Green Misallocation: Allocative Efficiency and Air Pollution*

Jefferson da Hora† Paulo Vaz‡

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Abstract

Understanding productivity differences between countries is one of the central problems of Economic Science, given its influence on living standards. Simultaneously, environmental issues have become more relevant due to the need to mitigate greenhouse gas emissions and combat global warming, requiring the integration of economic development and environmental preservation. This article aims to incorporate pollutant emissions into the investigation of resource misallocation, evaluating potential output gains and variations in pollutants emitted when production factors are efficiently allocated. Based on the financial information of publicly traded Brazilian companies and air pollution data, the methodology of Hsieh and Klenow (2009) is extended to incorporate pollutant emissions. It is observed that the degree of misallocation in 2022 is at the same level as in 2010, however, there has been an improvement in efficient allocation in recent years, with potential output gains, after the equalization of firms' marginal products, decreasing from 181% to 71% between 2015 and 2022. In the evaluation of pollutant emissions, with the elimination of misallocation, it was found that, in 2022, not only would there be output gains of 30%, but the amount of pollutants per product would also decrease by 31%.

Keywords: Total Factor Productivity - TFP. Misallocation. Efficient Allocation. Pollutant Emissions. Air Pollution. Firm-level Data.
JEL Codes: O1, O4, Q5.

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†Department of Economics - Federal University of Pernambuco (UFPE). E-mail: jefferson.vieira@ufpe.br.
‡Department of Economics - Federal University of Pernambuco (UFPE). E-mail: paulo.vaz@ufpe.br.

1 Introduction

Seeking to understand the productivity differences between countries, and their causes, is one of the fundamental issues in Economic Science, given the consensus in development literature that these differences are key determinants of disparities in living standards among countries (Restuccia and Rogerson, 2017).

Environmental concerns have received considerable attention in society, generating particular interest among policymakers. In this context, as we confront challenges such as the imperative reduction of greenhouse gas emissions, adaptation to climate change and mitigation of global warming, it is essential to understand how resource allocation and the pursuit of productivity can impact the environment.

There is an extensive literature that employs Data Envelopment Analysis (DEA) to integrate environmental aspects into the calculation of Total Factor Productivity (TFP), nevertheless, this integration is conducted in a non-parametric manner, i.e., without using the classical production functions of economic theory. Recently, several studies have used the method developed in Hsieh and Klenow (2009) to examine inefficient resource allocation in specific sectors, such as extractive industries and notably the electric power sector. However, such studies do not take into account pollutant emissions, which could affect the conclusion that efficient allocation of capital and labor would be ideal for society, as such "efficient" allocation could lead to increased pollution. In Acemoglu et al. (2010), the authors reach a similar conclusion, showing that there exists a laissez-faire equilibrium, decentralized and without any policy intervention, in which innovation occurs only in the polluting sector, and the growth rate of productivity using polluting inputs is increasing, while productivity using clean inputs is constant. On the other hand, considering pollutant emissions may also reveal that efficient allocation not only increases economic production but also reduces emissions.

In Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), the effects of resource misallocation among firms are evaluated, emphasizing that this inefficient allocation significantly contributes to productivity differences between countries, and highlighting potential gains from more effective allocation. Using firm-level microdata from the United States, China, and India, quantitative evidence was provided on the impact of misallocation on productivity levels, revealing that distortions in resource allocation could explain up to 60% of the observed productivity loss in China and India compared to the United States (Hsieh and Klenow, 2009). Subsequently, this theme has been addressed in specific studies for other countries, such as Vasconcelos (2017), which identified evidence of misallocation in the Brazilian manufacturing sector.

Using a similar approach to Hsieh and Klenow (2009), Yu et al. (2021), Yu et al. (2022), and Wu et al. (2022) also investigate the impacts of misallocation in the electric power generation sector in China, including renewable electric power plants, while Zhang and Kong (2022) examine the relationships between Total Factor Productivity of electric power firms and energy transition policies in China.

In the Brazilian context, environmental issues assume even greater importance due to the abundance of natural resources and its unique biodiversity. The country faces the challenge of reconciling economic development with environmental preservation, especially in sectors highly dependent on natural resources such as mining, oil exploration and refining and energy generation. In light of this, this paper aims to integrate firm pollutant emissions into the investigation of (green) misallocation, as well as to assess their impacts on Total Factor Productivity (TFP), potential output gains with efficient allocation, and variations in pollutant quantities generated.

In addition to this introduction, the second section covers concepts related to resource misallocation and its role in the economic growth of countries. Moreover, an overview of pollutant generation by human activity is provided. Furthermore, guidelines are presented that standardize the disclosure of air pollution emissions by firms, known as the Greenhouse Gas (GHG) Protocol.

The methodologies used in the calculations and evaluations of the degree of misallocation, potential output gains, and behavior of pollutant emissions are detailed in the third section. The traditional model of Hsieh and Klenow (2009) is derived, and modifications are implemented that incorporate firm pollutant emissions into the analyses, allowing for examination of pollutant releases per product if production factors were allocated efficiently.

The acquisition of the necessary data for the analyses conducted in this paper is detailed in the fourth section, where the main sources of data are presented, both from financial information (value added, capital, and labor) and from firm air pollution data. The methodologies applied in constructing the panels of financial information and pollutant emissions, as well as the cross-sectional analysis for the year 2022, are displayed and statistical summaries of the constructed databases are also presented.

The fifth section is dedicated to presenting results and conducting analyses. It presents the degree of misallocation for publicly traded companies from 2010 to 2022, including the distribution of productivities and their measures of dispersion. Additionally, the behavior of pollutant emissions for the year 2022 is exhibited under efficient resource allocation assumptions. In the sixth section, to test the robustness of the findings, calculations are redone by varying the values of parameters σ (elasticity of substitution between differentiated goods) and α_s (elasticity of output with respect to capital).

Finally, the seventh section presents the concluding remarks, aiming to synthesize the main ideas, procedures and results obtained, as well as to establish, based on the analyses conducted, additional points of discussion for the topic.

2 Misallocation and Pollutant Emissions

This Section is divided into two subsections: (i) Misallocation and (ii) Pollutant Emissions. Subsection 2.1 presents concepts related to resource misallocation, explaining its role in Economics, particularly in development literature, and its significance for economic growth in countries. In Subsection 2.2, an overview of pollutant emissions is provided, detailing the concepts of Scope 1, 2, and 3 emissions as defined by the GHG Protocol.

2.1 Misallocation

In Restuccia and Rogerson (2013) and Restuccia and Rogerson (2017), the authors provide an overview of recent literature linking productivity and misallocation, presenting the concepts and studies developed in this area. They emphasize that countries with lower productivity rates may be less efficient in allocating available production factors.

A significant portion of the per capita income differences between countries is explained by differences in total factor productivity, where aggregate productivity depends not only on TFP of individual production units but also on the inefficient allocation of resources among heterogeneous production units. The inefficient allocation of resources has been widely addressed in various streams of economic literature, with significant effects on the study of firm and overall economic productivity. One way to quantify the impact of misallocation is to measure how much output could be gained by reallocating capital and labor among firms.

Two main approaches are used in the literature to provide answers to questions related to productivity disparities between countries and inefficient resource allocation. These approaches are known as direct and indirect.

A primary feature of the direct approach is the selection of factors deemed relevant to inefficient allocation, aiming to find direct measures of these factors so that they can subsequently be used in economic models to quantitatively assess their impact on misallocation and aggregate TFP. An example of a widely studied factor is distortions in the credit market.

Direct methods face several challenges, such as obtaining measures of sources of misallocation, which can be very difficult if they reflect discretionary provisions. Moreover, the impacts of any specific factor are relatively minor when compared to the disparities observed between developed and developing economies.

The indirect approach in analyzing efficient allocation seeks to examine the aggregate effect of the complete set of underlying factors without necessarily identifying each specific source of misallocation. In contrast to the direct approach, which aims to quantify the effects of individual factors, the indirect approach focuses on the overall assessment of misallocation. Thus, the indirect approach provides a broader and more holistic view of misallocation, allowing for a comprehensive understanding of its effects on productivity.

Efficient allocation of inputs results in equal marginal products across all producers at a given level of aggregation. Therefore, by directly analyzing the variation in marginal products, it becomes possible to quantify the extent of misallocation without explicitly identifying its source. While this approach requires a certain structure, unlike the direct approach, it does not necessitate the specification of a complete model.

Several caveats should be considered regarding the indirect approach. Firstly, it pertains to the nature of heterogeneity in production functions among producers. Any variation in the ratios of capital and labor is interpreted as misallocation, although it may simply reflect technological differences between firms. Secondly, it involves adjustment costs, as literature indicates significant costs associated with adjusting both labor and capital at the firm level. This suggests that marginal products of capital and labor may vary among producers due to these adjustment costs and specific firm-level transitory shocks. Additionally, measurement errors in firm data will lead to differences in marginal products among firms, even when misallocation is absent. Moreover, productivity losses stemming from misallocation reported using the indirect approach are typically much higher than those reported using the direct approach.

The resource misallocation can stem from various sources, including legal decisions such as features of the tax code, as well as discretionary provisions by the government or other entities like banks, which favor or penalize specific companies, influencing even the decisions of entry and exit of these firms. In addition, market imperfections such as regulation, property rights, financial frictions, and information asymmetry can also contribute to misallocation. Recently, the literature on resource misallocation has also incorporated aspects such as discrimination, culture, and social norms, which can result in inefficient allocation of talent in the labor market.

2.2 Pollutant Emissions

From pre-industrial times to the present, the role of human activities in generating pollutants is undeniable. According to Masson-Delmotte et al. (2021), human influence on climate warming is observable in various spheres, encompassing the atmosphere, oceans, and terrestrial surface.

Carbon dioxide stands out as the main catalyst for global climate changes, with human activities playing a significant role, both in its production and in the emission of other greenhouse gases. These emissions have experienced a continuous increase since the early stages of the industrial era, causing modifications in the Earth's energy balance and triggering substantial climatic consequences (Johnson et al., 2007).

Given this scenario, the urgent need to reduce pollutant emissions has been widely recognized, and sustainability has become an increasingly important topic worldwide, mobilizing governments, civil society, and companies to adopt a wide range of new practices. Following the call of the United Nations Sustainable Development Goals (SDGs) and the growing focus of investors on non-financial reporting, a growing number of companies are measuring, disclosing, and managing sustainability risks and opportunities. As highlighted by Apergis et al. (2022), performance in environmental, social, and governance (ESG) metrics is considered an important factor reflecting companies' ability to generate value and execute effective strategies.

One of the goals is to standardize the disclosure of air pollution information. The Greenhouse Gas Protocol (GHG Protocol) is a globally recognized standard for measuring and managing greenhouse gas emissions. Established in 1990 to meet the need for a consistent reporting framework in this field, the GHG Protocol collaborates with governments, industrial associations, NGOs, corporations, and other entities to provide the most widely used emissions calculation guidelines worldwide. Playing a central role in promoting decarbonization in public and private operations, the GHG Protocol provides a unified framework for emissions management. Thus, organizations seeking carbon accounting solutions should ensure the adoption of a decarbonization platform aligned with the protocol's principles and guidelines.

In Monzoni (2008), the author explains that, seeking to delineate the sources of direct and indirect emissions, improve transparency, and be applicable to various types of organizations and different types of climate-related policies, three types of emission scopes were defined for greenhouse gas accounting and inventory preparation: Scope 1, 2, and 3.

Scope 1 covers direct greenhouse gas emissions originating from sources owned or directly controlled by an organization. These emissions are generated by a variety of activities, such as the generation of electricity, heat, or steam in stationary sources like boilers, furnaces, and turbines, as well as the processing or manufacturing of chemicals and materials. Additionally, emissions associated with the transportation of materials, products, waste, and employees in vehicles owned by the organization are also considered in Scope 1. Other sources of direct emissions include intentional or unintentional leaks from owned equipment, such as gas discharges in equipment operation, covers, packaging, and tanks, methane emissions in coal mines and ventilation, as well as hydrofluorocarbon (HFC) emissions from refrigeration and air conditioning equipment and methane leaks related to gas transportation.

Scope 2 concerns indirect greenhouse gas emissions arising from the acquisition of electricity and thermal energy consumed by a company, encompassing energy purchased or brought into the organization's operational boundaries. For many companies, this energy acquisition represents one of the main sources of pollutant emissions and, consequently, a significant opportunity for reducing their emissions. Accounting for these emissions allows for an assessment of risks and opportunities associated with variations in energy costs and emissions.

Indirect emissions originating from activities preceding the company's energy supplier, such as prospecting, well drilling, flaring, and transportation in the energy chain, are categorized in Scope 3. This category addresses all other indirect emissions related to the company's activities that occur in sources not owned or controlled by the company. Also known as value chain emissions, Scope 3 emissions encompass all indirect emissions that occur upstream and downstream in the reporting company's supply chain, categorized into 15 different categories as defined by the GHG Protocol, including business travel, waste disposal, and the acquisition of goods and services.

Therefore, the public disclosure of emission inventories, aggregating Scope 1, 2, and 3 emissions, is crucial for promoting corporate transparency and accountability. It provides relevant information about companies' carbon footprints, contributing to their credibility and image. Additionally, broad access to this data is essential since emissions affect the entire society, thus ensuring a basic right for citizens and public and private managers.

3 Misallocation Framework

3.1 Canonical Model

The theoretical model developed by Hsieh and Klenow (2009) was pioneering in calculating the misallocation originating from the existence of firm-level distortions, which affect the optimal allocation of resources (capital and labor) within sectors. Subsequent studies, such as Oberfield (2013), Dias et al. (2016), Vasconcelos (2017), and Chen et al. (2023), incorporated additional components into the theory, including the possibility of resource reallocation between sectors, the inclusion of inputs as a production factor, sectoral complementarities, and climate shocks.

This paper will follow the canonical model, which assumes an economy with a single final good Y, produced by a representative firm in a perfectly competitive market. The representative firm combines the product Y_s of S industries using a Cobb-Douglas production function:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} \tag{1}$$

Considering that $\sum_{s=1}^{S} \theta_s = 1$, cost minimization implies that the share of each sector in the economy is given by:

$$\theta_s = \frac{P_s Y_s}{PY} \tag{2}$$

The price of the product Y_s in each sector is denoted by P_s, while P is the price of the final good in the economy, which is assumed to be the numéraire, hence $P = 1$. At the firm level, the market is defined as monopolistic competition, where the intermediate good Y_s is a CES aggregate function with M_s differentiated products:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

The parameter σ measures the elasticity of substitution between differentiated goods, and Y_{si} is the output of firm i in sector s. Profit maximization in the sector implies the inverse demand for each variety:

$$P_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}} \tag{4}$$

The term $P_s Y_s^{\frac{1}{\sigma}}$ is unobserved and therefore can be set equal to 1. This equality does not affect relative productivities and output gains, as there is no reallocation of resources between sectors Dias et al. (2016). This premise is equivalent to setting $\kappa_s = \frac{(P_s Y_s)^{-\frac{1}{\sigma-1}}}{P_s} = 1$ in Hsieh and Klenow (2009).

Each differentiated good is determined by a Cobb-Douglas production function at the firm level:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s} \tag{5}$$

The firm's capital stock, labor, and total factor productivity are determined by K_{si}, L_{si}, and A_{si}, respectively. The capital share α_s may vary across sectors but remains the same within each sector.

It is possible to separately identify distortions affecting both capital and labor simultaneously and those affecting the marginal product of one resource relative to the other. Thus, two types of distortions are introduced in the model: (i) a product distortion $\tau_{Y_{si}}$, which increases the marginal products of capital and labor in the same proportion, and (ii) a capital distortion $\tau_{K_{si}}$, which increases the marginal product of capital relative to labor.

For example, the capital wedge $\tau_{K_{si}}$ is expected to be high for firms facing credit constraints, while $\tau_{Y_{si}}$ is likely high for firms constrained by size and low for firms receiving subsidies. Considering these distortions, the profit of firm i is given by:

$$\pi_{si} = (1 - \tau_{Y_{si}})P_{si}Y_{si} - \omega L_{si} - (1 + \tau_{K_{si}})RK_{si} \tag{6}$$

Where R and ω are the costs of capital and wages, respectively. Maximizing the firm's profit leads to the standard condition where its price is a fixed markup over its marginal cost

$$P_{si} = \frac{\sigma}{\sigma - 1}\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{\omega}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{K_{si}})^{\alpha_s}}{A_{si}(1-\tau_{Y_{si}})} \tag{7}$$

The first-order conditions for profit maximization also imply:

$$\frac{K_{si}}{L_{si}} = \frac{\alpha_s}{1-\alpha_s}\,\frac{\omega}{R}\,\frac{1}{(1+\tau_{K_{si}})} \tag{8}$$

$$L_{si} \propto \frac{A_{si}^{\sigma-1}(1-\tau_{Y_{si}})^{\sigma}}{(1+\tau_{K_{si}})^{\alpha_s(\sigma-1)}} \tag{9}$$

$$Y_{si} \propto \frac{A_{si}^{\sigma}(1-\tau_{Y_{si}})^{\sigma}}{(1+\tau_{K_{si}})^{\alpha_s(\sigma)}} \tag{10}$$

It is noted that the allocation of resources depends on both the firm's TFP levels and the distortions it faces. This fact leads to differences in marginal revenues of labor and capital across firms, which would be equalized in the absence of distortions.

From the first-order conditions, it is also possible to observe that the marginal revenues of capital and labor ($MRPK_{si}$ and $MRPL_{si}$) are proportional to the revenue ($P_{si}Y_{si}$) per unit of capital and labor, respectively:

$$MRPK_{si} = \alpha_s\,\frac{\sigma-1}{\sigma}\,\frac{P_{si}Y_{si}}{K_{si}} = R\,\frac{1+\tau_{K_{si}}}{1-\tau_{Y_{si}}} \tag{11}$$

$$MRPL_{si} = (1-\alpha_s)\,\frac{\sigma-1}{\sigma}\,\frac{P_{si}Y_{si}}{L_{si}} = \omega\,\frac{1}{(1-\tau_{Y_{si}})} \tag{12}$$

The equations above are also used to determine the distortions faced by each firm:

$$1+\tau_{K_{si}} = \frac{\alpha_s}{1-\alpha_s}\frac{\omega L_{si}}{RK_{si}} \tag{13}$$

$$1-\tau_{Y_{si}} = \frac{\sigma}{\sigma-1}\frac{\omega L_{si}}{(1-\alpha_s)P_{si}Y_{si}} \tag{14}$$

Within industries, firms equalize their marginal revenues from capital and labor after tax payments, therefore, prior to payment, there is dispersion in marginal revenues due to distortions. These equations enable the estimation of wedges from revenue (or value added) information, production factors, and the parameters σ and α_s.

According to Foster et al. (2008), there is a distinction between two measures of total factor productivity at the firm level. One is the traditional Solow residual, which indicates how productive a firm is in terms of "physical" output, denoted as $TFPQ$. The other is $TFPR$, which measures how productive the firm is in terms of revenue:

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \tag{15}$$

$$TFPR_{si} = P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \tag{16}$$

In the absence of distortions, more capital and labor are allocated to plants with higher physical productivity until the point where higher output Y_{si} results in a lower price P_{si}, equalizing its revenue productivity to that of plants with lower output, thus $TFPR$ does not vary among firms within the same sector. From Equations 7, 11 e 12, we can show that $TFPR_{si}$ is proportional to the geometric mean of the marginal revenue products of capital and labor:

$$\begin{aligned} TFPR_{si} = P_{si} A_{si} &= \frac{\sigma}{\sigma - 1} \left(\frac{R}{\alpha_s} \right)^{\alpha_s} \left(\frac{\omega}{1-\alpha_s} \right)^{1-\alpha_s} \frac{(1+\tau_{K_{si}})^{\alpha_s}}{1-\tau_{Y_{si}}} \\ &= \frac{\sigma}{\sigma - 1} \left(\frac{MRPK_{si}}{\alpha_s} \right)^{\alpha_s} \left(\frac{MRPL_{si}}{1-\alpha_s} \right)^{1-\alpha_s} \\ &\propto (MRPK_{si})^{\alpha_s} (MRPL_{si})^{(1-\alpha_s)} \propto \frac{(1+\tau_{K_{si}})^{\alpha_s}}{1-\tau_{Y_{si}}} \end{aligned} \tag{17}$$

Except for the wedges, the components of $TFPR_{si}$ are sector-specific fixed parameters, therefore, without distortions, revenue productivity does not vary within the same sector, as expected.

Given that, for an industry, $K_s = \sum_{i=1}^{M_s} K_{si}$ and $L_s = \sum_{i=1}^{M_s} L_{si}$ represent the aggregated capital and labor, and that TFP_s represents its total factor productivity, and considering that the sector is a representative firm whose production function is also Cobb-Douglas, with some algebra, we obtain:

$$Y_s = TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s} \tag{18}$$

$$TFP_s = \left[\sum_{i=1}^{M_s} \left(A_{si} \, \frac{\overline{TFPR_s}}{TFPR_{si}} \right)^{\sigma - 1} \right]^{\frac{1}{\sigma-1}} \tag{19}$$

The term $\overline{TFPR_s}$ is proportional to the geometric mean of the weighted average of the sector's marginal revenue products of capital and labor and represents the observed $TFPR$:

$$\begin{aligned}\overline{TFPR_s} &= \frac{\sigma}{\sigma-1}\left[\frac{R}{\left(\alpha_s \sum_{i=1}^{M_s} \frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}} \frac{P_{si}Y_{si}}{P_sY_s}\right)}\right]^{\alpha_s}\left[\frac{\omega}{\left((1-\alpha_s)\sum_{i=1}^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_sY_s}\right)}\right]^{1-\alpha_s} \\ &= \frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK_s}}{\alpha_s}\right)^{\alpha_s}\left(\frac{\overline{MRPL_s}}{1-\alpha_s}\right)^{1-\alpha_s} \propto (\overline{MRPK_s})^{\alpha_s}(\overline{MRPL_s})^{(1-\alpha_s)}\end{aligned} \tag{20}$$

With the values of $\overline{MRPK_s}$ and $\overline{MRPL_s}$ being determined by:

$$\overline{MRPK_s} = \frac{R}{\left(\sum_{i=1}^{M_s} \frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}} \frac{P_{si}Y_{si}}{P_sY_s}\right)} \tag{21}$$

$$\overline{MRPL_s} = \frac{\omega}{\left(\sum_{i=1}^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_sY_s}\right)} \tag{22}$$

If the marginal revenues of capital and labor are equalized among firms within the same sector and, consequently, their revenue productivities, we get $\overline{TFPR_s} = TFPR_{si}$. Thus, from Equation 19, we have that the industry's TFP will be:

$$\overline{A}_s = \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{23}$$

As the firms' production Y_{si} is not observed, but rather their revenue $P_{si}Y_{si}$, it is not possible to obtain the value of A_{si} directly from Equation 15. Using Equation 4 in 15, we obtain an expression that allows us to find the value of A_{si}:

$$A_{si} = \kappa \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \tag{24}$$

The scalar $\kappa_s = \frac{(P_sY_s)^{-\frac{1}{\sigma-1}}}{P_s}$ is not observed and can be set to 1, preserving the relative productivities and reallocation gains.

Combining Equations 1 and 18, it is possible to obtain the economy's aggregate product as a function of the aggregate sectoral production factors and their TFP:

$$Y = \prod_{s=1}^{S}(TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s})^{\theta_s} \tag{25}$$

Finally, for each sector, the ratio between the observed product Y and the efficient product Y^*, obtained when the $TFPR$ of the firms is equalized, thus with $TFP_s = \overline{A}_s$, provides the degree of misallocation in the industry and, consequently, the potential output gains when resources are allocated efficiently. The aggregation of all sectors, using the Cobb-Douglas function in 1, provides the misallocation and output gains for the economy.

$$\frac{Y}{Y^*} = \prod_{s=1}^{S} \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{\overline{A}_s} \frac{\overline{TFPR_s}}{TFPR_{si}} \right)^{\sigma-1} \right]^{\frac{\theta_s}{\sigma-1}} \tag{26}$$

3.2 Green Misallocation: Behavior of Pollutant Emissions

In the context of evaluating the behavior of air pollution when misallocation is eliminated, it will be considered that the firms' emissions are proportional to their output. Thus, to obtain the new value of the firm's emissions, it is also necessary to calculate what its efficient output would be:

$$e_{si}^* = e_{si} \left(\frac{Y_{si}^*}{Y_{si}} \right) \tag{27}$$

The firm's observed pollutant emissions and output are represented by e_{si} and Y_{si}, respectively, while the emissions when resources are efficiently allocated are represented by e_{si}^* and its efficient output by Y_{si}^*.

In the canonical model, the value of $\overline{TFPR_s}$ represents the observed average revenue productivity of a sector. However, an important question is how to find the value of the sector's productivity when distortions are eliminated and $TFPR_{si}$ are equalized, which will be represented by $TFPR_s^*$. According to Dias et al. (2016), one possibility is to use the value of the industry's $TFPR$ obtained when the wedges are zero, however, this does not guarantee that the quantities of K_s and L_s will be the same after the reallocation of resources among firms.

The alternative proposed by the authors is that all firms will face the same distortions, and such distortions will be such that the demand for capital and labor in the sector remains the same after the reallocation of production factors. Analogous to obtaining the distortions faced by firms, according to Equations 13 and 14, the sector's capital and labor wedges, which in this case will also be faced by firms, are given by:

$$1 + \overline{\tau}_{K_s} = \frac{\alpha_s}{1-\alpha_s} \frac{\omega L_s}{R K_s} \tag{28}$$

$$1 - \overline{\tau}_{Y_s} = \frac{\sigma}{\sigma-1} \frac{\omega L_s}{(1-\alpha_s) P_s^* Y_s^*} \tag{29}$$

Based on Equations 4, 15, and 16, it is possible to find the values of the firms' output and revenue as functions of their physical productivity and revenue productivity:

$$Y_{si} = \left(\frac{A_{si}}{TFPR_{si}}\right)^{\sigma} \tag{30}$$

$$P_{si}Y_{si} = \left(\frac{A_{si}}{TFPR_{si}}\right)^{\sigma-1} \tag{31}$$

The above equations allow estimating the efficient levels of firms' output and revenue by simply replacing the observed $TFPR_{si}$ with the efficient one:

$$Y_{si}^{*} = \left(\frac{A_{si}}{TFPR_{s}^{*}}\right)^{\sigma} = Y_{si}\left(\frac{TFPR_{si}}{TFPR_{s}^{*}}\right)^{\sigma} \tag{32}$$

$$P_{si}^{*}Y_{si}^{*} = \left(\frac{A_{si}}{TFPR_{s}^{*}}\right)^{\sigma-1} = P_{si}Y_{si}\left(\frac{TFPR_{si}}{TFPR_{s}^{*}}\right)^{\sigma-1} \tag{33}$$

Given the equalized sector distortions, when allocation is efficient, they can be substituted into Equation 17:

$$TFPR_{s}^{*} = \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{\omega}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\overline{\tau}_{K_s})^{\alpha_s}}{1-\overline{\tau}_{Y_s}} \tag{34}$$

The $TFPR_{s}^{*}$ is obtained by combining the above equation with the equalized sector distortions values in 28 and 29 and the firm's efficient revenue value, according to equation 33:

$$TFPR_{s}^{*} = \left(\frac{\sum_{i=1}^{M_s} A_{si}^{\sigma-1}}{K_s^{\alpha_s}L_s^{1-\alpha_s}}\right)^{\frac{1}{\sigma}} \tag{35}$$

Now, with the calculated $TFPR_s$ value, it is possible to find the firm's optimal production from equation 32 and then estimate the pollutant emissions value e_{si}^{*} from equation 27.

In evaluating emission behavior, even more relevant than the total amount of pollutants emitted is the amount emitted per unit of output. Therefore, let $E = \sum_{s=1}^{S}\sum_{i=1}^{M_s} e_{si}$ be the total emissions of the economy, E/Y provides the amount of emissions per output, called E_{prod}. Hence, the green misallocation index ϵ is defined, which measures the behavior of emissions when production factors are allocated efficiently:

$$\epsilon = \frac{E_{prod}^{*}}{E_{prod}} = \frac{E^{*}}{E}\frac{Y}{Y^{*}} \tag{36}$$

The index is interpreted as follows: when $\epsilon < 1$, the efficient allocation of factors not only increases production but also decreases the amount of pollutants emitted per unit of product. On the other hand, $\epsilon > 1$ means that the reallocation of factors, although increasing the economy's production, generates additional emissions per unit of product. The percentage change in emissions per unit of product is given by $100(\epsilon - 1)\%$. It should be noted that the index can also be used, analogously, for sectoral evaluation.

4 Data

The theoretical framework developed by Hsieh and Klenow (2009) for calculating misallocation involves firm-level data. The data for this study were primarily obtained through web scraping from two main sources: (i) the open database of the CVM (Comissão de Valores Mobiliários), Brazilian Securities and Exchange Commission and (ii) the Public Emissions Registry of the Brazilian GHG Protocol Program.

From these sources, panels of financial information and pollutant emissions were constructed. The intersection of these panels, supplemented with additional data from other sources, generated the 2022 cross-sectional database, containing aggregated financial and pollutant emission data.

4.1 Firm-Level Financial Information

The financial data regarding the firms were obtained through the open databases of CVM, the regulatory agency for the securities market, which provides standardized financial statements of listed companies on B3 S.A., the Brazilian stock exchange, during the period from 2010 to 2022. The databases are available in both individual and consolidated versions. Consolidated data were used due to their closer alignment with the firms' information in the emissions database, despite having fewer observations.

As, for productivity and misallocation calculations, firms' production is represented by a Cobb-Douglas function, as per Equation 5, the data sought includes values for capital stock, labor, and production. Traditionally used in this literature, including Hsieh and Klenow (2009), value added and personnel expenses represent production and labor, respectively.

Oberfield (2013) warns that measuring firms' capital stock poses significant challenges, potentially leading to an exaggerated misallocation indication if calculated inadequately. Therefore, the author employs the perpetual inventory method in constructing firms' capital stock. However, due to data limitations, especially for non-listed firms, this study adopts the original approach, using firms' fixed assets as capital stock, similar to Dias et al. (2016).

Table 1: Statistical Summary of the Listed Companies Panel

Year	Observations	Personnel Expenses (R$ billion)	Fixed Assets (R$ billion)	Value Added (R$ billion)	Revenue (R$ billion)
2010	296	129	1,001	787	1,704
2011	353	142	1,084	798	1,827
2012	284	160	1,211	745	1,940
2013	341	214	1,575	1,068	3,295
2014	277	177	1,382	764	2,101
2015	275	193	1,481	804	2,220
2016	276	199	1,396	929	2,286
2017	278	199	1,423	987	2,332
2018	283	216	1,503	1,159	2,679
2019	327	243	1,773	1,192	2,930
2020	376	261	1,852	1,265	3,311
2021	456	312	2,091	1,910	4,417
2022	466	354	2,406	2,033	5,140

Notes: Table constructed from open data provided by the CVM (Securities and Exchange Commission). Financial data are in nominal values. Each observation represents a company in a specific year.

The database created is an unbalanced panel with 4288 observations over 13 years, excluding financial sector firms. The number of companies per year ranges from 275 in 2015 to 466 in 2022, distributed across 24 distinct sectors. Table 1 presents the statistical summary.

The parameters α_s for the industries, capital share - elasticity of output with respect to capital, are calculated as 1 minus the elasticity of output with respect to labor, estimated using data on personnel expenses and value added from the panel of financial information of listed companies, as shown in Table 2. This table also includes data on the elasticities of American industries for the period between 2008 and 2022, published by the Bureau of Economic Analysis (BEA), which will also be used to calculate the degree of misallocation in Section 6 - Robustness.

4.2 Firm-Level Pollution Emissions

The data on pollutant emissions were obtained from the Public Emissions Registry, organized by the Center for Sustainability Studies (FGVces) at the School of Business Administration of Fundação Getulio Vargas (FGV EAESP). This registry includes information from the corporate greenhouse gas (GHG) emission inventories of organizations participating in the Brazilian GHG Protocol Program. The registry boasts the largest database of public organizational inventories in Latin America, with more than 2,300 inventories. The data cover the period from 2008 to 2022.

Table 2: Sectoral Capital Shares α_s

Sectors	BR Capital Shares	US Capital Shares
Agriculture (Sugar, Alcohol, and Sugarcane)	0.82	0.81
Food	0.54	0.59
Beverages and Tobacco	0.90	0.59
Toys and Leisure	0.78	0.41
Communication and IT	0.34	0.45
Trade (Wholesale and Retail)	0.73	0.50
Construction, Building Materials, and Decoration	0.67	0.37
Education	0.53	0.15
Packaging	0.70	0.47
Electric Power	0.88	0.74
Mining	0.87	0.72
Pharmaceuticals and Hygiene	0.60	0.73
Printing and Publishers	0.52	0.33
Lodging and Tourism	0.57	0.37
Metallurgy and Steel	0.68	0.48
Machinery, Equipment, Vehicles, and Parts	0.53	0.41
Paper and Pulp	0.85	0.47
Petrochemicals and Rubber	0.87	0.86
Oil and Gas	0.89	0.85
Sanitation, Water, and Gas	0.73	0.74
Transportation and Logistics	0.77	0.42
Medical Services	0.56	0.17
Telecommunications	0.89	0.73
Textile and Clothing	0.58	0.28

Notes: Capital shares α_s obtained from data provided by the Brazilian Securities and Exchange Commission (CVM) and the Bureau of Economic Analysis (BEA). CVM data cover the period 2010-2022, while BEA data cover the period 2008-2022.

The firms' inventories are qualified with three different seals: gold, silver, and bronze. The gold seal represents a complete publication, including third-party verification and validation of information by a Verification Body. The silver seal signifies a complete publication, and the bronze seal indicates a partial publication. Of the total sample, about 52% of the inventories are classified as gold and 41% as silver, providing robustness to the pollutant emission data.

Although the emission information is published on the program's website, the database is not available for download, necessitating data scraping. This effort resulted in a panel comprising 2,368 observations over 15 years. The number of firms per year ranges from 23 in 2008 to 432 in 2022, spread across up to 21 sectors. Emissions are detailed by scope type, including the source and type of gas, totaling up to 37 different variables, with information on Scopes 1 or 2 being mandatory. Table 3 presents the statistical summary of the panel.

Table 3: Statistical Summary of the Emissions Panel

Year	Observations	Total Emissions ($ktCO_2e$)	Average Emissions ($ktCO_2e$)	Maximum Emissions ($ktCO_2e$)
2008	23	85,230	3,706	51,273
2009	39	87,193	2,236	51,558
2010	79	117,324	1,485	53,867
2011	99	900,080	9,092	561,608
2012	115	355,629	3,092	229,039
2013	133	387,743	2,915	227,938
2014	142	423,786	2,984	253,241
2015	147	420,571	2,861	262,806
2016	153	1,014,659	6,632	541,005
2017	147	709,573	4,827	522,810
2018	151	651,982	4,318	492,277
2019	168	844,418	5,026	563,273
2020	217	926,271	4,269	566,683
2021	315	1,109,796	3,523	585,347
2022	440	1,156,192	2,628	586,859

Notes: Table constructed from data provided by the Public Emissions Registry. Scope 1, 2, and 3 values were summed to generate the emissions value per company. Each observation represents a company in a specific year.

4.3 Integration of Financial and Emission Data

To calculate the behavior of emissions if resources were optimally allocated, it is necessary for the observations to include both financial and emissions data of the companies. The initial merging of the panels constructed in subsections 4.1 and 4.2 generally results in a low number of matches. However, it is noted that sectors with the highest quantities (both in percentage and absolute terms) of observations are capital-intensive industries. Therefore, calculations and analyses considering pollutant emissions were performed for the following sectors: Capital-Intensive Agribusiness, Electric Power, Metallurgy and Steel, Oil, Gas and Derivatives and Others. These sectors are the result of classifications by the CVM Registry.

In addition to the initial merging of the panels, supplementary data on emissions from listed companies whose inventories were not included in the emissions panel, as well as financial information from companies that reported their emission inventories but are not publicly traded, were added. Finally, a cross-section for the year 2022 was generated. It is noteworthy that, since larger companies tend to report their emissions data, the firms in

Table 4: Cross-sectional Summary of 2022

Observations			Total Value Added (%)		Sector Value Added (%)			
Total	CVM	RPE	$\frac{\text{CVM Emissions}}{\text{Total CVM}}$	$\frac{\text{CVM Emissions}}{\text{Total Emissions}}$	Agribusiness	Electric Power	Metallurgy and Steelmaking	Oil, Gas and Derivatives
112	70	78	57.2	93.0	64.6	86.1	86.0	99.0

Notes: Table constructed from financial information panel and pollutant emissions panel. CVM observations refer to listed companies, RPE observations refer to firms that publish in the Public Emissions Registry. $\frac{\text{CVM Emissions}}{\text{Total CVM}}$ corresponds to the ratio of value added of cross-sectional listed companies to total value added of listed companies. $\frac{\text{CVM Emissions}}{\text{Total Emissions}}$ corresponds to the ratio of value added of cross-sectional listed companies to total value added of the cross-sectional. Sectoral values are relative to the ratio of value added of cross-sectional listed companies belonging to the sector to total value added of listed companies in the sector. All data is for the year 2022.

this database account for more than half of the value added of the stock market, reaching over 80% in some sectors such as Electric Power, Metallurgy and Steel, and Oil, Gas and Derivatives. Table 4 provides information related to the 2022 cross-section.

5 Empirical Analysis

In this section, we will present the main results found regarding output gains from resource reallocation, as well as the behavior of emissions with efficient allocation.

The choices of the parameter σ and the capital rental price R for the reference scenario will follow the values traditionally used in this literature, namely, $\sigma = 3$ and $R = 0.1$. In Hsieh and Klenow (2009), the authors show that the gains from reallocation increase with the elasticity of substitution, making this choice conservative, given that previous research estimates the value of σ to be between 3 and 10. Regarding the cost of capital, 5% is considered for the interest rate and 5% for depreciation. However, a different choice has few implications since R only affects the average capital distortion and not the relative distortions between firms, thus not impacting the calculation of output gains (Ziebarth, 2013).

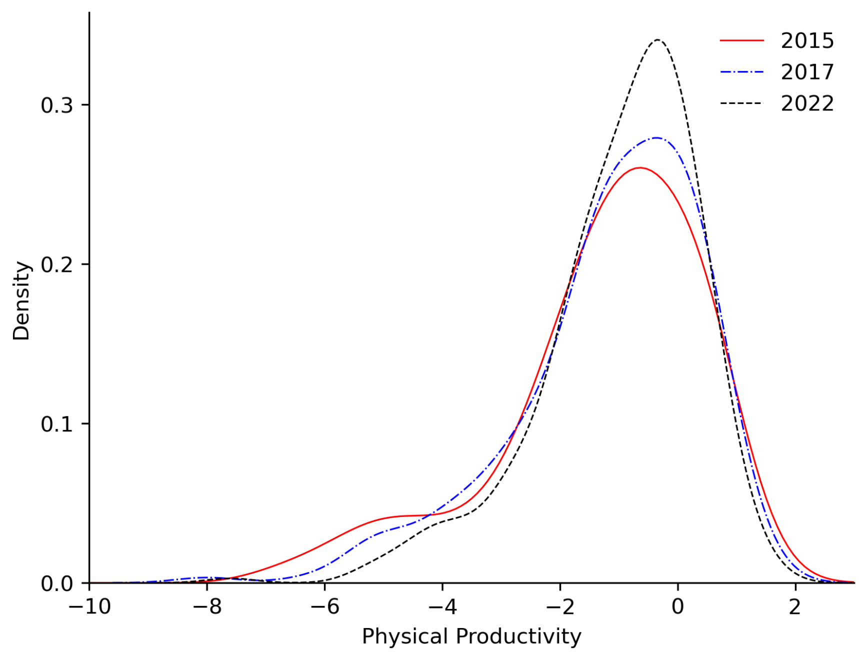
Total personnel expenses will be used as the measure of labor for the firms, resulting in $\omega = 1$ and $L_{si} = w_{si}N_{si}$, with w_{si} being the wage paid by the firm and N_{si} the number of employees. According to Dias et al. (2016), this assumption implies that differences in hours worked per employee and differences in human capital are already implicitly considered.

5.1 Misallocation in Publicly Traded Companies

The results presented in this Section were obtained using the financial information panel of publicly traded companies developed in Subsection 4.1. Before calculating the variables of interest, we excluded the two companies with the extreme values of $TFPR_{si}$ and the two companies with the extreme values of A_{si} for each sector and year. Although the panel does not have a large number of observations, this procedure is carried out to eliminate outliers and control for potential measurement errors.

Figure 1 shows the distribution of $TFPQ$ for selected years 2015, 2017 and 2022. The distribution of physical productivity is adjusted by sector productivity in the absence of distortions, $log(A_{si}M^{\frac{1}{\sigma-1}}s/\overline{A}s)$. It can be observed that the left tail of the distribution is thicker for 2015, indicating possible policies that favored the survival of inefficient firms (with relatively lower $TFPQ$), with its thickness decreasing in 2017 and 2022.

Figure 1: Distribution of $TFPQ$



Notes: The figure shows the physical productivity adjusted by the sector's productivity in the absence of distortions, $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$, of publicly traded companies for the years 2015, 2017, and 2022.

Table 5 shows that this pattern is consistent across other measures of $TFPQ$ dispersion: the standard deviation, the subtraction of the 75% and 25% percentiles, and the subtraction of the 90% and 10% percentiles. The standard deviation decreases from 1.36 in 2015 to 1.04 in 2022, while the difference between the 75% and 25% percentiles fell from 1.98 in 2015 to 1.59 in 2022. The pattern repeats for the differences between the 90% and 10% percentiles.

Figure 2 presents the distribution of $TFPR$ for selected years 2015, 2017 and 2022. If the allocation were efficient the $TFPR$ dispersion would be zero, hence greater productivity dispersion suggests a higher degree of misallocation. The revenue productivity distribution

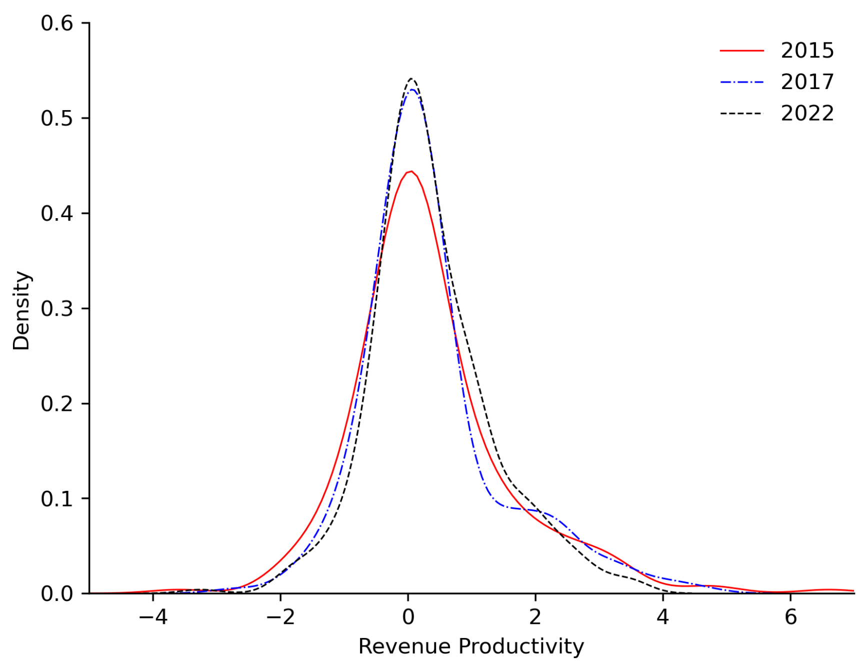
Table 5: Dispersion of $TFPQ$

	2015	2017	2022
Standard Deviation	1.36	1.20	1.04
75 - 25	1.98	1.88	1.59
90 - 10	4.55	3.84	3.25

Notes: Statistics are for $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$. Standard deviation is weighted by value added. 75 - 25 is the difference between the 75th and 25th percentiles, and 90 - 10 is the difference between the 90th and 10th percentiles.

is adjusted by $\overline{TFPR_s}$, $log(TFPR_{si}/\overline{TFPR_s})$. The year of 2015 has a higher degree of dispersion, with longer tails and lower density of firms centralized in the distribution. By 2017, a shift in the distribution shape is observed, which by 2022, is clearly less dispersed.

Figure 2: Distribution of $TFPR$



Notes: The figure shows the revenue productivity adjusted by the sector's observed $\overline{TFPR_s}$, $log(TFPR_{si}/\overline{TFPR_s})$, of publicly traded companies for the years 2015, 2017, and 2022.

Similar to $TFPQ$, Table 6 reinforces the consistency of the misallocation pattern shown in the $TFPR$ distribution through dispersion measures. The standard deviation decreases from 0.83 in 2015 to 0.63 in 2022, while the difference between the 90% and 10% percentiles fell from 2.82 in 2015 to 2.31 in 2022. The difference between the 75% and 25% percentiles, despite decreasing between 2015 and 2017, returns to the same level in 2022.

Table 6: Dispersion of $TFPR$

	2015	2017	2022
Standard Deviation	0.83	0.77	0.63
75 - 25	1.04	0.84	1.05
90 - 10	2.82	2.74	2.31

Notes: Statistics are for $log(TFPR_{si}/\overline{TFPR_s})$. Standard deviation is weighted by value added. 75 - 25 is the difference between the 75th and 25th percentiles, and 90 - 10 is the difference between the 90th and 10th percentiles.

Another point to be noted regarding the efficient allocation of production factors is how revenue productivity correlates with physical productivity. As pointed out by Dias et al. (2016), this relationship is particularly harmful because, when positive, it means that the more productive firms also face the greatest distortions.

Table 7 shows that the $TFPQ$ and $TFPR$ of firms are positively correlated, indicating that more productive companies face higher distortions and thus tend to produce less, while less productive companies tend to become larger than their efficient size. In addition to the complete sample calculations, values were also estimated for the Electric Power sector, which, as observed in Figure 3, shows a high correlation pattern.

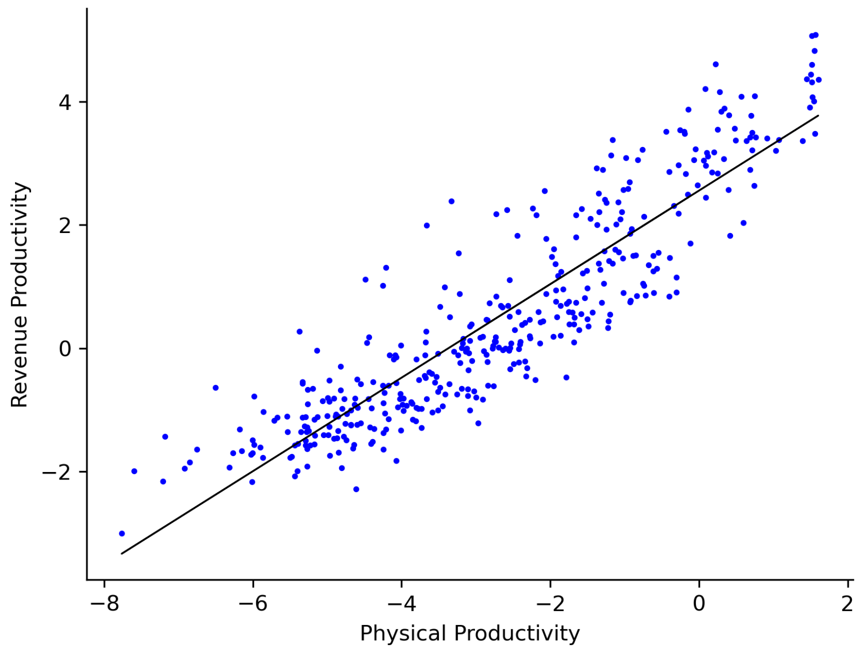
Table 7: Correlation between $TFPQ$ and $TFPR$

	2010-2022	2015	2017	2022
Total	0.60	0.67	0.57	0.65
Electric Power	0.90	0.95	0.92	0.91

Notes: Correlations are between $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$ and $log(TFPR_{si}/\overline{TFPR_s})$ for the selected years of 2015, 2017 and 2022, and for the entire period of 2010-2022. Values in the Total row correspond to all sectors, while Electric Power considers only companies in this sector.

Following the observations of Chen et al. (2023), another interesting illustration of the degree of misallocation is the comparison between how the production factors should be allocated, when associated with their physical productivities, and their actual allocations. In other words, considering the total capital and labor of a sector, in an efficient allocation, the shares of the factors for each firm should be proportional to $A^{\sigma-1}si$, as per Equations 8 and 9. Therefore, the correlation between $log(A_{si})$ and $log(K_{si})$ or $log(L_{si})$ is positive when distortions are equalized. In the case of marginal revenues of capital and labor, in an efficient allocation, $MRPK_{si}$ and $MRPL_{si}$ should be equalized across firms, so the correlation between physical productivity and marginal revenues is null.

Figure 3: Correlation between $TFPQ$ and $TFPR$ in the Electric Power Sector



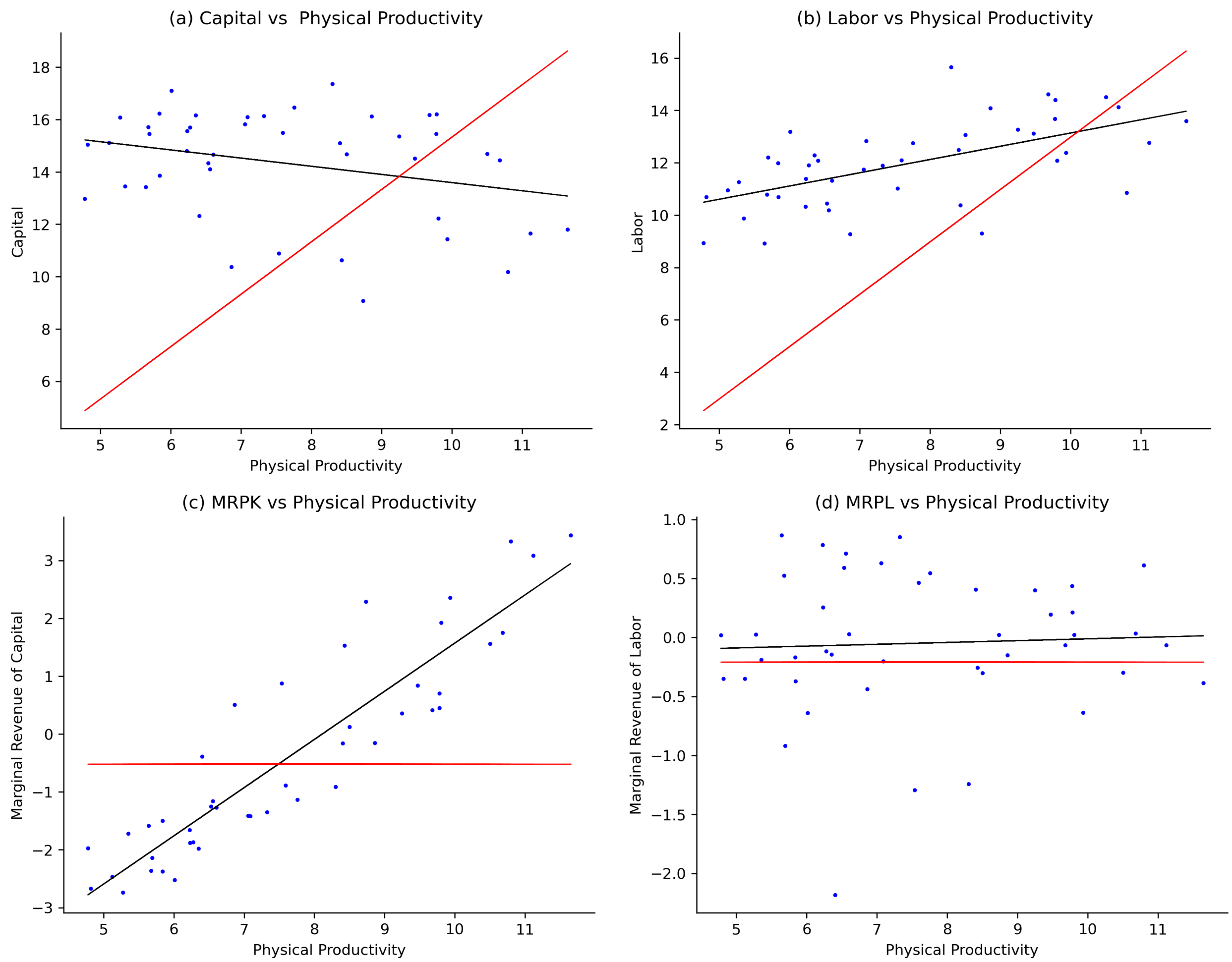
Notes: The figure shows the correlation between $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$ and $log(TFPR_{si}/\overline{TFPR_s})$ in the Electric Power sector for the period 2010-2022.

Figure 4 - (a) shows the amount of capital each firm possesses against its $TFPQ$, with the red line representing how the sector's capital allocation should be, contrasted with the observed trend in black, which has a slightly negative correlation. In the case of labor, as shown in Figure 4 - (b), a positive correlation is observed, but not in the same proportion as efficient allocation. Figure 4 - (c) documents the Marginal Revenue of Capital, which is strongly positive in relation to physical productivity, whereas, in an efficient allocation, as per the red line, it should be equalized among firms. Figure 4 - (d) shows that the observed Marginal Revenue of Labor is only slightly correlated with the firm's $TFPQ$, indicating a lower degree of misallocation for this factor.

The charts in Figure 4 are related to the Electric Power sector for the year 2022. It is important to note that there are sectors with positive correlations between production factors and productivity, as well as marginal revenues that are not correlated with productivity, such as the Metallurgy and Steel industry. However, the Electric Power sector provides graphics that more clearly illustrate the issues discussed in this study.

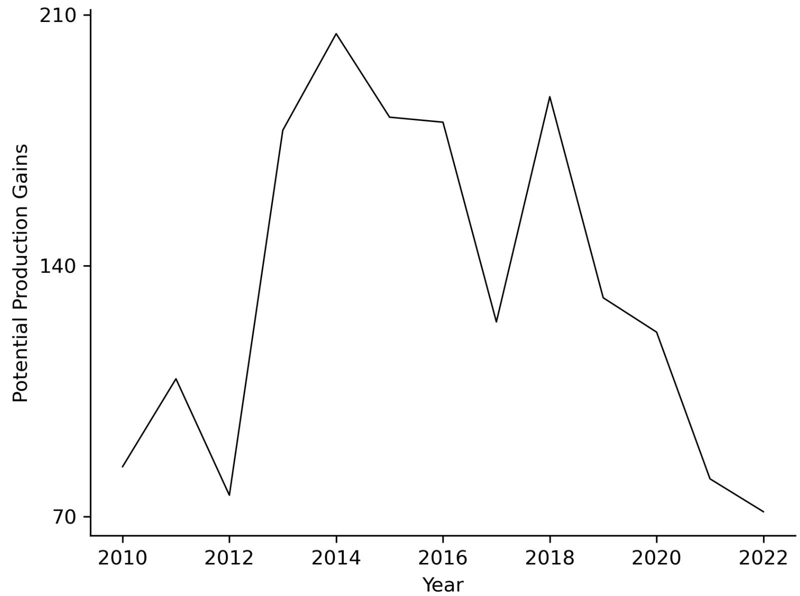
Figure 5 shows the potential output gains with the equalization of revenue productivities among firms within the same sector, for the period from 2010 to 2022. Despite the variations, a trend of increasing misallocation and, consequently, potential output gains can be observed between 2010-2012 and 2014-2018, with a trend of improvement in the efficient allocation of resources from then on. In Hsieh and Klenow (2009), the authors use the potential gains from the beginning and end of the evaluated period to estimate the behavior of the allocation of production factors. From this perspective, there were no advances in the efficient allocation of Brazilian publicly traded companies between 2010 and 2022, as their potential gains remain the same (around 70%).

Figure 4: Capital, Labor, $MRPK$, and $MRPL$: Observed and Efficient Allocation



Notes: (a) and (b) reports the efficient and actual allocation of capital and labor with respect to physical productivity. (c) and (d) reports the real and efficient marginal revenue product of capital and labor with respect to physical productivity. All values are in *log*. Each point represents a company, and the red line indicates the efficient allocation. The data are for the Electric Power sector, referring to the year 2022.

Figure 5: Potential Output Gains from Intra-Sectoral $TFPR$ Equalization



Notes: The output gains are in percentage. The values are for $100(Y^*/Y - 1)$, where $\frac{Y}{Y^*} = \prod_{s=1}^{S}\left[\sum_{i=1}^{M_s}\left(\frac{A_{si}}{\bar{A}_s}\frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}}$ and $TFPR_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{(1-\alpha_s)}}$.

Table 8: Potential Output Gains for the Years 2015, 2017 and 2022

	2015	2017	2022
Output Gains	181.41	124.27	71.31

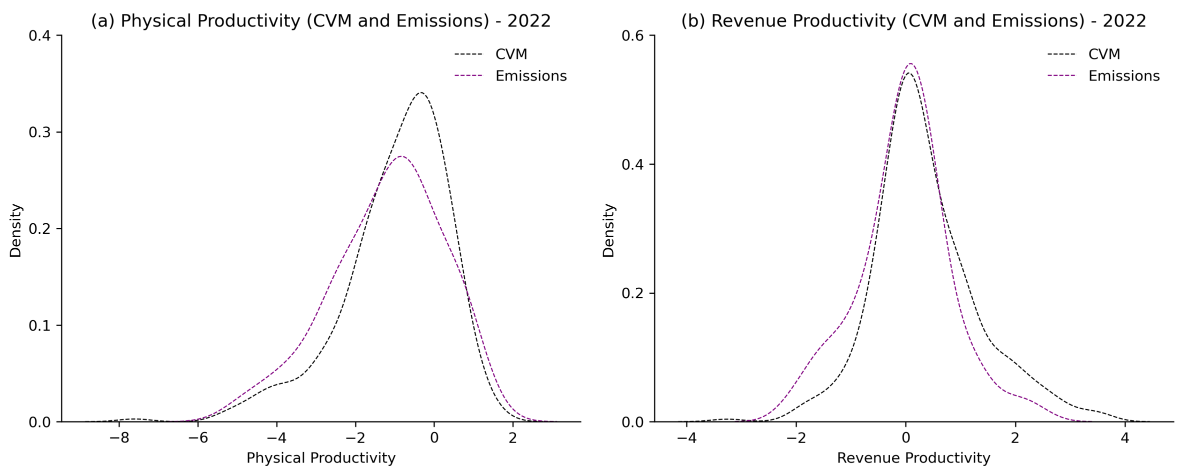
Notes: Output gains with intra-sectoral equalization of $TFPR$ for the selected years of 2015, 2017 and 2022. Gains are in percentage values.

According to this theory, it is not possible to determine the causes of the decline in the efficient allocation of resources between 2010 and 2018 (or the possible stagnation between 2010 and 2022). However, there is a consensus that Brazil went through a period of difficulties in the last decade, which could either reflect or have reflected the degree of misallocation observed. On the other hand, evaluating a more recent period using selected years, there was a 64% improvement in resource allocation, averaging 7% per year, between 2015 and 2022, as shown in Table 8. Considering the period from the last inflection point in 2018, the allocation is 68% more efficient in 2022, with an average improvement of 14% per year.

5.2 Green Misallocation Analysis

The results for this section were obtained using the cross-sectional data developed in Subsection 4.3, which considers not only the financial information of the firms but also their pollutant emissions. The initial part of this section will be similar to the previous section, including the exclusion of the two companies that show extreme values of $TFPR_{si}$ per sector, followed by the calculations of the variables of interest.

Figure 6: Distribution of Physical Productivities and Revenue Productivities



Note: (a) shows the physical productivities of listed companies (CVM) and companies from the cross-section (Emissions) adjusted by sector productivities in the absence of distortions, $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$, for the year 2022. (b) shows the revenue productivities of listed companies (CVM) and companies from the cross-section (Emissions) adjusted by observed sector productivities, $log(TFPR_{si}/\overline{TFPR_s})$, for the year 2022.

Figure 6 (a) shows the distribution of the $TFPQ$ of firms from the 2022 cross-section (Emissions), in contrast with the distribution of publicly traded companies (CVM) for the same year. The distributions of physical productivities are adjusted by the productivity of the sector in the absence of distortions, $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$. It is observed that the cross-sectional distribution is less centralized and has a slightly more pronounced left tail compared to the distribution of listed companies. Figure 6 (b) shows the distribution of the $TFPR$ of firms from the 2022 cross-section (Emissions), in contrast with the distribution of publicly traded companies (CVM) for the same year. The distributions of revenue productivities are adjusted by $\overline{TFPR_s}$, $log(TFPR_{si}/\overline{TFPR_s})$. The samples show a similar density of firms centralized in the distribution, with distinct behaviors in the tails.

Table 9 reinforces the patterns observed in Figure 6 through dispersion measures. For physical productivities, the standard deviations of the samples are similar, while the differences between the 75%-25% and 90%-10% percentiles are larger for the cross-section, thus adhering to Figure 6 (a). In the case of revenue productivities, the three measures of

Table 9: Dispersion of Physical and Revenue Productivities

	A_{si} (Emissions)	A_{si} (CVM)	$TFPR_{si}$ (Emissions)	$TFPR_{si}$ (CVM)
Standard Deviation	0.95	1.04	0.44	0.63
75 - 25	1.97	1.59	0.81	1.05
90 - 10	3.74	3.25	2.22	2.31

Notes: Statistics are for $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$ and $log(TFPR_{si}/\overline{TFPR_s})$ for listed companies (CVM) and cross-sectional companies (Emissions). Standard deviation is weighted by value added. 75th - 25th is the difference between the 75th and 25th percentiles, and 90th - 10th is the difference between the 90th and 10th percentiles. Results are for the year 2022.

the cross-section are lower than those observed in companies listed on the Brazilian stock exchange, which may indicate a lower degree of misallocation for the cross-section.

As shown in Table 4, more than 60% of the firms in the cross-section are listed, whose added value represents more than 90% of the total sample. Thus, it was expected that the distribution and dispersion measures would be similar to those presented in Subsection 5.1.

The correlation between revenue productivity and physical productivity is also evaluated since, as pointed out in the previous section, if it is positive, it indicates that the more productive firms are those that face the greatest distortions. Table 10 shows that the $TFPQ$ and $TFPR$ of firms are positively correlated. All sectors show a correlation between productivities, with the Electric Power sector presenting the highest degree.

Table 10: Correlation between $TFPQ$ and $TFPR$

	Total	Capital-Intensive Agribusiness	Electric Power	Metallurgy and Steelmaking	Oil, Gas and Derivatives	Others
2022	0.78	0.27	0.91	0.47	0.66	0.52

Notes: Correlations are between $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$ and $log(TFPR_{si}/\overline{TFPR_s})$ for the year 2022. The value in the Total row corresponds to the entire cross-sectional.

Table 11 shows the potential output gains from equalizing revenue productivities among firms in the same sector for the year 2022. Additionally, it provides the total emissions observed, i.e., with the presence of misallocation, and what the total emissions would be with the efficient allocation of resources. It is observed that even with a positive variation in production, the total value of emissions does not increase proportionally to the variation in production. The green misallocation index ϵ, which measures the behavior of emissions, is less than one, indicating that the efficient reallocation of factors not only increases the economy's production but also decreases the amount of pollutants emitted per product, as can be seen from the negative variation in emissions per product.

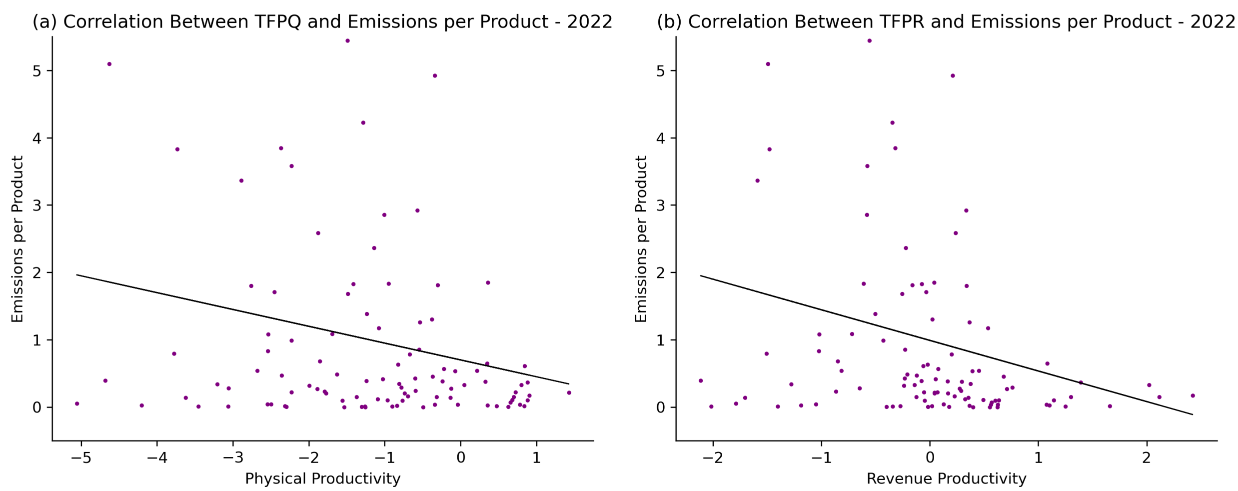
Table 11: Behavior of Emissions with Efficient Allocation

	2022
Output Gains	30.74
Total Observed Emissions	1,365,840
Total Emissions with Efficient Allocation	1,243,760
Epsilon	0.70
Emissions Variation per Product	-30.35

Notes: Output gains, with intra-sectoral TFPR equalization, in percentage terms. Total emissions are in ktonCO$_2$e. Emissions variation is given by $100(\epsilon - 1)\%$ where $\epsilon = \frac{E^*_{prod}}{E_{prod}} = \frac{E^*}{E}\frac{Y}{Y^*}$.

Figure 7, which correlates physical and revenue productivities with emissions per product, shows behavior consistent with the previously found results. Given that efficient allocation reduces emissions per product, it was expected that the more productive firms would be less polluting. This situation occurs for both $TFPQ$ and $TFPR$, corroborating the correlation found between productivities, as shown in Table 10.

Figure 7: Correlation of Productivities and Emissions per Product



Notes: (a) shows the correlation of physical productivity adjusted by sector productivity in the absence of distortions, $log(A_{si}M_s^{\frac{1}{\sigma-1}}/\overline{A}_s)$, with emissions per product, for the year 2022. (b) shows the correlation of revenue productivity adjusted by observed sector productivity, $log(TFPR_{si}/\overline{TFPR_s})$, with emissions per product, for the year 2022.

Table 12 details, by sector, the behavior of emissions for the year 2022. It is noted that for all sectors, the green misallocation index ϵ was less than one, indicating that potential output gains were accompanied by a reduction in emissions per product.

For the Electric Power sector, the decrease in emissions was proportionally less than the output gains (-57.08% versus 181.92%), similar to the Oil, Gas, and Derivatives industry, but to a lesser degree, with emissions per product and output gains variation of around -3%

and 4.4% , respectively. For the other sectors, efficient allocation not only generated potential output gains but also reduced emissions more than proportionally, i.e., a positive variation of 1% in production decreased emissions by more than 1%.

The Oil, Gas, and Derivatives sector accounts for about 50% of the added value of the 2022 cross-section, so its relatively low output gains and emissions per product variation have a strong impact on the overall sample results. Notably, this industry has one of the most negative images concerning environmental issues, as its products are intrinsically polluting. However, this fact leads the sector to seek alignment with global ESG practices, both to improve its image with the public and due to market demand, which already considers ESG performance in credit granting and cost, as detailed by Apergis et al. (2022). Additionally, the sector faces stricter environmental oversight, which may be one of the reasons for the low variation in emissions.

Table 12: Behavior of Emissions with Efficient Allocation - 2022

Sector	Production Gains	Observed Emissions	Emissions Efficient Allocation	ϵ Index	Emissions Variation per Product
Total	30.74	1,365,840	1,243,760	0.70	-30.35
Agribusiness	19.80	23,388	18,800	0.67	-32.90
Electric Power	181.92	75,451	91,305	0.43	-57.08
Metallurgy and Steelmaking	13.70	641,310	500,946	0.69	-31.30
Oil, Gas and Derivatives	4.42	622,927	630,727	0.97	-3.03
Others	21.46	2,764	1,982	0.59	-40.95

Notes: Output gains with intra-sector TFPR equalization in percentage values. Total emissions are in ktonCO$_2$e. Emissions variation is given by $100(\epsilon - 1)\%$ where green misallocation index $\epsilon = \dfrac{E^*_{prod}}{E_{prod}} = \dfrac{E^*}{E}\dfrac{Y}{Y^*}$.

It is not the intention of this work to seek explanations for why efficient resource allocation, in addition to generating potential output gains, leads to a reduction in emissions per product. However, it is noteworthy that this occurs for all sectors.

6 Robustness

The results presented in Section 5 consider the reference scenario, where the parameter value σ is equal to 3 and the capital shares α_s are constructed using the financial information panel of Brazilian publicly traded companies for the period 2010-2022. Additionally, it is assumed that the variation in firms' emissions is linear with output. In this Section, the parameter values will be altered, and emissions will not have constant returns to scale, in order to evaluate how the results are affected by these modifications.

6.1 Variation of the Elasticity of Substitution Between Differentiated Goods

The definition of the parameter σ is a strong simplification assumption for the analysis, as the elasticity of substitution between differentiated goods is not the same across different industries. Additionally, since the firm's production is not observed, the parameter is crucial for obtaining the product from the value added.

Traditionally, in the empirical literature on efficient resource allocation, a value of $\sigma = 3$ is used. However, in Dias et al. (2016), the authors also consider other values in their misallocation calculations, such as 5.6 for the United States and 6.8 for Portugal, based on more recent research.

Table 13 shows how the potential output gains of companies listed on the Brazilian stock exchange and the 2022 cross-section vary with changes in σ. It can be observed that the estimates of the degree of misallocation are highly sensitive to elasticity, with the 2017 gain more than tripling when σ is changed from 3 to 7. It is noted that the variation in emissions per product in the 2022 cross-section follows the same pattern, meaning a higher elasticity amplifies its decrease as the production gain has increased in magnitude.

Table 13: Robustness Test σ: Potential Output Gains and Emissions Variation

	2015	2017	2022	Cross-Sectional 2022	Emissions Variation per Product
$\sigma = 3$	181.41	124.27	71.31	30.74	-30.35
$\sigma = 5$	379.51	277.21	119.71	50.10	-49.24
$\sigma = 7$	526.25	406.65	152.03	68.59	-64.77

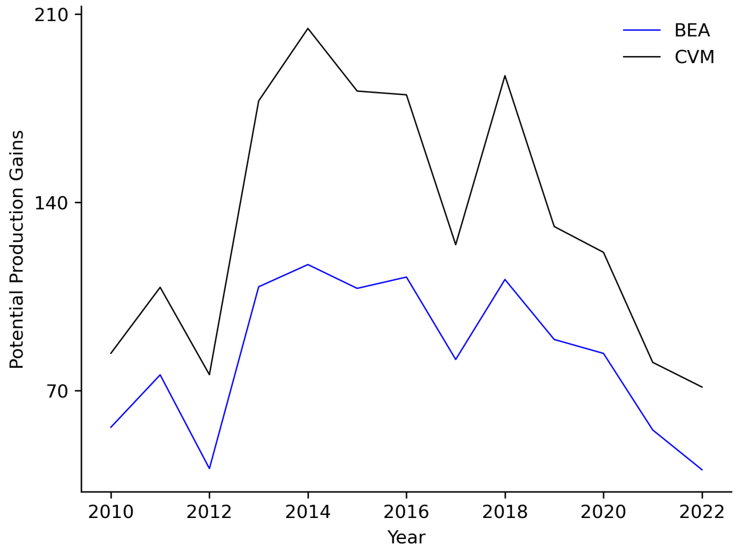
Notes: Robustness test, varying the elasticity of substitution between differentiated goods, for potential output gains with equalization of intra-sector TFPRs for selected years 2015, 2017 and 2022. Emissions variation per product is for the 2022 cross-sectional. All values are in percent.

As observed by Vasconcelos (2017), the increase in the degree of misallocation was expected, as more substitutable products imply that the effect of relative prices is intensified. Moreover, the results illustrate that the reference scenario with $\sigma = 3$ can be interpreted as a conservative estimate of the extent of the degree of misallocation.

6.2 Variation of the Output Elasticity with Respect to Capital

Hsieh and Klenow (2009) adopt American factor shares as a reference, assuming that elasticities are less distorted compared to other countries, both within and between sectors. For the reference scenario of this paper, α_s was calculated using the Personnel Expenses

Figure 8: Robustness Test α_s: Potential Output Gains from Equalizing Intra-sectoral $TFPR$



Notes: The output gains are in percentage terms. The values are for $100(Y^*/Y - 1)$, where $\frac{Y}{Y^*} = \prod_{s=1}^{S}\left[\sum_{i=1}^{M_s}\left(\frac{A_{si}}{\overline{A}_s}\frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}}$ and $TFPR_{si} = \dfrac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{(1-\alpha_s)}}$.

and Added Value data from the financial information panel of Brazilian companies listed on the stock exchange, assuming that any differences in elasticities compared to the United States are due to intrinsic country characteristics rather than distortions, as pointed out by Oberfield (2013).

To calculate the capital shares of American industries, sectoral data from the Bureau of Economic Analysis (BEA) for the period 2008-2022 were used. Since the sector classifications of the CVM and BEA are different, an approximate correspondence between the two classifications was made. An important difference compared to Hsieh and Klenow (2009) is that the financial information from CVM and BEA reflects not only salaries but also other personnel expenses such as social security and additional benefits, while the database used by the authors (NBER Productivity Database) does not consider these, thus requiring the additional assumption of a multiplicative factor for labor costs.

Figure 8 compares the potential output gains from equalizing revenue productivity among firms within the same sector, using the product elasticities relative to American and Brazilian capital for the period 2010 to 2022. It can be observed that the behavior of the degree of misallocation is extremely similar, with the difference manifesting in the level of potential output gains, which are lower when American capital shares are used.

Similarly, when using the Brazilian elasticity, there is a trend of improvement in efficient allocation for the more recent period of 2015-2022. When using the American elasticity, however, the growth is lower, at approximately 5% per year, compared to the 7% presented in the reference scenario. In the case of the 2022 cross-section, the magnitudes of output gains and emission variations per product also decrease when using the American α_s, as shown in Table 14.

Table 14: Robustness Test α_s: Potential Output Gains and Emissions Variation

	2015	2017	2022	Cross-Sectional 2022	Emissions Variation per Product
BR Capital Shares	181.41	124.27	71.31	30.74	-30.35
EUA Capital Shares	108.01	81.57	40.55	25.8	-20.35

Notes: Robustness test, varying the elasticity of output with respect to capital, for potential output gains with equalization of intra-sector TFPRs for selected years 2015, 2017 and 2022. Emissions variation per product is for the 2022 cross-sectional. All values are in percent.

6.3 Returns to Scale in Pollutant Emissions

In evaluating the behavior of companies' pollutant emissions when production factors are allocated efficiently, it was considered that emissions vary linearly with the firm's product, as per Equation 27. However, it is possible that technological factors impact companies' efficiency concerning pollutant emissions, meaning there may be returns to scale in pollutant emissions.

In Qi et al. (2021), the authors present evidence that larger industries are more likely to adopt cleaner technologies and are relatively less polluting, while Dasgupta et al. (1998) find that, for Brazil and Mexico, smaller firms generate more toxic gas emissions. Therefore, Equation 27 can be modified so that firm size is considered in projecting new emissions after eliminating misallocation:

$$e_{si}^* = e_{si} \left(\frac{Y_{si}^*}{Y_{si}} \right)^{\psi} \tag{37}$$

If ψ is less than 1, as the firm grows, its emissions per product decrease, conversely, if the firm shrinks, its emissions are proportionally higher. With ψ greater than 1, the larger the firm's product, the higher the pollutant emissions per product. If $\psi = 1$, the emissions variation is linear, which is the reference scenario.

As shown in Table 15, after eliminating misallocation, when emissions have decreasing returns to scale, there is a reduction in pollutants emitted per product. In contrast, when returns (of pollution) are increasing, emissions per product rise, causing the variation in emissions per product to change from -31.48% when $\psi = 0.8$ to -26.55% when $\psi = 1.2$.

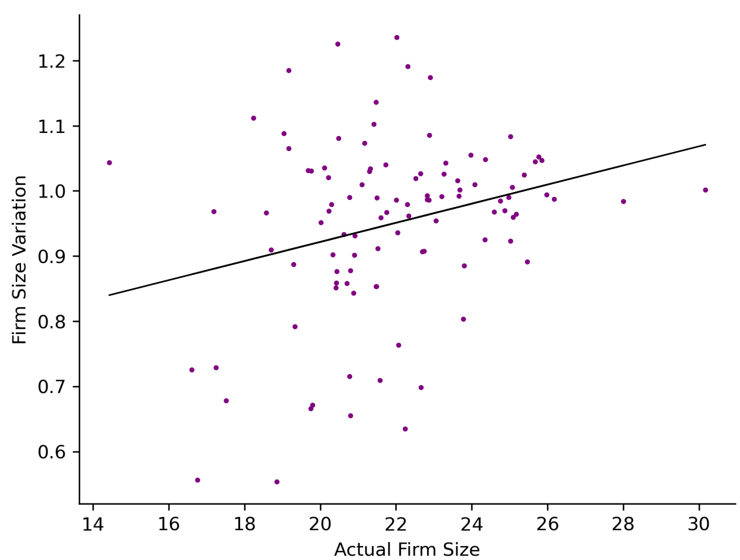
Table 15: Robustness Test ψ: Returns to Scale in Pollutant Emissions

	$\psi = 0.8$	$\psi = 0.9$	$\psi = 1.0$	$\psi = 1.1$	$\psi = 1.2$
Output Gains	30.74	30.74	30.74	30.74	30.74
Total Observed Emissions	1,365,840	1,365,840	1,365,840	1,365,840	1,365,840
Total Emissions with Efficient Allocation	1,223,629	1,229,462	1,243,760	1,269,554	1,311,646
Epsilon	0.69	0.69	0.70	0.71	0.73
Emissions Variation per Product	-31.48	-31.15	-30.35	-28.91	-26.55

Notes: Output gains with intra-sector TFPR equalization in percentage terms. Total emissions are in ktonCO$_2$e. Emissions variation is given by $100(\epsilon - 1)\%$ where $\epsilon = \frac{E^*_{prod}}{E_{prod}} = \frac{E^*}{E}\frac{Y}{Y^*}$.

These results imply that firms that have grown are less polluting than those that have shrunk, which aligns with the findings throughout this work. As observed in Figure 7, emissions per product are negatively correlated with physical productivity. Therefore, in an efficient allocation, firms with higher physical productivity (which are also less polluting) grow.

Figure 9: Correlation Between Firm Size Variation and Actual Firm Size



Notes: The figure shows the correlation between $log(Y_{si})$ and $log(Y^*_{si})/log(Y_{si})$ of firms from the 2022 cross-section.

Figure 9 reinforces the findings of Qi et al. (2021) and Dasgupta et al. (1998), as a positive correlation is observed between the firm's observed size and its size variation after the reallocation of production factors. In other words, for the observed sample, larger companies are more productive and less polluting.

7 Conclusion

The objective of this paper was to calculate the degree of resource misallocation among publicly traded Brazilian companies, the potential output gains when resources are efficiently reallocated and the change in pollutant emission levels. To investigate these objectives, concepts and the importance of analyzing efficient resource allocation in the economy were presented, as well as the main guidelines standardizing the emission inventories disclosed by firms, established by the GHG Protocol. Furthermore, pollutant emissions of companies were incorporated into the methodology developed by Hsieh and Klenow (2009) by considering the variation in firms' pollutant emissions as proportional to the ratio between the optimal output (without the presence of misallocation) and the observed output.

The theoretical framework developed requires the use of firm-level information. Therefore, using data provided by the Brazilian Securities Commission (CVM), a panel of firms' financial information was created. The pollutant emission data were obtained from the Public Emissions Registry, which covers the air pollution inventories of organizations participating in the Brazilian GHG Protocol Program. These were used to create a panel of firms' pollutant emissions, while the cross-section used in the calculation of emission behavior with efficient allocation was obtained by merging the two created panels.

The results presented for the reference scenario were consistent. Two distinct trends were observed: an increase in the degree of misallocation between 2010-2012 and 2014-2018, followed by an inflection and trend towards improved efficient allocation from the end of this period. For the selected years 2015, 2017 and 2022, whose potential output gains decreased from 181% to 71%, the distributions and their dispersion measures reflected the improvement in resource allocation.

An interesting point, observed when evaluating green misallocation for the 2022 cross-section, was that the efficient allocation of factors not only increases the economy's production but also decreases the amount of pollutants emitted per product. This situation occurred across all analyzed sectors, with the aggregate result being a potential production gain of 31% and a 30% reduction in emissions per product.

Robustness tests confirmed the values found in the reference scenario, as the observed variations were only in level, not in behavior. As the elasticity of substitution between differentiated goods increases, the degree of misallocation also increases, being highly sensitive to σ. This trend is also observed in emissions per product, meaning that increased elasticity amplifies the (negative) variation in air pollution. When using American capital shares, the results obtained were extremely similar to the reference scenario, with differences manifesting only in the level of potential output gains, whose values are higher when using Brazilian capital shares' product elasticities. Considering scale returns in firms' pollutant emissions, when emissions per product grow with firm size, an increase in total emissions is observed, while when emissions decrease with firm size, a reduction in per-product pollution is observed, corroborating the fact that larger firms are more productive and less polluting.

The objective of the indirect approach used in this work is to analyze the aggregate effect of misallocation, without needing to identify specific sources causing resource misallocation. However, the results obtained allow for some analyses when contextualized with the Brazilian scenario. The degrees of misallocation at the beginning and end of the observed period (2010-2022) are similar, around 70%, meaning no improvement in efficient resource allocation is observed during this period, which coincides with the difficulties faced by Brazil in the last decade.

The dynamic observed of increased output gains and reduced emissions per product with efficient resource allocation is perhaps the result that generates the greatest potential for further investigations, as it provides a starting point to ascertain iif there is causality in this relationship, obviously accompanied by a more robust database and well-defined identification strategy.

Finally, the biggest challenge in expanding the study of the green misallocation, i.e., links between productivity, resource misallocation and pollutant emissions, is precisely firm-level data, especially emissions data. In Brazil, where the universe of publicly traded companies or those present in the Annual Industrial Survey (PIA) numbers in the hundreds of thousands, the number of firms reporting their inventories in the Public Emissions Registry is modest. Thus, alternatives must be sought to obtain more representative data samples, whether such data pertain to specific sectors whose firms are legally required to report their emissions or from countries whose firm-level emissions data represent a larger share of the total companies.

With the growing need for environmental preservation, it becomes urgent and imperative to reconcile economic development with the reduction of pollutant emissions.

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Appendix A - Canonical Model Solution

Representative Firm's Problem

The representative firm combines the product Y_s of S industries using a Cobb-Douglas production function:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} \tag{A.38}$$

Considering that $\sum_{s=1}^{S} \theta_s = 1$, the cost minimization problem is given by:

$$\min_{\{Y_s\}_{s=1}^{S}} \sum_{s=1}^{S} P_s Y_s \tag{A.39}$$

Subject to the constraint in Equation A.38. The Lagrangian is given by:

$$\mathcal{L} = \sum_{s=1}^{S} P_s Y_s - \lambda(\prod_{s=1}^{S} Y_s^{\theta_s} - Y) \tag{A.40}$$

The first-order conditions imply that:

$$P_{s^*} = \lambda \theta_{s^*} Y_{s^*}^{\theta_{s^*}-1} \prod_{s=1}^{S-\{s^*\}} Y_s^{\theta_s}$$

$$P_{s^*} Y_{s^*} = \lambda \theta_s^* \prod_{s=1}^{S} Y_s^{\theta_s} = \lambda \theta_s^* Y \tag{A.41}$$

From the relationship between two sectors:

$$Y_s = Y_j \frac{P_j}{P_s} \frac{\theta_s}{\theta_j}$$

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} = \prod_{s=1}^{S} \left(Y_j \frac{P_j}{P_s} \frac{\theta_s}{\theta_j} \right)^{\theta_s}$$

$$Y = Y_j \frac{P_j}{\theta_j} \prod_{s=1}^{S} \left(\frac{\theta_s}{P_s} \right)^{\theta_s}$$

$$Y_j \frac{P_j}{\theta_j} = Y \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s} \tag{A.42}$$

Therefore, defining $P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s}\right)^{\theta_s}$, cost minimization implies that the share of each sector in the economy is given by:

$$\theta_s = \frac{P_s Y_s}{PY} \tag{A.43}$$

Intermediate Sector Problem

The market at the firm level is defined as monopolistic competition, with intermediate good Y_s and M_s differentiated products:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.44}$$

The demand for intermediate goods is derived from the profit maximization problem of industry s, where the profit is given by:

$$\pi_s = P_s Y_s - \sum_{i=1}^{M_s} P_{si} Y_{si} \tag{A.45}$$

Therefore, the profit maximization problem is given by:

$$\max_{Y_{si}} P_s \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \sum_{i=1}^{M_s} P_{si} Y_{si} \tag{A.46}$$

The first-order conditions imply the inverse demand for each variety:

$$\begin{aligned} P_{si} &= P_s \left(\frac{\sigma}{\sigma-1}\right) \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}} \\ P_{si} &= P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}} \end{aligned} \tag{A.47}$$

Firm-level Profit Maximization Problem

Each differentiated product is determined by a firm-level Cobb-Douglas production function:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s} \tag{A.48}$$

Using Equations A.47 and A.48, the profit maximization problem of firm i is given by:

$$
\begin{aligned}
\max_{K_{si},L_{si}} \pi_{si} &= (1-\tau_{Y_{si}})P_{si}Y_{si} - \omega L_{si} - (1+\tau_{K_{si}})RK_{si} \qquad \text{(A.49)}\\
&= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}} Y_{si} - \omega L_{si} - (1+\tau_{K_{si}})RK_{si}\\
&= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} - \omega L_{si} - (1+\tau_{K_{si}})RK_{si}\\
&= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\frac{\sigma-1}{\sigma}} - \omega L_{si} - (1+\tau_{K_{si}})RK_{si}
\end{aligned}
$$

From the first-order conditions with respect to K_{si} and L_{si}, the capital-labor ratio is obtained:

$$(1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s} = \omega \qquad \text{(A.50)}$$

$$(1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}\alpha_s A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s-1} = (1+\tau_{K_{si}})R \qquad \text{(A.51)}$$

$$\frac{K_{si}}{L_{si}} = \frac{\alpha_s}{1-\alpha_s}\,\frac{\omega}{R}\,\frac{1}{(1+\tau_{K_{si}})} \qquad \text{(A.52)}$$

Using Equations A.47, A.50, and A.52, we find P_{si}:

$$
\begin{aligned}
(1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\,\frac{\omega}{R}\,\frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s} &= \omega\\
(1-\tau_{Y_{si}})P_{si} Y_{si}^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\,\frac{\omega}{R}\,\frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s} &= \omega
\end{aligned}
$$

$$P_{si} = \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{\omega}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{K_{si}})^{\alpha_s}}{A_{si}(1-\tau_{Y_{si}})} \qquad \text{(A.53)}$$

Also from the first-order conditions, it is found that resource allocation depends both on the firm's TFP levels and the distortions it faces:

$$
\begin{aligned}
\omega &= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s}\\
L_{si}\omega &= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)Y_{si}\\
L_{si}\omega &= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) \left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\frac{\sigma-1}{\sigma}}(1-\alpha_s)\\
L_{si}L_{si}^{\frac{-(\sigma-1)}{\sigma}}\omega &= (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right) \left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\frac{\sigma-1}{\sigma}}(1-\alpha_s)L_{si}^{\frac{-(\sigma-1)}{\sigma}}\\
L_{si}^{\frac{1}{\sigma}} &= \frac{1}{\omega}(1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}} \left(\frac{\sigma-1}{\sigma}\right)(1-\alpha_s)\left[A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\,\frac{\omega}{R}\,\frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s}\right]^{\frac{\sigma-1}{\sigma}}
\end{aligned}
$$

$$L_{si} = \frac{1}{\omega^\sigma}\left[(1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha_s)\right]^\sigma \left[A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\ \frac{\omega}{R}\ \frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s}\right]^{\sigma-1}$$

$$L_{si} = \frac{A_{si}^{\sigma-1}(1-\tau_{Y_{si}})^\sigma}{(1+\tau_{K_{si}})^{\alpha_s(\sigma-1)}}\ \frac{1}{\omega^\sigma}\left[P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha_s)\right]^\sigma \left(\frac{\alpha_s}{1-\alpha_s}\ \omega\right)^{\alpha_s(\sigma-1)}$$

$$L_{si} \propto \frac{A_{si}^{\sigma-1}(1-\tau_{Y_{si}})^\sigma}{(1+\tau_{K_{si}})^{\alpha_s(\sigma-1)}} \tag{A.54}$$

Using the firm's production function, we get:

$$Y_{si} = A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s} L_{si}$$

$$Y_{si} = A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\ \frac{\omega}{R}\ \frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s} L_{si}$$

$$Y_{si} \propto A_{si}\left(\frac{\alpha_s}{1-\alpha_s}\ \frac{\omega}{R}\ \frac{1}{(1+\tau_{K_{si}})}\right)^{\alpha_s} \frac{A_{si}^{\sigma-1}(1-\tau_{Y_{si}})^\sigma}{(1+\tau_{K_{si}})^{\alpha_s(\sigma-1)}}$$

$$Y_{si} \propto \frac{A_{si}^{\sigma}(1-\tau_{Y_{si}})^\sigma}{(1+\tau_{K_{si}})^{\alpha_s\sigma}}\left(\frac{\alpha_s}{1-\alpha_s}\ \frac{\omega}{R}\right)^{\alpha_s}$$

$$Y_{si} \propto \frac{A_{si}^{\sigma}(1-\tau_{Y_{si}})^\sigma}{(1+\tau_{K_{si}})^{\alpha_s\sigma}} \tag{A.55}$$

The marginal revenues of capital and labor ($MRPK_{si}$ and $MRPL_{si}$) are proportional to the revenue ($P_{si}Y_{si}$) per unit of capital and labor, respectively:

$$(1+\tau_{K_{si}})R = (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}\alpha_s A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s-1}$$

$$(1+\tau_{K_{si}})R = (1-\tau_{Y_{si}})P_{si}\left(\frac{\sigma-1}{\sigma}\right)\alpha_s\frac{Y_{si}}{K_{si}}$$

$$MRPK_{si} = \alpha_s\ \frac{\sigma-1}{\sigma}\ \frac{P_{si}Y_{si}}{K_{si}} = R\ \frac{1+\tau_{K_{si}}}{1-\tau_{Y_{si}}} \tag{A.56}$$

$$\omega = (1-\tau_{Y_{si}})P_s Y_s^{\frac{1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right) Y_{si}^{-\frac{1}{\sigma}}(1-\alpha_s)A_{si}\left(\frac{K_{si}}{L_{si}}\right)^{\alpha_s}$$

$$\omega = (1-\tau_{Y_{si}})P_{si}\left(\frac{\sigma-1}{\sigma}\right)(1-\alpha_s)\frac{Y_{si}}{L_{si}}$$

$$MRPL_{si} = (1-\alpha_s)\ \frac{\sigma-1}{\sigma}\ \frac{P_{si}Y_{si}}{L_{si}} = \omega\ \frac{1}{(1-\tau_{Y_{si}})} \tag{A.57}$$

The above equations are also used to find the values of the distortions faced by each firm:

$$1 + \tau_{K_{si}} = \frac{\alpha_s}{1-\alpha_s}\frac{\omega L_{si}}{RK_{si}} \tag{A.58}$$

$$1 - \tau_{Y_{si}} = \frac{\sigma}{\sigma - 1}\frac{\omega L_{si}}{(1-\alpha_s)P_{si}Y_{si}} \tag{A.59}$$

From the firm's production function, the physical and revenue productivity are obtained:

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \tag{A.60}$$

$$TFPR_{si} = P_{si}A_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \tag{A.61}$$

From Equations A.53, A.56, and A.57, we can show that $TFPR_{si}$ is proportional to the geometric mean of the marginal products of revenue of capital and labor:

$$\begin{aligned}
TFPR_{si} &= P_{si}A_{si} = \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{\omega}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{K_{si}})^{\alpha_s}}{A_{si}(1-\tau_{Y_{si}})}A_{si} \\
TFPR_{si} &= \frac{\sigma}{\sigma-1}\left(\frac{R(1+\tau_{K_{si}})}{\alpha_s(1-\tau_{Y_{si}})}\right)^{\alpha_s}\left(\frac{\omega}{(1-\alpha_s)(1-\tau_{Y_{si}})}\right)^{1-\alpha_s} \\
TFPR_{si} &= \frac{\sigma}{\sigma-1}\left(\frac{MRPK_{si}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{si}}{1-\alpha_s}\right)^{1-\alpha_s} \\
TFPR_{si} &\propto (MRPK_{si})^{\alpha_s}(MRPL_{si})^{(1-\alpha_s)} \propto \frac{(1+\tau_{K_{si}})^{\alpha_s}}{1-\tau_{Y_{si}}}
\end{aligned}$$

Using Equation A.47 in A.60, we get an expression that allows us to find the value of A_{si}:

$$\begin{aligned}
P_{si} &= P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}} \\
P_{si}Y_{si} &= P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} \\
Y_{si} &= (P_s Y_s^{\frac{1}{\sigma}})^{\frac{-\sigma}{\sigma-1}}(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}} \\
A_{si} &= \frac{(P_s Y_s^{\frac{1}{\sigma}})^{\frac{-\sigma}{\sigma-1}}(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \\
A_{si} &= \kappa_s \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{(1-\alpha_s)}} \qquad (A.62)
\end{aligned}$$

The scalar $\kappa_s = \frac{(P_s Y_s)^{-\frac{1}{\sigma-1}}}{P_s}$ is not observed and can be set to 1.

Aggregate Calculations

$\overline{MRPK_s}$ and $\overline{MRPL_s}$ are the weighted averages of the marginal revenues of capital and labor, respectively:

$$\begin{aligned} MRPK_{si} &= R\frac{1+\tau_{K_{si}}}{1-\tau_{Y_{si}}} \\ \overline{MRPK_s} &= \frac{R}{\left(\sum_{i=1}^{M_s} \frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}} \frac{P_{si}Y_{si}}{P_s Y_s}\right)} \end{aligned} \tag{A.63}$$

$$\begin{aligned} MRPL_{si} &= \omega \, \frac{1}{(1-\tau_{Y_{si}})} \\ \overline{MRPL_s} &= \frac{\omega}{\left(\sum_{i=1}^{M_s} (1-\tau_{Y_{si}}) \frac{P_{si}Y_{si}}{P_s Y_s}\right)} \end{aligned} \tag{A.64}$$

Given that, for an industry, $K_s = \sum_{i=1}^{M_s} K_{si}$ and $L_s = \sum_{i=1}^{M_s} L_{si}$ and, for the representative firm, $K = \sum_{i=1}^{S} K_s$ and $L = \sum_{i=1}^{S} L_s$, from Equation A.43, with $P = 1$, and the distortions and marginal revenues of capital and labor obtained, we get:

$$\begin{aligned} K_{si} &= \frac{\sigma-1}{\sigma}\alpha_s \left(\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\right) \frac{P_{si}Y_{si}}{R} \\ K_s = \sum_{i=1}^{M_s} K_{si} &= \frac{\sigma-1}{\sigma}\alpha_s \sum_{i=1}^{M_s} \left[\left(\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\right) \frac{P_{si}Y_{si}}{R} \left(\frac{\theta_s Y}{P_s Y_s}\right)\right] \\ K_s &= \frac{\sigma-1}{\sigma}\alpha_s \theta_s Y \overline{MRPK_s}^{-1} \\ K = \sum_{i=1}^{S} K_s &= \frac{\sigma-1}{\sigma} Y \sum_{i=s'}^{S} \alpha_{s'} \theta_{s'} \overline{MRPK_{s'}}^{-1} \\ \frac{K_s}{K} &= \frac{\alpha_s \theta_s \overline{MRPK_s}^{-1}}{\sum_{i=s'}^{S} (1-\alpha_{s'})\theta_{s'} \overline{MRPK_{s'}}^{-1}} \\ K_s = \sum_{i=1}^{M_s} K_{si} &= K \frac{\alpha_s \theta_s / \overline{MRPK_s}}{\sum_{i=s'}^{S} \alpha_{s'} \theta_{s'} / \overline{MRPK_{s'}}} \end{aligned} \tag{A.65}$$

Similarly:

$$\begin{aligned}
L_{si} &= \frac{\sigma-1}{\sigma}(1-\alpha_s)(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{\omega} \\
L_s = \sum_{i=1}^{M_s} L_{si} &= \frac{\sigma-1}{\sigma}(1-\alpha_s)\sum_{i=1}^{M_s}\left[(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{\omega}\left(\frac{\theta_s Y}{P_s Y_s}\right)\right] \\
L_s &= \frac{\sigma-1}{\sigma}(1-\alpha_s)\theta_s Y \overline{MRPL_s}^{-1} \\
L = \sum_{i=1}^{S} L_s &= \frac{\sigma-1}{\sigma} Y \sum_{i=s'}^{S}(1-\alpha_{s'})\theta_{s'}\overline{MRPL_{s'}}^{-1} \\
\frac{L_s}{L} &= \frac{(1-\alpha_s)\theta_s\overline{MRPL_s}^{-1}}{\sum_{i=s'}^{S}(1-\alpha_{s'})\theta_{s'}\overline{MRPL_{s'}}^{-1}} \\
L_s = \sum_{i=1}^{M_s} L_{si} &= L\frac{(1-\alpha_s)\theta_s/\overline{MRPL_s}}{\sum_{i=s'}^{S}(1-\alpha_{s'})\theta_{s'}/\overline{MRPL_{s'}}} \qquad \text{(A.66)}
\end{aligned}$$

The $\overline{TFPR_s}$ is proportional to the geometric mean of the weighted average marginal revenues of capital and labor for a sector and represents the observed $TFPR$:

$$\overline{TFPR_s} = \frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK_s}}{\alpha_s}\right)^{\alpha_s}\left(\frac{\overline{MRPL_s}}{1-\alpha_s}\right)^{1-\alpha_s} \qquad \text{(A.67)}$$

Considering that the sector is represented by a Cobb-Douglas production function:

$$\begin{aligned}
Y_s &= TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s} \qquad \text{(A.68)} \\
TFP_s &= \frac{Y_s}{K_s^{\alpha_s} L_s^{1-\alpha_s}} \\
TFP_s &= \frac{\left[\sum_{i=1}^{M_s}(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{i=1}^{M_s} K_{si}\right)^{\alpha_s}\left(\sum_{i=1}^{M_s} L_{si}\right)^{1-\alpha_s}} \\
TFP_s &= \frac{\left[\sum_{i=1}^{M_s}(A_{si}\frac{1-\tau_{Y_{si}}}{(1+\tau_{K_{si}})^{\alpha_s}})^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\left(\sum_{i=1}^{M_s}\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\frac{P_{si}Y_{si}}{P_s Y_s}\right)^{\alpha_s}\left(\sum_{i=1}^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_s Y_s}\right)^{1-\alpha_s}} \\
TFP_s &= \left[\sum_{i=1}^{M_s}\left(A_{si}\,\frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \qquad \text{(A.69)}
\end{aligned}$$

If the marginal revenues of capital and labor are equalized among firms within the same sector and, consequently, their revenue productivities, we obtain $\overline{TFPR_s} = TFPR_{si}$.

Thus, from Equation A.69, we have that the efficient TFP of the industry will be:

$$TFP_s = \left[\sum_{i=1}^{M_s} \left(A_{si}\, \frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$$

$$TFP_s = \left[\sum_{i=1}^{M_s} \left(A_{si}\, \frac{\frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK_s}}{\alpha_s}\right)^{\alpha_s}\left(\frac{\overline{MRPL_s}}{1-\alpha_s}\right)^{1-\alpha_s}}{\frac{\sigma}{\sigma-1}\left(\frac{MRPK_{si}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{si}}{1-\alpha_s}\right)^{1-\alpha_s}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$$

$$\overline{A}_s = \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{A.70}$$

Combining Equations A.38 and A.68, the aggregate output of the economy can be obtained as a function of the aggregate sectoral production factors and their TFP:

$$Y = \prod_{s=1}^{S} (TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s})^{\theta_s} \tag{A.71}$$

The aggregation of all sectors, using the Cobb-Douglas function in A.38, provides the misallocation and allows for calculating the output gains for the economy:

$$TFP_s = \left[\sum_{i=1}^{M_s} \left(A_{si}\, \frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$$

$$\frac{TFP_s}{TFP_s^*} = \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{\overline{A}_s}\, \frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$$

$$\frac{Y}{Y^*} = \prod_{s=1}^{S} \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{\overline{A}_s} \frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{\theta_s}{\sigma-1}} \tag{A.72}$$