The Impact of Settlement Currency on Foreign Exchange Forward Contracts

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Abstract This paper investigates the distinctive dynamics of Brazil's domestic currency derivatives market, which exhibits remarkable activity compared to other emerging markets. Specifically, we examine the consequences of this market structure by contrasting Deliverable Forward markets, Offshore Non-Deliverable Forward markets, and the prevalent Domestic Non-Deliverable Forward markets in Brazil. Our model incorporates interactions between domestic and foreign consumers in spot and forward markets, alongside financial intermediaries and a government constrained by foreign currency debts and obligations. We find that under controlled external debt and minimal external risk, these markets function equivalently. However, the emergence of convertibility risk disrupts this equivalence, particularly evident in scenarios similar to Brazil's experiences in 2002.

Keywords: international finance; macroeconomics; exchange rate; forward markets.
JEL Code: F3, E44, F41, G15.

1. Introduction

We construct a model aimed at elucidating spot and future exchange rates determination, accounting for contracts featuring diverse settlement types. Existing open-economy macro models often confine themselves to the spot foreign exchange market, dictated by the interplay of supply and demand for foreign currency and subject to various frictions. While this market holds evident importance, acknowledging the existence of forward currency markets becomes imperative, particularly for countries like Brazil, where such markets may hold even greater significance.

Existing literature addressing currency futures markets often overlooks their specific operational intricacies. Instead, the existence of forward exchange rates is typically assumed, either explicitly or implicitly, within models aimed at elucidating phenomena such as deviations from covered interest rate parity (Du and Schreger, 2022; Coffey et al., 2009; Du et al., 2018; Cerutti et al., 2021), or currency hedging in international investments (Hagelin and Pramborg, 2004; Brown, 2001). Our study reveals that under conditions of low risk of external crises, typical of most developed markets, various types

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of future currency markets tend to be equivalent in terms of pricing and functioning. This equivalence allows for abstraction in models where this market is not the primary focus. However, certain operational nuances, particularly involving the settlement of contracts, can wield critical differences in specific contexts, notably in emerging countries, thus warranting attention that has hitherto been lacking in the literature.

The fundamental distinction in forward markets, extending beyond currencies, lies in Deliverable Forward (DF) and Non-deliverable Forward (NDF) markets. The distinction stems from the settlement model: in DF markets, contracts are settled at maturity through physical delivery of goods accompanied by predetermined payments between parties, whereas in NDF markets, settlement is solely financial. In the realm of currency markets, the modus operandi remains analogous, wherein for DF contracts, a forex transaction occurs at maturity for settlement, whereas in NDF markets, occurs only a net settlement based on the differences in prices, without actual forex transactions. If the buying party intends to acquire the foreign currency, they can simply purchase it in the spot market using the settlement difference, effectively akin to receiving the foreign currency directly.

Empirical evidence underscores the prevalence and significance of NDF markets in currency dynamics, especially in emerging markets (McCauley et al., 2014), most of them in Asia. Typically situated offshore with respect to the country whose currency is traded, these over-the-counter (OTC) markets primarily operate in major financial centers such as the United States and England. Schmittmann and Teng (2020) points out that the presence of such markets becomes attractive to investors by presenting an alternative to those countries' domestic markets, avoiding domestic regulation, convertibility risk, and credit risk. Thus, it becomes advantageous for investors who wish get exposure to or get protection against currency fluctuations risks of currency variations in such countries to use NDF markets instead of subjecting themselves to bureaucracy, costs, and risks related to external capital flows to those countries. Effectively, Debelle et al. (2006) presents the case of Australia that presented a scenario of high capital controls in the 1970s and 1980s, similar to Asian markets currently and with the prevalence of NDF markets. In that country there was a transition to a deliverable market following a series of incentives and regulatory changes from the mid-1980s until the NDF market practically disappeared compared to the DF market in terms of trading volume. In the same direction, McCauley et al. (2014) argue that NDF markets tend to disappear only after non-residents gain complete access to domestic markets and the assets achieve convertibility.

Despite the wealth of this literature elucidating the genesis and significance of NDF currency markets, particularly in emerging economies, the rationale for their presence in the Brazilian context is somewhat different. Unlike in other emerging markets where the impetus for such markets originated from external investors seeking exposure, in Brazil, the demand primarily emanated from domestic agents.

In Brazil, the legislation has historically restricted participation in the foreign exchange market. The Regulation of the Foreign Exchange Market and International Capital (RMCCI), overseen by the Central Bank of Brazil (BCB) until 2014, authorized only a select few entities, primarily financial institutions, to hold foreign currency accounts, with tightly controlled movements. Garcia and Urban (2005) and Prates (2015) argue that this institutional framework resulted in the characteristic that foreign exchange transactions within the country are practically nonexistent, and this internal non-convertibility of the currency was the motivating factor for the emergence of the Brazilian NDF market with its specificities, the main one being precisely the currency of settlement of transactions.

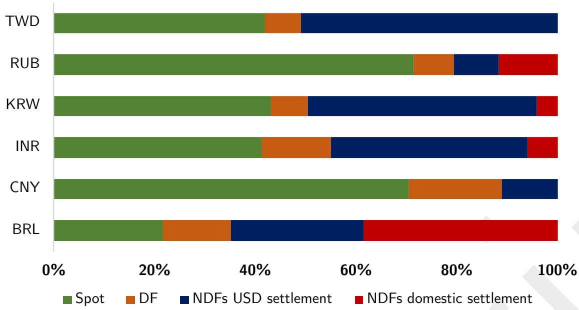
This regulatory backdrop delineates the functioning of the Brazilian market distinctively from prevalent Non-Deliverable Forward markets observed in other emerging economies. While offshore NDF markets settled in a strong foreign currency, typically the US dollar, operate beyond national jurisdiction, Brazil's relevant future market operates domestically (onshore), subject to local jurisdiction, regulation, and intervention by national authorities, with settlements in the domestic currency (BRL). Hence, in this exposition, we follow Garcia and Volpon (2014) employing the term Domestic Non-Deliverable Forward (DNDF) to denote this market, reserving the term NDF for offshore markets settled in strong foreign currency. In a broader sense, we can assert that despite operational similarities, these two markets possess contrasting natures: in NDF markets, the underlying asset of derivatives pertains to various emerging currencies, whereas in the case of Brazilian DNDF markets the underlying asset is the US dollar itself and not the Brazilian Real (BRL).

Figure 1 illustrates the comparative traded volume in various emerging currency markets (McCauley and Shu, 2016).

Observably, for most emerging markets, NDF markets settled in the local currency, albeit existing, do not constitute the dominant share in terms of traded volume. Spot markets and offshore NDFs settled in foreign currency collectively account for over 80% of total transactions across most countries. In contrast, for Brazil, the DNDF market assumes primacy, accounting for nearly 40% of total traded volume, surpassing the spot market by twofold.

Motivated by regulatory incentives, the Brazilian DNDF market emerged as a pragmatic substitute for the spot foreign exchange market. Its lower reg-

Figure 1
Currency Market Turnover



Source: McCauley and Shu (2016)

ulatory burden and exclusively financial settlements, devoid of the necessity for foreign currency accounts, render it accessible to individuals and entities for hedging, speculation, and protection purposes. Ventura and Garcia (2012) shows that operations that are typical of spot markets have been transferred to the future market, while Garcia and Urban (2005), Garcia et al. (2015) and BCB (2019) point out that the price formation in the Brazilian foreign exchange market actually occurs in the future market and is only later transmitted to the spot market. As a comparison, Wang et al. (2014) shows that for Taiwan, the flow of information is from the spot market to the DF and NDF derivatives markets, while for Korea, the stronger direction is from the offshore NDF market to the spot, which was also found by Park (2001).

While extant literature has elucidated the genesis and significance of NDF and DNDF markets, research on their consequences and practical implications vis-à-vis commonly modeled Deliverable Forward (DF) futures markets is much scarcer: Doukas and Zhang (2013) empirically compares the performance of carry trade operations in currencies with DFs and NDFs, and Garcia and Volpon (2014) is the only one to evaluate a specific scenario with the presence of DNDFs, such as in Brazil, indicating potential convertibility risk and demonstrating how the market's presence allows for an alternative exchange rate intervention strategy accessible to the monetary authority. Moreover, the existence of DNDFs could, at principle, potentially enhance the government's intervention capacity, given that all operations are settled in domestic currency.

Thus, to date, the literature lacks studies modeling economies featuring such markets and characteristics, as well as theoretically assessing the implications of DNDF markets compared to DF and NDF markets. This includes examining and reproducing the occurrence of apparently anomalous phenomena discussed in section 2 that may arise in this type of environment and that have not been fully explained by the existing literature. The proposed model, while apt for explicating the Brazilian context, holds also broader relevance for analyzing emerging markets, particularly amidst active attempts to develop domestic currency derivatives markets and increase market segmentation. This research addresses this gap, offering insights into the potential consequences of such developments in the exchange rate determination, thereby enriching our understanding of contemporary currency market dynamics.

2. Empirical Evidences

While the proposed model holds applicability across nations featuring such markets, its primary emphasis naturally gravitates towards Brazil, given the nation's distinctive characteristics concerning the settlement of foreign exchange futures contracts. Thus, it becomes imperative to delineate key stylized facts from Brazil's recent history to demonstrate their relationship with the analyzed markets and provide an explanation through our model. Particularly, the episode under scrutiny pertains to the economic context surrounding the year 2002, marked by certain ostensibly anomalous outcomes. It is noteworthy that this article's aim is not to undertake a specific case study of this episode but rather to devise a model capable of elucidating both the ostensibly normal patterns, observed predominantly, and this specific stress scenario.

Primarily, it's pertinent to note that the events scrutinized, commencing from mid-2001 and notably 2002, are predominantly politically driven. They stem from apprehensions among market participants regarding the potential election of Luiz Inácio Lula da Silva, colloquially known as Lula, to the presidency, an eventuality that indeed occurred in October 2002. Lula, a prominent union leader and a perennial candidate in preceding elections, had a history of radical statements regarding economic and financial issues.

In addition to political uncertainties, the economic landscape was fraught with financial fragilities, particularly concerning external debt. Brazil had grappled with successive episodes of crises stemming from external debt burdens throughout the 20th century, necessitating frequent negotiations and restructurings (Cerqueira, 2003). Moreover, the exigencies of balance of payments adjustments during crises primarily burdened the state (Werneck, 1986). Despite the stabilization efforts stemming from the Real Plan in 1994, Brazil's external debt levels and composition in the early 2000s continued to render the nation susceptible to external vulnerabilities (Werneck, 2014).

Hence, the phenomena and market reactions witnessed amidst the electoral context reflect the confluence of political risks amid an already precarious external debt scenario. Unsurprisingly, Brazilian external debt assumed center stage in public discourse, often depicted as a "national enemy" by significant segments of society. Illustratively, in 2000, an unofficial plebiscite organized by religious and civil society organizations, with the participation of about 5% of the national electorate, questioned the population about the payment of the external debt. The result, obviously biased, showed over 90% in favorable to default (Nova, 2000).

Among the prominent vocal critics of the external debt situation was the candidate Lula. Some of his statements compiled by Silva (2006) on the topic include: "We can not, we want not and will not pay the external debt" (Jul, 1985), "Our position is clear: we have to suspend the payment of the external debt" (Jan, 1989), and "The Workers' Party is not proposing default on external and internal debts. We do want, indeed, an audit of the external debt." (Feb, 2000). Despite moderate shifts in discourse over the years leading up to 2002, considerable market uncertainty, especially on this front, was inevitable, translating into heightened expectations of a crisis.

Ultimately, despite market apprehensions, the majority of anticipated risks failed to materialize following Lula's election. In 2002, he issued a document titled "Letter to the Brazilian People," pledging to honor contractual obligations. Subsequently, during the government's formation, individuals trusted by the market were appointed, including Henrique Meirelles as the Governor of the Brazilian Central Bank. Upon assuming office in 2003, it became evident that the government's agenda was significantly more moderate than initially feared. Ironically, all this aversion to external debt from the elected president resulted in him fulfilling his promises of truly getting rid of the issue, not through default as feared by the market, but through progressive repayment, resulting in its near elimination by the end of the 2000s.

We commence by presenting the exchange rate's behavior around the election period, as depicted in Figure 2.

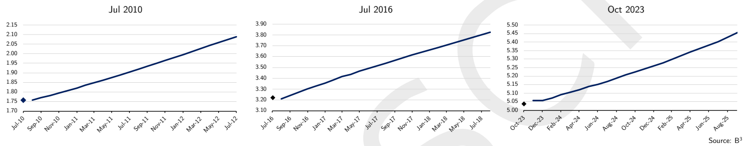
A nominal currency depreciation of 104% unfolds within just under two years, spanning from the onset of 2001 to the zenith of the crisis in the final quarter of 2002. In real terms, this period witnesses a substantial depreciation of 81%.[1]

Regarding the term structure of the exchange rate, we notice an event that is initially challenging to explain. In a country traditionally having higher interest rates than its peers, particularly vis-à-vis the US, the term structure traditionally shows a depreciation trend, in a structure denominated by Keynes (1930) as contango, consistent with virtually any juncture in the country's recent economic history (Figure 3).

[1] These values refer to the BRL/USD exchange rate. Rate depreciated from 1.94 in the first business day in 2001 to 3.96 in its peak in Oct 22, 2002.

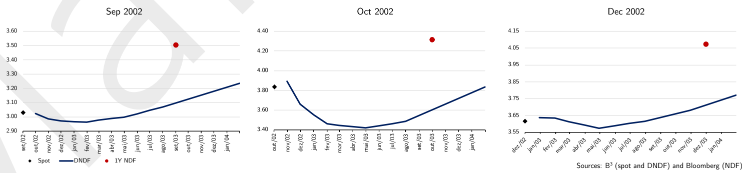
Figure 2
Exchange Rate BRL/USD

Figure 3
BRL/USD Term Structure in Selected Dates



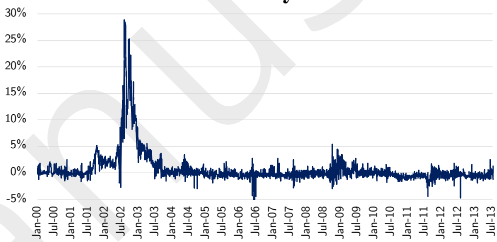
However, during the latter half of 2002, amidst the apex of the country's vulnerability, the traded term structure of the exchange rate exhibited a distinctive U-shape pattern (Figure 4). Notably, a short-term backwardation structure emerged, signaling an anticipated currency appreciation in the ensuing months, succeeded by a resumption of the depreciation trajectory thereafter.

Figure 4
BRL/USD Term Structure in 2002



The ostensibly intuitive explanation for this phenomenon might suggest that market participants anticipated a slight retraction in the exchange rate following an over-depreciation during the election period. However, we contend that such reasoning is inherently flawed. Firstly, if the precipitous depreciation observed in 2002 stemmed from political uncertainties surrounding the elections and their aftermath, there would be no compelling rationale to anticipate an improvement in economic conditions in subsequent periods. Under such circumstances, if agents were truly expecting an exchange rate appreciation in following months it would lead to an opportunity to earn (expected) profit by buying Brazilian currency at present in order to sell it in a more appreciated rate later, thus the consequence of this higher demand for Brazilian Real would be an immediate appreciation, in order to expected depreciation be compatible with the interest rate differential following the uncovered interest rate parity (UIP) condition. Moreover, our analysis offers stronger evidence that the curve reflecting currency appreciation solely pertains to the domestic Non-Deliverable Forward (DNDF) market. By sourcing data from prices in Over-The-Counter (OTC) Non-Deliverable Forward (NDF) markets in the United States for a one-year horizon (illustrated by the red point on the charts in Figure 2.3), a discernible divergence becomes evident. Specifically, a forecast of even greater currency depreciation lies ahead. This disparity in the anticipated term structures between NDF and DNDF markets represents an anomalous occurrence unique to this juncture in the nation's recent economic history. Defining convertibility risk as the percentage difference between forward exchange rate priced by the NDF and DNDF markets we obtain the trajectory depicted in Figure 5.

Figure 5
Convertibility Risk



Convertibility risk = (price NDF)/(price DNDF)- 1. Source: Bloomberg

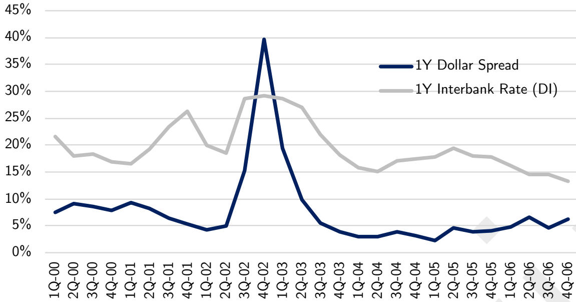
Furthermore, two other distinctive phenomena during this period, seemingly unrelated, ostensibly trace their roots to the foreign exchange futures markets in Brazil. Nevertheless, we shall demonstrate through our model that they stem from the same convertibility risk phenomenon. The first of these anomalies pertains to a spread, which, through arbitrage, should not exist but surfaced during this period, as highlighted by Fraletti (2003), involving government bonds with payments contingent on exchange rate fluctuations. Brazil had previously issued two bonds with this characteristic: the Special Series Central Bank Note (NBC-E) issued by the Central Bank until the enactment of Complementary Law 101 in 2002, known as the Fiscal Responsibility Law, which prohibited the central bank from issuing public debt securities from 2002 onwards. Additionally, there is the National Treasury Note Series D (NTN-D), still provided for in current legislation (Decree 11,301 of 2022), but it is no longer in pratice issued by the National Treasury.

The anomaly in 2002 arises from an financial strategy wherein an agent concurrently invests in one of the currency-indexed government bonds, thereby getting exposure to exchange rate risk, while also entering into a currency swap, exchanging payments contingent on exchange rate fluctuations for remuneration linked to the basic interest rate of the economy. Consequently, the impact of exchange rate fluctuations on the investor's balance sheet is nullified, rendering the final portfolio, in practical terms, equivalent to a government bond indexed to the economy's interest rate. Thus, through arbitrage, the expected returns of these investments should theoretically be identical. However, the anomaly in 2002 pertains to the breakdown of this equivalence, wherein an exchange-rate-linked bond with a swap exhibited higher expected returns than the Treasury Financial Note (LFT), which is the Brazilian government bond indexed to the basic interest rate (Selic rate). We shall demonstrate in section 4 that our model successfully predicts the feasibility of such outcomes during times of crisis.

Finally, in Brazil, there exists another highly liquid derivative market known as the DI X U.S. Dollar Spread or Onshore Dollar Rate (referred to as "Cupom Cambial" in Portuguese). The Dollar spread essentially represents the return on an investment in Brazil originating from external sources, calculated by considering the domestic interest rate discounted by currency depreciation, akin to a Brazilian interest rate available in dollars. As the nominal exchange rate in the country has a long-term depreciation trend due to higher nominal interest rates, the Dollar Spread is consistently lower than domestic rates (see Figure 6). However, during the latter half of 2002, the most critical period in the country's recent financial history, the dollar spread surpassed domestic rates.

As evidence presented indicates, the concurrent manifestation of vulnerability stemming from external debt alongside heightened uncertainty due to political reasons, resulted in a market stress situation, generating a set of very specific and uncommon economic implications in subsequent moments of Brazilian economic history. Through our model, we aim to elucidate the interrelatedness of these phenomena, illustrating how external debt levels and the attendant risk exposure of the economy serve as pivotal determinants for the emergence of such episodes.

Figure 6
Dollar Spread vs Domestic Interest Rate



Source: B³

3. The Model

Our framework extends the model proposed by Gabaix and Maggiori (2015) and Maggiori (2022) to account for imperfect financial markets within a two-country framework: domestic (Brazil) and foreign (US). The model features two representative households and global financial intermediaries that (imperfectly) arbitrate the Uncovered Interest Parity (UIP) differential between the countries, subject to financial constraints.

Representative households in both nations optimize their consumption baskets across intertemporal and intratemporal dimensions. The economies encompass four types of goods derived from endowments: domestic nontradable goods, foreign nontradable goods, domestic tradable goods, and foreign tradable goods. Each household selects from only three: the nontradable goods of its own country and the two tradable goods.

The distinctive additions in our model are the incorporation of a government sector, and households possibility to also engage in the following three exchange markets:

- **Spot market:** Direct conversion between domestic and foreign currencies at the current exchange rate.

- **Deliverable Forward (DF) market:** Households agree to buy (sell) a determined amount Z^n_{t-n} of exchange rate contracts with settlement in n periods ahead at some price $p^{Z,n}_{t-n}$. On the settlement date, the contract buyer receives 1 unit of foreign currency per contract, and the seller receives $p^{Z,n}_{t-n}$ units of domestic currency, independently of the current spot exchange rate at that date.

- **Domestic Non-Deliverable Forward (DNDF) market:** Households agree to buy (sell) a determined amount W_{t-n}^{n} of exchange rate contracts with settlement in n periods ahead at some price $p_{t-n}^{W,n}$. On the settlement date, if the current spot rate is more depreciated (appreciated), the domestic household receives (pays) the difference in domestic currency between the current spot rate and the agreed price. It is worth noting that the settlement does not include a true currency exchange but rather a financial transaction settled entirely in domestic currency.

Our decision not to model the offshore NDF market with settlement in foreign currency stems from its equivalence to the DF market within our framework. We focus on modeling the DNDF market, which hinges on the official domestic exchange rate level set by the government to determine settlement payments. Consequently, it could be indirectly influenced by this agent. As the DF market evolves with effective currency exchange and NDFs settle in offshore markets outside the domestic jurisdiction, they are, for all intents and purposes, equivalent in our model, especially since we do not incorporate specific frictions about NDF and DF markets, such as convenience yields. Therefore, we opt to model only the DF market.

3.1 Multiple Exchange Rates

In a standard monetary system devoid of financial frictions, the exchange rate serves as a relative price between two currencies, similar to regular prices in the economy. It emerges from the interplay between supply and demand forces, allowing the current exchange rate level to adjust to achieve market equilibrium. Additionally, the average spot rate level is crucial in determining payments in derivative assets, indexing contracts contingent to exchange rate variation.

However, across monetary history in several countries, including Brazil for decades in the 20th century, the existence of two or more simultaneous exchange rates was prevalent. These episodes can be directly linked to legal or regulatory constraints on agents in the economy seeking to buy or sell foreign currency. The simplest arrangement involves one “official” market with high regulatory oversight, where prices and/or quantities are tightly controlled by the government, and one “parallel” market operating without regulation or official institution approval. In some instances, participation in such markets might constitute a legal offense, with penalties that may include detention in certain countries.[2] In others, liquidity and participation are so widespread that it becomes the standard market for exchange transactions, with its existence

[2] In Brazil, Law 7492/86 stipulates a penalty of 2 to 6 years of imprisonment for individuals engaging in unauthorized foreign exchange transactions.

practically acknowledged and not repressed by the state.[3] These exchange market control measures typically aim to forestall undesirable exchange rate depreciation. The typical scenario is that the parallel market equilibrium reflects a more depreciated rate than the official one.

In our model, we incorporate two exchange rates: the official *de jure* nominal rate that indexes contracts and is officially used by the government and agents in official markets, defined as h_t; and the parallel or market *de facto* rate, denoted as e_t, which represents the simple equilibrium between the supply and demand for foreign currency[4]. For many countries and most of the time, in the absence of external crises and currency centralization, both rates are essentially the same. In other words, the official exchange rate genuinely reflects the equilibrium rate, and the relationship $e_t = h_t$ holds, with the official rate floating as usual in a flexible exchange rate regime. However, we include in our model the possibility of the government arbitrarily setting and fixing the official rate h_t (discussed in section 3.5), with only the market rate e_t floating according to supply and demand forces.

3.2 Balance of Payments

We propose the following formulation for the balance of payments of the economy (evaluated in domestic currency):

$$X_t - M_t - R_{t-1}Q_{t-1} + Q_t + \sum_{n=1}^{N} Z_{t-n}^{n}(e_t - p_{t-n}^{Z,n}) + \sum_{n=1}^{N} W_{t-n}^{n}(h_t - p_{t-n}^{W,n}) - F_t = 0 \tag{1}$$

The first two terms in the equation above $X_t - M_t$ represent the trade balance where X_t is the value of domestic exports and M_t is the value of imports. The remaining terms represent the capital account, where Q_t the optimal inflows (outflows) of resources from the intermediaries problem (discussed at section 3.4), and the term $\sum_{n=1}^{N} Z_{t-n}^{n}(e_t - p_{t-n}^{Z,n})$ represents the net flows from the settlement of DF contracts agreed in the previous periods.

For instance, in period $t = 10$, assuming constant availability of DF contracts for the subsequent two periods ($N = 2$), the DF contracts purchased at period $t = 8$ for two periods ahead Z_8^2 at price $p_8^{Z,2}$ are settled, as also the DF contracts purchased at period $t = 9$ for one period ahead Z_9^1 at price $p_9^{Z,1}$.

[3] Notice Argentina's current situation where the parallel 'Blue Dollar' market becomes the prominent exchange rate market for households (Wolffelt, 2023).

[4] We define both exchange rates as units of domestic currency per US dollar (in the case of Brazil, BRL/USD), implying that an increase in e_t and h_t means depreciation of the domestic currency

Notice that DF contract are settled with an effective currency exchange, so the short agent should pay in foreign currency the amount that could be sold at market at market rate e_t. It's worth noting that DF contracts are settled with an effective currency exchange, necessitating the short agent to remit payment in foreign currency equivalent to the amount tradable at the market rate e_t. Additionally, given our assumption of an endowment economy and the households' inability to engage in foreign bond markets, the agents utilize the derivatives market solely as a hedge against currency depreciation, which could inflate the cost of their consumption baskets. Consequently, any net gains from the derivatives market are promptly repatriated to the home country as capital flows, thereby necessitating inclusion in the balance of payments.

Similarly, the term $\sum_{n=1}^{N} W_{t-n}^{n}(h_t - p_{t-n}^{W,n})$ is interpreted analogously, albeit concerning DNDF contracts, which are settled financially without actual currency exchange, predicated on the difference between the prevailing spot rate set by the government (h_t) and the contracted price.

Lastly, F_t denotes an exogenously determined capital outflow from global markets, constituting the sole source of uncertainty in our model, with $E_{t-1}[F_t] = 0$. This stochastic variable encapsulates all political and economic uncertainties emanating from domestic and external sources, precipitating capital inflows or outflows from the country. A positive (negative) value for F_t signifies a net exogenous outflow (inflow) of resources from (to) the country, exerting downward (upward) pressure on the exchange rate.

3.3 Households

We assume for household problem a generalization of the one in Gabaix and Maggiori (2015), adding forward markets and extending for an arbitrary number of periods. That consumption basket to domestic and foreign households are given by:

$$C_t = \left[(C_{NT,t})^{\chi} (C_{H,t})^{a} (C_{F,t})^{\iota} \right]^{\frac{1}{\chi + a + \iota}} \tag{2}$$

$$C_t^* = \left[(C_{NT,t}^*)^{\chi^*} (C_{H,t}^*)^{\xi} (C_{F,t}^*)^{a^*} \right]^{\frac{1}{\chi^* + \xi + a^*}} \tag{3}$$

Where $C_{NT,t}$ and $C_{NT,t}^*$ are the consumption of nontradable goods for domestic and foreign households, $C_{H,t}$ and $C_{F,t}^*$ are the consumption of local goods for both households, and $C_{F,t}$ and $C_{H,t}^*$ are the consumption of imported goods.

The domestic household solves the following problem:

$$\max_{\{C_{NT,t},C_{H,t},C_{F,t},Z_t^n,W_t^n\}_{t\geqslant 0,n=1,2,\ldots,N]}} E_0\left[\sum_{t=0}^{T}\beta^t(\chi\ln C_{NT,t}+a\ln C_{H,t}+\iota\ln C_{F,t})\right]$$

Subject to a sequence of budget constraints:

$$P_tY_{NT,t}+p_{H,t}Y_{H,t}+R_{t-1}P_{t-1}B_{t-1}+\sum_{n=1}^{N}\left(Z_{t-n}^n(e_t-p_{t-n}^{Z,n})+W_{t-n}^n(h_t-p_{t-n}^{W,n})\right)=$$
$$P_tC_{NT,t}+p_{H,t}C_{H,t}+p_{F,t}C_{F,t}+P_tB_t+T_t=0 \tag{4}$$

We also define that exists some regulatory control that determine that agents operating in forward exchange rate markets have a maximum limit for their short uncovered positions:

$$Z_t^n\geqslant -\bar{Z}^n \qquad W_t^n\geqslant -\bar{W}^n \tag{5}$$

Notice that agents chooses their consumption baskets composition at each period $(C_{NT,t},C_{H,t},C_{F,t})$ as also their positions in DF (Z_{t-n}^n) and DNDF (W_{t-n}^n) future markets. We also include some net government taxes T_t paid in units of nontradable goods. The price index P_t are supposed as exogenous such that $P_t/P_{t+1}=\pi_t$, and as there is no nominal frictions in the model this price level has no impact on quantities.

We assume the analogous problem for foreign household

$$\max_{\{C_{NT,t}^*,C_{H,t}^*,C_{F,t}^*,Z_t^{n*},W_t^{n*}\}_{t\geqslant 0,n=1,2,\ldots,N]}} E_0\left[\sum_{t=0}^{T}\beta^t(\chi^*\ln C_{NT,t}^*+\xi\ln C_{H,t}^*+a^*\ln C_{F,t}^*)\right]$$

Subject to budget constraints written in foreign currency units, where we normalized the price of foreign nontradable good $P_t^*=1$:

$$Y_{NT,t}^*+p_{F,t}^*Y_{F,t}^*+R_{t-1}^*P_{t-1}^*B_{t-1}^*+\sum_{n=1}^{N}\left(Z_{t-n}^{n*}(1-p_{t-n}^{Z,n*})+W_{t-n}^{n*}\left(\frac{h_t}{e_t}-p_{t-n}^{W,n*}\right)\right)=$$
$$C_{NT,t}^*+p_{F,t}^*C_{F,t}^*+p_{H,t}^*C_{H,t}^*+B_t^*T_t^*=0 \tag{6}$$

And to short positions constraints:

$$Z_t^{n*} \geqslant -\bar{Z}^n \qquad W_t^{n*} \geqslant -\bar{W}^n \tag{7}$$

As our model does not include production, the currency depreciation is always negative for the domestic household due to the increase in the price of imported goods, and the opposite for foreign household. Given this, households will engage in the forward exchange rate market to hedge against currency depreciation. Consequently, domestic agents would prefer to take a positive (long) position $Z_t^n, W_t^n > 0$, while the foreign household would take a negative (short) position on these contracts $Z_t^{n*}, W_t^{n*} < 0$. As we assumed only short position limits, the constraints will be binding only for foreign households, while for domestic ones it will not be binding for sure, and thus can be ignored in their optimization problem.

Assuming the sequence of Lagrange multipliers μ_t and μ_t^* for the budget constraints we derive the optimality conditions:

$$
\begin{aligned}
(C_{NT,t}) &\quad : \quad \beta^t \chi \frac{1}{C_{NT,t}} - \mu_t P_t = 0 \\
(C_{H,t}) &\quad : \quad \beta^t a \frac{1}{C_{H,t}} - \mu_t p_{H,t} = 0 \\
(C_{F,t}) &\quad : \quad \beta^t \iota \frac{1}{C_{F,t}} - \mu_t p_{F,t} = 0 \\
(B_t) &\quad : \quad -\mu_t P_t + \mu_{t+1} R_t P_t = 0 \\
(Z_{t-n}^n) &\quad : \quad p_{t-n}^{Z,n} - E_{t-n}[e_t] = 0 \\
(W_{t-n}^n) &\quad : \quad p_{t-n}^{W,n} - E_{t-n}[h_t] = 0 \\
(\mu_t) &\quad : \quad P_t Y_{NT,t} + p_{H,t} Y_{H,t} + R_{t-1} P_{t-1} B_{t-1} + \sum_{n=1}^{N} \Big(Z_{t-n}^n (e_t - p_{t-n}^{Z,n}) \\
&\qquad + W_{t-n}^n (h_t - p_{t-n}^{W,n}) \Big) = P_t C_{NT,t} + p_{H,t} C_{H,t} + p_{F,t} C_{F,t} + P_t B_t + T_t = 0 \\
(C_{NT,t}^*) &\quad : \quad \beta^t \chi \frac{1}{C_{NT,t}^*} - \mu_t^* = 0 \\
(C_{F,t}^*) &\quad : \quad \beta^t a \frac{1}{C_{F,t}^*} - \mu_t^* p_{F,t}^* = 0 \\
(C_{H,t}^*) &\quad : \quad \beta^t \xi \frac{1}{C_{H,t}^*} - \mu_t^* p_{H,t}^* = 0
\end{aligned}
$$

$$
\begin{aligned}
(B_t^*) \quad &: \quad -\mu_t^* + \mu_{t+1}^* R_t^* = 0 \\
(Z_{t-n}^{n*}) \quad &: \quad Z_{t-n}^{n*} = -\bar{Z}^n \\
(W_{t-n}^{n*}) \quad &: \quad W_{t-n}^{n*} = -\bar{W}^n \\
(\mu_t^*) \quad &: \quad Y_{NT,t}^* + p_{F,t}^* Y_{F,t}^* + R_{t-1}^* B_{t-1}^* + \sum_{n=1}^{N} \left(Z_{t-n}^{n*}(1 - p_{t-n}^{Z,n*}) \right. \\
&\qquad \left. + W_{t-n}^{n*} \left(\frac{h_t}{e_t} - p_{t-n}^{W,n*} \right) \right) = C_{NT,t}^* + p_{F,t}^* C_{F,t}^* + p_{H,t}^* C_{H,t}^* + B_t^* + T_t^* = 0
\end{aligned}
$$

Considering we can adjust the scale of utility parameters (χ, a, ι), and (χ^*, ξ, a^*), and supposing that endowments and taxes are constants over time we set them conveniently such that $\chi = Y_{NT} - T$ and $\chi^* = Y_{NT}^* - T^*$, implying that $\chi = C_{NT,t}$ and $\chi^* = C_{NT,t}^*$. Under these adjustment we conclude that $\mu_t = \frac{\beta^t}{P_t}$ and $\mu_t^* = \beta^t$. Solving the relevant equations we obtain:

$$\iota P_t = C_{F,t} p_{F,t} \equiv M_t \tag{8a}$$

$$R_t = \frac{P_{t+1}}{P_t} \frac{1}{\beta} \tag{8b}$$

$$p_{t-n}^{Z,n} = E_{t-n}[e_t] \tag{8c}$$

$$p_{t-n}^{W,n} = E_{t-n}[h_t] \tag{8d}$$

$$\xi = C_{H,t}^* p_{H,t}^* \equiv X_t^* \tag{8e}$$

$$R_t^* = \frac{1}{\beta} \tag{8f}$$

$$\bar{Z}^n = -Z_{t-n}^{n*} = Z_{t-n}^n \tag{8g}$$

$$\bar{W}^n = -W_{t-n}^{n*} = W_{t-n}^n \tag{8h}$$

3.4 Financial Intermediaries

We assume the existence of global financial intermediaries following exactly the same specification as in Gabaix and Maggiori (2015). These firms aim to obtain profit based on the interest rate differential between the countries, considering the expected exchange rate variations, but are subject to limited capacity to bear risk. The choice to include intermediaries is made because they act as an intermediary between holding uncovered interest parity (UIP) and financial autarchy, allowing expectations about future movements in the exchange rate to impact the current rate in some way but not perfectly.

Essentially, financial intermediaries profit by borrowing funds in countries with lower interest rates and investing them in those with higher rates (adjusted for expected currency appreciation/depreciation). We assume each financier expected value function:

$$V_t = E_t \left[\beta \left(R_t - R_t^* \frac{e_{t+1}}{e_t} \right) \right] \tag{9}$$

Financial intermediaries proceed with their maximization problem subject to a limited commitment constraint akin to credit models: we assume that there is a possibility for such intermediaries to divert funds, such that only a portion $1 - \Gamma$ is recoverable. Given that lenders anticipate this possibility, in equilibrium the resultant problem is that intermediaries maximize their value function subject to the following constraint:

$$\frac{V_t}{e_t} \geqslant \Gamma \left(\frac{q_0}{e_0} \right)^2 \tag{10}$$

Using the same interpretation ans solution steps from Gabaix and Maggiori (2015) we obtain that aggregate optimal policy for intermediaries is given by:

$$Q_t = \frac{1}{\Gamma_t} E_t \left[e_t - e_{t+1} \frac{R_t^*}{R_t} \right] \tag{11}$$

Where Q_t is the total quantity of capital, in foreign currency, borrowed/invested by the intermediaries. Replacing the values for R_t and R_t^* obtained at (8b) and (8f) :

$$Q_t = \frac{1}{\Gamma_t} E_t \left[e_t - e_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{12}$$

3.5 Government

The government faces the following nominal budget constraint:

$$G_t^M + G_t^D + B_{t-1} R_{t-1} + h_t D_t^* = T_t + B_t \tag{13}$$

Where G_t^M is the nominal government mandatory spending, G_t^D is the discretionary government spending, B_t is the domestic issued debt and T_t is the tax revenue (which we assume that is paid in units of nontradable domestic good). All these variables are nominal ones and evaluated at domestic currency. D_t^* is the external debt to be paid at each period denominated in external currency (in our case US dollars). Dividing this constraint by P_t to calculate the real constraint:

$$g_t^M + g_t^D + b_{t-1}\frac{P_{t-1}}{P_t}R_{t-1} + \frac{h_t}{P_t}D_t^* = \tau_t + b_t \tag{14}$$

We assume that, at least in the short term, real mandatory spending and tax collection are constant, and the government will simply roll over the nominal domestic debt to keep it constant. Considering that discretionary spending must be non-negative, we can write:

$$g^M + b\frac{P_{t-1}}{P_t}R_{t-1} + \frac{h_t}{P_t}D_t^* \leqslant \tau + b \tag{15}$$

Using the result for R_t obtained in (8b), defining $g \equiv \tau - g^M - b(1-1/\beta)$ as the net domestic government surplus and manipulating we reach:

$$\frac{h_t}{P_t} \leqslant \left(\frac{\tau - g^M - b\left(1-\frac{1}{\beta}\right)}{D_t^*}\right) \equiv \frac{g}{D_t^*} \tag{16}$$

This outcome underscores that for the government to adhere to the budget constraint, there must exist a threshold for the real exchange rate delineated by the domestic surplus and external obligations. Now, we make an additional (and very plausible) assumption that, ex ante, the fragmentation of exchange rates, leading to the emergence of multiple rates, is undesirable for the government. Therefore, it will keep the official rate (h_t) and the market rate (e_t) identical unless it is strictly necessary to break the equality, by fixing the exchange rate h_t at a different level from its "shadow" (market) value e_t, in order to satisfy the government´s budget constraint:

$$\frac{h_t}{P_t} = \min\left\{\frac{e_t}{P_t}\ ;\ \frac{g}{D_t^*}\right\} \tag{17}$$

The interpretation of these findings is as follows: in the absence of external debt or any obligations denominated in foreign currency, the ratio $g/D_t^* \rightarrow \infty$, implying that the government will permit the exchange rate to freely adjust according to market fundamentals. However, as the level of external obligations increases, the maximum exchange rate feasible by the government to ensure solvency decreases. If the equilibrium market exchange rate surpasses this limit, the government will intervene in the currency market, fixing the official rate at a more depreciated level that guarantees its ability to meet payment obligations.

Lastly, we must impose an additional constraint on the level of external debt D_t^*. A government burdened with external indebtedness faces dual challenges: firstly, the necessity of generating a surplus adequate to service the debt. In this scenario, currency depreciation poses a risk as it elevates the domestic surplus required to fulfill commitments. However, there exists another dimension: even if sufficient resources in domestic currency are available, the government must have an ample supply of foreign currency accessible for purchase in the market. If external obligations denominated in foreign currencies exceed the foreign currency revenue from exports, the government will be unable to service its external debt, irrespective of the domestic surplus. In our model, we abstract from this risk, assuming that condition $D_t^* < \xi$ always holds.[5]

3.6 Solution

Considering that exports revenues are bring to domestic country at official exchange rate we have:

$$p_{H,t} = p_{H,t}^* h_t \tag{18}$$

Using (8e) we obtain that exports nominal value in domestic currency is given by:

$$X_t = h_t X_t^* \equiv C_{H,t}^* p_{H,t} = h_t C_{H,t}^* p_{H,t}^* = h_t \xi \tag{19}$$

Joining the result about with imports value in (8a), the price conditions (8c) and (8d) in the Balance of Payments equation (1) we get:

$$h_t \xi - P_t \iota - \frac{P_t}{P_{t-1}} \frac{1}{\beta} Q_{t-1} + Q_t + \sum_{n=1}^{N} \bar{Z}^n (e_t - E_{t-n}[e_t]) + \sum_{n=1}^{N} \bar{W}^n (h_t - E_{t-n}[h_t]) - F_t = 0 \tag{20}$$

By simplification reasons, we assume than $N = 1$ or, equivalently, that $\bar{Z}_t^n \to 0$ e $\bar{Z}_t^n \to 0$ for $n > 1$, implying that DF and DNDF contracts are negotiated in a relevant volume only for the next period. In order to solve we rewrite the equation above in terms of real exchange rates: $\tilde{e}_t = e_t / P_t$ and $\tilde{h}_t = h_t / P_t$ and replace the intermediaries solutions from (12):

$$\tilde{h}_t(\xi + \bar{W}) + \tilde{e}_t \left(\frac{1}{\Gamma_t} + \bar{Z} \right) - \tilde{e}_{t-1} \frac{1}{\beta} \frac{1}{\Gamma_{t-1}} + E_{t-1}[\tilde{e}_t] \left(\frac{1}{\beta} \frac{1}{\Gamma_{t-1}} - \bar{Z} \right) - E_t[\tilde{e}_{t+1}] \frac{1}{\Gamma_t} - E_{t-1}[\tilde{h}_t] \bar{W} - \iota - \tilde{F}_t = 0 \tag{21}$$

[5] The occurrence of this circumstance could precipitate a situation similar to that analyzed by Keynes (2019), concerning post-World War I reparations. In such a scenario, excessive outflows of resources would erode the terms of trade, rendering external obligations unmanageable regardless of the exchange rate level.

The equation above only holds for $t > 1$, if $t = 0$ there is no Q_{-1} or DF or DNDF contracts agreed in previous periods to be settled, thus assuming that there is no external crisis with centralization $e_0 = h_0$ and we can write:

$$\tilde{e}_0(\xi + \frac{1}{\Gamma_0}) + E_0[\tilde{e}_1]\frac{1}{\Gamma_0} - \iota = 0 \tag{22}$$

The final solution system is given by the equations (21) and (22) above, and the government policy for h_t from (17):

$$\tilde{h}_t = \min\left\{\tilde{e}_t \; ; \; \frac{g}{D_t^*}\right\} \tag{23}$$

3.7 Calibration

We calibrate the government foreign denominated external obligations as constant over the periods such that $D_t^* = D^* \; \forall t$. We also assume that flows risk are concentrated in the period $t = 1$, due to some economic or political stressful moment. $\tilde{F}_1$ follows a normal distribution:

$$\tilde{F}_1 = \mathcal{N}(0, \sigma^2) \tag{24}$$

For simplicity we define the following values for model parameters. As there is a finite time model we also set the intertemporal discount factor $\beta = 1$. For time horizon we set $T = 15$ as it is enough to show all relevant part of forward structure of exchange rate market.

Parameter	Description	Value
ι	Domestic household utility parameter for foreign goods	1
ξ	Foreign household utility parameter for imported goods	1
Γ	Friction level faced by financial intermediaries	1
g	Government domestic surplus	1
β	Intertemporal discount factor	1
$\bar{Z}$	Limit to short position in DF markets	0.1
$\bar{W}$	Limit to short position in DNDF markets	0.1
σ^2	Variance of stochastic financial flows	0.1
T	Household time horizon	15
π_t	Domestic inflation rate	0.02
P_0	Initial domestic price level	1

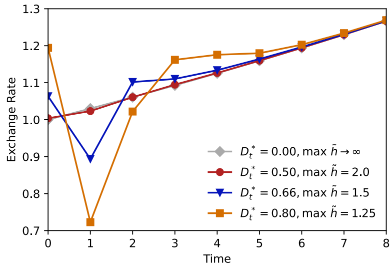
Table 1: Calibration Parameters

4. Results

4.1 Convertibility Risk

Solving the described model we can derive the term structure for DNDF contracts according to external obligations level D_t^*.

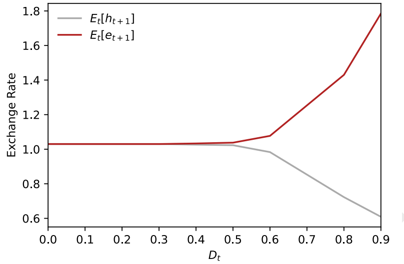
Figure 7
Exchange Rate Term Structure



Under low levels of external debt, the government encounters minimal challenges, as even in the event of pronounced currency depreciation, it retains the capacity to meet its obligations. However, as external liabilities escalate, the government's ability to make payments becomes contingent upon the exchange rate: with a positive realization of capital flows or a minor outflow, the government can manage its debts. Yet, in instances of substantial outflows resulting in significant currency depreciation, the government may be compelled to intervene in the exchange market, fixing the official exchange rate at its maximum feasible level. Since the price of DNDF contracts reflects the average for this official rate, and only one side bears the risks, this leads to an anticipated decrease in $E_t[h_{t+1}]$. This relationship is evident in Figure 8, depicting the prices of 1-period maturity DF and DNDF contracts relative to the external debt level. Notably, as external debt levels climb and the ceiling for the real official exchange rate diminishes, there is a more pronounced depreciation in the current period followed by a larger anticipated decline in the subsequent period. However, in all scenarios, the exchange rate term structure eventually reverts to its natural trend, characterized by constant depreciation in high-yield countries like Brazil.

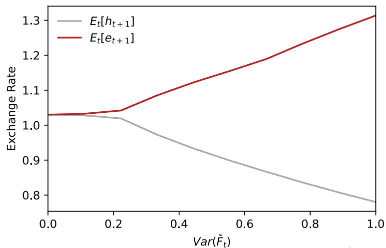
Furthermore, we present a noteworthy observation regarding the relationship between DNDF prices and capital flows risk in Figure 9. We observe that under conditions of significant external debt, heightened risk associated with exogenous capital flows impacting the country's balance of payments leads to an appreciation of the official exchange rate, as determined by market

Figure 8
DF and DNDF Prices for 1 Period Ahead vs External Debt Level



agents. This finding may appear counterintuitive, particularly in the context of emerging markets, where the prevailing expectation would be for higher external risk to induce currency depreciation. However, this apparent contradiction arises from the unique features of our model, wherein the DNDF market solely prices the official exchange rate, a variable subject to government control. By contrast, if we examine the DF market, we find that the traditional relationship between currency depreciation in emerging markets and external risk remains valid.

Figure 9
DF and DNDF Prices for 1 Period Ahead vs Capital Flows Risk



Assuming $D_t^* = 2/3$

Drawing from these findings, we posit that our model successfully explain the empirical findings outlined in section 2 concerning convertibility risk. Specifically, we contend that while DF and DNDF prices are expected to align under normal circumstances, instances of substantial external debt and heightened external risk, as witnessed in Brazil in 2002, can trigger a decoupling of these values. In such scenarios, it is the DF market that accurately reflects the genuine market exchange rate expectation, as the DNDF market becomes susceptible to the influence of centralization risk.

4.2 Synthetic Assets

We introduce two novel features into the model. The first enhancement entails domestic households now having the option to save using two distinct bonds: B_t^d represents the conventional domestic bond, offering a return equivalent to the domestic interest rate R_t; and B_t^c, a government bond offering returns corresponding to exchange rate fluctuations until the subsequent period. Additionally, we incorporate a risk premium associated with the perceived probability by agents that, in certain plausible scenarios, the government may default on public debt obligations.

For exchange rate-indexed bonds, households purchase an amount B_t^c at period t, with a return of $B_t^c \frac{h_{t+1}(s_{t+1}^i)}{h_t}$ expected in period $t+1$. Concerning the risk premium, we introduce a binary function $I_\delta(s_t^i)$ denoting whether the government defaults on public debt in such a scenario. Reformulating the household problem under this revised framework yields:

$$\max_{\{C_{NT,t},C_{H,t},C_{F,t},Z_t^n,W_t^n,B_t,B_t^c\}_{t\geqslant 0,n=1,2,\ldots,N]}} \sum_{t=0}^{T} \sum_{s_t^i}^{S_t} \beta^t (\chi \ln C_{NT,t}(s_t^i) + a \ln C_{H,t} + \iota \ln C_{F,t}) \pi(s_t^i)$$

Subject to a sequence of budget constraints:

$$\begin{aligned} & P_t Y_{NT,t} + p_{H,t}(s_t^i) Y_{H,t}(s_t^i) + \left(R_{t-1}(s_{t-1}^i) P_{t-1} B_{t-1}^d(s_{t-1}^i) + P_{t-1} B_{t-1}^c(s_{t-1}^i) \frac{h_t(s_t^i)}{h_{t-1}(s_{t-1}^i)} \right) (1 - I_\delta(s_t^i)) + \\ & \sum_{n=1}^{N} \left(Z_{t-n}^n(s_{t-n}^i)(e_t(s_t^i) - p_{t-n}^{Z,n}(s_{t-n}^i)) + W_{t-n}^n(s_{t-n}^i)(h_t(s_t^i) - p_{t-n}^{W,n}(s_{t-n}^i)) \right) \\ & = P_t C_{NT,t}(s_t^i) + p_{H,t}(s_t^i) C_{H,t}(s_t^i) + p_{F,t}(s_t^i) C_{F,t}(s_t^i) + P_t p_{B,t}^d B_t^d(s_t^i) + P_t p_{B,t}^c(s_t^i) B_t^c + T_t = 0 \end{aligned} \tag{25}$$

Notice that now bond assets have prices $p_{B,t}^d$ and $p_{B,t}^c$ for domestic and currency indexed bonds, respectively. Solving the problem following the same steps of section 3.3 we found that

$$\frac{1}{p_{B,t}^d} = R_t^d = \frac{1}{\beta} \frac{P_{t+1}}{P_t} \frac{1}{R_t} \frac{1}{E_t[1 - I_{\delta,t+1}^i]} \tag{26}$$

$$R_t^c = \frac{1}{p_{B,t}^c} = \frac{1}{\beta} \frac{P_{t+1}}{P_t} \frac{h_t}{E_t[h_{t+1}(1 - I_{\delta,t+1}^i)]} \tag{27}$$

4.3 Currency Swaps

With our model, we have the capability to create and price synthetic assets and derivatives. As an illustration, we consider a currency swap. According to the IMF definition (Baliño and Zamalloa, 1997), in a currency swap, the parties agree to exchange two currencies for a specified period to swap the exchange streams of payment until the maturity date. Synthetically, the asset could be decomposed into simultaneous spot and forward transactions with opposite directions.

We construct a swap where the agents trade exchange rate variation against the domestic interest rate. The expected return of this asset from the perspective of an agent short in exchange rate variation and long in the domestic rate, with maturity one period ahead, is:

$$R_t^{swap} = \frac{1}{R_t} \frac{E_t[h_{t+1}]}{h_t} \tag{28}$$

As detailed in the contracts of swaps traded in Brazil at B3[6], the swap settlement does not occur with actual currency transactions but is settled using the official rate, thus we calculate the return using h_t.

Now, let's consider the following financial strategy: an agent could purchase at the same time an amount of this swap contract and the same amount of the exchange rate-indexed bonds described in the previous section with the exact same maturity. By doing so, the agent actually builds a synthetic domestic title, as any positive (negative) variations in the exchange rate would increase (decrease) the bond return but decrease (increase) the return from the swap at the exact same amount, and the net return will be the domestic interest rate. By arbitrage, the price and return of this synthetic bond and the regular domestic bond should be identical. However, in the 2002 scenario described in section 2, the market faced an apparent violation of such parity, with synthetic bonds with swaps providing higher returns compared to domestic bonds. Our model can also successfully provide an explanation for that. Calculating the ratio of the return of synthetic and domestic bonds using the return equations above, we obtain:

$$\frac{R_t^c R_t^{swap}}{R_t^d} = \frac{E_t[1 - I_{\delta,t+1}^i] E_t[h_{t+1}]}{E_t[h_{t+1}(1 - I_{\delta,t+1}^i)]} \tag{29}$$

It is reasonable to assume that external and default crisis episodes are positively correlated:

[6] B3 is the derivatives (and many other assets) exchange in Brazil: www.b3.com.br.

$$cov_t(I^i_{\delta,t+1}, h_{t+1}) > 0 \Rightarrow cov_t(1 - I^i_{\delta,t+1}, h_{t+1}) < 0 \Rightarrow \frac{E_t[1 - I^i_{\delta,t+1}]E_t[h_{t+1}]}{E_t[(1 - I^i_{\delta,t+1})h_{t+1}]} > 1 \tag{30}$$

Thus we conclude that if there is a positive risk of default risk in the next period ($I^i_{\delta,t+1} = 1$ for some possible history realization s^i_{t+1}) then $R^c_t R^{swap}_t > R^d_t$ as observed in the showed Brazilian case. If agents are not pricing that risk and do not believe that there is default risk next period thus $I^i_{\delta,t+1} = 0$, implying $R^c_t R^{swap}_t = R^d_t$ and the arbitrage between these returns holds.

4.4 Dollar spread (onshore dollar rate)

We revisit the empirical evidence on the dollar spread (onshore dollar rate) presented in section 2, where we demonstrated that despite the dollar spread consistently remaining at lower levels compared to domestic interest rates, this relationship reversed during the most critical period for the country in 2002. We contend that this phenomenon can also be elucidated using our framework incorporating two possible exchange rates.

By construction the expected return of the dollar spread with 1 period maturity is given by:

$$R^{DS}_t = R_t \frac{h_t}{E_t[h_{t+1}]} \tag{31}$$

Suppose we are in the first period with initial state where $e_0 = h_0$. Manipulating the equation in (22) we obtain:

$$h_0 = \frac{E_0[e_1] + \Gamma_0 \iota}{(\Gamma_0 \xi + 1)(1 + \pi_t)} \tag{32}$$

Thus:

$$\frac{R^{DS}_t}{R_t} = \underbrace{\left(\frac{1}{(\Gamma_0 \xi + 1)(1 + \pi_t)} \right)}_{<1} \underbrace{\left(\frac{E_0[e_1]}{E_0[h_1]} + \frac{\Gamma_0 \iota}{E_0[h_1]} \right)}_{>1 \text{ as } e_1 \geqslant h_1} \tag{33}$$

We conclude that the ratio between the Dollar Spread and the domestic interest rate could be greater or lower than 1, depending on the interaction of the two terms. As we normalized external prices as 1, the inflation term π_t reflects the inflation differential, thus in countries as Brazil, with higher inflation in comparison with US, the denominator in the first term will be higher, which can lead to a result where $\frac{R_t^{DS}}{R_t} < 1$, especially when there is no external crisis risk ($E_0[h_1] = E_0[e_1]$), which is the typical case for Brazil. However, when there is external risk, the ratio $\frac{E_0[e_1]}{E_0[h_1]}$ will be higher than 1. If the gap between the official and market exchange rates is large enough, this term could compensate for the first term and lead to a scenario where $R_t^{DS} > R_t$, a possible counterintuitive result where interest rates in dollars exceed the domestic ones in the most critical moments for the country.[7]

5. Conclusion

Our framework elucidates the role of settlement methods in foreign exchange futures contracts within economies like Brazil, where settlement occurs mainly in domestic currency. We demonstrate that under controlled external debt, the settlement method becomes inconsequential due to the absence of convertibility risk. Consequently, the distinct market structure in Brazil does not significantly diverge from global implications regarding exchange rate determination. However, in contexts with substantial government external debt and economic uncertainty, the potential for external crises introduces exchange rate centralization risks. This risk perception alone can trigger market anomalies, as observed in Brazil in the mid-2002 period.

In our theoretical framework, the manifestation of crisis phenomena does not hinge on the actual occurrence of a crisis episode. The mere anticipation and formation of expectations among market agents regarding the potential emergence of a crisis state in future scenarios are adequate catalysts for such phenomena. Moreover, this effect intensifies with escalating probabilities of a crisis. For instance, in the aftermath of the 2002 election, despite market apprehensions, the anticipated risks failed to materialize. Consequently, there was an absence of exchange rate centralization or control, multiple exchange rate formation, and unofficial parallel currency markets. Subsequently, as market agents grew convinced that the perceived risks were unfounded, the market trended towards normalization, with these anomalies dissipating by mid-2003.

Despite the notable improvement in Brazil's external debt trajectory since 2002 and the prevailing absence of immediate crisis expectations, our model retains its analytical relevance for potential future scenarios. This is particularly pertinent within the context of countries sharing a similar institutional framework. Moreover, while Brazil's current economic situation may diverge

[7] In the second half of 2002 the EMBI+Br reached its historical maximum at 2443 points (century XXI average is 393 points). Concurrently, CDS soared to a peak of 3790 basis-points within the same period.

from the critical scenario under scrutiny, this may not hold true for other emerging economies. The prevailing situation in Argentina stands as a pertinent exemplar, with very significant convertibility risk nowadays. However, it is crucial to acknowledge that our model's applicability is primarily limited to sporadic crisis events linked to external debt and currency convertibility risk. Should these episodes recur with greater frequency, it is reasonable to anticipate a migration of market agents towards alternative mechanisms and markets, similarly to their shift from spot to futures markets in response to regulatory constraints.

In closing, we delineate potential avenues for future research, commencing with an exploration of modeling the diverse participation dynamics of market agents. While our model predominantly hinges on household participation in futures markets driven by protection against currency pass-through into domestic prices, empirical realities unveil a broader spectrum of motivations guiding market engagement. Indeed, agents, encompassing households and corporations alike, exhibit a multifaceted pursuit of wealth preservation and risk minimization, in both domestic and foreign currencies. Moreover, companies employ derivatives also to hedge against foreign investment risks but, manage imported inputs and remit profits. Consequently, these varied motivations engender divergent market behaviors, rendering market equivalency elusive even in the absence of crises.

Furthermore, advancing on the empirical front holds promise for refining our model's predictive prowess. Beyond qualitative alignment with observed episodes, a quantitative assessment of our model's performance could offer insights into the efficacy of various policy interventions and the identification of potential triggers for mitigating the onset of similar crises. By scrutinizing the model's ability to accurately replicate historical episodes, researchers can more precisely ascertain thresholds for external debt levels to forecast and preempt the emergence of new crises. Hence, an empirical validation not only bolsters the credibility of our model but also augments its utility as a predictive tool for preemptive crisis mitigation strategies.

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Appendix A: Replication Code

The following code replicates the model presented in this paper. The code running requires a python compiler (v. 3.11.5) and the external packages numpy (v. 1.24.4), scipy (v. 1.11.2) and matplotlib (v. 3.8.2), which can be found in the official Python Package Index (PyPI).

```
# coding: utf-8

import numpy as np
from scipy.optimize import minimize
import matplotlib.pyplot as plt
from scipy.stats import norm

np.set_printoptions(suppress = True)


T = 20
bar_W = 0.1
bar_Z = 0.1
beta = 1
Gamma = 1
iota = 1
xi = 1
g = 1
pi = 0.03
P0 = 1
sigma2 = 1


def discrete_normal(n, mu, sigma2):
    width = 4
    sigma = sigma2**0.5
    x = np.linspace(mu - width * sigma, mu + width * sigma, n)

    if n == 2:
        p = 0.5 * np.ones(n)
    elif n > 2:
        p = np.zeros(n)
        p[0] = norm.cdf(x[0] + 0.5 * (x[1] - x[0]), mu, sigma)
        for i in range(1, n - 1):
```

```
            p[i] = norm.cdf(x[i] + 0.5 * (x[i + 1] - x[i]), mu, sigma) - norm.cdf(x[i] - 0.5 * (x[i] - x[i - 1]), mu, sigma)
        p[n - 1] = 1 - np.sum(p[:n - 1])
    return x, p


def create_matrix(limited, F_t, h_bar, E0_e1, E0_h1, e0):
    mat = np.zeros((2*T-2, 2*T-2))
    cte = np.zeros((2*T-2))
    for t in range(1, T):

        if t == 1:
            cte[t-1] = E0_e1*(1/(beta*Gamma) - bar_Z) - E0_h1*bar_W - F_t - iota - e0*1/(beta*Gamma)
            mat[t-1, t-1+T-1] = xi + bar_W
            mat[t-1, t-1] = 1/Gamma + bar_Z
            mat[t-1, t] = - 1/Gamma
        else:
            mat[t-1, t-2] = - 1/(beta*Gamma)
            mat[t-1, t-1+T-1] = xi
            cte[t-1] = - iota
            if t < T-1:
                mat[t-1, t-1] = 1/Gamma + 1/(beta*Gamma)
                mat[t-1, t] = - 1/Gamma
            else:
                mat[t-1, t-1] = 1/(beta*Gamma)
        if limited[t-1]:
            mat[T-2+t,T+t-2] = 1
            cte[T-2+t] = - h_bar
        else:
            mat[T-2+t,t-1] = 1
            mat[T-2+t,T+t-2] = -1
    sol = np.round(np.linalg.solve(mat, -cte),5)

    return sol


def solve_specific(h_bar, F_t, E0_e1, E0_h1, e0):
    limited=[False for t in range(T-1)]
    ind_limited = 0
    for ind_limited in range(T-1):
```

```
        sol = create_matrix(limited, F_t, h_bar, E0_e1, E0_h1, e0)
        if (sol[T-1:] > h_bar).any():
            limited[ind_limited] = True
        else:
            break
    return sol


def solve_average(h_bar, E0_e1, E0_h1, e0, probab_Ft, values_Ft):
    avg = np.zeros(2*T-2)
    for f_t in range(len(values_Ft)):
        F_t = values_Ft[f_t]
        sol = solve_specific(h_bar, F_t, E0_e1, E0_h1, e0)
        avg = avg + sol*probab_Ft[f_t]
    return avg



def period_1(E0_e1):
    return (E0_e1 + Gamma*iota)/(Gamma*xi  + 1)



def iterate_avg(h_bar, e0, probab_Ft, values_Ft):
    E0_e1 = 1
    E0_h1 = 1
    tol = 0.0001
    max_iter = 100
    iter = 0
    while True:
        avg = solve_average(h_bar, E0_e1, E0_h1, e0, probab_Ft, values_Ft)
        if (np.abs(E0_e1 - avg[0]) < tol) and (np.abs(E0_h1 - avg[T-1]) < tol):
            break
        else:
            newE0_e1 = (E0_e1 + avg[0])/2
            newE0_h1 = (E0_h1 + avg[T-1])/2
            E0_e1 = newE0_e1
            E0_h1 = newE0_h1
        iter = iter + 1
```

```
        if iter > max_iter:
            raise Exception("No convergence")
    return avg


def solve_system(h_bar, sigma2):

    values_Ft, probab_Ft = discrete_normal(n=4, mu=0,
    sigma2=sigma2)
    phi = 0.5
    e0 = 1
    iter = 0
    max_iter = 100
    tol = 0.00001
    while True:
        avg = iterate_avg(h_bar, e0, probab_Ft,
    values_Ft)
        e0_inv = period_1(avg[0])
        diff = np.abs(e0-e0_inv)
        if diff < tol:
            break

        e0 = e0_inv*phi + e0*(1-phi)
        iter = iter + 1
        if iter > max_iter:
            raise Exception("No convergence")
    results = np.zeros((T,2))
    results[0,:] = e0
    results[1:,0] = avg[:T-1]
    results[1:,1] = avg[T-1:]
    return results

def nominal_rate(P0, pi, results):
    P = P0
    for t in range(T):
        results[t,:] = results[t,:]*P
        P = P*(1+pi)
    return results



D_t = 0.00001
results000 = nominal_rate(P0=P0, pi=pi, results=
    solve_system(h_bar=g/D_t, sigma2=1))
```

```
D_t = 0.50
results050 = nominal_rate(P0=P0, pi=pi, results=solve_system(h_bar=g/D_t, sigma2=1))
D_t = 0.66
results066 = nominal_rate(P0=P0, pi=pi, results=solve_system(h_bar=g/D_t, sigma2=1))
D_t = 0.80
results080 = nominal_rate(P0=P0, pi=pi, results=solve_system(h_bar=g/D_t, sigma2=1))



figure = plt.figure(figsize=(5,3.5))
chart1 = figure.add_subplot(1,1,1)

x_axis = [i for i in range(T)]

chart1.plot(x_axis, results000[:,1], color="#AAAAAA", label=r'$D_t^*=0.00, \max \ \tilde h \to \infty$', marker="D")
chart1.plot(x_axis, results050[:,1], color="#B22222", label=r'$D_t^*=0.50, \max \ \tilde h = 2.0$', marker="o")
chart1.plot(x_axis, results066[:,1], color="#0514BA", label=r'$D_t^*=0.66, \max \ \tilde h = 1.5$', marker="v")
chart1.plot(x_axis, results080[:,1], color="#D46E01", label=r'$D_t^*=0.80, \max \ \tilde h = 1.25$', marker="s")
chart1.set_xlim([0, 8])
chart1.set_ylim([0.7, 1.3])
chart1.legend(frameon=False)
chart1.set_xlabel("Time")
chart1.set_ylabel("Exchange Rate")

plt.ticklabel_format(style='plain')
plt.tight_layout()
plt.draw()
plt.show()


n_points = 10
Dt_range = np.linspace(0.0001, 0.9, n_points)
series_D = np.empty((n_points, 2))
```

```
series_D [:] = np.nan
for d in range(n_points):
    D_t = Dt_range[d]
    try:
        results = nominal_rate(P0=P0, pi=pi, results=
    solve_system(h_bar=g/D_t, sigma2=1))
        series_D[d,0] = results[1,0]
        series_D[d,1] = results[1,1]
    except:
        pass


figure2 = plt.figure(figsize=(5,3.5))
chart1 = figure2.add_subplot(1,1,1)

x_axis = Dt_range[~np.isnan(series_D[:,0])]

chart1.plot(x_axis, series_D[:,1][~np.isnan(series_D
    [:,1])], color="#AAAAAA", label=r'$E_t[h_{t+1}]$')
chart1.plot(x_axis, series_D[:,0][~np.isnan(series_D
    [:,0])], color="#B22222", label=r'$E_t[e_{t+1}]$')

chart1.set_xlim([0, 0.9])
chart1.legend(frameon=False)
chart1.set_xlabel(r"$D_t$")
chart1.set_ylabel("Exchange Rate")

plt.ticklabel_format(style='plain')
plt.tight_layout()
plt.draw()
plt.show()


sigma_F_range = np.linspace(0.0001, 1, n_points)
series_sigma = np.empty((n_points, 2))
series_sigma[:] = np.nan
D_t = 0.75
for d in range(n_points):
    sigma2 = sigma_F_range[d]
    results = nominal_rate(P0=P0, pi=pi, results=
    solve_system(h_bar=g/D_t, sigma2=sigma2))
    series_sigma[d,0] = results[1,0]
    series_sigma[d,1] = results[1,1]
```

```
figure3 = plt.figure(figsize=(5,3.5))
chart1 = figure3.add_subplot(1,1,1)


chart1.plot(sigma_F_range, series_sigma[:,1], color="#AAAAAA", label=r'$E_t[h_{t+1}]$')
chart1.plot(sigma_F_range, series_sigma[:,0], color="#B22222", label=r'$E_t[e_{t+1}]$')

chart1.set_xlim([0, 1])
chart1.legend(frameon=False)
chart1.set_xlabel(r"$Var(\tilde F_t)$")
chart1.set_ylabel("Exchange Rate")
chart1.text(0.7,0.62,r\"Assuming $D_t^* = 2/3$")

plt.ticklabel_format(style='plain')
plt.tight_layout()
plt.draw()
plt.show()


figure.savefig("figure_model_1.svg", format="svg")
figure2.savefig("figure_model_2.svg", format="svg")
figure3.savefig("figure_model_3.svg", format="svg")
```