Fiscal Federalism and Monetary Unions*

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Abstract

We apply ideas from fiscal federalism to reassess how fiscal authority should be delegated within a monetary union. In a real-economy model with no fiscal externalities, in which local fiscal authorities have an informational advantage about the preferences of their citizens for public spending relative to a fiscal union, a natural generalization of the classic decentralization result by Oates (1999) applies. Namely, a decentralized fiscal regime dominates a fiscal union, and the degree of dominance increases as the information of the fiscal union worsens in quality. In the presence of direct fiscal externalities across countries, however, a decentralized regime is optimal for small federations of countries, whereas a centralized regime is optimal for large ones. We then consider a monetary-economy model, in which governments finance their expenditures with nominal debt and inflation has a negative impact on aggregate productivity. If the monetary authority can commit to its inflation policy, then a version of Oates (1999)'s decentralization result holds. By contrast, when the monetary authority lacks commitment power, the resulting time-inconsistency problem generates an indirect *endogenous* fiscal externality. In this case, when a country-level fiscal authority chooses a higher level of nominal debt, it induces the monetary authority to inflate more to reduce the level of distortionary taxes needed to finance the higher debt. Because country-level fiscal authorities do not take into account the costs of the induced inflation in other countries that their fiscal policies induce, a negative fiscal externality arises. This externality naturally becomes more severe as the number of countries in the monetary union increases. Hence, as in the real-economy model, a decentralized fiscal regime is optimal for small monetary unions, whereas a fiscal union is optimal for sufficiently large ones. Our key result is that as the size of a monetary union increases, it becomes relatively more desirable to centralize fiscal authority. We conclude by discussing the implications of our results for the debate on the integration of fiscal policy within the EU and its enlargement.

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1 Introduction

"We should know over which matters several local tribunals are to have jurisdiction, and in which authority should be centralized." (Aristotle, Politics 4.15)

How decision-making power over fiscal policy should be allocated between a central authority and multiple local ones has been debated for millennia. At present, this question is especially relevant in the context of the European Union (EU), which is a monetary union whose member countries retain a large degree of independence in determining national fiscal policy. Many have argued that the issue of optimal delegation of fiscal decisions—namely, whether such decisions should be centralized within a union-wide fiscal authority or left instead to member states—is one of the most important ones for the future of the EU (Tabellini (2002)). In a similar vein, Saiegh and Tommasi (1999), Nicolini et al. (2002), and Cooper and Kempf (2004) have emphasized how the complex system governing the rules of fiscal federalism in Argentina is in large part responsible for its poor performance in terms of both fiscal and monetary policy.

The purpose of this paper is to adapt some of the ideas central to the doctrine of fiscal federalism in order to reassess standard results in the macroeconomics literature on how fiscal authority in general should be allocated within a monetary union and on when a fiscal union in particular is desirable for a monetary union. Canonical models such as that in Aguiar et al. (2015) imply that it is always (weakly) optimal to delegate fiscal policy to a central fiscal authority. The reason is twofold. First, a central fiscal authority internalizes any possible fiscal externality that local fiscal authorities do not have an incentive to take into account. Second, local fiscal authorities are typically thought of as having no advantage in fiscal matters over a central one. This approach to the fiscal delegation problem contrasts sharply with the common approach in the literature on fiscal federalism, which presumes that centralized authorities of fiscal unions are less capable of designing their policies in accord with member states' preferences over local public spending than a local authority is. That is, under centralization, policies are more uniform across states than is desirable. Given this premise, an important insight from the fiscal federalism literature is that absent large fiscal externalities in the provision of public goods, as is the case for infrastructure or defense, it is optimal to delegate fiscal decisions to local authorities.

Our paper proposes both a real-economy and a monetary-economy model that microfound this approach in a parsimonious way and, through the lens of these models, illustrates key forces governing the optimal delegation of fiscal policy within a monetary union. The idea underlying both models is that when the size of a federation of countries is small, free-riding type of fiscal externalities are also limited. Thus, any natural advantage that a decentralized regime may have in adapting policies to each member state's characteristics and preferences outweighs the costs of fiscal externalities across countries, thus making a fiscal union unappealing. But as the size of a federation increases, so do the free-riding incentives arising from any fiscal externality—we assume throughout that as the size of the group of countries considered increases, the informational disadvantage of a central fiscal authority over local ones remains constant.[1]

In light of these conflicting forces, our novel result is that the optimal delegation of fiscal authority implies that a decentralized fiscal regime is desirable for small monetary unions, whereas a fiscal union is desirable for large ones. Namely, there exists a threshold size for a monetary union such that a decentralized regime of fiscal authority is optimal up to such a size, but a centralized regime, that is, a fiscal union, is optimal above it. We characterize in detail the forces that determine this threshold size, which intuitively relate to the magnitude of fiscal externalities across countries and the quality of the information that a central fiscal authority has access to about the preferences of member states' citizens over public spending.

Our reading of the work on fiscal federalism is that in the absence of other countervailing forces, fiscal authority should be delegated to individual states. Indeed, Article 5(3) of the Treaty on European Union (TEU) signed in 1992 formalizes precisely this idea by enshrining the principle of *subsidiarity* of fiscal delegation, which defines the circumstances under which it is preferable for an action to be undertaken by the EU rather than by member states as instances in which the authority or competence of member states are inadequate or insufficient.[2] Although the EU is not technically a federation, many have argued that several of the ideas from the classic theory of fiscal federalism can be applied to it to determine the conditions under which various types of fiscal authority should be delegated to member states and those in which they should be instead centralized within a union-wide authority (Tabellini (2002)). This paper pursues this strategy by developing a framework that incorporates these ideas and is applicable to both real and monetary economies to evaluate the benefits of fiscal centralization.

One of the seminal studies on fiscal federalism is Oates (1972). A key tenet of Oates's theory is that a centralized authority tends to be less responsive to the different preferences for public spending of the residents of different local communities and so has a natural tendency towards uniformity in fiscal policy across communities (Oates, 1972, p. 11). Oates approaches the issue of delegation by focusing on what he terms an ideal special case, in which all individuals in a specific geographic area are immobile and have identical preferences for public goods. Moreover, any local fiscal authority enjoys an advantage over a central authority in that it has complete knowledge of the tastes of its constituents and so it

[1]Throughout, we maintain that as a monetary union grows, the informational disadvantage of a central fiscal authority relative to local ones remains constant. If, instead, the informational disadvantage of a fiscal union decreased as a monetary union grows, say, because of increasing returns to scale to information gathering, then the centralization of fiscal authority would be optimal for even smaller monetary unions. By contrast, if the informational disadvantage of a fiscal union increased at a faster rate than the rate at which fiscal externalities increase as a monetary union grows, then the centralization of fiscal authority would be optimal only for a larger monetary union. Indeed, a fiscal union may never be optimal regardless of the size of a monetary union.

[2]Instances include when: *i*) the competence in an area is shared between the Union and member states (*non-exclusive competence*); *ii*) the objectives of the proposed action cannot be sufficiently achieved by member states (*necessity*); and *iii*) the action can therefore, by reason of its scale or effects, be implemented more successfully by the Union (*added value*).

can finely tailor its policies to local needs. By contrast, Oates simply imposes that any fiscal decision taken by the central government must be homogeneous across regions. Under this assumption and a few others, Oates (1972) argues that a general decentralization theorem holds in that "*[f]or a public good—the consumption of which is defined over geographical subsets of the total population, and for which the costs of providing each level of output of the good in each jurisdiction are the same for the central or the respective local government—it will always be more efficient (or at least as efficient) for local governments to produce Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions*" (p. 35).

This influential perspective has been challenged, though. In a survey of the benefits of decentralization, Lockwood (2005, p. 2) characterizes Oates (1972)'s thesis as a preference-matching argument whereby "*goods provided by governments in localities will be better matched to the preferences of residents in those localities.*" Lockwood (2005, p. 3) questions such a benefit of decentralization since it rests on the "*ad-hoc assumption of policy uniformity: the central government is assumed to set a uniform level of local good provision in all regions.*" Lockwood (2005) further argues that although local governments may have better information about local preferences than a central government has, a benevolent central authority could design an incentive mechanism to elicit these preferences from each region and implement accordingly the efficient outcome. Besley and Coate (2003, p. 2612) similarly dispute the idea that the centralization of authority implies policy uniformity. They argue that empirically, many examples exist of goods that are unequally provided by a federal government to local regions, whereas theoretically, it is unclear why a government charged with providing public goods within a centralized system cannot differentiate their provision according to the "*heterogeneous tastes in each district.*"

In this paper, we propose models that are consistent with the idea that local authorities have a natural advantage in tailoring their policies to the preferences of their constituents but that are immune from these criticisms. Namely, we depart from Oates' ad-hoc assumption of policy uniformity and instead assume that local authorities have more precise information about their citizens' tastes for local public goods than does a central authority. In particular, in a decentralized regime, each local government observes the preferences of its citizens, whereas in a centralized regime, the union-wide fiscal authority only observes a noisy signal about the preferences of the citizens of each member state. We then formalize the notion that although member states have no incentive to conceal their information and instead attempt to communicate their true preferences to a central authority, this communication process is imperfect.

Some empirical support for the notion that even well-meaning agents often find it difficult to convey their preferences to others has been documented by Waldfogel (1993). Waldfogel (1993) argues that an important economic aspect of gift giving, say, around the holidays, is that gifts may be mismatched to their recipients' tastes. Intuitively, according to standard consumer choice logic, the best a gift giver can do is to duplicate the choice that the recipient would have made with an equal amount of resources available. But if the gift giver is less than perfectly informed about the recipient's preferences, it is likely that the gift will make the recipient worse off than if the recipient had directly made the consumption choice. Hence, gift-giving is a potential source of deadweight loss. Based on this idea, Waldfogel (1993) documents that holiday gift-giving from significant others destroys about 10 percent of the value of gifts, whereas gifts from members of the extended family destroy about a third of their value. Hence, if even close family members do not seem to be able to convey well to each other their preferences for typically simple goods and services exchanged as gifts, virtually in the absence of any conflict of interests, difficulties in communication may well arise for officials of a member state trying to communicate to a central fiscal authority its citizens' preferences for complex goods and services, especially in the presence of conflicting interests.

We begin our analysis with a model of a real multi-country economy that highlights the ideas behind Oates's decentralization theorem. The economy features no externalities across countries but a central fiscal authority receives only a noisy signal about countries' preferences for public goods. A natural generalization of Oates's decentralization theorem then holds: a decentralized regime dominates a fiscal union and the degree of dominance increases—namely, the number of countries for which a fiscal union is optimal increases—as the preference signal of the central authority becomes less informative. We then augment this real-economy model with a direct fiscal externality by assuming that any country's government spending has one component that contributes to the public goods of every other country and another component that contributes solely to its own public goods.[3] Our main result for this augmented model is that since the fiscal externality worsens as the number of countries grows, a threshold number of countries exists such that a decentralized fiscal regime is optimal for a small enough group of countries but a centralized fiscal regime is optimal for any large enough group.

We then turn to a simple two-period monetary-economy model in which countries finance their government spending with nominal debt, inflation has a negative effect on aggregate productivity, and labor income taxes are distortionary. We find that when the monetary authority has the ability to commit to an inflation policy, inflation is zero in equilibrium and the economy reduces to a real one for which our generalized decentralization theorem holds. The more interesting case arises when the monetary authority lacks the ability to commit to inflation decisions. In this case, whenever the monetary authority is faced with a larger amount of nominal government debt, it finds it optimal to inflate more so as to lessen the need to raise distortionary taxes on labor income. But when deciding on its own level of public spending

[3] Draghi (2023) argues that "*[i]n Europe today we have never faced so many shared supranational goals, by which I mean goals that cannot be managed by countries acting alone. We are undergoing a series of major transitions that will require vast common investments. The European Commission puts the investment needs for the green transition at more than 600 billion euro annually until 2030—and between a quarter and a fifth of this will have to be funded by the public sector.*"

and debt, each member state only considers the adverse consequences of the induced inflation on its own productivity, thus ignoring its effect on other member states. Hence, in a decentralized regime, a fiscal externality arises that is both endogenous and indirect: although the spending of any member state does not affect the utility or production of any other state, the resulting increase in inflation that such spending induces impacts all countries. This *indirect endogenous* negative externality in this monetary economy has parallels with the *direct exogenous* positive externality in the real economy described earlier. Indeed, our main result is that since this negative fiscal externality becomes more severe as the size of a monetary union increases, here too a cutoff size for the union exists such that a decentralized fiscal regime is optimal for smaller-sized unions, whereas a centralized fiscal regime is optimal for larger-sized ones.

The consensus in the macroeconomic literature, on the contrary, has long been that as long as cross-country externalities are present, a centralized fiscal regime is strictly preferred to a decentralized one. The reason is that this literature abstracts from key aspects of fiscal federalism that literature on it has instead emphasized. In our view, a critical missing ingredient is the idea that local policymakers have a natural advantage in tailoring policies to their constituents' interests. As we show, once we incorporate this dimension into an otherwise standard macroeconomic model, a novel result emerges. Namely, a centralized fiscal regime is no longer always optimal. Rather, it is often optimal to pair a small monetary union with a decentralized fiscal regime and a large monetary union with a centralized one. This delegation principle, which accounts for potential benefits to decentralization, may be worth serious consideration when contemplating fiscal delegation in practice.

One established branch of the macroeconomics literature is typified by the work of Aguiar et al. (2015), Beetsma and Uhlig (1997), and Chari and Kehoe (2008). This work never explicitly addresses the question of delegation of fiscal authority. Instead, it simply imposes that fiscal authority is decentralized and examines whether constraining these decentralized authorities' fiscal instruments can overcome the inefficiencies in decentralized decision-making that arise from fiscal externalities. For example, decentralized fiscal authorities may be constrained by upper limits on debt and deficits similar to those specified by the EU's Maastricht Treaty of 1992 and punished for violations of them.

Although theoretically appealing, in practice sanctioning decentralized fiscal authorities ex post for violating such rules may be difficult. Indeed, even when some Eurozone countries have repeatedly violated the Maastricht Treaty by large margins in the past twenty years, the remaining Eurozone countries have been unwilling to enforce the punishments dictated by the treaty. Hence, we view the non-credibility of such fiscal rules as making them practically irrelevant. As a result, differently from the literature, we purposely set up the delegation problem so that under a centralized fiscal authority, individual countries do not have the power to decide on their own levels of public spending and debt in the first place, so that the issue of ex-post non-credibility of punishments does not arise.

A second branch of the macroeconomic literature examines the optimal delegation of fiscal authority in the context of cross-country insurance against aggregate shocks—an issue that we abstract from. One view associated with Kenen (1969) is that cross-regional fiscal transfers are critical to the functioning of a monetary union. An alternative view associated with Mundell (1973) is that the need for such transfers lessens in the presence of sophisticated international financial markets. Kehoe and Pastorino (2017) address this debate and prove the conjecture of Mundell (1973) for a simple environment, building on the framework of Fahri and Werning (2013). Specifically, they argue that in the presence of well-developed financial markets, a fiscal union is unnecessary in that a decentralized fiscal regime with such financial markets can achieve the same welfare as a centralized one without access to them, provided it has a rich set of cross-regional transfers at its disposal.[4] Here we abstract from these considerations in order to focus solely on the question of delegation in the presence of an informational advantage to decentralization.

Finally, note that a related political economy literature analyzes centralized decision making departing from the idea that central governments maximize the welfare of their citizens. This literature focuses on settings in which locally elected representatives are part of a central legislature that decides on the provision of public goods for each member state. Outcomes then depend on the specific assumptions of the political decision-making process, for instance, whether a majority rule applies whereby all regions share equally in the costs of provision, regardless of whether their region receives a funded project. As Lockwood (2002) shows, because of such forms of cost-sharing through uniform taxation, these setups tend to lead to legislatures biased towards minimizing the cost of projects, rather than aiming at maximizing their net benefits. Besley and Coate (2003) derive similar results under different assumptions. We view our work as complementary to this literature.

2 A Real Economy

We propose a simple static model that highlights the main idea behind Oates (1999)'s decentralization theorem that local fiscal authorities have a natural advantage in deciding on local fiscal matters, relative to a centralized fiscal authority. Specifically, Oates (1999) argues that a basic shortcoming of a centralized system is its "*probable insensitivity to varying preferences among the residents of different communities*" (Oates 1999, p. 11). By contrast, "*[a] decentralized form of government [.] offers the promise of increasing economic efficiency by providing a range of output of certain public goods that correspond more closely to the differing tastes of groups of consumers*" (Oates, 1999, p. 12).

In this seminal work, Oates simply assumes that a centralized fiscal authority must provide the same

[4]Note that Fahri and Werning (2013) do not focus on delegation but rather show that a fiscal union without access to insurance markets but with access to a rich set of transfers and other tax instruments dominates a pure *laissez-faire* equilibrium without any government but with a sophisticated financial markets.

level of public goods to all member states, although states have differing preferences for them. We both generalize and provide a microfoundation for this premise by assuming that a centralized fiscal authority observes a noisy signal about the preferences for public goods of each member state so it is unable to precisely tailor its policies to each member state's specific preferences. The case in which the signal from each member state is close to uninformative nests the case discussed by Oates (1999). We maintain throughout that the informativeness of each country's signal does not depend on the number of countries in the union. At an intuitive level, by doing so we abstract from either increasing or decreasing returns to scale in information gathering or processing. We later discuss the implications of relaxing this assumption; see the discussion after Proposition 2.

The decentralization theorem focuses on what Oates (1999) terms the case of *perfect correspondence* in the provision of public goods, which rests on several assumptions. The first is that individuals with the same tastes for local public goods are grouped into one geographic region and only one local government has jurisdiction over each such region. The second is that the only citizens who benefit from the local public good are those in this region. The third is that the cost of providing this good is the same at the local level and at the centralized level. The fourth is that no cross-country transfers take place in that local public goods are paid for by local taxes in both the decentralized and the centralized regimes. Finally, each local government possesses "*complete knowledge of the tastes of its constituents*." Under these assumptions, the theorem states that "*it will always be more efficient (or at least as efficient) for local governments to provide the Pareto-efficient levels of output for their respective jurisdictions than for the central government to provide any specified and uniform level of output across all jurisdictions*."

We set up our baseline real model to be consistent with all of these assumptions, except that we allow for a more general information structure for the centralized authority. We show that a *generalized decentralization theorem* holds in this case as well. In this sense, we address the criticism of Besley and Coate (2003, p. 2612), who challenge the notion that "*centralization implies uniformity*" and argue that this assumption is "*neither empirically nor theoretically satisfactory*," by showing that a version of the result applies to this richer environment. We then turn to a case in which externalities arise, namely, a case in which Oates (1999)'s second assumption is violated. We focus on a situation in which the externality emerges because for any given country, its government spending provides not only direct benefits to it but also indirect benefits to all other countries—consider, for instance, the case of roads or bridges connecting regions of different countries. We obtain a new result, namely, that a cutoff rule in the size of a federation of countries exists such that for a small enough federation, a decentralized regime is optimal but for a sufficiently large federation, a fiscal union is optimal.

In the model, the effective spending of the government of each member state depends on both its own spending and a fraction of the spending of the government of all other member states. We focus on this case because as the fraction of other states' spending increases from zero to one, the model subsumes the two extreme cases of a purely local public good and a purely union-wide public good as well as any case in between. For example, it covers the case of, say, French parks, which provide much larger benefits to French citizens of the EU than to non-French citizens of the EU. It also covers, say, the instance of French tanks provided for the common defense of the union, which offer relatively similar benefits to French and non-French citizens of the EU. As in Oates (1972, 1999), we also assume that each country's government budget is balanced in that its spending is entirely paid for by its own taxes. In this sense, we consider a model with no cross-country transfers.

2.1 A Model of a Real Economy

Consider a one-period model in which each of $i = 1, \ldots, I$ countries is populated by a representative consumer and a government. Consumers in different countries differ only in their preferences for government spending. The utility of a representative consumer in country i is

$$u(c_i) + \theta_i h(g_i), \tag{1}$$

where c_i is consumption and g_i is government spending in country i, and $u(\cdot)$ and $h(\cdot)$ are strictly increasing and strictly concave functions. The variable θ_i is the taste of country i's consumers for government spending (public goods), which is independently randomly drawn for each country at the beginning of the period. For notational simplicity only, we assume that $\theta_i \in \{\theta_H, \theta_L\}$, with $\theta_H > \theta_L$. Letting q be the probability of θ_H, we denote the mean of θ by $\mu_\theta = q\theta_H + (1-q)\theta_L$. Each consumer in country i has an endowment y of goods so country i's resource constraint is

$$c_i + g_i = y. \tag{2}$$

The budget constraints for the (representative) consumer and the government of country i are

$$c_i + T_i = y \text{ and } \quad g_i = T_i, \tag{3}$$

where T_i denotes lump-sum taxes. The consumer of country i maximizes utility subject to the budget constraint $c_i + T_i = y$. The authority on each country's government spending on public goods depends on the degree of centralization of fiscal authority. In a decentralized regime, the government of each country i chooses its amount of government spending g_i by maximizing the welfare of its citizens. In a centralized regime, also referred to as a *fiscal union*, a centralized authority chooses g_i for each country i in order to maximize the welfare of all citizens in the union. In both cases, fiscal authorities are subject to the same government budget constraint in (3). Hence, we abstract from any role that a centralized fiscal authority may play in redistributing resources across regions.

A key feature of this setup motivated by Oates (1999) is that the government of country i is assumed to have better information about its citizens' taste for public spending than does a central fiscal authority. Namely, the government of country i perfectly observes θ_i, whereas the central fiscal authority observes only a noisy signal about the taste of each country i's citizens, $s_i \in \{s_H, s_L\}$, which is symmetric in that

$$\phi = \Pr(s_H|\theta_H) = \Pr(s_L|\theta_L) \quad \text{and} \quad 1-\phi = \Pr(s_L|\theta_H) = \Pr(s_H|\theta_L), \tag{4}$$

where $\phi \in [1/2, 1]$ denotes the informativeness of the signal. In particular, $\phi = 1/2$ implies that the signal is uninformative in that $E(\theta|s_H) = E(\theta|s_L) = \mu_\theta$, whereas $\phi = 1$ implies that the signal is perfectly informative in that $E(\theta|s_H) = \theta_H$ and $E(\theta|s_L) = \theta_L$. More generally, Bayes's rule implies that given the prior q that citizens' taste is θ_H, the posterior probabilities that their taste is θ_H are

$$Q(q, s_H) \equiv \Pr(\theta_H|s_H) = \frac{q\phi}{p_H} \quad \text{and} \quad Q(q, s_L) \equiv \Pr(\theta_H|s_L) = \frac{q(1-\phi)}{1-p_H} \tag{5}$$

after a high and low signal, respectively, where the unconditional probability of receiving a high signal is

$$p_H = q\phi + (1-q)(1-\phi). \tag{6}$$

Note that this economy has been purposely designed to be consistent with Oates (1999)'s five assumptions. Specifically, the utility function in (1) implies that all the consumers in a given country have the same tastes for government spending and the jurisdiction of country i's government is only over these consumers. Moreover, there are no external benefits to country i from any other state's spending g_j, $j \neq i$. Next, from the resource constraint and the budget constraints, it is immediate that the cost of providing g_i is the same for country i and for a fiscal union. In both regimes, each government pays for its spending with locally raised taxes. Finally, the government of country i has complete knowledge of the tastes θ_i of its citizens. We extend Oates (1999)'s framework by allowing the central fiscal authority to receive a noisy signal about θ_i and to optimally choose g_i conditional on it, rather than simply imposing that g_i is the same for all countries regardless of their citizens' underlying tastes.

The idea behind this formulation is that although each local government wishes to communicate to the central authority its true preference parameter θ_i, the communication is imperfect because tastes are difficult to completely describe or specify. Hence, all that is perceived by the central authority is a noisy signal $s = (s_1, \ldots, s_I)$ about them. It is worth noting that in the fiscal federalism literature, the premise of the results on optimal decentralization is not that agents have private information about their tastes for public goods and choose not to truthfully reveal it, but instead that central authorities have a natural tendency towards policy uniformity across states. Hence, our formulation generalizes the standard case considered by Oates (1972, 1999), which can be thought of as the uninformative case in which policy uniformity is optimal. We now turn to evaluating two opposite scenarios of delegation of fiscal authority, which correspond to a *decentralized* regime and a *centralized* one (a fiscal union).

Decentralized Regime. In a decentralized regime, the government of each country i has perfect information about the tastes of its citizens for government spending. Hence, it solves the problem

$$W^D(\theta_i) = \max_{c_i, g_i} [u(c_i) + \theta_i h(g_i)], \tag{7}$$

subject to the constraint $c_i + g_i = y$. The ex-ante utility of country i is then

$$V^D = qW^D(\theta_H) + (1-q)W^D(\theta_L). \tag{8}$$

Note that the choices of country i do not depend on the choices of any other country $j \neq i$. As all countries are ex-ante symmetric, V^D is also the ex-ante welfare of the union under an equal weighting scheme.

Centralized Regime. In a fiscal union, the central fiscal authority observes the vector of signals $\bar{s} = (s_1, \ldots, s_I)$ and chooses the vector of government spending $\bar{g} = (g_1, \ldots, g_I)$ for each country. To derive the ex-ante welfare of the fiscal union, the first step consists of solving the central fiscal authority's problem of allocating spending among countries by determining the maximized value of welfare for any given vector of signals $\bar{s}$. The second step consists of calculating ex-ante welfare as the expected value of this maximized value over all possible signal realizations.

For the first step, consider a vector of signals $\bar{s}$ and let $W^C(\bar{s})$ denote the problem of maximizing an equally-weighted average of the expected utilities of the consumers of the I countries given this vector,

$$W^C(\bar{s}) = \max_{c_i, g_i} \frac{1}{I} \sum\nolimits_{i=1}^{I} E\left[u(c_i) + \theta_i h(g_i) | \bar{s}\right], \tag{9}$$

subject to the constraint $c_i + g_i = y$ for all i, where the expectation over θ_i in (9) is conditional on the vector of observed signals $\bar{s}$. The problem of the fiscal union in (9) reduces to one of maximizing the utility of each country separately, given the signal received about that country. This property holds because the utility of each country i's citizens does not depend on the actions or outcomes in any other country, and both preference types and signals about them are independently distributed across countries. Hence, $W^C(\bar{s}) = \sum_{i=1}^{I} W^C(s_i)/I$, where for each country i, $W^C(s_i) = \max_{\{c_i, g_i\}} E\left[u(c_i) + \theta_i h(g_i) | s_i\right]$, subject to the constraint $c_i + g_i = y$ for all i. Given the multiplicative effect of θ_i on the utility from consuming public goods, this problem further reduces to $W^C(s_i) = \max_{\{c_i, g_i\}}[u(c_i) + \tilde{\theta}_i h(g_i)]$, where $\tilde{\theta}_i = E(\theta_i|s_i)$. For the second step, we calculate the ex-ante utility for the fiscal union, which is given by

$$V^C = p_H W^C(s_H) + (1 - p_H) W^C(s_L), \tag{10}$$

where p_H defined in (6) and $1 - p_H$, respectively, are the probabilities of receiving a high and a low signal.

[FIGURE 1 HERE]

Oates (1999)'s decentralization theorem corresponds to the completely uninformative case when $\phi = 1/2$—by continuity, also whenever the preference signal is sufficiently uninformative. In this case, the fiscal authority optimally chooses a uniform level of government spending for each country. As ϕ increases from 1/2 to 1, the fiscal authority increasingly tailors the level of spending to the underlying tastes of each country's citizens and does so perfectly when ϕ reaches one. This result is shown in the left panel of Figure 1, which plots the optimal government policy functions in the decentralized and centralized regimes for different degrees of informativeness ϕ of the signal. Clearly, if $\phi < 1$, then the central fiscal authority has inferior information compared to the local fiscal authority, so the latter is able to better allocate resources. Ex-ante welfare as a function of ϕ in the two regimes is displayed in the right panel of Figure 1. Only when $\phi = 1$ and the preference signal is perfectly informative, welfare is equal in the two regimes—otherwise, a decentralized fiscal regime dominates a centralized one. We formalize these observations in the following proposition, which follows by Blackwell's informativeness theorem.

Proposition 1 (A Generalized Decentralization Theorem)**.** *When signals about countries' preferences for government spending are not perfectly informative in that $\phi < 1$, a decentralized regime yields higher ex-ante welfare than a centralized regime, namely, a fiscal union, does. The difference in welfare between the two regimes decreases with the informativeness of signals.*

Intuitively, the local fiscal authority has a natural advantage over the central fiscal authority because of its superior information about local preferences for government expenditure. Therefore, a fiscal union is never preferable. Although the proposition does not depend on the assumption that types and signals are discrete or independently or identically distributed across countries, key to this result is the assumption that government spending in all other countries does not affect utility in any given country. Next, we relax this assumption by introducing a fiscal externality.

2.2 Adding a Fiscal Externality

Suppose now that the value of government spending to country i's citizens depends not only on country i's government expenditure but also on any other country j's government expenditure, $j \neq i$. We capture this feature by letting the function $h(\cdot)$ of government spending for country i's consumers also depend on the vector of government spending of all other countries $\bar{g}_{-i} = (g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_I)$. Accordingly, the utility function of a consumer in country i is now $u(c_i) + \theta_i h(g_i, \bar{g}_{-i})$. Preferences for government spending satisfy $\partial^2 h(\cdot)/\partial g_i \partial g_j \neq 0$ and thus give rise to *non-trivial* externalities across countries: the optimal level of government spending in country i is in general affected by the spending in any other country.[5] We find it convenient to focus on the case in which

$$h(g_i, \bar{g}_{-i}) = h\left(g_i + \gamma \sum\nolimits_{j \neq i} g_j\right), \tag{11}$$

so that total effective government spending in country i is the sum of its own spending g_i and a fraction γ of the total spending of all other countries. Intuitively, imagine that a fraction $1-\gamma$ of any country's spending benefits only that country and the remaining fraction γ benefits all countries. Since any country i's spending can be decomposed into these two components as $g_i = (1-\gamma)g_i + \gamma g_i$, country i's total effective spending can be expressed as $(1-\gamma)g_i + \gamma \sum_{j=1}^{I} g_j$ or $g_i + \gamma \sum_{j \neq i} g_j$. We denote total effective government spending in country i when countries' government spending are $\bar{g} = (g_1, \ldots, g_I)$ by $G_i(\bar{g}) = g_i + \gamma \sum_{j \neq i} g_j$.

We set up the economy so that it leads to a symmetric equilibrium in the relevant sense. That is, in the decentralized regime, all countries with the same realized preference θ_i for public goods choose the same level of government spending g_i. Likewise, in the centralized regime, all countries with realized preference signal s_i are assigned the same level of government spending g_i. We begin with the simple case in which signals are perfectly correlated across countries and then turn to the case in which signals are independently drawn in each country. Throughout, we consider utility functions with standard properties.

Assumption 1. *The utility function over the consumption of private goods satisfies the following properties:* $u'(x) > 0$, $u''(x) < 0$, $\lim_{x \to 0} u'(x) = \infty$, *and* $\lim_{x \to \infty} u'(x) = 0$. *The utility function over the consumption of public goods satisfies the following properties:* $h'(x) > 0$, $h''(x) < 0$, $\lim_{x \to \infty} h(x) = \infty$, $\lim_{x \to 0} h'(x) = \infty$, *and* $\lim_{x \to \infty} h'(x) = 0$.

The requirement that $\lim_{x \to \infty} h(x) = \infty$ ensures that ex-ante welfare under a fiscal union becomes arbitrarily large as the number of countries in the union progressively increases, which will be a crucial step in establishing the existence of a cutoff rule in Proposition 2.

[5] This assumption is violated when preferences for government spending are linear in spending. A setup with quasi-linear preferences would make the subsequent analysis much simpler but would not capture the key forces behind our motivating examples—in the case of tanks, we are interested in situations in which if all other countries in the union purchased a large number of tanks, then the last country's benefits from purchasing its own would be small. In our view, eliminating this type of free-riding behavior is at the heart of the benefits to centralization and central to the current debate within the EU about not only the gains from greater fiscal integration but also the purview of its common security and defense policy.

2.2.1 Perfectly Correlated Preferences across Countries

We examine first the simpler case in which countries draw the same preference type, either θ_H with probability q or θ_L with probability $1-q$.[6] We assume that in the decentralized regime, the local fiscal authority of each country observes the preferences of its citizens, whereas in the fiscal union, the central fiscal authority only observes the signal s about the common preference type of all countries, which satisfies assumptions (4) to (6). With correlated preferences, of course, the assumption that all countries have the same type but the central fiscal authority cannot observe it is less appealing than in the case of independent preferences. We interpret the correlated case as one in which the informational content of the preference signal that the central authority receives is fairly rich in that ϕ is high enough. In fact, the main result that we establish below—on how the benefits of centralization vary with the number of countries—holds as long as the local fiscal authority is at an informational advantage over the central one in that ϕ can be close to perfectly informative, as long as it is strictly less than 1.

Centralized Regime. By the assumed symmetry and concavity properties of the preferences for government spending, it is optimal for the fiscal union to treat all countries with the same preference signal symmetrically. Hence, we restrict attention to allocations such that for any given number of countries I, if the fiscal union observes signal s_H, then all countries receive the same level of government spending $g(s_H, I)$, whereas if it observes signal s_L, then all countries receive the same level of government spending $g(s_L, I)$. In contrast to the case without externalities, we cannot solve for allocations for each country separately. But by symmetry, we can first maximize ex-post utility over symmetric allocations in the class $\{g(s_H, I), g(s_L, I)\}$ for a given number of countries I, conditional on the signal s_i. We can then determine the expected value of these allocations over all possible preference signals to solve for the ex-ante welfare of the union. That is, given signal $s_i \in \{s_H, s_L\}$, the fiscal union maximizes the ex-post welfare

$$W^C(s_i, I) = \max_g [u(y-g) + \tilde{\theta}_i h(G(\bar{g}))], \tag{12}$$

where $\tilde{\theta}_i \equiv E(\theta|s_i)$ is the posterior mean of θ given s_i and $G(\bar{g}) = [1+\gamma(I-1)]g$. Ex-ante welfare is then

$$V^C(I) = p_H W^C(s_H, I) + p_L W^C(s_L, I), \tag{13}$$

where $p_H = q\phi + (1-q)(1-\phi)$ is the probability of signal s_H and $p_L = 1 - p_H$ is that of signal s_L. The following lemma states some key properties of ex-ante welfare under a central fiscal authority.

[6] Indeed, Draghi (2023) argues that more correlated shocks are becoming increasingly common in the Eurozone "*[t]he nature of the shocks we are facing is changing. With the pandemic, the energy crisis, and the war in Ukraine, we are increasingly confronting common, imported shocks rather than asymmetric, self-inflicted ones.*"

Lemma 1. *When preferences for government spending are perfectly correlated across countries, ex-ante welfare in a fiscal union strictly increases with the number of countries in the union and becomes arbitrarily large as the number of countries in the union grows arbitrarily large.*

To illustrate this result, consider the case of a specific utility function of the form

$$\log(c_i) + \theta_i \log(G(\bar{g})). \tag{14}$$

It is immediate that the solution to the problem of a fiscal union is then

$$c^C(s_i, I) = \frac{1}{1+\tilde{\theta}_i} y \text{ and } g^C(s_i) = \frac{\tilde{\theta}_i}{1+\tilde{\theta}_i} y. \tag{15}$$

Note for later that the optimal fraction of output that is devoted to the consumption of public goods does not vary with the number of countries but, importantly, total effective government spending

$$G^C(s_i, I) = [1 + \gamma(I-1)] g^C(s_i) \tag{16}$$

grows arbitrarily large as the number of countries in the union progressively increases. Substituting (15) into the expression for ex-ante welfare in (13) and using that $\tilde{\theta}_i \equiv E(\theta | s_i)$, we obtain that

$$V^C(I) = \sum\nolimits_{i=H,L} p_i \Big[\log(c^C(s_i, I)) + \tilde{\theta}_i \log(G^C(s_i, I)) \Big]. \tag{17}$$

Clearly, as the number of countries in the union increases, the union's ex-ante utility from private consumption is constant but its ex-ante utility from public consumption becomes larger and larger. Intuitively, although each country's government spends a constant amount of its endowment on public goods, the spillover from other countries' government spending through the externality term $\gamma(I-1)$ in (16) increases as the number of countries in the union increases, eventually making public spending infinitely valuable.

Decentralized Regime. The problem of the local fiscal authority of country i given the observed state $\theta_i = \{\theta_H, \theta_L\}$, when all other countries spend the same amount g_{-i}, is

$$W^D(\theta_i, g_{-i}, I) = \max_g [u(y-g) + \theta_i h(g + \gamma(I-1)g_{-i})]. \tag{18}$$

Ex-ante welfare is then defined as the expected value of $W^D(\theta_i, g_{-i}, I)$ with respect to θ_i,

$$V^D(I) = q W^D(\theta_H, I) + (1-q) W^D(\theta_L, I). \tag{19}$$

The main properties of the function $V^D(I)$ are stated in the following lemma.

Lemma 2. *When preferences for government spending are perfectly correlated across countries, ex-ante welfare in a decentralized fiscal regime strictly increases with the number of countries considered converging to the constant* $\bar{V}^D = u(y) + \mathbb{E}\{\theta_i[h'^{-1}(u'(y)/\theta_i)]\} < \infty$*, which is independent of* ϕ *and* γ*.*

As before, we consider the utility function in (14) to illustrate the results of Lemma 2, which will also prove useful when comparing welfare between the centralized and the decentralized regimes. The solution for the decentralized fiscal authority problem is

$$c^D(\theta_i, I) = \left[\frac{1+\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\right] y \text{ and } g^D(\theta_i, I) = \left[\frac{\theta_i}{1+\theta_i+\gamma(I-1)}\right] y, \tag{20}$$

after imposing symmetry. Note that in stark contrast to the centralized case, as the number of countries I becomes arbitrarily large, each country's fraction of output devoted to government spending becomes arbitrarily small, so private consumption for each country eventually absorbs all output ($c = y$). At the same time, total effective government spending

$$G^D(\theta_i, I) = [1+\gamma(I-1)]\, g^D(\theta_i, I) = \frac{1+\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\theta_i y$$

converges to the constant θy. Substituting these optimal policies into the ex-post welfare function $W^D(\theta_i, I) = \log(c^D(\theta_i, I)) + \theta_i \log(G^D(\theta_i, I))$ implies that as the number of countries I grows arbitrarily large, ex-post welfare in the decentralized regime converges to a constant

$$\lim_{I\to\infty} W^D(\theta_i, I) = \lim_{I\to\infty} [\log(c^D(\theta_i, I)) + \theta_i \log(G^D(\theta_i, I))] = \log(y) + \theta_i \log(\theta_i y).$$

Thus, ex-ante welfare under decentralization converges to the constant $\bar{V}^D = \log(y) + E[\theta_i \log(\theta_i y)]$, which is independent of ϕ and γ, when I arbitrarily increases.

Centralization vs. Decentralization. The comparison of these two regimes revolves around a fundamental trade-off. Given the superior information of local fiscal authorities relative to a central fiscal one, a decentralized fiscal regime can better adapt policies to each country's preferences for government spending. Such a regime, however, does not internalize any fiscal externalities across countries. By contrast, a fiscal union internalizes them, but it can only imperfectly design its policies in response to the preferences of its member states. Then, which regime is preferred depends on the strength of the informational advantage of local fiscal authorities and the magnitude of the fiscal externalities across countries. According to the balance of these two forces, we can show that for general utility functions, there exists a sufficiently small number of countries, denoted by $\underline{I}^\star(\phi, \gamma)$, such that a decentralized regime is preferred for any number of countries smaller than $\underline{I}^\star(\phi, \gamma)$. Likewise, there exists a sufficiently large number of countries, denoted by

$\bar{I}^{\star}(\phi, \gamma)$, such that a centralized regime is preferred for any number of countries greater than $\bar{I}^{\star}(\phi, \gamma)$. A sufficient condition for these lower and upper thresholds to coincide is that consumers' utility function is of the log form over consumption and government spending.

Both the quality of the information about countries' preferences over government spending and the size of fiscal externalities affect these cutoffs. Consider first the role of the quality of the information of a central fiscal authority about member countries' preferences for government spending, as measured by the informativeness ϕ of the signals it receives. As informational quality improves, namely, as ϕ increases towards 1, so does the central authority's ability to tailor its policies to member countries' preferences. In the limit when $\phi = 1$, the central authority has the same information as any country in a decentralized regime, so it is preferred to *any* decentralized regime—that is, for any $I \geq 2$. Intuitively, since the only disadvantage of a central fiscal authority is its inferior information, once this disadvantage is erased ($\phi = 1$), a centralized regime always dominates, as long as a fiscal externality exists that implies that the decisions of decentralized fiscal authorities do not coincide with those of a centralized fiscal authority.

Consider next the role of the size of fiscal externalities, as captured by the parameter γ. If the utility function over consumption and government spending have both the log form, then the larger is γ, the greater is the advantage of a central fiscal authority relative to a local one, because a central authority takes into account that greater government spending in any given country benefits all countries in the union. Hence, the cutoff number of countries for which a centralized regime is preferred decreases with γ. For more general utility functions, though, the cutoff number of countries for which a centralized regime is preferred could decrease or increase.[7]

Proposition 2 (Cutoff Rule for Optimal Delegation When Preference Types Are Correlated)**.** *For a given degree of informativeness $\phi \in [1/2, 1)$ of the preference signal and a given value of the fiscal externality parameter $\gamma > 0$, if $h(x)$ has the form in* (11)*, then: a) there exists a cutoff $\bar{I}^{\star}(\phi, \gamma)$ such that a centralized regime is preferred if $I > \bar{I}^{\star}(\phi, \gamma)$, with $\bar{I}^{\star}$ decreasing with ϕ; b) there exists a cutoff $\underline{I}^{\star}(\phi, \gamma)$ such that a decentralized regime is preferred if $I \leq \underline{I}^{\star}(\phi, \gamma)$, with $\underline{I}^{\star}$ decreasing with ϕ; and c) if $u(x) = h(x) = \log(x)$, then there exists a unique cutoff $I^{*}(\phi, \gamma)$ such that a centralized regime is preferred if $I > I^{*}(\phi, \gamma)$ and a decentralized regime is preferred if $I \leq I^{*}(\phi, \gamma)$, with $I^{*}(\phi, \gamma)$ decreasing with ϕ and γ.*

The results we have established depend on the nature of the fiscal externalities considered. In particular, the result that under a fiscal union, each country's ex-ante welfare becomes arbitrarily large, as the size of the union progressively increases, clearly depends on both the nature of the fiscal spillover encoded in total effective government spending and the form of the utility function over such spending. In terms of

[7] To see why, suppose that the current number of countries is such that ex-ante welfare is the same under a decentralized and under a centralized fiscal regime. Thus, as γ increases, whether the cutoff number of countries for centralization to be optimal increases or decreases depends on whether an increase in γ has a larger impact on welfare under decentralization or that under centralization. In the former case, the cutoff increases, whereas in the latter case, the cutoff decreases.

the fiscal spillover, holding fixed the utility function, if we assumed that a fraction of the *average spending in other countries* spills over to any given country, as in

$$h(g_i, \bar{g}_{-i}) = h\Big(g_i + \frac{\gamma}{I-1}\sum\nolimits_{j\neq i} g_j\Big), \tag{21}$$

rather than a fraction of the *total spending in other countries* as in (11), then ex-ante welfare under a fiscal union would converge to a finite value, as the union grows arbitrarily large. In terms of the utility function, holding fixed the form of the fiscal spillover, if we allowed for satiation at some finite value of the public good, then ex-ante welfare would typically converge to a finite value as the number of countries increased. Hence, in either case, as long as a local fiscal authority has a strict informational advantage over a centralized one ($\phi < 1$) and the fiscal externality is positive but small, a decentralized regime is in general preferred to a centralized one regardless of the number of countries.

Suppose now that we generalize the informational structure so as to allow for decreasing returns to scale to information acquisition for the centralized fiscal authority—namely, as the union grows, the quality of the signal about each country's preference for government spending deteriorates. Then, under the assumptions on preferences and fiscal externalities in Proposition 2, a threshold in the number of countries still exists such that a fiscal union is preferred for any number of countries above it.

To see why, consider the extreme case in which the centralized authority receives an uninformative signal about all countries. Welfare under centralization in this case is a lower bound on welfare under centralization for any arbitrary degree of informativeness of preference signals. Since the size of the fiscal externality grows arbitrarily large as the number of countries progressively increases, welfare under centralization must exceed welfare under decentralization whenever the number of countries is larger than some finite number. Thus, regardless of the information structure assumption, for a large enough number of countries, a centralized fiscal regime is preferred—and the scenario discussed in footnote 1 in which a fiscal union is never optimal regardless of the number of countries cannot arise.

2.2.2 Independent Preferences across Countries

Consider now the case in which each country draws its taste for government spending θ_i independently from any other country. A local fiscal authority in any country perfectly observes its preference type θ_i as well as those that of all other countries. The idea behind this common observability assumption is that any country can observe other countries' decisions on government spending at a preliminary planning stage and countries can commit to these decisions. In contrast, a central fiscal authority only observes a signal about each country's preference type, $\bar{s} = \{s_1, \ldots, s_I\}$. The same intuitions and results as for the case of perfectly correlated preferences apply here, with the difference that the problem of a fiscal union now involves an evaluation of the probability-weighted outcomes for all possible realizations of preference types and signals about them for each country.

Consider preferences for government spending as specified in (11), which satisfy the symmetry and concavity properties of Assumption 1. Hence, in the decentralized regime, we can restrict attention to allocations of the form $g_H = g(\theta_H)$ and $g_L = g(\theta_L)$ and let n denote the number of countries of type θ_H. In the centralized regime, we can restrict attention to allocations of the form $g_H = g(s_H)$ and $g_L = g(s_L)$ and let n denote the number of countries with signal s_H. As the total provision of public goods $\sum_{i=1}^{N} g_i$ depends on n under both regimes, we can express the function $h(g, \bar{g}_{-i})$ in the decentralized regime as

$$h_H(g; g_H, g_L, n, I) = h\left(g + \gamma\left[(n-1)g_H + (I-n)g_L\right]\right)$$

for a country of type θ_H, in which case there are $n-1$ other countries with $g_H = g(\theta_H)$ and $I-n$ other countries with $g_L = g(\theta_L)$, and as

$$h_L(g; g_H, g_L, n, I) = h\left(g + \gamma\left[n g_H + (I-n-1)g_L\right]\right) \tag{22}$$

for a country of type θ_L, in which case there are n other countries with $g_H = g(\theta_H)$ and $I-n-1$ other countries with $g_L = g(\theta_L)$. We use the same notation as before except that $g_H = g(s_H)$ and $g_L = g(s_L)$.

Centralized Regime. For a fiscal union, we can first solve for ex-post welfare by positing an arbitrary symmetric allocation in the class $\{g_H(n), g_L(n)\}$ corresponding to any realization of n high (s_H) signals and $I-n$ low (s_L) signals for the I countries, and solve such a problem for a fixed n. We can then calculate the relevant expectation over these allocations to obtain ex-ante welfare. The first step consists of solving the fiscal union's ex-post problem given I signals n of which are s_H, namely,

$$\begin{aligned} W^C(n, I) = \max_{g_H, g_L} \Big\{ & n\Big[u(y-g_H) + \tilde{\theta}_H h_H(g_H; g_H, g_L, n, I))\Big] \\ & +(I-n)\Big[u(y-g_L) + \tilde{\theta}_L h_L(g_L; g_H, g_L, n, I)\Big]\Big\}, \end{aligned} \tag{23}$$

where $\tilde{\theta}_H \equiv E(\theta|s_H)$ and $\tilde{\theta}_L = E(\theta|s_L)$. Denote the solution to this problem as $g_H^C(n, I)$ and $g_L^C(n, I)$.

The second step consists of calculating ex-ante welfare over all possible signals s_i, given a country's preference type, and over all countries' possible preference types. Recall that the unconditional probability of any country receiving a signal s_H is p_H defined in (6). Hence, the realization of n high signals for the I countries is governed by a Binomial distribution with parameters I and p_H. Ex-ante welfare is then

$$V^C(I) = \frac{1}{I}\sum\nolimits_{n=0}^{I} \binom{I}{n} p_H^n (1-p_H)^{I-n} W^C(n, I) \tag{24}$$

under a fiscal union. By the same logic as in Lemma 1, we can now state the following lemma.

Lemma 3. *When preferences for government spending are independent across countries, ex-ante welfare in a fiscal union becomes arbitrarily large as the number of countries grows arbitrarily large. If the preference signal is uninformative ($\phi = 1/2$), then ex-ante welfare in a fiscal union strictly increases with I.*

Decentralized Regime. Under a decentralized regime, we can first solve for ex-post welfare by positing an *almost symmetric* allocation $(g; g_H(n), g_L(n))$ such that all *other* types, besides that of the country i we consider, choose symmetric allocations. We do so as we just need to allow one country of either type to best respond to the actions of all others by examining the outcomes induced if such a country chooses an asymmetric action for its type, given the behavior of all other $I-1$ countries. The ex-post problem of the local fiscal authority of a country of type θ_H is then

$$W^D(n, I, g_H, g_L; \theta_H) = \max_g [u(y-g) + \theta_H h_H(g; g_H, g_L, n, I)]. \tag{25}$$

The solution to this problem, $G^D(n, I, g_H, g_L; \theta_H)$, yields the best response of a country of type θ_H in an almost symmetric candidate allocation, in which the $n-1$ other countries of type θ_H choose g_H and the $I-n$ other countries of type θ_L choose g_L. The analogous ex-post problem for a country of type θ_L is

$$W^D(n, I, g_H, g_L; \theta_L) = \max_g [u(y-g) + \theta_L h_L(g; g_H, g_L, n, I)], \tag{26}$$

which defines the best response $G^D(n, I, g_H, g_L; \theta_L)$. An *equilibrium under the decentralized regime* is a pair $(g^D(n, I; \theta_H), g^D(n, I; \theta_L))$ solution to the two-dimensional fixed point problem defined by

$$g^D(n, I; \theta_i) = G^D(n, I, g^D(n, I; \theta_H), g^D(n, I; \theta_L); \theta_i) \text{ for } i = H, L. \tag{27}$$

Substituting these equilibrium allocations back into the ex-post problems in (25) and (26) yields the ex-post values for types θ_H and θ_L, namely,

$$W^D(n, I; \theta_i) = W^D(n, I, g^D(n, I; \theta_H), g^D(n, I; \theta_L); \theta_i) \text{ for } i = H, L. \tag{28}$$

The last step consists of calculating ex-ante welfare under the decentralized regime, which accounts for the probability of any number $n \in \{1, \ldots, I\}$ of high preference types among the I countries,

$$V^D(I) = \frac{1}{I} \sum\nolimits_{n=0}^{I} \binom{I}{n} q^n (1-q)^{I-n} \left[q W^D(n, I; \theta_H) + (1-q) W^D(n, I; \theta_L) \right]. \tag{29}$$

By a similar logic as in Lemma 2, we can state a key property of this function in the following lemma.

Lemma 4. *When preferences for government spending are independent across countries, there exists an upper bound for ex-ante welfare $V^D(I)$ in a decentralized regime given by $\bar{V}_H^D = u(y)+\theta_H h\left[h'^{-1}(u'(y)/\theta_H)\right]$, which is independent of ϕ and γ.*

By Lemma 3, welfare under centralization grows unbounded with I, whereas by Lemma 4, welfare under decentralization is bounded above. Thus, for I sufficiently high, centralization must be preferred. By this intuition formalized in the Appendix A, we can establish the following result.

Proposition 3 (Cutoff Rule for Optimal Delegation When Preference Types Are Independent)**.** *For a given degree of informativeness $\phi \in [1/2, 1)$ of the preference signal and a given value of the fiscal externality parameter $\gamma > 0$, if $h(x)$ has the form in* (11)*, then: a) there exists a cutoff $\bar{I}^\star(\phi,\gamma)$ such that a centralized regime is preferred if $I > \bar{I}^\star(\phi,\gamma)$, with $\bar{I}^\star$ decreasing with ϕ; and b) there exists a cutoff $\underline{I}^\star(\phi,\gamma)$ such that a decentralized regime is preferred if $I \leq \underline{I}^\star(\phi,\gamma)$, with $\underline{I}^\star$ decreasing with ϕ.*

Proposition 3 is the analogue of Proposition 2 when preference types are independent across countries. However, with independent types, the log case is less tractable than when types are perfectly correlated due to the combinatorial problem that naturally arises. For this reason, case c) of Proposition 2 does not naturally arise with independent types. We illustrate this proposition by way of a simple example that compares ex-ante welfare under the centralized and the decentralized regimes in (24) and (29).

[FIGURE 2 HERE]

The left panel of Figure 2 shows welfare in both regimes as a function of the number of countries for a value of the externality parameter of $\gamma = 0.2$ and two values of the informativeness of the preference signal, namely, $\phi = 0.9$ (high informativeness) and $\phi = 0.5$ (lowest informativeness). Note that as the degree of informativeness of the signal increases, ex-ante welfare increases under centralization, whereas it is unchanged under decentralization. Hence, with a more informative signal, the cutoff value of I for which centralization is preferred decreases. In particular, the cutoff is $I^*(\phi = 0.5) = 5$ for an uninformative signal, but it decreases to $I^*(\phi = 0.9) = 3$ for a very informative signal. In the right panel of Figure 2, we fix the degree of informativeness of the preference signal to $\phi = 0.9$ and show how the cutoff number of countries for centralization to be preferred varies with the size of the fiscal externality induced by government spending, γ. As the externality increases from a low level of $\gamma = 0.2$ to a high level of $\gamma = 0.7$, the inability to internalize these spillovers makes the decentralized regime relatively unattractive. Accordingly, the cutoff for which centralization is preferred decreases from $I^*(\gamma = 0.2) = 3$ to $I^*(\gamma = 0.7) = 1$. Hence, a centralized regime is always preferred.

3 A Monetary Union

We now turn to embedding the fiscal union examined so far into a monetary union with no direct externalities, in which the single currency is formalized as a single price level in all countries. We derive two main results for this economy. First, if the monetary authority has the ability to commit to an inflation policy, then Oates (1999)'s logic on the benefits of decentralization applies: regardless of the number of countries in a monetary union, a fiscal union is never optimal. Second, if the monetary authority lacks commitment power, then a decentralized regime is optimal if the number of countries in the monetary union is small and a fiscal union is optimal if the number of countries in the union is large.

The logic behind these results for a monetary economy is similar to that for a real economy except for an important distinction. In a real economy, one country's actions impact another country's welfare if and only if a *direct* externality exists in that the actions of one country directly affect the payoffs of another country. In the monetary economy we analyze, no such direct interdependence exists across countries. Instead, all interactions across countries occur through the union-wide monetary authority, whose inflation policy is the conduit through which an *endogenous* and *indirect* fiscal externality arises. In particular, if a country's fiscal decisions, such as increasing nominal debt to finance public spending, induce the central monetary authority to set a higher inflation rate, then such a country's actions have an indirect *negative* externality on all other countries in the monetary union. In our monetary model, this externality tends to worsen as the size of the monetary union increases under natural conditions.

Depending on the strength of this externality, three general cases arise. At one extreme, this indirect externality is sufficiently small relative to the informational advantage of decentralization that a decentralized fiscal regime is preferred regardless of the number of countries in the monetary union. At the other extreme, this indirect externality is sufficiently large that a fiscal union is always preferred. The intermediate case occurs for a moderate externality, when a decentralized fiscal regime is preferred for small monetary unions, whereas a fiscal union is preferred for sufficiently large ones. We emphasize that these results stand in stark contrast with standard results in the macroeconomics literature, in which the consensus is that in the presence of fiscal externalities, a fiscal union is the ideal regime for a monetary union regardless of the size of the monetary union (see, for instance, Aguiar et al., 2015).

Finally, although the monetary economy without commitment on the part of the monetary authority largely parallels the real economy with direct fiscal externalities across countries, critical differences distinguish them. Since externalities in the monetary economy are induced solely by the equilibrium behavior of the monetary authority, they depend in general on all the forces directly and indirectly governing it, including the impact on labor supply of distortionary taxes that finance public spending and the effect on aggregate productivity of ex-post inflation. As such, externalities are not simply determined by a single parameter, such as the spillover parameter γ in the real-economy model that we have considered so far.

3.1 A Model of a Monetary Economy with a Monetary Union

Consider a two-period monetary economy with I countries in a monetary union. Denote periods by $t = 1, 2$. Each country $i = 1, \ldots, I$ is populated by a representative consumer, firms, and a government. As before, consumers in different countries differ only in their preferences for government spending. In particular, the utility of a representative consumer in country i is

$$u(c_{i1}) + \theta_i h(g_i) + \beta u(c_{i2} - v(\ell_i)), \tag{30}$$

where c_{i1} and c_{i2} are consumption in periods 1 and 2, g_i is government spending in period 1, ℓ_i is labor supply in period 2, and β is the consumer's subjective discount factor.

We assume that $u(\cdot)$ and $h(\cdot)$ are increasing and concave functions and $v(\cdot)$ is an increasing and convex function. We note that utility in the second period has the GHH form, which implies that no income effects on labor supply arise. As before, the taste θ_i for government spending of each country's citizens is stochastic. We consider the same two informational structures as in the real model, namely, the case in which the taste of a country's citizens for government spending is perfectly correlated across countries and the case in which it is independent across, them with the same notation.

We model monetary policy as the choice p_2 of the price level in period 2 that leads to the period-2 gross inflation rate $\pi = p_2/p_1$, with p_1 normalized to 1. Output in period 1 is the constant y_1, whereas output in period 2 is produced using labor ℓ_i according to the production function $A(x_i)\ell_i$, where x_i is an input bought by firms in country i from the rest of the world that enhances labor productivity in that $A'(x_i) > 0$. We capture the costs of inflation by assuming that each firm has an initial amount of money M that it can use to buy the input x_i at the nominal cost of πx_i units of money in period 2.

Firms. Competitive producers in country i maximize profits in that they solve the problem

$$\max_{x_i, \ell_i} [A(x_i)\ell_i - w_i \ell_i], \tag{31}$$

where w_i is the real wage paid per unit of labor, subject to the constraint $x_i \leq M/\pi$. This constraint can be thought of as a cash-in-advance constraint with the same timing as in Nicolini (1998). Thus,

$$w_i = A(x_i) \text{ and } \quad x_i = \frac{M}{\pi} \tag{32}$$

in a competitive equilibrium. The cash-in-advance constraint in (32) implies that the cost of inflation derives from the negative impact of inflation on aggregate productivity and, hence, output.

Government. As before, each country i's government observes its citizens' taste $\theta_i \in \{\theta_H, \theta_L\}$ for public spending, whereas a central authority only observes a noisy signal $s_i \in \{s_H, s_L\}$ about consumers' taste in that country. Here too, the informativeness ϕ of the signal satisfies (4), Bayes's rule in (5) holds, and the probability of a high signal is p_H in (6). For simplicity, we assume that government expenditure g_i is financed solely by *nominal* debt issued to country i's consumers in period 1 and paid for by distortionary labor taxes in period 2. In particular, each country i's government in period 1 issues to consumers claims to B_i units of currency payable in period 2. Let $1 + R$ denote the nominal interest rate on this debt. Then, government i's budget constraint in period 1 is

$$g_i = \frac{B_i}{1+R}. \tag{33}$$

Let τ_i be a proportional tax on labor income. In period 2, each government i collects real tax revenues T_i defined as $\tau_i A(x_i)\ell_i$ to repay the real value of its debt obligations B_i/π, subject to the budget constraint

$$T_i = \frac{B_i}{\pi}. \tag{34}$$

The source of the benefits from inflation, when the monetary authority lacks the ability to commit to an inflation policy, is apparent from (34): increasing inflation in period 2 reduces the value of the debt that a government must repay. No such benefit arises when the monetary authority pre-commits to an inflation policy before any other agent undertakes any action, since in this case, the monetary authority cannot affect the ex-post real interest rate on nominal debt.

Consumers. A consumer in country i can save by investing in either a real storage technology k_i with a real rate of return $1+r$, which is technologically fixed, or in the nominal debt d_i of country i's government. The consumer problem is to maximize (30) subject to the period budget constraints

$$c_{i1} = y_1 - k_i - d_i, \tag{35}$$

where y_1 is the endowment in period 1, which is common across countries, and

$$c_{i2} = (1-\tau_i)w_i\ell_i + (1+r)k_i + (1+R)\frac{d_i}{\pi}. \tag{36}$$

Substituting out these budget constraints, the consumer problem in country i is

$$\max_{k_i,d_i,\ell_i} \left[u(y_1 - k_i - d_i) + \theta_i h(g_i) + \beta u\Big((1-\tau_i)w_i\ell_i + (1+r)k_i + (1+R)\frac{d_i}{\pi} - v(\ell_i)\Big)\right], \tag{37}$$

with corresponding first-order conditions for real storage k_i, nominal government bonds d_i, and labor ℓ_i

$$u'(c_{i1}) = \beta(1 + r)u'(c_{i2} - v(\ell_i)), \tag{38}$$

$$u'(c_{i1}) = \beta \left(\frac{1 + R}{\pi}\right) u'(c_{i2} - v(\ell_i)), \tag{39}$$

and, by using (32),

$$v'(\ell_i) = (1 - \tau_i)A(M/\pi). \tag{40}$$

Throughout, we assume that y_1 and $1 + r$ are sufficiently large that the representative consumer in each country always saves, that is, $k_i > 0$ and $d_i > 0$. We also perform a convenient change of variable so that country i's government chooses tax revenues $T_i = \tau_i A(M/\pi)\ell_i$ instead of the tax rate τ_i, by manipulating the static first-order condition for labor to define equilibrium labor as a function of tax revenues. Formally, by multiplying the consumer's first-order condition for labor in (40) by ℓ_i, it follows that

$$T_i = [A(M/\pi) - v'(\ell_i)]\ell_i. \tag{41}$$

Suppose that the tax rate τ_i is small enough to be below the peak of the Laffer curve, so that tax revenues T_i increase with the tax rate τ_i. Since labor supply decreases with τ_i, then, over this range of values of τ_i, labor supply decreases with T_i, that is, higher tax revenues are associated with lower levels of labor supply. Hence, we can invert the expression in (41) to obtain the optimal labor ℓ_i supplied by a consumer in country i as a function of the tax revenues that the government raises and productivity, denoted by

$$\ell_i = \ell(T_i, A(M/\pi)). \tag{42}$$

From now on, we maintain that this function is constructed from the left side of the Laffer curve. The market-clearing constraints for goods are $c_{i1} + k_i + g_i = y_1$ and $c_{i2} = A(x_i)\ell_i + (1 + r)k_i$.

3.2 The Case with Commitment for the Monetary Authority

We interpret this case as one in which the monetary authority moves first and chooses (gross) inflation before any information or signals are realized. Throughout, we assume that the monetary authority can only select non-negative inflation $\pi \in [1, \infty)$. Consumers, firms, and fiscal authorities move after the monetary authority has made its choice and, hence, take π as given. We set up and solve for equilibrium by backward induction from the end of period 2. To do so, we first describe the equilibrium notion for both a decentralized fiscal regime and a centralized one, and for each regime, we solve for the continuation equilibrium in period 2 given π and some arbitrary fiscal policies chosen by either the local governments or the central fiscal authority in period 1. Then, we solve for the optimal fiscal policies in the two regimes and, after substituting them into consumers' and firms' problems, we derive the continuation equilibrium in period 1 given π. Finally, we consider the problem of the monetary authority at the beginning of period 1, which anticipates how the decisions of fiscal authorities and private agents as well as prices will depend on π. As will soon become clear, the optimal choice of a monetary authority that has commitment in either fiscal regime is to set inflation to zero in that $\pi = 1$.

3.2.1 Continuation Competitive Equilibrium in Period 1

Given some arbitrary policy π for the monetary authority, the realizations of types and signals for each country and fiscal authority, and fiscal policies $\{g_i, B_i, \tau_i\}$ for each country i, a *continuation competitive equilibrium in period* 1 is a nominal interest rate R and allocations $\{c_{i1}, c_{i2}, k_i, d_i, \ell_i, y_{i2}\}$ for all countries such that i) consumer policies in country i solve the consumer problem in (37); ii) firm policies solve the firm problem in (31) and the zero-profit condition

$$w_i = A(x_i) \tag{43}$$

holds, with $x_i = M/\pi$; iii) the government budget constraint holds in periods 1 and 2

$$g_i = \frac{B_i}{1+R} \text{ and } T_i = \frac{B_i}{\pi}; \tag{44}$$

iv) the no-arbitrage condition across storage and bonds holds

$$1 + r = \frac{1+R}{\pi}; \tag{45}$$

v) the resource constraints in periods 1 and 2 are satisfied

$$c_{i1} + k_i + g_i = y_1 \text{ and } c_{i2} = A(x_i)\ell_i + (1+r)k_i, \tag{46}$$

with $x_i = M/\pi$; and vi) the bond market clears

$$d_i = \frac{B_i}{1+R}. \tag{47}$$

In both the decentralized and the centralized regimes, we express the relevant fiscal authority's problem as a Ramsey problem in which the fiscal authority anticipates that the future will evolve according to the continuation equilibrium in period 1. Specifically, we exploit the equilibrium conditions described to set up the Ramsey problem of maximizing consumer utility in (37) as follows. First, we use conditions (44), (45), (47), and the period-1 consumer budget constraint to express consumption in period 1 as

$$c_{i1} = y_1 - k_i - \frac{T_i}{1+r}.$$

Then, we use (43), (44), (47), the definition of T as $\tau_i A(x_i)\ell_i$, and the period-2 consumer budget constraint to express consumption in period 2 as

$$c_{i2} = A(M/\pi)\ell(T_i, A(M/\pi)) + (1+r)k_i. \tag{48}$$

Finally, we use (44) and (45) to express government spending as $g_i = T_i/(1+r)$. Hence,

$$u\left(y_1 - k_i - T_i/(1+r)\right) + \theta_i h\left(T_i/(1+r)\right) + \beta u\left(A(M/\pi)\ell(T_i, A(M/\pi)) + (1+r)k_i - v(\ell(T_i, A(M/\pi)))\right) \tag{49}$$

is the value of consumer utility in a continuation equilibrium, with $T_i = B_i/\pi$ by (44). Note that we have used all the equilibrium conditions except for the first-order condition for savings in (38).

3.2.2 Decentralized Regime

Under a decentralized regime, the history that the fiscal authority of country i faces at the beginning of period 2 includes the inflation rate π chosen by the monetary authority as well as the realization of country i's preference type θ_i, which is observed by the country's consumers and fiscal authority. Importantly, given that the monetary authority has already set inflation, the problem of each country i's fiscal authority can be solved in isolation. The reason for this result is twofold. First, we have purposely abstracted from any direct externalities of the type considered in the real-economy model. Second, when the monetary authority can commit to an inflation policy, there are no indirect externalities arising from its actions. Because of these features, country i's government needs only to consider its own type θ_i and the already determined inflation rate π when choosing its fiscal policy. Taking as given each country's fiscal policy, the monetary authority then chooses the inflation rate at the start of the period.

Consider the problem of country i's government, which we express as a Ramsey-type problem by the logic described earlier. By (49), the fiscal authority's problem in the decentralized regime is given by

$$\begin{aligned} W^D(\theta_i, \pi) = \max_{k_i, T_i} \Bigg\{ & u\left(y_1 - k_i - \frac{T_i}{1+r}\right) + \theta_i h\left(\frac{T_i}{1+r}\right) \\ & + \beta u\left(A\left(\frac{M}{\pi}\right)\ell\left(T_i, A\left(\frac{M}{\pi}\right)\right) + (1+r)k_i - v\left(\ell\left(T_i, A\left(\frac{M}{\pi}\right)\right)\right)\right)\Bigg\}, \end{aligned} \tag{50}$$

subject to the first-order condition for savings in (38). It turns out that it is sufficient to focus on a *relaxed* version of this problem in which the first-order condition for savings is dropped. Intuitively, the first-order condition with respect to k_i for such a relaxed problem coincides with the first-order condition for savings in (38). Since a solution to the relaxed problem is then feasible for the original problem, it must solve the original problem. Note also that by (50) if productivity $A(x_i)$ did not depend on π, then inflation would have no effect on the value $W^D(\theta_i, \pi)$. The reason is simply that inflation affects only the nominal interest rate $1 + R = (1 + r)\pi$ for a given real interest rate $1 + r$. By (50), as π is raised, the nominal interest rate increases but the same amount of real revenues T_i is needed to finance the government spending $g_i = T_i/(1 + r)$ chosen in period 1. The following result is thus immediate.

Lemma 5. *When the monetary authority has commitment over its inflation policy, given an inflation rate π, the monetary-economy model is equivalent to a real-economy model in which real bonds carry a fixed real interest rate $1 + r$ and aggregate productivity is constant at level $A(M/\pi)$.*

In the case of interest in which $A(x_i)$ decreases with the input cost $x_i = M/\pi$, an increase in the net inflation rate $\pi - 1$ above zero decreases productivity, which lowers the value $W^D(\theta_i, \pi)$. Since it is only feasible for the monetary authority to induce non-negative net inflation—recall that the gross inflation rate π is (weakly) greater than 1—it follows that regardless of countries' realized preference types, ex-post welfare is maximized at $\pi = 1$. Given that this result holds for each possible preference type, ex-ante welfare is also maximized at $\pi = 1$. The next result summarizes this discussion.

Proposition 4. *When the monetary authority has commitment over its inflation policy, optimal (net) inflation in the decentralized regime is zero.*

3.2.3 Centralized Regime

The same argument underlying Proposition 4 applies to the centralized regime of a fiscal union: when the monetary authority has the ability to commit to an inflation policy, the inflation rate is $\pi = 1$ and so ex-ante welfare is maximized at zero net inflation. Hence, a result analogous to Proposition 4 holds.

Proposition 5. *When the monetary authority has commitment over its inflation policy, optimal (net) inflation in a fiscal union is zero.*

We can then reduce the monetary-economy model with noisy signals about countries' preferences for government spending to an equivalent real one with aggregate productivity $A = A(M)$, which is invariant to any fiscal choices by the monetary union's member countries. Since we have argued in Lemma 5 that under commitment by the monetary authority, a monetary economy under a decentralized fiscal regime reduces to its equivalent real counterpart with no fiscal externalities, a version of the earlier Oates (1999)'s decentralization result in Proposition 1 immediately applies to the monetary economy.

Proposition 6 (Generalized Decentralization Theorem in a Monetary Economy with Commitment)**.** *When signals about countries' preferences for government spending are not perfectly informative in that* $\phi < 1$*, a decentralized regime yields higher ex-ante welfare than a centralized regime, namely, a fiscal union, does. The difference in welfare between the two regimes decreases with the informativeness of signals.*

3.3 The Case without Commitment for the Monetary Authority

We now turn to the more subtle case in which the monetary authority does not have the ability to commit to an inflation rate. In this case, the equivalence between a real and a monetary economy that holds in the case with commitment, as established in Lemma 5, no longer applies. The interaction between fiscal and monetary policy will prove critical for welfare in this case.

The key difference between a monetary economy with commitment on the part of the monetary authority and one without it is the timing of the monetary authority's inflation decision. Without commitment, we can think of the monetary authority as moving at the beginning of period 2 and choosing inflation *after* all countries' fiscal authorities have chosen their levels of spending and nominal debt $\bar{B} = (B_1, \dots, B_I)$ and consumers have chosen their real savings $\bar{k} = (k_1, \dots, k_I)$.[8] Faced with the state variables $(\bar{B}, \bar{k})$, the monetary authority takes into account that for any history of such state variables and its choice of inflation, consumers', firms', and governments' choices in period 2 will constitute a continuation equilibrium in period 1. As before, we solve for equilibrium by backward induction starting from the end of period 2 to determine the monetary authority's inflation policy for any vector of state variables, $\pi(\bar{B}, \bar{k})$. We then proceed to the beginning of period 1 and determine fiscal policy under the two fiscal regimes of interest.

3.3.1 Continuation Competitive Equilibrium in Period 2

Consider the beginning of period 2. Since utility is additively separable across periods, the vector $(\bar{\theta}, \bar{s}, \bar{g})$ of preference types, signals about them, and government expenditures in all countries in period 1 have no direct effect on period-2 utility. Hence, they are irrelevant constants from the viewpoint of the monetary authority, fiscal authorities in period 2, and consumers in period 2. The state, then, reduces to $(\bar{B}, \bar{k})$.

Period-2 Fiscal Policy. By steps similar to those of the argument under commitment leading to (49), we can fold consumer and firm second-period choices into a Ramsey-type problem of maximizing country i's consumer utility in period 2. The only difference is that since period-1 values are just constants that can be omitted, the problem of government i in period 2 reduces to

$$\max_{T_i} u\left(A(M/\pi)\ell(T_i, A(M/\pi)) + (1+r)k_i - v(\ell(T_i, A(M/\pi)))\right), \tag{51}$$

[8] Throughout, in a slight abuse of notation, we refer to different values of $(B_1, \dots, B_I)$ by $\bar{B}$ to avoid defining new variables.

subject to $T_i = B_i/\pi$. Clearly, the solution to this problem amounts to choosing T_i on the left side of the Laffer curve in revenues so as to finance the real value of the outstanding debt, B_i/π.

Monetary Policy. The monetary authority chooses π to maximize an equally-weighted average of the continuation utility of each country's consumers. Since consumer utility is time separable and countries' preference types for government spending affect only period-1 utility, the monetary authority's inflation decision in period 2 given the state $(\bar{B}, \bar{k})$ does not depend on either preference types or signals about them. Substituting into the monetary authority's problem each country's optimal fiscal policies and associated distortions, as encoded in the function $\ell_i = \ell(B_i/\pi, A(M/\pi))$, this problem becomes

$$W_{MA}(\bar{B}, \bar{k}) = \max_{\pi \geq 1} \frac{1}{I} \sum\nolimits_{i=1}^{I} u(c_{i2} - v(\ell(B_i/\pi, A(M/\pi))), \tag{52}$$

with $c_{i2} = A(M/\pi)\ell(B_i/\pi, A(M/\pi)) + (1+r)k_i$, and defines the optimal monetary policy function $\pi(\bar{B}, \bar{k})$. Note that an indirect fiscal externality arises because of the monetary authority's response to the history $(\bar{B}, \bar{k})$ that it faces. Intuitively, under both a decentralized and a centralized fiscal regime, each fiscal authority anticipates how inflation will depend on its first-period choices and so understands that increasing spending and, hence, nominal debt in period 1 will induce the monetary authority to increase inflation in period 2. A spillover emerges in a decentralized regime because when country i's government chooses its spending and debt, it does not internalize the cost on other countries of the rate of inflation that its fiscal decisions induce the monetary authority to implement. For notational convenience, it will be useful to define the output function (adjusted for the disutility of labor supply)

$$F(B_i, \pi) = A(M/\pi)\ell(B_i/\pi, A(M/\pi)) - v(\ell(B_i/\pi, A(M/\pi))). \tag{53}$$

Then, for an arbitrary history $(\bar{B}, \bar{k})$, the problem of the monetary authority can be written as

$$W_{MA}(\bar{B}, \bar{k}) = \max_{\pi \geq 1} \frac{1}{I} \sum\nolimits_{i=1}^{I} u\left(F(B_i, \pi) + (1+r)k_i\right),$$

with first-order condition for the optimal inflation rule $\pi(\bar{B}, \bar{k}) = \pi(B_1, \ldots, B_I, k_1, \ldots, k_I)$ given by

$$\frac{1}{I} \sum\nolimits_{i=1}^{I} u'\left(F(B_i, \pi) + (1+r)k_i\right) F_\pi(B_i, \pi) = 0. \tag{54}$$

Letting $\bar{B}(B) = (B, \ldots, B)$ and $\bar{k}(k) = (k, \ldots, k)$ denote a *symmetric history* in which all countries choose the same B and k, the following lemma will help simplify the analysis that follows.

Lemma 6. *Given a symmetric history $(\bar{B}(B), \bar{k}(k))$ at the beginning of period 2, the monetary authority's inflation policy does not depend on k.*

The proof of this result uses the property that given a symmetric history, the first-order condition in (54) reduces to $F_\pi(B,\pi) = 0$, which does not depend on k. Using (54), we can also determine how a change in a single country's nominal debt B_i affects inflation, namely,

$$\sum\nolimits_{i=1}^{I} \left[u''(\cdot)\, F_\pi(B_i,\pi) + u'(\cdot)\, F_{\pi\pi}(B_i,\pi)\right] \frac{\partial \pi}{\partial B_i} + u'(\cdot)\, F_{B_i\pi}(B_i,\pi) = 0. \tag{55}$$

Thus, evaluating (55) at a symmetric history of debt (and capital choices) yields that

$$\left.\frac{\partial \pi(B_1,\ldots,B_I)}{\partial B_i}\right|_{B_1=\cdots=B_I=B} = -\frac{1}{I}\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)}. \tag{56}$$

3.3.2 Perfectly Correlated Preferences across Countries

We begin with the case in which preferences for government spending and signals about them are perfectly correlated across countries. For simplicity, we first consider a fiscal union and then a decentralized regime.

Centralized Regime. Consider a fiscal union in which the common signal about the preference type of all countries, $s \in \{s_H, s_L\}$, is observed at the beginning of period 1. Given the signal s, we can express the central fiscal authority's problem as a type of Ramsey problem with value

$$W^C(s,\pi(\cdot)) = \max_{B_i,k_i} \frac{1}{I}\sum\nolimits_{i=1}^{I}\left[u(c_{i1}) + E\left(\theta|s_i\right) h\left(\frac{B_i}{(1+r)\,\pi}\right) + \beta u\left(c_{i2} - v\left(\ell\left(\frac{B_i}{\pi}, A\left(\frac{M}{\pi}\right)\right)\right)\right)\right], \tag{57}$$

subject to the first-order condition for savings in each country, $u'(c_{i1}) = \beta(1+r)u'(c_{i2} - v(\ell_i))$, and non-negativity constraints on savings, $B_i \geq 0$ and $k_i \geq 0$, where $c_{i1} = y_1 - k_i - B_i/\left[(1+r)\pi\right]$, $c_{i2} = A(M/\pi)\ell(B_i/\pi, A(M/\pi)) + (1+r)k_i$, and $\pi = \pi(\bar{B},\bar{k})$. Since we focus on symmetric equilibria, Lemma 6 applies at an optimal allocation, so monetary policy does not depend on $\bar{k}$. Then, as before, the first-order condition for k_i for a relaxed version of (57) without the first-order condition for consumer savings coincides with the first-order condition for k_i for the original problem. Since a solution to the relaxed problem is feasible for the original problem, it must solve it.

We now turn to characterizing the centralized equilibrium. The first-order condition of (57) with respect to B_i, after some manipulation detailed in Appendix A, is

$$\left[\pi(\bar{B})F_B(B_i,\pi(\bar{B})) - 1\right] u'(c_{i1}) + E\left(\theta|s\right) h'(g_i) + \sum\nolimits_{j=1}^{I}\left[u'(c_{j1}) - E\left(\theta|s\right) h'(g_j)\right] \frac{B_j}{\pi(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B_j} = 0. \tag{58}$$

According to (58), the central authority balances the benefit to country i from a marginal increase in its spending and debt against the cost to all countries in the union of the induced inflation. Then,

$$\left[\pi(\bar{B})F_B(B,\pi(\bar{B}))-1\right]u'(c_1)+E\left(\theta|s\right)h'(g)+\left[u'(c_1)-E\left(\theta|s\right)h'(g)\right]I\frac{B}{\pi(\bar{B})}\frac{\partial\pi(\bar{B})}{\partial B}=0. \tag{59}$$

by imposing symmetry, with $\bar{B}=(B,\ldots,B)$. For later comparisons, it is useful to define the *centralized elasticity* of inflation with respect to a *joint* marginal increase in all countries' debt starting from a symmetric allocation. We then let $\pi^C(B)=\pi(\bar{B})=\pi(B,\ldots,B)$ and define the centralized elasticity as

$$\eta^C(B)\equiv\frac{B}{\pi^C(B)}\frac{\partial\pi^C(B)}{\partial B}=I\frac{B_i}{\pi(\bar{B})}\left.\frac{\partial\pi(\bar{B})}{\partial B_i}\right|_{B_1=\cdots=B_I=B}, \tag{60}$$

which is the last term on the right side of (59). Substituting (56) in (60), we obtain

$$\eta^C(B)=-\frac{B}{\pi^C(B)}\frac{F_{B\pi}(B,\pi^C(B))}{F_{\pi\pi}(B,\pi^C(B))}, \tag{61}$$

which does not depend on I. Finally, substituting (61) in (59) gives that

$$\left[\pi(\bar{B})F_B(B,\pi(\bar{B}))-1\right]u'(c_1)+E\left(\theta|s\right)h'(g)+\left[u'(c_1)-E\left(\theta|s\right)h'(g)\right]\eta^C(B)=0. \tag{62}$$

This condition implies that in a centralized regime, allocations are independent of the number of countries I in the union and, as a result, so is ex-ante welfare.[9] But then the allocation for each country in the union coincides with that in a trivial monetary and fiscal union composed of that country alone. Intuitively, since the central fiscal authority maximizes a weighted average of consumer utility for all the countries in the monetary union, it necessarily internalizes all the inflation spillovers induced by any change in the debt level of any one country. Also, by symmetry, such an authority essentially solves a representative country problem with its own monetary authority. Ex-ante value in a fiscal union of any size I is then

$$V^C=p_HW^C(s_H,\pi(\cdot))+(1-p_H)W^C(s_H,\pi(\cdot)). \tag{63}$$

We summarize this discussion in the following lemma, which parallels Lemma 1 in the real economy.

Lemma 7. *When preferences for government spending are perfectly correlated across countries, equilibrium allocations in a fiscal union with I countries are independent of I and coincide with those in a monetary and fiscal union with only one country.*

[9]Recall that in the real economy, centralized welfare increases with the number of countries I, whereas here it is constant. This result stems from the fact that the real economy features a direct externality that increases with I, whereas the monetary economy features a subtler externality mediated by the inflation rule, which is invariant to the number of countries.

Decentralized Regime. Consider the Ramsey problem for country i's government. We focus on symmetric allocations in that when all countries draw the preference type θ_H, fiscal policies are $(B_i(\theta_H), k_i(\theta_H))$-$= (B(\theta_H), k(\theta_H))$ for all i, whereas when all countries draw the preference type θ_L, fiscal policies are $(B_i(\theta_L), k_i(\theta_L)) = (B(\theta_L), k(\theta_L))$ for all i. Hence, in what follows, we denote θ_i simply by θ whenever possible. To define the problem of any given country i's government over the policies to choose, we just need to consider the problem of an individual country i's government, given that all others have chosen the same policies. That is, for each θ, we only need to consider *almost symmetric* allocations of the form $(B_i, k_i; \bar{B}_{-i}(B), \bar{k}_{-i}(k))$, where $\bar{B}_{-i}(B)$ and $\bar{k}_{-i}(k)$ denote period-2 histories such that all other countries except for i have chosen the same policies (B, k). As before, we can focus on a relaxed version of country i's government problem, in which the first-order condition for consumer savings is omitted as a constraint, and inflation does not depend on savings. With $\bar{B} = (B_i; \bar{B}_{-i}(B))$, the problem of country i is

$$W^D(\theta, B, I) = \max_{B_i, k_i} \left\{ u\left(y_1 - k_i - \frac{B_i}{(1+r)\pi(\bar{B})}\right) + \theta h\left(\frac{B_i}{(1+r)\,\pi(\bar{B})}\right) + \beta u\Big(F(B_i, \pi(\bar{B})) + (1+r)k_i\Big) \right\}. \tag{64}$$

The first-order condition of this problem with respect to B_i is

$$\left[\pi(\bar{B})F_B(B_i, \pi(\bar{B})) - 1\right] u'(c_{i1}) + \theta h'(g_i) + \left[u'(c_{i1}) - \theta h'(g_i)\right] \frac{B_i}{\pi(\bar{B})} \frac{\partial \pi(\bar{B})}{\partial B_i} = 0, \tag{65}$$

which, after imposing symmetry in that $B_i = B$, can be rewritten as

$$\left[\pi(\bar{B})F_B(B, \pi(\bar{B})) - 1\right] u'(c_1) + \theta h'(g) + \frac{1}{I}\left[u'(c_1) - \theta h'(g)\right] \eta^C(B) = 0, \tag{66}$$

with $\bar{B} = (B, \ldots, B)$, where the last term uses that the elasticity of inflation with respect to an increase in debt by a *single* country i by (60) satisfies

$$\eta^D(B, I) \equiv \frac{B_i}{\pi(\bar{B})} \frac{\partial \pi(\bar{B})}{\partial B_i}\Big|_{B_1 = \cdots = B_I = B} = \frac{1}{I}\eta^C(B). \tag{67}$$

By (67), starting from a common level of debt, the percentage change in inflation resulting from any one country increasing its nominal debt by 1% in the decentralized regime is just $1/I$ of the corresponding change in the centralized regime when all I countries simultaneously increase their debts by 1%. To understand why, note that when the decentralized fiscal authority of country i chooses its level of borrowing B_i in period 1, it takes into account that increasing *its own debt* will increase inflation in period 2. In the centralized regime, instead, the fiscal union takes into account the total effect on π of increasing debt in *all countries*. Clearly, then, the choice of debt in the decentralized regime depends on the number of countries in the monetary union. As this number progressively increases, the last term in (66) correspondingly decreases, eventually vanishing. In this case, when any local fiscal authority issues debt, it perceives the inflation rate as disconnected from its fiscal decisions and so it tends to issue a high level of debt. Whenever countries' fiscal authorities view inflation as such, a high inflation rate prevails in the union.

Comparison of Regimes. Two key differences distinguish the decentralized and the centralized regimes. First, as before, a local fiscal authority has an informational advantage over a fiscal union in that it observes the preference of its country's citizens for government spending. A fiscal union, instead, observes only a noisy signal about it. The second difference arises in terms of objectives. When calculating the direct benefit of an increase in country i's spending and debt, country i's the fiscal authority and a central fiscal authority calculate the same benefit for country i. However, when assessing the cost of the implied inflation, country i's government considers only its own cost (the last term in (65)), whereas a central fiscal authority takes into account the cost for all countries in the union (the last term in (58)).

As argued, a central fiscal authority's decisions do not depend on the number of countries in the monetary union. Hence, as formalized in Lemma 7, welfare in a fiscal union does not vary with I. In contrast, a local fiscal authority's decisions depend on the number of countries in the union, because the indirect fiscal externality from inflation increases as the number of countries increases. Under natural assumptions, we can then show that ex-ante welfare in a decentralized regime decreases as the number of countries in the union increases. Here we formulate assumptions directly on the function $F(B, \pi)$ defined in (53)—we later provide sufficient conditions on preferences and technology for them to be satisfied.

Assumption 2. *The function $F(B, \pi)$ defined in* (53) *has the following properties: $F_B(B, \pi) < 0$, $F_{BB}(B, \pi) < 0$, $F_{\pi\pi}(B, \pi) < 0$, and $F_{B\pi}(B, \pi) > 0$.*

The first condition that $F_B(B, \pi) < 0$ is a standard assumption ensuring that taxation is distortionary in the sense that increasing borrowing in period 1, and thus taxes in period 2, decreases utility in period 2 by decreasing labor supply and so total output in the economy. The condition that $F_{B\pi}(B, \pi) > 0$ guarantees that an increase in borrowing implies an increase in inflation. The remaining conditions are assumed for tractability, but they hold for the standard utility and production functions we consider. In what follows, it will be convenient to define the function

$$\tilde{\eta}(B, \pi, I) = -\frac{1}{I} \frac{B}{\pi} \frac{F_{B\pi}(B, \pi)}{F_{\pi\pi}(B, \pi)}. \tag{68}$$

It is clear from its definition and (61) that $\tilde{\eta}(B, \pi, I)$ corresponds to the elasticity of inflation to debt in equilibrium in a decentralized regime in that $\eta^D(B, I) = \tilde{\eta}(B, \pi^D(B, \ldots, B), I)$ by (67). The next result provides a characterization of how welfare in a decentralized regime varies with I.

Lemma 8. *When preferences for government spending are perfectly correlated across countries, under a decentralized optimal allocation, the following property holds*

$$\frac{\partial W^D(\theta, I)}{\partial I} = \beta(I-1)u'\left(F(B,\pi) + (1+r)k\right) F_B(B,\pi) \left[\frac{\eta^D(B,I)}{1-\eta^D(B,I)}\right] \frac{\partial B}{\partial I}. \tag{69}$$

Moreover, if government borrowing in equilibrium $B^*(\theta, I)$ increases with I and $\eta^D(B^*(\theta,I),I) < 1$ for all I, then ex-ante welfare $V^D(I)$ decreases with the number of countries in the monetary union.

To understand this result, note that since $F_B(B,\pi) < 0$, it follows from equation (69) that if $\eta^D(B^*(\theta,I),I) < 1$ for all I at the equilibrium level of debt $B^*(\theta,I)$ and $B^*(\theta,I)$ increases with I, then ex-post welfare $W^D(\theta,I)$ decreases with I. Thus, in a decentralized regime, ex-ante welfare decreases with I. Intuitively, the greater the number of countries in the monetary union, the less any given country internalizes that increasing its borrowing negatively affects the rest of the union by inducing an increase in inflation, and so the more each country desires to borrow. The monetary authority responds to the higher level of debt by raising inflation, but it does so less than one-for-one in percentage terms if $\eta^D(B^*(\theta,I),I) < 1$. Hence, as B increases, so does B/π, which implies that total tax revenues must also increase. Therefore, as the size of the union grows, so do the distortions from both inflation and taxes.

Lemmas 7 and 8 imply that as the number of countries in the monetary union increases, ex-ante welfare under centralization does not vary, whereas ex-ante welfare under decentralization decreases. Thus, welfare under centralization and welfare under decentralization must cross at most once as a function of I. Naturally, such a crossing point decreases with the quality of the central fiscal authority's information about countries' preferences for government spending, ϕ. Since a cutoff number of countries such that centralization is preferred for any larger monetary union exists and is equal to 1 when $\phi = 1$, if the informativeness of the signal is high enough, such a cutoff must be finite. Our next result, which is the analogue of Proposition 3 in the real economy, formalizes this argument.

Proposition 7. *(Cutoff Rule for Optimal Delegation for Correlated Preferences in a Monetary Economy) Assume that in a decentralized regime, government borrowing in equilibrium $B^*(\theta,I)$ increases with I and $\eta^D(B^*(\theta,I),I) < 1$ for all I. Then, for any given degree of informativeness ϕ of the preference signal, either there exists a finite cutoff $I^*(\phi)$ in the number of countries in the monetary union such that a centralized regime is preferred if $I > I^*(\phi)$ and a decentralized regime is preferred if $I \leq I^*(\phi)$ or a decentralized regime is preferred for all I. Moreover, there exists a degree of informativeness $\bar{\phi} \in [1/2, 1)$ of the preference signal such that the cutoff $I^*(\phi)$ is finite for any $\phi \geq \bar{\phi}$, with $I^*(\phi)$ decreasing with ϕ.*

3.3.3 Primitive Features of the Economy with Perfectly Correlated Preferences

Key features of the results derived so fare are that government borrowing increases as the number of countries in the monetary union increases, $\partial B^*(\theta, I)/\partial I > 0$, and that the elasticity of inflation to debt in a decentralized regime is smaller than one, $\eta^D(B^*(\theta, I), I) < 1$, in equilibrium. We now present sufficient conditions on the primitives of the economy for these properties to hold.

Assumption 3. *There exists* $\underline{A} > 0$ *such that* $A(M/\pi) \geq \underline{A}$ *for all* π *and* $\theta_L h'(\bar{x}/(1+r)) - \beta(1+r)u'(\underline{A}\, l(\bar{x}, \underline{A}) - v(l(\bar{x}, \underline{A}))) > 0$*, where* $\bar{x} \equiv \min\{(1+r)y_1, \bar{T}\}$ *and* $\bar{T}$ *is the maximal revenue from the Laffer curve under* $A(M/\pi) = \underline{A}$*.*

We can then then establish the following result.

Lemma 9. *Under Assumption 3,* $\eta^D(B^*(\theta, I), I) < 1$ *at the equilibrium level of government borrowing* $B^*(\theta, I)$ *in a decentralized regime.*

Trivilally, we could have simply assumed that $\tilde{\eta} < 1$, implying that $\eta^D(B^*(\theta, I), I) < 1$. The analytical case provides an example under which such a condition holds. Showing that Assumption 3 implies this same result is more involved and so we relegate this argument to Appendix A. Nonetheless, these latter conditions are easy to satisfy. For example, it is sufficient that θ_L is large enough and that aggregate productivity is bounded from below. To establish that welfare in the decentralized regime decreases with the number of countries I in the monetary union, we also need to show that borrowing increases with I. A sufficient condition for this property is the following.

Assumption 4. *Suppose that* $u(c) = c$*,* $u(c - v(\ell)) = c - v(\ell)$*, and* $\beta(1+r) > 1$*. Letting* $\gamma \equiv \theta h'(B/[(1+r)\pi])/(1+r) - \beta > 0$*,* $F(B, \pi)$ *satisfies*

$$\frac{\beta\pi}{B}\{I\tilde{\eta}(B,\pi,I)[F_B(B,\pi) + \pi F_{B\pi}(B,\pi)] + BF_{BB}(B,\pi)\} - \gamma\left[\frac{\partial\tilde{\eta}(B,\pi,I)}{\partial B} + \frac{I\pi\tilde{\eta}(B,\pi,I)}{B}\frac{\partial\tilde{\eta}(B,\pi,I)}{\partial\pi}\right] < 0.$$

In Section 3.4.2, we provide an example of a function $F(\cdot)$, derived from assumptions on preferences and technology, that satisfies Assumptions 2 to 4. We then have the following result.

Lemma 10. *Under Assumption 4, government borrowing in equilibrium* $B^*(\theta, I)$ *in a decentralized regime increases with the number* I *of countries in the monetary union.*

Lemmas 9 and 10 provide sufficient conditions on primitives under which Proposition 7 holds, which allows us to establish the existence of a cutoff rule for optimal delegation in the monetary economy without commitment on the part of the monetary authority.

3.4 Illustrative Cases

We illustrate the workings of the monetary model by first presenting a few numerical examples of it and then considering a case that can be analytically solved.

3.4.1 Numerical Examples

Suppose that $h(g) = \log(g)$, $v(\ell) = \chi\ell^{1+1/\zeta}/(1+1/\zeta)$, and $A(M/\pi) = a + d\,(M/\pi - 1) - e\,(M/\pi - 1)^2/2$, with parameters such that $\partial A(M/\pi)/\partial\pi \leq 0$ for $\pi \geq 1$—we do not model the disutility of labor as a log function to allow for a variable elasticity of labor supply in the comparative statics exercises that follow. We assume that the disutility of work is $\chi = 2.6$, the Frisch elasticity of labor supply is $\zeta = 1$, and the production function parameters are $a = 3$, $d = 0.15$, and $e = 3$. The discount factor is set to $\beta = 1$ and the interest rate to $r = 0.01$. Throughout, we maintain that the initial endowment is $y_1 = 10$ and the initial amount of money available to firms is $M = 1$.

Figure 3 plots the inflation rate chosen by the monetary authority as a function of government debt, which illustrates the source of the indirect fiscal externality discussed in both the centralized and the decentralized regimes. Recall that a central fiscal authority takes into account the policy function $\pi^C(B) = \pi(B, \ldots, B)$ when choosing B for each country. For B small enough, (net) inflation is set to zero, as the cost of even a small amount of inflation outweighs the cost of repaying a small amount of debt in terms of the required distortionary taxation. As borrowing increases, the cost of repaying the outstanding debt increases, and thus the monetary authority finds it optimal to increase inflation so as to decrease the amount of the debt to be repaid. Country i's fiscal authority, instead, chooses the level of debt B_i taking into account the monetary authority's policy function $\pi^D(B_i, B, I)$ for a given amount of symmetric debt B for the $I - 1$ remaining countries in the monetary union. In Figure 3, we show such a policy when the other $I - 1$ countries' debt is fixed at the common level $B = B^C$ in the centralized regime and exhibit cases in which the size of a monetary union is $I = 3$ and $I = 10$.

[FIGURE 3 HERE]

Key to the comparison of the centralized and decentralized regimes is the slope of the monetary authority's inflation policy as a function of the debt that fiscal authorities contemplate in each regime. In particular, when a local fiscal authority increases its own debt B_i, it anticipates only a small increase in the inflation rate, as apparent from the lower slope of the function $\pi^D(\cdot)$ compared with that of the function $\pi^C(\cdot)$. By (67), as the number of countries in the monetary union increases, the slope of the function $\pi^D(\cdot)$ decreases, because any country accounts for a smaller and smaller fraction of the members in the union. Hence, country i's borrowing B_i becomes progressively less important for the monetary authority's inflation decision. Due to this indirect fiscal externality, the value of a decentralized regime decreases with the number of countries in the monetary union. By contrast, no such externalities arises in a centralized regime, so its value is independent of the number of countries in the union.

Proposition 7 isolates three general cases of interest for this comparison of regimes. At one extreme, the indirect fiscal externality in the decentralized regime is sufficiently strong, relative to the informational disadvantage of a fiscal union, that a fiscal union is preferred regardless of the size of the monetary union, that is, $I^*(\phi) = 1$. At the other extreme, the fiscal externality is sufficiently small, relative to the informational disadvantage of a fiscal union, that a decentralized regime is preferred regardless of the size of the monetary union, this is, $I^*(\phi) = \infty$. We find most interesting the remaining case, in which it is optimal to pair a small monetary union with a decentralized fiscal regime but a large monetary union with a fiscal union, namely, the case when $I^*(\phi) > 1$ and $I^*(\phi) < \infty$.

[FIGURE 4 HERE]

Figure 4 shows these three scenarios. In the left panel, the informational problem is not too severe in that the high preference parameter θ_H is close enough to the low preference parameter θ_L that a centralized regime is always preferred to a decentralized one (for $I^*(\phi) \geq 2$). In the middle panel, the informational problem is sufficiently severe, that is, the difference between θ_H and θ_L is large enough, that a decentralized regime is always preferred to a centralized one. Finally, in the right panel, the information problem is of an intermediate degree of severity, which leads to a cutoff rule in the number of countries for optimal delegation: with fewer than 5 countries in the monetary union, ex-ante welfare is greater under decentralization, whereas with 5 or more countries, ex-ante welfare is greater under centralization.

[FIGURE 5 HERE]

We now turn to consider this intermediate case in greater detail. Proposition 7 states that as the informational content of signals about countries' preferences for government spending worsens, that is, as ϕ decreases, the cutoff level of countries for which a fiscal union is preferred increases. The intuition for this result is that as the quality of the information available to the central fiscal authority deteriorates, the value of a fiscal union declines, but since countries in a decentralized regime observe their preference type, the value of such a regime remains unchanged. Figure 5 shows this result. With a moderately informative signal ($\phi = 3/4$), a fiscal union is optimal for any monetary union with more than 4 countries, but with an uninformative signal ($\phi = 1/2$), it takes more than 8 countries for a fiscal union to be optimal.

[FIGURE 6 HERE]

Consider next how government spending and inflation in the two fiscal regimes vary with the informativeness of signals about countries' preferences for government spending and the number of countries in the monetary union. Figure 6 shows how these policies change in a fiscal union as the informativeness of signals improves—as ϕ increases from 1/2 to 1. When $\phi = 1/2$, policies are uniform across member states, as consistent with the premise of Oates (1999). The left panel of Figure 6 shows that as ϕ increases, government spending becomes increasingly better tailored to the preference signal and, as the right panel of Figure 6 illustrates, the resulting inflation rate becomes increasingly different after high and low signals.

[FIGURE 7 HERE]

Figure 7 depicts how government spending and inflation in a decentralized regime vary with the size of the monetary union, I. The left panel of Figure 7 shows that government spending increases with the number of countries with both high and low preferences for government spending. The right panel shows how such an increase in spending leads to higher inflation as the size of the monetary union increases. Of course, this force is the main reason why welfare falls in a decentralized fiscal regime as a monetary union grows in size: the free-riding problem of fiscal policy worsens. In particular, the induced inflation cost for each country of its government's fiscal policy decreases, so governments increase their spending and hence their debt levels lead to higher inflation, which hurts all countries.

In Appendix C, we consider the role for our results of the Frisch elasticity of labor supply and the parameter d of the production function governing the cost of inflation. Intuitively, a smaller labor supply elasticity implies that inflation is less desirable, as government debt can be repaid at a lower distortionary cost for the economy in terms of labor income taxation. Thus, fiscal unions are preferable for relatively larger monetary unions. On the contrary, a lower distortionary cost of inflation on productivity implies that the monetary authority has a greater incentive to choose a high level of inflation, which benefits countries ex post and so leads to fiscal unions to be preferable for relatively smaller monetary unions.

3.4.2 An Analytical Example

We now turn to an analytical example in which the elasticity of inflation with respect to borrowing is constant in both the centralized and the decentralized regimes. Such a framework allows us to obtain a full characterization of equilibrium, which makes transparent how the desirability of the two fiscal regimes depends on the number of countries in a monetary union.

So far, we have maintained that aggregate productivity A decreases with inflation. Here, for analytical convenience, we treat A as constant and follow Aguiar et al. (2015) by assuming that inflation entails a negative disutility cost of $\psi\pi$. We assume that utility is linear over consumption and government spending, namely, $u(x) = h(x) = x$. We let $(1 + r)\beta > 1$ so that consumers save their entire endowments in period 1 in that $c_1 = 0$ and $k = y_1 - B/[(1 + r)\pi]$. In Appendix B, we specify the disutility of labor supply $v(\ell)$

such that the function $F(B, \pi)$ defined in (53) takes the form

$$F\left(B,\pi\right)=\kappa_0-\frac{\kappa_1}{\alpha}\left(\frac{B}{\pi}\right)^{\alpha}-\psi\pi, \tag{70}$$

with $\kappa_0 > 0$, $\kappa_1 > 0$, $\alpha > 1$, and $\psi > 0$. We also maintain that $\theta_L/(1+r) - \beta > 0$.[10] All conditions in Assumptions 2 to 4 are then satisfied—see Appendix B for all omitted details.

Recall that the monetary authority solves the problem $W_{MA}(B, k) = \max_{\pi \geq 1} \frac{1}{I} \sum_i \left[F(B, \pi) + (1 + r)k\right]$, whose first-order condition implies that the inflation rule for an arbitrary vector of debt $(B_1, \ldots, B_I)$ is

$$\pi(B_1,\ldots,B_I)=\left(\frac{\kappa_1}{\psi}\right)^{\frac{1}{1+\alpha}}\left[\frac{\sum_i B_i^{\alpha}}{I}\right]^{\frac{1}{1+\alpha}}. \tag{71}$$

Then, the inflation rule in the centralized and decentralized regimes are

$$\pi^C(B)=\left(\frac{\kappa_1}{\psi}\right)^{\frac{1}{1+\alpha}}B^{\frac{\alpha}{1+\alpha}} \text{ and } \pi^D(B_i,B,I)=\left(\frac{\kappa_1}{\psi}\right)^{\frac{1}{1+\alpha}}\left[\frac{B_i^{\alpha}+(I-1)B^{\alpha}}{I}\right]^{\frac{1}{1+\alpha}}, \tag{72}$$

respectively, which imply the following elasticities of inflation with respect to debt in the two regimes

$$\eta^C\equiv\frac{B}{\pi^C(B)}\frac{\partial\pi^C(B)}{\partial B}=\frac{\alpha}{\alpha+1} \text{ and } \eta^D(I)\equiv\frac{B_i}{\pi^D(B_i,B,I)}\frac{\partial\pi^D(B_i,B,I)}{\partial B_i}\Big|_{B_i=B}=\frac{1}{I}\frac{\alpha}{\alpha+1}=\frac{\eta^C}{I}. \tag{73}$$

Note that the elasticity under centralization is constant in the number of countries in the monetary union, whereas the elasticity under decentralization decreases with it. To understand why, note that when the decentralized fiscal authority of country i chooses its level of borrowing B_i in period 1, it takes into account that its borrowing will increase inflation in period 2. Whereas in the decentralized regime, then, a fiscal authority only considers the effect on π of its increase in B_i, in the centralized regime, the fiscal union takes into account the total effect on π of an increase in borrowing by all countries.

The best-response problem of a decentralized fiscal authority when all other countries choose B is

$$W^D(\theta_i,B,I)=\max_{B_i}\left\{\frac{\theta_i B_i}{(1+r)\,\pi}+\beta\left[F(B_i,\pi)+(1+r)y_1-\frac{B_i}{\pi}\right]\right\},$$

with $\pi = \pi^D(B_i, B, I)$. Given the optimal level of borrowing $B^D(\theta_i, I) = \kappa_1 \left\{\gamma_i[1 - \eta^D(I)]/(\beta\kappa_1)\right\}^{\frac{1+\alpha}{\alpha-1}} /\psi$,

[10] This condition ensures that governments issue bonds. Throughout, we assume that the Lagrange multiplier associated with $\pi \geq 1$ is zero, a sufficient condition for which is that ψ is sufficiently low.

with $\gamma_i \equiv \theta_i/(1+r) - \beta$, ex-post welfare in the decentralized regime at the optimal allocation is[11]

$$W^D(\theta_i, I) = \gamma_i^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\beta\kappa_1}\right)^{\frac{1}{\alpha-1}} \left[(1-\eta^C/I)^{\frac{1}{\alpha-1}} - \left(\frac{\alpha+1}{\alpha}\right)(1-\eta^C/I)^{\frac{\alpha}{\alpha-1}}\right] + \chi, \tag{74}$$

with $\chi \equiv \beta[\kappa_0 + (1+r)y_0]$ and strictly decreases with I. Since welfare under centralization does not depend on I, welfare under centralization and welfare under decentralization as a function of I must cross at most once and the crossing point naturally decreases with the quality of the information. The next Corollary formalizes this discussion.

Corollary 1. *In the parameterized monetary economy without commitment by the monetary authority and perfect correlated preferences for government spending across countries, there exists a cutoff number of countries $I^*(\phi)$, which decreases with the degree of informativeness ϕ of the preference signal and is potentially infinite, such that a centralized regime is preferred if and only if $I > I^*(\phi)$.*

3.5 Independent Preferences across Countries

Consider now the case in which preferences for government spending are independent across countries. Although the same three cases as before arise in terms of the optimality of a fiscal union as the number of countries in the monetary union increases, we focus attention on the most interesting case, in which a cutoff rule in the number of countries is optimal. The main difference with respect to the case of perfectly correlated preferences just analyzed is that in period 2, depending on the fiscal regime, equilibrium allocations depend on the number of countries with either high preference types or high signals.

Centralized Regime. At the beginning of period 1, the central fiscal authority receives a high signal s_H about the preferences for government spending of n countries in the monetary union and a low signal s_L about the preferences of the remaining $I-n$ countries, with $n \in \{1, \ldots, I\}$. Clearly, it is optimal for the fiscal authority to choose the same allocation for the countries that receive the same signal so that we can record the relevant history as $(B_H(n, I), k_H(n, I))$ for the n countries for which the central fiscal authority receives a high signal and by $(B_L(n, I), k_L(n, I))$ for the $I-n$ countries for which the central

[11] Unlike in the centralized regime, borrowing under a decentralized regime increases with the number of countries in the union. The reason is that as I increases, the inflation spillover becomes worse in the sense that a decentralized fiscal authority only takes into account a fraction $1/I$ of the total effect of increasing B on inflation.

fiscal authority receives a low signal. Hence, the central fiscal authority's problem in period 1 is

$$W^C(n,I) = \max_{B_H,k_H,B_L,k_L} \frac{1}{I}\Bigg\{ n\left[u(c_{H1}) + E(\theta|s_H)h\left(\frac{B_H}{(1+r)\,\pi}\right) + \beta u\left(c_{H2} - v\left(\ell\left(\frac{B_H}{\pi}, A\left(\frac{M}{\pi}\right)\right)\right)\right)\right]$$
$$+ (I-n)\left[u(c_{L1}) + E(\theta|s_L)h\left(\frac{B_L}{(1+r)\,\pi}\right) + \beta u\left(c_{L2} - v\left(\ell\left(\frac{B_L}{\pi}, A\left(\frac{M}{\pi}\right)\right)\right)\right)\right]\Bigg\}, \quad (75)$$

subject to the first-order condition for savings in each country in (38), with $c_{i1} = y_1 - k_i - B_i/[(1+r)\pi]$, $c_{i2} = A(M/\pi)\ell(B_i/\pi, A(M/\pi)) + (1+r)k_i$, and $\pi = \pi(B_H(n,I), k_H(n,I), B_L(n,I), k_L(n,I))$. In general, differently from the case of perfectly correlated preferences across countries, the monetary authority's inflation choice depends on both the level of debt and capital in each type of country. To see why, note that the first-order condition for the monetary authority's problem is

$$nu'(c_{H2} - v(\ell_H))F_{H\pi} + (I-n)u'(c_{L2} - v(\ell_L))F_{L\pi} = 0, \quad (76)$$

where $F_{H\pi} = F_\pi(B_H(n,I), k_H(n,I))$ and $F_{L\pi} = F_\pi(B_L(n,I), k_L(n,I))$. Hence, when $k_H(n,I)$ does not equal $k_L(n,I)$, the monetary authority' choice of inflation depends on countries' savings decisions, so the savings constraint cannot be omitted. Ex-ante welfare in a fiscal union is given by

$$V^C(I) = \frac{1}{I}\sum\nolimits_{n=0}^{I} \binom{I}{n} p_H^n (1-p_H)^{I-n} W^C(n,I), \quad (77)$$

which accounts for all possible realizations of high and low preference signals for the I countries.

Decentralized Regime. When stating the problem of a local fiscal authority, we need to consider almost symmetric histories that differ by both the total number of countries characterized by a high preference type and the realized type of the country considered. Formally, consider a country that draws θ_H at the beginning of period 1 and chooses (B_H, k_H). The history that it faces in period 2 is that of the other $n-1$ countries that drew θ_H and chose $(B_H(n,I;\theta), k_H(n,I;\theta))$ and of the $I-n$ countries that drew θ_L and chose $(B_L(n,I;\theta), k_L(n,I;\theta))$. Conversely, if a country draws θ_L and chooses (B_L, k_L) in period 1, then the history that it faces in period 2 is that of the other $I-n-1$ countries that drew θ_L and chose $(B_L(n,I;\theta), k_L(n,I;\theta))$ and the n countries that drew θ_H and chose $(B_H(n,I;\theta), k_H(n,I;\theta))$.

Each country's fiscal authority takes into account that the monetary authority will face an almost symmetric history, if the country's levels of debt and savings differ from those chosen by any other country. In particular, the fiscal authority of a country i whose preference parameter is θ_H considers the first-order condition for this almost-symmetric monetary authority's problem,

$$u'(c_{i2} - v(\ell_i))F_{i\pi} + (n-1)u'(c_{H2} - v(\ell_H))F_{H\pi} + (I-n)u'(c_{L2} - v(\ell_L))F_{L\pi} = 0, \quad (78)$$

which implies a policy for the monetary union that we denote by $\pi_H(B_i, k_i, B_H(\cdot), k_H(\cdot), B_L(\cdot), k_L(\cdot))$, where $B_H(\cdot) = B_H(n, I; \theta)$, $k_H(\cdot) = k_H(n, I; \theta)$, $B_L(\cdot) = B_L(n, I; \theta)$, and $k_L(\cdot) = k_L(n, I; \theta))$. Similarly, a country i whose preference parameter is θ_L takes into account the first-order condition

$$u'(c_{i2} - v(\ell_i))F_{i\pi} + nu'(c_{H2} - v(\ell_H))F_{H\pi} + (I - n - 1)u'(c_{L2} - v(\ell_L))F_{L\pi} = 0, \tag{79}$$

which implies a policy for the monetary authority that we denote by $\pi_L(B_i, k_i, B_H(\cdot), k_H(\cdot), B_L(\cdot), k_L(\cdot))$, where $B_H(\cdot) = B_H(n, I; \theta)$, $k_H(\cdot) = k_H(n, I; \theta)$, $B_L(\cdot) = B_L(n, I; \theta)$, and $k_L(\cdot) = k_L(n, I; \theta))$.

Given the monetary authority's inflation policy and all other countries' choices, the best response of the local fiscal authority of a country of type θ_H solves the problem with value $W^D(n, I, B_{-i}, k_{-i}; \theta_H)$,

$$W^D(\cdot; \theta_H) = \max_{B_i, k_i} \left[u\left(y_1 - k_i - \frac{B_i}{(1+r)\pi_H}\right) + \theta_H h\left(\frac{B_i}{(1+r)\,\pi_H}\right) + \beta u\left(F(B_i, \pi_H) + (1+r)k_i\right)\right],$$

after suppressing the dependence of π_H on the fiscal authority's choices, subject to the first-order condition for savings. The best response of the local fiscal authority of a country of type θ_L solves an analogous problem. An equilibrium is a (vector) fixed point of these best-response functions, which implies the ex-post welfare $W^D(n, I; \theta_H)$ and $W^D(n, I; \theta_L)$ for a country of type θ_H and θ_L, respectively, for n realizations of θ_H. Ex-ante welfare, which simply averages ex-post welfare over all possible number of realizations of a high preference type θ_H among the I countries in the monetary union, is

$$V^D(I) = \frac{1}{I} \sum\nolimits_{n=0}^{I} \binom{I}{n} q^n (1-q)^{I-n} \left[qW^D(n, I; \theta_H) + (1-q)W^D(n, I; \theta_L)\right].$$

In Figure 8, we graph ex-ante welfare. In contrast to the case of perfectly correlated preferences, ex-ante welfare in a fiscal union increases with the number of countries in the monetary union, I. Intuitively, as the number of countries increases, the number of "mixed" histories of high and low signals also increases, which lead to higher welfare than histories of only high or only low signals for any degree of informativeness of the preference signal.[12] As in the perfectly correlated case, the value of decentralization instead declines with the number of countries. Hence, a cutoff rule in the number of countries in the union for the optimal fiscal regime arises, with a central regime preferable only if the number of countries is sufficiently large. In general, the more informative the signal, the smaller the minimal number of countries for a fiscal union to be preferred. In particular, with a very informative signal of $\phi = 0.9$, a fiscal union is preferable for any monetary union with 2 or more member countries, whereas with an uninformative signal of $\phi = 0.5$, a fiscal union is preferable for a monetary union with at least 7 member countries.

[12]Intuitively, welfare under a mixed signal such as (s_H, s_L) is larger than a convex combination of welfare under s_H alone and welfare under s_L alone because [ADD].

[FIGURE 8 HERE]

In Appendix C, we examine the role for our results of the Frisch elasticity of labor supply and the parameter d of the production function for the cost of inflation. As before, a smaller labor supply elasticity decreases the distortions from labor income taxation, which lowers the inflationary impact of government debt and so leads to fiscal unions to be preferable for relatively larger monetary unions. By contrast, a lower distortionary cost of inflation on productivity implies that the monetary authority is more willing to raise inflation ex post, which leads to fiscal unions to be preferable for relatively smaller monetary unions.

4 Conclusion

Which fiscal regime is appropriate for a monetary union? This question is at the heart of salient policy debates, for instance, the current ones concerning the desirability of greater fiscal integration among EU countries and the benefits of an enlargement of the EU. We have proposed a simple framework to illustrate how ideas from the fiscal federalism literature about the design of an optimal regime for fiscal policy can be usefully applied to examine the optimal degree of fiscal coordination within a monetary union. A robust finding of our analysis is that small monetary unions should be paired with *decentralized* fiscal regimes, whereas large monetary unions should be paired with *centralized* fiscal ones. In particular, as a monetary union grows in size, a centralized fiscal regime is likely to be preferable as better suited at internalizing the impact of countries' government spending on the union-wide inflation rate.

Underlying our exposition are two key features that emerge as the size of a monetary union increases. First, both direct and indirect fiscal externalities become more pronounced under the decentralization of fiscal authority. Second, the informational disadvantage of a central fiscal authority does not worsen too rapidly. We have purposely constructed such a minimal model so as to make the countervailing forces at play most transparent and this way highlight the different premises of the literature on fiscal federalism, which emphasizes the benefits of decentralization, and the literature on monetary unions, which emphasizes the benefits of centralization.

Throughout, we have focused on one type of fiscal policy—a single country-wide level of public spending in a union—with only one dimension of heterogeneity across countries—the desirability of such spending. A fruitful avenue of future research would be to extend some of the ideas explored here to richer policy environments with heterogeneity both within and across regions of a monetary union, which could inform practical decisions on the appropriate degree of fiscal decentralization in existing monetary unions.

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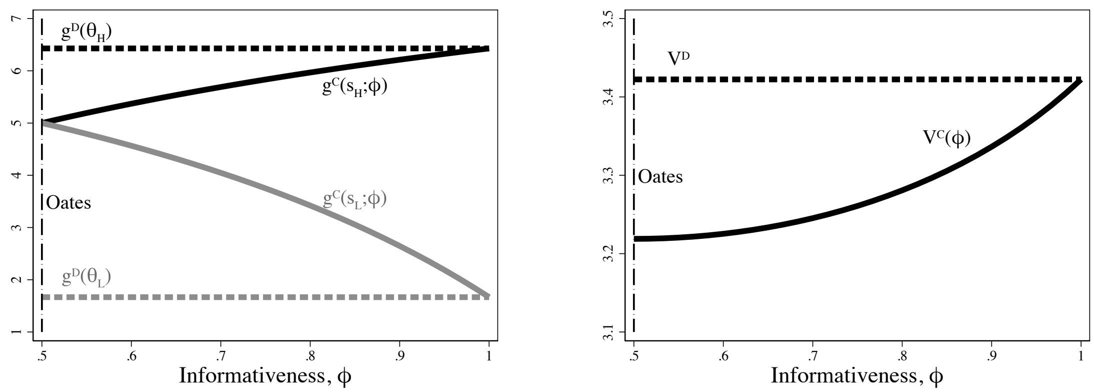
Figures

Figure 1: A Generalized Decentralization Theorem with Perfectly Correlated Preferences

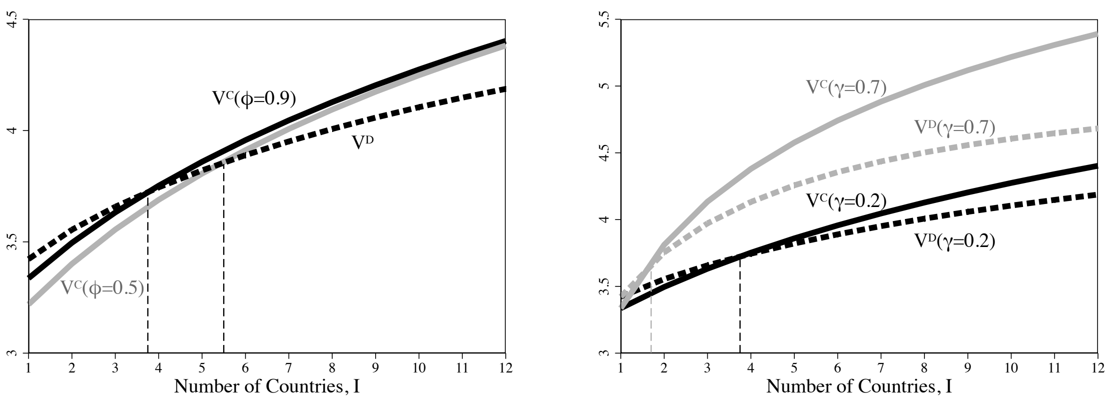
(a) Government Spending (b) Ex-Ante Welfare



Note: The preference parameters are $\theta_H = 1.8$ and $\theta_L = 0.2$, the probability of drawing a high preference is $q = 0.5$, and the endowment is $y = 10$. The functional forms for consumer utility are $u(c) = \log(c)$ and $h(g) = \log(g)$. In the left panel, g_H^D and g_L^D (respectively, $g_H^C(\phi)$ and $g_L^C(\phi)$) denote optimal government spending as a function of the informativeness ϕ of the preference signal in the decentralized (respectively, centralized) case. In the right panel, V^D and $V^C(\phi)$ denote ex-ante welfare in the decentralized and centralized case, respectively.

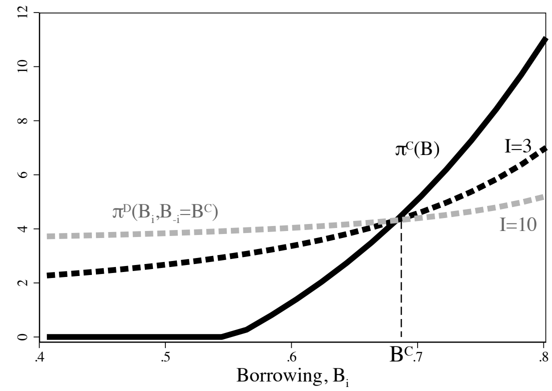
Figure 2: Welfare Comparisons with Perfectly Correlated Preferences

(a) Ex-Ante Welfare for Varying Informativeness (b) Ex-Ante Welfare for Varying Externality



Note: The preference parameters are $\theta_H = 1.8$ and $\theta_L = 0.2$, the probability of drawing a high preference is $q = 0.5$, and the endowment is $y = 10$. The functional forms for consumer utility are $u(c) = \log(c)$ and $h(g) = \log(g)$. In the left panel, the externality parameter γ is equal to 0.2, and the value of ϕ is displayed in the graph. In the right panel, the informativeness of the preference signal ϕ is equal to 0.9 and the value of γ is displayed in the graph.

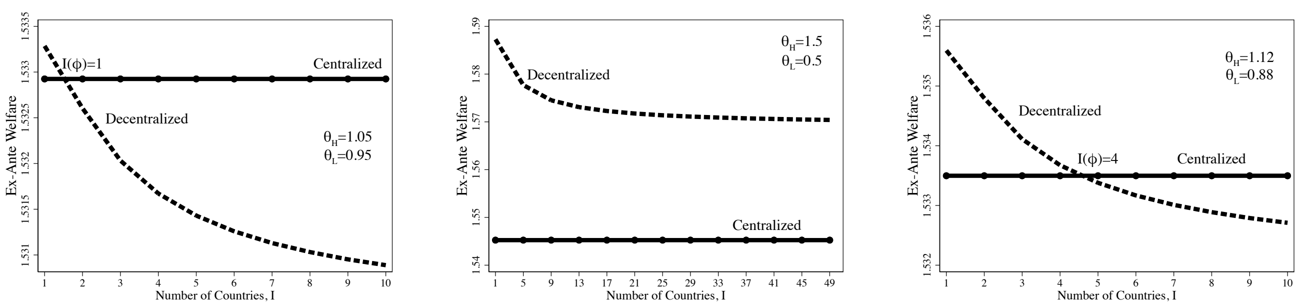
Figure 3: Best Response of Monetary Authority with Perfectly Correlated Preferences



Note: $\pi^C(B) = \pi(B,\ldots,B)$ is the monetary authority's best response when the fiscal union raises the debt of all I countries by an equal amount and $\pi^D(B_i, B^C) = \pi(B^C,\ldots,B_i,\ldots,B^C)$ is the best response of the monetary authority when country i alone raises its debt and the other $I-1$ countries hold their debt fixed at B^C. The preference parameters are $\theta_H = 1.12$ and $\theta_L = 0.88$ with probability $q = 0.5$ of drawing a high preference. The informativeness of the signal is $\phi = 0.75$.

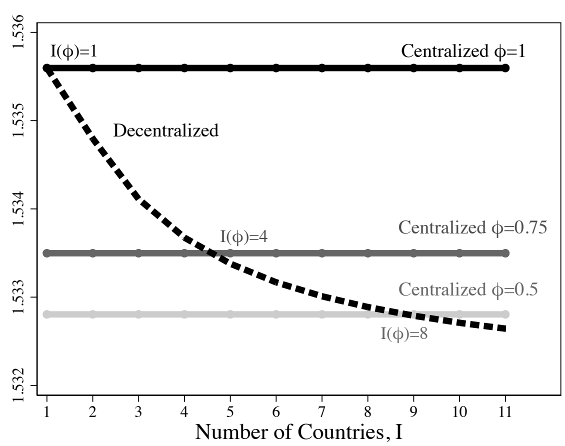
Figure 4: Comparison of Regimes with Perfectly Correlated Preferences

(a) Centralization Dominates (b) Decentralization Dominates (c) Cutoff Rule



Note: The preference parameters are displayed in the graph and the probability of drawing a high preference is $q = 0.5$. The informativeness of the signal is $\phi = 0.75$.

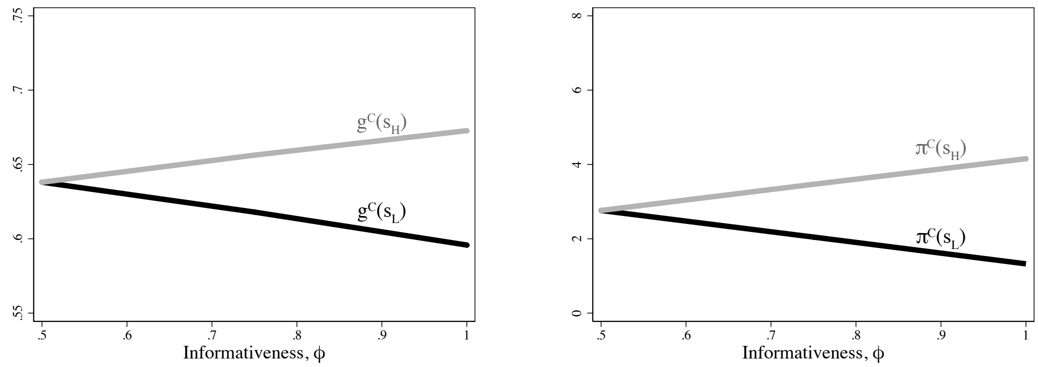
Figure 5: The Role of Information with Perfectly Correlated Preferences



Note: The preference parameters are θ_H=1.12 and θ_L=0.88 with probability $q = 0.5$ of drawing a high preference.

Figure 6: Information and Centralized Policy Functions with Perfectly Correlated Preferences

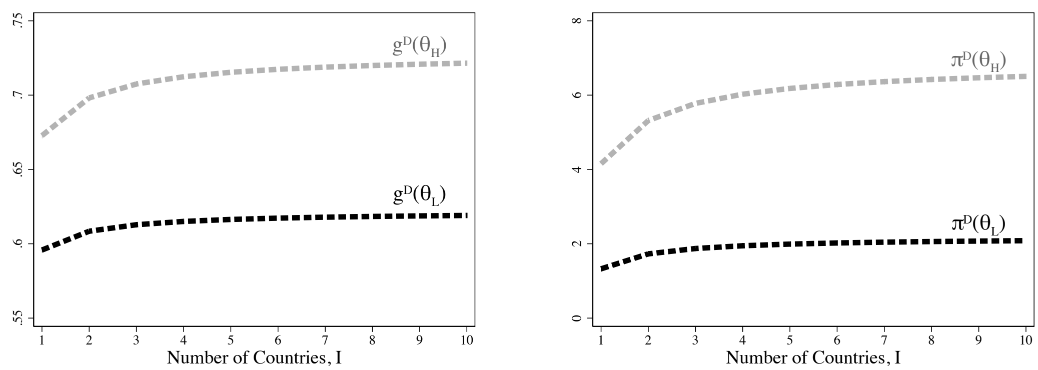
(a) Spending under Centralization (b) Inflation under Centralization



Note: The preference parameters are $\theta_H = 1.12$ and $\theta_L = 0.88$ with probability $q = 0.5$ of drawing a high preference.

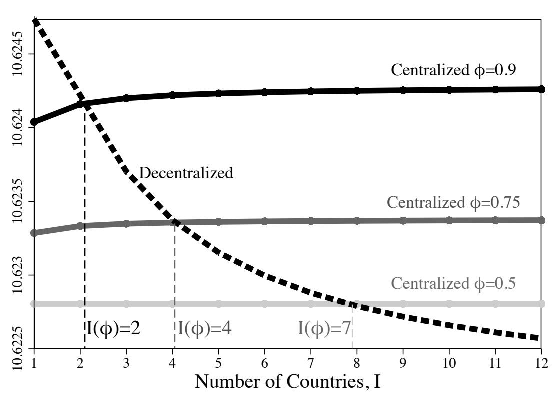
Figure 7: Number of Countries and Decentralized Policy Functions with Perfectly Correlated Preferences

(a) Spending under Decentralization (b) Inflation under Decentralization



Note: The preference parameters are $\theta_H = 1.12$ and $\theta_L = 0.88$ with probability $q = 0.5$ of drawing a high preference.

Figure 8: Comparison of Regimes with Independent Preferences



Note: The preference parameters are $\theta_H = 1.1$ and $\theta_L = 0.9$ with probability $q = 0.5$ of drawing a high preference.

A Omitted Proofs

Throughout the proofs, we write interchangeably $V^C(I,\phi)$ and $V^C(I)$ whenever we want to highlight the dependency with respect to ϕ.

Proof of Proposition 1: To prove this result we use Blackwell's theorem on the information structures (Blackwell (1951)) twice: first to show that the welfare in the centralized regime is strictly lower than that in the decentralized regime and second to show that the welfare in the centralized regime worsens as the signal in that regime becomes less informative. To this end, we set up some notation in order to map our economy into that considered in the theorem. Consider two information structures σ_1 and σ_2 represented by maps $\sigma_k : \Theta \to \Delta(S)$ for $k = 1, 2$ where $\theta_i \in \Theta = \{\theta_H, \theta_L\}$ are the set of states, $s \in S = \{s_H, s_L\}$ are the set of signals, and $\sigma_k(s|\theta)$ is the conditional probability of observed signal s given state θ under information structure k. Recall that σ_2 is a *garbling* of σ_1 if an agent who knows σ_1 could replicate σ_2 by randomly drawing a signal $s' \in S$ after each observation $s \in S$, that is, there exists a *garbling function* $\gamma : S \to \Delta(S)$ such that

$$\sigma_2(s'|\theta) = \sum\nolimits_{s\in S} \gamma(s'|s)\sigma_1(s|\theta), \tag{80}$$

where $\gamma(s'|s) < 1$ for either $\gamma(s_H|s_H)$ or $\gamma(s_L|s_L)$. That is, to (strictly) garble the signal the garbling function must sometimes report that the signal is high when it is actually low or report that the signal is low when it is actually high. Blackwell's theorem states that if σ_2 is a garbling of σ_1 then any Bayesian decision maker prefers σ_1 to σ_2.

To apply this result in our context, let α_k denote the (symmetric) signal structure associated with a signal with informativeness parameter $\phi_k \in [1/2, 1]$ where

$$\phi_k = \sigma_k(s_H|\theta_H) = \sigma_k(s_L|\theta_L) \text{ and } 1 - \phi_k = \sigma_k(s_L|\theta_H) = \sigma_k(s_H|\theta_L)$$

and denote ex-ante welfare of an agent in the centralized regime under information structure σ_k as

$$V_k^C = p_{Hk} W_k^C(s_H) + (1 - p_{Hk}) W_k^C(s_L), \tag{81}$$

where $W_k^C(\bar{s}) = \sum_{i=1}^{I} W_k^C(s_i)$ and for each i,

$$W_k^C(s) = \max\nolimits_{c_i,g_i} Q_k(q,s_H)) \left[u(c_i(s_H) + \theta_H h(g_i(s_H))\right] + Q_k(q,s_L)) \left[u(c_i(s_L) + \theta_L h(g_i(s_L))\right],$$

subject to $c_i + g_i = y$ where $Q_k(q, s_H) = q\phi_k/p_{Hk}, Q_k(q, s_L) = q(1 - \phi_k)/(1 - p_{Hk})$, and $p_{Hk} = q\phi_k + (1 - q)(1 - \phi_k)$. To show that welfare in the fiscal union decreases when the informativeness of the signal falls from ϕ_1 to ϕ_2 where $\frac{1}{2} \leq \phi_2 < \phi_1$, we need only show that the associated information structures satisfy the garbling condition (80). To do so define the symmetric garbling function γ that takes original signals s_H and s_L in S into the garbled signals $\tilde{s}_H$ and $\tilde{s}_L$ in S via

$$\gamma = P(\tilde{s}_H|s_H) = P(\tilde{s}_L|s_L) \text{ and } 1-\gamma = P(\tilde{s}_L|s_H) = P(\tilde{s}_H|s_L).$$

Then we can write the less informative signal as a garbled version of the original signal by

$$\phi_2 = P(s_H|\theta_H)P(\tilde{s}_H|s_H) + P(s_L|\theta_H)P(\tilde{s}_H|s_L) = \phi_1\gamma + (1-\phi_1)(1-\gamma).$$

Solving for γ gives that it is a strict garbling in that

$$\gamma = \frac{\phi_1+\phi_2-1}{2\phi_1-1} < 1 \text{ if } \frac{1}{2} < \phi_2 < \phi_1. \tag{82}$$

So by Blackwell's theorem, the ex-ante welfare in the centralized regime satisfies $V_2^C < V_1^C$.

To show that welfare in the decentralized regime is strictly greater than that in the fiscal union, note that the value of the centralized regime equals that of the fiscal union when the fiscal union has a perfectly informative signal. Next, to show that the difference in welfare in the two regimes increases as the informativeness of the signal decreases note first that the welfare in the decentralized regime is independent of the informativeness of the signal. Second, since the value in the fiscal union strictly decreases when the informativeness of the signal fall, this result follows. □

Proof of Lemma 1: We start by considering the ex-post welfare under a centralized regime for a given realization s_i of the signal, as in (12). The first-order condition of this problem is

$$u'(y-g) = \tilde{\theta}_i(1+\gamma(I-1))h'(g+\gamma(I-1)g). \tag{83}$$

Then, taking the derivative of the ex-post welfare (12) with respect to the number of countries in the union,

$$\frac{\partial W^C(s_i,I)}{\partial I} = -\frac{\partial g}{\partial I}\Big[u'(y-g) + \tilde{\theta}_i(1+\gamma(I-1))h'(g+\gamma(I-1)g)\Big] + \tilde{\theta}_i\gamma g h'(g+\gamma(I-1)g),$$

and using the first-order condition (83), we get that

$$\frac{\partial W^C(s_i,I)}{\partial I} = \tilde{\theta}_i\gamma g h'(g+\gamma(I-1)g) > 0. \tag{84}$$

This shows that W^C is strictly increasing in I. Moreover, since W^C is evaluated at the optimal level of spending, for any arbitrary level of spending $\bar{g} \in (0, y)$ we must have that

$$W^C(s_i, I) \geq u(y - \bar{g}) + \tilde{\theta}_i h(\bar{g} + \gamma(I-1)\bar{g}),$$

and so taking expectations on both sides over signals, the following also holds

$$V^C(I) \geq u(y - \bar{g}) + \mu_\theta h(\bar{g} + \gamma(I-1)\bar{g}), \tag{85}$$

where $V^C(I) = p_H W^C(s_H, I) + p_L W^C(s_L, I)$. Then, as I tends to infinity the right-hand side of (85) goes to infinity, so it must be that $V^C(I)$ also tends to infinity. Also, from (84) we know that $W^C(s_i, I)$ is strictly increasing in I for a given signal. Hence, $V^C(I)$, which is the expectation over $W^C(s_i, I)$ is also strictly increasing in I. □

Proof of Lemma 2: To show that V^D is strictly increasing in the number of countries in the union, we start by showing that the optimal amount of public goods chosen by the decentralized authority is strictly decreasing in the number of countries and that it tends to zero as the number of countries tends to infinity. Then, using these properties of the optimal allocation, we can prove that the ex-post welfare is strictly increasing in I, and thus so is the ex-ante welfare, V^D. Finally, taking limits as I grows to infinity we can derive that V^D is bounded above by a constant. Consider the problem of the decentralized fiscal authority,

$$W^D(\theta_i, I) = \max_{g_i} u(y - g_i) + \theta_i h(g_i + \gamma \sum_{j \neq i} g_j),$$

and taking first-order conditions and then imposing symmetry, we get

$$u'(y - g) = \theta_i h'(g + \gamma(I-1)g). \tag{86}$$

Let $g^D(\theta_i, I)$ be the solution to this problem. Then, it must be that $g^D(\theta_i, I)$ is strictly decreasing in I. To see why, suppose by way of contradiction that it is increasing. Then, as we increase I, the right-hand side of (86) must decrease locally because we are assuming $h'' < 0$ in Assumption 1, which would imply that the left-hand side of that equation must also decrease. But that is a contradiction because $u'' < 0$. Moreover, as I tends to infinity, we must have that $\lim_{I \to \infty} g^D(\theta_i, I) = 0$, otherwise (86) cannot be satisfied.

Substituting the solution $g^D(\theta_i, I)$ from the first-order condition into the ex-post welfare value we get,

$$W^D(\theta_i, I) = u(y - g^D(\theta_i, I)) + \theta_i h\left[h'^{-1}\left(\frac{1}{\theta_i} u'(y - g^D(\theta_i, I))\right)\right].$$

Since g^D is strictly decreasing in I, using the inverse function rule by which $(h'^{-1})'(x) = \frac{1}{h''(h'^{-1}(x))} < 0$, and using also the properties of u and h functions stated in Assumption 1, then we have that $W^D(\theta_i, I)$ is strictly increasing in I.[13] Finally, using $\lim_{I\to\infty} g^D(\theta_i, I) = 0$, and taking the limit as I tends to infinity of the ex-post welfare under decentralization, we have that

$$\lim_{I\to\infty} W^D(\theta_i, I) = u(y) + \theta_i h\left[h'^{-1}\left(\frac{1}{\theta_i}u'(y)\right)\right] < \infty.$$

Therefore we obtain that the ex-post value in the decentralized regime is bounded above by a constant. Then, taking expectations over θ to get the ex-ante welfare we have that $\partial V^D(I)/\partial I > 0$ because $W^D(\theta_i, I)$ is increasing in I for each θ_i, and

$$\lim_{I\to\infty} V^D(I) = u(y) + \mathbb{E}\left\{\theta_i\left[h'^{-1}\left(\frac{1}{\theta_i}u'(y)\right)\right]\right\} \equiv \bar{V}^D < \infty, \tag{87}$$

as stated in the lemma. Clearly, the constant $\bar{V}^D$ is independent of both γ and ϕ. □

Proof of Proposition 2: First, to show the existence of $\bar{I}^*$ and $\underline{I}^*$, we argue that the centralized and decentralized ex-ante welfare functions $V^C(I,\phi)$ and $V^D(I)$, must cross at least once. Note first that at $I = 1$ the value under decentralization is strictly higher than under decentralization because there is better information in the decentralized equilibrium and no externality. That the value under centralization is eventually higher than that under decentralization follows from Lemmas 1 and 2 since they imply that both V^C and V^D are increasing in I (and continuous) and that as the number of countries in the union tends to infinity, the ex-ante value of the centralized regime also tends to infinity, but the ex-ante value under decentralization is bounded above by a constant $\bar{V}^D$ that is less than infinity. Hence, $V^C(I,\phi)$ and $V^D(I)$ must cross at least once. Thus, we can define the cutoffs above and below which each type of regime is preferred as follows[14]

$$\bar{I}^*(\phi,\gamma) = \sup\{I \in [1,\infty) |\, V^C(I;\phi,\gamma) \leq V^D(I;\gamma)\}$$
$$\underline{I}^*(\phi,\gamma) = \inf\{I \in [1,\infty) |\, V^C(I;\phi,\gamma) \geq V^D(I;\gamma)\}.$$

Note that $V^D(I)$ does not vary with ϕ but Blackwell's Theorem implies that V^C increases with ϕ and also increases with I by Lemma 1. Thus, the cutoffs decrease with ϕ. This proves part a) and b) of the proposition. Note that $\bar{I}^*(\phi,\gamma) \geq \underline{I}^*(\phi,\gamma)$, so that our characterization leaves open which regime is preferred in between these values. However, as we show next if utility has the log form then $\bar{I}^*(\phi,\gamma) =$

[13]To see the derivation of the inverse function rule let $y = f'(x)^{-1}$ so that $x = f(y)$. Now differentiating both sides of the latter formula with respect to x and applying the chain rule we have $1 = f'(y)\frac{dy}{dx}$, and so rearranging $\frac{dy}{dx} = \frac{1}{f'(y)}$. Then, using that $y = f'(x)^{-1}$ and substituting in the previous expression, we get $\frac{d(f'(x)^{-1})}{dx} = \frac{1}{f'(f'(x)^{-1})}$ which is the formula of the inverse function rule used in the proof.

[14]Note that here we treat I as a real variable rather than an integer one, so cutoffs are defined as the smallest and largest indifference points. In practice, any such cutoff can be rounded up to the closest integer.

$\underline{I}^*(\phi, \gamma)$.

Next, under the assumption that utility has the log form, we show that there is a unique cutoff $I^*(\phi, \gamma)$ such that if $I > I^*(\phi, \gamma)$ centralization is preferred and if $I \leq I^*(\phi, \gamma)$ then decentralization is preferred. Given the results a) and b) from the proposition that we just proved, now to prove the existence of a unique cutoff $I^\star(\phi, \gamma)$ we just need to show that V^C increases at a faster rate with the number of countries in the union than V^D, so that the centralized ex-ante welfare crosses the decentralized payoff only once. To do so, using the assumed utility functions, we derive the derivatives of V^C and V^D with respect to the number of countries in the union, I, and compare them.

Consider first the problem of the fiscal union. Substituting the equilibrium allocations (15) and (16), which are

$$c^C(s_i) = \frac{1}{1+\tilde{\theta}_i} y \quad \text{and} \quad g^C(s_i) = \frac{\tilde{\theta}_i}{1+\tilde{\theta}_i} y,$$

into the ex-ante value for the union (13) we have that

$$\begin{aligned} V^C(I) &= \mathbb{E}\left[\log\left(\frac{1}{1+\tilde{\theta}_i} y\right) + \tilde{\theta}_i \log\left([1+\gamma(I-1)]\frac{\tilde{\theta}_i}{1+\tilde{\theta}_i} y\right)\right] \\ &= \mathbb{E}\left[\tilde{\theta}_i \log\left([1+\gamma(I-1)]\right)\right] + \mathbb{E}\left[\log\left(\frac{1}{1+\tilde{\theta}_i} y\right) + \tilde{\theta}_i \log \frac{\tilde{\theta}_i}{1+\tilde{\theta}_i} y\right] \\ &= \mu_\theta \log\left(1+\gamma(I-1)\right) + \mathbb{E}(\lambda_i), \end{aligned} \tag{88}$$

where $\lambda_i \equiv \log\left(\frac{1}{1+\tilde{\theta}_i} y\right) + \tilde{\theta}_i \log\left(\frac{\tilde{\theta}_i}{1+\tilde{\theta}_i} y\right)$, and the expectations are taken with respect to the realization of the signals s_i. We also used in the previous derivation that $\mu_\theta \equiv q\theta_H + (1-q)\theta_L = p_H\tilde{\theta}_H + p_L\tilde{\theta}_L$ Then, taking the derivative with respect to I we get

$$\frac{\partial V^C(I)}{\partial I} = \frac{\mu_\theta \gamma}{1+\gamma(I-1)} > 0. \tag{89}$$

Similarly, using the equilibrium allocations under decentralization given by (20), that is,

$$c^D(\theta_i, I) = \left[\frac{1+\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\right] y \text{and} \quad g^D(\theta_i, I) = \left[\frac{\theta_i}{1+\theta_i+\gamma(I-1)}\right] y,$$

the ex-ante value in the decentralized regime is

$$\begin{aligned} V^D(I) &= \mathbb{E}\left[\log\left(\frac{1+\gamma(I-1)}{1+\theta_i+\gamma(I-1)} y\right) + \theta_i \log\left(\frac{[1+\gamma(I-1)]\,\theta_i}{1+\theta_i+\gamma(I-1)} y\right)\right] \\ &= \mathbb{E}\left[(1+\theta_i)\log\left(\frac{1+\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\right) + \theta_i \log \theta_i\right] + \left[(1+\mu_\theta)\log y\right] \end{aligned} \tag{90}$$

where the expectation is taken with respect to θ_i. Then, taking the derivative with respect to I we get

$$\begin{aligned}\frac{\partial V^D(I)}{\partial I} &= \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\gamma(I-1)} - \frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right] \\ &= (1+\mu_\theta)\frac{\gamma}{1+\gamma(I-1)} - \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right],\end{aligned}$$

and, using equation (89) we can write it as follows

$$\frac{\partial V^D(I)}{\partial I} = \frac{1+\mu_\theta}{\mu_\theta}\frac{\partial V^C(I)}{\partial I} - \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right] > 0. \tag{91}$$

Now, to compare the rates at which the centralized and decentralized values increase with I, using (91) we have that

$$\begin{aligned}\frac{\partial V^C(I)}{\partial I} - \frac{\partial V^D(I)}{\partial I} &= \frac{\partial V^C(I)}{\partial I} - \frac{1+\mu_\theta}{\mu_\theta}\frac{\partial V^C(I)}{\partial I} + \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right] \\ &= \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right] - \frac{1}{\mu_\theta}\frac{\partial V^C(I)}{\partial I}.\end{aligned} \tag{92}$$

Now, substituting (89) into (92),

$$\begin{aligned}\frac{\partial V^C(I)}{\partial I} - \frac{\partial V^D(I)}{\partial I} &= \mathbb{E}\left[\frac{(1+\theta_i)\gamma}{1+\theta_i+\gamma(I-1)}\right] - \frac{\gamma}{1+\gamma(I-1)} \\ &= \gamma\mathbb{E}\left[\frac{1+\theta_i}{1+\theta_i+\gamma(I-1)} - \frac{1}{1+\gamma(I-1)}\right] \\ &= \mathbb{E}\left[\frac{\gamma^2\theta_i(I-1)}{(1+\theta_i+\gamma(I-1))(1+\gamma(I-1))}\right] > 0.\end{aligned}$$

This shows that ex-ante utility in the centralized regime increases faster with I than in the decentralized regime. Formally, we have i) $V^C(1) \geq V^D(1)$ with equality only if $\phi = 1$, ii) $\lim_{I\to\infty} V^C(I) > \lim_{I\to\infty} V^D(I)$, iii) both $V^C(I)$ and $V^D(I)$ are increasing in I, and iv) $V^C(I)$ increases with I at a faster rate than $V^D(I)$. Hence, there exists a unique cutoff $I^*(\phi, \gamma) \geq 1$ such that if the number of countries in the union exceeds this cutoff, then the centralized regime is better, and if it is below this cutoff the decentralized regime is better.

Finally, we show that the cutoff rule $I^*(\gamma, \phi)$ is decreasing in both arguments. First, to prove that the cutoff is decreasing in ϕ, notice from (88) that V^C is increasing in ϕ. This is intuitive as higher ϕ corresponds to better information for the centralized fiscal authority about the type θ. Since the decentralized authorities perfectly observe the taste for the public good, the value of the decentralized fiscal authority, V^D, is independent of the informativeness of the signal. It then follows that as ϕ increases, the fiscal union becomes relatively better compared to the decentralized regime and therefore the threshold above which it is preferable to have a fiscal union decreases.

Second, to prove that the cutoff is decreasing in the externality parameter γ, using (88) and (90) we obtain

$$\frac{\partial V^C(I)}{\partial \gamma} = \frac{\mu_\theta (I-1)}{1+\gamma(I-1)} > 0,$$

and,

$$\begin{aligned}\frac{\partial V^D(I)}{\partial \gamma} &= \mathbb{E}\left[(1+\theta_i)\left(\frac{I-1}{1+\gamma(I-1)} - \frac{I-1}{1+\theta_i+\gamma(I-1)}\right)\right] \\ &= \mathbb{E}\left[(1+\theta_i)\left(\frac{1}{\mu_{\theta_i}}\frac{\partial V^C(I)}{\partial \gamma} - \frac{I-1}{1+\theta_i+\gamma(I-1)}\right)\right] \\ &= \frac{1+\mu_\theta}{\mu_\theta}\frac{\partial V^C(I)}{\partial \gamma} - \mathbb{E}\left[\frac{(1+\theta_i)(I-1)}{1+\theta_i+\gamma(I-1)}\right],\end{aligned}$$

where the second line uses equation (89), and expectations are taken with respect to the realization of θ_i. Then, the difference between how fast ex-ante welfare increases with the externality parameter γ in a centralized compared to a decentralized regime is

$$\begin{aligned}\frac{\partial V^C(I)}{\partial \gamma} - \frac{\partial V^D(I)}{\partial \gamma} &= \mathbb{E}\left[\frac{(1+\theta_i)\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\right] - \frac{1}{\mu_\theta}\frac{\partial V^C(I)}{\partial \gamma} \\ &= \mathbb{E}\left[\frac{(1+\theta_i)\gamma(I-1)}{1+\theta_i+\gamma(I-1)}\right] - \frac{I-1}{1+\gamma(I-1)} \\ &= \mathbb{E}\left[\frac{\theta_i\gamma(I-1)^2}{(1+\theta_i+\gamma(I-1))(1+\gamma(I-1))}\right] > 0.\end{aligned}$$

Thus, for a given I, as we increase γ, the welfare in the centralized regime increases by more than in the decentralized regime. This implies that as γ increases the centralized becomes relatively better than the decentralized regime as γ increases, and thus the cutoff for the number of countries above which the centralized is better decreases. □

Proof of Lemma 4: The first-order condition in the problem of the decentralized fiscal authority (25) and (26), for a given country i and for a given realization of preferences $\theta = \{\theta_H, \theta_L\}$ is

$$u'(y-g_i) = \theta_i h'(g_i + \gamma \sum\nolimits_{j\neq i} g_j),$$

so, letting $\{g_j^D\}$ denote the set of optimal decisions on public goods for the I countries in the union, and so dividing by θ_i and taking h'^{-1} to both sides, we get that

$$h'^{-1}\left(\frac{1}{\theta_i}u'(y-g_i^D)\right) = g_i^D + \gamma \sum\nolimits_{j\neq i} g_j^D.$$

Then, the ex-post welfare if the number of θ_H realizations in the union is n is

$$W^D(\theta_i; n, I) = u(y - g_i^D) + \theta_i h\left[h'^{-1}\left(\frac{1}{\theta_i}u'(y - g_i^D)\right)\right].$$

Recall from the proof of Lemma 2 that using the inverse function rule we have that $(h'^{-1})'(x) = \frac{1}{h''(h'^{-1}(x))} < 0$ so that $h'^{-1}(x)$ is decreasing in x. Using the conditions in Assumption 1, $u(y - g_i^D) \leq u(y)$, and using $\theta_H > \theta_L$, we have that for $i = \{H, L\}$ so $\frac{1}{\theta_i}u'(y - g_i^D) \geq \frac{1}{\theta_H}u'(y)$. Then, since $h'^{-1}(x)$ is decreasing in x and $\theta_H > \theta_L$ we get that,

$$W^D(\theta_i; n, I) \leq u(y) + \theta_H h\left[h'^{-1}\left(\frac{1}{\theta_H}u'(y)\right)\right] \equiv \bar{V}_H^D. \tag{93}$$

Then, we can bound above the decentralized ex-ante welfare defined in (29), as

$$V^D(I) \leq \bar{V}_H^D, \tag{94}$$

which completes the argument. □

Proof of Proposition 3: Let us define $\bar{I}^*(\phi)$ as follows,

$$\bar{I}^*(\phi) = \sup\{I \in [1, \infty) |\, V^C(I; \phi) \leq V^D(I)\}, \tag{95}$$

so that $\bar{I}^*(\phi)$ is the largest number of countries in a union for which the centralized ex-ante welfare crosses the decentralized one from below.

Next, we show that this object is well-defined. Define the set $\mathfrak{I}(\phi) \equiv \left\{I \in [1, \infty) :\, V^C(I; \phi) \leq V^D(I)\right\}$ to be the size of the union I at informativeness level ϕ for which the centralized welfare is lower than decentralized welfare. Since with $I = 1$ the decentralized regime is always weakly preferred, the set $\mathfrak{I}(\phi)$ is non-empty, because $I = 1$ belongs to it. To show that $\bar{I}^*(\phi)$ is well-defined, it suffices to show that, for any ϕ, the set $\mathfrak{I}(\phi)$ has an upper bound, hence the supremum exists.

We use Lemma 3 and 4 to construct such an upper bound. From these lemmas, we have that if the signal is completely uninformative, $V^C(I; \phi = 1/2)$ is strictly increasing in I, $V^C(I)$ tends to infinity as I goes to infinity, and decentralized welfare $V^D(I)$ is bounded above by the constant $\bar{V}_H^D$ defined in (93) that is less than infinity. Let $\tilde{I}$ be such that: $V^C(\tilde{I}, \phi = 1/2) = \bar{V}_H^D$. $\tilde{I}$ is our upper-bound candidate. Note that $V^D(\tilde{I}, \phi) \geq V^D(\tilde{I}, \phi = 1/2) = \bar{V}_H^D \geq V^D(I)$. Thus, $\tilde{I}$ is an upper-bound for $\mathfrak{I}(\phi)$, the supremum exists, and hence $\bar{I}^*(\phi)$ is well-defined.

Next, we show that $\bar{I}^*(\phi)$ is decreasing in ϕ. Let $\phi' > \phi$ and let I and I' be the associated cutoffs under ϕ and ϕ'. Hence, $V^D(I') = V^C(I', \phi') > V^C(I', \phi)$ where the equality follows from I' being the cutoff value at ϕ' and the inequality follows from Blackwell's theorem which implies that the value under centralization is increasing in ϕ. Thus, $I' = \bar{I}^*(\phi')$ is an element of the set $\mathfrak{I}(\phi)$. Since $\bar{I}^*(\phi)$ is, by definition, the largest element of such set, we get that $\bar{I}^*(\phi) \geq \bar{I}^*(\phi')$.

This concludes the proof of part a) of the Proposition. The proof for part b) will proceed in very similar steps to the one for part a), and can be skipped without greater loss. First, define $\underline{I}^*(\phi)$ as follows,

$$\underline{I}^*(\phi) = \inf\{I \in [1, \infty) |\, V^C(I; \phi) \geq V^D(I)\}. \tag{96}$$

Since, by Blackwell, $V^C(I = 1; \phi) < V^D(I = 1)$, we can interpret $\underline{I}^*(\phi)$ is the smallest number of countries in a union for which the centralized ex-ante welfare crosses the decentralized one from below.

Next, we show that this object is well-defined. Define the set $\mathfrak{L}(\phi) \equiv \left\{I \in [1, \infty) : V^C(I; \phi) \geq V^D(I)\right\}$ to be the size of the union I at informativeness level ϕ for which the centralized welfare is greater than decentralized welfare. Since with $I = \bar{I}^*(\phi)$ the centralized regime welfare equates to the decentralized one, the set $\mathfrak{I}(\phi)$ is non-empty, because $I = \bar{I}^*(\phi)$ belongs to it. To show that $\underline{I}^*(\phi)$ is well-defined, it suffices to show that, for any ϕ, the set $\mathfrak{L}(\phi)$ has a lower bound, hence the infimum exists. This follows immediately from the observation that, by the definition of $\mathfrak{L}(\phi)$, $I = 1$ is always a lower bound for such a set.

Lastly, we show that $\underline{I}^*(\phi)$ is decreasing in ϕ. Let $\phi' > \phi$ and let I and I' be the associated cutoffs under ϕ and ϕ'. Hence, $V^D(I) = V^C(I, \phi) < V^C(I, \phi')$ where the equality follows from I being the cutoff value at ϕ and the inequality follows from Blackwell's theorem which implies that the value under centralization is increasing in ϕ. Thus, $I = \underline{I}^*(\phi)$ is an element of the set $\mathfrak{L}(\phi')$. Since $\underline{I}^*(\phi')$ is, by definition, the smallest element of such set, we get that $\underline{I}^*(\phi') \leq \underline{I}^*(\phi)$, which completes the proof.

□

Proof of Lemma 6: To show that the optimal policy of the monetary authority does not depend on k, we differentiate the first-order condition defining the optimal policy function $\pi(\bar{B}, \bar{k})$ for a given vector of borrowing, $\bar{B}$ and savings $\bar{k}$, with respect to k_i, and show that when it is evaluated at a symmetric allocation it is indeed zero.

Recall that the first-order condition for the monetary authority (54) is

$$\frac{1}{I} \sum\nolimits_{i=1}^{I} u'(c_{i2} - v(\ell_i)) F_\pi(B_i, \pi) = 0,$$

where $c_{i2} - v(\ell_i) = F(B_i, \pi) + (1 + r)k_i$. Then, totally differentiating this equation with respect to k_i and π, we obtain

$$u''(c_{i2} - v(\ell_i))(1+r)F_\pi(B_i, \pi)dk_i + \sum\nolimits_{j=1}^{I} \left\{u''(c_{i2} - v(\ell_i)) \left[F_\pi(B_j, \pi)\right]^2 + u'(c_{i2} - v(\ell_i))F_{\pi\pi}(B_j, \pi)\right\} d\pi = 0,$$

and, therefore,

$$\frac{\partial \pi(\bar{B},\bar{k})}{\partial k_i} = -\frac{u''(c_{i2}-v(\ell_i))(1+r)F_\pi(B_i,\pi)}{\sum_{j=1}^{I}\left\{u''(c_{i2}-v(\ell_i))\left[F_\pi(B_j,\pi)\right]^2 + u'(c_{i2}-v(\ell_i))F_{\pi\pi}(B_j,\pi)\right\}},$$

which is equal to zero at any symmetric allocation, because evaluated at any symmetric allocation, the first-order condition (54) implies that $F_\pi(B_i,\pi)=0$. □

Derivation of Equation (58): Consider the centralized equilibrium. The first-order condition of (57) with respect to B_i is

$$\frac{-u'(c_{i1}) + E(\theta|s)\,h'(g_i)}{(1+r)\pi(\bar{B})} + \beta u'(c_{i2})F_B(B_i,\pi(\bar{B}))$$
$$+ \sum_j \left[u'(c_{j1}) - E(\theta|s)\,h'(g_j)\right]\frac{B_j}{(1+r)\pi^2(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B_i} + \beta\sum_j u'(c_{j2})F_\pi(B_j,\pi(\bar{B}))\frac{\partial \pi(\bar{B})}{\partial B_i} = 0.$$

Using that at the symmetric equilibrium allocation $F_\pi(B,\pi(\bar{B}))=0$, $u'(c_1)=\beta(1+r)u'(c_2)$, and multiplying both sides by $(1+r)\pi(\bar{B})$, this first-order condition reduces to

$$-u'(c_1) + E(\theta|s)\,h'(g) + \beta(1+r)\pi(\bar{B})u'(c_2)F_B(B,\pi(\bar{B})) + I\left[u'(c_1) - E(\theta|s)\,h'(g)\right]\frac{B}{\pi(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B} = 0.$$

Rearranging terms and again using $u'(c_1)=\beta(1+r)u'(c_2)$ gives the desired expression, namely,

$$[\pi(\bar{B})F_B(B,\pi(\bar{B}))-1]u'(c_1) + E(\theta|s)\,h'(g) + I\left[u'(c_1) - E(\theta|s)\,h'(g)\right]\frac{B}{\pi(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B} = 0. \qquad \square$$

Derivation of Equation (66): Consider the decentralized equilibrium. The first-order condition of (64) with respect to B_i is

$$\frac{-u'(c_{i1}) + E(\theta|s)\,h'(g_i)}{(1+r)\pi(\bar{B})} + \beta u'(c_{i2})F_B(B_i,\pi(\bar{B}))$$
$$+ \left[u'(c_{i1}) - E(\theta|s)\,h'(g_i)\right]\frac{B_i}{(1+r)\pi^2(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B_i} + \beta(c_{i2})F_\pi(B_i,\pi(\bar{B}))\frac{\partial \pi(\bar{B})}{\partial B_i} = 0.$$

Using that at the symmetric equilibrium allocation $F_\pi(B,\pi(\bar{B}))=0$, $u'(c_1)=\beta(1+r)u'(c_2)$, and multiplying both sides by $(1+r)\pi(\bar{B})$, this first-order condition reduces to

$$-u'(c_1) + E(\theta|s)\,h'(g) + \beta(1+r)\pi(\bar{B})u'(c_2)F_B(B,\pi(\bar{B})) + \left[u'(c_1) - E(\theta|s)\,h'(g)\right]\frac{B}{\pi(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B} = 0,$$

. Rearranging terms and using $u'(c_1)=\beta(1+r)u'(c_2)$, gives the desired expression, namely,

$$[\pi(\bar{B})F_B(B,\pi(\bar{B}))-1]u'(c_1) + E(\theta|s)\,h'(g) + \left[u'(c_1) - E(\theta|s)\,h'(g)\right]\frac{B}{\pi(\bar{B})}\frac{\partial \pi(\bar{B})}{\partial B} = 0. \qquad \square$$

Proof of Lemma 8: To see how changes in the number of countries affect the ex-post welfare under a decentralized regime, we first need to show how inflation changes with both the amount of borrowing of countries and the number of countries in the union. To do this, fix a country i and consider an almost symmetric equilibrium, in which country i is borrowing B and the other $I-1$ countries are borrowing $\bar{B}$. This leads to an optimal inflation policy $\pi^D(B,\bar{B})$ from the monetary authority problem, which solves the following first-order condition

$$(I-1)F_\pi(\bar{B},\pi)+F_\pi(B,\pi)=0.$$

Then, implicitly differentiating this first-order condition and imposing symmetry, we get the following derivatives

$$\begin{aligned}
\frac{\partial\pi(B,\bar{B})}{\partial B}|_{\bar{B}=B} &= -\frac{1}{I}\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)},\\
\frac{\partial\pi(B,\bar{B})}{\partial \bar{B}}|_{\bar{B}=B} &= -\frac{I-1}{I}\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)},\\
\frac{\partial\pi(B,\bar{B})}{\partial I}|_{\bar{B}=B} &= 0.
\end{aligned} \tag{97}$$

Now using the definition of $\tilde{\eta}$ as in (68) and the equilibrium result that $\eta^D(B,I)=\tilde{\eta}$, and substituting it into (97), we get that the change in inflation if all other countries increase their borrowing $\bar{B}$ is

$$\frac{\partial\pi(B,\bar{B})}{\partial \bar{B}}|_{\bar{B}=B} = (I-1)\frac{\pi}{B}\eta^D(B,I). \tag{98}$$

Also, rearranging (66) and using $\eta^D=\eta^C/I$, we get

$$u'(c_1)-\theta h'(g_i)=\frac{u'(c_1)\pi F_B}{1-\eta^D}. \tag{99}$$

Define the ex-post welfare of the decentralized authority that chooses a borrowing-capital allocation (B,k), taking as given that the other $I-1$ countries are borrowing $\bar{B}$ as

$$\tilde{W}^D(\theta,B,k,\bar{B})\equiv u\left(y_1-\frac{B}{(1+r)\pi(B,\bar{B})}-k\right)+\theta h\left(\frac{B}{(1+r)\,\pi(B,\bar{B})}\right)+\beta u\left(F(B,\pi(B,B))+(1+r)k\right).$$

Notice that evaluating (64) at the symmetric equilibrium, we get $W^D(\theta,I)=\tilde{W}^D(\theta,B(I),k(I),B(I))$, and then differentiating it with respect to I gives

$$\begin{aligned}
\frac{\partial W^D}{\partial I} &= \frac{\partial \tilde{W}^D}{\partial B}\frac{\partial B}{\partial I} + \frac{\partial \tilde{W}^D}{\partial k}\frac{\partial k}{\partial I} + \frac{\partial \tilde{W}^D}{\partial \bar{B}}\frac{\partial \bar{B}}{\partial I}\bigg|_{\bar{B}=B} = \frac{\partial \tilde{W}^D}{\partial \bar{B}}\frac{\partial \bar{B}}{\partial I}\bigg|_{\bar{B}=B} \\
&= \left\{\frac{1}{(1+r)\pi}u'(c_1)\frac{B}{\pi} - \theta h'(g_i)\frac{1}{(1+r)\pi}\frac{B}{\pi} + \beta u'(c_2 - v(l))F_\pi\right\}\frac{\partial \pi}{\partial \bar{B}}\frac{\partial \bar{B}}{\partial I}\bigg|_{\bar{B}=B} \\
&= \frac{1}{(1+r)\pi}\left\{u'(c_1) - \theta h'(g_i)\right\}\frac{B}{\pi}\frac{\partial \pi}{\partial \bar{B}}\frac{\partial B}{\partial I}
\end{aligned}$$

In the first line we used that the first-order condition of $\tilde{W}^D$ with respect to B and k imply $\partial\tilde{W}^D/\partial B = 0$ and $\partial\tilde{W}^D/\partial k = 0$. The second line uses the definition of $\tilde{W}^D$, and the third line follows from $F_\pi = 0$. Substituting (98) and (99) in the above equation we have

$$\begin{aligned}
\frac{\partial W^D}{\partial I} &= \frac{1}{(1+r)\pi}\left\{u'(c_1) - \theta h'(g_i)\right\}(I-1)\eta^D\frac{\partial B}{\partial I} \\
&= (I-1)\frac{u'(c_1)}{1+r}F_B\frac{\eta^D}{1-\eta^D}\frac{\partial B}{\partial I} \\
&= \beta(I-1)u'(c_2 - v(\ell))F_B\frac{\eta^D}{1-\eta^D}\frac{\partial B}{\partial I},
\end{aligned}$$

where the last equality follows from imposing the household's first-order condition for savings. Rewriting the last equation using the notation $c_2 - v\ell = F(B,k) + (1+r)k$, yields the desired equation (69). □

Proof of Proposition 7: Lemmas 7 and 8 show that as the number of countries in the union increases, the ex-ante welfare under centralization does not vary, whereas the ex-ante welfare under decentralization decreases. Thus, either the decentralized regime is always preferred or there is a finite $I(\phi) \in \{1, 2 \ldots\}$, such that $V^C(\phi) \geq V^D(I,\phi)$ if and only if $I \geq I(\phi)$. From Blackwell's theorem, it is then immediate that $I(\phi)$ is decreasing in I. To see that there exists a $\bar{\phi} \in [1/2, 1)$ such that $I(\phi) < \infty$, for all $\phi \geq \bar{\phi}$, note that the maximum theorem implies that $V^C(\phi)$ is continuous in ϕ. Since $I(\phi = 1) = 1$ and $I(\phi)$ is decreasing in ϕ, then by continuity in ϕ there exists a $\bar{\phi} \in [1/2, 1)$ such that $I(\phi) < \infty$, for all $\phi \geq \bar{\phi}$. □

Proof of Lemma 9: If the first condition in Assumption 3 holds, then trivially, given that in equilibrium $\eta^D = \tilde{\eta}$ we get that $\eta^D < 1$. Here we show that if the second condition in Assumption 3 holds, then the result also follows. To show that $\eta^D(B^*(\theta, I), I) < 1$ in that case, we use the optimality conditions that determine $B^*(\theta, I)$, together with the equilibrium capital allocation and inflation. The conditions that determine the equilibrium are the three equations consisting of the first-order conditions for B and k for the fiscal authority and the first-order condition for π for the monetary authority, namely

$$\left[-u'\left(y_1-\frac{B}{(1+r)\pi}-k\right)+\theta h'\left(\frac{B}{(1+r)\pi}\right)\right]\left[1-\eta^D(I,B,\pi)\right]=-u'\left(y_1-\frac{B}{(1+r)\pi}-k\right)\pi F_B,$$
$$u'\left(y_1-\frac{B}{(1+r)\pi}-k\right)=\beta(1+r)u'(F(B,\pi)+(1+r)k),$$
$$F_\pi(B,\pi)=0.$$

It is convenient to let $x\equiv B/\pi$ denote the real value of debt and rewrite this system as

$$\left[-u'\left(y_1-\frac{x}{1+r}-k\right)+\theta h'\left(\frac{x}{1+r}\right)\right]\left[1-\eta^D(I,\pi x,\pi)\right]=-u'\left(y_1-\frac{x}{1+r}-k\right)\pi F_B, \tag{100}$$
$$u'\left(y_1-\frac{x}{1+r}-k\right)=\beta(1+r)u'(F(x\pi,\pi)+(1+r)k), \tag{101}$$
$$F_\pi(x\pi,\pi)=0. \tag{102}$$

Substituting (101) in (100), we get

$$\left[\theta h'\left(\frac{x}{1+r}\right)-\beta(1+r)u'(F(x\pi,\pi)+(1+r)k)\right]\left[1-\eta^D(I,\pi x,\pi)\right]=-u'\left(y_1-\frac{x}{1+r}-k\right)\pi F_B.$$

Since $F_B<0$ by Assumption 2, the right side of this equation is positive, so to show that $\eta^D<1$, we need only to show that the first term on the left side is positive. Let us define the first term on the left side as

$$\gamma(x,\pi,k)\equiv\theta h'\left(\frac{x}{1+r}\right)-\beta(1+r)u'(F(x\pi,\pi)+(1+r)k).$$

To show that $\gamma(x,\pi,k)>0$, we use that $\gamma(x,\pi,k)\geq\gamma(x,\pi,0)$ and show that $\gamma(x,\pi,0)\geq 0$. To show that $\gamma(x,\pi,0)\geq 0$, we first note that for each π, $\gamma(x,\pi,0)$ is decreasing in x. Next, we construct an upper bound for x. Since $c_1,k_1\geq 0$, it follows that $x\leq(1+r)y_1$. Under productivity $\underline{A}$, the real value of debt x is also bounded above by the maximum revenue that can be raised, which corresponds to the peak of the Laffer curve, namely $\bar{T}$. Let $\bar{x}\equiv\min\{(1+r)y_1,\bar{T}\}$. Thus, it follows that

$$\begin{aligned}
\gamma(x,\pi,0) &\geq \theta_L h'\left(\frac{x}{1+r}\right)-\beta(1+r)u'(F(x\pi,\pi))\\
&= \theta_L h'\left(\frac{x}{1+r}\right)-\beta(1+r)u'(A\left(M/\pi\right)l(x,A\left(M/\pi\right))-v(l(x,A\left(M/\pi\right))))\\
&\geq \theta_L h'\left(\frac{x}{1+r}\right)-\beta(1+r)u'(\underline{A}l(x,\underline{A})-v(l(x,\underline{A}))\\
&\geq \theta_L h'\left(\frac{\bar{x}}{1+r}\right)-\beta(1+r)u'(\underline{A}l(\bar{x},\underline{A})-v(l(\bar{x},\underline{A}))>0,
\end{aligned}$$

where the second line follows from the definition of F and the last line follows from $\gamma(x,\pi,0)$ being decreasing in x and Assumption 3. This proves that $\gamma(x,\pi,k) \geq \gamma(x,\pi,0) > 0$, and hence $\eta^D < 1$. □

Proof of Lemma 10: Using Assumption 4 we can solve the problem of the decentralized fiscal authority and take derivatives of the equilibrium allocation of borrowing with respect to the number of countries to see that it is positive. Notice that under Assumption 4, the problem of the fiscal authority is simplified because, since $\beta(1+r) > 1$, the fiscal authority always finds it optimal to choose $c_1 = 0$. Then, its problem becomes

$$W^D(\theta, B_{-i}, k_{-i}, I) = \max_{B_i} \theta h\left(\frac{B_i}{(1+r)\,\pi(\bar{B})}\right) + \beta\left(F(B_i, \pi(\bar{B})) - \frac{B}{\pi(\bar{B})} + (1+r)y_1\right).$$

Taking the first-order condition with respect to B_i, using that $B_i = B$ for all i and $F_\pi = 0$ from the monetary authority first-order condition, gives

$$\left(\frac{\theta}{1+r}h'\left(\frac{B}{(1+r)\,\pi}\right) - \beta\right)\left(\frac{1}{\pi} - \frac{B}{\pi^2}\frac{\partial \pi^D}{\partial B}\right) + \beta F_B(B,\pi) = 0,$$

and, using that $\eta^D = \tilde{\eta}$ in equilibrium from (67),

$$\left(\frac{\theta}{1+r}h'\left(\frac{B}{(1+r)\,\pi}\right) - \beta\right)(1 - \tilde{\eta}(B,\pi,I)) + \beta\pi F_B(B,\pi) = 0.$$

Define $\gamma(B,\pi) \equiv \frac{\theta}{1+r}h'\left(\frac{B}{(1+r)\pi}\right) - \beta$. Then, the first-order condition becomes $\gamma(B,\pi)\left[1 - \tilde{\eta}(B,\pi,I)\right] + \beta\pi F_B(B,\pi) = 0$. Differentiating with respect to I, we obtain that

$$\begin{aligned}
&\left\{\frac{\partial\gamma(B,\pi)}{\partial B} + \frac{\partial\gamma(B,\pi)}{\partial\pi}\left[\frac{\partial\pi}{\partial B} + \frac{\partial\pi}{\partial\bar{B}}\right]\right\}(1 - \tilde{\eta}(B,\pi,I))\frac{\partial B}{\partial I} \\
&-\gamma(B,\pi)\frac{\partial\tilde{\eta}(B,\pi,I)}{\partial I} - \gamma(B,\pi)\left\{\frac{\partial\tilde{\eta}(B,\pi,I)}{\partial B} + \frac{\partial\tilde{\eta}(B,\pi,I)}{\partial\pi}\left[\frac{\partial\pi}{\partial B} + \frac{\partial\pi}{\partial\bar{B}}\right]\right\}\frac{\partial B}{\partial I} \\
&+\beta\left\{\left[\frac{\partial\pi}{\partial B} + \frac{\partial\pi}{\partial\bar{B}}\right]F_B(B,\pi) + \pi F_{BB}(B,\pi) + \pi F_{B\pi}(B,\pi)\left[\frac{\partial\pi}{\partial B} + \frac{\partial\pi}{\partial\bar{B}}\right]\right\}\frac{\partial B}{\partial I} = 0.
\end{aligned}$$

To simplify the expression above, notice that in the proof of Lemma 8 we have derived the derivatives of inflation with respect to borrowing as shown in (97), which implies

$$\frac{\partial\pi}{\partial B} + \frac{\partial\pi}{\partial\bar{B}} = -\frac{1}{I}\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)} - \frac{I-1}{I}\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)} = -\frac{F_{B\pi}(B,\pi)}{F_{\pi\pi}(B,\pi)} = I\frac{\pi}{B}\tilde{\eta}(\pi,B,I).$$

Then, substituting in the above expression, and omitting the arguments (B,π,I) from the functions for notational simplicity, we have that

$$\left\{\frac{\partial\gamma}{\partial B}+I\frac{\pi}{B}\tilde{\eta}\frac{\partial\gamma}{\partial\pi}\right\}(1-\tilde{\eta})\frac{\partial B}{\partial I}-\gamma\frac{\partial\tilde{\eta}}{\partial I}-\gamma\left\{\frac{\partial\tilde{\eta}}{\partial B}+I\frac{\pi}{B}\tilde{\eta}\frac{\partial\tilde{\eta}}{\partial\pi}\right\}\frac{\partial B}{\partial I}$$
$$+\beta\left\{I\frac{\pi}{B}\tilde{\eta}\left[F_B+\pi F_{B\pi}\right]+\pi F_{BB}\right\}\frac{\partial B}{\partial I}=0. \tag{103}$$

Finally, we use the definition of $\gamma(B,\pi)$, to get its derivative with respect to I,

$$\frac{\partial\gamma}{\partial B}+I\frac{\pi}{B}\tilde{\eta}\frac{\partial\gamma}{\partial\pi}=\frac{\theta}{(1+r)^2}h''\left(\frac{B}{(1+r)\,\pi}\right)\left[\frac{1}{\pi}-I\frac{\pi}{B}\tilde{\eta}\frac{B}{\pi^2}\right]=\frac{\theta\,(1-I\tilde{\eta})}{(1+r)^2\pi}h''\left(\frac{B}{(1+r)\,\pi}\right).$$

And, plugging it back into (103), we obtain

$$\frac{\partial B}{\partial I}=\frac{\gamma\frac{\partial\tilde{\eta}}{\partial I}}{\frac{\theta(1-I\tilde{\eta})(1-\tilde{\eta})}{(1+r)^2\pi}h''\left(\frac{B}{(1+r)\pi}\right)+\beta\frac{\pi}{B}\left\{I\tilde{\eta}\left[F_B+\pi F_{B\pi}\right]+BF_{BB}\right\}-\gamma\left\{\frac{\partial\tilde{\eta}}{\partial B}+I\frac{\pi}{B}\tilde{\eta}\frac{\partial\tilde{\eta}}{\partial\pi}\right\}}. \tag{104}$$

We know that $\partial\tilde{\eta}/\partial I<0$.By Assumption 4, the denominator of (104) is also negative, so $\partial B/\partial I>0$. □

Sufficient Conditions for an Interior Solution under Assumption 4. Here we provide conditions that ensure that the decentralized allocation is interior under Assumption 4. Recall that under Assumption 4, $c_1=0$ and $k=y_1-\frac{B}{(1+r)\pi(\bar{B})}$. Thus, to rule out the corner cases $B=0$ or $k=0$, we assume that $h'(0)=\infty$, so that the desired amount of public good has to be positive, $g>0$ and hence so does B, and that y_1 is sufficiently large, so that $k>0$. The argument to ensure that the Lagrange multiplier in the monetary authority's problem associated with $\pi\geq 1$ is zero in equilibrium is more involved and is formally proved in the following lemma. After the lemma, we also provide an example of a functional form for F under which the conditions of the corresponding lemma are satisfied.

Lemma 11. *Define $B^*(\theta)$ as the allocation that solves*

$$\frac{\theta}{1+r}h'\left(\frac{B^*}{1+r}\right)+\beta F_B(B^*,1)-\beta=0. \tag{105}$$

and, assume that $y_1>\frac{B^*(\theta_H)}{1+r}$ and $F_\pi(B^*(\theta_L),1)>0$. Then, the Lagrange multiplier associated with $\pi\geq 1$ is zero in the symmetric decentralized equilibrium.

Proof. To prove that the Lagrange multiplier associated with $\pi\geq 1$ is zero we use a contradiction argument. First, we construct a set for the allocations of country i, B_i together with the symmetric allocation of the remaining countries $\bar{B}$, such that the Lagrange multiplier is strictly positive if and only if $(B,\bar{B})$ belong to this set. Second, we assume that a symmetric solution of the decentralized problem, say B^* belongs to such a set, so that the Lagrange multiplier is strictly positive. We show that this allocation satisfies condition (105), and therefore by the assumption $F_\pi(B^*,1)>0$ of Lemma 11. Finally, we establish a contradiction by showing that for strictly positive Lagrange multipliers we must have $F_\pi(B^*, 1) < 0$. This argument implies that the Lagrange multiplier in the decentralized symmetric equilibrium is zero. Notice that any solution of the decentralized fiscal authority depends on the realization of the preference for public goods θ. For notational simplicity, in the remainder of the proof we omit such dependency.

We start by constructing a set $\mathcal{B}$ such that the Lagrange multiplier is strictly positive if and only if $(B, \bar{B})$ belongs to such set. Recall that the first-order condition of the monetary authority problem is

$$(I-1)F_\pi(\bar{B}, \pi) + F_\pi(B_i, \pi) + \lambda = 0,$$

where λ is the Lagrange multiplier associated with $\pi \geq 1$. Since complementary slackness condition is $\lambda(\pi - 1) = 0$, this implies that the solution is associated with $\lambda > 0$ if and only if

$$(I-1)F_\pi(\bar{B}, 1) + F_\pi(B_i, 1) < 0. \tag{106}$$

Now, we show that there exists a function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that the Lagrange multiplier λ associated with $\pi \geq 1$ is strictly positive if and only if $\bar{B} < f(B_i)$. Define such function f as the one that solves

$$(I-1)F_\pi(f(B_i), 1) + F_\pi(B_i, 1) = 0, \tag{107}$$

that is, the first-order condition is exactly zero when evaluated at $(B_i, f(B_i))$. Given that we are assuming $F_{B\pi} > 0$ from Assumption 2, then for any $\bar{B} < f(B_i)$ we must have

$$(I-1)F_\pi(\bar{B}, 1) + F_\pi(B_i, 1) < (I-1)F_\pi(f(B_i), 1) + F_\pi(B_i, 1) = 0,$$

which is condition (106) for the solution to be associated with a strictly positive multiplier λ on the constraint $\pi \geq 1$. Thus, we just showed that for any $\bar{B} < f(B_i)$, where f is defined as in (107), the solution for inflation is associated with a strictly positive multiplier on the constraint $\pi \geq 1$, and therefore with the solution $\pi(B_i, \bar{B}) = 1$.

Using this definition of $f(B_i)$ we can construct the following set. Let $\mathcal{B} = \{(B, \bar{B}) \in \mathbb{R}^2_+ : \bar{B} < f(B)\}$. Note that by the definition of $f(B)$, we have a strictly positive Lagrange multiplier if and only if, $(B_i, \bar{B}) \in \mathcal{B}$. Since we want to prove that any decentralized symmetric equilibrium, $B_i = B^*$ and $\bar{B} = B^*$, has a Lagrange multiplier equal to zero, it's sufficient to prove that the equilibrium symmetric allocation does not belong to the set, that is, $(B^*, B^*) \notin \mathcal{B}$.

To setup the contradiction argument denote the solution of the decentralized problem be $B_i = \bar{B} = B^*$, and assume that $(B^*, B^*) \in \mathcal{B}$. We want to establish that this assumption leads to a contradiction and therefore we must have $(B^*, B^*) \notin \mathcal{B}$, which implies that the Lagrange multiplier is zero. First, using a perturbation argument to show that the solution B^* satisfies condition (105). Consider that the fiscal authority of country i does an arbitrarily small perturbation in B^* of size ϵ. Since $\mathcal{B}$ is an open set, for ϵ small enough, $(B^* + \epsilon, B^*) \in \mathcal{B}$. Thus, we still have $\pi(B^* + \epsilon, \bar{B}^*) = 1$ and the decentralized fiscal authority understands that this perturbation will not affect π, so we don't need to compute the effects of this perturbation on π, which are zero. Now, note the decentralized fiscal authority welfare, under $\pi = 1$, is

$$W^D(\theta, B^*, \epsilon) = \theta h\left(\frac{B^* + \epsilon}{1+r}\right) + \beta\left(F(B^* + \epsilon, 1) - B^* - \epsilon + (1+r)y_1\right). \tag{108}$$

Given that B^* is the solution to the symmetric decentralized problem, it must be the case that there is not another allocation that increases welfare. In other words, any change in the perturbation ϵ should not increase W^D. By taking the derivative of (108) with respect to ϵ, we can see that a perturbation in ϵ will not increase welfare if and only if

$$\frac{\theta}{1+r}h'\left(\frac{B^*}{1+r}\right) + \beta F_B(B^*, 1) - \beta = 0,$$

which is the same condition as in (105). Therefore, B^* is defined as in the condition in Lemma 11, and so by assumption we have that $F_\pi(B^*(\theta_L), 1) > 0$. However, equation (106) implies that if $\lambda > 0$, as we assumed was the case for B^*, then we must have $F_\pi(B^*, 1) < 0$. Given that F_π is decreasing by Assumption 2 and that B^* is increasing in θ, if condition $F_\pi(B^*(\theta_L), 1) > 0$ holds for the lowest realization of the shock, then it must hold for all.[15] Hence this is a contradiction by which we cannot have the solution to the decentralized problem belonging to the set $\mathcal{B}$. That is, we must have that the Lagrange multiplier is zero.

To conclude the proof, we are left to show that the ϵ perturbation is feasible to the decentralized fiscal authority, namely, that $B^* + \epsilon \in (0, (1+r)y_1)$. This means that by applying the perturbation to the solution B^* we are not implying a negative allocation for borrowing, B, or for saving, $k = y_1 - \frac{B}{1+r}$. As we prove next, these conditions are satisfied following from the assumptions in Lemma 11. First, we assume that at the highest possible realization of the preference for public good we have that $B^*(\theta_H) < (1+r)y_1$. If this is satisfied at θ_H then given that B^* is increasing in θ, it is also satisfied at $\theta_L < \theta_H$. Second, by the Inada condition on the utility over public goods, h, any symmetric solution $B^*(\theta)$ will be such that $B^* > 0$. Thus, $B^* \in (0, (1+r)y_1)$ and for ϵ small enough, $B^* + \epsilon \in (0, (1+r)y_1)$, so the perturbation that we have considered in the proof is feasible. □

To conclude with the sufficient conditions for an interior solution, here we provide an example of a

[15] That B^* is increasing in θ follows immediately from the first-order condition of decentralized fiscal authority which is equivalent to condition (106). Then, given that both h and F are concave in B by Assumption 2, it must be that an increase in θ implies a decrease in the optimal B^*.

functional form for $F(\cdot)$ that satisfies Lemma 11. Assume the following functional form, which is the same as in the analytical solution shown in Appendix B: $F(B,\pi) = \kappa_0 - \frac{\kappa}{\alpha} B^{\alpha}\pi^{-\alpha} - \psi\pi$, where $\kappa_0 > 0$, $\kappa > 0$, $\alpha > 1$, and $\psi > 0$. Thus, $F_B(B,1) = -\kappa B^{\alpha-1}$ and $F_\pi(B,1) = \kappa B^{\alpha} - \psi$. Let B^* be the unique solution defined in Lemma 11. Then, one intuitive way to satisfy our assumption that $F_\pi(B^*,1) > 0$ is $\psi < \kappa {B^*}^{\alpha}$. This is intuitive: the central bank will accept some positive inflation only if the cost of inflation is not too high. □

B Monetary Economy: An Analytical Case

Functional form assumption in the disutility of over labour $v(\ell)$ to obtain (70). We assume that disutility over labor is given by

$$v(\ell) = A\ell - \kappa_0 + \frac{(\alpha-1)\bar{\ell}}{\alpha}\left[\frac{1}{\bar{\ell}}\left(\log\bar{\ell} - \log\ell\right)\right]^{\frac{\alpha}{\alpha-1}}, \tag{109}$$

where $\alpha > 1$ and $\bar{\ell}$ is a parameter indicating the most labor a consumer can supply. As in (41), using the first-order condition of the consumer for labor, we can solve for the level of labor associated with raising T in revenues with a linear income tax to obtain

$$T = [A - v'(\ell)]\ell = \left[\frac{1}{\bar{\ell}}\left(\log\bar{\ell} - \log\ell\right)\right]^{\frac{1}{\alpha-1}},$$

and so,

$$\bar{\ell}T^{\alpha-1} = \log\left(\bar{\ell}/\ell\right).$$

Then, rearranging this equation we can express labor as a function of tax revenues

$$\ell(T) = \bar{\ell}\exp(-\bar{\ell}T^{\alpha-1}). \tag{110}$$

As in the main text, it will be useful to define the output function in terms of the function $F(B,\pi)$ described in (53). Now, including the direct negative effect on output of inflation, $\psi\pi$, into this function we have

$$F(B,\pi) \equiv A\ell - \psi\pi - v(\ell), \tag{111}$$

where $\ell = \ell(T) = \ell(B/\pi)$, so utility in period 2 is $u(c_{i2} - v(\ell_i)) = F(B,\pi) + (1+r)k_i$. Substituting our functional form for $v(\cdot)$ from equation(109), and the equilibrium labor allocation (110), we can write (111) as

$$F(B,\pi) = \kappa_0 - \frac{\kappa_1}{\alpha}\left(\frac{B}{\pi}\right)^{\alpha} - \psi\pi, \tag{112}$$

where $\kappa_1 = (\alpha-1)\bar{\ell}$. Note that (112) corresponds to (70) in the main text.

Derivation of Equation (71). The monetary authority solves

$$W_{MA}(\bar{B},\bar{k}) = \max_{\pi\geq 1}\frac{1}{I}\sum_i \left[F(B_i,\pi) + (1+r)k_i\right],$$

with first-order condition given by

$$\kappa_1 \sum_i B_i^{\alpha}\pi^{-\alpha-1} - \phi I = 0.$$

This implies that the optimal inflation rule is

$$\pi(B_1,\ldots,B_I) = \left(\frac{\kappa_1}{\psi}\right)^{\frac{1}{1+\alpha}}\left[\frac{\sum_i B_i^{\alpha}}{I}\right]^{\frac{1}{1+\alpha}}. \tag{113}$$

Note that (113) corresponds to (71) in the main text.

Derivation of Optimal Borrowing under Centralization. The fiscal union solves

$$W^C(s,I) = \frac{1}{I}\max_B \sum\left\{\frac{\tilde{\theta}_i B}{(1+r)\,\pi} + \beta\left[F(B,\pi) + (1+r)y_1 - \frac{B}{\pi}\right]\right\}, \tag{114}$$

where $\pi = \pi^C(B)$. The first-order condition in the centralized fiscal authority problem is then

$$\frac{\tilde{\theta}_i}{(1+r)}\left[\frac{1}{\pi} - \frac{B}{\pi^2}\frac{\partial\pi}{\partial B}\right] + \beta\left[F_B(B,\pi) + F_\pi(B,\pi)\frac{\partial\pi}{\partial B} - \left(\frac{1}{\pi} - \frac{B}{\pi^2}\frac{\partial\pi}{\partial B}\right)\right] = 0. \tag{115}$$

Where $\tilde{\gamma}_i \equiv \tilde{\theta}_i/(1+r) - \beta$. Then, we can rewrite the previous expression as

$$\frac{\tilde{\gamma}_i}{\pi} - \tilde{\gamma}_i\frac{B}{\pi^2}\frac{\partial\pi}{\partial B} + \beta\left[F_B(B,\pi) + F_\pi(B,\pi)\frac{\partial\pi}{\partial B}\right] = 0, \tag{116}$$

and using the derived constant elasticity of inflation with respect to borrowing from $\eta^C = \alpha/(\alpha+1)$ and the first-order condition of the monetary authority that implies $F_\pi = 0$, we get

$$\tilde{\gamma}_i(1-\eta^C) + \beta\pi F_B(B,\pi) = 0. \tag{117}$$

Finally, from (70) we have that $\pi F_B = -\kappa_1 \left(\frac{B}{\pi}\right)^{\alpha-1}$, and using the solution for π^C from (72), we get

$$\pi^C F_B(B, \pi^C) = -\kappa_1 \left(\frac{\psi}{\kappa_1} B\right)^{\frac{\alpha-1}{1+\alpha}},$$

so substituting it in the first-order condition (117) we can solve for the optimal borrowing in the centralized regime,

$$B^C = \left(\frac{\kappa_1}{\psi}\right) \left[\frac{\tilde{\gamma}_i(1-\eta^C)}{\beta\kappa_1}\right]^{\frac{1+\alpha}{\alpha-1}}. \tag{118}$$

Derivation of Centralized Welfare. Given the solution for borrowing, B^C and inflation π^C in the centralized fiscal authority regime, we can now solve for the ex-post value for the fiscal authority. For a given signal s_i about the preferences over public expenditures, the ex-post value is

$$W^C(s) = \tilde{\gamma}_i \frac{B^C}{\pi^C(B^C)} + \beta \left[F(B^C, \pi^C(B^C)) + (1+r)y_1\right]. \tag{119}$$

To get a complete solution we need to substitute for π^C and B^C. First, substituting (118) in (114), we get

$$\pi^C(B^C) = \left(\frac{\kappa_1}{\psi}\right) \left[\frac{\tilde{\gamma}_i(1-\eta^c)}{\beta\kappa_1}\right]^{\frac{\alpha}{\alpha-1}}. \tag{120}$$

We can also use (118) and (120) to obtain the real revenues raised in equilibrium, namely

$$\frac{B^C}{\pi^C(B^C)} = \left[\frac{\tilde{\gamma}_i(1-\eta^c)}{\beta\kappa_1}\right]^{\frac{1}{\alpha-1}}. \tag{121}$$

And, to get an expression for $F(B, \pi)$, we use(120) and (121) in (70) so that

$$\begin{aligned} F(B^C, \pi^C(B^C)) &= \kappa_0 - \frac{\kappa_1}{\alpha} \left[\frac{\tilde{\gamma}_i(1-\eta^c)}{\beta\kappa_1}\right]^{\frac{\alpha}{\alpha-1}} - \kappa_1 \left[\frac{\tilde{\gamma}_i(1-\eta^c)}{\beta\kappa_1}\right]^{\frac{\alpha}{\alpha-1}} \\ &= \kappa_0 - \frac{1}{\beta}\tilde{\gamma}_i^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\beta\kappa_1}\right)^{\frac{1}{\alpha-1}} \left(\frac{\alpha+1}{\alpha}\right) (1-\eta^C)^{\frac{\alpha}{\alpha-1}}. \end{aligned} \tag{122}$$

Finally, we can substitute (121) and (122) in (119) to obtain an analytical solution for its ex-post welfare, that is,

$$\begin{aligned} W^C(s_i) &= \gamma_i \left[\frac{\tilde{\gamma}_i(1-\eta^C)}{\beta\kappa_1}\right]^{\frac{1}{\alpha-1}} + \beta \left[\kappa_0 - \frac{1}{\beta}\tilde{\gamma}_i^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\beta\kappa_1}\right)^{\frac{1}{\alpha-1}} \left(\frac{\alpha+1}{\alpha}\right) (1-\eta^C)^{\frac{\alpha}{\alpha-1}} + (1+r)y\right] \\ &= \tilde{\gamma}_i^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\beta\kappa_1}\right)^{\frac{1}{\alpha-1}} \left[(1-\eta^C)^{\frac{1}{\alpha-1}} - \left(\frac{\alpha+1}{\alpha}\right)(1-\eta^C)^{\frac{\alpha}{\alpha-1}}\right] + \chi, \end{aligned} \tag{123}$$

where $\chi \equiv \beta(\kappa_0 + (1+r)y_0)$.

Derivation of Optimal Borrowing under Decentralization. The best response of the decentralized fiscal authority when all other countries choose B is to solve

$$W^D(\theta_i, B, I) = \max_{B_i} \left\{ \frac{\theta_i B_i}{(1+r)\,\pi} + \beta \left[F(B_i, \pi) + (1+r) y_1 - \frac{B_i}{(1+r)\pi} \right] \right\},$$

subject to the monetary policy rule $\pi = \pi^D(B_i, B, I)$ as in (72). The first-order condition is

$$-\frac{\gamma_i B_i}{\pi^2} \frac{\partial \pi}{\partial B_i} + \frac{\gamma_i}{\pi} + \beta F_B + \beta F_\pi \frac{\partial \pi}{\partial B_i} = 0,$$

where $\gamma_i = \theta_i/(1+r) - \beta$. Using the constant elasticity formula (73) and that, after imposing symmetry, the first-order condition of the monetary authority becomes $F_\pi = 0$, we get

$$\gamma_i(1 - \eta^D(I)) + \beta \pi F_B(B, \pi) = 0. \tag{124}$$

Comparing equation (124) with the one derived in the centralized regime (117), we can see that there are only two differences. First, in the centralized regime the fiscal authority does not observe perfectly the preferences of countries, so it has to take expectations given a signal s_i. And second, the elasticity of inflation with respect to borrowing that the fiscal authority takes into account is $\eta^C = \alpha/(\alpha+1)$ in the centralized case, and is $\eta^D(I)$ given by (73) in the decentralized case. As shown before, this elasticity only depends on the number of countries in the decentralized regime. Then, following the same steps than in the centralized case, we get analogous equations that solve for the optimal level of borrowing,

$$B^D(\theta_i, I) = \left(\frac{\kappa_1}{\psi}\right) \left[\frac{\gamma_i(1 - \eta^D(I))}{\beta \kappa_1}\right]^{\frac{1+\alpha}{\alpha-1}}. \tag{125}$$

Derivation of Equation (74). Using (125) and (72), we obtain inflation under the optimal decentralized borrowing,

$$\pi^D(I) = \left(\frac{\kappa_1}{\psi}\right) \left[\frac{\gamma_i(1 - \eta^D(I))}{\beta \kappa_1}\right]^{\frac{\alpha}{\alpha-1}}. \tag{126}$$

Unlike in the centralized regime, borrowing under a decentralized regime is increasing in the number of countries in the union. The reason is that as I increases, the externality becomes worse in the sense that the decentralized fiscal authority only takes into account a fraction $1/I$ of the total effect of increasing B on inflation.

Finally, evaluating ex-post welfare in the decentralized regime at its optimal allocations gives

$$W^D(\theta_i, I) = \gamma_i^{\frac{\alpha}{\alpha-1}} \left(\frac{1}{\beta \kappa_1}\right)^{\frac{1}{\alpha-1}} \left[(1 - \eta^C/I)^{\frac{1}{\alpha-1}} - \left(\frac{\alpha+1}{\alpha}\right)(1 - \eta^C/I)^{\frac{\alpha}{\alpha-1}}\right] + \chi, \tag{127}$$

where we used that $\eta^D(I) = \eta^C/I$ for comparison with (123). Ex-ante welfare in the decentralized regime is then defined as $V^D(I) = qW^D(\theta_H, I) + (1-q)W^D(\theta_L, I)$. Note that (127) corresponds to (74) in the main text. □

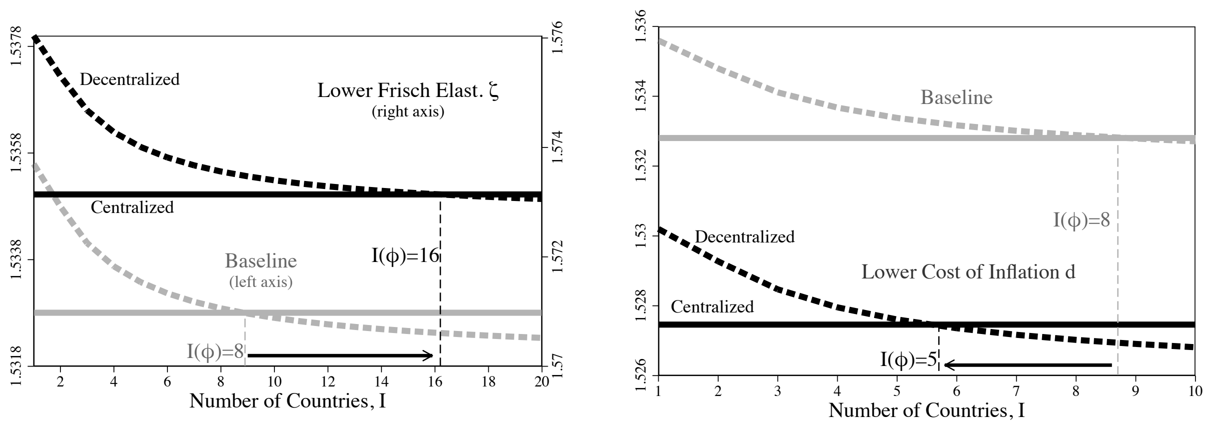
C Omitted Numerical Results

We present here details of the comparative statics results discussed in Section

Comparative Statics with Perfectly Correlated Preferences across Countries. We examine how the results in Figure 5 change as we change the values of two key parameters: the parameter ζ for the Frisch elasticity of labor supply and the parameter d of the production function governing the cost of inflation in the economy. In the left panel of Figure 10, we repeat Figure 5 when the Frisch elasticity ζ is decreased from its baseline value of 1 to 0.95. Contrasting the two figures reveals that when the signal is uninformative, it takes more than 16 countries before a fiscal union is preferred, as opposed to 8 countries when ζ was higher. Why? First, a smaller labor supply elasticity decreases the distortions from labor income taxation, whereas the impact of inflation on productivity remains the same. Hence, for any given level of debt, the monetary authority is less willing to raise inflation, which in turn makes the fiscal externality induced by debt smaller, thus narrowing the advantage of a fiscal union over a decentralized regime. Thus, it takes a larger monetary union for a fiscal union is preferable.

Figure 9: Comparative Statics on Ex-Ante Welfare with Perfectly Correlated Preferences

(a) Lower Frisch Elasticity ζ (b) Lower Cost of Inflation d



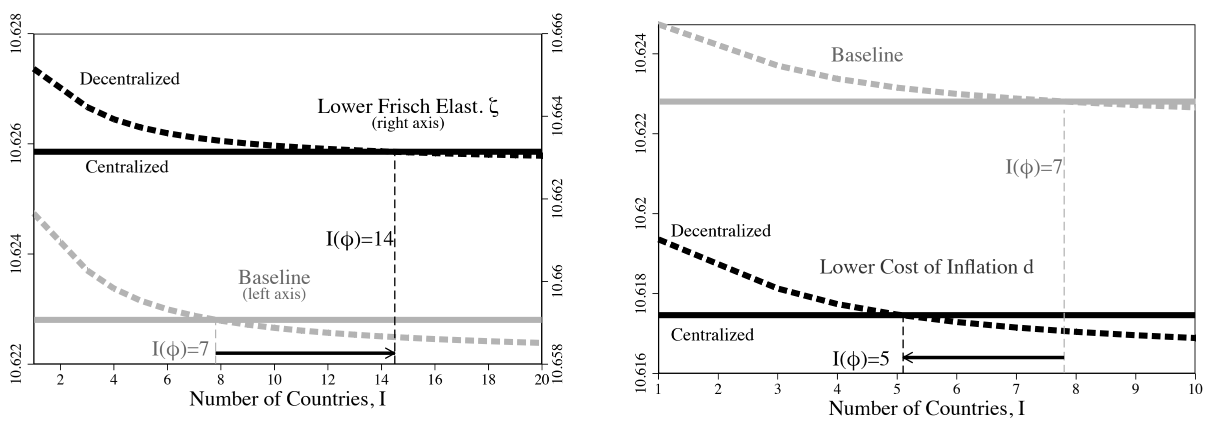
Notes: The initial endowment is $y = 10$, the production function parameters are $a = 3$, $e = 3$, and $M = 1$, the disutility of work is $\chi = 2.6$, and the preference parameters are $\theta_H = 1.12$ and $\theta_L = 0.88$ with probability of drawing a high preference $q = 0.5$. The discount factor is set to $\beta = 1$ and the interest rate to $r = 0.01$. The information parameter ϕ is set to 0.5. In the left panel we set $d = 0.15$, and the Frisch elasticity of labor supply is $\zeta = 1$ in the baseline and $\zeta = 0.95$ in the lower Frisch elasticity economy. In the right panel, the Frisch elasticity of labor supply is set to $\zeta = 1$ and the parameter d in the production function is kept to 0.15 in the baseline and set to 0.05 in the lower cost of inflation economy.

In the right panel of Figure 10, we repeat Figure 5 with a lower distortionary cost of inflation in terms of productivity, as captured by $d = 0.05$, relative to the baseline case in which $d = 0.15$. This lower cost of inflation implies that for the same level of debt, the monetary authority is relatively more willing to raise inflation ex post. This monetary authority's incentive amplifies the fiscal externality. Thus, it takes a smaller monetary union for a fiscal union to be preferable. For instance, in the uninformative signal case with the lower cost of inflation, it takes only more than 5 countries for a fiscal union to be preferred rather than 8 countries, as was the case with the higher cost of inflation. □

Comparative Statics with Independent Preferences across Countries. Here we do the same exercise as in Figure 10, that is, changing the values of the Frisch elasticity and cost of inflation parameters, but now in the case where preferences across countries are independent.

Figure 10: Comparative Statics on Ex-Ante Welfare with Independent Preferences

(a) Lower Frisch Elasticity ζ (b) Lower Cost of Inflation d



Notes: The initial endowment is $y = 10$, the production function parameters are $a = 3$, $e = 3$, and $M = 1$, the disutility of work is $\chi = 2.6$, and the preference parameters are $\theta_H = 1.1$ and $\theta_L = 0.9$ with probability of drawing a high preference $q = 0.5$. The discount factor is set to $\beta = 1$ and the interest rate to $r = 0.01$. The information parameter ϕ is set to 0.5. In the left panel we set $d = 0.15$, and the Frisch elasticity of labor supply is $\zeta = 1$ in the baseline and $\zeta = 0.95$ in the lower Frisch elasticity economy. In the right panel, the Frisch elasticity of labor supply is set to $\zeta = 1$ and the parameter d in the production function is kept to 0.15 in the baseline and set to 0.05 in the lower cost of inflation economy.

The baseline economy in this case is the one displayed in Figure 8. We find that the results are in the same direction than the ones in which we consider perfectly correlated preferences across countries. First, as shown in the left panel of Figure 9, a decrease in the Frisch elasticity gives an advantage to the decentralized regime and thus increases the threshold on the number of countries above which a centralized regime is preferred. And second, a decrease in the cost of inflation gives an advantage to the centralized regime, so the threshold con the number of countries above which a centralized regime is preferred decreases. This is displayed on the right panel of Figure 9. □