Decomposition of the nominal and real yield curve, term premium dynamics, and inflation forecast in Brazil

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Abstract

In this paper, we use the dynamic and arbitrage-free affine models for the term structure of interest rates AFTSM′s to model nominal and real interest rates jointly. The approach allows decomposing interest rates into expectations for future interest rates and the risk premium investors compensate for buying long-term bonds. In addition, we analyze its ability to capture risk-adjusted inflation expectations using it for inflation forecasting. The results suggest that the real and nominal term premiums are time-varying and increase along maturities. Also, the risk-adjusted inflation expectations outperform the FOCUS survey in long forecasting horizons.

JEL classification: C53; E43; G17.

Keywords: yield curve; inflation risk premium; affine term-structure model; expected inflation; Break-even inflation rate.

1 Introduction

The estimation of the inflation risk premium has proved to be a challenging problem (Breach *et al.* , 2020). Models with different specifications or analyzing different periods have found different results. For instance, with data before the 2008 financial crisis, estimations involving structural models obtained results with a high magnitude of inflation risk premium (Ang *et al.* , 2008; Bekaert & Wang, 2010; Chernov & Mueller, 2012). On the other hand, studies using more recent data suggest premiums for the risk of inflation of smaller magnitude and sometimes even negative (Grishchenko & Huang, 2013; Abrahams *et al.* , 2016; Breach *et al.* , 2020). Since the relation of yield curves and the macroeconomics (Litterman & Scheinkman, 1991; Cochrane & Piazzesi, 2005; Ang *et al.* , 2006; Cochrane & Piazzesi, 2009; Cieslak & Povala, 2011; Crump *et al.* , 2018; Bernanke, 1990), it is essential to understand of the movements the term structure to improve forecasting, derivatives pricing, hedging, and fiscal and monetary policy.

In this paper, we use an arbitrage-free affine Gaussian model for the term structure (ATSM) to jointly model nominal and real interest rates, decompose the breakeven inflation, analyze the term premium dynamics, and forecast inflation. For model estimation, we use the recent approach for asset pricing based on linear regressions proposed by Adrian *et al.* (2015) and Abrahams *et al.* (2016). They present a method that allows computational gains in estimating factor models for the term structure while allowing the term premium to vary over time and serial dependence on the factors. Several other studies that estimated ATSM-class models for other economies used maximum likelihood (Joyce *et al.* , 2010; Kaminska, 2013; d'Amico *et al.* , 2018), which involves high-dimensional nonlinear optimization over a maximum likelihood function that can have many maxima locations (Hamilton & Wu, 2012). The approach proposed by Abrahams *et al.* (2016) considerably reduces these difficulties in estimating models of this class. To our knowledge, no study uses this procedure to address this question in the Brazilian economy.

The literature on the Brazilian economy suggests that the risk premium varies over time; see Lima & Issler, 2003; Tabak & Andrade, 2003; Marçal & Pereira, 2007; Tabak, 2009 for early references. Vicente & Graminho (2015) and Caldeira (2020), for instance, suggest that the inflation risk premium is time-varying. However, Vicente & Graminho (2015) does not find evidence of a liquidity premium in Brazil. Also, They suggest that the inflation risk premium is small for short horizons and is time-varying for long horizons, and inflation expectations are the main component of breakeven inflation. Surveys of expected inflation naturally emerge as a predictor of future inflation. See, among other (Ang *et al.* , 2007; Chun, 2012). The FOCUS survey, conducted by the Central Bank of Brazil, emerges as the main competitor to forecasting inflation in our research, see (de Carvalho *et al.* , 2009). Also, breakeven inflation is naturally a competitor; see (Vicente & Guillen, 2013; Caldeira & Furlani, 2013).

Our innovation is in the method used for the estimation, which lies in the approach of Abrahams *et al.* (2016). Our main findings can be summarized as follows. First, we disentangle the influence of term premiums on nominal and real rates. Then, we show the decomposition of the BEIR in expected inflation and inflation risk premium. Lastly, we use the derivation of the expected inflation to predict the IPCA. Following the literature, the results suggest that the premiums are time-varying and increase along maturities. The inflation risk premium is also time-varying, with negative values in specific periods. The expected inflation and the Focus survey outperform the RW forecasts; however, the Focus approach is a major workhorse. The second section introduces the Abrahams *et al.* (2016) AFNS model estimation following this introductory section. In the third section, we present in-sample results, term premium results, and out-of-sample inflation forecasts. In the fourth section, we conclude.

The paper is organized as follows. Section 2 describes the factor models used for modeling the term structure and shows how to convert yield forecasts into bond return forecasts. Section 3 discusses a two-step estimation procedure for expected bond returns and for the conditional covariance matrix of bond returns. Section 4 discusses the empirical applications to both portfolio optimization and to VaR computations. Finally, Section 5 concludes.

2 Gaussian Affine Term Structure Models

Affine term structure models (ATSMs), since Duffee (2002), are the most commonly used class of models in the literature for decomposing interest rates on government bonds. More recently, the approach developed by Adrian *et al.* (2015) has been widely used to decompose interest rates into their components: expectation and forward premium. This section presents an ATMS model specification following the exposition of Abrahams *et al.* (2015) and Abrahams *et al.* (2016).

The price, at time t, of a zero-coupon bond with maturity n is denoted by $P_t^{(n)}$. As is common in Gaussian models for the term structure, it is assumed that the vector of state variables is governed by an autoregressive process of the type VAR(1):

$$X_{t+1} - \mu_X = \Phi\left(X_t - \mu_X\right) + \nu_{t+1}, \qquad \nu_{t+1} \sim \mathcal{N}(0, \Sigma) \tag{1}$$

where the shocks ν_{t+1} are conditionally Gaussian, homoscedastic and independent over time. A single pricing mechanism is introduced to enforce the absence of arbitrage which governs all traded assets:

$$P_t^{(n)} = \mathbb{E}\left\{M_{t+1} P_{t+1}^{(n-1)}\right\}. \tag{2}$$

The stochastic discount factor M_t (pricing kernel) is a function of the short-term interest rate and the risk perceived by the market:

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}\nu_{t+1}\right), \tag{3}$$

where $r_t = \ln P_t^{(n)}$ denotes the risk-free interest rate that is continuously compounded. In Gaussian ATSMs the log price, $P_t^{(n)}$, of a risk-free discount bond with remaining time to maturity n follows $\log P_t^{(n)} = A_n + B_n^{prime} X_t$ which implies that:

$$r_t = \delta_0 + \delta_1' X_t. \tag{4}$$

The risk market price vector, λ_t, is an essentially affine function of the factors, as in Duffee (2002):

$$\lambda_t = \Sigma^{-1/2} \left(\lambda_0 + \lambda_1 X_t\right), \tag{5}$$

where λ_0 and λ_1 have dimensions $K \times 1$ and $K \times K$, respectively. Further defines:

$$\tilde{\mu} = \left(I_K - \Phi\right) \mu_X - \lambda_0, \tag{6}$$

$$\tilde{\Phi} = \Phi - \lambda_1. \tag{7}$$

These parameters govern the dynamics of the pricing factors under the risk-neutral and feature prominently in the recursive pricing relationships derived below.

Given the above assumptions, it can be shown that interest rates on zero-coupon bonds are affine functions of the factors (Ang & Piazzesi, 2003):

$$y_t^{(n)} = -\frac{1}{n} \left(A_n + B_n' X_t\right), \tag{8}$$

where the coefficients A_n and B_n follow the recursive equations:

$$A_n = A_{n-1} + B_{n-1}' \tilde{\mu} + \frac{1}{2} B_{n-1}' \Sigma B_{n-1} - \delta_0, \quad A_0 = 0 \tag{9}$$

$$B_n' = B_{n-1}' \tilde{\Phi} - \delta_1', \quad B_0 = 0_{K \times 1}. \tag{10}$$

Recently, there has been a growing interest in the literature in recovering expectations about future inflation rates from the nominal and real term structure of interest rates (Abrahams *et al.* , 2016; Breach *et al.* , 2020). Let Q_t be a time price index t and let $P_{t,R}^{(n)}$ be the price in t of an inflation-indexed bond with face value 1, which pays the amount $\frac{Q_{t+n}}{Q_t}$ at maturity, $t+n$. The price of such a title satisfies the following:

$$P_{t,R}^{(n)} = \mathbb{E}_t \left\{ \exp\left(-r_t - \ldots - r_{t+n-1}\right) \frac{Q_{t+n}}{Q_t} \right\}. \tag{11}$$

Denote the log-inflation for one period by $\pi_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right)$, therefore:

$$\frac{Q_{t+n}}{Q_t} = \exp\left(\sum_{i=1}^{n} \pi_{t+i}\right). \tag{12}$$

As in the case of nominal bonds, the prices of inflation-indexed bonds are exponentially affine in terms of pricing factors:

$$\log P_{t,R}^{(n)} = A_{n,R} + B_{n,R}' X_t. \tag{13}$$

Thus, one-period inflation is also a linear function of the state variables:

$$\pi_t = \pi_0 + \pi_1' X_t,$$

where π_0 is a scalar and π_1 is a vector of dimension $(K \times 1)$. According to Abrahams *et al.* (2016), it is possible to derive recursions for the prices of inflation-linked bonds by rewriting the equation (11) in terms of the price of another inflation-linked bond traded one period ahead:

$$P_{t,n}^{R} = \mathbb{E}_t \left\{ \exp\left(-r_t + \pi_{t+1}\right) P_{t+1,R}^{(n-1)} \right\}. \tag{14}$$

Solving this equation and combining the coefficients, we arrive at the coefficients of Equation (13), which are determined by the following system of equations in differences:

$$A_{n,R} = A_{n-1,R} + B_{n-1,R}^{\pi\prime}\tilde{\mu} + \frac{1}{2}B_{n-1,R}^{\pi\prime}\Sigma B_{n-1,R}^{\pi} - \delta_{0,R}, \quad A_{0,R} = 0 \tag{15}$$

$$B_{n,R}' = B_{n-1,R}^{\pi\prime}\tilde{\Phi} - \delta_1', \quad B_{0,R} = 0_{K\times 1}. \tag{16}$$

where $\delta_{0,R} = \delta_0 - \pi_0$ and $B_{n,R}^{\pi} = (B_{n,R} + \pi_1)\ \forall n$. Making the parameters referring to the risk market price, λ_0 and λ_1, equal to zero in the systems of equations (9)-(10) and (**??**)-(16), we obtain the risk-adjusted pricing parameters (makes the mapping of the risk-neutral measure, $\mathbb{Q}$, to the physical measure, $\mathbb{P}$).

3 Estimation

3.1 Nominal bonds returns

Recall that log excess one-period holding returns are defined as

$$rx_{t+1}^{(n-1)} = \log P_{t+1}^{(n-1)} - \log P_t^{(n)} - r_t. \tag{17}$$

Plugging equation (13) we obtain

$$rx_{t+1}^{(n-1)} = (A_{n-1} - A_n - \delta_0) - \left(B_n' + \delta_1'\right) X_t + B_{n-1}' X_{t+1} \tag{18}$$

Thus, imposing the recursive equations yields (9) and (10)

$$rx_{t+1}^{(n-1)} = \alpha_{n-1} - B_{n-1}'\tilde{\Phi} X_t + B_{n-1}' X_{t+1}, \tag{19}$$

where

$$\alpha_{n-1} = -\left(B_{n-1}'\tilde{\mu} + \frac{1}{2}B_{n-1}'\Sigma B_{n-1}\right). \tag{20}$$

3.2 Inflation-linked bonds returns

Log excess one period holding returns on inflation indexed securities are then given by

$$rx_{t+1,R}^{(n-1)} = \log P_{t+1,R}^{(n-1)} - \log P_{t,R}^{(n)} - r_t. \tag{21}$$

Thus, imposing the recursive equations yields (15) and (16)

$$rx_{t+1,R}^{(n-1)} = \alpha_{n-1,R} - (B_{n-1,R} + \pi_1)' \tilde{\Phi} X_t + B'_{n-1,R} X_{t+1}, \tag{22}$$

where

$$\alpha_{n-1,R} = -\left(\pi_0 + (B_{n-1,R} + \pi_1)' \tilde{\mu} + \frac{1}{2} (B_{n-1,R} + \pi_1)' \Sigma (B_{n-1,R} + \pi_1)\right). \tag{23}$$

3.3 Initial Conditions

To obtain initial conditions note that adding inflation to both sides of equation (21) and combining with equations (15), and (16), we obtain

$$rx_{t+1,R}^{(n-1)} + \pi_{t+1} = \alpha^{\pi}_{n-1,R} - (B_{n-1,R} + \pi_1)' \tilde{\Phi} X_t + (B_{n-1,R} + \pi_1)' X_{t+1}, \tag{24}$$

where

$$\alpha^{\pi}_{n-1,R} = -\left((B_{n-1,R} + \pi_1)' \tilde{\mu} + \frac{1}{2} (B_{n-1,R} + \pi_1)' \Sigma (B_{n-1,R} + \pi_1)\right). \tag{25}$$

Stacking log excess holding period returns on nominal bonds from equation (19) and on inflation-indexed bonds from equation (24) into the vector R^{π}, we thus obtain

$$R^{\pi}_{t+1} = \alpha - B\tilde{\Phi} X_t + B X_{t+1}, \tag{26}$$

where

$$\alpha = -\left(B\tilde{\mu} + \frac{1}{2}\gamma\right), \tag{27}$$

$$B = (B_1, \ldots, B_{N_N}, B_{1,R} + \pi_1, \ldots, B_{N_R,R} + \pi_1)', \tag{28}$$

$$\gamma = \left(B'_1 \Sigma B_1, \ldots, B'_{N_N} \Sigma B_{N_N}, (B_{1,R} + \pi_1)' \Sigma (B_{1,R} + \pi_1), \ldots, (B_{N_R,R} + \pi_1)' \Sigma (B_{N_R,R} + \pi_1)\right)'. \tag{29}$$

For initial conditions we use an approach similar to Adrian *et al.* (2015). To provide initial estimates of our parameters we stack the observed return data as

$$\mathbf{R}^{\pi} = \alpha^{\pi} \iota'_T - B\tilde{\Phi} X_{-} + B X + E \tag{30}$$

where R^{π} is $N \times T, X_{-}$and X are $K \times T$ matrices of the stacked X_{t-1} 's and X_t 's, respectively, and ι_T is a $T \times 1$ vector of ones. Using the estimated residuals, $\hat{E}_{\text{ols}}$, from this regression we obtain $\hat{\Sigma}_e = T^{-1} \cdot \hat{E}_{\text{ols}} \hat{E}'_{\text{ols}}$. Our initial value for $\tilde{\Phi}$ is

$$\hat{\tilde{\Phi}}_{\text{gls}} = -\left(\hat{B}'_{\text{ols}} \hat{\Sigma}_e^{-1} \hat{B}_{\text{ols}}\right)^{-1} \hat{B}'_{\text{ols}} \hat{\Sigma}_e^{-1} \widehat{B\tilde{\Phi}_{\text{ols}}} \tag{31}$$

We then run an additional SUR on ι_T and $\left(-\hat{\tilde{\Phi}}_{\text{gls}} X_- + X\right)$ to obtain initial values for α and B which we label $\hat{\alpha}_{\text{gls}}$ and $\hat{B}_{\text{gls}}$. Finally, we produce an initial value for $\tilde{\mu}$ as

$$\hat{\tilde{\mu}}_{\text{gls}} = -\left(\hat{B}'_{\text{gls}}\hat{\Sigma}_e^{-1}\hat{B}_{\text{gls}}\right)^{-1}\hat{B}'_{\text{gls}}\hat{\Sigma}_e^{-1}\left(\hat{\alpha}_{\text{gls}} + \frac{1}{2}\hat{\gamma}_{\text{gls}}\right), \tag{32}$$

where $\hat{\gamma}_{\text{gls}}$ is formed using $\hat{B}_{\text{gls}}$ and $\hat{\Sigma}$ (see equation (29)). We also need initial values for the parameters (δ_0, δ_1') governing the nominal short rate. Since the nominal short rate is directly observed, this is simply achieved by performing an OLS regression of the short rate onto a constant and the vector of pricing factors as in Adrian *et al.* (2015).

The parameters $\tilde{\mu}$ and $\tilde{\Phi}$ are related to the market price of risk parameters λ_0 and λ_1 via the relationships $\tilde{\mu} = (I_K - \Phi)\,\mu_X - \lambda_0$ and $\tilde{\Phi} = \Phi - \lambda_1$. Since the pricing factors X are observed and follow the joint vector autoregression given by equation (1), the OLS estimator of μ_X is simply given by the sample mean of the factors X and the OLS estimator of Φ is obtained by regressing the demeaned observations of X on their one period lags equation by equation. We stack the estimated innovations into the matrix $\hat{V}$ and construct an estimator of the state variable variance-covariance matrix $\hat{\Sigma} = T^{-1} \cdot \hat{V}\hat{V}'$. Given estimates $\hat{\mu}_X$ and $\hat{\Phi}$, we then obtain estimates of the market price of risk parameters via

$$\begin{aligned} \hat{\lambda}_0 &= \left(I_K - \hat{\Phi}\right)\hat{\mu}_X - \hat{\tilde{\mu}}_{\text{gls}}, \\ \hat{\lambda}_1 &= \hat{\Phi} - \hat{\tilde{\Phi}}_{\text{gls}}. \end{aligned} \tag{33}$$

In our empirical application we skip the estimation of parameters via numerical maximization of the likelihood as per Abrahams *et al.* (2016) and use the values of OLS initial conditions estimation. Also we use the sum of squared real return fitting errors as the criterion function to estimate π_0 and π_1 as per Abrahams *et al.* (2015). We provide explicit expressions for real yields as linear-quadratic functions of π_0 and π_1 (given estimates for $\hat{\tilde{\Phi}}_{\text{gls}}$, $\hat{\tilde{\mu}}_{\text{gls}}$, $\hat{\delta}_{0,\text{ols}}$, $\hat{\delta}_{1,\text{ols}}$) which may be used for numerical optimization. We then solve for the estimated π_0 and π_1 with the initial conditions via,

$$\left(\hat{\pi}_0, \hat{\pi}_1'\right)' = \arg\min_{\pi_0,\pi_1} \sum_{i=1}^{N_R}\sum_{t=1}^{T}\left(rx_{t+1,R}^{(n-1)} - g\left(\pi_0, \pi_1; \hat{\tilde{\Phi}}_{\text{gls}}, \hat{\tilde{\mu}}_{\text{gls}}, \hat{\delta}_{0,\text{ols}}, \hat{\delta}_{1,\text{ols}}, n_i, t\right)\right)^2 \tag{34}$$

where $g(\cdot)$ can be found by using the recursive equations (15) and (16). Next section presents the dataset and results.

4 Data and Results

We use end-of-month values from 2006:01 to 2022:04 for a total of $T = 196$ monthly observations. In the estimation, a cross-section of $N_N = 11$ one-month excess holding period returns for nominal rates with maturities $n = 6, 12, 24, \cdots, 120$ months and $N_R = 9$ excess returns on NTNB′s with maturities $n = 24, \cdots, 120$ months is used. The SELIC rate is used as the nominal risk-free rate. The price index Q_t used to calculate NTNB′s payouts is IPCA index, which is available from the

IBGE site. See Table 1 for statistics of interest rates.

Table 1: **Descriptive Statistics**

Maturities	$n = 12$	$n = 24$	$n = 36$	$n = 60$	$n = 120$
Nominal Interest Rates					
Mean	0.103	0.107	0.110	0.114	0.117
Std. Dev.	0.034	0.030	0.027	0.024	0.022
Skewness	−0.514	−0.485	−0.404	−0.237	−0.052
kurtosis	2.484	2.671	2.828	3.053	3.205
$\rho(1)$	0.977	0.970	0.961	0.948	0.935
$\rho(6)$	0.775	0.767	0.751	0.713	0.649
Real Interest Rates					
Avg.	0.048	0.053	0.055	0.057	0.058
Std. Dev.	0.027	0.024	0.022	0.018	0.014
Skewness	−0.097	0.019	0.064	0.108	0.076
kurtosis	2.783	2.791	2.899	2.950	2.839
$\rho(1)$	0.962	0.964	0.964	0.966	0.962
$\rho(6)$	0.732	0.773	0.778	0.774	0.758

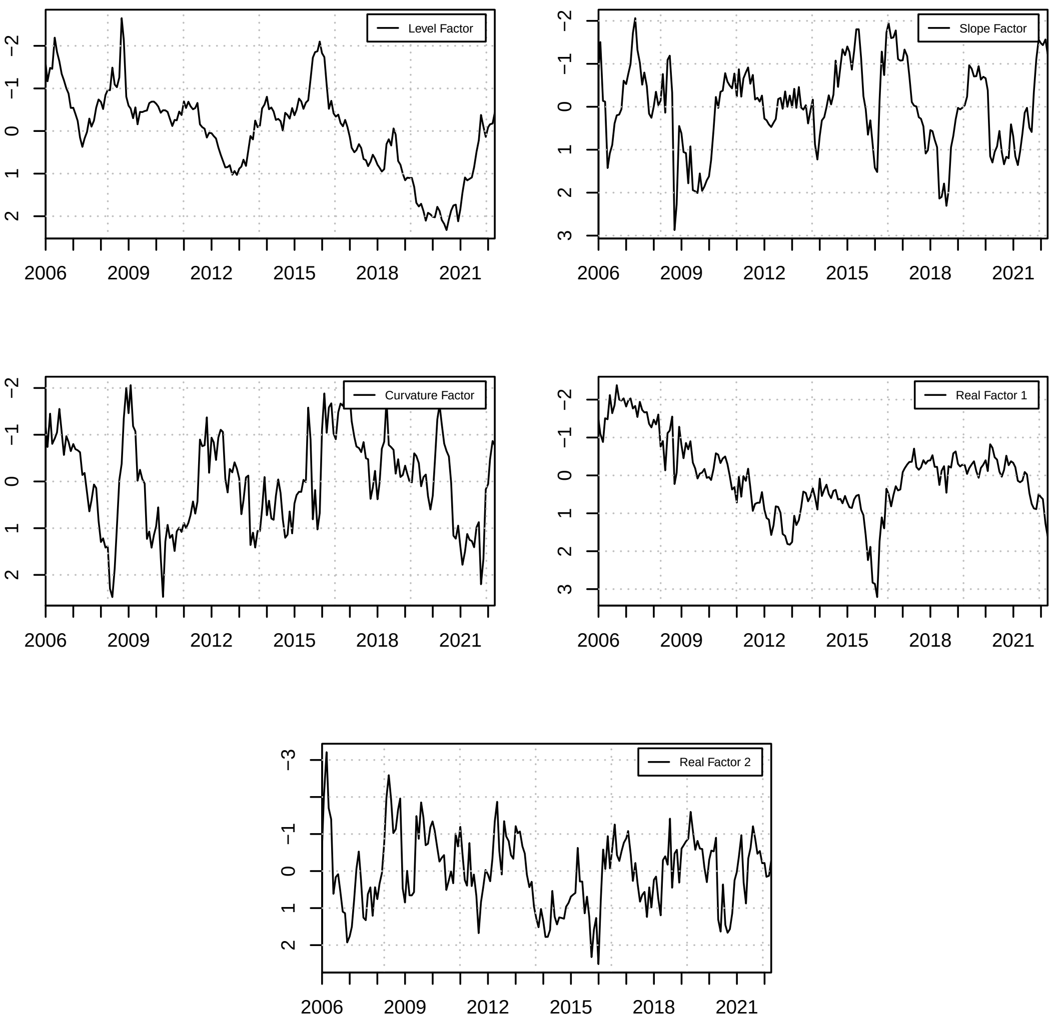
Following Abrahams *et al.* (2016) and various other authors (Adrian *et al.* , 2015; Joslin *et al.* , 2011; Wright, 2011), we calculate principal components from yields and used as pricing factors in the model. Specifically, two sets of principal components are used. First, $K_N = 3$ principal components are extracted from nominal yields of maturities $n = 6, 12, 24, \cdots, 120$ months. Then additional factors are obtained as the first $K_N = 2$ principal components from the residuals of regressions of NTNB′s yields of maturities $n = 24, \cdots, 120$ months on the K_N nominal principal components. This orthogonalization step reduces the unconditional collinearity among the pricing factors. In sum, $K = K_N + K_R = 5$ model factors. See Figure 1.

We show the in-sample results in Table 2 and Figures 2 and 3 . In general, the model fits better to long maturities. The BEIR decomposition suggests that the real and nominal term premiums increase along maturities. Also, the same happens with inflation risk premiums, which account for the most movements of the BEIR in long maturities. Thus the expected inflation is quite flat for long maturities and is highly correlated to BEIR in short maturities. See Figures 4, 5, and 6.

In Inflation forecasting, we use the model-implied inflation expectations as a predictor, representing breakeven inflation rates adjusted for risk premia. For instance, we use the six-month maturity to predict inflation 6-months ahead, and so on. The same is done to unadjusted NTNB breakevens, which is a predictor of future inflation as well. The third is a simple random walk forecast, which takes the average realized inflation over the prior n months as a prediction of average inflation over the next n months. Forecasts are performed over horizons from 6 to 36 months, and forecasting errors are computed using overlapping observations. The panel reports out-of-sample results, using an eleven-year "learning period" over the period 2006:01–2016:06 and forecasting over the period 20016:07–2022:04. So, 6-months ahead has 70 forecasts, 12-months ahead has 64 forecasts, 24 months ahead has 52 forecasts, and 24-months ahead has 40 forecasts. See Table

Figure 1: **Pricing factors: observed time series**

Note: This figure plots the time series of the factors of our model. These are the first three principal components extracted from the cross-section of end-of-month observations of nominal yields of maturities $n = 6, 12, 24, \ldots, 120$ months. The fourth and fifth factors are the first two principal components extracted from the cross-section of orthogonalized real yields of maturities $n = 24, \ldots, 120$, the residuals from regressing real yields on the first three principal components of the nominal yield curve.



3 and Figure 7. The results suggest it is difficult to outperform the Focus survey; however, the model-implied forecast follows closely. The next section presents concluding remarks.

Table 2: **In-Sample Results**

Note: This table compares the root mean squared error and mean absolute error of nominal and real yield curves at one-year, three-year, five-year, and ten-year maturities. The first panel reports in-sample results for the entire sample from 2006:01 to 2022:04

	Maturities			
Measure	n = 12	n = 36	n = 60	n = 120
RMSE : Nominal	0.157	0.073	0.135	0.148
RMSE : Real	0.472	0.096	0.132	0.070
MAE : Nominal	0.123	0.058	0.104	0.117
MAE : Real	0.375	0.075	0.099	0.057

Figure 2: **Observed and Nominal Model-Implied Time Series**

Note: This figure provides time series plots of observed and model-implied nominal yields at one-year, three-year, five-year, and ten-year maturities. The observed yields are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields.

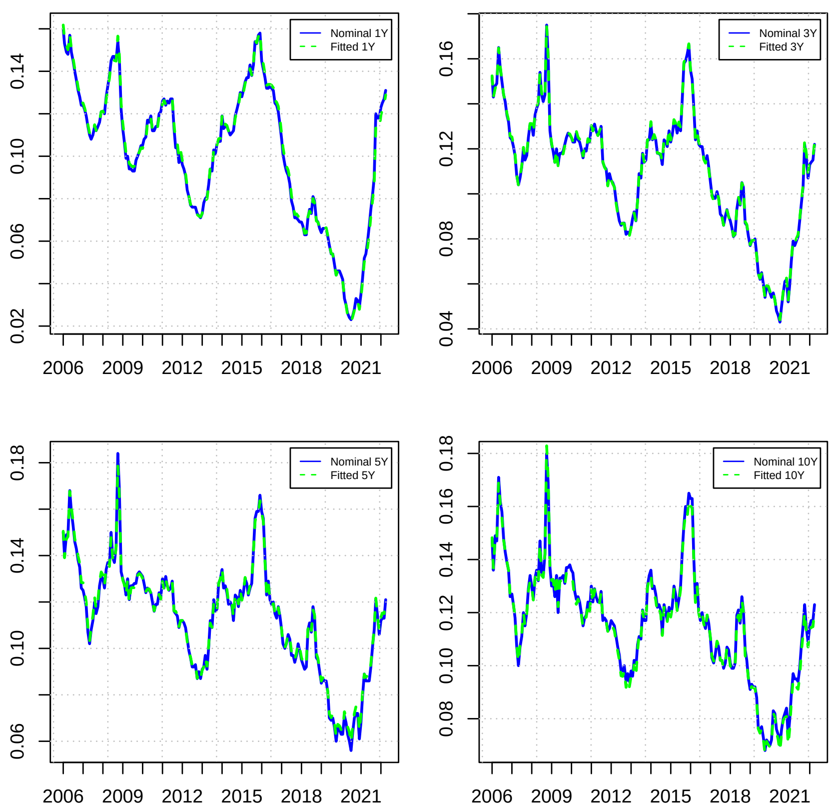


Figure 3: **Observed and Real Model-Implied Time Series**

Note: This figure provides time series plots of observed and model-implied real yields at one-year, three-year, five-year, and ten-year maturities. The observed yields are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields.

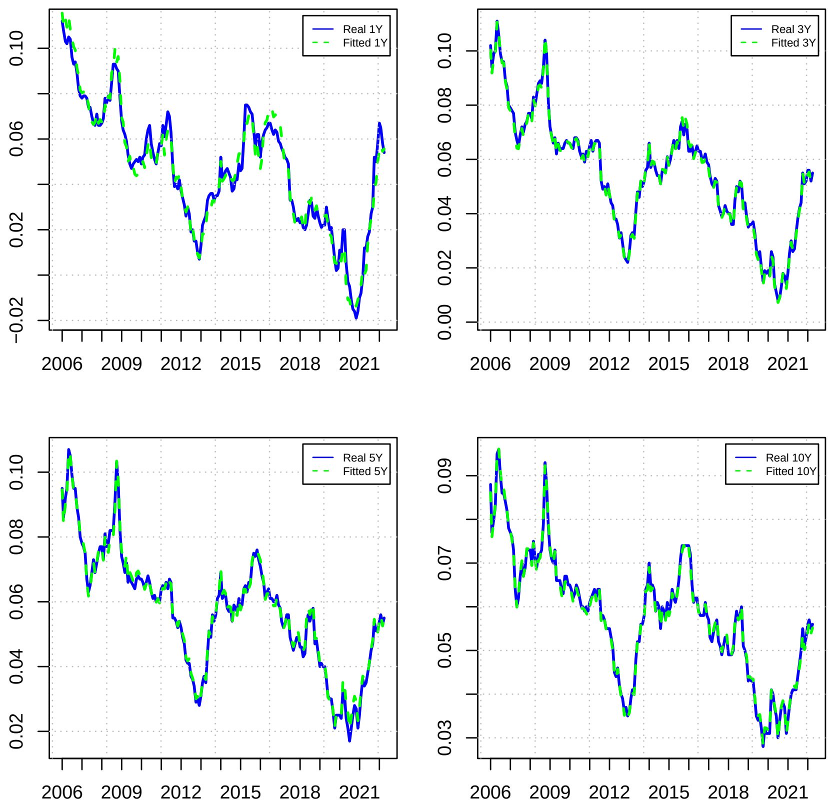


Figure 4: **Nominal Term Premium**

Note: This figure provides time series plots of the decomposition of the observed nominal yield curves in risk-neutral yield and yield term premium at one-year, three-year, five-year, and ten-year maturities.

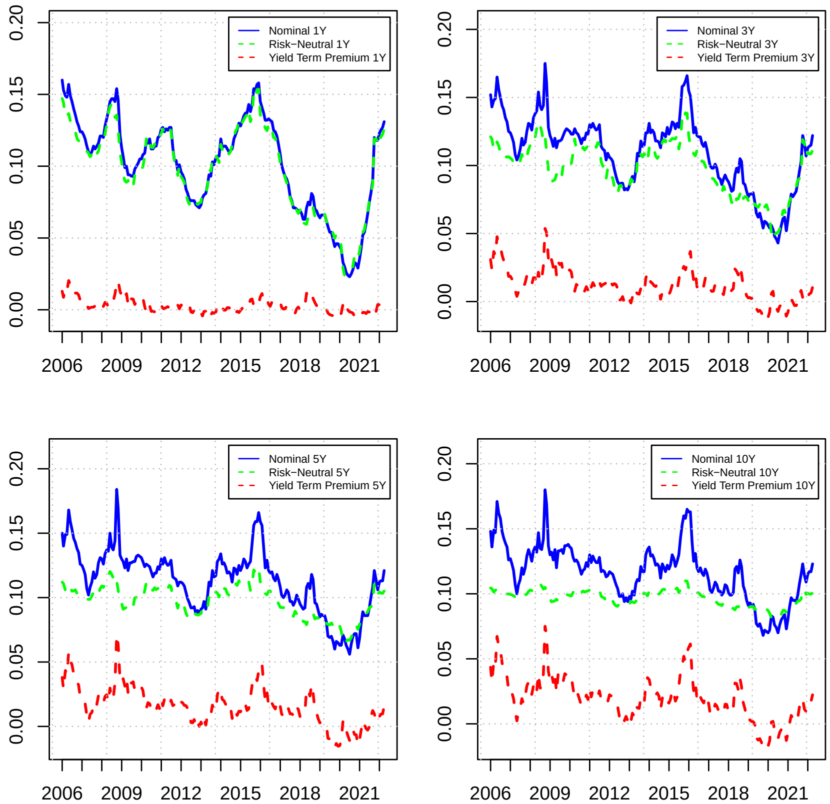


Figure 5: **Real Term Premium**

Note: This figure provides time series plots of the decomposition of the observed nominal yield curves in risk-neutral yield and yield term premium at one-year, three-year, five-year, and ten-year maturities.

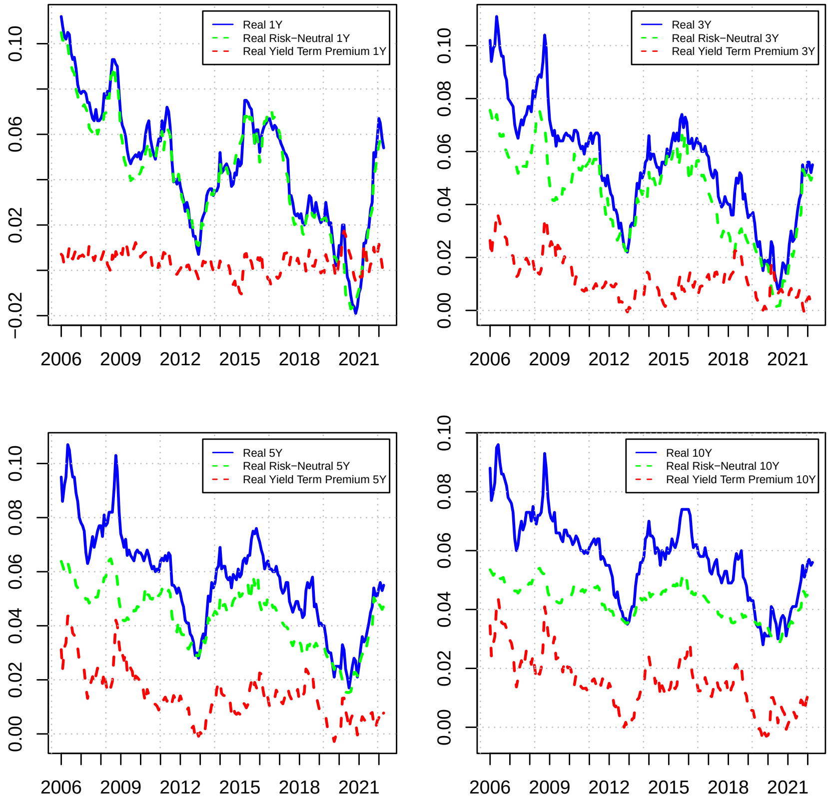


Figure 6: **BEIR Decomposition**

Note: This figure shows the decomposition of breakeven inflation rates into the model-implied expected inflation and the inflation risk premium. The panels show this decomposition at one-year, three-year, five-year, and ten-year maturities.

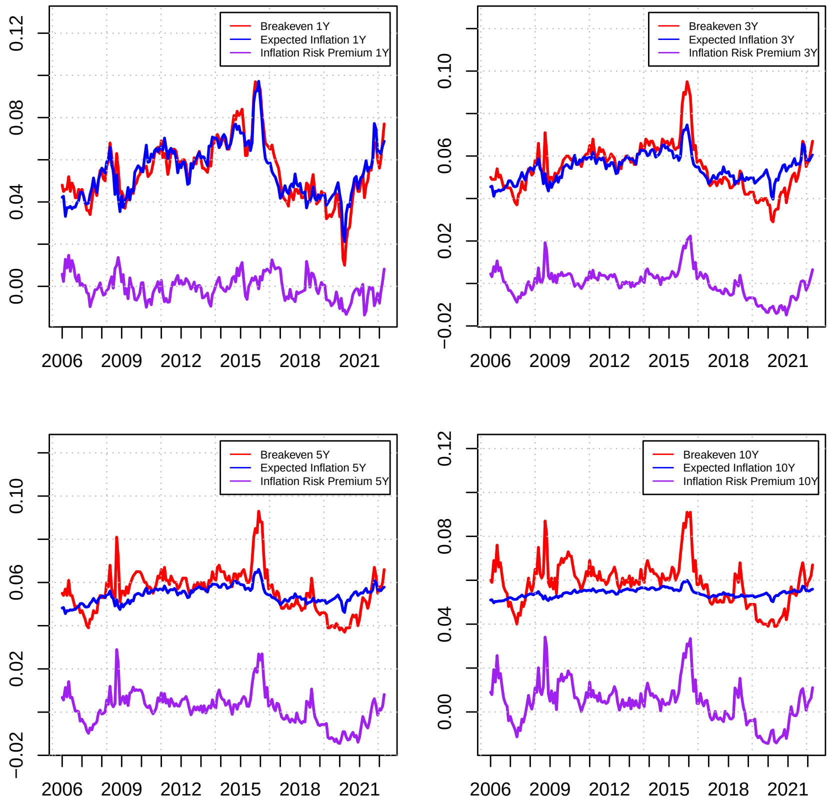


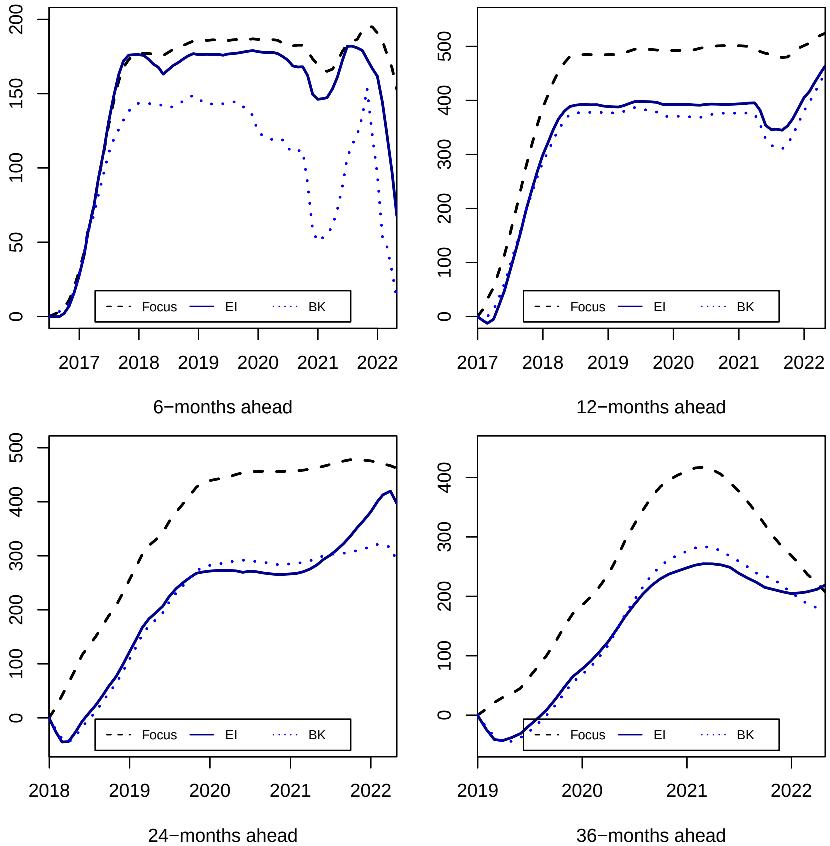
Table 3: **Inflation Forecasting**

Note: This table compares three models' root mean squared error for predicting future inflation (IPCA). The first uses the model-implied inflation expectations derived in Section 2. These represent breakeven inflation rates adjusted for risk premia. The second method takes unadjusted breakevens as a predictor of future inflation. The third is a simple random walk forecast, i.e., it takes the average realized inflation over the prior n months as a prediction of average inflation over the next n months. Forecasts are performed over horizons from 6 to 36 months, and forecasting errors are computed using overlapping observations. The panel reports out-of-sample results, utilizing an eleven-year "learning period" over the period 2006:01–2016:06 and forecasting over the period 20016:07–2022:04. So, 6-months ahead has 70 forecasts, 12-months ahead has 64 forecasts, 24-months ahead has 52 forecasts, and 24-months ahead has 40 forecasts. Bold values are statistically significant by at least 5%, according to Giacomini & White (2006) test.

	Horizons			
Model	n = 12	n = 36	n = 60	n = 120
RandonWalk	2.743	4.320	4.487	4.295
Focus	**0.842**	**0.749**	**0.747**	**0.848**
Modelforecast	0.933	0.782	**0.788**	0.839
Breakevens	0.989	0.788	0.850	0.873

Figure 7: **Cumulative Squared Prediction Error**

Note: This figure shows the cumulative squared prediction error of Random Walk, Focus, Model-Implied Expected Inflation, and BEIR forecasts at one-year, three-year, five-year, and ten-year maturities.



5 Concluding remarks

We estimate an arbitrage-free Gaussian model for the term structure of the yield curve that allows joint modeling of nominal and real interest rates. The model enables the decomposition of BEIR into expectations for inflation and risk premium. In-sample results suggest that the term premiums are time-varying and increase along maturities, which include negative values. The risk-adjusted inflation expectations outweigh unadjusted BEIRs and a Random Walk in the out-of-sample inflation forecast. The Focus survey is a benchmark challenging to outperform. However, the model-implied predictions have better results in long horizons.

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