Forecasting the exchange rate in Brazil: Fundamentals models versus random walk

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Resumo

Analisamos o poder preditivo dos modelos baseados em fundamentos em comparação com os modelos de passeio aleatório para horizontes de 1 a 24 meses no Brasil. Especificamente, investigamos quais modelos baseados em fundamentos superam o modelo de passeio aleatório durante períodos de valorização e desvalorização da taxa de câmbio. Além disso, analisamos se os modelos baseados em fundamentos que superam o passeio aleatório contêm informações não consideradas pelas expectativas de mercado. Os resultados indicam que alguns modelos baseados em fundamentos são úteis para prever a taxa de câmbio. O poder preditivo dos modelos baseados em fundamentos aumenta em períodos marcados por uma tendência de valorização ou desvalorização da moeda. Em particular, os modelos baseados em fundamentos do tipo Paridade do Poder de Compra têm um poder preditivo maior do que o modelo de passeio aleatório e adicionam informações às expectativas de mercado para diferentes horizontes de tempo e períodos de valorização e desvalorização da taxa de câmbio.

Palavras-chave: Previsão da taxa de câmbio, modelos baseados em fundamentos, poder preditivo, expectativas de mercado, conteúdo informativo.

Abstract

We analyze the predictive power of fundamentals versus random walk models for horizons from 1 to 24 months in Brazil. Specifically, we investigate what fundamentals models outperform random walk during periods of appreciation and depreciation of the exchange rate. Furthermore, we analyze whether the fundamentals models that beat random walk contain information not considered by market expectations. The findings point out that some fundamentals models are useful for forecasting the exchange rate. The predictive power of fundamentals models increases in periods marked by a trend of currency appreciation or depreciation. In particular, the PPP-type fundamentals models have greater predictive power than the random walk and add information to market expectations for different time horizons and periods of exchange rate appreciation and depreciation.

Key words: Exchange rate forecast, fundamentals models, predictive power, market expectations, informational content.

Área SBE: Macroeconomia Aplicada
JEL code: F47, F31, C53.

1. Introduction

Forecasting the exchange rate is one of the main challenges for decision-making by economic agents. The well-known "Meese-Rogoff puzzle" indicates that "atheoretical" models, especially the random walk ("naive no change model"), perform better than those that consider economic fundamentals.[1] Despite this, the use of economic models for forecasting the exchange rate represents a useful tool (Cheung et al., 2019). In particular, using models that identify the main determinants of the exchange rate allows the construction of more reliable economic scenarios. It should be noted that the analysis of the exchange rate is crucial, for example, to understand the dynamics of inflation through the pass-through effect and restrictions on economic growth via the performance of the trade balance. Hence, identifying the main fundamentals models that explain the exchange rate permits better planning both for companies and economic policy decisions by the government.

Although exchange rate forecasting models have received great momentum with advances in "machine learning" techniques, the literature has been concerned with developing several models based on economic fundamentals. In general, the leading forecasters in the models consider the difference between domestic and international variables referring to: interest rate, output gap, monetary aggregates, inflation, and prices. Furthermore, the most relevant exchange rate forecasting models can be classified into five types: "sticky price monetary models" (SPMM), "behavioral equilibrium exchange rate" (BEER) model, purchasing power parity" (PPP), "uncovered interest rate parity" (UIRP) model, and "Taylor rule" (TR) models. One consequence of this diversity of models is that the results found in the literature are not unanimous regarding the best model to predict the exchange rate (Ren, Liang, and Wang, 2021; Colombo, Pelagatti, 2020; Ca'Zorzi and Rubaszek, 2020; Engel et al., 2019; Ince, 2014; Wu and Wang, 2013).

This study uses monthly data from an emerging market with more than two decades of history under a floating exchange rate regime. Specifically, we consider information from the Brazilian economy from 1999 to 2021. The twenty-two years of the sample allow us to investigate which model with fundamentals performs best against the random walk for forecasting the exchange rate from one to twenty-four months ahead. Furthermore, we investigate whether the fundamentals models that outperform the

[1] For an analysis of the literature regarding the use of theoretical and "atheoretical" models for forecasting the exchange rate, see Rossi (2013).

random walk for each horizon under consideration represent increased information for market agents. Notably, the period under analysis is marked by shocks that led to a change in the trend from appreciation to devaluation of the exchange rate in Brazil. Therefore, analyzing which models are most suitable for forecasting the exchange rate in a specific context is possible.

In order to assess which fundamentals models have better predictive accuracy than the random walk for horizons from 1 to 24 months, we used statistics traditionally applied in exchange rate forecasting studies. Precisely, we used the ratio between the mean squared error of the predictions of the fundamentals models and the mean squared error of the random walk; and the "out-of-sample" R^2 statistic.[2] In addition, we applied the Clark and West (2007) test to assert the statistical significance of the superiority of the fundamentals models compared to the random walk ones. After identifying the models with greater predictive power, we verified, through the application of the Fair and Shiller (1989) test, whether the forecasts made by such models add informational content to the market expectations. Overall, the results show that fundamentals models are superior to the random walk for all horizons under consideration. Furthermore, there is evidence that the forecasts generated by fundamentals models have information that is neglected by market expectations.

Our analysis belongs to the literature branch that assesses the fundamentals models' ability to outperform random walk models for exchange rate forecasting. Meese and Rogoff (1983) identify the existence of a puzzle for forecasting the exchange rate, which refers to the fact that fundamentals models are not superior to random walk models ("atheoretical" models).[3] However, the literature that relies on economic theory to forecast the exchange rate has evolved and offers a wide range of models. Nevertheless, there is no consensus in the literature regarding which models would be the best for making exchange rate forecasts when considering different time horizons.[4] This study helps to identify which fundamentals models have greater predictive power in the context of an emerging economy for periods marked by a trend of currency appreciation and

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[2] See, e.g., for the model's mean squared error ratio - Cheung et al. (2019), and for the "out-of-sample" R^2 - Jamali and Yamani (2019).

[3] The so-called "Meese-Rogoff puzzle" became known as one of the six biggest puzzles of international macroeconomics (Obstfeld and Rogoff, 2001). Another name for "Meese-Rogoff puzzle" is the "exchange rate disconnect puzzle".

[4] Regarding the diversity of fundamentals models with the greatest predictive power of the exchange rate, see Ren, Liang, and Wang, 2021; Ribeiro, 2017; Li, Tsiakas, and Wang, 2015; Berge, 2014; Cheung, Chinn, and Pascual, 2005.

depreciation. Furthermore, our analysis shows whether expectations arising from fundamentals models have informational content different from that contained in market expectations.

Concerning fundamentals models, “monetary models” are the forerunners in exchange rate forecast analysis (Meese and Rogoff, 1983). In a simplified way, we can say that the “monetary models” use a structure that takes into account the demand and supply of relative money between two countries to explain fluctuations in the nominal exchange rate (Molodtsova and Papell, 2009). There are two types of “monetary models”: (i) the “flexible price monetary model”, in which exchange rate fluctuations should be proportional to price fluctuations over time (see Frenkel, 1976; Mussa, 1976); and (ii) the “sticky price monetary model”, in which exchange rates and interest rates are subject to strong fluctuations to compensate for rigidity in the prices of goods and services (see Dornbusch, 1976).

In recent times, exchange rate forecasting models known as the “Behavioral Equilibrium Exchange Rate” (BEER) have come to be used more frequently (see, for example, Clark and Macdonald, 1998). A common feature in this modeling strategy is using a set of variables that represent the fundamentals and are useful to explain the current behavior of the exchange rate. Unlike the “fundamental equilibrium exchange rate” models (FEER, see Williamson, 1994), the relevant notion of “equilibrium” to explain the behavior of the exchange rate is associated with an appropriate set of explanatory variables rather than the idea of macroeconomic equilibrium.

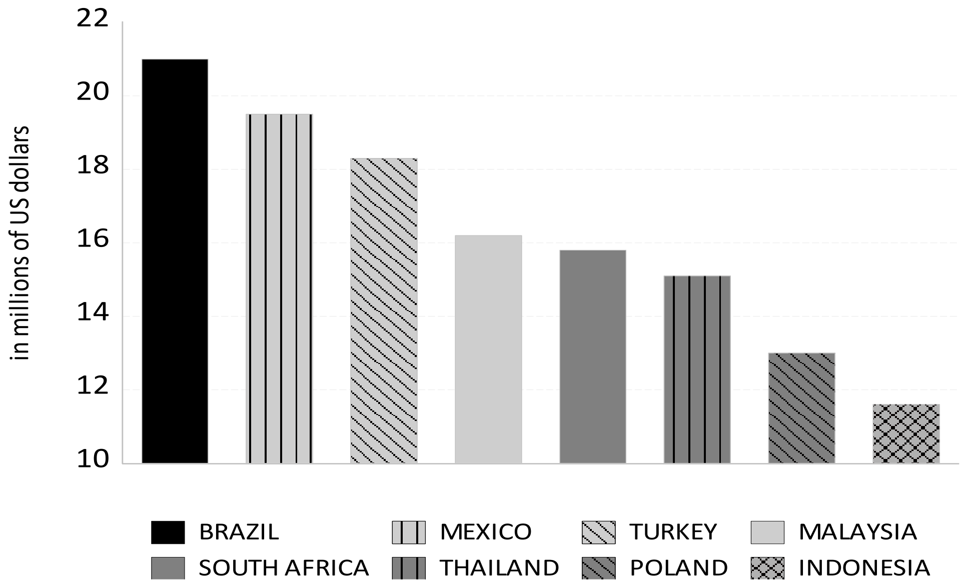
Another type of fundamentals model refers to “purchasing power parity” (PPP). According to the “purchasing power parity” hypothesis, the price of a basket of goods and services in a given economy should be equal to that of another when converted to the same monetary unit (see, for example, Rossi, 2013). In short, the PPP-type models have, in essence, the differential between the domestic and the international price to explain the exchange rate. In addition to the price parity relationship in the aforementioned model, the “uncovered interest rate parity” (UIRP) is another type of fundamentals model. In general, “uncovered interest rate parity” represents a situation in which there is a short-term equilibrium between the expected exchange rate depreciation and the equality of the differential between the domestic and international interest rates (see, for example, Sarno and Taylor, 2003).

With the increase in the use of central bank reaction rules for determining the interest rate based on the deviation of inflation from the target and the output gap, a model that has come to be considered for the exchange rate forecast is the one that considers the "Taylor rule" fundamentals. According to Molodtsova and Papell (2009), there are two possible specifications: the "symmetric model" - assumes that the foreign central bank follows a rule similar to that used by the country under consideration, and the "asymmetric model" – in this case, the foreign central bank adds to the Taylor rule the difference between the exchange rate and the target defined by the PPP.

Specifically for the Brazilian case, few studies analyze fundamentals models for forecasting the exchange rate. It is important to highlight that Brazil has a total volume of foreign exchange traded representative to other emerging markets (see figure 1). Therefore, identifying which fundamentals models are more accurate and can add information to market expectations can contribute to studies of other emerging markets. Gaglianone and Martins (2017) compared fundamentals models ("monetary" and the "Taylor rule") and random walk for the Brazilian case, based on monthly data, referring to the period from January 2000 to March 2015. The findings show that the fundamentals models have good performance for forecasting the exchange rate. Felício and Rossi Júnior (2014), through the use of common factors extracted from a set of eighteen countries, analyzed the dynamics of the Brazilian exchange rate (BRL/USD) after adopting the floating exchange rate regime from 1999 to 2011. The authors' main conclusion is that the common factors are useful in improving the predictive power of the fundamentals models.

Figure 1

Foreign exchange market turnover



Notes: Foreign Exchange market turnover, the source is the triennial survey of the Bank for International Settlements. (BIS, 2022). Daily averages, in millions of US dollars.

Besides this introduction, this article is structured as follows. Section 2 introduces each fundamentals model and its respective specifications. In addition, we present the tests for predictive power and informational content that we use to examine the exchange rate forecast models. Section 3, considering horizons from 1 to 24 months, assesses the accuracy and informational content of fundamentals and random walk models. Moreover, we extended the investigation by analyzing periods with a trend of appreciation and depreciation of the exchange rate. Lastly, section 4 concludes.

2. Data and methodology

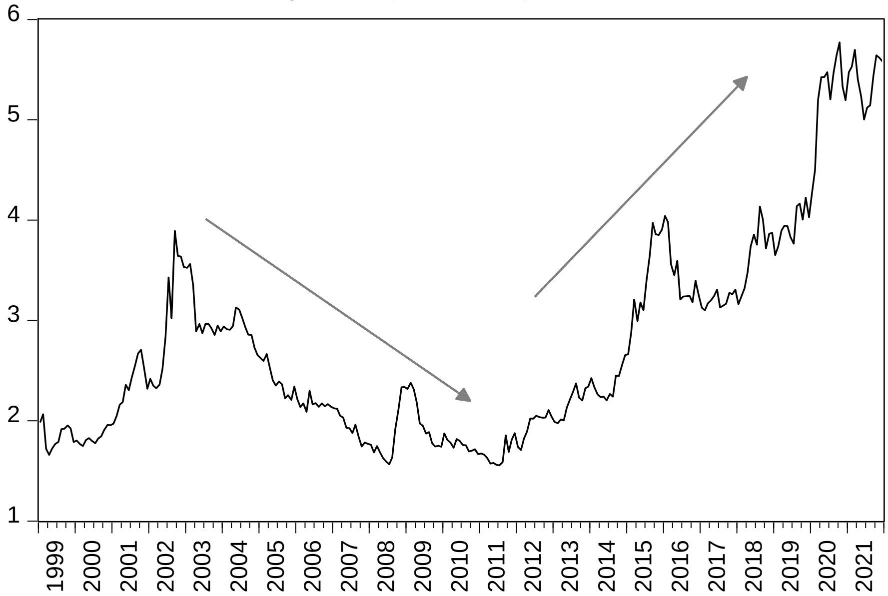
The main objective of this study is to identify which models based on economic fundamentals have the best predictive power for the exchange rate compared to the random walk. In short, we first perform several regressions considering economic fundamentals as explanatory variables for forecasting the exchange rate considering different time horizons. From the results found, we performed several analyses to identify the models with the best predictive power. In addition, we evaluated the "informational content" of the fundamentals models with the best accuracy performance compared to the professional forecasters' expectations.

In the same way as Rossi (2013), for example, we evaluated out-of-sample forecasts considering rolling windows regressions. Therefore, the sample is divided into two parts. The first corresponds to the "in-sample" portion, consisting of the observations from 1 to N. The second part is the "out-of-sample" portion, comprising the observations from $N+\tau$ to $T+\tau$, of size $Z \equiv T-N+1$. Specifically, the estimation window size is given by N, and the exchange rate forecast is made for τ periods (months) ahead.

We considered different short and medium-term time horizons (τ = 1, 3, 6, 9, 12, and 24 months) to capture the evolution of the predictive power of the fundamentals models. In addition, we use a window corresponding to 60 months of the sample. The window size represents the minimum time to examine the relationships between the variables and, therefore, to evaluate the predictive power of the fundamentals models. Hence, it is possible to identify which models are suitable for each time horizon.

Based on a monthly database, the analysis period extends from January 1999 to December 2021. The sample's beginning coincides with the adoption of the flexible exchange rate regime in Brazil. The exchange rate generally showed an appreciation trend between 2003 and 2011 (see figure 2). On the other hand, there is a reversal in this trend from 2011 onwards. A possible explanation for this phenomenon can be justified by the occurrence of a "perfect storm". In other words, a period in which there was a combination of economic policies that neglected fiscal balance and low and stable inflation with stimuli to economic growth without taking into account the side effects (see de Mendonça and Valpassos, 2022). In short, in addition to the analysis for the total period, we considered two subsamples to verify whether the performance of the models changes: an appreciation period (2002M09-2011M06) – marked by the elimination of the exchange rate "overshooting" caused by the election of the first mandate of president Luiz Inácio Lula da Silva and the abundance of international liquidity; and a depreciation period (2011M07-2021M12) – resulting from the "perfect storm" effect, reduction in international liquidity due to the 2008-2009 global financial crisis, and the increased risk arising from the impeachment of President Dilma Rousseff and the economic crisis generated by Covid-19.

Figure 2
Exchange rate (*BRL/USD*) *– 1999-2021*



Note: End of the month exchange rate, the source is the Central Bank of Brazil.

We used the Fully Modified Ordinary Least Squares (FMOLS) method. One of the advantages of using this method is that it considers an estimator that employs a semi-parametric correction to eliminate the problems caused by the long-term correlation between the cointegrating equation and the stochastic regressors innovations (Phillips and Hansen, 1990). It is worth mentioning that the literature on exchange rate forecasting points out that it is a common feature in the models that the exchange rate and the economic fundamentals are I(1) and cointegrated (Cheung, Chinn, and Pascual, 2005; Engel and West, 2005). Moreover, we checked the integration order of the series, and the results indicate that they are I(1) – see table A.3 (appendix). In addition, we verified that a cointegrating relationship exists among the variables in all models (see table A.4 - appendix).

Therefore, using the error correction mechanism becomes appropriate since it assumes the existence of a long-term relationship that captures the imbalances between the level of the exchange rate and the level of the fundamentals. In particular, the application of the error correction mechanism has two steps: the first establishes the long-term cointegration relationship, in level, between the exchange rate and the fundamentals; the second incorporates the estimated cointegrated vector ($\widehat{\beta_t}$) into the difference equation derived from the relationship, in level, between fundamentals and the exchange rate.[5]

The relationship, in level, between the current exchange rate (s_t) and the fundamentals (f_t) can be represented as follows:

(1) $$s_t = \beta_0 + \beta_1 f_t + u_t.$$

The exchange rate forecast model includes an error correction term at time *t*, that is:

(2) $$s_{t+\tau} - s_t = \gamma_0 + \gamma_1(s_t - \widehat{\beta_0} - \widehat{\beta_1} f_t) + u_t.$$

Equation 2 is estimated via FMOLS. The error correction term captures exchange rate deviations from its "fundamental value" (Rossi, 2013). Because $\gamma_1 < 1$, the exchange rate returns to its "fundamental value" over time. In other words, the predictive power of fundamentals models should be greater for longer horizons. Furthermore, as Cheung et al. (2019), we consider long-term time-varying relationships through rolling windows.

To analyze the importance of fundamentals in exchange rate forecasting models, we consider five types and fourteen specifications (see Cheung et al., 2019; Jamali and Yamani, 2019; Rossi, 2013; Molodtsova and Papell, 2009). Specifically, the types used are: "sticky price monetary models", "behavioral equilibrium exchange rate", "purchasing power parity", "uncovered interest rate parity", and "Taylor rule" models.[6] With the exception of the behavioral equilibrium and Taylor rule exchange rate models, our estimations move from a more parsimonious specification to a more general one.

[5] We consider estimates only with "ex-ante" information using lagged data to forecast the exchange rate ahead. Furthermore, the specifications of each category detailed in equations from 3 to 7 are "in level".

[6] We used "out of sample" forecasts for the exchange rate to analyze the model's predictive performance.

The first model considered in our analysis is the most traditional for forecasting based on the fundamentals models, that is, the monetary models. The premise for this rationale is that bilateral fluctuations in exchange rate movement should represent relative currency demand across the countries. We used sticky price monetary models (SPMM) in our analysis because they are superior to flexible price monetary models for forecasting the exchange rate.[7] The most parsimonious model is based on Mark (1995); and takes into account to forecast the exchange rate the differential between the domestic (Brazil) and international (United States) – represented by "~" - of the following variables: money supply ($\widetilde{m}_t$), product ($\tilde{y}_t$), monetary policy interest rate ($\tilde{\imath}_t$), and inflation rate ($\tilde{\pi}_t$). In addition to this specification, we considered the inclusion of other variables that the literature points out as relevant. For example, the use of proxies for: international uncertainty (Volatility of S&P 500 index options - VIX_t), risk aversion (ICE BofA US High-Yield Market Index Option-Adjusted Spread - HYS_t), international gold price (US$ - $GOLD_t$), commodity prices (All Commodity Price Index - $COMM_t$), and the strength of the US dollar (US$) against a basket of influential currencies (DXY_t).[8] Hence, a general specification for our models is a result of:

(3) $$s_t = \alpha_0 + \alpha_1 \widetilde{m}_t + \alpha_2 \tilde{y}_t + \alpha_3 \tilde{\imath}_t + \alpha_4 \tilde{\pi}_t + \alpha_5 X_t + u_t,$$

where X_t is a vector of control variables (VIX_t, HYS_t, $GOLD_t$, $COMM_t$, DXY_t) and u_t is the error term.[9]

The second model in our analysis refers to the behavioral equilibrium exchange rate theory (*BEER*). Except for the variables related to macroeconomic balance, models of this type consider a broad set of fundamentals for forecasting the exchange rate. Therefore, we use in our specifications the domestic and international differential of the following variables (see Cheung et al., 2019): money supply ($\widetilde{m}_t$), price level ($\tilde{p}_t$), Balassa-Samuelson effect of the relative price of non-tradable goods ($\tilde{z}_t$), and real interest rate ($\tilde{r}_t$). Furthermore, we include the following variables in the models (see Ren, Liang, and Wang, 2021; Itskhoki and Mukhin, 2021; Ferraro, Rogoff, and Rossi, 2015): terms of trade (TOT_t), commodity prices ($COMM_t$), and an "uncertainty index" (VIX_t).

[7] For an analysis of the poor performance of flexible pricing monetary models, see Jamali and Yamani (2019) and Molodtsova and Papell (2009).
[8] As an example of the application of the variables mentioned in the explanation of the exchange rate, see Cheung and Wang (2022); Ren, Liang, and Wang (2021); Zhang, Dufour, and Galbraith (2016); Felicio and Rossi Júnior (2014); Chen, Rogoff, and Rossi (2010); Chen and Rogoff (2003). For a definition of variables, sources, and descriptive statistics, see table A.1 (appendix).
[9] The inclusion of control variables in the specifications considers the prediction performance of the fundamentals model in relation to the random walk model (mean squared error ratio). All specifications are shown in table A.2 (appendix).

Therefore, the baseline model referring to the behavioral equilibrium exchange rate model corresponds to:

(4) $$s_t = \delta_0 + \delta_1\tilde{m}_t + \delta_2\tilde{p}_t + \delta_3\tilde{z}_t + \delta_4\tilde{r}_t + \delta_5 X_t + u_t,$$

Where X_t is a vector of control variables (TOT_t, $COMM_t$, VIX_t).

The third type of model considered in this study refers to purchasing power parity (PPP). The basic premise is the validity of the Law of One Price. In other words, even when considering different countries, a homogeneous product has the same price when converted to the same currency. Similar to Cheung et al. (2019), Jamali and Yamani (2019), Engel, Mark, and West (2007), the specification for forecasting the exchange rate related to the PPP fundamentals is given by:

(5) $$s_t = \varphi_0 + \varphi_1\tilde{p}_t + u_t,$$

where $\tilde{p}_t$ is the differential in prices between Brazil and the USA.

The "uncovered interest rate parity" (*UIRP*) is the fourth fundamentals models for forecasting the exchange rate. The essence of this fundamental is that changes in the exchange rate are equivalent between the differential of the domestic interest rate and the international interest rate.[10] The baseline specification of this type of fundamental takes into account only the interest rate differential of the domestic monetary policy in relation to the international one. We use a second specification because there are other relevant variables related to the UIRP for forecasting the exchange rate (mainly risk-related). In short, the general specification corresponds to:

(6) $$s_t = \omega_0 + \omega_1\tilde{\imath}_t + \omega_2 X_t + u_t,$$

where X_t is a vector of control variables ($EMBI_t$, VIX_t, HYS_t).

The introduction of the VIX, HYS, and EMBI in the model permits us to consider aspects related to market volatility, risk aversion, and investors' behavior. The VIX captures shocks to the U.S. stock market and liquidity conditions. Due to the relevance of the U.S. financial market in the global market, there is a natural spillover effect on exchange rates (Cheung and Wang, 2022). The HYS represents the yield difference between high-yield and government bonds. When the HYS increases, signaling a wider gap between these yields, it reflects a diminished investor appetite for risk (Felício and Rossi Júnior, 2014). The increased risk aversion often leads investors to favor safer assets, impacting capital flows and potentially strengthening more secure currencies,

[10] Some essential assumptions related to the UIRP are rational expectations and risk neutrality of economic agents (Sarno, Taylor, and Frankel, 2003). For the validity of the UIRP, see Bussière et al. (2022).

consequently influencing exchange rates. Finally, the EMBI shows the perceived credit risk of investing in emerging market sovereign debt compared to more secure U.S. Treasury bonds. According to Thomas (2012), the exchange rates have responded significantly to changes in the EMBI since mid-2008.

In order to consider interest rate maturity, we use two additional specifications for the UIRP. The basic specification, such as Chen and Tsang (2013), considers the informational content in the yield curve's slope. The full specification uses the same covariates as in equation 6 and adds the strength of the US dollar (US$) against a basket of influential currencies (DXY_t). Therefore:

(7) $s_t = \vartheta_0 + \vartheta_1 \tilde{\imath}_t + \vartheta_2 \tilde{\imath}_t^k + \vartheta_3 X_t + u_t,$

where i^k is the fixed interest rate term structure - LTN - 1 month - (% p.a.) and X_t is a vector of control variables ($EMBI_t$, VIX_t, DXY_t, HYS_t).

The last type of fundamentals in our analysis relates to using the Taylor rule as a model for forecasting the exchange rate. In general, the Taylor rule assumes that central banks adjust the short-term interest rate in response to changes in inflation ($\tilde{\pi}_t$) and the output gap ($\widetilde{gap}_t$). Since Molodtsova and Papell (2009), the use of the Taylor rule to model the determination of the exchange rate has become frequent. Because we are considering the exchange rate forecast in an emerging market, we include the commodity index ($COMM_t$) in the baseline specification to capture the effect of international shocks. Furthermore, due to the existence of asymmetric models in relation to the rule used by the domestic and foreign central banks, we introduced the interest rate differential of the domestic and international monetary policy and the real exchange rate into the model (see Rossi, 2013). As a result, we have the following general specification:

(8) $s_t = \psi_0 + \psi_1 \tilde{\pi}_t + \psi_2 \widetilde{gap}_t + \psi_3 X_t + u_t,$

where X_t is a vector of control variables ($\tilde{\imath}_t$, q_t, $COMM_t$).

2.1. Accuracy and informational content of exchange rate forecast models

In order to evaluate the predictive power of the exchange rate forecast models with the aforementioned fundamentals, we use the exchange rate forecast from two sources: forecasts generated by a random walk model (without drift) and market expectations provided by the Time Series Management System/ Central Bank of Brazil (TSMS/CBB). Specifically, to analyze the accuracy of forecasts, we considered the ratio of the mean squared error (MSE) between the fundamentals models and the reference model (random walk), R² out of sample (R^2_{oos} – see Anatolyev et al., 2017; Li, Tsiakas, and Wang, 2015) and the Clark and West test (2007). Regarding the analysis of the informational content of the fundamentals models and professional forecasters' expectations, we used the Fair and Shiller (1989) test - "FS test".

The first accuracy indicator used to verify the performance of the exchange rate forecasting models (MSE_{ratio}) is the ratio between the mean squared errors of the fundamentals models (MSE_f) with the mean squared error of the random walk model (MSE_{rw}):[11]

(8) $$MSE_{ratio} = \frac{MSE_f}{MSE_{rw}},$$

The MSE_{ratio} interpretation is straightforward. When the ratio is less (greater) than 1, the fundamentals model has a better (inferior) predictive performance than the random walk. Fundamentals models follow the structure presented in the previous section. In general, the literature considers the random walk without drift as the benchmark model and has the exchange rate at time *t* as the best predictor for the exchange rate at time *t+1* (Rossi, 2013).

The second indicator for evaluating the accuracy of exchange rate forecasts is the "out-of-sample" R² statistic - R^2_{oos}. As in Della Corte and Tsiakas (2012), we use two sources of exchange rate forecasting: the first corresponds to the expectations generated by the random walk model for τ periods ahead ($\hat{s}_{rw,t+\tau}$); and the second refers to the expectations obtained from the fundamentals model ($\hat{s}_{f,t+\tau}$) for τ periods ahead. Therefore:

(9) $$R^2_{oos} = 1 - \frac{\sum_{t=N+\tau}^{T+\tau}(s_{t+\tau} - \hat{s}_{f,t+\tau})^2}{\sum_{t=N+\tau}^{T+\tau}(s_{t+\tau} - \hat{s}_{rw,t+\tau})^2}.$$

As in the case of MSE_{ratio}, the indicator's interpretation is straightforward. A positive (negative) R^2_{oos} indicates that the forecast from the fundamentals models performs better (inferior) than the random walk model.

In addition to the MSE_{ratio} and R^2oos, we applied the Clark and West (2007) test ("CW test") to ascertain the statistical significance of the superiority of the forecasts generated by the fundamentals models in relation to the random walk. Although the Diebold and Mariano (1995) test is widely used for analyzing the accuracy of forecast models, the test may not be suitable for analyzing the exchange rate forecast for out-of-

[11] For examples of the application of the MSE_{ratio} as an indicator of forecast accuracy, see Cheung et al., (2019), Clark and McCracken, (2006), and Cheung, Chinn, and Pascual (2005).

sample models because it has a bias due to the comparison of the mean squared error of two nested models (Molodtsova and Papell, 2009). A feature of the CW test is that it takes into account the aforementioned bias, as it assumes that the mean squared error referring to the alternative model (fundamentals models) is greater than the reference model (random walk) due to the presence of noise in the alternative forecasting model (Della Corte and Tsiakas, 2012).

The CW test has equal forecast accuracy as the null hypothesis and considers a quadratic loss function defined as the square of the prediction errors. Specifically, we consider the forecast errors between the realized exchange rate and the forecasts coming from the fundamentals models for the period $t+\tau$. Suppose the fundamentals model has low predictive power. In that case, it means that the exchange rate follows a martingale sequence in differences.[12] In other words, the exchange rate follows a random walk (a martingale, in the broadest sense).

If the best predictor of the exchange rate at time $t+1$ is the observed value at time t (random walk model), we have a martingale process with zero mean. Hence, as recommended by Clark and West (2007), it is necessary to adjust the mean squared error of the predictions of the fundamentals models ($\hat{s}_{f,t+\tau}$) in relation to the reference models ($\hat{s}_{rw,t+\tau}$):

(10) $$MSE_f - adj. = \left(\sum (s_{t+\tau} - \hat{s}_{f,t,t+\tau})^2/P\right) - \sum \left(\hat{s}_{rw,t,t+\tau} - \hat{s}_{f,t,t+\tau}\right)^2/P,$$

where: $MSE_f = \sum (s_{t+\tau} - \hat{s}_{f,t,t+\tau})^2/P$; "adj." is a "fit" term as in Clark and West (2007), that is, adj.= $\left(\hat{s}_{rw,t,t+\tau} - \hat{s}_{f,t,t+\tau}\right)$; P is the number of forecasts used for calculating the averages.

Therefore, the difference in mean squared errors between the exchange rate forecasting models ($\widehat{Ck}_{t+\tau} = MSE_j - (MSE_f - adj.)$) is a result of:

(11) $$\widehat{Ck}_{t+\tau} = (s_{t+\tau} - \hat{s}_{rw,t,t+\tau})^2 - \left[(s_{t+\tau} - \hat{s}_{f,t,t+\tau})^2 - \left(\hat{s}_{rw,t,t+\tau} - \hat{s}_{f,t,t+\tau}\right)\right].$$

Therefore, in order to test for the equal forecast accuracy of the models under consideration (CW test, that is, H_0: $MSE_j - (MSE_f - adj.) = 0$), we regress the $\widehat{Ck}_{t+\tau}$ on a constant term and check if the t-statistic of the estimated coefficient in the regression (OLS) is equal to zero (one-tailed test).

Because we expect fundamentals models to have a good predictive power of the

[12] The "martingale" process refers to a sequence of error terms that generally are not independent and identically distributed. In other words, the conditional expectation of the next value is equal to the current value of the variable, regardless of previously assumed values (Clark and West, 2006).

exchange rate, we extend our analysis to assess whether expectations generated from these models have informational content beyond what is contained in professional forecasters' expectations. Regarding market expectations, the Central Bank of Brazil provides the median exchange rate expectations compiled from up to 140 institutions (banks, consultancies, asset managers, and academia) in the TSMS/CBB. It is important to note that participants in the Central Bank of Brazil's Market Expectations System have incentive mechanisms to keep their expectations updated. In particular, institutions need to ratify or update their forecasts within a period of up to thirty days to remain in the system (see de Mendonça, Vereda, and Araujo, 2022).

The fact that we are considering several fundamentals models means that we have a diversity of forecasts for the exchange rate. Consequently, we can identify which fundamentals are relevant even when considering market expectations to explain the exchange rate. Therefore, we performed an "encompassing" test proposed by Fair and Shiller (1989). In general, the test can be performed using an OLS regression based on the following equation:[13]

(12) $s_{t+\tau} = \theta_0 + \theta_1 \hat{s}_{f,t+\tau} + \theta_2 \hat{s}_{me,t+\tau} + e_{t+\tau}.$

If the fundamentals models forecasts ($\hat{s}_{f,t}$) and the professional forecasters' expectations ($\hat{s}_{me,t}$) have informational content, the coefficients associated with them (θ_1 e θ_2) must have statistical significance. In particular, if only θ_1 is significant, market expectations do not add relevant information to the exchange rate forecast that is not contained in fundamentals models. On the other hand, if there is statistical significance only for θ_2, we can say that fundamentals models do not contribute additional information to that contained in the market expectations. Finally, suppose the two coefficients are statistically significant. In that case, the informational content of both sources should not be neglected, and, therefore, the combination of both expectations improves the predictive power of the exchange rate.

3. Accuracy and informational content of fundamentals and random walk models

Based on the MSE_{ratio} and R^2_{oos} statistics results, we evaluated the accuracy of exchange rate forecasts realized from February 2004 to December 2021 for different

[13] The regressions have a robust standard error by applying the covariance matrix of Newey and West (1987).

horizons (τ=1,3,6,9,12, 24 months). The results presented in table 1 show that in only two cases (UIRP – models 2 and 4 for a 24-month horizon), the MSE_{ratio} indicates that the random walk has lower predictive power than the fundamentals models (MSE_{ratio}<1). The R^2_{oos} results confirm the evidence pointed out by the MSE_{ratio}. In other words, the random walk is superior to the vast majority of the fundamentals models for forecasting the exchange rate (R^2_{oos}<0).[14] In short, both statistics align with the literature on exchange rate forecasting that indicates that the random walk can be considered a dominant model (see Cheung et al., 2019; Alquist and Chinn, 2008; Engel and West, 2005).

In addition to the MSE_{ratio} and R^2_{oos} statistics, we performed the CW test to assess whether there is a statistically significant difference between the exchange rate forecasts from the fundamentals and random walk models. Regarding the CW test, the test statistic must be positive and significant for the fundamentals model to have an accuracy greater than that of the random walk. Except for the case of the BEER-type models referring to the 12- and 24-month horizons, the results in table 2 confirm the evidence pointed out by the MSE_{ratio} and R^2_{oos} statistics that the random walk is superior to the fundamentals models for the period under consideration.[15]

Although the results referring to the total sample point to the greater predictive power of the exchange rate through the random walk, it is important to assess whether this result is maintained when considering periods in which there is a tendency for currency appreciation and depreciation. As shown in the previous section, the Brazilian case presents two distinct periods regarding the evolution of the exchange rate: a currency appreciation period from September 2002 to June 2011 and a depreciation period from July 2011 to December 2021. In other words, there may be more adequate fundamentals to explain the exchange rate in specific environments (Bacchetta and van Wincoop, 2013).

[14] A possible explanation for the lack of predictive power, especially in the short term by the UIRP, is due to temporary and permanent monetary policy shocks producing opposite signal effects on the nominal exchange rate (see, for example, Grohé and Uribe, 2022).

[15] The good performance of the BEER-type models in comparison to random walk is in consonance with results found by, for example, Cheung et al. (2019).

Table 1

Exchange rate forecast accuracy statistics (MSE_{ratio} and R^2_{oos}): fundamentals models versus random walk

	SPMM							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.090	-0.090	1.133	-0.133	1.169	-0.169	1.156	-0.156
3 months	1.233	-0.233	1.261	-0.261	1.229	-0.229	1.232	-0.232
6 months	1.581	-0.581	1.561	-0.561	1.454	-0.454	1.305	-0.305
9 months	1.808	-0.808	1.754	-0.745	1.627	-0.627	1.430	-0.430
12 months	1.718	-0.718	1.583	-0.583	1.428	-0.428	1.331	-0.331
24 months	1.652	-0.652	1.422	-0.422	1.212	-0.212	1.240	-0.240

	BEER						**PPP**	
	Model 1		Model 2		Model 3		Model 1	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.142	-0.142	1.108	-0.108	1.095	-0.095	1.190	-0.190
3 months	1.305	-0.305	1.195	-0.195	1.173	-0.173	1.480	-0.480
6 months	1.251	-0.251	1.202	-0.202	1.188	-0.188	1.821	-0.821
9 months	1.247	-0.247	1.213	-0.213	1.169	-0.169	2.122	-1.122
12 months	1.182	-0.182	1.148	-0.148	1.106	-0.106	2.160	-1.160
24 months	1.237	-0.237	1.159	-0.159	1.141	-0.141	2.467	-1.467

	UIRP							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.077	-0.077	1.103	-0.103	1.040	-0.040	1.099	-0.099
3 months	1.323	-0.323	1.338	-0.338	1.182	-0.182	1.141	-0.141
6 months	1.777	-0.777	1.490	-0.490	1.524	-0.524	1.217	-0.217
9 months	2.076	-1.076	1.599	-0.599	1.721	-0.721	1.297	-0.297
12 months	2.114	-1.114	1.633	-0.633	1.786	-0.786	1.245	-0.245
24 months	1.837	-0.837	0.856	0.144	2.366	-1.366	0.985	0.015

	TR			
	Model 1		Model 2	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.137	-0.137	1.059	-0.059
3 months	1.548	-0.548	1.075	-0.075
6 months	1.767	-0.767	1.088	-0.088
9 months	1.755	-0.755	1.105	-0.105
12 months	1.637	-0.637	1.100	-0.100
24 months	1.377	-0.377	1.072	-0.072

Notes: Full sample period - 2004M01 to 2021M12. The random walk model has a higher predictive power than fundamentals models if MSE_{ratio}>1 and R^2oos<0. See equations from (3) to (8) for the baseline specification of each model. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

Table 2
Exchange rate forecast accuracy test (*Clark-West test*):
fundamentals models versus random walk

Fundamentals	Model	τ = 1	τ = 3	τ = 6	τ = 9	τ = 12	τ = 24
SPMM	1	0.546	-0.094	-1.816	-2.628	-2.218	0.920
		(0.293)	(0.462)	(0.035)	(0.005)	(0.014)	(0.179)
	2	0.569	0.043	-1.581	-2.011	-1.239	2.028
		(0.285)	(0.483)	(0.058)	(0.023)	(0.108)	(0.022)
	3	0.530	-0.676	-2.484	-2.977	-0.941	3.270
		(0.298)	(0.250)	(0.069)	(0.002)	(0.174)	(0.001)
	4	0.424	-0.037	-0.094	-0.202	-0.393	2.988
		(0.336)	(0.485)	(0.463)	(0.420)	(0.347)	(0.002)
BEER	1	-0.353	-0.650	0.681	1.034	1.715	3.223
		(0.362)	(0.258)	(0.248)	(0.151)	(0.044)	(0.001)
	2	0.462	-0.038	0.599	1.127	1.696	3.357
		(0.322)	(0.485)	(0.275)	(0.131)	(0.046)	(0.000)
	3	0.643	0.123	0.581	1.303	1.926	3.462
		(0.261)	(0.451)	(0.281)	(0.097)	(0.028)	(0.000)
PPP	1	-1.062	-0.942	-0.449	-0.145	0.574	1.584
		(0.145)	(0.174)	(0.327)	(0.442)	(0.283)	(0.057)
UIRP	1	-0.059	-0.972	-2.994	-5.089	-7.411	-3.974
		(0.476)	(0.166)	(0.002)	(0.000)	(0.000)	(0.000)
	2	0.611	-0.186	-0.939	-1.845	-2.282	4.963
		(0.271)	(0.426)	(0.174)	(0.033)	(0.012)	(0.000)
	3	0.331	-0.385	-2.470	-3.810	-4.428	-3.520
		(0.370)	(0.350)	(0.007)	(0.000)	(0.000)	(0.000)
	4	-0.193	-0.061	-0.702	-1.690	-0.170	4.334
		(0.423)	(0.476)	(0.242)	(0.046)	(0.432)	(0.000)
TR	1	0.300	-1.097	-3.184	-3.968	-4.183	-1.883
		(0.382)	(0.137)	(0.001)	(0.000)	(0.000)	(0.031)
	2	-0.011	0.461	0.855	1.193	1.775	3.647
		(0.496)	(0.323)	(0.197)	(0.117)	(0.039)	(0.000)

Notes: Full sample period - 2004M01 to 2021M12. τ = 1,3,6,9,12,24 months. The table reports the predictive power based on Clark and West (2007) test (CW test). The tests assess the equal predictive ability between the martingale process (random walk) and fundamentals models. The CW test indicates two values. The first is the test statistics under the one-sided p-value. When the statistics of the CW test is greater than +1.282 (10%) or +1.645 (5%), the fundamentals models outperform random walk. When the statistic has negative values means that the null hypothesis is not rejected directly. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

In short, it is necessary to reassess the predictive power of fundamentals in relation to random walk models for periods with a tendency for currency appreciation and depreciation. Given that we are considering a 60-month window for exchange rate forecasts in fundamentals models, we present the results referring to MSE_{ratio} and R^2_{oos} for two subsamples: the appreciation period – from September 2007 to June 2011 and the depreciation period from July 2016 to December 2021.[16]

[16] The beginning of the subsamples referring to the periods of appreciation and depreciation of the exchange

Table 3

Exchange rate forecast accuracy statistics (MSE_{ratio} and R^2_{oos}): fundamentals models versus random walk – exchange rate appreciation period

	SPMM							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}
1 month	1.234	-0.205	1.211	-0.182	1.204	-0.176	1.327	-0.295
3 months	1.344	-0.330	1.218	-0.205	1.214	-0.201	1.200	-0.187
6 months	1.245	-0.241	1.312	-0.308	1.293	-0.289	1.194	-0.190
9 months	1.214	-0.205	1.378	-0.368	1.325	-0.316	1.241	-0.232
12 months	1.365	-0.336	1.503	-0.471	1.447	-0.416	1.391	-0.362
24 months	0.916	0.084	1.047	-0.047	0.931	0.069	0.923	0.077
	BEER						PPP	
	Model 1		Model 2		Model 3		Model 1	
Horizon	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}
1 month	1.610	-0.572	1.502	-0.466	1.433	-0.399	1.235	-0.206
3 months	1.744	-0.726	1.428	-0.413	1.343	-0.329	1.266	-0.253
6 months	1.248	-0.244	1.353	-0.349	1.335	-0.331	0.892	0.111
9 months	1.267	-0.257	1.341	-0.332	1.346	-0.336	0.919	0.088
12 months	1.353	-0.325	1.465	-0.435	1.451	-0.420	1.018	0.003
24 months	0.947	0.053	0.992	0.008	0.961	0.039	0.656	0.344
	UIRP							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}
1 month	0.919	0.103	1.066	-0.041	0.940	0.083	1.087	-0.061
3 months	0.974	0.036	0.957	0.053	0.978	0.032	1.025	-0.014
6 months	0.895	0.108	0.967	0.036	1.051	-0.047	1.051	-0.047
9 months	0.760	0.245	1.005	0.002	0.974	0.033	1.071	-0.063
12 months	0.764	0.252	1.141	-0.117	0.834	0.183	1.202	-0.177
24 months	0.626	0.374	0.869	0.131	0.574	0.426	0.930	0.070
	TR							
	Model 1		Model 2					
Horizon	MSE_{ratio}	R^2_{oos}	MSE_{ratio}	R^2_{oos}				
1 month	1.134	-0.107	1.143	-0.116				
3 months	1.365	-0.351	1.134	-0.122				
6 months	1.811	-0.805	1.082	-0.078				
9 months	1.801	-0.788	1.154	-0.146				
12 months	1.686	-0.650	1.246	-0.220				
24 months	0.871	0.129	0.888	0.112				

Notes: The appreciation period - 2007M09 to 2011M06. The random walk model has a higher predictive power than fundamentals models if $MSE_{ratio}>1$ and $R^2oos< 0$. See equations from (3) to (8) for the baseline specification of each model. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

rate have a lag of 5 years about the dates mentioned in the previous section. One reason is that we consider a window of 60 months (5 years) to generate the first forecast of the models with fundamentals within each regime (currency appreciation and depreciation).

For the period of exchange rate appreciation, based on the MSE_{ratio} and R^2_{oos}, we identify which fundamentals models outperform the random walk. Differently from the results found for the total sample, the statistics show that some fundamentals models have a predictive power superior to the random walk for all time horizons. It should be noted that the predictive power of almost all fundamentals models is superior to the random walk for longer horizons (24 months). In particular, the results presented in table 3 show that the models belonging to the UIRP type (mainly models 1 and 3) are those with superior performance than the random walk for almost all horizons under consideration (MSE_{ratio}<1 e R^2_{oos}>0). Furthermore, the results show that, in general, the model referring to the PPP proved to be relevant for horizons greater than 6 months.

The results of the CW test confirm the evidence indicated by the MSE_{ratio} and R^2_{oos} statistics that fundamentals models have a higher predictive power of the exchange rate than random walk models for the 24-month horizon (see table 4). In addition, the CW test confirms that fundamentals models of the UIRP type (model 1) and PPP stand out for forecasting the exchange rate in different time horizons. It is worth mentioning that although UIRP-type models are typically rejected in empirical studies, there is evidence in the literature that they are useful for forecasting the exchange rate for both short and long horizons (see Cuestas, Filipozzi and Staehr, 2015; Alquist and Chinn, 2008). Regarding the good accuracy of PPP-type models, see Ca'Zorzi and Rubazek (2020), Ince (2014), Engel, Mark, and West (2007).

As in the previous cases, we use the MS_{ratio} and R^2_{oos} statistics to analyze the period marked by an exchange rate depreciation. In general, the results indicate that some fundamentals models have a predictive power greater than the random walk. As for the period of currency appreciation, virtually all fundamentals models for the 24-month horizon outperform the random walk (see table 5). However, unlike the exchange rate appreciation period, the number of fundamentals models that outperform the random walk for the 12-month horizon is not negligible (9 models).

Table 4

Exchange rate forecast accuracy test (*Clark-West test*):

fundamentals models versus random walk – exchange rate appreciation period

Fundamentals	Model	τ = 1	τ = 3	τ = 6	τ = 9	τ = 12	τ = 24
SPMM	1	-0.925	-1.337	-0.065	0.173	0.047	2.986
		(0.183)	(0.097)	(0.474)	(0.432)	(0.482)	(0.003)
	2	-0.181	-0.554	-1.135	-0.909	-0.479	2.885
		(0.429)	(0.293)	(0.134)	(0.186)	(0.318)	(0.004)
	3	-0.745	-1.281	-1.531	-1.332	-1.485	2.904
		(0.232)	(0.107)	(0.070)	(0.098)	(0.076)	(0.004)
	4	-1.242	-1.151	-0.994	-0.793	-0.845	2.951
		(0.114)	(0.131)	(0.165)	(0.218)	(0.204)	(0.004)
BEER	1	-1.275	-1.044	0.595	0.154	0.781	2.911
		(0.108)	(0.154)	(0.279)	(0.440)	(0.221)	(0.004)
	2	-1.494	-1.692	-1.424	-0.841	-0.662	2.899
		(0.075)	(0.052)	(0.084)	(0.205)	(0.258)	(0.004)
	3	-1.448	-1.443	-1.408	-0.986	-0.715	2.921
		(0.081)	(0.082)	(0.087)	(0.167)	(0.241)	(0.004)
PPP	1	-0.730	1.389	2.508	2.271	2.318	3.109
		(0.237)	(0.089)	(0.010)	(0.017)	(0.015)	(0.003)
UIRP	1	2.890	0.803	2.901	4.246	3.064	3.162
		(0.004)	(0.215)	(0.004)	(0.000)	(0.003)	(0.002)
	2	-0.018	1.039	1.010	1.101	0.941	2.968
		(0.493)	(0.155)	(0.162)	(0.142)	(0.178)	(0.004)
	3	2.380	1.168	-0.126	0.856	2.074	3.215
		(0.013)	(0.128)	(0.450)	(0.201)	(0.025)	(0.002)
	4	-0.551	0.043	0.009	0.500	0.423	2.952
		(0.293)	(0.483)	(0.497)	(0.311)	(0.338)	(0.004)
TR	1	-1.340	-2.697	-4.219	-4.969	-4.843	2.781
		(0.097)	(0.007)	(0.000)	(0.000)	(0.000)	(0.005)
	2	-0.791	-0.715	-0.281	-0.320	-0.035	2.993
		(0.219)	(0.241)	(0.391)	(0.376)	(0.486)	(0.003)

Notes: The appreciation period - 2007M09 to 2011M06. τ = 1,3,6,9,12,24 months. The table reports the predictive power based on Clark and West (2007) test (CW test). The tests assess the equal predictive ability between the martingale process (random walk) and fundamentals models. The CW test indicates two values. The first is the test statistics under the one-sided p-value. When the statistics of the CW test is greater than +1.282 (10%) or +1.645 (5%), the fundamentals models outperform random walk. When the statistic has negative values, it means that the null hypothesis is not rejected directly. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

In addition to the aspects mentioned above, it is important to highlight that unlike the period marked by exchange rate appreciation, the UIRP-type models (models 1 and 3) are those that have the worst performance among the fundamentals models (they do not beat the random walk in any horizon in consideration – see table 5). The same can be said about the PPP-type models, whose performance is inferior to the random walk for horizons from 1 to 12 months. The highlight for the period of currency devaluation is the BEER-type fundamentals models. All specifications of the BEER fundamentals have a superior performance for forecasting the exchange rate compared to the random walk. In addition, the TR-type 2 model also has a noteworthy performance, as the random walk outperforms it only on the 1-month horizon. Finally, it should be noted that the forecast accuracy of the SPMM-type models (models 2 and 4) outperforms that of the random walk for horizons longer than 9 months.

Table 5

Exchange rate forecast accuracy statistics (MSE_{ratio} and R^2_{oos}): fundamentals models versus random walk – exchange rate depreciation period

	SPMM							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.055	-0.055	1.110	-0.110	1.333	-0.333	1.225	-0.225
3 months	1.128	-0.128	1.041	-0.041	1.120	-0.120	1.261	-0.261
6 months	1.339	-0.339	1.064	-0.064	1.225	-0.225	1.186	-0.186
9 months	1.111	-0.111	0.906	0.094	1.143	-0.143	1.002	-0.002
12 months	0.810	0.190	0.629	0.371	0.777	0.210	0.710	0.290
24 months	0.395	0.605	0.414	0.586	0.251	0.749	0.363	0.637

	BEER						PPP	
	Model 1		Model 2		Model 3		Model 1	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	0.965	0.035	0.975	0.025	0.981	0.019	1.043	-0.043
3 months	0.870	0.130	0.822	0.178	0.820	0.180	1.178	-0.179
6 months	0.729	0.271	0.708	0.292	0.726	0.274	1.530	-0.530
9 months	0.628	0.372	0.641	0.359	0.657	0.343	1.846	-0.846
12 months	0.548	0.443	0.555	0.436	0.570	0.420	1.760	-0.790
24 months	0.298	0.702	0.280	0.720	0.304	0.696	0.631	0.369

	UIRP							
	Model 1		Model 2		Model 3		Model 4	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.140	-0.140	1.066	-0.066	1.075	-0.075	1.134	-0.134
3 months	1.577	-0.577	1.427	-0.427	1.329	-0.329	1.105	-0.105
6 months	2.141	-1.141	1.644	-0.644	1.774	-0.774	1.148	-0.148
9 months	2.416	-1.416	1.804	-0.804	1.954	-0.954	1.198	-0.198
12 months	2.263	-1.301	1.877	-0.908	1.915	-0.947	0.928	0.057
24 months	1.437	-0.437	0.364	0.636	1.229	-0.229	0.259	0.741

	TR			
	Model 1		Model 2	
Horizon	MSE_{ratio}	R^2oos	MSE_{ratio}	R^2oos
1 month	1.328	-0.328	1.019	-0.019
3 months	2.222	-1.221	0.929	0.071
6 months	2.197	-1.197	0.789	0.211
9 months	2.075	-1.074	0.667	0.333
12 months	1.768	-0.798	0.551	0.440
24 months	0.991	0.009	0.252	0.748

Notes: The depreciation period - 2011M07 to 2021M12. The random walk model has a higher predictive power than fundamentals models if $MSE_{ratio}>1$ and $R^2oos< 0$. See equations from (3) to (8) for the baseline specification of each model. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

The result of the CW test is consistent with the perspective that fundamentals models are good predictors of the exchange rate when considering a 24 months horizon (see table 6).

Table 6

Exchange rate forecast accuracy test (*Clark-West test*):

fundamentals models versus random walk - exchange rate depreciation period

Fundamentals	Model	τ = 1	τ = 3	τ = 6	τ = 9	τ = 12	τ = 24
SPMM	1	1.287	1.511	-0.554	0.839	3.013	16.987
		(0.103)	(0.069)	(0.291)	(0.203)	(0.002)	(0.000)
	2	0.858	1.663	2.350	2.740	4.453	17.150
		(0.198)	(0.519)	(0.012)	(0.004)	(0.000)	(0.000)
	3	0.079	0.708	-0.372	0.532	4.284	19.375
		(0.469)	(0.241)	(0.356)	(0.299)	(0.000)	(0.000)
	4	0.646	0.720	1.473	2.272	3.758	13.442
		(0.261)	(0.238)	(0.074)	(0.014)	(0.000)	(0.000)
BEER	1	1.182	2.367	3.447	4.481	5.916	17.368
		(0.122)	(0.011)	(0.001)	(0.000)	(0.000)	(0.000)
	2	1.139	2.356	3.289	4.045	5.535	16.193
		(0.131)	(0.012)	(0.001)	(0.000)	(0.000)	(0.000)
	3	1.138	2.464	3.320	4.226	5.541	16.618
		(0.131)	(0.009)	(0.001)	(0.000)	(0.000)	(0.000)
PPP	1	1.254	2.251	2.823	3.864	5.428	14.244
		(0.108)	(0.015)	(0.004)	(0.000)	(0.000)	(0.000)
UIRP	1	-0.678	-2.119	-3.567	-4.036	-5.943	-0.275
		(0.251)	(0.020)	(0.000)	(0.000)	(0.000)	(0.392)
	2	0.877	0.499	-0.589	-1.573	-1.748	13.588
		(0.193)	(0.310)	(0.280)	(0.062)	(0.044)	(0.000)
	3	-0.316	-1.902	-3.395	-4.005	-5.916	0.787
		(0.377)	(0.032)	(0.001)	(0.000)	(0.000)	(0.218)
	4	-0.524	1.036	0.802	0.421	5.035	17.118
		(0.302)	(0.153)	(0.213)	(0.338)	(0.000)	(0.000)
TR	1	0.083	-0.630	-1.784	-2.250	-2.278	2.418
		(0.467)	(0.266)	(0.041)	(0.015)	(0.014)	(0.010)
	2	0.606	2.038	3.383	4.846	6.305	17.529
		(0.274)	(0.024)	(0.001)	(0.000)	(0.000)	(0.000)

Notes: The depreciation period - 2011M07 to 2021M12. τ = 1,3,6,9,12,24 months. The table reports the predictive power based on Clark and West (2007) test (CW test). The tests assess the equal predictive ability between the martingale process (random walk) and fundamentals models. The CW test indicates two values. The first is the test statistics under the one-sided p-value. When the statistics of the CW test is greater than +1.282 (10%) or +1.645 (5%), the fundamentals models outperform random walk. When the statistic has negative values means that the null hypothesis is not rejected directly. SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule.

The CW test ratifies the evidence from the MSE_{ratio} and R^2_{oos} statistics that BEER and TR-type fundamentals (model 2) outperform the random walk. On the other hand, the CW test also shows that the predictive power of the SPMM-type models (models 2 and 4) is greater than that of the random walk for shorter horizons and that the PPP-type models should not be neglected. This result is important, as it suggests that the range of fundamentals models capable of beating the random walk in the depreciation period is greater than the period marked by currency appreciation. Notably, the good accuracy of the mentioned fundamentals models for forecasting the exchange rate is not an exception in the literature. See, for example: BEER - Kharrat, Hammami, and Fatnassi (2020); Cheung et al. (2019), Takizawa, Hauner, and Lee (2010), TR – Jamali and Yamani (2019), Molodtsova and Papell (2009), SPMM – Ren, Wang, and Zhang (2019), Moosa and Burns (2014), and PPP – Ca'Zorzi and Rubaszek (2020), Ca'Zorzi, Muck, and Rubaszek (2016), Ince (2014), Engel, Mark, and West (2007).

Based on the results found from the CW test, table 7 shows the models with the greatest predictive power of the exchange rate for horizons from 1 to 24 months. In general, regardless of the sample under consideration, the results show that the fundamentals models (all types) outperform the random walk model when considering the 24 months horizon. For the case of the full sample, the random walk model beats the fundamentals for forecasts from 1 to 6 months. However, regarding the fundamentals models, the highlights are the BEER-type models for horizons from 9 to 24 months. Concerning the appreciation period subsample, the performance of the fundamentals models is highlighted when compared to the total sample. In particular, the PPP and UIRP models have the highest accuracy. Almost all fundamentals models outperform the random walk for horizons greater than 3 months for the period marked by exchange rate depreciation.

After identifying fundamentals models with a predictive power superior to the random walk, we perform an FS test to assess whether the forecasts generated by such models can add information to market expectations. In other words, the FS test allows us to verify whether professional forecasters' expectations neglect useful information for exchange rate forecasts that are present in fundamentals models. It is important to highlight that, differently from what was done for the CW test, in which we were identifying which fundamentals models beat the random walk accuracy, in the case of the FS test, we are evaluating whether the forecasts of the identified models have relevant informational content when considering market expectations (see the results in table 8).

Table 7
Best accuracy models for the exchange rate

Horizon	Periods: Full sample	Exchange rate appreciation	Exchange rate depreciation
1 month	Random Walk	UIRP (models 1 and 3)	Random Walk
3 months	Random Walk	PPP (model 1)	SPMM (models 1 and 2) BEER (models 1, 2, and 3) PPP (model 1) TR (model 2)
6 months	Random Walk	PPP (model 1) UIRP (model 1)	SPMM (models 2 and 4) BEER (models 1, 2, and 3) PPP (model 1) TR (model 2)
9 months	BEER (model 3)	PPP (model 1) UIRP (model 1)	SPMM (models 2 and 4) BEER (models 1, 2, and 3) PPP (model 1) TR (model 2)
12 months	BEER (models 1, 2 and 3) TR (model 2)	PPP (model 1) UIRP (models 1 and 3)	SPMM (models 1, 2, 3, and 4) BEER (models 1, 2 and 3) PPP (model 1) UIRP (model 4) TR (model 2)
24 months	SPMM (models 2, 3 and 4) BEER (models 1, 2 and 3) PPP (model 1) UIRP (models 2 and 4) TR (model 2)	SPMM (models 1, 2, 3 and 4) BEER (models 1, 2 and 3) PPP (model 1) UIRP (models 1, 2, 3 and 4) TR (models 1 and 2)	SPMM (models 1, 2, 3, and 4) BEER (models 1, 2, and 3) PPP (model 1) UIRP (models 2 and 4) TR (models 1 and 2)

Notes: Based on Clark and West's (2007) test result. Each cell contains the information of the fundamentals models that outperform the random walk for each horizon.

Specifically, in the FS test, for the fundamentals models to add information to the professional forecasters' expectations, it is necessary that the test (regression result) shows that the coefficient associated with the forecast of the models has statistical significance.[17] The results of the full-sample FS test show that none of the selected fundamentals models has incremental information to market expectations for all horizons under consideration (see table 8). However, regarding the period marked by appreciation, the FS test shows that most fundamentals models have additional informational content compared to professional forecasters' expectations (7 specifications out of a total of 10 have significant coefficients for the forecasts from fundamentals models). Finally, in the

[17] Unfortunately, market expectations for the 24-month horizon are only available from January 2021in the TSMS/CBB, which, therefore, prevents us from carrying out the test for the aforementioned horizon.

period marked by depreciation, the result of the FS test shows that among all the models with previously selected fundamentals, the one that adds information to market expectations for all the time horizons under consideration is the PPP-type models.

Table 8

The difference in the informational content of fundamentals models and market expectations

Sample	Fund.	Model	τ = 1	τ = 3	τ = 6	τ = 9	τ = 12
Full	BEER	1	n.a.	n.a.	n.a.	n.a.	0.489 (0.626)
		2	n.a.	n.a.	n.a.	n.a.	0.342 (0.733)
		3	n.a.	n.a.	n.a.	0.099 (0.921)	0.594 (0.553)
	TR	2	n.a.	n.a.	n.a.	n.a.	1.393 (0.165)
Apprec.	PPP	1	n.a.	-2.154 (0.044)	-2.229 (0.038)	-3.737 (0.001)	-0.288 (0.777)
	UIRP	1	5.272 (0.000)	n.a.	-4.137 (0.001)	1.028 (0.317)	-2.028 (0.057)
		3	4.776 (0.000)	n.a.	n.a.	n.a.	-1.596 (0.127)
Deprec.	SPMM	1	n.a.	0.754 (0.455)	n.a.	n.a.	-0.034 (0.973)
		2	n.a.	-0.245 (0.808)	-1.347 (0.185)	-0.882 (0.383)	0.096 (0.924)
		3	n.a.	n.a.	n.a.	n.a.	-0.827 (0.413)
		4	n.a.	n.a.	-2.618 (0.012)	-2.206 (0.033)	-1.079 (0.287)
	BEER	1	n.a.	0.072 (0.943)	0.479 (0.634)	-0.779 (0.440)	1.016 (0.316)
		2	n.a.	0.894 (0.377)	1.205 (0.235)	1.391 (0.172)	1.327 (0.192)
		3	n.a.	0.906 (0.370)	1.072 (0.290)	1.243 (0.221)	1.059 (0.296)
	PPP	1	n.a.	-2.337 (0.025)	-3.606 (0.001)	-5.305 (0.000)	-5.462 (0.000)
	UIRP	4	n.a.	n.a.	n.a.	n.a.	-4.097 (0.000)
	TR	2	n.a.	-0.269 (0.790)	0.262 (0.794)	-0.476 (0.637)	0.713 (0.480)

Notes: τ = 1,3,6,9,12,24 months. The table reports the statistics based on Fair and Shiller's (1989) test. The test assesses the informational content between the fundamentals models and market expectations. The p-value is reported in parentheses. When the statistics have significance, the fundamentals models have additional information not contained in the market expectation. Full – denotes "full period". Apprec – denotes "appreciation period". Deprec – denotes "depreciation period". "n.a." – non-applicable.

4. Concluding remarks

The literature on exchange rate forecasting casts doubt on whether fundamentals models can outperform the accuracy of random walk models (see, for example, Engel, Mark, and West, 2007). However, there is empirical evidence that economic models can be useful for predicting the exchange rate when considering different time horizons (see Cheung et al., 2019; Rossi, 2013; Berge, 2014). Although the literature has a great number of fundamentals models, there is little evidence regarding which model performs better considering different horizons. In addition, most studies do not take into account if the economy is subject to periods of exchange rate depreciation or appreciation. Another aspect that is neglected by the literature is whether the market expectations consider or not economic fundamentals. Finally, there is a lack of analyses based on emerging market data.

Answering the above questions is crucial for market professionals and policymakers to make decisions. Specifically, using economic models permits the discovery of the main variables that affect the exchange rate considering a specific horizon (e.g., one month, six months, one year) and regime (appreciation or depreciation). As Sarno and Valente (2009) pointed out, it seems illogical that the knowledge of the state of the economy at a point in time is useless information to forecast exchange rate movements. Notwithstanding, the ability to select the best model from the various sets of models is not an easy task. In short, a map indicating what model performs better for each horizon and regime is a valuable tool for anyone who needs to predict the exchange rate.

This paper helps to fill some of the gaps mentioned above. Specifically, this study takes up the argument of the "Meese and Rogoff puzzle" by presenting empirical evidence for the Brazilian case during the largest part of the flexible exchange rate regime (January 1999 to December 2021). In particular, we evaluate from a comprehensive number of fundamentals models which ones outperform the random walk for horizons from 1 to 24 months. Furthermore, we checked whether the fundamentals models' exchange rate forecasts have additional information not contained in market expectations.

The study's main outcomes reveal that fundamental models are advantageous in forecasting exchange rates for all horizons under consideration. The predictive power of these models increases when we examine periods characterized by exchange rate appreciation or depreciation. Despite several fundamentals models surpassing the random walk, PPP-type models stand out in particular. The evidence suggests that PPP-type fundamental models demonstrate superior predictive power compared to the random walk and provide additional information to market expectations for different time horizons and periods of exchange rate appreciation and depreciation. This finding is consistent with earlier research studies such as Zorzi et al. (2022), Ca'Zorzi and Rubaszek (2020), Ca'Zorzi, Muck, and Rubaszek (2016), Ince (2014), Engel, Mark, and West (2007), which have also highlighted the efficacy of PPP-type models in exchange rate forecasting.

Specifically, the UIRP-type models perform well under exchange rate appreciation. This result cannot be considered awkward since some studies, e.g., Cuestas, Filipozzi, and Staehr (2015) and Alquist and Chinn (2008), documented the usefulness of the UIRP-type models. Furthermore, SPMM, BEER, and TR-type models have good predictive power when considering a period of exchange rate depreciation. Again, our findings are in line with studies about empirical exchange rate models, which highlight the performance of SPMM-type models (e.g., Ren, Wang, and Zhang, 2019; Moosa and Burns, 2014), BEER-type models (e.g., Kharrat, Hammami, and Fatnassi, 2020; Cheung et al., 2019; Takizawa, Hauner, and Lee, 2010), and TR-type models (e.g., Jamali and Yamani, 2019; Molodtsova and Papell, 2009).

Finally, the results show that professional forecasters' expectations have neglected information from economic theory, which is useful for forecasting the exchange rate. Undoubtedly, the advancement of technologies promoted with the advancement of, for example, "machine learning", has increased the predictive power (Amat, Michalski, and Stoltz, 2018). However, the "fundamentals" are still helpful for developing and explaining new exchange rate forecasting models. In short, it is necessary to find a balance between fundamentals and "atheoreticals" models to predict the exchange rate and explain its movements. Particularly, considering if the economy is living under a period of exchange rate appreciation or depreciation is crucial for correctly selecting the model.

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Table A.1

Description of the variables, sources of data, and descriptive statistics

Variable	Description	Data source	Mean	Std. dev	Min	Max
COMM	All Commodity Price Index, 2016 = 100, includes both Fuel and Non-Fuel Price Indices	IMF DATA	115.34	41.77	42.01	202.80
DXY	The U.S. Dollar Index (USDX) is a relative measure of the strength of U.S. dollar (USD) against a basket of six influential currencies, including the Euro, Pound, Yen, Canadian Dollar, Swedish Korner, and Swiss Franc.	Yahoo Finance	91.35	11.20	71.87	120.21
EMBI	Emerging Markets Bond Index Plus estimates the performance of emerging market debt relative to US Treasuries	IPEADATA	433.77	341.38	146.00	2039.00
gap	Difference between GDP and the potential output (HP filter)	TSMS/CBB	101.18	18.04	53.90	164.18
		FRED	100.05	5.27	70.64	113.77
GOLD	The LBMA Gold price is an auction independently operated and administered by ICE Benchmark Administration (IBA). The price is set in US dollars per fine troy ounce (price taken at 10:30).	LBMA	980.22	518.18	256.20	1971.17
HYS	The ICE BofA US High Yield Index Option-Adjusted Spread is a market capitalization-weighted and is designed to measure the performance of U.S. dollar denominated below investment grade (commonly referred to as "junk") corporate debt publicly issued in the U.S. domestic market.	FRED	5.64	2.65	2.57	20.31
π	Inflation rate (Brazil – based on Extended National Consumer Price Index (IPCA); and for the USA – based on Consumer Price Index for All Urban Consumers)	TSMS/CBB	0.52	0.39	-0.38	3.02
		FRED	0.19	0.37	-1.92	1.22
i	Monetary policy interest rate (SELIC and FEDFUNDS)	TSMS/CBB	12.83	6.18	1.9	43.25
		FRED	1.79	1.97	0.05	6.54
i_{y1}	Interest rate - one-year treasury constant maturity	IPEADATA	12.88	5.99	2.41	33.75
		FRED	1.87	1.87	0.05	6.33
m	Money supply (M1)	TSMS/CBB	242.64	143.99	45.13	627.90
		FRED	3272.73	4362.20	1085.20	20599.40
s_{me}	Exchange rate market expectations (median)	TSMS/CBB	2.86	1.06	1.57	5.57
p	Price level (Brazil – based on Extended National Consumer Price Index (IPCA); and for the USA – based on Consumer Price Index for All Urban Consumers)	TSMS/CBB	2.32	0.88	1.01	4.20
		FRED	1.33	0.18	1.00	1.70
s	Exchange rate - end of period (BRL/USD)	TSMS/CBB	2.76	1.06	1.56	5.77
q	Real exchange rate	TSMS/CBB and FRED	2.91	1.09	1.60	5.84
r	Real interest rate	TSMS/CBB	6.19	5.29	-8.32	29.25
		FRED	-0.38	2.08	-6.71	4.67
TOT	The terms of trade are defined as the relationship between the prices of the country's exports and those of its imports.	IPEADATA	97.70	10.78	80.82	122.73
VIX	The volatility of S&P 500 index options measures market expectation of near-term volatility conveyed by stock index option prices	FRED	20.14	8.27	10.13	62.64
Y	Output (industrial production total index for Brazil and the USA)	TSMS/CBB	89.26	10.93	59.70	112.60
		FRED	96.35	5.12	82.10	106.22
z	The relative price of non-tradable (Brazil – based on the Extended National Consumer Price for non-tradable goods. USA – based on the Consumer Price Index for All Urban Consumers: Services Less Energy Services)	TSMS/CBB	2.12	0.86	1.01	3.72
		FRED	1.41	0.24	1.00	1.87

Notes: Time Series Management System/Central Bank of Brazil (TSMS/CBB); London Bullion Market Association (LBMA); Federal Reserve Economic Data (FRED); Institute of Applied Economic Research (IPEADATA); and International Monetary Fund - Data (IMF DATA).

Table A.2

Exchange rate fundamentals models

Fundamentals	Model	Specification
SPMM	1	$s_t = \alpha_0 + \alpha_1\widetilde{m}_t + \alpha_2\tilde{y}_t + \alpha_3\tilde{\imath}_t + \alpha_4\tilde{\pi}_t + u_t$
	2	$s_t = \alpha_5 + \alpha_6\widetilde{m}_t + \alpha_7\tilde{y}_t + \alpha_8\tilde{\imath}_t + \alpha_9\tilde{\pi}_t + \alpha_{10}COMM_t + u_t$
	3	$s_t = \alpha_{11} + \alpha_{12}\widetilde{m}_t + \alpha_{13}\tilde{y}_t + \alpha_{14}\tilde{\imath}_t + \alpha_{15}\tilde{\pi}_t + \alpha_{16}COMM_t + \alpha_{17}GOLD_t + \alpha_{18}VIX_t + \alpha_{19}HYS_t + u_t$
	4	$s_t = \alpha_{20} + \alpha_{21}\widetilde{m}_t + \alpha_{22}\tilde{y}_t + \alpha_{23}\tilde{\imath}_t + \alpha_{24}\tilde{\pi}_t + \alpha_{25}COMM_t + \alpha_{26}HYS_t + \alpha_{27}DXY_t + u_t$
BEER	1	$s_t = \delta_0 + \delta_1\widetilde{m}_t + \delta_2\tilde{p}_t + \delta_3\tilde{z}_t + \delta_4\tilde{r}_t + \delta_5TOT_t + u_t$
	2	$s_t = \delta_6 + \delta_7\widetilde{m}_t + \delta_8\tilde{p}_t + \delta_9\tilde{z}_t + \delta_{10}\tilde{r}_t + \delta_{11}TOT_t + \delta_{12}COMM_t + u_t$
	3	$s_t = \delta_{13} + \delta_{14}\widetilde{m}_t + \delta_{15}\tilde{p}_t + \delta_{16}\tilde{z}_t + \delta_{17}\tilde{r}_t + \delta_{18}TOT_t + \delta_{19}COMM_t + \delta_{20}VIX_t + u_t$
PPP	1	$s_t = \varphi_0 + \varphi_1\tilde{p}_t + u_t$
UIRP	1	$s_t = \omega_0 + \omega_1\tilde{\imath}_t + u_t$
	2	$s_t = \omega_2 + \omega_3\tilde{\imath}_t + \omega_4EMBI_t + \omega_5VIX_t + \omega_6HYS_t + u_t$
	3	$s_t = \vartheta_0 + \vartheta_1\tilde{\imath}_t + \vartheta_2\tilde{\imath}_t^k + u_t$
	4	$s_t = \vartheta_3 + \vartheta_4\tilde{\imath}_t + \vartheta_5\tilde{\imath}_t^k + \vartheta_6EMBI_t + \vartheta_7DXY_t + \vartheta_8VIX_t + \vartheta_9HYS_t + u_t$
TR	1	$s_t = \psi_0 + \psi_1\tilde{\pi}_t + \psi_2\widetilde{gap}_t + u_t$
	2	$s_t = \psi_3 + \psi_4\tilde{\pi}_t + \psi_5\widetilde{gap}_t + \psi_6\tilde{\imath}_t + \psi_7\tilde{q}_t + \psi_7COMM_t + u_t$

Notes: SPMM: Sticky price monetary model; BEER: Behavioral equilibrium exchange rate; PPP: Relative purchasing power parity; UIP: Uncovered interest rate parity; TR: Taylor Rule. The inclusion of control variables in the specifications considers the fundamentals models' performance compared to the random walk (MSE_{ratio} and R^2oos).

Table A.3

Unit root test (*Ng-Perron*)

Variable:	Lag	MZa	MZt
s_t	2	-5.271	-1.552
Δs_t	10	-15.752	-2.796
$\widetilde{m}_t$	0	-0.486	-0.1967
$\Delta\widetilde{m}_t$	1	-135.035	-8.217
$\tilde{y}_t$	10	0.408	0.344
$\Delta\tilde{y}_t$	10	-6.424	-1.702
$\tilde{\imath}_t$	4	-6.729	-1.710
$\Delta\tilde{\imath}_t$	1	-15.118	-2.694
$\tilde{\pi}_t$	13	-12.441	-2.494
$\Delta\tilde{\pi}_t$	3	-76.390	-6.177
$COMM_t$	1	-5.220	-1.615
$\Delta COMM_t$	10	-14.532	-2.678
$GOLD_t$	2	-2.746	-1.124
$\Delta GOLD_t$	10	-19.260	-3.099
VIX_t	13	-8.974	-2.029
ΔVIX_t	0	-137.992	-8.302
HYS_t	0	-8.686	-2.073
ΔHYS_t	10	-33.915	-4.093
DXY_t	0	-4.665	-1.485
ΔDXY_t	9	-17.152	-2.927
$\tilde{p}_t$	2	-4.962	-1.482
$\Delta\tilde{p}_t$	8	-25.595	-3.577
$\tilde{z}_t$	12	-3.351	-1.284
$\Delta\tilde{z}_t$	4	-16.039	-2.827
$\tilde{r}$	13	-11.509	-2.399
$\Delta\tilde{r}$	1	-20.670	-3.215
TOT_t	1	-8.969	-2.117
ΔTOT_t	3	-14.463	-2.689
$\tilde{\imath}_t^k$	18	-9.702	-2.199
$\Delta\tilde{\imath}_t^k$	2	-23.780	-3.442
$EMBI_t$	1	-7.344	-1.825
$\Delta EMBI_t$	0	-114.799	-7.514
$\widetilde{gap}_t$	34	-12.951	-2.541
$\Delta\widetilde{gap}_t$	8	-21.428	-3.211
$\tilde{q}_t$	1	-5.565	-1.616
$\Delta\tilde{q}_t$	4	-48.941	-4.885

Notes: C.V. = critical value (Ng-Perron, 2001). Asymptotic critical values 10%: MZa = -14.200 MZt = -2.620. The number of lags is based on the Modified Akaike criterion. Constant and linear trend included in the test equation.

Table A.4

Johansen's Cointegration Test

Fundamentals	Model:	Hyp. No. CE(s)	Eigenvalue	Trace Statistic	C. V. (0.05)	Prob.
SPMM	1	None *	0.243	183.425	159.530	0.001
		At most 1	0.093	111.293	125.615	0.266
		At most 2	0.090	86.045	95.754	0.193
		At most 3	0.243	183.425	159.530	0.001
	2	None *	0.182	106.033	95.754	0.008
		At most 1	0.074	53.920	69.819	0.465
		At most 2	0.056	34.050	47.856	0.499
		At most 3	0.049	19.192	29.797	0.479
	3	None *	0.293	243.190	197.371	0.000
		At most 1	0.152	153.120	159.530	0.106
		At most 2	0.114	110.192	125.615	0.294
		At most 3	0.087	78.715	95.754	0.409
	4	None *	0.242	183.425	159.530	0.001
		At most 1	0.093	111.293	125.615	0.266
		At most 2	0.090	86.045	95.754	0.193
		At most 3	0.078	61.544	69.819	0.191
BEER	1	None *	0.160	123.865	95.754	0.000
		At most 1	0.120	78.634	69.819	0.008
		At most 2	0.087	45.323	47.856	0.085
		At most 3	0.054	21.677	29.797	0.317
	2	None *	0.196	173.704	125.615	0.000
		At most 1	0.159	116.866	95.754	0.001
		At most 2	0.107	71.818	69.819	0.034
		At most 3	0.080	42.509	47.856	0.145
	3	None *	0.211	212.110	159.530	0.000
		At most 1	0.182	150.384	125.615	0.001
		At most 2	0.109	98.306	95.754	0.033
		At most 3	0.090	68.276	69.819	0.066
PPP	4	None *	0.074	22.539	12.321	0.001
		At most 1	0.005	1.484	4.130	0.262
UIRP	1	None *	0.063	19.379	15.495	0.012
		At most 1	0.006	1.597	3.841	0.206
	2	None *	0.136	109.950	95.754	0.004
		At most 1	0.107	69.929	69.819	0.049
		At most 2	0.071	38.902	47.856	0.264
		At most 3	0.042	18.782	29.797	0.509
	3	None *	0.171	54.126	29.797	0.000
		At most 1	0.020	5.595	15.495	0.743
		At most 2	0.002	0.470	3.841	0.493
	4	None *	0.265	171.192	125.615	0.000
		At most 1	0.100	91.894	95.754	0.089
		At most 2	0.089	64.642	69.819	0.121
		At most 3	0.076	40.674	47.856	0.199
TR	1	None *	0.105	39.626	29.797	0.003
		At most 1	0.039	10.419	15.495	0.250
		At most 2	0.000	0.023	3.841	0.880
	2	None *	0.289	160.950	95.754	0.000
		At most 1	0.123	71.523	69.819	0.036
		At most 2	0.080	37.230	47.856	0.337
		At most 3	0.041	15.318	29.797	0.759

Note: CE = cointegrating equations. (*) denotes rejection of H_0 at the 0.05 level. MacKinnon-Haug-Michelis (1999) p-values. Deterministic assumptions: Cointegrating relationship includes a constant. Short-run dynamics include a constant.