and Gruet, P. (2021). A survey of electricity spot and futures price models for risk management applications, *Energy Economics* **102**: 105504.

Deschatre, T., Féron, O. and Hoffmann, M. (2020). Estimating fast mean-reverting jumps in electricity market models, *ESAIM: Probability and Statistics* **24**: 963–1002.

Di Poto, G. and Fanone, E. (2011). Estimating a lévy multifactor market model for electricity futures market by using ica, *The Journal of Energy Markets, Winter* (2012).

Diko, P., Lawford, S. and Limpens, V. (2006). Risk premia in electricity forward prices, *Studies in Nonlinear Dynamics & Econometrics* **10**(3).

Doran, H. E. and Schmidt, P. (2006). Gmm estimators with improved finite sample properties using principal components of the weighting matrix, with an application to the dynamic panel data model, *Journal of econometrics* **133**(1): 387–409.

Duffie, D. and Singleton, K. J. (1993). Simulated moments estimation of markov models of asset prices, *Econometrica* **61**(4): 929–952.
URL: http://www.jstor.org/stable/2951768

Eraker, B., Johannes, M. and Polson, N. (2003). The impact of jumps in volatility and returns, *The Journal of Finance* **58**(3): 1269–1300.

Erdemlioglu, D., Laurent, S. and Neely, C. J. (2015). Which continuous-time model is most appropriate for exchange rates?, *Journal of Banking & Finance* **61**: S256–S268.

Fanelli, V. and Schmeck, M. D. (2019). On the seasonality in the implied volatility of electricity options, *Quantitative Finance* **19**(8): 1321–1337.

Fanelli, V., Maddalena, L. and Musti, S. (2016). Modelling electricity futures prices using seasonal path-dependent volatility, *Applied energy* **173**: 92–102.

Figueroa-López, J. E. and Mancini, C. (2019). Optimum thresholding using mean and conditional mean squared error, *Journal of Econometrics* **208**(1): 179–210.

Figueroa-López, J. E. and Nisen, J. (2013). Optimally thresholded realized power variations for lévy jump diffusion models, *Stochastic Processes and their Applications* **123**(7): 2648–2677.

Fleten, S.-E. and Lemming, J. (2003). Constructing forward price curves in electricity markets, *Energy Economics* **25**(5): 409–424.

Frazier, D. T. and Renault, E. (2019). Indirect inference: which moments to match?, *Econometrics* **7**(1): 14.

Frestad, D. (2012). Liquidity and dirty hedging in the nordic electricity market, *Energy economics* **34**(5): 1341–1355.

Füss, R., Mahringer, S. and Prokopczuk, M. (2015). Electricity derivatives pricing with forward-looking information, *Journal of Economic dynamics and control* **58**: 34–57.

Galanos, A. (2024). *rugarch: Univariate GARCH models.* R package version 1.5-3.

Geman, H. and Roncoroni, A. (2006). Understanding the fine structure of electricity prices, *The Journal of Business* **79**(3): 1225–1261.

Genaro, A. d. and Astorino, P. (2022). A tutorial on the generalized method of moments (gmm) in finance, *Revista de Administração Contemporânea* **26**: e210287.

Gibson, R. and Schwartz, E. S. (1990). Stochastic convenience yield and the pricing of oil contingent claims, *The Journal of Finance* **45**(3): 959–976.

Glasserman, P. (2004). *Monte Carlo methods in financial engineering*, Vol. 53, Springer.

Gudkov, N. and Ignatieva, K. (2021). Electricity price modelling with stochastic volatility and jumps: An empirical investigation, *Energy Economics* **98**: 105260.

Hall, A. R. (2003). *Generalized Method of Moments:A Companion to Theoretical Econometrics*, John Wiley Sons, Ltd.

Hall, A. R. (2015). Econometricians have their moments: Gmm at 32, *Economic Record* **91**: 1–24.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators, *Econometrica: Journal of the econometric society* **50**(4): 1029–1054.

Hansen, L. P., Heaton, J. and Yaron, A. (1996). Finite-sample properties of some alternative gmm estimators, *Journal of Business & Economic Statistics* **14**(3): 262–280.

Hansen, P. R., Lunde, A. and Nason, J. M. (2011). The model confidence set, *Econometrica* **79**(2): 453–497.

Hawkes, A. G. (2018). Hawkes processes and their applications to finance: a review, *Quantitative Finance* **18**(2): 193–198.

Heath, D., Jarrow, R. and Morton, A. (1992). Claims valuation', *Econometrica* **60**(1): 77–105.

Hinz, J., Von Grafenstein, L., Verschuere, M. and Wilhelm, M. (2005). Pricing electricity risk by interest rate methods, *Quantitative Finance* **5**(1): 49–60.

Hirshleifer, D. (1991). Seasonal patterns of futures hedging and the resolution of output uncertainty, *Journal of economic theory* **53**(2): 304–327.

Hirukawa, M. (2023). Robust covariance matrix estimation in time series: A review, *Econometrics and Statistics* **27**: 36–61.

Jaeck, E. and Lautier, D. (2016). Volatility in electricity derivative markets: The samuelson effect revisited, *Energy Economics* **59**: 300–313.

Keles, D., Genoese, M., Möst, D. and Fichtner, W. (2012). Comparison of extended mean-reversion and time series models for electricity spot price simulation considering negative prices, *Energy economics* **34**(4): 1012–1032.

Kiesel, R., Schindlmayr, G. and Börger, R. H. (2009). A two-factor model for the electricity forward market, *Quantitative Finance* **9**(3): 279–287.

Knittel, C. R. and Roberts, M. R. (2005). An empirical examination of restructured electricity prices, *Energy Economics* **27**(5): 791–817.

Kougias, I. and Szabó, S. (2017). Pumped hydroelectric storage utilization assessment: Forerunner of renewable energy integration or trojan horse?, *Energy* **140**: 318–329.

Laudagé, C., Aichinger, F. and Desmettre, S. (2024). A comparative study of factor models for different periods of the electricity spot price market, *Journal of Commodity Markets* **36**: 100435.

Liao, Z. (2013). Adaptive gmm shrinkage estimation with consistent moment selection, *Econometric Theory* **29**(5): 857–904.

Liu, H. and Shi, J. (2013). Applying arma–garch approaches to forecasting short-term electricity prices, *Energy Economics* **37**: 152–166.

Lucia, J. J. and Schwartz, E. S. (2002). Electricity prices and power derivatives: Evidence from the nordic power exchange, *Review of derivatives research* **5**: 5–50.

Mancini, C. (2004). Estimation of the characteristics of the jumps of a general poisson-diffusion model, *Scandinavian Actuarial Journal* **2004**(1): 42–52.

Mancini, C. (2009). Non-parametric threshold estimation for models with stochastic diffusion coefficient and jumps, *Scandinavian Journal of Statistics* **36**(2): 270–296.

Miltersen, K. R. and Schwartz, E. S. (1998). Pricing of options on commodity futures with stochastic term structures of convenience yields and interest rates, *Journal of financial and quantitative analysis* **33**(1): 33–59.

Nasdaq, O. (2024). Contract specifications–commodity derivatives.

Newey, W. K. and Smith, R. J. (2004). Higher order properties of gmm and generalized empirical likelihood estimators, *Econometrica* **72**(1): 219–255.

Nomikos, N. and Soldatos, O. (2008). Using affine jump diffusion models for modelling and pricing electricity derivatives, *Applied Mathematical Finance* **15**(1): 41–71.

Paulsen, S. K. and Bergsholm, A. S. (2020). *Efficiency in the nordic futures power market: an empirical study of the nordic futures power market*, Master's thesis, Norwegian School of Economics.

Pepermans, G. (2019). European energy market liberalization: Experiences and challenges, *International Journal of Economic Policy Studies* **13**(1): 3–26.

Phillips, P. C. and Yu, J. (2009). Maximum likelihood and gaussian estimation of continuous time models in finance, *Handbook of financial time series*, Springer, pp. 497–530.

Pirrong, C. (2011). *Commodity price dynamics: A structural approach*, Cambridge University Press.

Rasmussen, P. B. H. and Stensland, G. (2000). Valuation and risk management in the norwegian electricity market.

Salmela, V. et al. (2020). *The impact of european emission allowance prices on nordic electricity wholesale prices*, Master's thesis, Hanken School of Economics.

Sanda, G. E., Olsen, E. T. and Fleten, S.-E. (2013). Selective hedging in hydro-based electricity companies, *Energy Economics* **40**: 326–338.

Sauer, T. (2011). Numerical solution of stochastic differential equations in finance, *Handbook of computational finance*, Springer, pp. 529–550.

Størdal, S., Ewald, C.-O., Lien, G. and Haugom, E. (2023). Trading time seasonality in electricity futures, *Journal of Commodity Markets* **31**: 100291.

Tankov, P. (2003). *Financial modelling with jump processes*, Chapman and Hall/CRC.

Wang, Q. and Wu, N. (2012). Long-run covariance and its applications in cointegration regression, *The Stata Journal* **12**(3): 515–542.

Wei, W. and Lunde, A. (2023). Identifying risk factors and their premia: a study on electricity prices, *Journal of Financial Econometrics* **21**(5): 1647–1679.

10. Appendix

10.1 Forecast performance - BIC

On this subsection we report the forecast performance comparison using the mean absolute error of the ARMA-Garch[13] model whose orders are selected using the information criteria BIC.

[13]The ARMA-Garch orders selected according to BIC criterion for P_Y, P_{Q1}, P_{Q2}, P_{Q3} and P_{Q4} were respectively (2,4,1,1), (5,7,1,1), (3,3,1,1), (4,3,1,1) and (10,5,1,1)

Table 24
Results of the confidence set test results for 1 day-ahead forecasts from 02/01/2020 to 29/12/2023

1D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	1.975	3.911	-3.908^{***}	< 0.01
P_{Q1}	3.570	5.519	-3.857^{***}	< 0.01
P_{Q2}	2.010	2.570	-3.870^{***}	< 0.01
P_{Q3}	1.179	1.367	-3.528^{***}	< 0.01
P_{Q4}	1.554	1.650	-1.245	0.213
Curv	2.058	3.003	-4.079^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 1 day-ahead forecasts of forward electricity contracts prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 02/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The model orders for the ARMA-Garch are chosen using BIC information criteria.

Table 25
Results of the confidence set test results for 5 days-ahead forecasts from 08/01/2020 to 29/12/2023

5D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	5.895	14.289	-6.860^{***}	< 0.01
P_{Q1}	11.014	22.563	-4.254^{***}	< 0.01
P_{Q2}	6.060	8.870	-5.584^{***}	< 0.01
P_{Q3}	3.193	4.332	-3.967^{***}	< 0.01
P_{Q4}	4.877	4.296	1.898^{*}	0.052
Curv	6.208	10.870	-4.748^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 5 days-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 08/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year.$***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The ARMA-Garch order is chosen using BIC information criteria

Table 26
Results of the confidence set test results for 10 days-ahead forecasts from 15/01/2020 to 29/12/2023

10D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	9.462	22.266	-5.228^{***}	< 0.01
P_{Q1}	18.043	36.342	-6.053^{***}	< 0.01
P_{Q2}	9.565	14.389	-5.030^{***}	< 0.01
P_{Q3}	4.916	6.915	-4.580^{***}	< 0.01
P_{Q4}	7.730	6.387	-2.459^{**}	0.014
Curv	9.943	17.260	-5.177^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 10 days-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 15/01/2020 until 29/12/2023. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The ARMA-Garch are chosen using BIC information criteria.

Table 27
Results of the confidence set test results for 1 day-ahead forecasts from 02/01/2020 to 30/12/2021

1D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	0.858	0.993	-5.978^{***}	< 0.01
P_{Q1}	1.444	1.529	-2.581^{**}	0.010
P_{Q2}	0.874	0.939	-5.430^{***}	< 0.01
P_{Q3}	0.658	0.709	-6.657^{***}	< 0.01
P_{Q4}	0.542	0.539	0.316	0.759
Curv	0.875	0.942	-6.668^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 1 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 02/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in ith quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The ARMA-Garch orders are chosen using BIC information criteria.

Table 28
Results of the confidence set test results for 5 days-ahead forecasts from 08/01/2020 to 30/12/2021

5D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	2.189	3.212	-7.796^{***}	< 0.01
P_{Q1}	3.799	4.613	-4.236^{***}	< 0.01
P_{Q2}	2.282	2.868	-5.520^{***}	< 0.01
P_{Q3}	1.737	2.158	-6.335^{***}	< 0.01
P_{Q4}	1.454	1.452	0.051	0.959
Curv	2.292	2.861	-7.940^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 5 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 08/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in *ith* quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The ARMA-Garch order is chosen using BIC information criteria.

Table 29
Results of the confidence set test results for 10 days-ahead forecasts from 15/01/2020 to 30/12/2021

10D	Cont.	ARMA-Garch	Test. Stat	p-value
P_Y	3.194	5.138	-8.761^{***}	< 0.01
P_{Q1}	5.511	6.992	-5.318^{***}	< 0.01
P_{Q2}	3.312	4.614	-4.972^{***}	< 0.01
P_{Q3}	2.623	3.549	-10.196^{***}	< 0.01
P_{Q4}	2.071	2.097	-0.280	0.778
Curv	3.342	4.478	-11.307^{***}	< 0.01

This table reports estimates of the mean absolute errors (MAE) of 10 day-ahead forecasts of forward electricity contract prices and test statistic values of confidence set test developed by (Hansen et al., 2011). Model parameters are estimated using daily log returns of quarterly and one year-ahead forward electricity from 03/01/2005 until 30/12/2019 and the forecasting period is from 15/01/2020 until 30/12/2021. P_{Qi} is the price of the forward electricity contract with delivery in *ith* quarter of the following year and P_Y is the price of the forward electricity contract with delivery in the following year. $***$, $**$, and $*$ denote statistical significance at the 1%, 5% and 10% levels, respectively. The ARMA-Garch order is chosen using BIC information criteria.