Enhancing Grain Portfolio Risk Management with GAMLSS and MSGARCH

Aprimorando a Gestão de Risco de Portfólio de Grãos com GAMLSS e MSGARCH

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Abstract This paper presents a novel method integrating Generalized Additive Models for Location, Scale, and Shape (GAMLSS) with Bayesian Markov-Switching GARCH (MSGARCH) models to enhance forecasting in commodity price returns, focusing on grain portfolios. We leverage GAMLSS to model non-normal distributions of return series, crucial for accurately simulating real options. These models then inform the Bayesian MSGARCH framework, improving projections of returns and volatility, essential for effective financial planning and risk management. This innovative approach not only advances practical portfolio management but also contributes to the theoretical development of real options theory. Demonstrating its efficacy, our methodology offers a more informed, strategic approach to the complex world of commodity trading, bridging the gap between theoretical models and practical financial applications. **Keywords**: Commodity Portfolio Management; GAMLSS; Monte Carlo Simulation; Bayesian MSGARCH; Real Options Theory; Financial Time Series
JEL Code: E3, C41, C43.

1. Introduction

The intricate world of financial portfolio management, particularly in the realm of commodity grain portfolios, presents unique challenges and opportunities for financial analysts and portfolio managers. In such a context, the selection of individual assets is often constrained by external factors, leading to a focus on optimizing the existing portfolio mix. This paper delves into the application of real options theory in financial planning for commodity grain

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portfolios, proposing an innovative approach that leverages the true probability distributions of individual return series to enhance the efficacy of Monte Carlo simulations.

The cornerstone of our approach lies in the utilization of Generalized Additive Models for Location, Scale, and Shape (GAMLSS) (Stasinopoulos and Rigby, 2007) to accurately model the probability distribution of each return series. This methodological choice is driven by the need to capture the unique characteristics of grain commodity returns, which often exhibit non-normal distributions with significant skewness and kurtosis. By employing GAMLSS, we can obtain a more realistic and nuanced understanding of each commodity's return distribution, which is crucial for effective risk assessment and decision-making in portfolio management.

Incorporating these individualized probability distributions into computational simulations, particularly within the framework of Monte Carlo simulations via Markov chains, marks a significant advancement in the application of real options theory. This integration allows for a more precise and tailored analysis of the portfolio, taking into account the distinct volatility and risk profiles of each commodity. The use of a Bayesian Markov-Switching GARCH (MSGARCH) (Ardia et al., 2019) model further enhances this approach. By setting the distributions identified by GAMLSS as priors in the Bayesian MSGARCH model, we can create a robust and dynamic framework that accurately projects returns and volatilities, thereby optimizing the financial planning process.

This methodology not only contributes to the practical aspects of portfolio management but also enriches the theoretical underpinnings of real options theory. It provides a novel perspective on how real options can be effectively utilized in the context of grain commodity portfolios, where traditional asset selection strategies may not be feasible. By integrating advanced statistical modeling techniques with computational simulations, this research offers a comprehensive and scientifically rigorous approach to portfolio optimization, risk management, and strategic decision-making in the complex and often unpredictable world of commodity trading.

In essence, this paper aims to bridge the gap between theoretical financial models and the practical realities of commodity portfolio management. By leveraging the latest advancements in statistical modeling and computational simulations, we endeavor to provide portfolio managers with a powerful toolset for navigating the challenges of the grain commodity market, ultimately contributing to the advancement of real options theory and its application in the field of financial portfolio management.

2. Last findings in distributional discussion and financial time series returns forecasting

Recent advancements in financial time series analysis encompass a range of sophisticated methodologies, each contributing unique insights into market dynamics, risk assessment, and portfolio optimization.

The application of Generalized Additive Models for Location, Scale, and Shape (GAMLSS) in financial time series has been a significant development. (Rigby and Stasinopoulos, 2005) introduced GAMLSS to model the probability distribution of return series, offering flexibility in distribution selection and handling censored data. (Stasinopoulos and Rigby, 2007) further demonstrated the GAMLSS framework's efficacy in fitting highly skew and kurtotic distributions in financial time series.

In the realm of real options theory, Monte Carlo simulations have been instrumental. (Amédée-Manesme et al., 2013) showcased the effectiveness of combining Monte Carlo simulations with options theory in real estate portfolio valuations and risk management, enhancing financial assessment accuracy.

Forecasting financial return series, particularly for commodities, has seen notable advancements. (Cotter et al., 2020) developed models for accurately capturing the dynamics of commodity prices and their return distributions. (Chandrasekara et al., 2016) provided insights into modeling return distributions for financial indices like Oil, Gold, and Cocoa using the scaled *t* distribution.

The exploration of Markov Switching GARCH (MSGARCH) models in financial time series has opened new avenues for understanding market volatilities. (Ardia et al., 2019) introduced the MSGARCH package in R, facilitating effective simulations, estimations, and risk management in financial markets. This tool has been instrumental in analyzing exchange rate and stock market return data.

Moreover, the application of Markov regime-switching models in financial time series has shed light on the significant impact of regime shifts on market properties. (Cai, 1994) and (Otranto, 2016) have demonstrated how these shifts profoundly affect the characteristics of financial time series, offering a more comprehensive perspective on market volatility and its implications for investment strategies.

In conclusion, the integration of these diverse methodologies—from GAMLSS and Monte Carlo simulations to MSGARCH and Markov regime-switching models—has significantly advanced the field of financial research. These developments not only enhance our understanding of market dynamics but also equip financial analysts and investors with sophisticated tools for portfolio optimization and risk management. The collective insights from these studies lay a solid foundation for future research in financial time series modeling, paving the way for more informed and strategic investment decisions.

3. Methodology

In this section, we introduce the Generalized Additive Models for Location, Scale, and Shape (GAMLSS) framework (Stasinopoulos and Rigby, 2007)). This advanced statistical modeling technique extends beyond traditional regression analysis by allowing for the explicit modeling of not only the mean (location) but also other distribution parameters such as scale, skewness, and kurtosis. The GAMLSS framework is particularly suited for data sets with complex, non-normal error distributions and offers enhanced flexibility over traditional models.

This research utilizes the Generalized Additive Models for Location, Scale, and Shape (GAMLSS) framework, a sophisticated approach for statistical analysis that accommodates non-normal and complex error distributions. GAMLSS extends traditional regression models, enabling the modeling of the entire distribution of the response variable, encompassing the location (μ), scale (σ), and shape parameters (ν, τ).

$$
\begin{aligned}
Y &\sim \mathrm{D}(\mu, \sigma, \nu, \tau), \\
\mu &= g_1^{-1}(\mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{s}_1(\mathbf{x}_1)), \\
\sigma &= g_2^{-1}(\mathbf{X}_2 \boldsymbol{\beta}_2 + \mathbf{s}_2(\mathbf{x}_2)), \\
\nu &= g_3^{-1}(\mathbf{X}_3 \boldsymbol{\beta}_3 + \mathbf{s}_3(\mathbf{x}_3)), \\
\tau &= g_4^{-1}(\mathbf{X}_4 \boldsymbol{\beta}_4 + \mathbf{s}_4(\mathbf{x}_4)),
\end{aligned}
$$

In GAMLSS, selecting the appropriate probability distribution for the response variable is crucial. The selection process involves:

1. Considering a range of distributions (Normal, Binomial, Poisson, etc.).
2. Fitting the GAMLSS model to the data using each distribution.
3. Comparing the model fits using criteria such as the Generalized Akaike Information Criterion (GAIC), Bayesian Information Criterion (BIC) or Deviance.
4. Choosing the distribution that provides the best fit based on these criteria.

The GAMLSS-estimated distributions are integral as priors in the Bayesian Markov-Switching GARCH (MSGARCH) model (Ardia et al., 2019)). This synergetic approach, combining GAMLSS and Bayesian MSGARCH, allows us to construct a more comprehensive and nuanced model of financial series volatility.

3.1 Markov-Switching GARCH Model

In our study, we explore a novel methodological synthesis that integrates the probabilistic findings from the GAMLSS framework into the Bayesian MSGARCH model. This approach aims to utilize the estimated distributions from GAMLSS as informative priors in our Bayesian MSGARCH model, enhancing the analysis of financial time series, particularly in assessing volatility dynamics.

Prior Setting in Bayesian MSGARCH:

$$\text{Prior}(\theta) = \text{GAMLSS-Estimated Distribution}(\mu, \sigma, \nu, \tau),$$

where θ represents the parameter set of the MSGARCH model. The prior distribution is informed by the GAMLSS estimates for μ (location), σ (scale), ν, and τ (shape parameters). It is important to note that while GAMLSS provides a comprehensive analysis of the distributional characteristics of financial returns, its direct application as priors in MSGARCH requires careful consideration. The parameters in GAMLSS do not directly correspond to the volatility dynamics typically modeled in MSGARCH. Therefore, our approach focuses on leveraging the distributional insights from GAMLSS to inform the broader structure and parameterization of the MSGARCH model, rather than a direct parameter-to-parameter translation.

Bayesian Inference:

$$\text{Posterior}(\theta|\text{Data}) \propto \text{Likelihood}(\text{Data}|\theta) \times \text{Prior}(\theta),$$

emphasizing the integration of GAMLSS-estimated distributions within the Bayesian framework of MSGARCH. This methodological integration is intended to enrich the MSGARCH analysis by providing a more nuanced understanding of the underlying distributional properties of the financial time series.

The Markov-Switching GARCH (MSGARCH) model represents a significant advancement in financial time series analysis by incorporating regime shifts, essential for capturing the dynamic nature of financial markets. The MSGARCH model is adept at modeling periods of varying volatility, reflecting the real-world behavior of financial markets more accurately.

Model Dynamics:

$$y_t|(s_t = k, I_{t-1}) \sim D(0, h_{k,t}, \xi_k),$$

where $D(0, h_{k,t}, \xi_k)$ denotes a distribution with zero mean and regime-dependent variance and shape parameters. The variable s_t evolves according to a Markov chain, capturing the state-dependent nature of financial volatility.

Regime Transition:

$$P(s_t = j|s_{t-1} = i) = p_{i,j},$$

where $p_{i,j}$ represents the transition probability from state i to state j. This Markovian structure enables the model to adapt dynamically to changes in market conditions.

Volatility Dynamics:

$$h_{k,t} = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 h_{k,t-1} + \ldots + \beta_p h_{k,t-p},$$

allowing for a flexible representation of volatility dynamics within each regime.

This sophisticated approach to volatility modeling is crucial for effective risk management and option pricing in financial markets, as it accounts for the non-linear and regime-dependent nature of market volatility. Our methodology acknowledges the complexities and potential limitations of integrating GAMLSS and MSGARCH models, aiming to utilize the strengths of both to provide a more comprehensive understanding of financial time series behavior.

4. Results and discussion

In this section of our study, we initially focus on a portfolio of grain commodities, specifically examining the series of returns for corn. This choice stems from the understanding that, contrary to Markowitz's theory of inverse covariance in portfolio diversification, commodities in the same category might not always exhibit such inverse relationships. By concentrating on corn, a key commodity within this sector, we aim to shed light on specific market behaviors and trends that might differ from those observed in a more diversified portfolio context. This targeted analysis allows for a nuanced understanding of the corn market dynamics.

First of all, let´s inspect all selected time series in our portfolio:

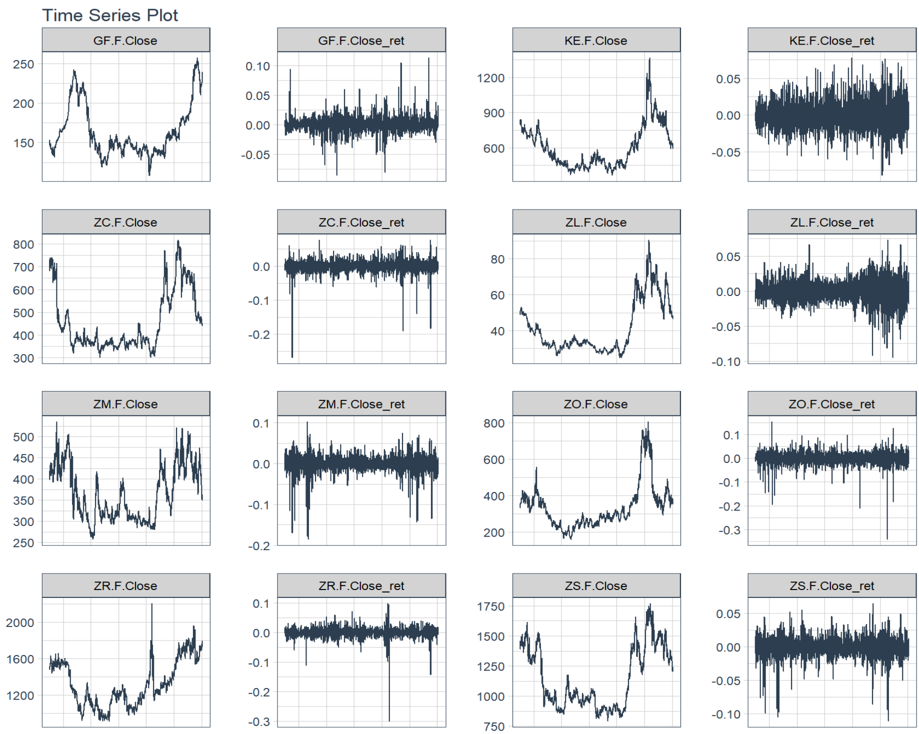

Figure 1

Closing prices and respective returns for each time series data in selected portfolio

Figure 1 displays the time series plots for the closing prices and returns of a selection of grain and livestock commodities. The commodities included are corn (ZC=F), oats (ZO=F), Kansas City wheat (KE=F), rough rice (ZR=F), feeder cattle (GF=F), soybeans (ZS=F), soybean meal (ZM=F), and soybean oil (ZL=F). Each plot is divided into two sections: the left side depicts the closing prices, while the right side shows the returns.

The price plots exhibit volatility clustering, a common characteristic in commodity markets, where periods of high volatility are followed by periods of low volatility. Notably, corn (ZC.F.Close) shows significant price spikes, which could be indicative of market responses to external factors such as weather patterns, trade policies, and global demand shifts.

When examining the returns, corn (ZC.F.Close_ret) and others displays periods of heightened activity interspersed with more stable phases, suggesting a non-constant variance in the return series—a phenomenon that may be effectively captured by our proposed GAMLSS and MSGARCH modeling approach.

The statistical analysis of the return series for each commodity in our portfolio is summarized in Table 1. This table presents the results of the Jarque-Bera test, which assesses the normality of the distribution of returns, alongside measures of skewness and kurtosis. Across the board, the p-values indicate a rejection of the null hypothesis for normal distribution, confirming that the returns are not normally distributed and exhibit characteristics of leptokurtosis.

Table 1

Distributional pattern of price returns of selected grains portfolio

Time Series	Jarque-Bera	p-value	Skewness	Kurtosis	Distribution Type
GF=F	17180.20	0.00	0.58	12.19	Leptokurtic
KE=F	226.80	0.00	0.24	1.33	Leptokurtic
ZC=F	139867.89	0.00	-2.54	34.56	Leptokurtic
ZL=F	870.51	0.00	-0.28	2.70	Leptokurtic
ZM=F	34661.20	0.00	-1.77	17.03	Leptokurtic
ZO=F	66153.75	0.00	-1.84	23.74	Leptokurtic
ZR=F	374155.67	0.00	-3.01	56.81	Leptokurtic
ZS=F	7481.51	0.00	-1.06	7.79	Leptokurtic

Following the rejection of normality in our return series, as evidenced by the Jarque-Bera test results, we proceeded to identify the most fitting distributional patterns for each commodity in the portfolio. This step is crucial as it informs the selection of appropriate priors for our Bayesian MSGARCH models, ensuring that our volatility forecasts are grounded in empirically observed behaviors of return series.

In our analysis, various distributions were fitted to the price returns of a selected portfolio of grains. Table 2 presents the best-fit distributions for the return series of each commodity, chosen based on the distributional characteristics revealed in Table 1. We employed various distribution families, including Johnson SU, Generalized t, and Skew t types, to match the observed leptokurtosis and skewness. Parameters such as location (μ), scale (σ), and shape (ν, τ) were estimated to fine-tune the distributions to the empirical data.

Table 2

Distributional pattern of price returns of selected grains portfolio

	Ticker	Family	μ	σ	ν	τ
GF=F ⇒	Feeder Cattle	Johnson SU original	0.00	-4.88	0.02	0.07
KE=F ⇒	Hard Red Winter Wheat	Exp. gen. β 2 (2nd kind)	-0.00	-4.71	-0.04	-0.27
ZC=F ⇒	Corn	Generalized t	0.00	-4.11	0.28	0.90
ZL=F ⇒	Soybean Oil	Generalized t	0.00	-4.02	0.34	0.99
ZM=F ⇒	Soybean Meal	skew t type 3	-0.00	-4.47	0.07	1.21
ZO=F ⇒	Oats	skew t type 4	0.00	-4.14	1.28	1.50
ZR=F ⇒	Rough Rice	SST	0.00	-4.20	0.05	0.22
ZS=F ⇒	Soybeans	skew t type 4	0.00	-4.60	1.31	1.70

Each distribution is described by a set of parameters that collectively define its shape and scale. The parameter μ represents the location parameter, analogous to the mean in a normal distribution, which indicates the central tendency of the returns. The σ parameter is related to the scale of the distribution, influencing the spread or variability of the returns.Each distribution is described by a set of parameters that collectively define its overall behavior and characteristics, providing a comprehensive framework for understanding the statistical properties of the price returns for each commodity.

The parameters ν and τ are shape parameters. ν influences the heaviness of the tails of the distribution, where lower values indicate heavier tails, suggesting a higher probability of extreme returns. This is particularly relevant in financial contexts where the risk of extreme market movements is a significant concern. The τ parameter affects the peakedness or kurtosis of the distribution, with higher values indicating a more pronounced peak, suggesting a higher concentration of returns close to the mean.The parameters ν and τ are shape parameters. ν influences the heaviness of the tails of the distribution, where lower values indicate more substantial tails. This feature is crucial for modeling the probability of extreme return values, which is particularly relevant in financial markets where tail risks cannot be ignored.

The specific families of distributions used – such as the Johnson SU, Exponential Generalized Beta 2, Generalized t, Skew t type 3, Skew t type 4, and SST – were chosen based on their ability to capture the unique characteristics of the return distributions for each commodity. These distributions provide a nuanced view of the underlying dynamics of commodity prices, which is essential for informed decision-making in commodity markets and financial investment strategies.

This narrative ensures that the reader is aware of where to find detailed information on the probability density functions, highlighting the importance of these details to the overall integrity of your econometric modeling. It also emphasizes the rigor of your methodological approach.

The Bayesian estimation further refines our model, capturing the nuances of volatility in the corn returns. The Markov-switching specification named gjrGARCH_std is a testament to the complexity and the heterogeneity of the regimes within the corn market. The Bayesian approach reveals a subtle interplay between the states, as evidenced by the posterior mean transition matrix, which reflects a high degree of persistence within each regime.

The estimated parameters from the posterior sample underscore the dynamics within the corn market:

Posterior mean of parameters:

$$\begin{aligned}
\alpha_{0,1} &= 0.0000, \quad (\text{SD} = 0.0000) \\
\alpha_{1,1} &= 0.0474, \quad (\text{SD} = 0.0271) \\
\beta_1 &= 0.8648, \quad (\text{SD} = 0.0830) \\
\nu_1 &= 5.5070, \quad (\text{SD} = 0.5896)
\end{aligned}$$

The transition probabilities for remaining in the same state or switching to another state are as follows:

$$\mathbf{P} = \begin{bmatrix} 0.9953 & 0.0047 \\ 0.0269 & 0.9731 \end{bmatrix}$$

With stable probabilities of:

$$\pi = \begin{bmatrix} 0.8504 & 0.1496 \end{bmatrix}$$

The acceptance rate of the MCMC sampler at 27.4% and the DIC score of -15502.9513 collectively affirm the robustness of the Bayesian estimation, providing a reliable framework for volatility forecasting and risk assessment in the corn market.

Parameter estimates indicate that the evolution of the volatility process is heterogeneous across the two regimes. Indeed, we first note that the two regimes report different unconditional volatility level by around 24,01287% and 25,97128%.

The intricacies of the corn market's volatility are captured through the regime-switching model, where the estimated volatility and regime probabilities are visually represented in the attached graph. This graphical representation provides a clear depiction of the market's dynamic nature, highlighting the transitions between periods of low and high volatility.

Figure 2 offers a compelling visualization of the volatility regimes over time, depicted by the red line indicating smoothed probabilities of the high-volatility state. The black dots represent the actual volatility percentages observed in the corn market. It is evident from the graph that the high-volatility states correspond with periods of market turbulence, which are crucial for traders and risk managers to identify and navigate.

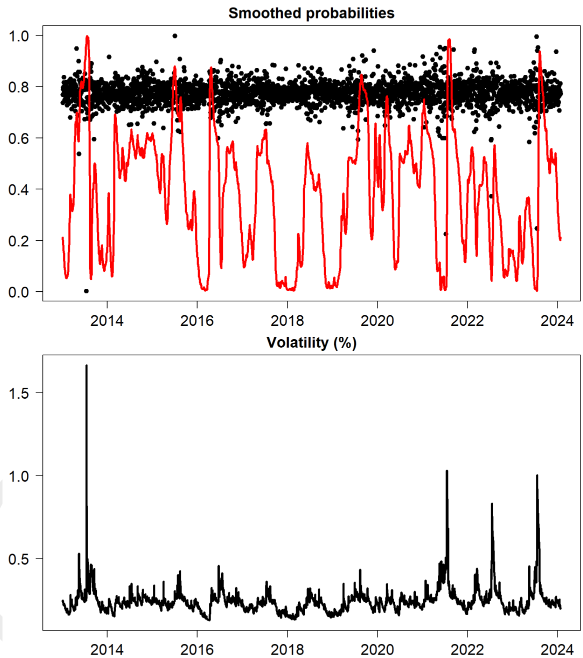

Figure 2
Estimated volatility and regime probabilities for corn

The lower panel of Figure 2 illustrates the persistence and the frequency of volatility spikes, with the *y*-axis reflecting the probability of being in a high-volatility state. The clustering of spikes suggests that volatility is not randomly distributed over time but tends to occur in bursts, followed by relatively calmer periods.

This behavior is consistent with the phenomenon of volatility clustering commonly observed in financial time series and specifically modeled by our GARCH-based approach.

Analyzing the upper panel, we notice that the smoothed probabilities (red line) align closely with the observed volatility (black dots), indicating that our model effectively captures the transition between regimes. This alignment also suggests that the model could serve as a predictive tool, providing foresight into potential shifts in market conditions.

The posterior distribution of mixture and Markov-switching models often exhibits non-elliptical shapes which lead to non-reliable estimation of the uncertainty of model parameters (see, e.g., (Ardia et al., 2009)). This invalidates the use of the Gaussian asymptotic distribution for inferential purposes in finite samples.

Our results display this characteristic as shown in Figure 3 where we plot 2,500 draws of the posterior sample for the parameters $\alpha_{1,1}$ and $\alpha_{1,2}$.

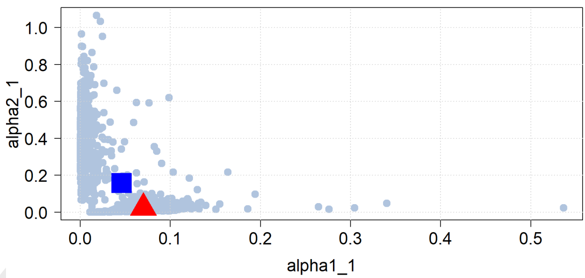

Figure 3

Scatter plot of posterior draws from the marginal distribution of $(\alpha_{1,1}, \alpha_{1,2})$**>** ***obtained with the adaptive random walk strategy. The blue square reports the posterior mean, and the red triangle reports the ML estimate.***

The blue square reports the posterior mean while the red triangle reports the ML estimate. An interesting aspect of the Bayesian estimation is that we can make distributional (probabilistic) statements on any (possibly nonlinear) function of the model parameters. This is achieved by simulation. For instance, for each draw in the posterior sample we can compute the unconditional volatility in each regime, to get its posterior distribution.

Figure 4 displays the posterior distributions of the unconditional annualized volatility in each regime.

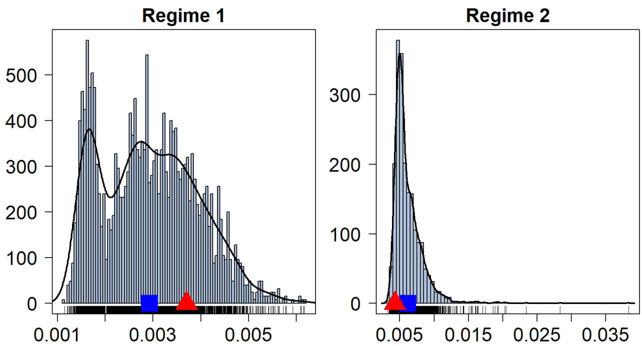

Figure 4
Histograms of the posterior distribution for the unconditional volatility in each regime.

In the low-volatility regime, the distribution is centered around 10% per annum. For the high-volatility regime, the distribution is centered around 80% per annum. The 95% confidence bands given by the Bayesian approach are [1.4%, 3.9%] and [4.8%, 11.1%], respectively. Notice that both distributions exhibit positive skewness. Hence, relying on the asymptotic normal approximation with the delta method would yield erroneous estimates of the 95% confidence band of the unconditional volatility in each regime.

Having delved into the Bayesian estimation's intricate details and its implications for understanding volatility regimes, our analysis now progresses to the Monte Carlo Markov Chain (MCMC) method. The MCMC technique stands as a cornerstone of computational Bayesian statistics, facilitating the estimation of complex models where traditional analytical approaches may falter.

The MCMC method, particularly its application through the Metropolis-Hastings algorithm, allows us to draw samples from the posterior distribution of our model's parameters, even when the distribution cannot be explicitly defined. These samples form the basis of our Bayesian inference, providing a numerical approximation to the posterior distributions that are otherwise analytically intractable.

Building upon the robust framework established through Bayesian estimation and MCMC methodologies, we turn our attention to the practical outcomes of these simulations. Specifically, we examine the results pertaining to the log-returns and conditional volatilities of corn prices, as projected by our model.

Figure5 presents the observed and out-of-sample (OOS) forecasted log-returns for corn prices. The observed log-returns, depicted in blue, illustrate the historical volatility experienced within the market. In contrast, the OOS forecasted log-returns, represented in red, offer a window into the future through the lens of our model's probabilistic predictions.

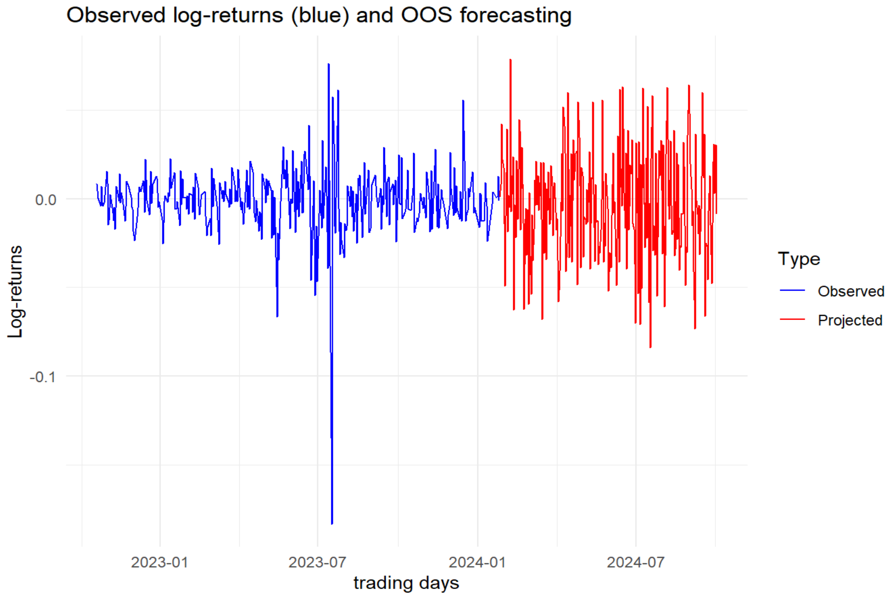

Figure 5

Observed and OOS forecasting of log-returns for corn prices

It is evident from Figure 5 that the projected log-returns capture the essence of market behavior, closely following the trends and fluctuations observed in historical data. This congruence between observed and projected values underpins the model's capability to accurately reflect the market's complexities and to serve as a reliable tool for anticipating future movements.

Moving on to the conditional volatilities, Figure 6 contrasts the observed volatilities against the forecasted volatilities, providing an assessment of how well our model captures the dynamic risk profile of the corn market. The observed conditional volatilities, shown in blue, highlight the periods of market stress, which are critical for risk management purposes.

Figure 6 juxtaposes the observed conditional volatilities against those forecasted by our model, offering a comparative view of the market's anticipated risk profile. The blue line traces the historical volatilities, reflecting the market's realized fluctuations over time. The red line, denoting projected volatilities, provides insights into future expectations as inferred by the model.

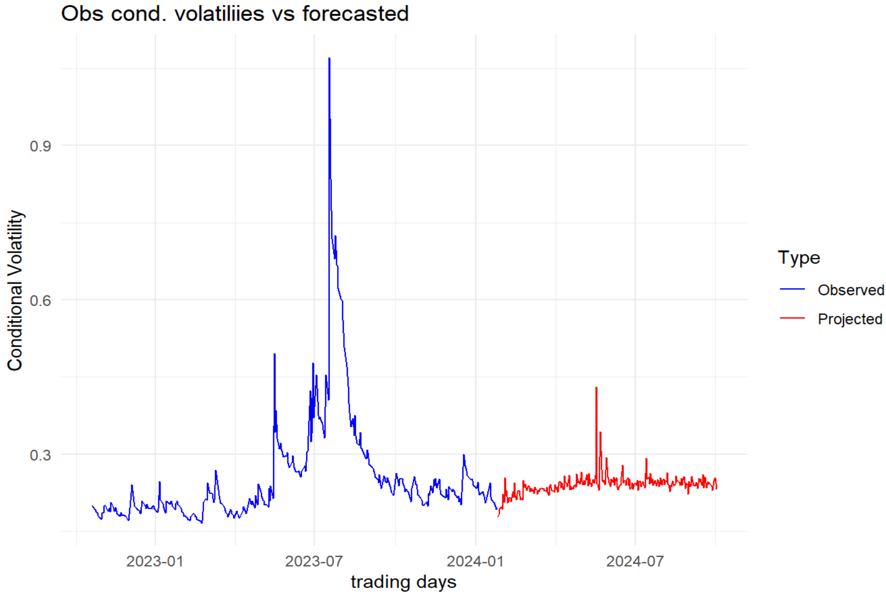

Figure 6

Observed and forecasted conditional volatilities for corn prices

The historical volatility pattern, characterized by pronounced spikes, serves as a testament to the corn market's susceptibility to sudden shifts in sentiment and fundamentals. In contrast, the forecasted volatility suggests a relatively calmer market outlook, potentially indicative of a period of stabilization or the successful implementation of market-calming strategies.

However, it is imperative to approach these projections with cautious optimism. While the forecasted reduction in volatility is encouraging, it is crucial to acknowledge the inherent uncertainty in such predictions. Market conditions can change rapidly due to a multitude of unforeseen factors, and thus, the actual path of future volatility may deviate from these forecasts.

The MCMC simulation process underpinning these forecasts has been carefully calibrated to capture the complex dynamics of the corn market. The resulting projections are not merely statistical outputs; they embody a comprehensive analysis that considers the stochastic nature of market movements.

The insights gained from Figure 4 are invaluable for risk management purposes. The observed conditional volatilities underscore the need for robust hedging mechanisms to protect against market turbulence. Meanwhile, the projected conditional volatilities can inform strategic decisions, such as the timing of entries and exits in the market, and the allocation of resources to different risk management instruments.

Ultimately, the conditional volatility forecasts derived from our Monte Carlo MCMC simulations serve as a critical input for portfolio optimization and risk assessment. By providing a probabilistic view of future volatilities, our model aids in the construction of resilient portfolios capable of withstanding the vicissitudes of the commodities market.

In conclusion, the detailed analysis of both observed and projected volatilities enriches our understanding of the corn market's behavior. It highlights the integral role of advanced econometric techniques in deciphering the complexities of financial markets, thereby enabling more informed decision-making for practitioners and stakeholders in the agricultural commodities trading space.

5. Final Considerations and Future Research Directions

This paper has unveiled a synergistic integration of Generalized Additive Models for Location, Scale, and Shape (GAMLSS) with Bayesian Markov-Switching Generalized Autoregressive Conditional Heteroskedasticity (MS-GARCH) models, aimed at refining the forecasting accuracy of commodity price returns. By applying these advanced statistical tools within the specific context of grain portfolio management, we have illustrated their collective power in offering profound insights into financial time series.

The utilization of the GAMLSS framework has enabled us to accurately model the non-standard distributional attributes characteristic of grain commodity returns. Concurrently, the Bayesian MSGARCH models have been pivotal in offering dynamic and probabilistic forecasts of returns and volatility. The empirical validation of these methodologies showcases a significant enhancement in simulation and prediction precision over traditional modeling approaches.

In the practical application of these models, particularly within the corn market, we have observed compelling market dynamics such as volatility clustering and regime shifts. These phenomena underscore the necessity for traders and risk managers to possess a nuanced understanding of these models. The Bayesian methodology, with its capacity for probabilistic forecasting, emerges as a vital asset in the financial econometrics toolkit.

As our research journey concludes, the pathway for future investigation unfurls with promising prospects. One such direction entails the exploration of Bayesian simulation models, specifically assessing the variability in asset distributions through Markov Chain Monte Carlo (MCMC) techniques. This approach offers a fascinating avenue to examine the models' adeptness at capturing and projecting the erratic peaks and troughs within return series, thereby enhancing our comprehension of market dynamics and the predictive reliability of financial models.

Further research could also extend the current modeling framework to a broader spectrum of commodities and financial instruments, probing the models' efficacy across varied markets and asset classes. This exploration would illuminate how these models fare in environments characterized by differing liquidity and volatility levels.

Advancements in computational algorithms, spurred by the evolution of machine learning and artificial intelligence, present another fertile ground for research. There exists an opportunity to craft more refined and scalable algorithms capable of managing extensive datasets, which, in turn, could yield forecasts of unparalleled accuracy. Additionally, weaving macroeconomic indicators and news sentiment into the modeling framework can provide a more comprehensive perspective on the forces propelling market dynamics. This holistic approach promises to substantially augment the predictive capabilities of financial models, offering a richer analysis of market behavior.

Finally, with the financial landscape continually reshaped by regulatory shifts and the escalating emphasis on sustainability and ethical investment practices, it becomes imperative to assess their impact on commodity markets. Future studies could delineate how financial models might adapt to these evolving paradigms, ensuring their ongoing relevance and utility.

In essence, our work not only furnishes the financial sector with pragmatic tools for planning and risk management but also propels the theoretical development of real options theory forward. We invite the academic and professional finance communities to build upon our findings, venturing into the untapped territories of financial econometrics that lie ahead.

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A. Appendix

Table A1
Summary of Key Research in Financial Time Serfies Analysis

Authors (Year)	Focus Area	Methodology	Key Contributions
(Rigby and Stasinopoulos, 2005)	GAMLSS in Financial Time Series	Statistical Modeling	Introduced GAMLSS for flexible distribution modeling and handling censored data
(Stasinopoulos and Rigby, 2007)	GAMLSS in Financial Time Series	Statistical Modeling	Demonstrated efficacy in fitting skew and kurtotic distributions
(Amédée-Manesme et al., 2013)	Monte Carlo Simulations in Real Options	Simulation	Enhanced real estate portfolio valuations and risk management
(Cotter et al., 2020)	Forecasting Commodity Prices	Time Series Analysis	Developed models for commodity price dynamics and return distributions
(Chandrasekara et al., 2016)	Forecasting Financial Indices	Time Series Analysis	Used Scaled t distribution for modeling financial indices like Oil, Gold, and Cocoa
(Ardia et al., 2019)	MSGARCH Models in Financial Time Series	Statistical Modeling	Introduced MSGARCH package in R for market volatility analysis
(Cai, 1994)	Markov Regime-Switching Models	Statistical Modeling	Analyzed the impact of regime shifts on financial time series properties
(Otranto, 2016)	Markov Regime-Switching Models	Statistical Modeling	Explored flexible Markov regime-switching models for economic time series

Distributional PDFs for table 2

According to the work from (Rigby et al., 2017)

Johnson SU original:

$$f(x;\gamma,\delta,\lambda,\xi)=\frac{\delta}{\lambda\sqrt{2\pi}\sqrt{1+\left(\frac{x-\xi}{\lambda}\right)^2}}\exp\left(-\frac{1}{2}\left[\gamma+\delta\sinh^{-1}\left(\frac{x-\xi}{\lambda}\right)\right]^2\right)$$

where

$$z=\nu+\tau\sinh^{-1}(s)=\nu+\tau\log\left[s+(s^2+1)^{1/2}\right],$$

Exponential generalized beta 2 (EGB2):

$$f_Y(y|\mu,\sigma,\nu,\tau)=e^{z\nu}\left[\sigma|\tau B(\nu,\tau)|(1+e^z)^{\nu+\tau}\right]^{-1}$$

where

$$z=\frac{y-\mu}{\sigma},$$

and $B(\nu,\tau)$ is the Beta function. The parameters μ, σ, ν, and τ represent the location, scale, and shape parameters, respectively.

Generalized-*t*

The probability density function (pdf) of the Generalized t distribution, denoted by $GT(\mu,\sigma,\nu,\tau)$, is defined by:

$$f_Y(y|\mu,\sigma,\nu,\tau)=\tau\left[\frac{2\nu^{1/\tau}\Gamma(1/\tau,\nu)}{\sigma(1+|z|^\tau/\nu)^{(\nu+1/\tau)}}\right]$$

for $-\infty<y<\infty$, where $-\infty<\mu<\infty$, $\sigma>0$, $\nu>0$ and $\tau>0$, and where

$$z=\frac{y-\mu}{\sigma},$$

and $\Gamma(\cdot)$ is the gamma function. The $GT(\mu,\sigma,\nu,\tau)$ distribution is symmetric about $y=\mu$.

Skewed *t* type 3

The probability density function (pdf) for the Skew t type 3 distribution, denoted by $ST3(\mu,\sigma,\nu,\tau)$, is defined as:

$$f_Y(y|\mu,\sigma,\nu,\tau) = \begin{cases} \frac{c}{\sigma}\left[1+\left(\frac{z^2}{\nu}\right)^{-\frac{\tau+1}{2}}\right], & \text{if } y<\mu \\ \frac{c}{\sigma}\left[1+\left(\frac{z^2}{\nu\tau}\right)^{-\frac{\tau+1}{2}}\right], & \text{if } y \geqslant \mu \end{cases}$$

where

$$z = \frac{y-\mu}{\sigma}, \quad c = 2\nu^{1/2}B\left(\frac{1}{2},\frac{\nu}{2}\right)^{-1}.$$

Skewed *t* type 4

The probability density function (pdf) of the Skew t type 4 distribution, denoted by $ST4(\mu,\sigma,\nu,\tau)$, is defined as:

$$f_Y(y|\mu,\sigma,\nu,\tau) = \begin{cases} \frac{c}{\sigma}\left[1+\left(\frac{z}{\tau}\right)^2\right]^{-\frac{\nu+1}{2}}, & \text{if } y<\mu \\ \frac{c}{\sigma}\left[1+\left(\frac{z}{\tau}\right)^2\right]^{-\frac{\tau+1}{2}}, & \text{if } y \geqslant \mu \end{cases}$$

where

$$z = \frac{y-\mu}{\sigma}, \quad c = 2\left[\nu^{1/2}B\left(\frac{1}{2},\frac{\nu}{2}\right)\right]^{-1}$$

SST (Skewed Student-*t*)

The probability density function (pdf) for the Skew *t* type 3 distribution, denoted by $ST3(\mu,\sigma,\nu,\tau)$, is defined as:

$$f_Y(y|\mu,\sigma,\nu,\tau) = \begin{cases} \frac{c}{\sigma}\left[\frac{\nu^2}{\tau}\right]^{-\frac{(\tau+1)}{2}}\left[1+\frac{z^2}{\nu^2\tau}\right]^{-\frac{(\tau+1)}{2}}, & \text{if } y<\mu \\ \frac{c}{\sigma}\left[\frac{\tau}{\nu^2}\right]^{-\frac{(\tau+1)}{2}}\left[1+\frac{z^2}{\nu^2\tau}\right]^{-\frac{(\tau+1)}{2}}, & \text{if } y \geqslant \mu \end{cases}$$

where

$$z = \frac{y-\mu}{\sigma}, \quad c = 2\left[\nu^{1/2}B\left(\frac{1}{2},\frac{\nu}{2}\right)\right]^{-1}$$