

**ÁREA TEMÁTICA:  
FINANÇAS**

**PRICE DISCOVERY OVER TIME: AN  
APPLICATION TO THE BITCOIN MAR-  
KET**

## Price discovery over time: an application to the Bitcoin market

### Resumo

Este artigo tem como foco analisar o processo de formação de preços com base na informação filtrada entre negociações de microestrutura, que permitirá indicar qual mercado, entre os analisados, garante a liderança no processo de price-discovery. A ideia é aplicar a metodologia de kernel least squares (KLS) para indicar qual vetor de cointegração possui componente de maior variância que explica a volatilidade do preço do ativo analisado. A aplicação é feita no mercado de cryptoativos, em especial, o Bitcoin, e analisamos dados de microestrutura de mais de 17 casas de negociação destes ativos, conhecidas como exchanges. Após filtramos quais teriam alguma informação relevante, e volume suficiente para se tornar um agente de custódia e negociação relevante. Os resultados indicam que mesmo o Bitcoin, ainda está em fase de transformação no mercado, com playeres e exchanges alterando sua importância relativa, o que contraria a hipótese de mercados eficientes, em que todas as informações disponíveis estariam acessíveis a todos os agentes, dado que, a escolha da exchange ainda é uma forma de poder arbitrar neste mercado, a partir de high-frequency-trading (HFT).

**keywords:** HFT, cryptoativos, KLS, information-share.

### Abstract

This article aims to analyze the price formation process based on filtered information between microstructure trades, which will indicate the market among those analyzed that ensures leadership in the price discovery process. The idea is to apply the kernel least squares methodology to indicate which cointegration vector has a larger variance component that explains the volatility of the asset price being analyzed. The application is made in the cryptocurrency market, especially Bitcoin, and we analyze microstructure data from more than 17 trading platforms for these assets, known as exchanges. After filtering which ones would have relevant information and sufficient volume to become a relevant custodial and trading agent, the results indicate that even Bitcoin is still in a transformation phase in the market, with players and exchanges altering their relative importance, which contradicts the hypothesis of efficient markets, in which all available information would be accessible to all agents, given that the choice of the exchange is still a way to arbitrage in this market, based on high-frequency trading.

**keywords:** HFT, cryptocurrencies, KLS, information-share.

# 1 Introduction

In this article, we will promote a study applied to the time-varying in cryptocurrency. In light of the foregoing, the parameters associated with price formation are also associated with a time cutting, so we will undertake a statistical test that corroborates or refutes the temporal dependency hypothesis on the parameters associated with Component Share (CS) concerning the Price Discovery (PD) process (see, among others, Brandvold, Molnár, Vagstad and Valstad, 2015; Urquhart, 2016; Brauneis and Mestel, 2018; Baur, Dimpfl and Kuck, 2019; Makarov and Schoar, 2020). In a market, in particular, still growing and with low maturity, the parameters might be even more time-dependent than the parameters associated with better-established markets such as the stock market.

The same methodology was tested by Fernandes and Scherrer (2018) and Fruet Dias, Fernandes and Scherrer (2020) in the stock market, which gives evidence to our research on Price Discovery under time-varying. Specifically in this article, we will focus on the PD process and analyze the CS of this process based on (De Jong, 2002 ;Putniņš, 2013), and we will also estimate using the temporal dependency hypothesis using a Giraitis, Kapetanios and Yates (2013) kernel least squares estimator (KLS), in order to address this estimate appropriately. Some articles used this hypothesis to test volatility. Bohte and Rossini (2019) show the estimation by TV using an Bayesian model. Durham (2019) used quantile regression to describe volatility, with a Dynamic M-GARCH approach.

The cryptocurrency market allows people who are still outside the traditional market, whether it be financial or banking, trade and negotiated with speed, anonymity and decentralized. At the same time, large technology companies offering non-centralized information services and data records can use cryptocurrencies as a money transfer tool, without using the traditional financial system. There are numerous initiatives from companies like Facebook, Google, and Twitter that intend to use cryptos or similar system to allow the transfer of money between its users. Larger companies investing share of their assets on cryptos, like Tesla did. This point might guarantee the expansion of demand in the cryptocurrency market. Thus, we understand that studying the PD over CS is necessary to support the correct academic debate on digital financial assets (Härdle, Harvey and Reule, 2020).

The regulation of cryptocurrency markets can promote security, stability, and credibility for this market. However, otherwise coming can expel many traders who are looking for precisely this characteristic of non-regulation. The stock market is opposed to the crypto market, because of the considerable legal regulatory framework that mediates the negotiation process. Thus, our methodology that analyzes the CS on temporal dependency may provide theoretical subsidies for this discussion of reduction or increase regulation of the markets. Thus, we promote the theoretical subsidy for TD discussion and analysis of the efficiency of the cryptocurrency markets. This includes all the factors that depend on time and may vary over it. To that end, we will use a model based on the vector error correction, as the methodology that allows decomposing both CS and IS.

The current analysis depends on the fact that the same asset is negotiated at different exchanges, which are homogeneous. This predicate is sensitive for applications over the stock markets, because any asset and its ETF or mini-contract or future-contract is not in essence the same. However, the degree of homogeneity of the same cryptocurrency traded in different exchanges is substantially higher, which allows us to guarantee more robust results.

Our analysis is composed of a collection of high-frequency trading data from 4 bitcoin exchanges, between April and October 2018, a period in which there was a total variation in

prices of 100%, and there is a price difference among exchanges that reached 10%. Thus, we intend to answer what is the leading exchange in the price formation process, taking into account the minute-by-minute data to subsidize this answer, and we estimated the KLS that provides the daily curve of this process.

Our results demonstrate a time dependency, through the rejection of the Elliott and Müller (2006), a CS behavior over time for each exchange that demonstrated a pattern that depends on the volume traded, costs and fees, security and price. As already tested for the stock market, the test for bitcoin demonstrated the importance of analyzing the coefficients from their time dynamics and dependency.

The remainder of this paper is organized as follows. Section 2 describes the finance literature of price discovery, and the modelling of our methodology. Section 3 describes the price informativeness at cryptocurrency market of Bitcoins, and describes it's exchanges and how exchanges working as a third-party intermediary, which allow the same asset (Bitcoin) be negotiated at singular price at every exchange, and every transaction been registered. Section 4 presents our data set and all exchange price details over the 6 months period. Section 5 discusses the results of our estimations and we offer some concluding remarks in Section 6.

## 2 Finance Price Discovery Theory

In this section, we discuss the continuous-time model for price discovery, proposed by Fruet Dias, Fernandes and Scherrer (2020) and our application at cryptocurrency market.

The VECM framework is the base of the information share (IS) originally proposed by Hasbrouck and the componente share (CS) proposed by Chu, Hsieh and Tse. The IS is the simple decomposition of variance of the efficient price innovation equation, if the price series represent the same asset. Usually, the IS are build by the use of the spot and futures contracts of the same asset. In our essay, we can use the same asset (bitcoin) that is trade at different exchanges and the same nominal value (US\$). CS it's the ratio between  $\alpha_{\perp}$  and  $\beta_{\perp}$ , which are the two components of the  $I(1)$ <sup>1</sup> parts of the price discovery.

We assume the following equation for the prices of the financial assets:

$$dP_t = \Pi dP_t + C dW_t, \quad \text{with } P_0 = p_0, \quad (1)$$

where  $P_t$  is a  $k \times 1$  vector of log prices with  $k$  as the number of trading venues,  $\Pi = \alpha\beta'$  is a  $k \times k$  reduced-rank matrix with  $r = k - 1$  rank,  $\alpha$  and  $\beta$  are  $k \times r$  full-rank matrices,  $W$  is a  $k \times 1$  vector of Brownian motion, and  $C$  is a  $k \times k$  matrix with  $\Sigma = CC'$  as a positive definite. Therefore, there are  $r = k - 1$  cointegration vectors, with log prices sharing the asset's efficient price as the single common stochastic trend. Supported by Fruet Dias, Fernandes and Scherrer (2020) we assume that  $\beta = (I_r, l_r)$  is know, where  $l_r$  is a  $r \times 1$  unit vector. Accordingly,  $\alpha$  determines the reaction to deviations from the long-run equilibria  $\beta'P_t$ . The solution to (1) is a homogenous Gaussian Markov process given by

$$P_t = \exp(t\Pi) \left[ P_0 + \int_0^t \exp(-u\Pi) C dW_u \right], \quad (2)$$

Using the same structure of Fruet Dias, Fernandes and Scherrer (2020) We assume prices are observed regularly and equidistantly over the unit interval  $[0, 1]$  that characterizes,

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say, one trading day (calendar-time sampling, as discussed in Hansen and Lunde, 2006). Denote each interval in  $[0, 1]$  as  $[t_{i-1}, t_i]$ , where  $i = 1, 2, \dots, n$  and  $n$  is the total number of intervals such that  $0 = t_0 < t_1 < \dots < t_n = 1$ . The length of each interval is  $\delta = t_i - t_{i-1} = 1/n$  in  $[0, 1]$ . For instance, the usual trading day in the U.S. market lasts for 6.5 hours (23,400 seconds), and thus, sampling one observation per minute yields  $n = 390$  intraday observations, with  $\delta = 1/390$ . Denoting by  $\exp(A)$  the matrix exponential of a  $k \times k$  matrix  $A$  such that  $\exp(A) = \sum_{\ell=0}^{\infty} \frac{1}{\ell!} A^\ell$ , the exact discretization of (1) at interval length  $\delta$  reads

$$\Delta P_{t_i} = \Pi_\delta P_{t_{i-1}} + \varepsilon_{t_i}, \quad (3)$$

where  $\Pi_\delta = \alpha_\delta \beta'$  and  $\alpha_\delta = \alpha(\beta' \alpha)^{-1} [\exp(\delta \beta' \alpha) - I_r]$ , with  $I_r$  denoting a  $r$ -dimensional identity matrix, and  $P_{t_i}$  is a  $k \times 1$  vector of log-prices observed at discrete time. The innovation  $\varepsilon_{t_i}$  is iid Gaussian with zero mean and covariance matrix given by  $\Sigma_\delta = \int_0^\delta \exp(u \Pi) \Sigma \exp(u \Pi') du$ .

Kessler and Rahbek (2004) provide the conditions under which the mapping given  $\theta = (\Pi, \Sigma) \xrightarrow{\psi} \psi(\theta) = (\Pi_\delta, \Sigma_\delta)$  is unique,  $\theta$  is identifiable, and the space spanned by the columns of  $\alpha$  is equal to the one spanned by the columns of  $\alpha_\delta$ . Specifically, if all eigenvalues of  $\Pi$  are real and no elementary divisor of  $\Pi$  occurs more than once, Proposition 1 in Kessler and Rahbek (2004) shows that the mapping  $\psi$  is injective and  $\theta$  is identifiable. It is important to note that temporal aggregation preserves the cointegration rank, i.e.,  $\text{rank } \Pi_\delta = \text{rank } \Pi$ , and that the definition of (co)integration for Ornstein-Uhlenbeck OU processes in continuous time is consistent with the definition in discrete time (Kessler and Rahbek, 2004). This means that one may conduct inference about rank and cointegrating space using discrete-time procedures and then interpret the results in the continuous-time setting.

## 2.1 Component share

The component share relies on the orthogonal complement of  $\alpha_\delta$ , namely,  $\alpha_{\delta,\perp}$  such that  $\alpha'_{\delta,\perp} \alpha_\delta = 0$  (see, among others, Booth, So and Tseh, 1999; Chu, Hsieh and Tse, 1999; Harris, McNish and Wood, 2002; Hansen and Lunde, 2006). Because  $\alpha_{\delta,\perp}$  is not unique, one typically imposes  $\alpha_{\delta,\perp,1} + \alpha_{\delta,\perp,2} = 1$ . While  $\alpha_\delta$  corresponds to the stationary direction of the process in (3),  $\alpha_{\delta,\perp}$  relates to the non-stationary direction. This makes  $\alpha_{\delta,\perp}$  a natural quantity to assess how the efficient price relates to each market innovation. The market with the highest  $\alpha_{\delta,\perp}$  has the least need of adjustment towards the latent efficient price and hence it is the one that leads the price discovery process.

Using the normalization  $\alpha_{\delta,\perp,1} + \alpha_{\delta,\perp,2} = 1$ , it follows from the exact discretization of the reduced-rank OU process in (3) that allows to calculate the  $\alpha_\perp$ ,

$$\alpha_{\delta,\perp} = \left( \frac{\alpha_{\delta,2}}{\alpha_{\delta,2} - \alpha_{\delta,1}}, -\frac{\alpha_{\delta,1}}{\alpha_{\delta,2} - \alpha_{\delta,1}} \right)' = \left( \frac{\alpha_2}{\alpha_2 - \alpha_1}, -\frac{\alpha_1}{\alpha_2 - \alpha_1} \right)', \quad (4)$$

given that  $(\beta' \alpha)^{-1} [\exp(\delta \beta' \alpha) - I_r]$  cancels out for appearing in both numerators and denominators. It is now clear that  $\alpha_{\delta,\perp}$  is invariant to the sampling frequency in that  $\alpha_{\delta,\perp} = \alpha_\perp$  for any  $0 < \delta < 1$ . This means that identification and inference of the continuous-time price discovery measure arises directly from estimating  $\alpha_{\delta,\perp}$  at any sampling frequency. From an empirical perspective, (4) it allows us to learn about the continuous-time price discovery mechanism even if using data at a lower frequency (and hence less prone to market microstructure noise).

## 2.2 Information share

Here, we introduce the Hasbrouck's (1995) information share (Baillie, Booth, Tse and Zobotina, 2002; De Jong, 2002; Grammig, Melvin and Schlag, 2005; and Yan and Zivot, 2010). In short, the IS measure gives the share of each market contribution to the total variance of the efficient price ( $IS_{\delta,1} + IS_{\delta,2} = 1$ ). Using the exact discretization of (1), the IS measure of a given market  $m \in \{1, 2\}$  for  $0 < \delta < 1$  is

$$IS_{\delta,m} = \frac{[\xi_{\delta} C_{\delta}]_m^2}{\xi_{\delta} \Sigma_{\delta} \xi_{\delta}'}, \quad (5)$$

where  $\Sigma_{\delta} = C_{\delta} C_{\delta}' = \int_0^{\delta} \exp(u\Pi) \Sigma \exp(u\Pi') du$ ,  $\xi_{\delta}$  is the common row of  $\Xi_{\delta}$  in (??) that follows from  $\beta_{\perp} = (1, 1)'$ , and  $[\cdot]_m$  denotes the  $m$ th element of a vector. Using the fact that  $\alpha_{\delta,\perp,m} = \alpha_{\perp,m}$  for any  $0 < \delta < 1$ , the average IS measure in a given market  $m \in \{1, 2\}$  for  $0 < \delta < 1$  then reads

$$IS_{\delta,m} = \frac{1}{2} \left( \frac{[\xi_{\delta} C_{\delta}]_m^2}{\xi_{\delta} \Sigma_{\delta} \xi_{\delta}'} + \frac{[\xi_{\delta} \bar{C}_{\delta}]_m^2}{\xi_{\delta} \Sigma_{\delta} \xi_{\delta}'} \right) = \begin{cases} \frac{(\alpha_{\perp,1} \sigma_{\delta,1} + \alpha_{\perp,2} \sigma_{\delta,2} \rho_{\delta})^2 + \alpha_{\perp,1}^2 \sigma_{\delta,1}^2 (1 - \rho_{\delta}^2)}{2(\alpha_{\perp,1}^2 \sigma_{\delta,1}^2 + \alpha_{\perp,2}^2 \sigma_{\delta,2}^2 + 2\alpha_{\perp,1} \alpha_{\perp,2} \sigma_{\delta,1} \sigma_{\delta,2} \rho_{\delta})}, & \text{if } m = 1, \\ \frac{(\alpha_{\perp,2} \sigma_{\delta,2} + \alpha_{\perp,1} \sigma_{\delta,1} \rho_{\delta})^2 + \alpha_{\perp,2}^2 \sigma_{\delta,2}^2 (1 - \rho_{\delta}^2)}{2(\alpha_{\perp,1}^2 \sigma_{\delta,1}^2 + \alpha_{\perp,2}^2 \sigma_{\delta,2}^2 + 2\alpha_{\perp,1} \alpha_{\perp,2} \sigma_{\delta,1} \sigma_{\delta,2} \rho_{\delta})}, & \text{if } m = 2. \end{cases} \quad (6)$$

As opposed to the component share,  $IS_{\delta,m}$  is not invariant to the sampling frequency because the market-specific variances and correlation across markets in (6) depend on  $\delta$ . In particular, the contemporaneous correlation absorbs most of the lead-lag patterns as  $\delta$  increases because both markets have now sufficient time to impound the news. In fact, exact discretization yields  $|\rho_{\delta}| \rightarrow 1$  as  $\delta \rightarrow 1$ , and thus,  $\lim_{\delta \rightarrow 1} IS_{\delta,1} = \lim_{\delta \rightarrow 1} IS_{\delta,2} = 1/2$ .

Besides, there is an important and unique issue of the bitcoin market. There are endless possibilities for new exchanges to enter. The formation process of each one, as we will see in session 3, takes place in a decentralized way. Accordingly, instead of having contracts and mini contracts traded on a single exchange, we will have the same asset - identical - but traded on multiple platforms, where price detachment informs exactly issues of cost, liquidity, and security of the exchange system. Such characteristics also differ between exchanges, but they could never be compared since, in any exchange, there is precisely the same asset.

Therefore, the methodology had to be able to decompose the  $\alpha_{\delta,\perp}$  in multiples markets and finite samples. This means that a fair comparison of IS measures must take into consideration not only the sampling frequency but also the contemporaneous correlation across markets in continuous time. However, this is not straightforward. Teasing out the continuous-time covariance matrix from estimates of  $\Sigma_{\delta} = \int_0^{\delta} \exp(u\Pi) \Sigma \exp(u\Pi') du$  tends to produce poor results in finite samples, typically resulting in negative semi-definite estimates of  $\Sigma$  for prices sampled at frequencies lower than 10 seconds. Moreover, markets are currently very fast and interconnected given the rise of high-frequency trading and statistical arbitrage across and within markets (see, among others, Menkveld, 2014, 2016; O'Hara, 2015), implying higher contemporaneous correlation across markets even at the very high frequency and, in turn, IS measures that converge to 1/2.

### 3 Price Informativeness of Crypto Market

In this section, we describe the cryptocurrency Bitcoin, formation, intermediaries and participants.

#### 3.1 Cryptocurrencies

Since 2008, after the white paper from "Satoshi Nakamoto", which describes a form of stack digital time-stamp information, and using a cryptography protection in order to maintain the information public but non re-writable. Such mechanism allows trustfully and reliable transfer of money between peers without a third party, such as a Bank, in order to maintain the register of such transaction.

The name cryptocurrency comes from the cryptography security technology that protects the old information, and does not allow anyone, instead of the key-holder, to write a transfer of that key to another. None of those technologies are new, but they are used to maintain a finance book system, in order to provide security and fastness is quite revolutionary. Adding, also, an algorithm that allows the users of the system discovery new keys, these actions are named as: "mine a block"<sup>2</sup>. The mechanism behind such cryptography was described by Penard and van Werkhoven (2008); and Lamberger and Mendel (2011).

The literature appoint to three major aspects of cryptomarket that drives the demand: fast, safety and anonymity. The anonymity is positive correlated with tax avoidance and criminal activity (Luther and White, 2014). The supply side of this market is based on the above already described algorithm, and there is 21 millions upper-bound limit for units of Bitcoin. Until June 2020 18,413,350 (87.6% of total) is already been mined. There also some competitive cryptocurrencies that enlarge the market, and even Bitcoin being the biggest and most know, other cryptos and ICO (initial coin offering) process can provide more and more agents and traders to this market (Catalini and Gans, 2018).

#### 3.2 Exchanges and Historical Information

Acting as the most crucial third-party intermediary at Bitcoin market, exchanges prevail as a gateway for Bitcoin traders. The users had a preference for size or volume, fewer fees and better services Bhaskar and Chuen (2015). At the same time, when Bitcoin acceptance rises, the demand for exchange might decrease, and the numbers of those deal only Bitcoin also.

The differences among exchanges are based on three aspects: i. origin country, ii. currencies that are accepted, iii. fees (transactional, deposit, and withdrawal). We choose Binance, Bitfinex, Bitflyer, and Bitstamp.

The first one (Binance) had originated in China in July 2017. Only one year of working puts the Binance as leader crypto exchange worldwide. Binance trading more than 100 cryptocurrencies, and accepted fiat currencies of most countries in the world. Before China's govern prohibition of crypto trading, the company moved its HQ to Japan in September in the same year. Nowadays (2020) Binance had more than five times the trade dominance of the second one, bitFlyer.

Bitflyer history begins in 2014, with Yuzo Kano, a former Goldman Sachs trader. Hosted in Tokyo, and further spread at São Francisco and Luxembourg. Bitflyer is the number one exchange in Japan, and Bitcoin represents 94% of all volume of crypto trading.

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<sup>2</sup>The algorithm name is SHA-256

Bitifnex is the more old exchange present in our list. Founded in December 2012, start only trading Bitcoin and further added more cryptocurrencies to its portfolio. Its HQ is in Hong Kong, and now had representation in more than 50 countries. Its portfolio of cryptocurrencies had Ethereum, Litecoin, Ripple, and more.

The last one is the Bitstamp, which is the only based outside Asia. Its HQ is in London. The exchange also uses the European Union's Single Euro Payments Area, a mechanism for transferring money between European bank accounts. This could be a facilitator in order to enlarge its operational and volume in Europe. However, the cryptocurrency market has a large demand for operations under the radar of fiscal authorities. So, this advantage apparently does not put the exchange as a leader even in European soil.

### **3.3 Trading Bitcoin with Exchanges**

As a third party of bitcoin trading, the exchanges working as a regular stock exchange operate. Using the same system, but with much less regulation. For a foreign investor citizen trades in NY exchange, it is necessary to open an account at a local brokerage, and an investment account at a local or national bank, to move their investment.

Accordingly, the local and the regulatory framework of each country is not a concern in Bitcoin investors. They can operate using Paypal, credit card, electronic transfer, and every exchange had several lists of fiat currency that they accepted. So we expected that the Timezone and location of the traders do not interfere at the time-varying coefficient of the price-discovery process. The process of choose and specific exchange involves fees, size, and liquidity. We provide a description of our dataset.

## **4 Data**

Our data set consists of 4 exchanges, gathering by API on a minute level and calculate the midquote as the average of the best bid and best ask. The data was collected from April until October. The price difference between highest and lowest was higher than 45%. All four exchanges showed the same pattern, but we pursued a way to describe what is most relevant to the price-discovery process. In the ?? we describe the dataset and its descriptive statistics.



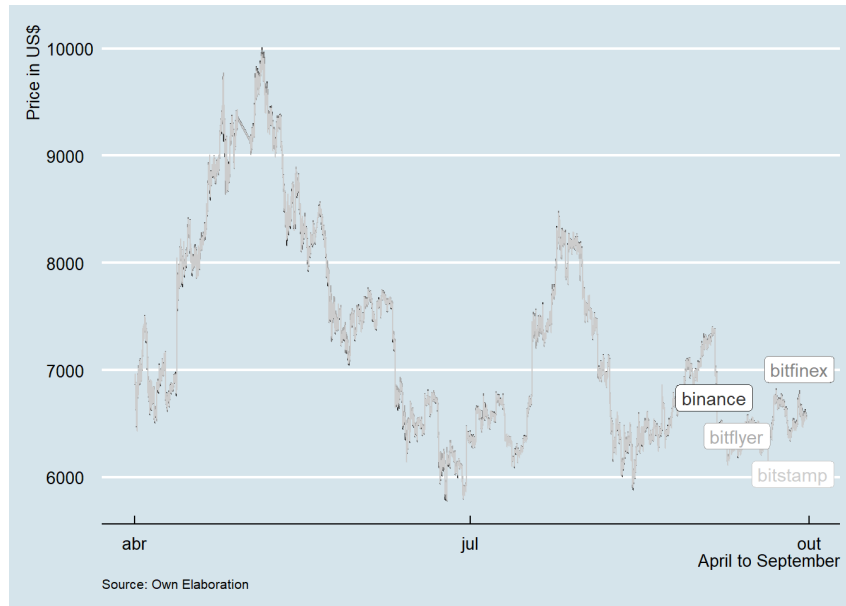


Figure 1: Price of Bitcoin in Selected Exchanges(US\$)

1 Represents our disposable data-set. With quasi-linearity, we choose these 4 exchanges in order to perform our KLS based estimator. With 95% of all trading in China, it is not a surprise that on the big 4 exchange, three of them are located in Asia and only the fourth place is located outside. In the same hand, the volume and the fee cost of trade are negative correlated; and, for this, the average price in the less costly exchange is slightly higher.

Table 1: Descriptive Statistics

	Exchange	Count	Bid	Ask	Price	sd	fee	location
1	binance	63423	7226.84	7229.27	7228.05	943.86	0.1	Asia
2	bitfinex	63281	7229.16	7229.39	7229.27	944.34	0.2	Asia
3	bitflyer	63245	7220.31	7235.22	7227.77	935.44	0.15	Asia
4	bitstamp	63037	7222.32	7226.69	7224.50	941.81	0.25	Europa

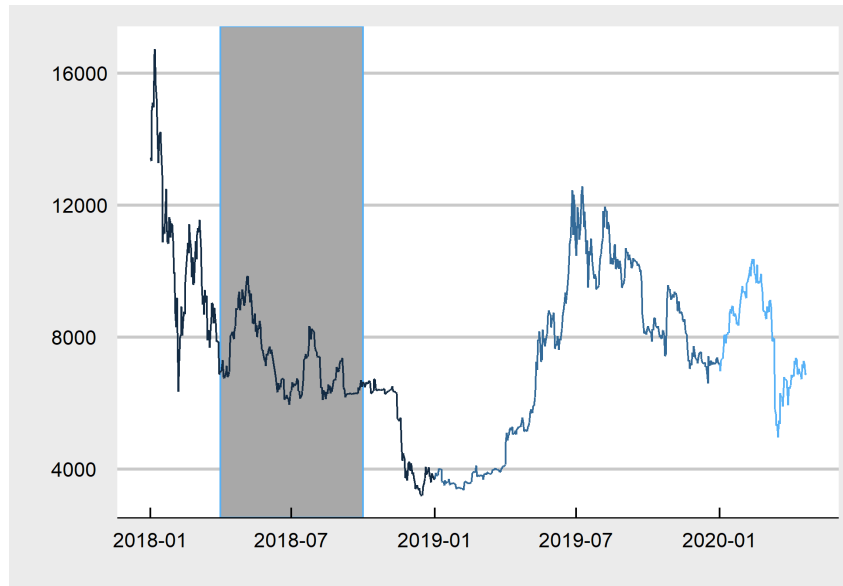


Figure 2: Price of Bitcoin 2018-2020(US\$)

The graph 2 allow us to understand the size of disposable when compares only 2 years window. Of course, we can promote a continuous methodology for implement the results of time-varying estimator. However, we show that price formation process, specifically the CS can be a good indicator for the price formation leader exchange. Moreover, even with great volatility, and  $I(1)$  series, we can provide some valid and updated information.

## 5 Time variation in the continuous-time component shares

There is seemingly a consensus in the literature that the price discovery processes change over time, with many studies running daily VECM specifications to address this issue (see, among others, Hasbrouck, 2003; Chakravarty, Gulen and Mayhew, 2004; Hansen and Lunde, 2006; Mizraeh and Neely, 2008). Additionally, empirical evidence suggests that the price discovery changes with some highly persistent market indicators such as trading volume and volatility. For instance, Figuerola-Ferretti and Gonzalo (2010) posit an equilibrium model of commodity spot and future prices in which the speed-of-adjustment parameters of a discrete-time VECM depend on the relative number of market participants. As a result, they establish a direct link between component shares and market activity indicators, such as relative volume or trade intensity.

We start with a formal test of whether component shares change over time. In particular, we employ Elliott and Müller's (2006) test for the null hypothesis of constant speed-of-adjustment parameters against the alternative hypothesis that they display persistent variation in time. In the context of one asset trading at two markets, time-varying speed-of-adjustment parameters automatically imply that the CS measures also change over time. The Elliott-Müller test is convenient because it accommodates well enough the sort of variation we describe in Section 2.

The price discovery process change over time, and factors such as trading volume, volatility, market participants, are responsible, according to the literature finance, to promote this possible change. Furthermore, these factors varying with time. So, the won price discovery process can be time-varying. Accordingly, in order to explore such a possibility, we implement the Elliott-Müller test.

Table 2: Elliot-Müller test

Exchange	Statistic	Critic Value	Decision
Binance	-54.0122	-19.8400	Rej. H0
Bitfinex	-116.9970	-19.8400	Rej. H0
Bitflyer	-141.4942	-19.8400	Rej. H0
Bitstamp	-513.1238	-19.8400	Rej. H0

Table 2 provides that for all exchanges we had the same decision. Reject the null hypothesis of absence of time-varying. With those results, it was implemented the continuous-time CS estimator.

## 5.1 Daily evolution of the continuous-time CS measures

Table 3: Today's Data

Exchange	Vol. (US\$ B.)	Perc
Coinbase	428	52,4%
Kraken	76	9,3%
Bitstamp	55	6,7%
Bitfinex	51	6,2%
Bitflyer	40	4,9%
Others	167	20,5%

Table 3 presents the descriptive statistics of today biggest exchanges. A fast looking is sufficient to answer what is the leader exchange in price-discovery process. On the other hand, in a volatile market like Bitcoin, all this information can change a lot in just a short period of time. Thus, in this table, we look for 2020-04-21 data. And we can compare this information with our results, that use data from April to September 2018.

Figure 3 exhibits the daily component share estimates for the four exchanges selected, plotting the estimates of the daily component shares in continuous time and their respective 95% confidence intervals for each exchange. Figure 3 shows us the daily time-dependency component share, which represents each exchange has more or less relevance. The European one, had its importance stable around 20%. On the other hand, the Binance improves its importance. If we take time, this exchange in September 2017, launch the Binance Coin, that represents 1.19% of all cryptocurrency markets. This one has more than four times the value of the second exchange, Coinbase, over the bitcoin market<sup>3</sup>. The Bitstamp come in fourth place, and Bitfinex in fifth, followed by Bitfinex. If we sum all fives ones, the volume is less than the first one. Our estimates capture this data, as show below:

<sup>3</sup>Those data was collected a <https://www.bitcointradevolume.com/> access at 2020-04-21

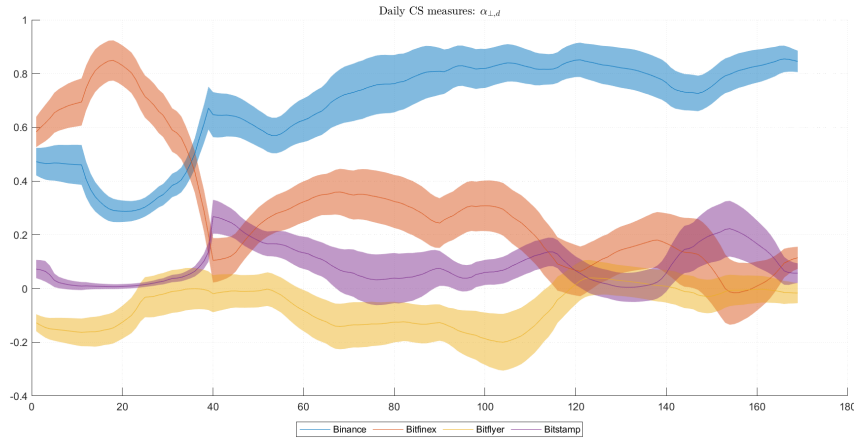


Figure 3: CS results in Time-Varying estimator

Our estimate captures the time dependency of the leading exchange on all possible variant conditions. Thus, issues such as volume, momentum, volatility, and global and regional economic growth affect the CS trend and the orthogonal decomposition for the different exchanges. Also, there may be time variations among exchanges in terms of reliability, costs and fees, risks, and issues of confidentiality and data protection that affect the trend of the PD process.

## 5.2 Time-Varying under Different Time Zones

In this section, we estimate the continuous-time CS measures a set of bitcoin exchanges, adopting time zone breaks, using high-frequency data from April to October 2018. This breaks for time zone had the objective to test the hypothesis of there is some liquidity or geographic characteristic correlated with the continuous-time CS measures. We estimate the continuous-time CS measures by KLS.

We promoted three breaks into our data set. The timestamps had to lie in one of three sets, Asia time zone: 20:00:00 to 03:00:00, EURO time zone: 04:00 to 11:30, and EUA time zone: 09:00:00 to 16:00:00. The intent is to test the geography and time dependency for our estimates. Bitcoin markets tend to have less geography (and regulation) dependency than usual stock-markets, for factors that we discuss at section 3.3.

Therefore, we open the possibility of three different patterns for the component-share time-varying. We expected previously that this pattern did not diverge from the previous, because the time zone is an intrinsically regional variable. Moreover, exchanges and trading bitcoins are not subject to local regulation or locals aspects. Accordingly, there is no theoretical evidence that allows us to conclude that some exchange because of HQ location had to be more important than another.

## 5.3 Results of TV under different Time Zones

As we can see, there is a great methodology question about the ranges of CS estimated by KLS. As long as, theoretically, there is possible to decompose the CS into  $n-1$  vectors orthonormal. However, the computational cost of such a task will be huge. Although our estimates for I.C's day by day respect the boundary of  $[0,1]$  for each orthogonal vector.

As we describe in 3.3, trading with Bitcoin exchange is not correlated of local regulation, size of markets, and local economic activity. Although, an exchange could choose to

establish itself at a remote place in order to promote some economy in terms of operational costs, for example. On the other hand, every trader can use those four exchanges as a tool to intermediate their negotiation, even he or she is living in the opposite global position.

Our estimates indicate what is the leading exchange in the price-formation and innovative process. We find that there is a non-negligible change between CS by exchanges. The pattern of our estimates indicate that Binance emerges as a leader of CS estimated by time-varying after 60 days of the beginning of data series. After the 60th day, Binance remains as the leader exchange in terms of CS.

Suppose we adopt the conventional approach for CS estimate, all these movements and changes among exchanges will be lost or negligible. The same empirical results were found by (Fruet Dias, Fernandes and Scherrer, 2020) for stock markets. Our findings are entirely new, especially for the Bitcoin market.

The results indicate a very similar decomposition between each cutting; however, with variation in the sample period, thus peremptorily indicating the need to adopt the methodology of estimating the CS is time-varying. Also, part of the movement reflects the increased importance of a specific Exchange over the others. It is indicating that there are characteristics linked to the markets that also reflect the CS issue and that in traditional asset markets, it would not be possible to be decomposed. Bearing in mind that in all traditional exchanges, there is no precisely the same asset, with precisely the same characteristics, negotiated in parallel, in a decentralized manner, and potentially by different selling and buying agents as we had in crypto exchanges.

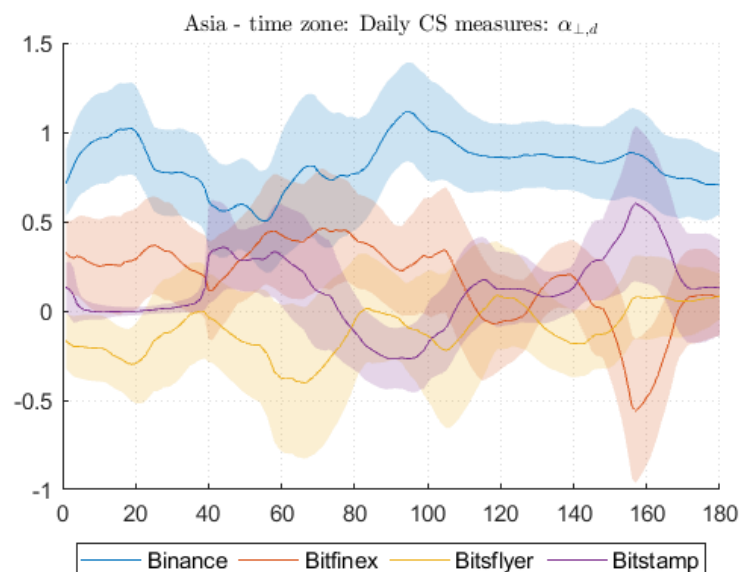


Figure 4: CS results in Time-Varying estimator: Asia Time Zone

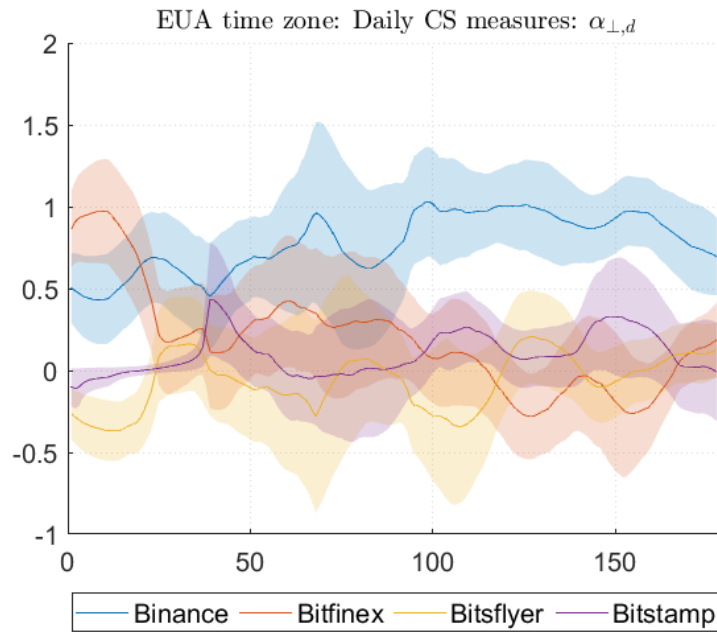


Figure 5: CS results in Time-Varying estimator: EUA Time Zone

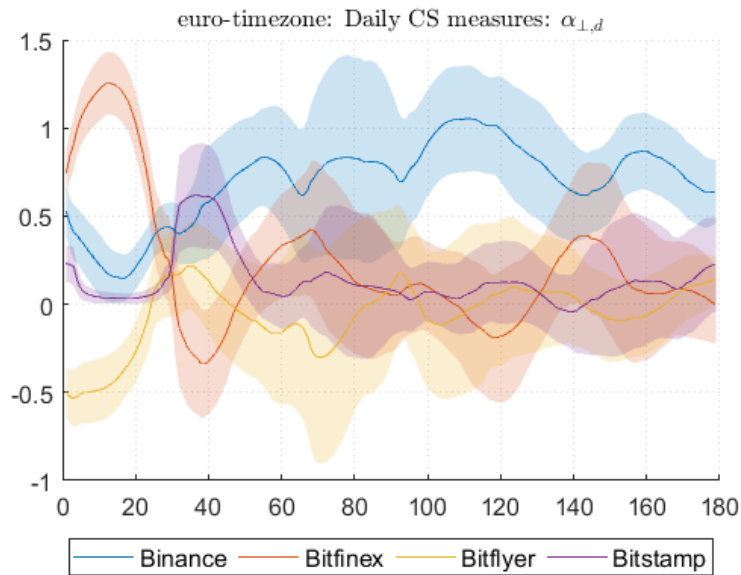


Figure 6: CS results in Time-Varying estimator: EUA Time Zone

## 6 Conclusion

This paper using a time-varying approach for price-discovery in continuous time for estimating the component-share. Using the KLS estimator applied at bitcoin market. We first show that the component share measure of price discovery is invariant to the discretization frequency, allowing us to make inference on the continuous-time price discovery mechanism from discrete sampled prices. This is in contrast with Hasbrouck's (1995) information

share, which depends on the contemporaneous correlation across markets, which naturally increases in magnitude as the sampling frequency decreases.

We then make use of Giraitis, Kapetanios and Yates's (2013) KLS method to estimate daily component shares. By exploiting the inter-dependency across days, the KLS approach yields more efficient estimates than we would otherwise obtain by treating the daily variation in the VECM parameters as independent over time.

Empirically, we take 17 bitcoins exchanges, using for exchanges to apply your method. Only four of them provided good informational to allow us to estimate the VECM vector. We find statistical evidence that the component shares indeed change over time for virtually every exchange in our sample. Our estimates indicate that market leadership alternates over time, and we have captured these phenomena that are possibly linked to the trading fee and volume.

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