Carry Trade Returns and Portfolio Holdings*

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Abstract

We find in aggregate portfolio holdings data that carry traders are outweighed by other investors in short-term debt markets. We interpret this finding as evidence of investor heterogeneity. We study the joint determination of individual and aggregate carry trade positions and carry trade returns in a general equilibrium model with investor heterogeneity in risk aversion. We study the asset pricing implications of the model using standard asset pricing tests, and find empirical support for key asset pricing and portfolio predictions that the model makes.

Keywords: ***carry trade, portfolio holdings, risk aversion, heterogeneity, asset pricing***

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1 Introduction

The carry trade has two parts: an investor takes a carry trade position, then the investor earns a carry trade return. Much of the existing finance research on the carry trade seeks to rationalize the return, but a satisfying theory of the carry trade should rationalize both the position and the return. When the carry trade position is brought into the scope of analysis alongside the return, the analysis becomes more challenging. Empirically, carry traders appear to be outweighed by other investors in short-term debt markets, and the proposition that carry traders are representative agents becomes difficult to support.

Consumption-based theories of the carry trade often see the carry trader as a representative agent whose consumption patterns coincide with the consumption patterns of the aggregate economy, but our results suggest that carry traders consume in a ways that differ systemically from the majority of investors in most countries. In settings where investors choose between portfolio positions that promise higher average but more volatile income and portfolio positions that promise lower average but less volatile income, we show that investors with greater risk tolerance will maximize utility by choosing portfolios that offer higher average but more volatile income. This fact remains true in theory even in economies where all investors are strictly risk averse. In empirically-relevant calibrations of our theoretical framework, the risk-tolerant investors are carry traders, and we believe, therefore, that carry traders have higher average and more volatile consumption than the average investor in most countries.

Heterogeneity in consumption can cause quantity-of-risk estimates to vary greatly in consumption CAPM regressions, depending on the particular consumption aggregate that appears in the regression. For the most economically-meaningful quantity-of-risk estimates, we recommend using a consumption aggregate for the group of investors that is most likely to take carry trade positions, and our theoretical framework identifies these investors as high-income households.

To illustrate simply the effect of heterogeneity on quantity-of-risk estimates in a consumption CAPM framework, consider two investor types who live in the same economy, earn the same wage, but take opposite investment positions in home and foreign real bonds. We present a richer model in Section 3 where investors choose these positions endogenously, but here we fix ideas in a simpler setting.[1] Investors of the first type, risk-tolerant investors with index ρ_H, take carry trade positions that are expected to earn, and that do earn on average positive real returns. Investors of the second type, risk-averse investors with index ρ_L, take the opposite of carry trade positions—what might be call *drop trade* positions—that are expected to earn, and that do earn on average negative real returns. For concreteness, define the carry trade position as a long position in one unit of the foreign bond and a short position in one unit of the home bond, and a define a drop trade position

[1]The setting here can be derived from a log-linear approximation to standard budget constraints for investors with heterogeneous coefficients of relative risk aversion in the fully-articulated DSGE model that we present in Section 3, by assuming exogenous and fixed real portfolio holdings that have been chosen for convenience to equal investors' inverse subjective discount factors. We will soon provide an appendix with a fuller description of the simplifications that lead to the setting that we present here.

as the opposite of that. Notice that risk-tolerant investors have higher average real income than risk-averse investors, because of the positive average carry trade real return. Consumption for individual investors of each type and for the aggregate economy are given below in log-deviations from steady-state values:

$$C_t(\rho_H) \approx Y_t + R_{Xt}\,, \quad C_t(\rho_L) \approx Y_t - R_{Xt}\,, \quad \text{and} \quad C_t \approx \omega_H C_t(\rho_H) + \omega_L C_t(\rho_L)\,, \tag{1}$$

where Y_t denotes the log-deviation of real income from its steady-state value, R_{Xt} denotes the carry trade real return, defined as the home real return minus the foreign, and ω_H and ω_L denote weights for the risk-tolerant and risk-averse investor types, respectively. The weights sum to one. We assume that the carry trade real return is positive on average, so that risk tolerant investors of type ρ_H are indeed carry traders. We also assume that real income and the carry trade real return have positive covariance.

The consumption-based capital asset pricing model applied to the carry trade often uses aggregate consumption growth to estimate the quantity of risk that a representative carry trader experiences. The simplest model can be written as

$$R_{Xt} = \beta C_t + \epsilon_t\,.$$

Even in a setting with only two investor types, the theoretical relationship between returns and aggregate consumption growth that underlies this regression relies on an approximation, if standard household preferences are assumed. The approximation may still provide valid asset pricing predictions, but particularly in the context of the carry trade, care should be taken in assigning economic interpretations to the quantity-of-risk estimates that such a regression provides. The regression provides quantity-of-risk estimates for a fictional composite investor who would experience a quantity of risk from the carry trade that may bear little resemblance to the quantity of risk that true carry traders experience. To take an extreme case, if the weights of each investor type satisfy $\omega_L - \omega_H = \text{var}(R_{Xt}) / \text{cov}(Y_t, R_{Xt})$, then the standard consumption-based asset pricing model will estimate a quantity-of-risk parameter $\beta = 0$. In the empirically-relevant case where carry traders are outweighed by other investors, $\omega_L > \omega_H$, the quantity-of-risk parameter $\beta = 0$ obtains despite the fact that carry traders experience a positive quantity of risk $\beta_H > 0$.

The standard consumption CAPM regression still prices the carry trade correctly in our model with heterogeneity in risk aversion, but the heterogeneity changes the economic interpretation of the quantity-of-risk parameter β that the regression estimates. We caution against interpreting the quantity-of-risk estimate from standard regression with aggregate consumption growth as the quantity of risk that carry traders experience, because the consumption of carry traders differs systematically from the consumption of other investors. By introducing heterogeneity in risk aversion into the standard framework, we can study formally how an investor's optimal portfolio position and quantity of risk vary with the investor's risk aversion, we can identify specific investors for whom carry trades are optimal, and we can study how these carry traders differ from other investors in the economy in terms of income and consumption.

Importantly, individual and aggregate investment behavior can differ meaningfully in our model. By introducing heterogeneity, we are able to study individual carry traders in economies that feature no aggregate carry trade positions. This feature of our model allows us to bridge a gap between the finance literature on carry trade returns and the international macroeconomics literature on aggregate portfolio holdings. The finance literature commonly uses representative-agent models to study carry trade returns, where the representative agent is assumed to be a carry trader because otherwise no carry trade would occur. In contrast, the international macroeconomics literature uses representative-agent models to study the conditions under which home bias in aggregate portfolio holdings arises, where the representative agent must be a home-bias investor irrespective of the relative real returns on home and foreign assets. In both literatures, no meaningful distinction can be made between individual and aggregate investment positions. In our setting with investor heterogeneity, individual carry traders can be outweighed by other investors within country, and the individual carry trade—and indeed a range of other individual portfolio positions—can be studied jointly with the aggregate portfolio position of the economy.

We make our case for investor heterogeneity in Section 2 by examining short-term debt holdings in country-level data. The fundamental premise of our analysis is that at least some carry traders exist in most countries. Starting from this premise, we conclude that carry traders must be outweighed by other investors in short-term debt markets, because carry trades are rarely discernible in aggregate data on short-term debt holdings at the country level. In Section 3 we construct a theoretical two-country general equilibrium endowment economy with household heterogeneity in risk aversion, and we study the carry trade in this setting. The model is tractable, and allows us to derive approximate closed-form solutions for individual and aggregate consumption, for the investment positions of individual investors and the aggregate economy, and for the carry trade real return. We find in the model that consumption growth for carry traders differs systematically from consumption growth for the aggregate economy. In the model, carry trade real returns are driven by disturbances to total factor productivity and money supply in each country. In summary, we argue empirically that carry traders represent a minority of investors, we model them as such, we derive the portfolio and pricing implications of our carry trade model, and test the implications empirically.

2 Aggregate Short-Term Debt Positions

In this section we study aggregate short-term debt holdings at the country level. We start from the following observation: that if the majority of a country's individual investors were to take carry trade positions using the same few funding currencies, then we would see negative aggregate net positions in short-term debt in these funding currencies. In the data we almost never see this.

Because aggregate net positions are the sum of individual gross short and gross long positions, and aggregate net positions are essentially always positive, it seems that individual investors taking long positions either out-number or out-weigh the individual investors within country taking short positions in the same short-term debt markets. Hence, we argue that carry traders cannot represent a majority of investors and should not be modeled as representative agents. Before making this argument in detail, we present empirical evidence on aggregate short-term debt holdings at the country level that leads us to believe that most investors are not carry traders.

We begin by defining several terms related to the investment positions that investors take in short-term debt markets. We define an investment position as the value of a portfolio of assets. We distinguish between aggregate and individual positions, and between net and gross positions. Aggregate positions are the sum of individual positions, and net positions are the sum of gross long and gross short positions. Long positions are always positive or zero, and short positions are always negative or zero.

Country i's aggregate net position in short-term debt issued by country j in currency k, denoted B^k_{ijt}, is defined as the sum of country i's aggregate gross long position in the short-term debt of country j, denoted $B^{(+)}_{ijt}$, and country i's aggregate gross short position in the short-term debt of country j, denoted $B^{(-)}_{ijt}$. The aggregate net position of country i in short-term debt of country j can therefore be written as

$$B^k_{ijt} = \sum_{\rho \in \mathcal{H}_i} B^k_{ijt}(\rho) = \sum_{\rho \in \mathcal{H}_i} B^{(-)}_{ijt}(\rho) + \sum_{\rho \in \mathcal{H}_i} B^{(+)}_{ijt}(\rho) = B^{(-)}_{ijt} + B^{(+)}_{ijt} \,, \tag{2}$$

where $\mathcal{H}_i$ denotes the set of investors in country i, $B^k_{ijt}(\rho)$ denotes the individual position of investor ρ from country i in short-term debt of country j, and $B^{(-)}_{ijt}(\rho)$ and $B^{(+)}_{ijt}(\rho)$ denote the individual gross short and aggregate gross long positions of investor ρ from country i in short-term debt of country j, respectively. For countries that report aggregate gross short and aggregate net positions, we can impute their aggregate gross long position in the short-term debt of countries. We can then compute short ratios for these countries. We define the short ratio for country i's position in the short-term debt of country j as

$$SR_{ijt} = \frac{-B^{(-)}_{ijt}}{B^{(+)}_{ijt} - B^{(-)}_{ijt}} \,. \tag{3}$$

Our first piece of evidence comes from a set of six countries that report, as an unbalanced panel, their aggregate gross short positions and their aggregate net positions in short-term debt with 236 counterparty countries over an eight-year period in the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS). From the reported aggregate gross short and aggregate net positions, we can compute aggregate gross long positions, denoted $B^{(+)}_{ijt}$. Of the six reporting countries, only Germany and the Cayman Islands report substantial aggregate gross short positions; the remaining countries report little to no short positions over the sample period. We find that short ratios often equal one-half, indicating that individual investors within a country often take mutually offsetting short and long positions with a particular counterparty country, such that the aggregate net position is zero with that counterparty country. These cases of mutually offsetting long and short positions suggest that within-country investor heterogeneity may be common. We argue in Section 1 and later in our theoretical model in Section 3 that exactly this type of heterogeneity changes the interpretation of quantity-of-risk estimates from consumption-CAPM regressions that use aggregate consumption.

Our second piece of evidence comes from a broader set of 86 countries that report their aggregate net positions in short-term debt with 236 counterparty countries, but not their aggregate gross short positions. For this broader set of countries, we find strikingly few cases of negative aggregate net holdings of short-term debt—only 183 out of 181 731 reported aggregate net positions in short-term debt are negative in our data. Again, we reason that if a majority of individual investors were to take carry trade positions in the same few funding currencies, then we would see negative aggregate net positions with the counterparty countries that issue short-term debt in these funding currencies. But we nearly never observe negative aggregate net positions in the data. One concern is that countries misreport or under-reported short positions, which could explain why negative aggregate net positions are so rare. But we know that many countries updated their reporting methodology for short positions upon the release in 2009 of the sixth edition Balance of Payments Manual (International Monetary Fund, 2009), and in the post-2009 period we still observe strikingly few negative aggregate net holdings of short-term debt.

Our third piece of evidence comes from the strong home currency bias in short-term debt holdings that we estimate for an even broader cross section of 202 countries with 229 counterparty countries. The home-currency bias that we estimate from the aggregate country-level data corroborates the bias that Maggiori et al. (2018) document in granular security-level data. Currency holdings must be estimated in our case, because holdings are not reported by currency in the CPIS data. The home-currency bias that we estimate is strong, so most countries' aggregate net holdings of foreign-currency short-term debt are small relative to their aggregate net holdings of domestic-currency short-term debt. Because the carry trade is a net-zero position requiring gross short and gross long positions of equal absolute value, the carry trade alone cannot produce the home-currency bias that we estimate from the data. We also highlight a potentially new stylized fact in our estimates of aggregate net holdings by currency, namely a tendency for countries with lower domestic interest rates, like Japan, to exhibit greater home-currency bias than countries with higher domestic interest rates, like Australia. If this fact proves robust, it would further suggest that factors other than carry trade returns influence the portfolio positions that investors take.

Our fourth piece of evidence comes from the signs of "centered" aggregate net positions in short-term debt. Theoretical models of international investment in bond markets often assume two symmetric countries, each with a tradable domestic bond in zero net supply, so that aggregate short positions arise naturally whenever countries trade bonds. In order to connect these two-country theoretical models to our empirical estimates, we collapse our estimates of country-level short-term debt holdings by currency into a two-by-two matrix of bilateral holdings of short-term debt denominated in a "home" currency and a composite "foreign" currency. We designate the United States as home country, and we designate an aggregate of all other countries as the foreign country. From the collapsed two-by-two matrix of bilateral short-term debt holdings, we then compute a model-consistent "centered" matrix of bilateral short-term debt holdings for home and foreign. We derive our centering procedure in Appendix C. We show that, after centering, the United States takes a positive centered aggregate position in domestic short-term debt and a negative centered aggregate position in foreign short-term debt. Because foreign short-term debt offers a higher real return, the centered position of the United States is exactly the opposite of an aggregate carry trade, again suggesting that carry traders should not be modeled as a homogeneous majority of investors. Appendix C is complete, but the subsection here in the body of the paper that applies the procedure outlined in Appendix C is incomplete.

In the following four subsections, we present in greater detail the evidence on aggregate portfolio positions in short-term debt. To summarize the evidence, we find that gross short positions in short-term debt nearly never exceed gross long positions in short-term debt in absolute value, and therefore that aggregate net positions are nearly never negative. Starting from the premise that at least some investors are carry traders, and knowing that carry trades in debt markets entail short positions, we interpret the infrequency of negative aggregate net positions as evidence that carry traders represent a minority of investors. In our view, the evidence suggests heterogeneity in the investment positions that individual investors take, and this heterogeneity can cause problems for representative agent models that aim to explain carry trade portfolio positions and carry trade returns endogenously and simultaneously. At their root, these problems arise because representative agent models remove any meaningful distinction between individuals and aggregates, and aggregate portfolios do not resemble aggregate carry trades in the data.

2.1 Aggregate Gross Short Positions by Foreign Country of Issuance

Six countries report their aggregate gross short positions in foreign short-term debt separately from their aggregate net positions in CPIS. These countries report their positions annually, by counterparty country, for up to 236 counterparty countries over a period of up to eight years. However, most reported aggregate gross short positions are zero-valued for most counterparty countries in most years. In total, non-zero gross short positions are reported with only 61 of 236 of the counterparty countries in the data. Of the six countries that report, only two countries report any significant positions: the Cayman Islands and Germany. The remaining countries Aruba, Belgium, Bulgaria, and Cyprus report either zero or small and infrequent aggregate gross short positions in foreign short-term debt. The evidence from the Cayman Islands and Germany may nevertheless generalize to other offshore financial centers and other traditional developed economies.

Holding Country	Average Annual			
	Aggregate Gross Short Positions (in USD millions)	Short Ratio (in %)	Count of Aggregate Gross Short Positions	
			Non-Zero	Non-Missing
Cayman Islands	−1 095	15	28	154
Germany	−3	3	5	235

Table 1 – **Reported Aggregate Gross Short Positions in Short-Term Debt, 2015–2019.** The table shows information on aggregate gross short positions reported in the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS) by two countries over the years 2015 to 2019. The first column of the table lists the main countries that report holding short positions in CPIS. The second column shows the average size of the aggregate gross short positions that each country reports against counterparty countries each year, averaged over counterparties within years, then averaged over years. The average aggregate gross short positions are listed in USD millions. The third column shows each country's average short ratio, defined as $-1 \times short/(long - short)$, computed pairwise for each reporting country and counterparty, then averaged across counterparties and over years for each reporting country, where *short* denotes an aggregate gross short position and *long* denotes an aggregate gross long position. The fourth column shows the number of non-zero gross short positions that reporting countries report each year, averaged over years for each reporting country. The fifth column shows the number of non-missing gross short positions that reporting countries report each year, averaged over years for each reporting country. Most of the non-missing reported gross short positions are zero-valued. Values have been rounded to the nearest integer.

For example, one might expect the differences in short-taking activity between the Cayman Islands and Germany to resemble in some respects the differences between, say, Bermuda and France. Whether for an offshore financial center or a traditional developed economy, the evidence that we have on aggregate gross short positions suggests that negative aggregate *net* positions in foreign short-term debt are rare.

Table 1 summarizes the reported aggregate gross short positions in short-term debt of the Cayman Islands and Germany over the period 2015–2019. The aggregate gross short positions reported by the Cayman Islands stand out in terms of their value and their frequency. Averaging across counterparty countries and over years, we find that the Cayman Islands report an average annual aggregate gross short position that is 350 times greater in value than that of Germany. Averaging the total number of non-zero aggregate gross short positions reported by each country with all counterparty countries over years, we find that the Cayman Islands take on average six times as many aggregate gross short positions as Germany. Relative to their corresponding gross long positions, the aggregate gross short positions taken by the Cayman Islands are also much greater in value than those taken by Germany, as indicated by the short ratio in column two in Table 1. Recall from equation (3) that the short ratio takes values between zero and one, and measures the absolute value of a country's aggregate gross short position relative to the absolute value its short and long positions with a particular counterparty. If a country's aggregate gross short position with a particular counterparty exceeds its aggregate gross long position with that counterparty in absolute value, then the country's short ratio will exceed one half, and the country's aggregate net position will be negative with that counterparty. On average, the short ratios of

Rank	Cayman Islands' Top Destinations for Gross Short Positions		Germany's Top Destinations for Gross Short Positions	
	Issuing Country	Cumulative Gross Short Position (in USD millions)	Issuing Country	Cumulative Gross Short Position (in USD millions)
1	United States	−253 070	France	−1 778
2	United Kingdom	−42 899	United States	−692
3	Germany	−41 506	Netherlands	−564
4	Italy	−31 115	Italy	−341
5	France	−28 423	Spain	−213

Table 2 – Top Destinations for Short-Taking by Value of Reported Cumulative Gross Short Positions in Short-Term Debt for the Cayman Islands and Germany, 2015–2019. The table shows for the Cayman Islands and Germany the five top destinations for gross short positions, ranked by cumulative value of reported gross short positions over the period 2015–2019. Cumulative gross short positions are computed by summing over years the aggregate gross short positions with each counterparty country that the Cayman Islands and Germany report to the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS) between 2015 and 2019. The first column of the table ranks issuing countries by the cumulative value of reported gross short positions in short-term debt that the Cayman Island and Germany report. For the Cayman Islands, the second column names the top issuing countries and the third column shows the reported value of the reported cumulative gross short positions in USD millions. For Germany, the same information is shown in columns four and five.

the Cayman Islands are five times those of Germany, but far lower than one half. Hence, negative aggregate net positions are rare, suggesting that carry traders represent a minority of investors in the Cayman Islands and Germany.

Table 2 shows the top destination countries for the aggregate gross short positions that the Cayman Islands and Germany report in CPIS. The destination countries are ranked by the cumulative value of the aggregate gross short positions that the Cayman Islands and Germany report over the years 2015–2019. Japan is notably absent from the lists of top-five destinations for both countries, despite that attractiveness of Japanese short-term debt as a low-cost funding asset for carry trades. Gagnon and Chaboud (2007) find mixed evidence for the existence of substantial carry trade positions in Japanese sectoral data, and and we also find limited evidence of substantial short positions in Japanese short-term debt that are not offset by equal or larger long positions within country. This result is perhaps unsurprising, if you think that investors have motivations other than earning carry trade returns, and again points to investor heterogeneity, if we are to assume that at least some investors take carry trade positions funded by short positions in Japanese short-term debt.

Because the carry trade requires short and long positions of equal absolute value, we can gain additional insight into the importance of carry trading relative to other investment activity by comparing aggregate gross short positions with the corresponding aggregate gross long positions that the Cayman Islands and Germany report in CPIS, again using the short ratio defined in Equation (3). Short positions are negative or zero by definition, so the short ratio, where it exists, takes values between zero and one. The short ratio takes a value of zero when the short position

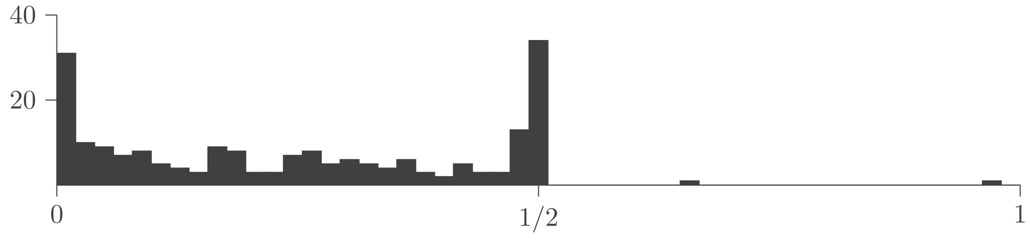

Figure 1 – **Short Ratios for Non-Zero Gross Short Positions in Short-Term Debt for All CPIS Reporting Countries, 2013–2020.** The figure shows a 51-bin histogram of short ratios computed from all non-zero gross short positions in short-term debt of the six countries that reported gross short positions in the International Monetary Fund's Coordinated Portfolio Investment Survey during the years 2013 to 2020. The six countries are Aruba, Belgium, Bulgaria, the Cayman Islands, Cyprus, and Germany. We define the short ratio in Equation (3) as the absolute value of an aggregate gross short position divided by the absolute values of the aggregate gross short position and corresponding aggregate gross long position, computed pairwise for each reporting country and counterparty country each year. Short positions are negative or zero, so the short ratio, when it exists, takes values between zero and one. A short ratio of zero implies a gross long position and no off-setting gross short position. A short ratio of one-half implies a gross long position and an off-setting gross short position of equal absolute value. A short ratio of greater than one-half implies a negative net position. We emphasize non-zero gross short positions by excluding from this histogram the point mass of short ratios that equal zero exactly; these cases represent a majority of cases.

is zero and the corresponding long position is positive, a value of one-half when short and long positions are non-zero and equal in absolute value, and a value of one when the short position is negative and the corresponding long position is zero. In the CPIS data, nearly all short ratios equal one-half or less.

Figure 1 shows a 51-bin histogram of short ratios computed from all non-zero gross short positions in short-term debt for the six countries that reported gross short positions in the International Monetary Fund's Coordinated Portfolio Investment Survey during the years 2013 to 2020. Note that zero-valued reported aggregate short positions are excluded from the computations that produce the histogram, and zero-valued positions are the majority of positions reported by the six reporting countries. Figure 1 shows that when short positions are taken, short ratios of around one-half are quite common, indicating that many of the gross short positions in short-term debt that countries report are approximately off-set within country by equal and opposite reported gross long positions. These cases indicate within-country heterogeneity in portfolio positions, and pose potential problems for the interpretation of asset pricing results in representative agent settings. We almost never observe short ratios above one-half, indicating that aggregate gross short positions without offsetting long are rare in the data.

2.2 Aggregate Net Positions by Foreign Country of Issuance

The CPIS data on aggregate gross short and aggregate gross long positions paint a clear picture of aggregate investment behavior at the country level, but only for a limited cross-section of reporting countries. In order to study a broader set of 86 reporting countries, we now consider aggregate net holdings of foreign short-term debt, rather than aggregate gross short and long positions.

Reported Aggregate Net Positions	Of Which, Positions With Negative Values			
	in % of Total Count		in % of Total Value	
	2001–09	2010–19	2001–09	2010–19
With Non-Missing Values	0.06	0.12	0.01	0.09
With Non-Zero Values	0.33	0.71	0.01	0.09
With Non-Zero Values in Japanese Debt	0.48	0.65	0.00	0.00

Table 3 – **Relative Counts and Relative Values of Reported Negative Aggregate Net Holdings of Foreign Short-Term Debt, 2001–2019.** The table describes relative counts and relative values of negative aggregate net positions in foreign short-term debt that countries report in CPIS over the period 2001 to 2019. Countries report their aggregate net positions in CPIS by counterparty country and year. Columns two and three show relative counts of reported positions. Row one in columns two and three shows the count of negative aggregate net positions as a percentage of the count of all non-missing aggregate net positions. Row two in columns two and three shows the count of negative aggregate net positions as a percentage of the count of all non-missing and non-zero aggregate net positions. Row three in columns two and three shows the count of negative aggregate net positions in Japanese short-term debt as a percentage of the count of all non-missing and non-zero aggregate net positions in Japanese short-term debt. Columns four and five show relative values of reported positions. Row one in columns four and five shows the absolute value of negative aggregate net positions as a percentage of the value of all non-missing aggregate net positions. Row two in columns four and five shows the absolute value of negative aggregate net positions as a percentage of the value of all non-missing and non-zero aggregate net positions. Row three in columns four and five shows the absolute value of negative aggregate net positions in Japanese short-term debt as a percentage of the value of all non-missing and non-zero aggregate net positions in Japanese short-term debt. Relative counts and relative values are each computed for the ten years before and after the publication of the Sixth Edition Balance of Payments Manual International Monetary Fund (2009) that establishes new reporting standards for short positions.

Fewer conclusions about individual investor behavior can be drawn from evidence on aggregate net portfolio holdings than from the evidence on aggregate gross short and aggregate gross long positions, but some patterns that we observe in aggregate net portfolio holdings would be difficult to reconcile with widespread carry trading among individual investors. In particular, for the broader set of 86 reporting countries, aggregate net portfolio holdings in foreign short-term debt are nearly never negative. This result suggests that individual carry traders represent a minority of investors in most countries. Again, we reason that if a majority of individual investors funded carry trades by short-selling short-term debt denominated in the same few funding currencies, then we would see negative aggregate net portfolio holdings with the counterparty countries that issue debt in funding currencies.

Table 3 shows relative counts and relative values of reported negative aggregate net holdings of foreign short-term debt over the period 2000 to 2019. The most important take-away from Table 3 is that negative aggregate net positions in foreign short-term debt are extremely rare in the country-level CPIS data. Over the full twenty-year sample period, countries report 169 732 non-missing aggregate net portfolio positions, of which 29 207 are non-zero, and only 163 are negative. The first two rows of Table 3 show that the number and absolute value of the negative aggregate net positions that countries report in CPIS equal a small percentage of the number and value of all non-missing or non-zero aggregate net positions that countries report—the percentages are less than 1% in all cases, and even less than 0.1% in some cases. The third row of Table 3 shows that negative aggregate net positions are small and rare even in Japanese short-term debt, where we would most expect to observe them, given the history of low interest rates in Japan.

One concern is that we observe infrequent negative aggregate net portfolio holdings in the CPIS data because short positions are inconsistently reported or under-reported. While the accurate reporting of short positions remains a concern, especially in countries that conduct surveys of custodian banks to collect information for CPIS (see Taub, 2008, for a discussion of related issues), we argue that improved reporting standards have lessened reporting problems in recent years. In particular, we identify the year 2010 as a turning-point, because in late 2009 the IMF introduced new standards for reporting short positions in international financial statistics. Moving from column two to column three and from column four to column five, Table 3 shows that the relative counts and relative values of negative aggregate net positions in foreign short-term debt increased noticeably after the introduction of the IMF reporting standard in 2009. We interpret the increase in relative counts and relative values as evidence of improved short position reporting in the second half of the sample period.

The main IMF reporting standard for international financial statistics are published in two documents: the Balance of Payments Manual (BPM) and the CPIS Survey Guide (CSG). The BPM is the more consequential and established of the two standards, it was first published in 1948, and it governs broadly the reporting of international financial statistics at the country level. The CSG is narrower in scope, and younger than the BPM. Until the turn of the century, no definitive reporting standard for short positions appeared in either of these two publications: the fifth edition BPM makes no mention of short positions whatsoever (International Monetary Fund, 1993), and the first edition CSG mentions and describes possible methods for reporting short positions, but provides no definitive standard (International Monetary Fund, 1997, papa. 93 and Box 4). After the turn of the century, the second edition CSG published clearer guidance: "If (when) the security is on-sold, the "borrower" of the security should record a "short" position" (International Monetary Fund, 2002, para. 3.78), and then, seven years later, the sixth edition BPM established the definitive standard that applies today:

> Short positions occur when an institutional unit sells securities for which it is not the economic owner. [...] Delivery to the purchaser is made through the use of a borrowed security. The party with the short position records a negative value for the holding of the asset. The short position is shown as a negative asset, rather than a liability. (International Monetary Fund, 2009, para. 7.28)

The most recent third edition CSG now cites the sixth edition BPM when describing the CPIS reporting standard for short positions (Josyula, 2018, para. 3.52).

While 2009 marks the introduction of a definitive standard for the reporting of short positions in international financial statistics, the reporting standard was not fully implemented immediately by all reporting countries. Press releases and country-level reporting guides from the years following

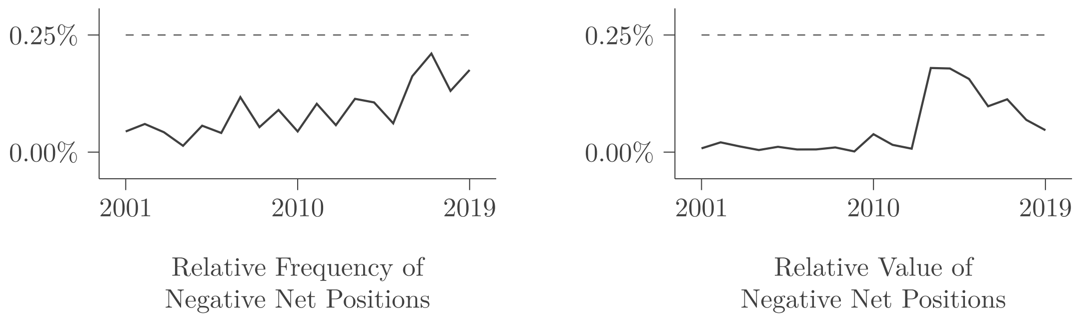

Figure 2 – Relative Count and Relative Value of Negative Net Positions in Foreign Short-Term Debt, 2000–2019. The figures show the relative count and relative value of reported negative net positions in short-term debt, aggregated across all countries that report to CPIS in each reporting year. We define the aggregate net position as the gross long position plus the gross short position, the latter being negative or zero. The relative frequency figure on the left shows the number of negative aggregate net positions that countries report each year, divided by the number of all aggregate net positions that countries report each year. The relative value figure on the right shows the total value of negative aggregate net positions that countries report each year, divided by the total value of all aggregate net positions that countries report each year, and multiplied by minus one.

the publication of the sixth edition BPM indicate that the standard had been implemented by many countries by 2015.[2] For example, the ECB announced that Eurostat would disseminate statistics under the new standard beginning in October 2014 (European Central Bank, 2014). Figure 2 shows a rise in relative counts and relative values of reported short positions in the years following the publication of the new reporting standard in 2009. In light of this evidence, we emphasize the period from 2015 to 2019 in the rest of our analysis of the CPIS data, the period when we are most confident that short positions are consistently and properly reported by the greatest number of CPIS countries.

We emphasize one important take-away from this discussion of changing reporting standards and the evolution of reported negative aggregate net positions over time: Before the change in reporting standards, when short positions were less likely to be consistently and properly reported, negative aggregate net positions were rare. After the change, when short positions were more likely to be consistently and properly reported, negative aggregate net positions were still rare. Indeed, even if the relative count and relative value of negative aggregate net positions were to increase one hundred fold, negative aggregate net positions would still represent fewer than one in four aggregate net positions. We take this evidence to indicate that not all investors, and perhaps not many, are carry traders.

2.3 Aggregate Net Positions in Domestic and Foreign Currency

The International Monetary Fund's Coordinated Portfolio Investment Survey is the most comprehensive dataset available on country-level holdings of short-term debt, but the survey has important

[2] Unfortunately, the United States is not among the countries that have fully adopted the BPM6 standard for reporting short positions. The United States reports gross long positions to CPIS, without netting gross short positions. In the United States . . . describe the TIC System, describe the instruction form, describe the custody survey process and problems with custody surveys, describe the Treasury consultation documents.

limitations: the survey provides no information on aggregate holdings of domestic short-term debt, no information on the currency composition of aggregate holdings of foreign short-term debt, and no information on the short-term debt holdings of non-reporting countries. These three shortcomings motivate our attempt in this section to estimate from CPIS and supplementary datasets the approximate currency composition of aggregate holdings of domestic and foreign short-term debt for a large cross section of reporting and non-reporting countries. Fortunately, we can build our estimation procedure up from separate procedures that researchers have already developed for estimating domestic debt holdings and for estimating the currency composition of foreign debt holdings.

Fidora et al. (2007) develop a procedure for estimating domestic holdings of domestic debt as a residual difference between issuance amounts outstanding of domestic debt and foreign holdings of domestic debt. Lane and Shambaugh (2010a) develop a procedure for estimating the currency composition of debt holdings reported by counterparty, as holdings are reported in CPIS, based on information about the currency composition of debt issuances. We largely follow the procedures developed by these authors, with minor modifications, and our marginal contribution to this literature is to integrate the procedures and to apply the procedures to short-term debt rather than to debt of all maturities, which is novel as far as we know, and useful because it produces currency composition estimates for complete country-level short-term debt portfolios, rather than for the foreign fraction of country-level short-term debt portfolios. Naturally, there are limitations to the estimation procedures we use, to the datasets that we use, and to the accuracy of the estimates that we produce. In Appendix B we describe our estimation procedures in detail and discuss some limitations of our data and our approach. In this section, we summarize our estimates of the currency composition of aggregate net short-term debt positions for a selection of countries, and describe in overview the estimation procedures used.

Our estimates suggest a strong home-currency bias in aggregate net positions in short-term debt, consistent with an established literature on home bias that documents a tendency for domestic investors to hold domestic assets disproportionately. The home bias literature includes studies of home country bias in debt holdings, such as those by Fidora et al. (2007) and Coeurdacier and Rey (2013), and studies of home currency bias in portfolio holdings, such as that by Maggiori et al. (2018). The strong home bias that we and these authors find suggests that a majority of investors are unlikely to engage in widespread carry trade activity at the individual level in most countries; if investors were engaged in such activity, then we would expect, for example, the home currency bias to be smaller in countries with lower domestic interest rates, and higher in countries with higher domestic interest rates, but we observe opposite relationship, and we would expect to observe frequent negative aggregate net positions in funding currencies, but we nearly never observe this.

Table 4 summarizes our currency composition estimates for the domestic and foreign short-term debt holdings of six selected countries: the United States, Japan, Germany, the United Kingdom,

Holding Country	Share of World Holdings (in %)	Estimated Net Portfolio Holdings of Short-Term Debt by Currency of Issuance (in % of total portfolio value)						
		USD	JPY	EUR	GBP	AUD	KYD	Not Allocated
United States	41.8	98.5	0.8	0.4	0.1	0.0	0.0	0.3
Japan	4.8	2.0	94.9	1.1	0.3	0.0	0.0	1.7
Germany	4.1	9.7	0.2	86.2	0.2	0.0	0.0	3.6
United Kingdom	4.1	8.1	1.9	13.0	71.3	0.2	0.0	5.6
Australia	1.3	7.3	4.1	2.3	3.4	69.5	0.0	13.4
Cayman Islands	1.2	61.6	8.3	17.3	5.9	0.2	−0.1	6.8

Table 4 – Estimated Holdings of Domestic and Foreign Short-Term Debt by Currency of Issuance for Selected Countries, 2015–2019. The table describes estimated aggregate holdings of domestic and foreign short-term debt, by currency of issuance, for selected countries. The estimates derive from reported foreign short-term debt holdings from the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS), reported foreign short-term debt holdings by currency for the U.S. from the U.S. Treasury's Report on U.S. Portfolio Holdings of Foreign Securities, and reported short-term debt issuances in domestic and international markets from the Bank for International Settlement's International Securities Statistics. Domestic holdings of domestic short-term debt are computed as total outstanding issuance minus estimated foreign holdings. We estimate foreign holdings for a broader set of countries using reported holdings from a narrower set of CPIS-reported countries in a gravity-style regression. In some cases, we estimate total domestic market issuance of short-term debt using information on domestic and international market short-term debt issuance for a narrower set of BIS-reported countries. The details of the estimation procedures and the construction the aggregate short-term debt holdings by country and currency are described in detail in Appendix B.

Australia, and the Cayman Islands. While we do estimate the currency composition of short-term debt holdings for the complete cross-section of 202 countries, we select these six countries for the tables in this subsection for several specific reasons: we want some countries with major international currencies, some countries that hold a large fraction of outstanding short-term debt, some countries that issue a large fraction of outstanding short-term debt, some countries that issue assets commonly use in carry trades, some traditional economies and some offshore financial centers, and ideally countries that have good coverage in the IMF, BIS, and U.S. Treasury datasets that we use in our estimate procedures. The six countries selected for Table 4 are the six countries that best met these criteria. Column two of Table 4 shows each country's share of estimated world holdings of short-term debt, expressed as a percentage of the estimated value of worldwide holdings of short-term debt. Columns three through eight show the currency composition of each country's holdings of short-term debt, where holdings in each currency are expressed as a percentage of the sum total value of each country's holdings across all currencies. All values in Table 4 are expressed as percentages and have been averaged over the years 2015 to 2019.

The percentages in the diagonal cells of columns three through eight in Table 4 display a strong home-currency bias. The diagonal percentages are large because countries tend to hold domestic debt disproportionately, and because countries tend to issue domestic debt in the domestic currency disproportionately. Also evident from the diagonal percentages is the fact that countries with higher domestic interest rates, such as Australia, display less home currency bias, while countries with lower domestic interest rates, such as Japan, display more home currency bias. As mentioned above, this stylized fact would be hard to reconcile with theories that involved a majority of individual investors within countries taking carry trade positions.

In the following paragraphs, we describe some details of the estimation procedures used to produce the estimates in Table 4. We produce these estimates in three steps. In step one, we fit a gravity equation to CPIS data on aggregate net foreign short-term debt holdings for reporting countries, and use the fitted gravity equation to predict the aggregate net foreign short-term debt holdings of non-reporting countries. In step two, we estimate short-term debt issuance amounts outstanding for a subset of countries using International Debt Statistics (IDS) data published by the Bank for International Settlements (BIS), and then estimate domestic holdings of domestic short-term debt as the residual difference between estimated issuance amounts outstanding and estimated rest-of-world holdings of each country's short-term debt. In step three, we use IDS data on the currency composition of short-term debt issuances to estimate the currency composition of short-term debt holdings by counterparty country for the subset of counterparty countries in our sample for which issuance-based currency weights are available. We sum each country's holdings across counterparty countries for each currency to obtain the currency composition of short-term debt holdings that we report in Table 4. Each step is described in greater detail below.

Step One. Our first step in generating these estimates is to fit a gravity equation to CPIS data on aggregate net holdings of foreign short-term debt. Table 5 summarizes for a selection of six countries the CPIS data that we use to fit the gravity equation. The table shows aggregate net positions in short-term debt by counterparty country that the six countries report in CPIS over the period 2015 to 2019. Values in the diagonal cells are missing in columns three through eight of the table because the values in these cells represent domestic holdings of domestic short-term debt, and countries only report holdings of foreign short-term debt in CPIS. The off-diagonal cells in columns three through eight show each country's holdings of short-term debt issued by the debt-issuing countries listed at the top of each column, expressed as percentages of the total value of each country's foreign short-term debt holdings. The Note that if countries were to consistently report negative aggregate net holdings with a particular counterparty, then the percentage weight for that counterparty would be negative. In the CPIS data, negative aggregate net positions in short-term debt are reported infrequently, and the counterparty weights are nearly never negative.

The number of reporting countries is smaller than the number of debt-issuing countries in CPIS data. In total, there are 86 that report holdings of debt issued by 236 counterparty countries. From the available CPIS data, we estimate a gravity equation and use the fitted equation to obtain estimates of portfolio holdings of foreign short-term debt for 202 reporting and non-reporting countries. We also use the fitted gravity equation to predict holdings of domestic short-term debt, but we use these gravity-based estimates only for countries with no available IDS data on short-term debt issuance amounts outstanding. For countries that do have IDS data available, we can estimate holdings of domestic short-term debt as the residual differences between total issuances and foreign

Holding Country	Share of World Holdings (in %)	Reported Net Portfolio Holdings of Foreign Short-Term Debt by Country of Issuance (in % of portfolio value)						
		USA	CYM	GBR	JPN	AUS	DEU	Rest of World
United States	49.2	–	0.8	12.3	17.8	9.3	3.9	55.8
Cayman Islands	25.4	59.6	–	6.2	8.7	0.3	6.5	18.6
United Kingdom	14.0	27.1	0.2	–	10.8	0.8	3.2	57.8
Japan	4.3	34.4	6.8	8.9	–	1.2	3.1	45.6
Australia	4.3	33.5	1.8	22.9	27.0	–	1.1	13.7
Germany	2.9	10.8	4.2	7.7	7.6	0.2	–	69.5

Table 5 – **Reported Holdings of Foreign Short-Term Debt by Country of Issuance for Selected Reporting Countries, 2015–2019.** The table describes the reported holdings of foreign short-term debt, reported by country of issuance, for selected countries that report such holdings to the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS). The first column of the table list CPIS reporting countries. The second column of the table lists each country's share of the total short-term debt holdings reported by all CPIS-reporting countries worldwide, where the share is computed yearly and then averaged over the years 2015–2019. The remaining columns show the portfolio weights for short-term debt issued by the United States of America (USA), the Cayman Islands (CYM), the United Kingdom (GBR), Japan (JPN), Australia (AUD), Germany (DEU), and the rest of the world (Rest). The diagonal cells in these columns are blank because countries do not report to CPIS their holdings of domestic debt.

holdings, and we prefer the residual difference estimates over the gravity estimates of domestic short-term debt holdings. The gravity estimates are used for XXX countries, but these countries are typically smaller economies, and they held only XXX% of the world short-term debt on average over the period 2015 to 2019.

Our gravity equation uses variables that are standard in the gravity literature: the geographic distance between countries, the time-zone gap, the gap in GPD, the gap in GDP per capita, the log population of the reporting country, and the log GDP per capita of the reporting country, as well as indicator variables for status as an offshore financial center, contiguity with counterparty country, common language with counterparty country, and common colonial past with counterparty country. The Centre d'Études Prospectives et d'Informations Internationales (CEPII) provide a convenient gravity dataset with the necessary variables. In our benchmark estimates of the gravity equation, we use the PPML method of Silva and Tenreyro (2006), because this method has been shown to better handle the heteroskedasticity in the CPIS data that the frequent zero-valued and infrequent large positive-valued observations cause. As a robustness check, we also estimate the gravity equation using the traditional log-one-plus method that Lane and Shambaugh (2010a) use. In the traditional log-one-plus regression, we include fixed effects for years and debt-issuing countries.

This section should probably be extended to include the actual gravity equation that we estimate and our regression results, or it should maybe link a subsection in Appendix B with this information. In general, and perhaps because short-term debt holdings are more sparsely reported in CPIS than are debt holdings of all maturity, the R^2 values from our gravity regressions—both the PPML and log-plus-one regressions—are lower than the R^2 that Lane and Shambaugh (2010a) report. I think our R^2 values are around 1/2.

Step Two. Our second step is to estimate domestic-market short-term debt issuance amounts outstanding for a subset of 67 countries using IDS issuance data, and then to compute domestic holdings of domestic short-term debt for 80 countries as the residual difference between domestic and international issuance amounts outstanding of short-term debt from IDS data and rest-of-world holdings of short-term debt from CPIS data. The IDS dataset provides information on short-term debt issuance by currency (USD, EUR, domestic currency, or other) and by market (domestic or international), but the country coverage in the IDS issuance data is worse than the coverage in the CPIS holdings data, because the coverage of domestic-market issuance is poor. IDS data cover international-markets issuances for 80 countries, but domestic-market issuances for only 13 of these 80 countries—hence our estimates of domestic-market short-term debt issuances for the remaining 67 countries. For countries in our full cross-section of 202 countries that have no IDS coverage, we use the gravity equation described above in step one of our estimation procedure to estimate domestic holdings of domestic short-term debt because residual difference estimates are unavailable.

Table 6 summarizes IDS data on international-markets short-term debt issuance amounts outstanding for a selection of six countries. The debt-issuing countries listed in Table 6 are sorted in column two from largest to smallest by their average annual shares of worldwide international-markets short-term debt issuance amounts outstanding reported in IDS. Columns three through eight of Table 6 show the currency composition of each country's international-markets short-term debt issuance amounts outstanding, where six currencies are indicated at the top of each column by three-letter codes: GBP for Pound Sterling, EUR for Euro, AUD for Australian Dollar, USD for U.S. Dollar, KYD for Cayman Islands Dollar, and JPY for Yen. Column nine, Not Allocated, shows issuance amounts outstanding that could not be allocated to a specific currency. The Not Allocated column in Table 6 shows the percentage of each country's short-term debt issuance denominated in a currency other than the six currencies listed in Table 6, or denominated in a currency designed as "other" in the IDS data. For many countries, the Not Allocated percentage is small, but for some countries, like Australia and the Cayman Islands, the percentage is substantial.

The United States and Japan occupy relatively low positions in the international-markets short-term debt issuance ranking in Table 6, both falling below Australia in the ranking—a country that produces less than one-tenth of the GDP of the United States, for example. The low positions of the United States and Japan reflect the fact that international short-term debt markets are relatively unimportant for these countries, because large domestic markets exist. Table 6 therefore gives an incomplete picture of country-level short-term debt issuance amounts outstanding.

For the 13 countries in our sample for which IDS data on both domestic-market and international-markets issuances are available, we find that domestic-market issuances represent XXX% of total issuances in domestic and international markets. Domestic market issuance is generally larger than international-markets issuance, and international investors can increasingly access domestic

Issuing Country	Share of World Issuance (in %)	Outstanding Short-Term Debt Issued in International Markets, Reported by Currency of Issuance (in % of total issuance face value)						
		GBP	EUR	AUD	USD	KYD	JPY	Not Allocated
United Kingdom	48.1	23.2	39.0	0.0	34.8	0.0	0.0	3.0
Germany	30.9	0.0	17.1	0.0	64.2	0.0	0.0	18.7
Australia	10.1	0.0	4.0	4.5	40.7	0.0	0.0	50.7
United States	6.5	0.0	78.8	0.0	8.3	0.0	0.0	12.9
Cayman Islands	2.3	0.0	26.8	0.0	31.8	0.0	0.0	41.4
Japan	2.2	0.0	12.0	0.0	54.4	0.0	20.4	13.2

Table 6 – Reported Amounts Outstanding of Short-Term Debt Issued in International Markets by Currency of Issuance for Selected Reporting Countries, 2015–2019. The table describes the face value of issuance amounts outstanding of short-term debt by currency of issuance for selected countries, from the Bank for International Settlements International Debt Statistics (IDS) dataset. The first column of the table lists countries of issuance. The second column lists each issuing country's share of total international-market short-term debt issuance amounts outstanding across all IDS countries worldwide. The remaining columns show the weights for the international-market short-term debt that each country issues in United States Dollar (USD), Cayman Islands Dollar (KYD), British Pound (GBP), Japanese Yen (JPY), Australian Dollar (AUD), and Euro (EUR), as well as the weight for each country's international-market short-term debt that we could not allocate to a currency because of incomplete information (Not Allocated). The shares and weights in the table are computed yearly for each debt-issuing country and then averaged over the years 2015–2019.

markets, as Lane and Shambaugh (2010a) note, so the domestic currency of short-term debt issued in domestic markets is an important determinant of the currency composition of the short-term debt that international investors hold. For countries with no IDS coverage of domestic-market short-term debt issuances, but with IDS coverage of international-markets issuances, we predict domestic market issuances using a fitted regression equation. Specifically, for countries with both domestic-market coverage and international-markets coverage in IDS, we regress the log value of domestic-market issuances on the log values of international-markets issuances denominated in the domestic currency of the issuer, in USD, in EUR, and in "other" currencies, and log GDP per capita and log population of the debt-issuing country. The regression has an R-squared of XXX. Using the IDS data for international-markets issuance together with our estimates of domestic-market issuance, and IDS data on domestic-market issuance where available, we compute estimates of the total short-term debt issuance amounts outstanding for 80 countries.

This section should probably also be extended to include exactly the regression equation that we estimate to predict missing values of domestic-market short-term debt issuance amounts outstanding, along with the table with regression results and etc, or we should like to Appendix B where we provide this information.

Step Three. Our third step is to apply the currency composition of short-term debt issuances that we estimated in the second step to the short-term debt holdings that we estimated in the first step. Countries do not report the currency composition of their short-term debt holdings in CPIS, and as columns three through eight in Table 6 indicate, we cannot assume that the short-term debt holdings that countries report are denominated in the domestic currencies of the debt-issuing countries. We follow Lane and Shambaugh (2010a) and instead assume that debt-holding countries hold short-term debt by currency in the same proportions as debt-issuing countries issue short-term debt by currency. We assume that international investors can access domestic markets for short-term debt in foreign countries, so we combine domestic-market and international-market issuances and use total issuances to estimate currency composition, rather than using international-markets issuances alone. In this respect, we differ from Lane and Shambaugh, who use international-markets issuances alone. Lane and Shambaugh themselves note that domestic-market participation by international investors was becoming more common even in their sample period, which ran from 1990 to 2004. Particularly for the United States, domestic-market short-term debt issuances are likely to be held by international investors, who are far more likely to hold USD-denominated short-term debt issued by the United States than EUR-denominated short-term debt issued by the United States, despite the fact that over two-thirds of U.S. international-markets short-term debt is EUR-denominated, as Table 6 shows.

We improve the accuracy of the currency composition estimates described above, by constructing rest-of-world issuance weights that subtract the short-term debt holdings of the United States and of global central banks from issuance amounts outstanding by currency, because these holdings are reported directly reported by currency denomination. Specifically, we use more granular data from the U.S. Treasury on foreign short-term debt holdings of the United States by counterparty country and currency, and we use aggregate data from the IMF Currency Composition of Official Foreign Exchange Reserves (COFER) survey on the currency composition of official foreign exchange reserves held globally by central banks. This information allows us to compute residual issuance amounts outstanding of the short-term debt issued by each country, after subtracting the currency-specific holdings of the United States and global central banks. We then use these residual amounts outstanding to compute a set of issuance-based rest-of-world currency composition weights that we then use to estimate the currency composition of foreign short-term debt holdings that Table 4 presents for six selected countries.

The main take-away from Table 4 is that the clear pattern of home-currency bias is incompatible with the notation of widespread carry trade investment behavior at the level of individual investors. Further details of our estimation procedures and results can be found in Appendix B.

2.4 Computing Centered Home and Foreign Portfolio Positions

In this section we collapse the estimated short-term debt holdings by currency described above into a two-by-two matrix of bilateral portfolio holdings for a designated home country and a composite foreign country. We then use the two-by-two matrix of bilateral portfolio holdings to compute centered home and foreign portfolio positions. The centered positions that we compute from the collapsed data satisfy two conditions: the assets in the centered positions are in zero net supply, and the centered positions are symmetric across countries. We describe our procedure for collapsing the data and for identifying "optimal" centered positions in Appendix C. This section is currently incomplete, and the implementation of the procedure outlined in Appendix C is incomplete, but we predict that the centered two-by-two matrix of bilateral holdings of short-term debt with the United States as home country will show a long position for the United States in low-interest domestic short-term debt and a short position for the United States in high-interest foreign short-term debt—exactly the opposite of an aggregate carry trade. The computations still need to be done, and this section still needs to be written.

The approach above will give us an optimal model-consistent component matrix **B** that maps cleanly to the theoretical portfolio holdings that we model. If the portfolio holdings in **B** constitute an aggregate carry trade for the home country (in the sense that the home position in **B** is long in the high-interest asset), then we can position ourselves as one of the first general equilibrium macro-finance papers to study both the carry trade position and the return in the theory and the data. If the portfolio holdings in **B** constitute an aggregate drop trade, as they did in the numerical example above, then we can advertise our results as a puzzle: why don't aggregate portfolio holdings constitute aggregate carry trades in the data? We can show in the model how a representative drop trader can arise naturally in a standard two-country setting. We can also ask what the empirical finding of an aggregate drop trade implies for Lustig and Verdelhan (2007): is consumption risk a sensible interpretation of their results, if investors in the data tend to be drop traders?

For predictions related to aggregate portfolio holdings, limited data are available. The IMF CPIS dataset and the BIS International Securities dataset provide this information. These datasets report portfolio holdings annually at multiple levels of aggregation and for multiple asset classes. Aggregate portfolio holdings are reported at the country level, and additionally at the level of large investor groups within countries (government, financials, non-financials, households and non-profits, and so on). The asset classes that are reported include long-term debt, short-term debt, and equity. As with many datasets in economics, these datasets may have gaps in coverage, and variable definitions may not correspond precisely with concepts in the model, but in principle these datasets should suffice to study aggregate portfolio holdings.

The theoretical model that we develop makes several important assumptions that do not hold in the data. We therefore develop a procedure for measuring the distance between the true data and a model-consistent Adjustments to the CPIS data are necessary. The first adjustment is necessary because the model assumes that domestic debt securities are always issued in the domestic currency, while domestic debt securities in the data are sometimes issued in foreign currency. The second adjustment is necessary because the model assumes that assets are in zero net supply while assets in the data are in positive net supply. Both adjustments are outlined in Lane and Shambaugh (2010a) and Amdur (2010).

3 Theoretical Model

In this section, we study a two-country dynamic stochastic general equilibrium model with incomplete markets and household heterogeneity in risk aversion. The model features a determinate non-stochastic steady state and stationary dynamics at the household and aggregate levels around the non-stochastic steady state. The model allows for exact aggregation across households in the non-stochastic steady state and approximate aggregation across households locally around the non-stochastic steady state. Using perturbation methods, we derive closed-form solutions from the model for the real exchange rate, real asset returns, household and aggregate consumption, household and aggregate wealth, and household and aggregate portfolio holdings. We show that households tailor their consumption to their heterogeneous risk preferences: households with lower risk aversion choose higher expected consumption and higher variance of consumption, while households with higher risk aversion choose lower expected consumption and lower variance of consumption. Households achieve these differing patterns in consumption by tailoring their portfolio positions to either raise or lower the variance of their incomes. Individual households in each country take a range of different portfolio positions, including carry trade positions, hand-to-mouth positions, and what we call drop trade positions, which are the opposite of carry trade positions.

Unlike representative agent models, our heterogeneous-household model allows household portfolio positions to differ from aggregate portfolio positions. Because many countries take aggregate drop-trade positions, we argue that heterogeneity is necessary for the study of the carry trade in these countries. Our heterogeneous-household model can be used to study the carry trade while simultaneously respecting the empirical evidence on aggregate portfolio holdings. In this way, our model helps to reconcile the finance literature on the carry trade with the international macroeconomics and macro-finance literature on aggregate portfolio holdings.

Simple two-country incomplete-market models are often indeterminate and non-stationary, and the international macroeconomics literature has developed a range of simple devices that can be added to such models to induce determinacy and stationarity (Schmitt-Grohé and Uribe, 2003). In our heterogeneous-household setting the problems of indeterminacy and non-stationarity are more severe than usual, because the problems can arise separately at the household level and at the aggregate level. Common stationarity-inducing devices from the international macroeconomics literature, such as endogenizing the subjective discount factor or introducing portfolio adjustment costs, are inconvenient in our setting because they interfere with aggregation or because they address the problems of indeterminacy and non-stationarity only at the aggregate level. We therefore turn to a less common device to induce determinacy and stationarity in our model: wealth in the utility function. We focus on a simple specification, and discus more complicated specifications in an appendix.

3.1 Model Primitives

Households. A continuum of households exists in a home country H and a second continuum of households exists in a foreign country F. We assume that each household in each country has constant relative risk aversion over consumption, and we assume that the coefficients of relative risk aversion, denoted ρ, differ across households within each country. The distribution of risk aversion across households in each country is identical across countries. Each household is therefore uniquely identified within country by their coefficient of relative risk aversion, and each household in one country has a counterpart in the other country with identical risk aversion. We use the coefficient of relative risk aversion to index households in each country.

An individual household ρ in country i maximizes the expected present value of lifetime utility over consumption and wealth. We use $U_{it}(\rho)$ to denote the expected present value of lifetime utility, and write it as

$$U_{it}(\rho) = \mathrm{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t}\big(U_{is}^{\mathrm{C}}(\rho) + U_{is}^{\mathrm{W}}(\rho)\big)\right], \qquad i \in \{\mathrm{H}, \mathrm{F}\}, \tag{4}$$

where $\mathrm{E}_t[\cdot]$ denotes the expectation operator conditioned on information available in period t, β denotes the subjective discount factor, $U_{is}^{\mathrm{C}}(\rho)$ and $U_{is}^{\mathrm{W}}(\rho)$ denote, respectively, the utility from consumption and the utility from real wealth that household ρ in country i receives in period s.

For consumption, we assume a constant-relative-risk-aversion period utility function,

$$U_{it}^{\mathrm{C}}(\rho) = \frac{1}{1-\rho}\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{1-\rho}, \qquad i \in \{\mathrm{H}, \mathrm{F}\}, \tag{5}$$

where $C_{it}(\rho)$ and $C_i(\rho)$ denote the consumption basket of household ρ in country i in period t and in the non-stochastic steady state, respectively. The parameter ρ indexes households and denotes each household's coefficient of relative risk aversion. For real wealth, we assume a constant-absolute-risk-aversion period utility function,

$$U_{it}^{\mathrm{W}}(\rho) = -\frac{\theta}{\rho}\left(\frac{e^{W_{it}^{i}(\rho)}}{e^{W_{i}^{i}(\rho)}}\right)^{-\rho}, \qquad i \in \{\mathrm{H}, \mathrm{F}\}, \tag{6}$$

where $W_{it}^{j}(\rho)$ and $W_{i}^{j}(\rho)$ denote the real wealth of household ρ in country i in period t and in the non-stochastic steady state, respectively, deflated in currency $j \in \{\mathrm{H}, \mathrm{F}\}$. The parameter θ denotes the wealth-in-utility parameter that governs the importance of real wealth as a source of utility for the household. We think of the wealth-in-utility parameter θ as a number near zero. If we set θ equal to zero exactly, the model becomes indeterminate and non-stationary.

Each household faces a budget constraint each period. Stated in real terms, the budget constraint requires that the real value of the household's bond holdings and consumption at the end of each period equals the real value of the household's bond holdings plus the household's endowment income at the beginning of each period:

$$B^i_{iit}(\rho) + B^i_{ijt}(\rho) + C_{it}(\rho) = R^i_{it} B^i_{iit-1}(\rho) + R^i_{jt} B^i_{ijt-1}(\rho) + Y_{it}(\rho)\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}, \tag{7}$$

where $B^k_{ijt}(\rho)$ denotes the real value of the nominal bond issued by country j, held by household ρ in country i in period t, deflated in currency $k \in \{\text{H},\text{F}\}$, where R^j_{it} denotes the gross real return in period t on the nominal bond issued by country i, deflated in currency j, and where $Y_{it}(\rho)$ denotes the real endowment of household ρ in country i in period t. The same real endowment is paid entirely in units of the domestic good to each household within country. That is,

$$Y_{it}(\rho) = Y_{it}\,, \quad i \in \{\text{H},\text{F}\}. \tag{8}$$

Household real wealth equals the real value of the household's holdings of the home and foreign nominal bonds,

$$W^i_{it}(\rho) = B^i_{iit}(\rho) + B^i_{ijt}(\rho)\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,, \tag{9}$$

and we model nominal bonds as single-period assets that pay one unit of the currency of the bond-issuing country at maturity with certainty. For nominal bonds issued in country i, we define the real return between period $t-1$ and t, with payments deflated in currency j, as

$$R^j_{it} = \frac{S_{jit}/P^j_{jt}}{P^j_{\text{B}it-1}/P^j_{jt-1}}\,, \quad i,j \in \{\text{H},\text{F}\}\,, \tag{10}$$

where S_{ijt} denotes the nominal exchange rate in period t, defined as units of currency i per unit of currency j, where P^j_{it} denotes the price of consumption basket i in currency j in period t, and where $P^j_{\text{B}it}$ denotes the price of bond i in currency j in period t. We define the real value of nominal bond holdings for household ρ in country i, deflated in currency j in period t as a quantity of nominal bonds $A_{ijt}(\rho)$ multiplied by a nominal bond price $P^j_{\text{B}it}$ and deflated in currency k,

$$B^k_{ijt}(\rho) = \frac{A_{ijt}(\rho) P^k_{\text{B}jt}}{P^k_{it}}\,, \quad i,j,k \in \{\text{H},\text{F}\}\,. \tag{11}$$

In Appendix D.1, we show that the real household budget constraint in (7), written in terms of real returns and real holdings of nominal bonds, is equivalent to a nominal budget constraint written in terms of nominal bond quantities and nominal bond prices.

Home and foreign countries produce differentiated consumption goods, and households trade these goods across countries. Households choose the quantities of each good to include in their consumption baskets, and we define the household consumption basket as

$$C_{it}(\rho) = \gamma C_{iit}(\rho)^{\alpha} C_{ijt}(\rho)^{1-\alpha}\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}, \tag{12}$$

where $C_{ijt}(\rho)$ denotes the quantity of good j that household ρ in country i consumes in period t, where the parameter α denotes the expenditure share on the domestic good, and where the parameter γ is a normalizing constant.[3] To obtain home bias in consumption, we assume that each household's expenditure share on the domestic good is greater than one-half in each country.[4]

Households maximize their consumption baskets by choosing quantities of the home and foreign goods. The total expenditure on consumption from the first-stage problem constrains this second-stage problem, and the second-stage expenditure constraint requires that

$$P^i_{it} C_{it}(\rho) = P^i_{\mathrm{C}it} C_{iit}(\rho) + P^i_{\mathrm{C}jt} C_{ijt}(\rho)\,, \quad i \neq j\,,\ i,j \in \{\mathrm{H},\mathrm{F}\}\,, \tag{13}$$

where $P^j_{\mathrm{C}it}$ denotes the price of good j in currency i at time t.

Aggregating Household Variables. The measure of households in each country equals one, and the coefficient of relative risk aversion that uniquely identifies each household within country follows a distribution described by density function $f(\rho)$. We assume positive support over the interval $(0, \rho_m)$, and that the density of households rises as ρ approaches ρ_m from below, forming a left tail of households with decreasing coefficients of relative risk aversion, according to the function

$$f(\rho) = \frac{\kappa}{\rho}\left(\frac{\rho}{\rho_m}\right)^{\kappa} \quad \text{with} \quad \kappa > 1,\ \rho_m > 0\,, \tag{14}$$

where κ and ρ_m are shape and scale parameters, respectively, for the density function $f(\rho)$. While we assume a specific functional form in (14) for concreteness, our model can be solved for any number of alternative density functions, and our main results with respect to portfolio holdings will go through with most density functions that yield reasonable aggregate coefficients of relative risk aversion and that feature positive support over sufficiently low values of the household coefficient of relative risk aversion.[5]

The consumption utility function in (5) ties the household coefficient of relative risk aversion

[3] We define the normalizing constant as $\gamma = [\alpha^\alpha (1-\alpha)^\alpha]^{-1}$ and include it to simplify our algebra; the constant is not necessary to solve the model and could be omitted at some cost to parsimony. Likewise, the Cobb-Douglas functional form is not necessary to solve the model, and could be replaced by a more general constant-elasticity-of-substitution basket at some cost to parsimony. The Cobb-Douglas basket obtains as a limiting case of the constant-elasticity-of-substitution basket in which the elasticity of substitution between home and foreign goods approaches one in the limit.

[4] Home bias in consumption can be obtained by alternative means, for example by modeling trade costs in goods markets (Obstfeld and Rogoff, 2000) or by modeling a non-tradable goods sector (Pesenti and Van Wincoop, 2002). We are interested in the effect that home bias in consumption has on portfolio decisions, and less interested in the fundamental source of the consumption home bias itself, so we confine our analysis to the case of home-biased expenditure shares.

[5] Empirical evidence on the distribution of risk aversion across households in the United States suggests that many households have coefficients of relative risk aversion near one, but that a smaller number of households have significantly higher and lower coefficients of relative risk aversion. Enkhbaatar (2020) studies households in the United States using data from the University of Michigan's Panel Study of Income Dynamics, and gives non-parametric estimates of the distribution of the coefficient of relative risk aversion across U.S. households with differing levels of wealth. Enkhbaatar estimates that the coefficient of relative risk aversion is generally falling in household wealth, and ranges from a high of around 1.15 for low-wealth households to a low of around 0.22 for high-wealth households in the United States. The density function that describes the distribution of household wealth in the United States is also generally falling in the level of household wealth, which motivates our assumption in (14) of a left tail of households with decreasing coefficients of relative risk aversion.

directly to the household intertemporal elasticity of substitution with respect to consumption (both with respect to consumption), so that heterogeneity in the former implies heterogeneity in the latter. Let $\sigma(\rho)$ denote the elasticity of intertemporal substitution with respect to consumption for household ρ, and note that the utility function in (5) implies

$$\sigma(\rho) = \frac{1}{\rho} \tag{15}$$

for any household ρ.

In general, the inverse relationship at the household level in (15) does not hold at the aggregate level. Instead, shape parameters from the distribution of ρ drive a wedge between the aggregate coefficient of relative risk aversion and the aggregate elasticity of intertemporal substitution. We define the aggregate coefficient of relative risk aversion, denoted $\tilde{\rho}$, and the aggregate intertemporal elasticity of substitution, denoted $\tilde{\sigma}$, as

$$\tilde{\rho} = \int_0^{\rho_m} \rho f(\rho)\,\mathrm{d}\rho \quad \text{and} \quad \tilde{\sigma} = \int_0^{\rho_m} \sigma(\rho) f(\rho)\,\mathrm{d}\rho\,, \tag{16}$$

and we use the density function in (14) to derive the following relationship between them:

$$\tilde{\sigma} = \frac{\kappa^2 - 1}{\kappa^2}\frac{1}{\tilde{\rho}}\,. \tag{17}$$

From (15) and (17), we see that heterogeneity across households drives an aggregation wedge between the coefficient of relative risk aversion and the intertemporal elasticity of substitution. The wedge vanishes as the shape parameter κ from the distribution of the coefficient of relative risk aversion across households approaches infinity and the distribution collapses to a point.

The density function for ρ in (14) implies a Pareto density function for $\sigma(\rho)$ with shape parameter κ and scale parameter $\sigma_m = 1/\rho_m$. The Pareto density function falls strictly in $\sigma(\rho)$ and forms a right tail of households with increasing intertemporal elasticities of substitution. We illustrate the density functions for ρ and $\sigma(\rho)$ in Figure 3. The two density functions can be used interchangeably to aggregate economic variables across households within countries, and we demonstrate this interchangeability in Appendix D.2.

We obtain aggregate variables in the same way that we obtained the aggregate coefficient of relative risk aversion and aggregate intertemporal elasticity of substitution, by integrating the product of a household variable and the density of households $f(\rho)$. Using the household consumption basket as an example,

$$C_{it} = \int_0^{\rho_m} C_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad i \in \{\text{H}, \text{F}\}\,. \tag{18}$$

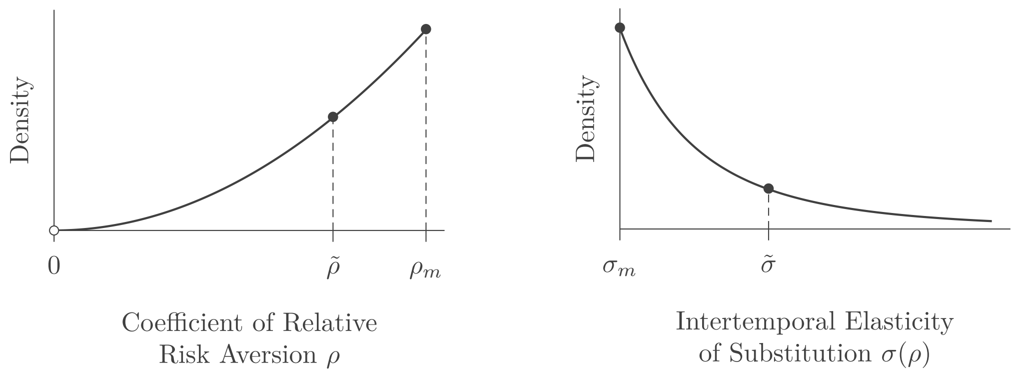

Figure 3 – **Density Functions for the Distributions of the Coefficient of Relative Risk Aversion and the Intertemporal Elasticity of Substitution Across Households.** Households are indexed by ρ, and ρ simultaneously denotes the household coefficient of relative risk aversion. In the model, $\rho = \sigma(\rho)^{-1}$, where $\sigma(\rho)$ denotes the intertemporal elasticity of substitution. The figure shows plots of the density functions $f_\rho(\rho)$ and $f_{\tilde{\sigma}}\big(\sigma(\rho)\big)$ defined in Equation (14), plotted for values $\rho \in (0, \rho_m)$ and $\sigma(\rho) \in (\sigma_m, \infty)$ for the illustrative calibration of $\kappa = 3$ and $\sigma_m = 2/3$. We assume the same distributions for home and foreign countries. For this illustrative calibration, the aggregate coefficient of relative risk aversion is given by $\tilde{\rho} = 9/8$ and the aggregate intertemporal elasticity of substitution is given by $\tilde{\sigma} = 1$. The calibration was chosen for analytical convenience, and alternative calibrations are also possible that yield $\tilde{\rho} < \tilde{\sigma}$. The density functions plotted here define equivalent heterogeneity across households, in the sense that in the sense that aggregation of household variables across households is possible with respect to either ρ or $\sigma(\rho)$, and the result aggregate will be the same in either case.

We aggregate household endowments $Y_{it}(\rho)$, goods demand $C_{ijt}(\rho)$, real bond holdings $B^k_{ijt}(\rho)$, and real wealth $W^j_{it}(\rho)$ in this way to obtain aggregate endowments Y_{it}, goods demands C_{ijt}, real bond holdings B^k_{ijt}, and real wealth W^j_{it} for each country $i \in \{\text{H}, \text{F}\}$. Because we assume a unit measure of households in each country, the aggregate variables that we obtain are simultaneously country averages. Superscripts and subscripts on aggregate variables carry the same meaning as on the corresponding household variables.

Closing The Model. Goods and nominal bonds are differentiated across countries and trade in separate markets. Both goods markets and both nominal bond markets must clear each period. Each country is endowed with a supply of the domestic good, and the supply is exogenous and stochastic. Bonds are in zero net supply. For each market, clearing requires that aggregate demand across countries equal aggregate supply,

$$C_{iit} + C_{jit} = Y_{it} \quad \text{and} \quad B^i_{iit} + B^i_{jit} = 0\,, \quad i \neq j\,,\ i, j \in \{\text{H}, \text{F}\}\,. \tag{19}$$

The law of one price holds for goods and nominal bonds, so the price of any good and or any nominal bond must be the same in both countries, after currency conversion. That is,

$$P^i_{\text{C}it} = S_{ijt} P^j_{\text{C}it}\,, \quad \text{and} \quad P^i_{\text{B}it} = S_{ijt} P^j_{\text{B}it}\,, \quad i, j \in \{\text{H}, \text{F}\}\,. \tag{20}$$

Because consumption baskets differ across countries, consumption basket prices also differ across countries and we define the real exchange rate as the ratio of consumption basket prices expressed in a common currency:

$$Q_{ijt} = \frac{S_{ijt} P^j_{jt}}{P^i_{it}}\,, \quad i, j \in \{\text{H}, \text{F}\}\,, \tag{21}$$

where Q_{ijt} denotes the real exchange rate, defined as units of consumption basket i per unit of consumption basket j.

We use a quantity equation in each country to determine price levels,

$$M^j_{it} = Y_{it} P^j_{it}\,, \quad i, j \in \{\text{H}, \text{F}\}\,, \tag{22}$$

where M^j_{it} is the supply of money in country i, expressed in currency j. We take money supply as exogenous and stochastic in each country, and we assume that money supplies are symmetric across countries in the non-stochastic steady state when expressed in a common currency. That is, we assume that $M^i_i = S_{ij} M^j_j$, where the absence of the time subscript indicates the steady state.

Finally, we assume that home and foreign goods endowments and home and foreign money supplies are the only exogenous sources of uncertainty in the model, and that random innovations to each of these variables are purely transitory. The assumption of transitory innovations is not necessary to solve the model, and we could alternatively assume auto-correlated innovations at some cost to parsimony. Because the model features four sources of uncertainty and only two nominal bonds, markets are incomplete.

In the next section, we solve for the non-stochastic steady state equilibrium, approximate the model locally around the non-stochastic steady state, aggregate the approximate model, and solve for the real exchange rate, real returns on nominal bonds, household and aggregate consumption, household and aggregate real wealth, and household and aggregate portfolio holdings. These solutions allow us to characterize the portfolio positions of households and countries as either carry trades or drop trades, and we show that the model can produce aggregate drop trade positions in countries where some but not all households take carry trade positions. We present our results in a sequence of propositions.

3.2 Propositions

3.2.1 Utility Maximization.

Households solve a two-stage utility maximization problem. In the first stage of the problem, households maximize expected discounted lifetime utility in (4) by choosing quantities of the consumption basket to consume, and by constructing a portfolio of home and foreign nominal bonds. For household ρ in country i, the first stage of the utility maximization problem yields the following Euler equation,

$$\beta\,\text{E}_t\left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} R^i_{jt+1}\right] = \left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho} \frac{1}{C_i(\rho)} - \theta \left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho}, \quad i, j \in \{\text{H}, \text{F}\}\,. \tag{23}$$

If the wealth-in-utility parameter θ is positive, real wealth raises household utility directly. This direct effect on utility comes in addition to the traditional indirect effect of real wealth on utility through consumption. A rise in present consumption that lowers real wealth contemporaneously will also lower utility from real wealth contemporaneously and households forming optimal consumption plans will account for this effect by subtracting the marginal utility of real wealth from the marginal utility of consumption in the household Euler equation. If, on the other hand, the wealth-in-utility parameter θ is negative, real wealth lowers household utility directly, but still raises household utility indirectly through consumption; if θ is sufficiently small, the indirect effect will exceed the direct effect, and a rise in real wealth will have a positive net effect on household utility. Hence, in the case where the wealth-in-utility parameter θ is negative, wealth may be a burden but wealthier households may still experience higher utility through greater consumption.[6]

In the second stage of the household utility maximization problem, households choose quantities of home and foreign goods to consume in their baskets, subject to the plans they formed in the first stage of the problem. The second-stage problem yields household demand functions for the home and foreign goods,

$$C_{iit}(\rho) = \alpha \frac{P^i_{it}}{P^i_{\mathrm{C}it}} C_{it}(\rho) \quad \text{and} \quad C_{ijt}(\rho) = (1-\alpha) \frac{P^i_{it}}{P^i_{\mathrm{C}jt}} C_{it}(\rho) \,, \quad i \neq j \,,\ i,j \in \{\mathrm{H}, \mathrm{F}\} \,. \tag{24}$$

The parameter α takes its name from the demand function for the domestic good in (24), because rearranging the demand function for the domestic good yields an expression for α that equals exactly the household's expenditure share on the domestic good.

Finally, the expenditure constraints in (13) and goods demands in (24) from the household's second-stage utility maximization problem, together with the definition of the consumption basket in (12), imply an optimal price index for the consumption basket in country i in currency k,

$$P^i_{it} = \left(P^i_{\mathrm{C}it}\right)^{\alpha} \left(P^i_{\mathrm{C}jt}\right)^{1-\alpha} \,, \quad i \neq j \,,\ i,j \in \{\mathrm{H}, \mathrm{F}\} \,. \tag{25}$$

3.2.2 Non-Stochastic Steady State.

Evaluating the household Euler (23) in the steady state, we obtain

$$\theta C_i(\rho) = 1 - \beta R^i_j \,, \quad i,j \in \{\mathrm{H}, \mathrm{F}\} \,, \tag{26}$$

[6] We motivate the burden of real wealth in two ways. First, higher real wealth may slightly lower household utility directly because higher real wealth may attract unwanted public attention, strain relationships or cause familial strife, alienate the household, or cause other forms of psychological stress. This first motivation is captured by the phrase "more money, more problems", popularized by the American rap artist Notorious B.I.G.'s 1997 *Billboard* Hot 100 chart-topping song "Mo Money Mo Problems." Second, higher real wealth may slightly lower household utility directly because higher real wealth may be accompanied by societal expectations of greater generosity and more social responsibility, and households with higher real wealth may find these expectations burdensome. This second motivation is captured by the phrase *noblesse oblige*. In the burden-of-wealth case, households in the model with higher real wealth will still have higher utility than households with lower real wealth when the wealth-in-utility parameter θ is negative, as long as θ is sufficiently small, because the indirect benefits of real wealth that accrue through future consumption will out-weigh the small direct burden of real wealth.

where R^i_j denotes the steady-state real return on the nominal bond issued by country j and deflated in currency i. Equation (26) implies that the steady-state real return on the home nominal bond equals the steady-state real return on the foreign nominal bond when returns are computed in a common currency, $R^i_i = R^i_j\,,\ i, j \in \{\text{HF}\}$.

If θ is set to zero, we see from (26) that the household Euler equation simplifies to $R^i = 1/\beta$ in the non-stochastic steady state and the Euler equation imposes no optimality condition on household consumption. Absent this condition, household consumption is constrained only by household real wealth in the steady-state budget constraint. The budget constraint alone cannot pin down both consumption and real wealth, so the steady state is indeterminate. Assuming an exogenous steady-state distribution of real wealth across households would pin down consumption, but would not fully repair the model because the model is also non-stationary when θ is set to zero, as we show below.

A literature on indeterminacy and non-stationarity in two-country macroeconomic models proposes several devices that modify the Euler equation such that determinacy and stationarity result. Endogenous subjective discount factors (sometimes called Uzawa preferences), portfolio holding or adjustment costs, overlapping generations, and wealth in the utility function are examples of such devices. Schmitt-Grohé and Uribe (2003) and Ghironi (2006) discuss these devices in detail. Of the available devices, we find that wealth in the utility function works well in our setting with heterogeneity in household risk aversion, while the other devices either fail to induce stationarity or determinacy at both the household and aggregate levels, interfere with aggregation, or add an unwanted second dimension of heterogeneity across households.

Wealth in the utility function modifies the household Euler equation by introducing an additional motivation for households to accumulate wealth. In traditional economic models, households forming optimal consumption plans must account for the link between consumption and real wealth that arises through the budget constraint: lower utility from consumption today means higher realwealth today and higher utility from consumption tomorrow. Wealth in the utility function modifies the link between consumption and real wealth: lower utility from consumption today means higher real wealth and higher utility from real wealth today and higher utility from consumption tomorrow. A new term in the household Euler equation captures the marginal utility of higher real wealth today. This term prevents consumption from dropping out of the equation in the non-stochastic steady state. The household Euler equation together with the budget constraint then determine the distribution of consumption and real wealth across households in the non-stochastic steady state.

Because households differ only in their aversion to risk, and because no risk features in the non-stochastic steady state, the steady-state distribution of consumption and real wealth across households is degenerate. We are, therefore, able to state the following proposition describing the non-stochastic steady state of the model.

Proposition 1 (Non-Stochastic Steady State)**.** *Household heterogeneity in risk aversion plays no* *role in the non-stochastic steady state, and the steady-state distribution of endogenous variables across households is degenerate. With steady-state endowments normalized to unity, household and aggregate steady-state consumption baskets are given by*

$$C_i(\rho) = C_j(\rho) = C_i = C_j = 1\,, \quad i,j \in \{\text{H},\text{F}\}\,, \tag{27}$$

and household and aggregate steady-state goods demands are given by

$$C_{ii}(\rho) = C_{jj}(\rho) = C_{ii} = C_{jj} = \alpha\,, \quad i,j \in \{\text{H},\text{F}\}\,, \tag{28}$$

$$C_{ij}(\rho) = C_{ji}(\rho) = C_{ij} = C_{ji} = 1-\alpha\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,. \tag{29}$$

With nominal bonds in zero net supply, household and aggregate steady-state real wealth are given by

$$W_i^i(\rho) = W_j^i(\rho) = W_i^i = W_j^i = 0\,, \quad i,j \in \{\text{H},\text{F}\}\,, \tag{30}$$

and household and aggregate real bond holdings are given by

$$B_{ii}^i(\rho) = -B_{ij}^i(\rho)\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,, \tag{31}$$

$$B_{ii}^i = B_{jj}^i = -B_{ij}^i\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,. \tag{32}$$

Steady-state real returns on home and foreign nominal bonds are equalized and given by

$$R_j^i = R_i^i = \frac{1-\theta}{\beta} \coloneqq R\,, \quad i,j \in \{\text{HF}\}\,. \tag{33}$$

3.2.3 Model Aggregability

We can aggregate individual household variables by integrating over the product of the household variable for household ρ and the household density function $f(\rho)$. This simple aggregation procedure works for individual household variables, but often fails for equilibrium conditions that depend on several household variables, especially when these conditions depend explicitly on the coefficient of relative risk aversion and when the coefficient of relative risk aversion is additively inseparable from household variables. This failure to aggregate arises because of heterogeneity in the coefficient of relative risk aversion and the endogenous household variables that depend on the coefficient of relative risk aversion.

We have seen that heterogeneity vanishes in the non-stochastic steady state: intuitively, heterogeneity enters the model only through the coefficient of relative risk aversion and there is no risk in the non-stochastic steady state. For this reason, steady-state equilibrium conditions aggregate trivially. This fact was convenient for deriving the aggregate steady-state equilibrium in Proposition 1 and it will also be convenient for deriving approximate aggregate equilibrium conditions locally around the non-stochastic steady-state equilibrium.

Definition 3.1 (Aggregability)**.** Let $X_{it},\ i \in \{1,\dots,m\}$ denote a generic aggregate variable, let $X_{it}(\rho),\ i \in \{m+1,\dots,n\}$ denote a generic household variable, and let ρ denote an exogenous parameter that indexes households, explicitly enters equilibrium conditions, and follows a density function $f(\rho;\kappa)$, where κ is a parameter of the density function. Let

$$g_t\big(X_{1t},\dots,X_{mt},X_{m+1t}(\rho),\dots,X_{nt}(\rho);\ \rho\big) = 0 \tag{34}$$

denote a generic household equilibrium condition. The condition is *aggregable* when a function G_t exists such that

$$G_t(X_{1t},\dots,X_{nt};\ \kappa) = \int_\rho g_t\big(X_{1t},\dots,X_{mt},X_{m+1t}(\rho),\dots,X_{nt}(\rho);\ \rho\big) f(\rho)\,\mathrm{d}\rho = 0\,, \tag{35}$$

where

$$X_{it} = \int_\rho x_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad i \in \{m+1,\dots,n\}\,. \tag{36}$$

A model is aggregable when all household equilibrium conditions in the model are aggregable. Model aggregability is a strong property, and many models that feature household heterogeneity are not aggregable.

Some household equilibrium conditions in our model are aggregable, like the household budget constraint in (7), but others are not aggregable, like the household Euler equation in (23), because the parameter ρ enters explicitly and cannot be additively separated from household variables. However, when exact equilibrium conditions are not aggregable, they may have approximate representations that are aggregable.

Definition 3.2 (Steady-State and Approximate Aggregability)**.** Dropping time subscripts to denote the non-stochastic steady state, the generic household equilibrium condition in (34) *steady-state aggregable* when a function G exists such that

$$G(X_1,\dots,X_n;\kappa) = \int_\rho g\big(X_1,\dots,X_m,X_{m+1}(\rho),\dots,X_n(\rho);\rho\big) f(\rho)\,\mathrm{d}\rho = 0\,, \tag{37}$$

where X_i and $X_i(\rho)$ are non-stochastic steady state values of X_{it} and $X_{it}(\rho)$, respectively.

Using hats to denote deviations from the non-stochastic steady state, we call the generic household equilibrium condition in (34) *first-order aggregable* when a function $\hat{G}_t$ exists such that

$$\hat{G}_t\Big(\hat{X}_{1t},\dots,\hat{X}_{nt};\ \kappa\Big) = \int_\rho \hat{g}_t\Big(\hat{X}_1,\dots,\hat{X}_m,\hat{X}_{m+1}(\rho),\dots,\hat{X}_n(\rho);\rho\Big) f(\rho)\,\mathrm{d}\rho = 0\,, \tag{38}$$

where $\hat{X}_{it} = h_i(X_{it}) - h_i(X_i)$ for $i \in \{1,\dots,m\}$, and $\hat{x}_{it}(\rho) = h_i\big(x_{it}(\rho)\big) - h_i\big(x_i(\rho)\big)$ for $i \in \{m+1,\dots n\}$, where h_i is a well-defined transformation function, for example the logarithmic function when the variable under transformation is strictly positive. Steady-state arguments of the function $\hat{g}_t^n(\rho)$ have been suppressed in (38) to keep expressions compact. First-order aggregability generalizes to nth-order aggregability in the obvious way.

The household Euler equation in (23) is not aggregable, but the steady-state household Euler equation in (26) is aggregable because the coefficient of relative risk aversion falls out of the steady state equation. The first-order approximate household Euler equation is aggregable, although the coefficient of relative risk aversion enters the equation explicitly, because the equation can be rearranged to separate the coefficient of relative risk aversion additively from the household variables in the equation.

The household Euler equations is the only equation in the model that that depends explicitly on the household coefficient of relative risk aversion. Aggregating the remaining equilibrium conditions is straight-forward and we therefore state the following proposition.

Proposition 2 (Aggregability)**.** *The household budget constraint in (7), Euler equation in (23), and goods demand in (24) are steady-state and first-order aggregable.*

The first-order approximate household and aggregate budget constraints are given by

$$\begin{aligned}\hat{W}_{it}^i(\rho) + \hat{C}_{it}(\rho) &= R\hat{W}_{it-1}^i(\rho) + RB_{ii}^i\Big(\hat{R}_{it}^i - \hat{R}_{jt}^i\Big) + \hat{Y}_{it} + \mathcal{O}(\epsilon^2)\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,,\\ \hat{W}_{it}^i + \hat{C}_{it} &= R\hat{W}_{it-1}^i + RB_{ii}^i\Big(\hat{R}_{it}^i - \hat{R}_{jt}^i\Big) + \hat{Y}_{it} + \mathcal{O}(\epsilon^2)\,, \quad i \neq j\,,\ i,j \in \{\text{H},\text{F}\}\,.\end{aligned} \tag{39}$$

The first-order approximate household and aggregate Euler equations are given by

$$\beta R\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho) - \sigma(\rho)\hat{R}_{jt+1}^i\Big] = \hat{C}_{it}(\rho) - \theta\hat{W}_{it}^i(\rho) + \mathcal{O}(\epsilon^2)\,, \quad i,j \in \{\text{H},\text{F}\}\,, \tag{40}$$

$$\beta R\,\mathrm{E}_t\Big[\hat{C}_{it+1} - \tilde{\sigma}\hat{R}_{it+1}^i\Big] = \hat{C}_{it} - \theta\hat{W}_{it}^i + \mathcal{O}(\epsilon^2)\,, \quad i,j \in \{\text{H},\text{F}\}\,. \tag{41}$$

The first-order aggregate demand functions for home and foreign goods in Models 1 through 4 are identical, and are given by

$$C_{ijt}(\rho) = \hat{P}_{it}^i - \hat{P}_{\text{C}jt}^i + \hat{C}_{it}(\rho)\,, \quad i,j \in \{\text{H},\text{F}\} \tag{42}$$

$$\hat{C}_{ijt} = \hat{P}_{it}^i - \hat{P}_{\text{C}jt}^i + \hat{C}_{it}\,, \quad i,j \in \{\text{H},\text{F}\}\,. \tag{43}$$

In the following section, we derive the linear rational expectations solution to the aggregate model, after discussing conditions under which household and aggregate variables in Models 2 through 4 display stationary dynamics around the non-stochastic steady state.

3.2.4 Solving for Aggregate Cross-Country Differences.

The aggregate equilibrium conditions that resulted from first-order aggregation of household equilibrium conditions in Proposition 2, together with first-order approximations of the remaining of equilibrium conditions of the model, produce a system of first-order approximate aggregate equilibrium conditions in which no household variables appear. This aggregate system can therefore be solved independently from the household equilibrium conditions of the model.

We solve the aggregate system in terms of cross-country differences between home and foreign aggregate variables, after transforming variables into log deviations from the non-stochastic steady state. We use a subscript X to denote cross-country differences defined as home minus foreign. For aggregate consumption and real wealth,

$$\hat{Y}_{\mathrm{X}t} = \hat{Y}_{\mathrm{H}t} - \hat{Y}_{\mathrm{F}t} \quad \text{and} \quad \hat{C}_{\mathrm{X}t} = \hat{C}_{\mathrm{H}t} - \hat{C}_{\mathrm{F}t}\,. \tag{44}$$

For cross-country differences in deflated nominal variables, we deflate using home currency. When we deflate foreign nominal variables using home currency, we introduce the real exchange rate into the system of aggregate equilibrium equations. To keep our notation compact, we define $\hat{Q}_t \coloneqq \hat{Q}_{\mathrm{HF}t}$. In view of the definitions of real wealth, real returns, and real holdings of nominal bonds in (9)–(11), the following first-order approximate relationships obtain:

$$\begin{aligned} \hat{W}_{\mathrm{X}t} - \hat{Q}_t &= \hat{W}^{\mathrm{H}}_{\mathrm{H}t} - \hat{W}^{\mathrm{F}}_{\mathrm{F}t} + \mathcal{O}(\epsilon^2)\,, \\ \hat{B}_{\mathrm{X}t} - \hat{Q}_t &= \hat{B}^{\mathrm{H}}_{\mathrm{HH}t} - \hat{B}^{\mathrm{F}}_{\mathrm{FF}t} + \mathcal{O}(\epsilon^2)\,, \\ \hat{R}_{\mathrm{X}t} + \hat{Q}_t - \hat{Q}_{t-1} &= \hat{R}^{\mathrm{H}}_{\mathrm{H}t} - \hat{R}^{\mathrm{F}}_{\mathrm{F}t} + \mathcal{O}(\epsilon^2)\,, \end{aligned} \tag{45}$$

where $\hat{R}_{\mathrm{X}t}$ denotes the cross-country difference in real returns on nominal bonds, deflated in home currency, $\hat{B}_{\mathrm{X}t}$ denotes the cross-country difference in real holdings of the domestic bond in home and foreign countries, deflated in home currency, and $\hat{W}_{\mathrm{X}t}$ denotes the cross-country difference in real wealth, deflated in home currency.

The real exchange rate enters in the first two equations in (45) because of the conversion from foreign to home currency in deflating the nominal values and payoffs of foreign nominal bonds. For compactness, we will suppress the subscripts on the real exchange rate in our notation for the remainder of the paper, and we write $\hat{Q}_t$ whenever $\hat{Q}_{\mathrm{HF}t}$ is intended.

The real exchange rate that appears in (45) can be viewed both as a source of real risk to households that trade internationally, and as an equilibrium real price that adjusts along with consumption to clear international goods markets. This second view as an international real price can best be seen in following condition that derives from goods market clearing in (19) and aggregate goods demands in (43):

$$\hat{Y}_{\mathrm{X}t} = \frac{\phi}{1-\phi^2}\hat{Q}_t - \frac{\phi^2}{1-\phi^2}\hat{C}_{\mathrm{X}t} + \mathcal{O}(\epsilon^2)\,, \tag{46}$$

where we define $\phi = 2\alpha - 1$ as a home bias index. We assume exogenous home bias, so the expenditure share on the domestic good lies between one half and one, $\alpha \in (1/2, 1)$, and the home bias index lies between zero and one, $\phi \in (0, 1)$. Equation (46) shows that the real exchange rate and the cross-country difference in consumption share the burden of adjustment to cross-country differences in production. Whether the burden of adjustment falls more on the real exchange rate or more on the cross-country difference in consumption will depend on the other aggregate equilibrium conditions of the model and on the calibration of parameters.

We combine the market clearing condition in (46) with the aggregate budget constraint in (7) and aggregate Euler equations in (41) in order to reduce the first-order aggregate equilibrium conditions to a system of two linear difference equations that depend on the cross-country difference in aggregate real wealth and the cross-country difference in aggregate consumption. For Models 1 through 4, we write the aggregate system as

$$\begin{bmatrix} \hat{W}_{\text{X}t} \\ \mathrm{E}_t\left[\hat{C}_{\text{X}t+1}\right] \end{bmatrix} = \mathbf{D}_{\text{WC}} \begin{bmatrix} \hat{W}_{\text{X}t-1} \\ \hat{C}_{\text{X}t} \end{bmatrix} + \mathbf{D}_{\text{V}} \hat{V}_{\text{X}t} + \mathbf{D}_{\text{Y}} \hat{Y}_{\text{X}t} + \mathcal{O}(\epsilon^2)\,, \tag{47}$$

where $\hat{V}_{\text{X}t} = B_{\text{X}} \hat{R}_{\text{X}t}$ denotes the cross-country difference in aggregate portfolio valuation effects, and where $\mathbf{D}_{\text{WC}}$ denotes a 2×2-matrix and $\mathbf{D}_{\text{V}}$ and $\mathbf{D}_{\text{Y}}$ denotes 2×1-matrices of model parameters. In Appendix E.1 we write the parameter matrices out explicitly for Models 1 through 4.

Devereux and Sutherland (2011) show that aggregate portfolio valuation effects can be treated initially as if they were exogenous, when using standard linear rational expectations solution procedures to solve the first-order aggregate system of equilibrium conditions in (47). Aggregate portfolio valuation effects can be treated this way initially, because they do not affect the eigenvalues of the first-order aggregate system. After solving the first-order aggregate system for real wealth and consumption in terms of portfolio valuation effects, a solution for portfolio valuation effects can then be derived from second-order approximations of Euler equations and auxiliary equilibrium conditions of the model.

The aggregate system in (47) produces unique solution for cross-country differences in real wealth and consumption (in terms of portfolio valuation effects), if matrix $\mathbf{D}_{\text{WC}}$ has as many eigenvalues outside the unit circle as the aggregate system of equilibrium conditions has non-predetermined variables. In this case, the models satisfy the well-known conditions of Blanchard and Kahn (1980). If additionally the models have no unit roots, then the models are stationary.

The matrix $\mathbf{D}_{\text{WC}}$ from Model 1 is particularly simple,

$$\text{Model 1:} \quad \mathbf{D}_{\text{WC}} = \begin{bmatrix} R & -1 \\ 0 & 1 \end{bmatrix}, \tag{48}$$

where, by inspection, the eigenvalues equal of the first-order aggregate system equal $\lambda_1 = R$ and $\lambda_2 = 1$. Model 1 satisfies the conditions of Blanchard and Kahn (1980) if the first $R > 1$, because the aggregate system has one non-predetermined variable, but Model 1 is non-stationary because of the unit root in the second eigenvalue. Models 2 through 4 approach Model 1 as the wealth-in-utility parameter θ approaches zero, so Model 1 provides a useful benchmark. We can establish that

Models 2 through 4 satisfy the conditions of Blanchard and Kahn (1980) and have no unit roots, simply by considering the slopes of the eigenvalues of matrix $\mathbf{D}_{\text{WC}}$ in Models 2 through 4 with respect to the parameter θ, evaluating the slopes at $\theta = 0$. We state the following proposition, and give a proof of the proposition in Appendix E.1 (not yet typed).

Proposition 3 (Stationarity of Cross-Country Differences for Aggregates)**.** *In the case of $\theta = 0$, matrix $\mathbf{D}_{\text{WC}}$ in Models 1 through 4 has eigenvalues $\lambda_1 = R$ and $\lambda_2 = 1$ that satisfy the conditions in Blanchard and Kahn (1980) for a unique rational expectations solution, but the solution is non-stationary because the second eigenvalue has a unit root.*

The eigenvalues in Models 2 through 4 have slopes with respect to θ that satisfy the inequalities

$$\left.\frac{\partial \lambda_1}{\partial \theta}\right|_{\theta=0} < 0\,, \qquad \left.\frac{\partial \lambda_2}{\partial \theta}\right|_{\theta=0} > 0\,. \tag{49}$$

In Models 2 through 4, there exists an $\epsilon < 0$ such that setting $\theta = \epsilon$ produces eigenvalues λ_1 and λ_2 that satisfy the conditions in Blanchard and Kahn (1980) for a unique rational expectations solution, and that satisfy the conditions for stationary.

Models 2 through 4 are stationary in other cases as well, not only in the neighborhood of $\theta = 0$, and not only in the more-money-more-problems case of $\theta < 0$. But we find it convenient to focus on the case of a sufficiently small negative value for θ in our analytical work because this case represents a small deviation from the benchmark model, and because this case allows us to solve analytically for real portfolio holdings in Model 2. The opposite case a sufficiently large positive value for θ also delivers stationary analytical solutions in Model 2, but leads to indeterminate steady-state real portfolio holdings. Indeterminate steady-state real portfolio holdings are a special case of Model 2 only; in Models 3 and 4, the case of a sufficiently large positive value for θ delivers both stationarity and determinacy of steady-state real portfolio holdings, but we do not solve Models 3 and 4 analytically.

For Models 1 and 2, we solve the first-order aggregate system analytically for cross-country differences in wealth and consumption, expressed as deviations from non-stochastic steady state values, and written in terms of exogenous variables, parameters, the lag of aggregate real wealth (a predetermined aggregate variable), and the aggregate portfolio valuation effect.[7] We use these solutions for cross-country differences in aggregate real wealth and consumption solutions to then derive solutions for the cross-country difference in aggregate portfolio valuation effects, the cross-country difference in real returns on nominal bonds, and the real exchange rate. This procedure allows us to state the following proposition.

Proposition 4 (First-Order Solution for Aggregate Cross-Country Differences)**.** *In Models 1*

[7] Devereux and Sutherland (2011) show that valuation effects can be treated as exogenous when using standard linear rational expectations solution procedures to solve the aggregate system in (47) in a first step, because to a first-order approximation, these effects behave as mean-zero i.i.d. random variables and do not affect the eigenvalues of the aggregate system. After solving the aggregate system in terms of portfolio valuation effects, we solve for portfolio valuation effects themselves using the definition of the real return on nominal bonds in (10).

through 4, the first-order aggregate system of equilibrium conditions in (47) yields the following first-order accurate solutions for cross-country differences in aggregate real wealth and aggregate consumption in terms of cross-country differences in lagged aggregate real wealth, aggregate portfolio valuation effects, and exogenous aggregate production:

$$\begin{bmatrix} \hat{W}_{\mathrm{X}t} \\ \hat{C}_{\mathrm{X}t} \end{bmatrix} = \boldsymbol{\eta}_{\mathrm{X}} \begin{bmatrix} \hat{W}_{\mathrm{X}t-1} \\ \hat{V}_{\mathrm{X}t} \\ \hat{Y}_{\mathrm{X}t} \end{bmatrix} + \mathcal{O}(\epsilon^2)\,, \tag{50}$$

where $\boldsymbol{\eta}_{\mathrm{X}}$ denotes a 2×3-matrix of partial elasticities with element $\eta_{\mathrm{X}ij}$, $i \in \{\mathrm{W}, \mathrm{C}\}$, $j \in \{\mathrm{W}, \mathrm{V}, \mathrm{Y}\}$, denoting the partial elasticity of variable i with respect to variable j. The partial elasticities $\eta_{\mathrm{X}ij}$ are functions of the parameters of the model.

Model 1 is non-stationary, and the partial elasticity $\eta_{\mathrm{WW}y}$ equals one. In the case of $\theta = \epsilon$, ϵ sufficiently small, Models 2, 3, and 4 are stationary, and the partial elasticity $\eta_{\mathrm{WW}y}$ lies between zero and one. Write this solution as a matrix instead, and be more rigorous with ϵ.

The elasticities in (50) are partial elasticities, in the sense that cross-country differences in aggregate portfolio valuation effects, like cross-country differences in aggregate real wealth and aggregate consumption, depend on cross-country differences in production and lagged aggregate real wealth in general equilibrium. Recall that the cross-country difference in portfolio valuation effects equals the cross-country difference between steady-state holdings of the domestic nominal bond multiplied by the difference in real returns on home and foreign nominal bonds. To obtain general elasticities from the partial elasticities in (50), we use first-order accurate expression for real returns on nominal bonds that derives from the definition of real returns on nominal bonds in (10) and from the quantity equations in (22):

$$\hat{R}_{\mathrm{X}t} = \eta_{\mathrm{X}RM} \hat{M}_{\mathrm{X}t} + \eta_{\mathrm{X}R\mathrm{V}} \hat{V}_{\mathrm{X}t} + \eta_{\mathrm{X}R\mathrm{Y}} \hat{Y}_{\mathrm{X}t} + \mathcal{O}(\epsilon^2)\,. \tag{51}$$

Using (51) for real returns still leaves the cross-country difference in steady state portfolio holdings undetermined, but the cross-country difference in steady state portfolio holdings can be found using the second-order aggregate Euler equation, as we show in below.

For Models 1 and 2, we derive analytical solutions to the first-order aggregate system of equilibrium conditions in Appendix **??**. For Models 3 and 4, the household utility function is non-linear in real wealth, which causes real wealth to appear as an additional variable in first-order household and aggregate Euler equations, where it does not appear in Models 1 and 2, as we show in (26). Because of this complication, we prefer to solve Models 3 and 4 numerically (to be completed).

3.2.5 Linear Rational Expectations Solution for Household Cross-Country Differences.

The solution to the first-order aggregate system in (50) can now be used to now solve the household system of equilibrium conditions. As with the aggregate system, we first solve the household system for cross-country differences in household real wealth and household consumption, and then use solutions for cross-country differences together with expressions for cross-country sums of household real wealth and household consumption to obtain expressions for household real wealth and household consumption for each country individually. At the household level, expressions for cross-country sums of household real wealth and household consumption require slightly more work to obtain than at the aggregate level, because a second system of household equations in cross-country sums must be solved at the household level, whereas market clearing conditions could be used straightforwardly at the aggregate level.

We let $\hat{B}_{\mathrm{X}t}(\rho)$, $\hat{W}_{\mathrm{X}t}(\rho)$ and $\hat{C}_{\mathrm{X}t}(\rho)$ denote cross-county differences in household real holdings of the domestic bond, household real wealth, and household consumption, where we compute differences for variables expressed as deviations from steady-state values, always computed as the home household variable minus the foreign household variable for home and foreign households with equal coefficients of relative risk aversion. We define these variables as

$$\begin{aligned}
\hat{C}_{\mathrm{X}t}(\rho) &= \hat{C}_{\mathrm{H}t}(\rho) - \hat{C}_{\mathrm{F}t}(\rho)\,,\\
\hat{B}_{\mathrm{X}t}(\rho) &= \hat{B}^{\mathrm{H}}_{\mathrm{HH}t}(\rho) - \hat{B}^{\mathrm{H}}_{\mathrm{FF}t}(\rho) = \hat{B}^{\mathrm{H}}_{\mathrm{HH}t}(\rho) - \hat{B}^{\mathrm{F}}_{\mathrm{FF}t}(\rho) - \hat{Q}_{\mathrm{HF}t} + \mathcal{O}(\epsilon^2)\,,\\
\hat{W}_{\mathrm{X}t}(\rho) &= \hat{W}^{\mathrm{H}}_{\mathrm{H}t}(\rho) - \hat{W}^{\mathrm{H}}_{\mathrm{F}t}(\rho)\,.
\end{aligned} \tag{52}$$

We write the household system in terms of the cross-country differences in home and foreign household real wealth and household consumption, aggregate real wealth, and aggregate and household portfolio valuation effects. Using a first-order approximation of the household budget constraint in (7), the first-order approximate household Euler equations in (39), and the first-order solution for aggregate real wealth in (50), we write the household system of equilibrium conditions as

$$\begin{bmatrix} \hat{W}_{\mathrm{X}t}(\rho) \\ \mathrm{E}_t\left[\hat{C}_{\mathrm{X}t+1}(\rho)\right] \\ \hat{W}_{\mathrm{X}t} \end{bmatrix} = \mathbf{D}_{\mathrm{WC}}(\rho) \begin{bmatrix} \hat{W}_{\mathrm{X}t-1}(\rho) \\ \hat{C}_{\mathrm{X}t}(\rho) \\ \hat{W}_{\mathrm{X}t-1} \end{bmatrix} + \mathbf{D}_{\mathrm{V}}(\rho) \begin{bmatrix} \hat{V}_{\mathrm{X}t}(\rho) \\ \hat{V}_{\mathrm{X}t} \end{bmatrix} + \mathbf{D}_{\mathrm{Y}}(\rho)\hat{Y}_{\mathrm{X}t} + \mathcal{O}(\epsilon^2)\,, \tag{53}$$

where $\hat{V}_{\mathrm{X}t}(\rho) = B_{\mathrm{X}}(\rho)\hat{R}_{\mathrm{X}t}$ denotes the cross-country difference in home portfolio valuation effects, and where $\mathbf{D}_{\mathrm{WC}}(\rho)$ denotes a 3×3-matrix, $\mathbf{D}_{\mathrm{V}}(\rho)$ denotes a 3×2-matrix, and $\mathbf{D}_{\mathrm{Y}}(\rho)$ denotes a 3×1-matrix of model parameters for home and foreign households with equal coefficients of relative risk aversion ρ.

The first-order household system of equilibrium conditions depends on aggregate real wealth as a second predetermined variable, alongside household real wealth. Aggregate real wealth enters the household problem through the real exchange rate. So far swe have stated household equilibrium conditions in the home and foreign countries in terms of real variables and deflated nominal variables, where we have deflated nominal variables in the domestic currency. Because we now consider cross-country differences in deflated nominal variables, it will be convenient to deflate all variables using a single currency, and we use the home currency. Deflating in a common currency rather than in the domestic currency requires the use of the real exchange rate. The real exchange rate in turn depends on lagged aggregate real wealth, so lagged aggregate real wealth enters the system of first-order household equilibrium conditions. The solution for aggregate real wealth in (50) must therefore be included in the first-order household system in (53).

To establish stationarity of the first-order household system of equilibrium conditions in (53) we follow the same strategy as with the first-order aggregate system. Namely, we show that the first-order household system in Model 1 displays non-stationary unit-root dynamics, then show that a marginal decrease in the wealth-in-utility parameter in the neighborhood of $\theta = 0$ is sufficient to induce stationarity in Models 2 through 4. For the household model,

In Model 1, where $\theta = 0$, the household parameter matrix $\mathbf{D}_{\text{WC}}(\rho)$ is again particularly simple,

$$\text{Model 1:} \quad \mathbf{D}_{\text{WC}}(\rho) = \begin{bmatrix} R & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{54}$$

where, by inspection, the eigenvalues of the first-order household system in Model 1 equal $\lambda_1(\rho) = R$, $\lambda_2(\rho) = 1$, and $\lambda_3(\rho) = 1$. The third eigenvalue $\lambda_3(\rho)$ equals the coefficient $\eta_{\text{WW}y}$ from the solution to the aggregate system in (50) for Models 1 through 4; we have seen above that this coefficient equals one for Model 1, which has a unit root.

The household problem in Models 1 through 4 has two predetermined variables, the cross-country difference in aggregate real wealth and the cross-country difference in household real wealth, and one non-predetermined variable, the cross-country difference in household consumption. For a unique and stationary solution to the first-order household system of equilibrium conditions, the models must have one eigenvalue outside the unit circle, the remaining two eigenvalues inside the unit circle. If the first-order aggregate system is stationary, then the third eigenvalue $\lambda_3(\rho) = \eta_{\text{WW}y}$ will lie inside the unit circle. Of the remaining remaining two eigenvalues, one must lie outside and one inside the unit circle. We state the following proposition.

Proposition 5 (Stationarity of Cross-Country Differences for Households)**.** *In the case of $\theta = 0$, matrix $\mathbf{D}_{\text{WC}}(\rho)$ has eigenvalues $\lambda_1(\rho) = R$, $\lambda_2(\rho) = 1$, and $\lambda_3(\rho) = 1$ that satisfy the conditions in Blanchard and Kahn (1980) for a unique rational expectations solution, but that fail to satisfy the conditions for stationary because the second and third eigenvalues have unit roots.*

The eigenvalues have slopes with respect to θ that satisfy the inequalities

$$\left.\frac{\partial\lambda_1(\rho)}{\partial\theta}\right|_{\theta=0} < 0\,, \quad \left.\frac{\partial\lambda_2(\rho)}{\partial\theta}\right|_{\theta=0} > 0\,, \quad \left.\frac{\partial\lambda_3(\rho)}{\partial\theta}\right|_{\theta=0} > 0\,. \tag{55}$$

In Models 2 through 4, there exists an $\epsilon < 0$ such that setting $\theta = \epsilon$ produces eigenvalues λ_1, λ_2, and λ_3 that satisfy the conditions in Blanchard and Kahn (1980) for a unique rational expectations solution, and that satisfy the conditions for stationary.

We solve the first-order household system in (53) analytically for cross-country differences in household real wealth and household consumption, expressed as deviations from steady state values, and written in terms of exogenous variables, parameters, lags of aggregate and household real wealth, and aggregate and household portfolio valuation effects. We state the following proposition:

Proposition 6 (First-Order Solution for Household Cross-Country Differences)**.** *The first-order household system of equilibrium conditions in (53) yields the following first-order accurate solutions for cross-country differences in household real wealth and household consumption in terms of cross-country differences in lagged household real wealth, lagged aggregate real wealth, aggregate and household portfolio valuation effects, and exogenous aggregate production:*

$$\begin{bmatrix} \hat{W}_{\mathrm{X}t}(\rho) \\ \hat{C}_{\mathrm{X}t}(\rho) \end{bmatrix} = \boldsymbol{\eta}_{\mathrm{X}_\rho}(\rho) \begin{bmatrix} \hat{W}_{\mathrm{X}t-1}(\rho) \\ \hat{V}_{\mathrm{X}t}(\rho) \end{bmatrix} + \boldsymbol{\eta}_{\mathrm{X}}(\rho) \begin{bmatrix} \hat{W}_{\mathrm{X}t-1} \\ \hat{V}_{\mathrm{X}t} \\ \hat{Y}_{\mathrm{X}t} \end{bmatrix} + \mathcal{O}(\epsilon^2)\,, \tag{56}$$

where $\boldsymbol{\eta}_{\mathrm{X}_\rho}(\rho)$ is a 2×2-matrix of partial elasticities with elements $\eta_{\mathrm{X}_\rho ij}(\rho)$, $i \in \{\mathrm{C}, \mathrm{W}\}$, $j \in \{\mathrm{W}, \mathrm{V}\}$, and where $\boldsymbol{\eta}_{\mathrm{X}}(\rho)$ is a 2×3-matrix of partial elasticities with elements $\eta_{\mathrm{X}ij}(\rho)$, $i \in \{\mathrm{C}, \mathrm{W}\}$, $j \in \{\mathrm{W}, \mathrm{V}, \mathrm{Y}\}$. For example, $\eta_{\mathrm{XWW}}(\rho)$ is the elasticity of household real wealth $\hat{W}_{\mathrm{X}t}(\rho)$ with respect to lagged aggregate real wealth $\hat{W}_{\mathrm{X}t-1}$ and $\eta_{\mathrm{X}_\rho\mathrm{WW}}(\rho)$ is the elasticity of household real wealth $\hat{W}_{\mathrm{X}t}(\rho)$ with respect to lagged household real wealth $\hat{W}_{\mathrm{X}t-1}(\rho)$.

Model 1 is non-stationary in cross-country differences, and the partial elasticity $\eta_{\mathrm{X}_\rho\mathrm{WW}}(\rho)$ equals one. There exists an $\epsilon < 0$ such that setting $\theta = \epsilon$ induces stationarity in Models 2, 3, and 4.

3.2.6 Linear Rational Expectations Solutions for Aggregates and Households.

From the cross-country differences that we have derived in the previous two sections for aggregate and household real wealth and consumption, we cannot yet characterize the investment and consumption behavior of home and foreign counties or households individually. We now solve for cross-country sums of household and aggregate variables, and, from these cross-country sums and the cross-country differences that we obtained in the previous sections, we back out solutions for home and foreign countries and households individually.

We adopt a compact notation for cross-country sums of household and aggregate variables that have been expressed as deviations from non-stochastic steady-state values. We use a subscript

"$_{\mathrm{G}}$" to denote these cross-country sums. For example, we denote cross-country sums of aggregate consumption and household consumption as $\hat{C}_{\mathrm{G}t}$ and $\hat{C}_{\mathrm{G}t}(\rho)$, respectively, and define these variables as

$$\hat{C}_{\mathrm{G}t} = \hat{C}_{\mathrm{H}t} + \hat{C}_{\mathrm{F}t} \quad \text{and} \quad \hat{C}_{\mathrm{G}t}(\rho) = \hat{C}_{\mathrm{H}t}(\rho) + \hat{C}_{\mathrm{F}t}(\rho)\,. \tag{57}$$

For cross-country sums of deflated nominal variables, we deflate always using home currency. For example, we denote cross-country sums of aggregate real wealth and household real wealth as $\hat{W}_{\mathrm{G}t}$ and $\hat{W}_{\mathrm{G}t}(\hat{\rho})$, respectively, and define these variables as

$$\hat{W}_{\mathrm{G}t} = \hat{W}^{\mathrm{H}}_{\mathrm{H}t} + \hat{W}^{\mathrm{H}}_{\mathrm{F}t} \quad \text{and} \quad \hat{W}_{\mathrm{G}t}(\hat{\rho}) = \hat{W}^{\mathrm{H}}_{\mathrm{H}t}(\rho) + \hat{W}^{\mathrm{H}}_{\mathrm{F}t}(\rho)\,. \tag{58}$$

We define analogous cross-country sums for aggregate production, $\hat{Y}_{\mathrm{G}t}$, and for household and aggregate portfolio valuation effects, $\hat{V}_{\mathrm{G}t}(\rho)$ and $\hat{V}_{\mathrm{G}t}$, respectively.

For aggregate real wealth and aggregate consumption in Models 1 through 4, cross-country sums can be derived straight-forwardly from market clearing conditions. For aggregate real wealth, we simply take first-order Taylor series expansions of the home and foreign bond market clearing conditions in (19) and then sum across countries. For aggregate consumption, we take first-order Taylor series expansions of the home and foreign goods market clearing conditions in (19), and combine with first-order aggregate demand functions in (43), and sum across countries. We obtain

$$\hat{W}_{\mathrm{G}t} = 0 \quad \text{and} \quad \hat{C}_{\mathrm{G}t} = \hat{Y}_{\mathrm{G}t}\,, \tag{59}$$

where the cross-country sum of aggregate real wealth must be zero, expressed as a deviation from the non-stochastic steady state, because bonds are in zero net supply. Combining this solution for cross-country sums with the solution for cross-country differences in (50) yields solutions for aggregate real wealth and aggregate consumption for home and foreign countries individually.

For aggregate cross-country differences, we established that the model is stationary when the wealth-in-utility parameter is non-zero, by considering the eigenvalues from the first-order aggregate system of equilibrium conditions for cross-country differences. For cross-country sums, stationarity is easier to establish. The cross-country sum of aggregate real wealth is zero at all times, expressed as a deviation from non-stochastic steady state, and is therefore stationary, and the cross-country sum of aggregate consumption equals exogenous production of home and foreign goods, which we assume to be stationary.

For household real wealth and household consumption, there are no market clearing conditions that can be used to obtain convenient expressions for cross-country sums. Instead, we consider a system of household equilibrium conditions formed by cross-country sums of the household Euler equation, and cross-country sums of the household budget constraint.

To obtain aggregate solutions for aggregate real wealth and aggregate consumption for each country individually, still in terms of aggregate portfolio valuation effects, we combine the solutions to the aggregate system in (50) with the market clearing conditions in (19).

We now solve the household system for cross-country sums of household real wealth and household consumption, denoted $\hat{W}_{\text{G}t}(\hat{\rho})$ and $\hat{C}_{\text{G}t}(\rho)$, respectively, for home and foreign households with equal coefficients of relative risk aversion. We define these cross-country sums as

$$\hat{W}_{\text{G}t}(\hat{\rho}) = \hat{W}^{\text{H}}_{\text{H}t}(\rho) + \hat{W}^{\text{H}}_{\text{F}t}(\rho) \quad \text{and} \quad \hat{C}_{\text{G}t}(\rho) = \hat{C}_{\text{H}t}(\rho) + \hat{C}_{\text{F}t}(\rho)\,. \tag{60}$$

and write the first-order household system in terms of cross-country sums of home and foreign household real wealth and household consumption as

$$\begin{bmatrix} \hat{W}_{\text{G}t}(\hat{\rho}) \\ \text{E}_t\left[\hat{C}_{\text{G}t+1}(\rho)\right] \end{bmatrix} = \mathbf{S}_{\text{WC}}(\rho) \begin{bmatrix} \hat{W}_{\text{G}t-1}(\hat{\rho}) \\ \hat{C}_{\text{G}t}(\rho) \end{bmatrix} + \mathbf{S}_{\text{V}}(\rho)\hat{V}_{\text{G}t}(\rho) + \mathbf{S}_{\text{Y}}(\rho)\hat{Y}_{\text{G}t} + \mathcal{O}(\epsilon^2)\,. \tag{61}$$

Household Portfolio Holdings. In particular, we derive solution can now be derived for the portfolio holdings of individual households. It is shown that individual portfolio holdings depend on the coefficient of relative risk aversion of the individual household, and for different levels of relative risk aversion, different portfolio holding patterns arise. Specifically, it becomes possible in this model with risk aversion heterogeneity across household that certain households in one country are take long positions in the domestic asset and short positions in the foreign asset, while other households in the same country take short positions in the domestic asset and long positions in the foreign asset. In this way, the portfolio holdings of individual households are decoupled from the net foreign asset position of the aggregate economy.

Following a procedure similar to that used to solve the aggregate system, and again using the second-order approximate Euler equations for home and foreign households, a solution for household portfolio holdings can be derived. The solution for household portfolio holdings is only second-order accurate if the aggregate portfolio holdings $\overline{A}^{H}$ on the right-hand side are exact. The expression for $\overline{A}^{H}$ is not exact, and cannot be used here without lowering the accuracy of the solution.

Notice three things about the result: first, the sign of the coefficient switches when risk aversion takes values on either side of the home bias index ϕ; second, as risk aversion becomes infinite the portfolio holdings converge to a finite number as long as aggregate portfolio holdings are finite; third, at the threshold value of $\rho = \phi$, portfolio holdings are zero. The figure below illustrates these properties of the solution.

Expected Real Returns on Nominal Bonds. To solve for the expected real return differential on home and foreign nominal bonds, recall the second-order approximate expressions for the second-order approximate home and foreign Euler equations for individual households. Instead of aggregating and then differencing the expressions, now aggregate and add the expressions to obtain a condition that will pin down the expected return differential, where the definition for $\hat{R}^{e}_{Xt}$

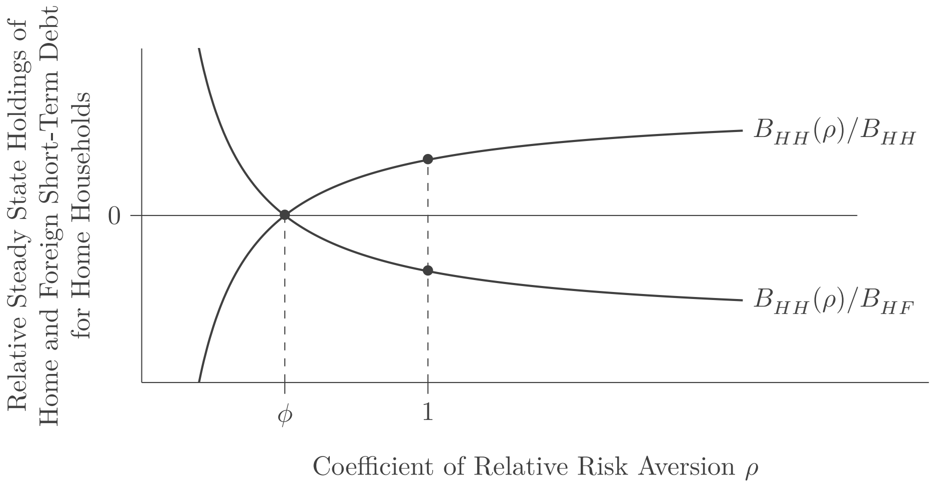

Figure 4 – Relative Steady State Holdings of Home and Foreign Bonds for Households in Country i as a Function of the Household Coefficient of Relative Risk Aversion. The figures plots household steady-state portfolio holdings of home and foreign bonds relative to aggregate steady-state portfolio holdings of home and foreign bonds as a function of the household coefficient of relative risk aversion for households in country H. The functions are sketched based on the solution for individual portfolio positions. The sketch assumes that steady-state aggregate home holdings of the home bond are positive, i.e. that $B_{HH} > 0$. The coefficient of relative risk aversion for household ρ is denoted ρ and drawn on the horizontal axis of the figure. The steady-state home and foreign bond holdings of home household ρ are denoted $B_{HH}(\rho)$ and $B_{HF}(\rho)$, respectively, are drawn on the vertical axis of the figure. The figure shows that a minority of individual investors have coefficients of relative risk aversion that lie below the value of ϕ and take short positions in the home bond and long positions in the foreign bond, whereas the majority of investors have coefficients of relative risk aversion that lie above the value of ϕ and take long positions in the home bond and short positions in the foreign bond. The figure also shows the portfolio position of the household with log utility (the case of $\rho = 1$). Figure 4 gives only a sketch of the solution. The solution and sketch will need to be double-checked and refined, and a careful calibration of the model will need to be undertaken.

assumes that deviations of log returns from steady state are log normal (and therefore assumes that shocks in this model are log normal). We follow Lettau (2003) in defining the return in this way, because the definition eliminates "Jensen" terms that would otherwise clutter the expression for expected returns.

From here, use the market clearing conditions for consumption goods together with aggregate version of the second-order approximate log-linear expressions for household consumption baskets to obtain

$$\hat{C}_{t+1} + \hat{C}^*_{t+1} = \hat{Z}_{t+1} + \hat{Z}^*_{t+1} + \mathcal{O}(\epsilon^2)\,. \tag{62}$$

Using (62) together with the expression for the real exchange rate in terms of consumption and productivity differentials and the solution for consumption, rewrite as

$$\begin{aligned} \hat{R}^e_{Xt+1} &= \mathrm{E}_t\left[\hat{R}_{Xt+1}\left(\hat{Z}_{t+1} + \hat{Z}^*_{t+1}\right)\right] \\ &\quad + \frac{\phi^2}{\phi^2-1}\left[1-\beta(1-\phi)-\phi^{-1}\right]\mathrm{E}_t\left[\hat{R}_{Xt+1}\Delta\hat{Z}_{t+1}\right] \\ &\quad + \frac{\phi^2}{\phi^2-1}(1-\beta)\beta^{-1}\overline{A}^{\scriptscriptstyle H}\,\mathrm{E}_t\left[\hat{R}^2_{Xt+1}\right] + \mathcal{O}(\epsilon^3)\,. \end{aligned} \tag{63}$$

The expression in (63) represents a nice second-order accurate partial equilibrium solution for the expected real return differential on home and foreign nominal bonds.

The expression represents a second-order accurate general equilibrium solution for the expected real return differential on home and foreign nominal bonds. However, notice that $\overline{A}^{\scriptscriptstyle H}$ appears on the right-hand side of the expression. Because the general equilibrium solution for $A^{\scriptscriptstyle H}_t$ is only "zero-order" accurate, using to eliminate $A^{\scriptscriptstyle H}_t$ here would reduce the accuracy of the solution for $\hat{R}^e_{Xt}$. However, notice that $A^{\scriptscriptstyle H}_t$ always appears with the coefficient $1-\beta$, which will tend to be small when the subjective discount factor β is calibrated to take a value close to one. In the limit, as the subjective discount factor approaches unity, the following much neater expression obtains:

$$\lim_{\beta\to 1}\hat{R}^e_{Xt+1} = 2(1-\phi)\,\mathrm{E}_t\left[\hat{Z}_{t+1}\Delta\hat{Z}_{t+1}\right] - (1-\phi)^2\,\mathrm{E}_t\left[\left(\Delta\hat{Z}_{t+1}\right)^2\right] + \mathcal{O}(\epsilon^3)\,. \tag{64}$$

This limiting result may also clarify why monetary shocks have seemed unimportant in empirical regressions of the carry trade return. The result suggests that monetary shocks will be unimportant whenever monetary shocks are equally volatile across countries but uncorrelated, or whenever the subjective discount factor approaches unity.

Proposition 7 (Expected Real Return on Nominal Bonds)**.** *In Models 1 and 2, to a second-order approximation, the expected difference between real returns on the home and foreign nominal bonds, defined as home minus foreign, will be negative in a stationary and determinate economy with positive steady-state holdings of the home bond, if wealth gives households slight* dis-*utility, and if the variance of foreign production is sufficiently greater than the variance of home production.*

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A Notation

We use superscripts on variables to indicate currency, and we use the subscript "x" to indicate cross-country differences in variables. We always define cross-country differences as home minus foreign. For nominal variables, cross-country differences are always deflated in home currency. We drop time subscripts to denote non-stochastic steady states. For the most part, hats above variables indicate log deviations from steady state; real wealth and real portfolio holdings are exceptions for which hats indicate level deviations from steady state.

The table below provides an overview of our notation. Definitions of variable are given in the main body of the text, as variables are introduced.

Category	**Symbol**	**Description**
Parameters:	α	Expenditure share on domestic goods
	β	Subjective discount factor
	θ	Wealth-in-utility weight
	ρ	Household coefficient of relative risk aversion and index
	$f(\rho)$	Density function for coefficient of relative risk aversion
	ρ_m	Density function scale parameter
	κ	Density function shape parameter
	ϕ	Home bias index, $\phi := 2\alpha - 1$
	γ	Consumption basket parameter, $\gamma := \alpha^\alpha(1-\alpha)^{1-\alpha}$
	$\sigma(\rho)$	Intertemporal elasticity of substitution, $\sigma(\rho) := 1/\rho$
	$\tilde{\sigma}$	Aggregate intertemporal elasticity of substitution
	$\tilde{\rho}$	Aggregate coefficient of relative risk aversion
	$\eta_{\mathrm{X}xy}(\rho)$	Household partial elasticity of x with respect to y
	$\eta_{\mathrm{X}xy}$	Aggregate partial elasticity of x with respect to y
	$\mathbf{D}_i$	Matrix of model parameters in first-order aggregate system of equilibrium conditions, $i \in \{WC, V, Y\}$
	D_{ij}	Element i, j of matrix $\mathbf{D}_k$
Bond Holdings:	$A_{ijt}(\rho)$	Country-i household-ρ holdings (in quantity) of country-j bonds
	A_{ijt}	Country-i aggregate holdings (in quantity) of country-j bonds
	A_{it}	Exogenous net supply (in quantity) of the country-i bond
	$B^k_{ijt}(\rho)$	Country-i household-ρ holdings (in real value) of the country-j bond, deflated in currency k

Continued ...

Category	Symbol	Description
	$B_{\text{x}t}(\rho)$	Cross-country difference in holdings (in real value) of the home bond by home and foreign households with identical risk aversion, deflated in home currency
	B_{ijt}^{k}	Country-i aggregate holdings (in real value) of the country-j bond, deflated in currency k
	$B_{\text{x}t}$	Cross-country difference in aggregate holdings (in real value) of the home bond, deflated in home currency
	B_{it}^{j}	Exogenous net supply (in real value) of the country-i bond, deflated in currency j
	$B_{ijt}^{(+)}(\rho)$	Country-i household ρ long position (in real value) in the country-j bond, deflated in USD
	$B_{ijt}^{(+)}$	Country-i aggregate long position (in real value) in the country-j bond, deflated in USD
	$B_{ijt}^{(-)}(\rho)$	Country-i household ρ short position (in real value) in the country-j bond, deflated in USD
	$B_{ijt}^{(-)}$	Country-i aggregate long position (in real value) in the country-j bond, deflated in USD
	SR_{ijt}	Short ratio for country i holdings of the country-j bond, defined as $SR_{ijt} := -B_{ijt}^{(-)}/(B_{ijt}^{(+)} - B_{ijt}^{(-)})$
Consumption:	$C_{it}(\rho)$	Country-i household-ρ consumption basket
	$C_{\text{x}t}(\rho)$	Cross-country difference in household consumption baskets for home and foreign households with identical risk aversion, defined as home minus foreign
	C_{it}	Country-i aggregate consumption basket
	$C_{\text{x}t}$	Cross-country difference in aggregate consumption baskets, defined as home minus foreign
	$C_{ijt}(\rho)$	Country-i consumption of good j for household id
	C_{ijt}	Country-i consumption of good j
Endowment:	$Y_{it}(\rho)$	Country-i household ρ endowment basket
	Y_{it}	Country-i aggregate endowment basket
	$Y_{ijt}(\rho)$	Country-i household-ρ good-j endowment
	Y_{ijt}	Country-i aggregate good-j endowment
Prices:	$P_{\text{B}it}^{j}$	Nominal price of bond i in currency j at time t

Continued . . .

Category	Symbol	Description
	$P_{\mathrm{c}it}^{j}$	Nominal price of good i in currency j at time t
	P_{it}^{j}	Nominal price level in country i in currency j
Exchange Rate:	Q_{ijt}	Real exchange rate, defined as units of home consumption basket per one unit of foreign consumption basket
	S_{ijt}	Nominal exchange rate, defined as units of home currency per one unit of foreign currency
Real Returns:	R_{t}^{i}	Gross risk-free real return, deflated in currency i
	R_{it}^{j}	Gross real return on bond i, deflated in currency j
	$R_{\mathrm{x}t}$	Cross-country difference in gross real bond returns, defined as home minus foreign, deflated in home currency
Valuation Effect:	$V_{it}^{j}(\rho)$	Country-i household-ρ real valuation effect, deflated in currency j
	$V_{\mathrm{x}t}(\rho)$	Cross-country difference in real valuation effects for home and foreign households with identical risk aversion, defined as home minus foreign
	V_{it}^{j}	Country-i aggregate real valuation effect, deflated in currency j
	$V_{\mathrm{x}t}$	Cross-country difference in aggregate real valuation effects, defined as home minus foreign
Wealth:	$W_{it}^{j}(\rho)$	Household real wealth for household ρ in country i, deflated in currency j
	$W_{\mathrm{x}t}(\rho)$	Cross-country difference in household wealth, defined as home minus foreign for home and foreign households with equal ρ, deflated in home currency
	W_{it}^{j}	Aggregate real wealth in country i, deflated in currency j
	$W_{\mathrm{x}t}$	Cross-country difference in aggregate wealth at time t, defined as home minus foreign, deflated in home currency
Money Supply:	M_{it}^{j}	Nominal money supply of country i in currency j
Tax:	$\tau_{it}^{j}(\rho)$	Country-i household-ρ real tax burden
	τ_{it}^{j}	Country-i aggregate real tax burden
Production:	Y_{it}	Country-i aggregate output of good i
	$Y_{\mathrm{x}t}$	Cross-country difference in output of home and foreign goods, defined as home minus foreign

Continued ...

Category	Symbol	Description
Productivity:	Z_{it}	Country-i aggregate total factor productivity
Utility:	$U_{it}(\rho)$	Country-i household-ρ expected discounted lifetime utility
	$U^{\mathrm{C}}_{it}(\rho)$	Country-i household-ρ period utility function over consumption
	$U^{\mathrm{W}}_{it}(\rho)$	Country-i household-ρ period utility function over real wealth

B Estimating Portfolio Positions in Short-Term Debt

In this appendix section, we will describe in detail our procedure for estimating country-level short-term debt holdings by currency. Basically, we integrate the empirical procedures that Fidora et al. (2007) and Lane and Shambaugh (2010a) develop, with a few modifications. This section needs to be written, but eventually it will give the specifications of the gravity equations that we use to predict bilateral portfolio positions for non-reporting countries, and the specifications of the regression equation that we use to predict domestic-market short-term debt issuances for countries with no available IDS data. It will also describe the procedure for handling world central bank holdings of official foreign exchange reserves, and our use of Treasury data on U.S. holdings of short-term debt by currency, and the problem with the Treasury data that it only reports gross long positions, and our procedure of identifying at least some of the missing U.S. short positions.

C Portfolio Holdings Decomposition

In the data there are N_t reporting countries each year t, with assets in non-zero net supply, while our theory assumes two symmetric countries with assets in zero net supply. Our theoretical assumptions introduce a gap between our model and the data, and we would like to better understand how large this gap is, and which assumptions contribute most to the gap between our model and the data. In this appendix we describe a procedure for collapsing the N_t countries in our data each year into two countries (home and foreign), and a procedure decomposing the collapsed 2×2 matrix of portfolio holdings into an optimal theory-consistent component of portfolio holdings, and three additional components that arise when the assumptions of the model fail in the data.

The procedure we present here can be viewed as an alternative to the procedure proposed in Lane and Shambaugh (2010b) and applied by Amdur (2010) in a context similar to ours. The procedure described here has the advantage of being optimal, in the sense that the model-consistent component is as close to the collapsed data as possible (measuring closeness by the Frobenius norm). It has the further advantage of being closely tied to specific assumptions that we make in the theoretical model.

C.1 Collapsing Portfolio Holdings and Returns from N Countries to 2

Let $\tilde{\tilde{\mathbf{B}}} \in \mathbb{R}^{N\times N}$ denote the portfolio holdings data, and let elements $\tilde{\tilde{B}}_{ij}$ represent assets owned by country i and issued by country j. Let $\tilde{\tilde{\mathbf{R}}} \in \mathbb{R}^{N\times 1}$ denote the real returns data, and let $\tilde{\tilde{R}}_{Bj}$ represent the real return on the asset issued by country j. Write the portfolio holdings and real return data as

$$\tilde{\tilde{\mathbf{B}}} = \begin{pmatrix} \tilde{\tilde{B}}_{11} & \tilde{\tilde{B}}_{12} & \dots & \tilde{\tilde{B}}_{1N} \\ \tilde{\tilde{B}}_{21} & \tilde{\tilde{B}}_{22} & \dots & \tilde{\tilde{B}}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{B}}_{N1} & \tilde{\tilde{B}}_{N2} & \dots & \tilde{\tilde{B}}_{NN} \end{pmatrix}, \quad \tilde{\tilde{\mathbf{R}}} = \begin{pmatrix} \tilde{\tilde{R}}_{B1} \\ \tilde{\tilde{R}}_{B2} \\ \vdots \\ \tilde{\tilde{R}}_{BN} \end{pmatrix}. \tag{65}$$

where we assume that the home country occupies the first row and column of $\tilde{\tilde{\mathbf{B}}}$ and first row of $\tilde{\tilde{\mathbf{R}}}$.

Portfolio holdings in the data are $N \times N$, but in our model they are 2×2, so we need to collapse the data in order to get something that we can map to the model. To collapse the data matrix $\tilde{\tilde{\mathbf{B}}}$ from $\mathbb{R}^{N\times N}$ to $\mathbb{R}^{2\times 2}$, treat the countries outside of row one as "foreign", and sum the appropriate foreign asset holdings to obtain a 2×2 collapsed matrix $\tilde{\mathbf{B}}$:

$$\tilde{\mathbf{B}} = \begin{pmatrix} \tilde{B}_{11} = \tilde{\tilde{B}}_{11} & \tilde{B}_{12} = \sum_{j=2}^{N} \tilde{\tilde{B}}_{1j} \\ \tilde{B}_{21} = \sum_{i=2}^{N} \tilde{\tilde{B}}_{i1} & \tilde{B}_{22} = \sum_{i=2}^{N} \sum_{j=2}^{N} \tilde{\tilde{B}}_{ij} \end{pmatrix}. \tag{66}$$

The elements in the collapsed data matrix $\tilde{\mathbf{B}}$ can now be interpreted as the home and foreign portfolio holdings of the home and foreign bond, where the foreign bond is a composite.

It will be necessary to also collapse the N real returns into a home and a foreign return, just as the N-country asset holdings were collapsed into home and foreign asset holdings. Let $\tilde{\mathbf{R}} \coloneqq \left(\tilde{\tilde{R}}_{B1} \quad \tilde{R}_{B2}\right)'$ denote USD real returns, and find the $\tilde{R}_{B2}$ that satisfies

$$\mathbf{1}_N' \tilde{\tilde{\mathbf{B}}} \tilde{\tilde{\mathbf{R}}} = \mathbf{1}_2' \tilde{\mathbf{B}} \tilde{\mathbf{R}} \,, \tag{67}$$

where $\mathbf{1}_N$ is a $(N \times 1)$-column vector of ones. Notice that the $\tilde{\tilde{R}}_{B1}$ in $\tilde{\mathbf{R}}$ comes directly from the original data. The $\tilde{R}_{B2}$ in $\tilde{\mathbf{R}}$ that satisfies (67) is then a weighted average of the real returns on the remaining assets in the data:

$$\tilde{R}_{B2} \coloneqq \sum_{i=1}^{N} \sum_{j=2}^{N} \omega_{ij} \tilde{\tilde{R}}_{Bj} \,, \quad \omega_{ij} \coloneqq \frac{\tilde{\tilde{B}}_{ij}}{\sum_{i=1}^{N} \sum_{j=2}^{N} \tilde{\tilde{B}}_{ij}} \,. \tag{68}$$

In the following sections, we decompose the collapsed matrix $\tilde{\mathbf{B}}$ of home and foreign asset holdings into an optimal model-consistent component, which we denote $\mathbf{B}$, and three additional components that arise when specific modelling assumptions fail in the data.

C.2 Optimal Model-Consistent Component of the Collapsed Data

We now seek a model-consistent component of the collapsed data. The model-consistent component must be consistent with two model assumptions: first, that countries are steady-state symmetric, and second, that assets are in zero net supply. These two assumptions imply that the model-consistent component can be represented as a (2×2) matrix that is bisymmetric with column sums that equal zero:

$$\mathbf{B}^* = B^* \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{69}$$

Notice that $\mathbf{B}^*$ is unique up to the arbitrary scalar value B^*. The scalar B^* can be chosen to minimize the distance between the collapsed data matrix $\tilde{\tilde{\mathbf{B}}}$ and an arbitrary model-consistent data matrix $\mathbf{B}^*$, using the Frobenius norm $\|\cdot\|$ to measure distance:

$$\begin{aligned} \left\|\tilde{\mathbf{B}} - \mathbf{B}^*\right\| &= \left\| \begin{pmatrix} \tilde{B}_{11} - B^* & \tilde{B}_{12} + B^* \\ \tilde{B}_{21} + B^* & \tilde{B}_{22} - B^* \end{pmatrix} \right\| \\ &= \left[(\tilde{B}_{11t} - B^*)^2 + (\tilde{B}_{12t} + B^*)^2 + (\tilde{B}_{21t} + B^*)^2 + (\tilde{B}_{22t} - B^*)^2 \right]^{\frac{1}{2}}. \end{aligned} \tag{70}$$

Now let $B = \arg\min_{B^*} \left\|\tilde{\mathbf{B}} - \mathbf{B}^*\right\|$ denote the optimal scalar implied by the first-order optimality condition from minimizing (70):

$$0 = \frac{\partial}{\partial B^*} \left\|\tilde{\mathbf{B}} - \mathbf{B}^*\right\| \Bigg|_{B^*=B} \quad \Leftrightarrow \quad B = \frac{1}{2}\left[\frac{1}{2}\left(\tilde{B}_{11} + \tilde{B}_{22}\right) - \frac{1}{2}\left(\tilde{B}_{12} + \tilde{B}_{21}\right)\right], \tag{71}$$

and write the optimal model-consistent matrix $\mathbf{B}$ as

$$\mathbf{B} = B \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{72}$$

Notice from (71) that B is one-half the difference between the diagonal and anti-diagonal element averages in the collapsed data matrix $\tilde{\mathbf{B}}$. The value of B will therefore be positive if home and foreign in the collapsed data hold their own assets in greater quantity than they hold each other's assets—in other words, if there is a tendency in the data towards home bias in asset holdings.

C.3 Decomposing the Collapsed Data Matrix

The collapsed data matrix $\tilde{\mathbf{B}}$ can now be decomposed into the optimal model-consistent component matrix $\mathbf{B}$ in (72) and three model-inconsistent component matrices:

$$\tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}^{\mathrm{b}} + \mathbf{B}^{\mathrm{z}} + \mathbf{B}^{\mathrm{bz}}, \tag{73}$$

where the model-inconsistent component matrices are given by

$$\mathbf{B}^{\mathrm{b}} = B^{\mathrm{b}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad \mathbf{B}^{\mathrm{z}} = B^{\mathrm{z}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{B}^{\mathrm{bz}} = B^{\mathrm{bz}} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \tag{74}$$

and where the coefficients are given by

$$\begin{aligned} B^{\mathrm{b}} &= \frac{1}{2}\left[\frac{1}{2}\left(\tilde{B}_{11} + \tilde{B}_{12}\right) - \frac{1}{2}\left(\tilde{B}_{21} + \tilde{B}_{22}\right)\right], \\ B^{\mathrm{z}} &= \frac{1}{2}\left[\frac{1}{2}\left(\tilde{B}_{11} + \tilde{B}_{22}\right) + \frac{1}{2}\left(\tilde{B}_{12} + \tilde{B}_{21}\right)\right], \\ B^{\mathrm{bz}} &= \frac{1}{2}\left[\frac{1}{2}\left(\tilde{B}_{11} + \tilde{B}_{21}\right) - \frac{1}{2}\left(\tilde{B}_{12} + \tilde{B}_{22}\right)\right]. \end{aligned} \tag{75}$$

This decomposition can be verified by substituting the expressions in (74) and (75) into the decomposition in (73). The decomposition works, and the components are intuitive (we describe the intuition below), but we have no nice mathematical justification for it. For instance, is the decomposition it unique? Are the model-inconsistent components optimal in some sense? Do the model-inconsistent components double-count anything? We do believe that the decomposition is useful and sensible, but we still aim to improve the mathematical justifications for it.

Each decomposition component does have an intuitive interpretation. The matrix $\mathbf{B}^{\mathrm{b}}$ captures deviations in the data from the assumption that asset holdings are symmetric across countries. The coefficient B^{b} equals half of the difference between the two row averages in the collapsed data, and therefore equals zero if asset holdings are symmetric in the collapsed data. The matrix $\mathbf{B}^{\mathrm{z}}$ captures deviations in the data from the assumption that assets are in zero net supply. The coefficient B^{z} equals the average asset holdings in the collapsed data, and therefore equals zero if assets are in zero net supply in the collapsed data. The matrix $\mathbf{B}^{\mathrm{bz}}$ captures simultaneous deviations in the collapsed data from the assumptions of symmetry of portfolio holdings and zero net supply of assets. The coefficient B^{bz} equals half of the difference between column averages in the collapsed data, and therefore equals zero if asset holdings are symmetric or if assets are in zero net supply in the collapsed data.

The matrix decomposition in (73) is useful because it helps measure the restrictiveness of the model's assumptions relative to the data. If model's two key assumptions held in the data, then the final three components of the decomposition would be null matrices, and the optimal model-consistent matrix would equal the collapsed data matrix: $\tilde{\mathbf{B}} = \mathbf{B}$. If asset holdings were asymmetric across countries but assets were in zero net supply in the collapsed data, then $\tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}^{\mathrm{b}}$. If assets were in non-zero net supply but asset holdings were symmetric across countries in the collapsed data, then $\tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}^{\mathrm{z}}$. If both assumptions failed in the collapsed data, then $\tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}^{\mathrm{b}} + \mathbf{B}^{\mathrm{z}} + \mathbf{B}^{\mathrm{bz}}$.

The matrix decomposition in (73) can be mapped to into real numbers using the Frobenius norm. The size of the gap between the data and the model is

$$\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\| = \left(\left\|\mathbf{B}^{\mathrm{b}}\right\|^2 + \left\|\mathbf{B}^{\mathrm{z}}\right\|^2 + \left\|\mathbf{B}^{\mathrm{bz}}\right\|^2\right)^{\frac{1}{2}}, \tag{76}$$

where the expression in (76) is simpler than it otherwise would be, because of the structure of the matrices on the right-hand sides of the definitions in (74). The expression in (76) can be further rearranged to yield an additively separable decomposition in real numbers. Because the units would be difficult to interpret directly, we write the gap size as a fraction of the collapsed data size, rearranging for additive separability:

$$\frac{\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}{\left\|\tilde{\mathbf{B}}\right\|} = \frac{\left\|\mathbf{B}^{\mathrm{b}}\right\|^2}{\left\|\tilde{\mathbf{B}}\right\|\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|} + \frac{\left\|\mathbf{B}^{\mathrm{z}}\right\|^2}{\left\|\tilde{\mathbf{B}}\right\|\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|} + \frac{\left\|\mathbf{B}^{\mathrm{bz}}\right\|^2}{\left\|\tilde{\mathbf{B}}\right\|\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}. \tag{77}$$

The relative gap size will equal one if the optimal model-consistent component is the null-matrix, as would be the case, for instance, if home and foreign in the collapsed data held the same positive amount of each asset. The relative gap size will equal zero if the optimal model-consistent component equals the collapsed data itself, as would be the case, for instance, if home and foreign in the collapsed data held symmetric holdings of assets in zero net supply.[8]

Alternatively, if the size of the gap components relative to the total gap size should be emphasized, then dividing both sides of (76) by the norm of the gap itself would be most useful:

$$1 = \left(\frac{\left\|\mathbf{B}^{\mathrm{b}}\right\|}{\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}\right)^2 + \left(\frac{\left\|\mathbf{B}^{\mathrm{z}}\right\|}{\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}\right)^2 + \left(\frac{\left\|\mathbf{B}^{\mathrm{bz}}\right\|}{\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}\right)^2. \tag{78}$$

C.4 A 3-Country Numerical Example

This section applies the procedure described above to a small numerical example for $N = 3$ countries. Table 8 below shows T-Bills hypothetically issued by and held by the United States, New Zealand, and Japan. The hypothetical T-Bill holdings have been expressed in USD to make comparisons easy. The table also shows hypothetical USD real returns on the T-Bills, where real return is defined as Lustig and Verdelhan (2007) define it: "the risky dollar return from buying a foreign T-Bill in country i, selling it after one period, and converting the proceeds back into [USD]" and then adjusting for inflation in the usual way.

[8]Note, however, that $\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\| \Big/ \left\|\tilde{\mathbf{B}}\right\|$ is not bounded above by one. Note also that $\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\| \Big/ \left\|\tilde{\mathbf{B}}\right\| + \left\|\mathbf{B}\right\| \Big/ \left\|\tilde{\mathbf{B}}\right\| \neq 1$ in general, because of the triangle inequality property of norms.

Table 8 – Hypothetical T-Bill Portfolio Holdings and Returns

	Issuer:		
Holder:	US	NZ	JP
US	150	95	−130
NZ	50	25	−80
JP	−75	60	150
Return:	3%	5%	1%

Note: The table shows hypothetical aggregate T-Bill holdings and returns for three countries: United States (US), New Zealand (NZ), and Japan (JP). The middle rows of column 1 show the names of the countries holding the T-Bills. The middle rows of column 2 show each country's holdings of the T-Bill issued by the United States. The middle rows of columns 3 and 4 show each country's holdings of the T-Bills issued by New Zealand and Japan, respectively. Portfolio holdings are in real USD, i.e. the real values of T-Bill holdings after converting all currency prices into USD and adjusting for the price level in the United States. The bottom row of the table shows the real rates of return that obtain after conversion of all currency prices and cash flows into USD and after adjusting for the price level in the United States. Note that USD real returns equal foreign currency real returns multiplied by the gross growth rate of the real exchange rate, so that foreign currency and USD real returns may differ in sign.

If Table 8 represents the full dataset, then we can write

$$\tilde{\tilde{\mathbf{B}}} = \begin{pmatrix} 150 & 95 & -130 \\ 50 & 25 & -80 \\ -75 & 60 & 150 \end{pmatrix}, \quad \tilde{\tilde{\mathbf{R}}} = \begin{pmatrix} 3\% \\ 5\% \\ 1\% \end{pmatrix}, \tag{79}$$

where an element $\tilde{\tilde{B}}_{ij}$ in matrix $\tilde{\tilde{\mathbf{B}}}$ represents country XXX's holdings of the T-Bill issued by country XXX, with $XXX, XXX \in \{\text{US}, \text{NZ}, \text{JP}\}$, and where an element $\tilde{\tilde{R}}_{Bj}$ in matrix $\tilde{\tilde{\mathbf{R}}}$ represents the USD real return on the bond issued by country XXX.

Choosing the United States (US) as the home country, the matrix $\tilde{\tilde{\mathbf{B}}}$ can now be collapsed by summing the elements representing the T-Bill holdings of New Zealand (NZ) and Japan (JP). Using equations (66) and (68), the collapsed matrices of real returns and portfolio holdings are computed as

$$\tilde{\mathbf{B}} = \begin{pmatrix} 150 & -35 \\ -25 & 155 \end{pmatrix}, \quad \tilde{\mathbf{R}} = \begin{pmatrix} 3\% \\ 7\% \end{pmatrix}. \tag{80}$$

Notice that the assets in the collapsed data matrix $\tilde{\mathbf{B}}$ are in non-zero net supply (the matrix columns don't sum to zero) and the United States is not symmetric with the collapsed foreign country created by summing the holdings of New Zealand and Japan (the matrix is not bisymmetric). Notice further that the real return of 7% on the collapsed foreign asset is higher than the returns on either the NZ T-Bill or the JP T-Bill. This magnification of returns arises from leverage effects in the computation of the foreign return: $\tilde{R}_{B2} = (180/120) \cdot 5\% - (60/120) \cdot 1\% = 7\%$.

The collapsed data matrix $\tilde{\mathbf{B}}$ can now be decomposed into an optimal model-consistent component $\mathbf{B}$ and three model-inconsistent components $\mathbf{B}^{\mathrm{b}}$, $\mathbf{B}^{\mathrm{z}}$, and $\mathbf{B}^{\mathrm{bz}}$. As shown in (74), these components are characterized partially by their scalar coefficients, here computed as:

$$B = 91.25\,, \qquad B^{\mathrm{b}} = -3.75\,,$$
$$B^{\mathrm{z}} = 61.25\,, \qquad B^{\mathrm{bz}} = -1.25\,.$$

The norms of the component matrices are then computed as

$$\|\mathbf{B}\| = 2 \cdot 91.25 = 182.5\,, \qquad \left\|\mathbf{B}^{\mathrm{b}}\right\| = 2 \cdot 3.75 = 7.5\,,$$
$$\left\|\mathbf{B}^{\mathrm{z}}\right\| = 2 \cdot 61.25 = 122.5\,, \qquad \left\|\mathbf{B}^{\mathrm{bz}}\right\| = 2 \cdot 1.25 = 2.5\,.$$

Using these results, the overall distance between the collapsed data and the model can now be computed using (76) as

$$\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\| = \left[7.5^2 + 122.5^2 + 2.5^2\right]^{\frac{1}{2}} = 122.75\,,$$

and the overall size of the collapsed data can be computed as

$$\left\|\tilde{\mathbf{B}}\right\| = \left[(150)^2 + (-35)^2 + (-25)^2 + (155)^2\right]^{\frac{1}{2}} = 219.94\,.$$

Using Equation (77) we find that the gap between the data and the model is nearly 56% of the size of the collapsed data:

$$\frac{\left\|\tilde{\mathbf{B}} - \mathbf{B}\right\|}{\left\|\tilde{\mathbf{B}}\right\|} = \frac{122.75}{219.94} = 55.81\%\,,$$

and using Equation (78) we find over 99% of the gap is driven by the non-zero net supply of assets in the data:

$$1 = \left(\frac{7.5}{122.75}\right)^2 + \left(\frac{122.5}{122.75}\right)^2 + \left(\frac{2.5}{122.75}\right)^2 = 0.37\% + 99.59\% + 0.04\%\,.$$

The numerical results are consistent with a visual inspection of the collapsed hypothetical data in (80). The collapsed hypothetical data are nearly bisymmetric, but the column sums are positive and large. Hence, home and foreign countries are nearly symmetric but each has assets in large positive net supply.

D Derivations of Theoretical Results

This section of the appendix contains details and discussions of derivations of some of the theoretical results that we state in Section 3. This section of the appendix is a work in progress.

D.1 Nominal Budget Constraint

In this subsection we describe the relationship between nominal and real budget constraints and between nominal and real carry trade returns. Nominal and real budget constraints are mathematically equivalent and lead to identical first-order conditions and still imply the second-order accurate expression (**??**) for the expected carry trade real return. Nominal and real carry trade returns are related through a version of the Fisher equation, and we use the Fisher equation to derive a first-order accurate expression for the nominal carry trade return in terms of the real carry trade return and inflation.

To begin, we define home real portfolio holdings in terms of quantities and domestic nominal prices as follows:

$$B^k_{ijt} := \frac{B_t}{} \tag{81}$$

$$A^{H}_t := B_t P_{Bt}/P_t \quad \text{and} \quad A^{F}_t := B^*_t P^*_{B^*t} S_{ijt}/P_t\,, \tag{82}$$

where B and B^* are the home household's quantity holdings of the home and foreign bonds, respectively, where P_B and $P^*_{B^*}$ are home and foreign nominal bond prices in home and foreign currency, respectively, and where the nominal exchange rate S_{jt} converts foreign currency to home currency. Note that purchasing power parity applies to this single-good frictionless model, so the following condition holds at all times: $P = P^* S_{jt}$.

Now recall the expression for the real budget constraint in (**??**), use the definitions of nominal bond real returns in (**??**), and use the definitions of real bond holdings in (82) to rewrite the real budget constraint in nominal terms:

$$\begin{aligned}
& & A^{H}_t + A^{F}_t + C_t &= R^{H}_t A^{H}_{t-1} + R^{F}_t A^{F}_{t-1} + W_t L_t \\
\Leftrightarrow & & \frac{B_t P_{Bt}}{P_t} + \frac{B^*_t P^*_{B^*t} S_{ijt}}{P_t} + C_t &= \frac{1/P_t}{P^{H}_{Bt-1}/P_{t-1}} \frac{B_{t-1} P_{Bt-1}}{P_{t-1}} \\
& & &\quad + \frac{1/P^*_t}{P^{F*}_{Bt-1}/P^*_{t-1}} \frac{B^*_{t-1} P^*_{B^*t-1} S_{t-1jt}}{P_{t-1}} + W_t L_t \\
\Leftrightarrow & & B_t P_{Bt} + B^*_t P^*_{B^*t} S_{ijt} + P_t C_t &= B_{t-1} + B^*_{t-1} S_{tjt} + P_t W_t L_t\,,
\end{aligned} \tag{83}$$

where simplifications arise in the last line because of the purchasing power parity condition. The home household's nominal budget constraint can thus be derived from the real budget constraint through equivalence relations. Because the constraints are mathematically equivalent, one should expect the first-order conditions of the household to be unchanged, no matter whether the real or the nominal constraint is used, and indeed they are unchanged.

Starting from the household maximization problem subject to the nominal budget constraint, first-order conditions with respect to home and foreign bond quantity holdings, combined with the first-order condition for consumption, yield the following optimality conditions:

$$\begin{aligned} C_t^{\rho} &= \mathrm{E}_t[t] C_{t+1}^{-\rho} \frac{1/P_{t+1}}{P_{Bt}^{H}/P_t} = \mathrm{E}_t[t] C_{t+1}^{-\rho} R_{t+1}^{H} \\ C_t^{\rho} &= \mathrm{E}_t[t] C_{t+1}^{-\rho} \frac{1/P_{t+1}^{*}}{P_{Bt}^{F*}/P_t^{*}} = \mathrm{E}_t[t] C_{t+1}^{-\rho} R_{t+1}^{F} \,, \end{aligned} \tag{84}$$

where the second equalities in these expressions use the definition of nominal bond real returns in (??). The second equalities make clear the equivalence between the household maximization problem subject to the nominal budget constraint and the household maximization problem subject to the real budget constraint.

Combining these expressions with the purchasing power parity condition, the following first-order approximate expressions obtain:

$$\begin{aligned} \mathrm{E}_t[t]\hat{\imath}_{Bt+1} &= \mathrm{E}_t[t]\hat{R}_{t+1}^{H} + \hat{\pi}_{t+1} + \mathcal{O}(\epsilon^2) \\ \mathrm{E}_t[t]\hat{\imath}_{B^*t+1} &= \mathrm{E}_t[t]\hat{R}_{t+1}^{F} + \hat{\pi}_{t+1} - \hat{I}_{t+1}^{s} + \mathcal{O}(\epsilon^2) \,, \end{aligned} \tag{85}$$

where $\hat{\imath}_{Bt+1} \coloneqq -\hat{P}_{Bt+1}^{H}$, $\hat{\imath}_{B^*t+1} \coloneqq -\hat{P}_{Bt+1}^{F*}$, $\hat{\pi}_{t+1} \coloneqq \hat{p}_{t+1} - \hat{p}_t$, and $\hat{I}_{t+1}^{s} \coloneqq \hat{S}_{t+1jt} - \hat{S}_{ijt}$. From these expressions for the nominal bond nominal returns, it follows that the carry trade nominal return is given by

$$\mathrm{E}_t[t]\hat{\imath}_{Xt+1} = \mathrm{E}_t[t]\hat{R}_{Xt+1} + \mathrm{E}_t[t]\hat{I}_{t+1}^{s} + \mathcal{O}(\epsilon^2) \,, \tag{86}$$

where $\hat{\imath}_{Xt+1} \coloneqq \hat{\imath}_{Bt+1} - \hat{\imath}_{B^*t+1}$. From here, one can simply substitute (a modified version of) expression (??) for the expected carry trade real return. Using the purchasing power parity condition it can further be shown that $\hat{I}_{t+1}^{s} = \hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{*}$, and the quantity equations in (??) can then be used to express $\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^{*}$ completely in terms of exogenous productivity and money supply variables.

I have not attempted to second-order approximate the first equalities in the Euler equations in (84), as I had done to obtain (??). I suspect that such second-order approximations would yield a pricing equation in terms of covariances between consumption and the carry trade nominal return, and between consumption and the currency return. However, although I have not confirmed this, I also suspect that such a pricing equation would be equivalent to the second-order approximation for carry trade real returns in (??), and lead to few new insights.

D.2 Equivalent Aggregation

The densities in Equation (14) define equivalent heterogeneity across households, and either density function, after suitable transformations of variables, can be used to aggregate household-level quantities across households within a country. To see this, let $T(\rho) = \rho^{-1} = \sigma(\rho)$, let $g(\rho)$ be a function on $(0, \rho_m]$, and note that

$$\int_E g(\rho) f_\rho(\rho)\,\mathrm{d}\lambda(\rho) = \int_{T^{-1}(F)} g\Big(T^{-1}\big(\sigma(\rho)\big)\Big) f_{\tilde{\sigma}}\big(\sigma(\rho)\big)\,\mathrm{d}\lambda(\sigma(\rho))\,, \tag{87}$$

where $T(E) = F$, $E \in (0, \rho_m]$, $F \in [\sigma_m, \infty)$, and where λ denotes the Lebesgue measure. Hence, household heterogeneity can be equivalently defined in terms of the coefficient of risk aversion or the intertemporal elasticity of substitution, and a suitable transformation makes aggregation possible with respect to either variable. See Halmos (1962, Chapter VIII, Theorems A.–D.) for a more detailed mathematical treatment.

Here, the transformation yields the densities

$$f_\rho(\rho) = \frac{\kappa}{\rho}\left(\frac{\rho}{\rho_m}\right)^\kappa \quad \text{and} \quad f_{\sigma(\rho)}\big(\sigma(\rho)\big) = \frac{\kappa}{\sigma(\rho)}\left(\frac{\sigma_m}{\sigma(\rho)}\right)^\kappa \quad \text{with} \quad \kappa > 1,\ \rho_m > 0\,. \tag{88}$$

E Proposition Proofs

E.1 Proof of Proposition 3: Aggregate Stationarity

The coefficient matrices in (47) are given by

$$\mathbf{D}_i = \begin{pmatrix} D_{WW} & D_{WC} \\ D_{CW} & D_{CC} \end{pmatrix}, \quad \mathbf{D}_2 = \begin{pmatrix} D_{VW} \\ D_{VC} \end{pmatrix}, \quad \mathbf{D}_3 = \begin{pmatrix} D_{YW} \\ D_{YC} \end{pmatrix}, \tag{89}$$

with elements D_{ij}, $i \in \{W, C, V, Y\}$ and $j \in \{W, C\}$, that depend on the parameters of Models 1 through 4. Coefficients D_{Wi}, $i \in \{W, C, V, Y\}$, are identical across Models 1 through 4, and the remaining coefficients differ across models, as long as the wealth-in-utility parameter is non-zero, $\theta \neq 0$.