Climate Policies, Production Networks and Inequality[*]

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Abstract

In this paper we develop a multi-sector general equilibrium model to study the distributional effects of carbon taxation. The model incorporates heterogeneous households, sectors, and factors interconnected by input-output linkages. The network propagates fuel price shocks through the production chains, triggering widespread intermediate and final consumption re-optimization due to substitution and income effects. We analytically characterize changes in welfare according to variations in: i) consumption costs, given relative price changes according to direct and indirect reliance on fuels through the production network; and ii) nominal income, steaming from factor demand shifts resulting from resource reallocation in the economy. Using matched Brazilian microdata, we find that carbon taxation has an aggregate regressive effect, with significant heterogeneous impacts across households.

Keywords: carbon tax, production networks, distributive policy, household heterogeneity

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1 Introduction

In response to the escalating environmental crisis, the adoption of carbon pricing schemes has emerged as a global trend in recent years. According to the World Bank, as of 2023, 24% of the greenhouse gases (GHG) emitted worldwide were subject to some form of pricing (World Bank 2024).[1] Theoretically, by setting a price on the carbon content of goods, governments aim to align private and social marginal costs, thereby providing efficient incentives to reduce emissions in the spirit of a Pigouvian tax.

However, in practice, these large-scale fiscal policies lead to widespread resource reallocation within the economy, inevitably creating winners and losers. Consequently, such policies may disproportionately impact economically vulnerable demographic groups, widening existing disparities in income and wealth. Alternatively, they may affect groups with sufficient political leverage to undermine support for the policy's adoption. It is, therefore, of fundamental importance to understand the heterogeneous implications of carbon tax reforms on individuals' welfare and income inequality.

In this paper, we develop a detailed and comprehensible structural model to examine the distributional effects of carbon pricing reforms. At the core of our model lies the heterogeneity of individuals regarding their consumption baskets, the production sector to which they supply their labor to, and the policy's indirect impacts from the propagation of tariffs through the supply chains and general equilibrium effects. Our contribution resides in simultaneously accounting for all of these key components when estimating the impacts of climate mitigation policies.

In our economy, firms produce a uniform sectoral good by combining fuels and non-fuel inputs produced by other firms and various types of labor characterized by segmentation in the labor market, such as different schooling levels and occupations. Each household inelastically supplies labor of a single type, which is fully mobile across sectors.[2] As mentioned earlier, consumers have heterogeneous consumption baskets. Additionally, consumers (both final and intermediate) have heterogeneous abilities to substitute across inputs so that the ability to substitute the consumption of a combustion fuel for more labor tends to be significantly lower than substituting it for another

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[1] Of the 24%, 6% are covered by carbon taxes and 19% by Emissions trading system (ETS).

[2] We also provide results for the case where there is limited worker mobility.

fuel, even if it is a cleaner one. A similar argument intuitively holds for the consumption of non-fuel inputs and labor, and so on.

Upon the implementation of a carbon pricing scheme, the consumption of polluting inputs — namely fuels in our baseline estimation — become relatively more expensive.[3] Competitive producers that rely on such inputs then pass on the marginal cost increases to their prices, transmitting the tariff shock *downstream* to their customers. These customers, in turn, will pass on their price increases in their output prices, thereby affecting the intermediate consumers they supply. The extent of the impact on downstream consumers' marginal costs depends on their exposure to fuels, both direct and indirect through the network, as measured by their expenditure shares.

As a response to the resulting changes in relative prices in the economy, consumers (final and intermediate) optimally reshape their consumption patterns according to their ability to substitute away from the affected good. If a good is consumed in a complementary(substitutable) way to other items in the consumption basket, the expenditure share allocated to it will increase(decrease) if it gets relatively more expensive. That constitutes the *upstream* propagation of carbon taxation shocks.

The model tracks down the complex reallocation of resources happening throughout the economy as a result of the joint interaction of the downstream and upstream propagation. Households are impacted by the reform via two distinct channels (Goulder et al. 2019). First, as the relative prices of goods shift, final consumers' purchasing power is affected according to the composition of their consumption baskets. This is what we call the price effect channel. Second, due to the reallocation of resources in the economy, the relative demand for factors changes, which will have heterogeneous impacts on households depending on their factor endowments (e.g. workers in more polluting sectors are likely to lose labor income). Moreover, introducing new taxes inherently creates additional tax revenues that must be split across households by the government, representing an opportunity for distributional policies that can help mitigate inequality (Metcalf 1999). This second channel is the income effect.

Additionally, we account for the Engel effect acting upon the demand for polluting inputs by embedding non-homotheticity into household preferences.[4] Notably, a substantial body of literature

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[3] We also provide results for the case where there are emissions from the production process other than from fuel combustion, such as in the metal and oil extraction industries.

[4] Our setup closely follows the one introduced by Comin et al. (2021) and applied to the production networks

has shown that lower-income households tend to allocate a larger proportion of their total expenditures to polluting goods, which are subject to taxation (e.g. Poterba 1991; Metcalf 2019; Chancel et al. 2023; and Känzig 2023).[5] In this setup, the income and price effects interact since income variations will affect the degree of exposure of the households to the relative price changes.

The welfare impact on households is the net effect between the price effects and income effects. In section 4 we analytically decompose the income and price channels. We show that the overall impact on the consumption cost of an individual hinges on its reliance, both directly and indirectly through the supply chain, on dirtier fuels that are more targeted by the tax. That is, the price channel effect is determined not only by an individual's exposure to the fuels, but instead it depends on the reliance of all the producers upstream of it. On the other hand, the effect on household income is the aggregation of all the network-adjusted streams of resources channeled towards the demand of the factors owned by it. This encodes the degree to which consumers (final and intermediate) switch their demand, measured by expenditure shares, towards inputs that employ the factors in their production process and the degree to which consumers particularly reliant on these inputs increase their overall demand level.

To quantitatively assess the implications of our framework, we parameterize our model using microdata of the Brazilian economy. We selected Brazil as the laboratory for our analysis because the country has very detailed data on the supply of labor according to workers' characteristics. Moreover, albeit being an economy deeply reliant on some of the most polluting domestic sectors (see section 2), and having committed to a significant reduction in its annual emissions of 50% from 2005 levels by 2030 (Brazil 2022), Brazil is yet to implement any form of carbon pricing policy .[6]

In subsection 2.2 we conduct an in-depth analysis of the Brazilian economy to investigate how households' expositions to both price and income effects vary according to income bins. Using the national household consumption survey, we document that poorer households allocate a significantly bigger share of consumption expenditure to more energy-intensive products, and wealthier households allocate larger shares of consumption to services.

Subsequently, we examine labor income exposure using administrative data on all formal Brazil-

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literature by Baqaee and Burstein (2023). It is further explained in section 3.

[5] We show in Table 1 in subsection 2.2 that this is also the case in the Brazilian economy.

[6] subsection 2.2 provides a detailed overview of the Brazilian economy and environmental context, focusing in households consumption patterns, income sources distribution, and historical emissions outcomes.

ian employer-employee contracts. Using schooling attainment as a proxy for per capita income, we show that more educated workers make up the largest share of the energy and extractive industries' payrolls. Conversely, individuals with lower educational attainment comprise the majority of the farming sector, while those with moderate schooling levels are prevalent in manufacturing and transportation industries.

Our quantitative analysis consists of increasing the consumption tax rates of fuels based on their carbon intensity,[7] with the additional tax revenues being rebated back to final consumers. The results support the intuition that the price effect has a regressive impact on households, disproportionately affecting those with lower levels of schooling. Conversely, the income effect works in the opposite direction, significantly impacting the factor incomes of better-educated individuals more than those with average schooling. The net impact on welfare is negative for all education levels, leading to a moderate decline in real GDP and aggregate welfare.

Moreover, redistributing the additional tax revenue equally across individuals significantly reduces income inequality and benefits less-educated workers. However, this approach imposes a greater overall economic cost to achieve the same emissions reduction, as households receiving larger rebates tend to spend more on polluting goods, requiring a higher carbon price.

Related Literature and Contribution

This paper connects to (and brings together) multiples strands of literature. First, there is a large literature estimating the economic and environmental consequences of climate policy while taking into consideration general equilibrium effects,[8] in particular the distributive effects of carbon pricing (eg. Ohlendorf et al. 2021; Grottera et al. 2017; Magalhaes 2013; Benmir and Roman (2022)). In a detailed computable general equilibrium (CGE) model tailored for the US economy, Goulder et al. (2019) finds that while the price effects of carbon taxes are regressive, the progressive nature of the income effects often entirely counterbalances these regressive impacts. They also show that the way the government redistributes the additional tax revenue plays an important part in determining the progressivity of the policy. Using high-frequency data,Känzig (2023) empirically demonstrates that the European ETS features short-term regressive implications from both price and income effects.

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[7] We also have exercises where sources of emissions other than fuel combustion are taxed.

[8] Hassler et al. (2018) is a review of this literature.

His results indicate that the regressivity from the income channels stems from the concentration of low-income households in sectors like retail, which are more susceptible to short-term aggregate economic fluctuations. Building on this insight, he develops an environmental HANK model to study the role of redistributing carbon revenues, finding that the regressive distributional nature of carbon pricing can be mitigated by progressively distributing carbon revenues.

Our framework differentiates from this literature because we explicitly address how the input-output network linking the producers can amplify the effects of carbon pricing reforms by propagating the shocks throughout the production chains. Methodologically, this approach situates our paper within the production networks literature.[9] Baqaee and Farhi (2019) was the first paper to develop a production networks model with multiple heterogeneous agents.[10] They introduce a flexible general equilibrium framework for measuring the aggregate and sectorial impacts of markup-like distortions that inefficiently shift resource allocation. Their setup allows arbitrary elasticities of substitution, returns to scale, factor mobility, and input-output network linkages and is able to disentangle the impacts in technical and allocative efficiency.

Using a similar framework to ours, but modeling an economy with multiple countries (Baqaee and Farhi 2024), Bachmann et al. (2022) quantitatively evaluate the impacts on Germany's economy of the cut on Russian gas supply resulting from the Ukraine War.[11] Baqaee and Burstein (2023) consider a production networks model with non-homothetic preferences, as ours, but with a representative consumer. They show that when preferences are non-homothetic, changes to real consumption and welfare, defined according to the equivalent variation, seize to coincide. Intuitively, as income effects affect the tastes of consumers, the preference relations are modified. They thereby propose a metric that uses ex-post expenditure shares, instead of ex-ante shares.

Carbon pricing policies have also been an object of previous research in the production networks literature. King et al. (2019) model an efficient Cobb-Douglas economy with representative final consumer and show that the optimal policy, in terms of emissions reduction, targets sectors according to their position in the network, rather than their total emissions. Devulder and Lisack (2020) model an open efficient economy with CES technologies and find that when France is the only

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[9] Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2023) are reviews of this literature.

[10] Rojas-Bernal 2023 builds a similar framework embedding I-O linkages and household heterogeneity.

[11] Contrary to the widespread public opinion at the time, which anticipated a substantial negative impact on Germany's GDP, their findings predicted only a modest impact – a projection that was subsequently corroborated by official data.

country adopting carbon taxes, the impact on its economy is considerably larger than if other countries also adopts climate mitigation policies. None of these papers, however, takes into account household heterogeneity. Closer to our paper, Cavalcanti et al. (2021) also investigate aggregate and distributional effects of taxing carbon in a model with worker heterogeneity, but their framework disregards the price effect channel, as there is no final consumption baskets differentiation.

Roadmap: This article is organized in five sections in addition to this introduction. In section 2, we delve into the data regarding the position of polluting sectors in the production networks of a selected group of countries. Then, section 3 lays down our setup, explaining the environment and some key notation. section 4 shows the main analytical results of this paper, while in section 5 we conduct quantitative exercises do assess the economic impacts of carbon taxes, both at the aggregate and distributional level, while section 6 concludes.

2 Empirical Motivation

2.1 GHGs Emissions and Centrality in the Production Chains

A central feature of our analysis is the propagation of carbon taxation throughout the production chains. Once a polluting product is taxed, the marginal cost for its downstream customers increases. This is a first-order effect, but as the affected buyers pass on the increase in marginal cost to their prices, and so forth, there are indirect cascading effects of higher orders. The impact on a buyer is greater the larger the expenditure share of the input that becomes more expensive. Consequently, the initial price shock's transmission potential to other sectors, and its ability to generate aggregate impacts on the economy, hinge on the relevance of the products initially targeted by the policy as inputs to the remaining sectors of the economy, both directly and indirectly.

To empirically evaluate the interplay between sectoral emissions intensity and relevance as input suppliers to the economy, we consider the definition of weighted outdegree (Acemoglu et al. 2012). Let x_{ij} be the intermediate consumption of sector j's product by sector i, and p_j be the price of product j, so that $\omega_{ij} \equiv \frac{p_j x_{ij}}{\sum_k x_{ik} p_k}$ is the share of product j consumption in sector i's expenditure. Then the first-order outdegree of sector j is the share of sector j's output in the input supply of the entire economy

$$\delta_j^1 \equiv \sum_i \omega_{ij} \tag{1}$$

One can then subsequently define the outdegrees of n-orders as the weighted sum of the outdegrees of a smaller order, with weights given by the corresponding input shares

$$\delta_j^n \equiv \sum_i \omega_{ij} \delta_i^{n-1} \tag{2}$$

Using input-output (Timmer et al. 2015) and environmental accounts data (Corsatea et al. 2019) for a sample spanning 56 sectors in 40 countries in 2014, we compute the outdegrees of each sector in each country and their respective shares in the national CO_2 emissions. Then, we compute the empirical cumulative distribution function (CDF) of each outdegree for each country, and depict the results for the first (1a) and tenth outdegrees (1b) in Figure 1. The farther to the right on the graph, the higher the percentage of national CO_2e emissions attributed to the sector, and the higher up, the lower the probability of any other sector in the country having a higher outdegree than the sector in question. The vast majority of sectors are responsible for a negligible amount of emissions, which can be seen by the cluster of points to the left of both charts. However, it is noteworthy that the observations to the right of the 20% emissions mark are almost all at least in the last quartile for first-outdegrees, and this pattern remains stable up to the tenth outdegree.[12] This indicates that the most polluting sectors, and therefore those most vulnerable to a carbon tax, are often among the most important suppliers in the economy.[13]

Specifically analyzing the Brazilian economy, Figure 2 shows the correlation between the value of the first and tenth-order outdegrees and their respective empirical CDFs. The size of the dots indicates the percentage of national emissions for which each sector is responsible. In Figure 2a, we observe that most of the larger points are concentrated at the top of the graph. Notably, among the four largest emission generators in the Brazilian economy, which together account for more than 55% of all annual emissions, three are in the last quintile of the first-order outdegree distribution,

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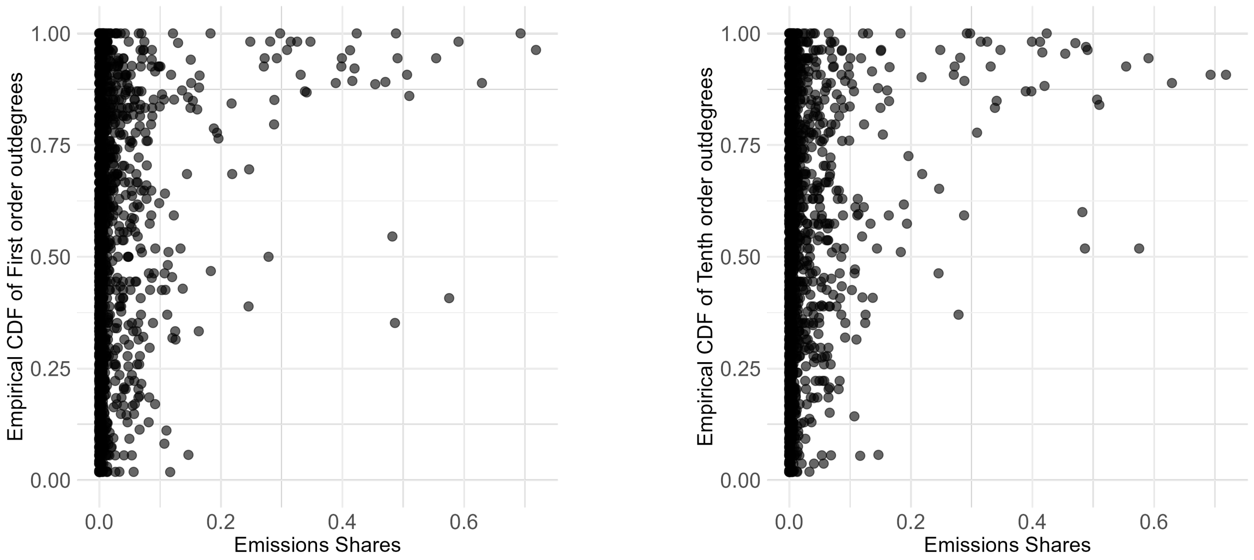
[12] An analogous pattern is observed between the first and tenth outdegrees.

[13] We demonstrate in Figure 1 within Appendix E the segmentation between two groups of countries clustered according to per capita GDP levels, showing that this pattern is consistent across economies at different stages of development.

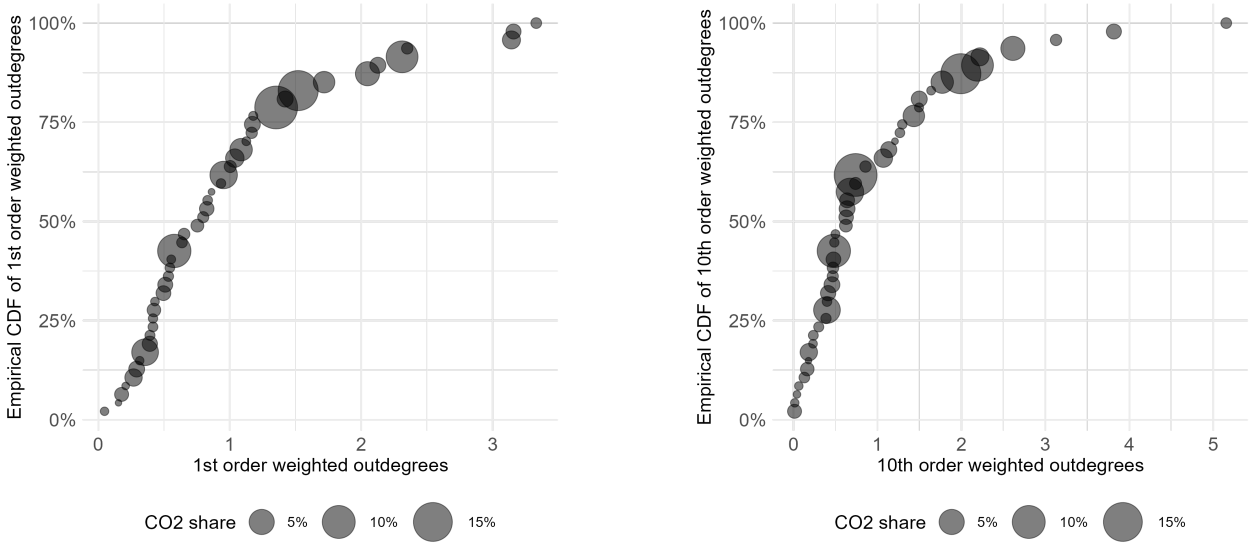
Figure 1: Largest emitters and downstream propagation potential



(a) First-order weighted outdegrees (b) Tenth-order weighted outdegrees

meaning they are among the top 20% most important suppliers to the economy to a first degree.[14] In Figure 2b, a similar pattern is noted.

Figure 2: Empirical cumulative distribution function of the first to fourth-order outdegrees



(a) First-order weighted outdegrees (b) Tenth-order weighted outdegrees

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[14] The only significant emitter outside the fifth quintile is air transportation.

2.2 Brazilian Context

In this section, we present a comprehensive analysis of Brazil's CO_2e emissions, examining both the patterns across broad emission sources and at the sectoral level. For this purpose, we utilize emissions data from the System for Estimating Greenhouse Gas Emissions, henceforth referred to as SEEG (De Azevedo et al. 2018).[15] SEEG compiles national annual GHG emissions data following the IPCC methodology across various aggregations and sources since 1990.

To further explore household heterogeneity in exposure to environmental policy, we draw on data from two official datasets. The first is the Household Budget Survey (POF), a cross-sectional household survey conducted by the Brazilian Official Bureau of Statistics (IBGE),[16] which provides detailed information on expenditure patterns. The second is the *Relação Anual de Informações Sociais* (RAIS), a matched employer-employee administrative dataset covering all formal workers in Brazil, containing comprehensive sources of labor income.

2.2.1 Emissions in Brazil

Brazil emitted a total of 1.5 gigatonnes of net CO_2e in 2019, an amount 36% higher than the limit pledged in the country's Nationally Determined Contribution for 2030. Illustrated in the left chart of Figure 3, the breakdown of emissions by aggregate sources reveals that over half of the total emissions come from livestock, primarily due to enteric fermentation, and Land Use Change & Forestry (LUCF). The latter occurs when the land cover is altered to a type of land use associated with lower carbon stock per area, particularly in the Amazon territory. These sources, including agriculture, waste, and others, have historically been excluded from carbon pricing schemes (World Bank 2024) and are therefore referred to as ”nontaxable”.

Particularly important to our analysis are the ”taxable” emissions sources that are further broken down in the right-hand chart of Figure 3. Taxable sources are comprised of fuel combustion, industrial processes - released during the manufacturing and chemical transformation of materials - and fugitive emissions - unintended gas leaks from the production, processing, transmission, storage, and use of fuels. Summing across these three sources, transportation emerges as the largest emitter

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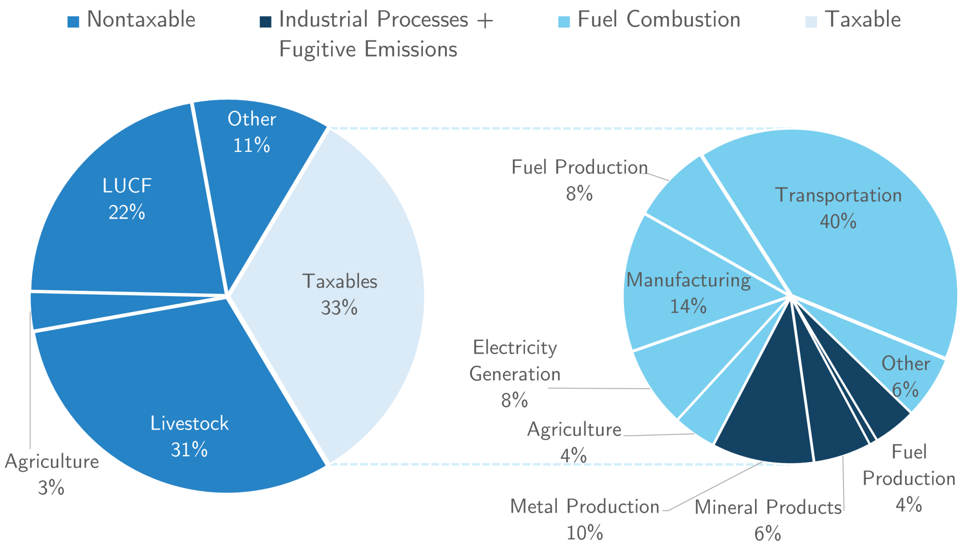
[15] SEEG is an initiative spearheaded by the *Observatório do Clima*, a network of civil society organizations in Brazil.

[16] POF's latest release covers the 2017–2018 period, including a sample of 52,917 households over 12 months across the entire national territory.

at 40%, followed by manufacturing, which accounts for 14%. Fuel production contributes 12%, while electricity generation and agriculture are responsible for 8% and 4% of the emissions, respectively.

Figure 3: Brazilian Net CO_2e Emissions Shares in 2019



Notes: "Others" is comprised of waste, product use and land management. "Nontaxable" emissions sources are those generally not included in carbon pricing schemes, while "taxable" sources are usually included.
Source: SEEG (De Azevedo et al. 2018).

2.2.2 The Income-elasticity of Energy-intensive Products

Table 1 presents the average household expenditure shares for each pre-tax per capita income decile. The data is from POF, but we reconciled the sectoral classification with the one in the Brazilian national accounts, and then further aggregated the 124 products into 9 for better visualization.[17] Consistent with literature documentation for other economies, as households' overall consumption increases, a lesser amount of expenditure is allocated to goods with a larger carbon footprint (in this case energy, transports, farming and food), and a higher share is allocated to cleaner products, specially services (Poterba 1991; Chancel 2022; Känzig 2023).[18] This indicates that poorer households are more exposed to price effects triggered by carbon pricing policies.

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[17] See section 5 for details of this process.

[18] In Table 9 in Appendix E we create the same table strictly from POF data, without the reconciliation with the national accounts. The exact same pattern is observed, although other items are considered such as housing, taxes and asset variation.

Table 1: Expenditure Shares by Income Deciles in Brazil

Decile	Farming	Energy	Foods Beverage	Other Industries	Chemical Industry	Mineral Industries	Transports	Services
1st	5.4%	12.6%	18.8%	19.1%	11.5%	1.4%	3.4%	27.7%
2nd	4.2%	12.5%	16.0%	19.2%	11.1%	1.2%	3.6%	32.2%
3rd	3.7%	12.4%	14.7%	19.3%	11.6%	1.6%	3.6%	32.9%
4th	3.5%	12.2%	14.0%	19.9%	10.9%	1.4%	3.6%	34.5%
5th	3.3%	12.7%	13.2%	19.3%	11.0%	1.4%	3.5%	35.6%
6th	3.0%	11.9%	12.0%	18.9%	10.3%	1.5%	3.2%	39.2%
7th	2.9%	12.1%	11.8%	20.1%	9.9%	1.5%	3.2%	38.4%
8th	2.6%	11.5%	10.6%	20.6%	9.2%	1.7%	2.6%	41.3%
9th	2.3%	11.2%	9.5%	20.4%	8.0%	1.4%	2.2%	45.0%
10th	1.7%	8.7%	6.7%	21.1%	6.0%	1.4%	2.6%	51.9%
Average	2.7%	11.0%	10.8%	20.2%	8.8%	1.4%	2.9%	42.1%

The table breaks down expenditure shares for each income per capita decile (columns add up to 100%. The "Average" row depicts the weighted average considering the entire sample. "Extractive Activities" was omitted because the final consumption expenditure shares were null. Source: POF data reconciled with the Brazilian national accounts sectoral classification and aggregated by the authors.

2.3 Sources of Labor Income Across Income Bins

To grasp the influence of carbon pricing on workers from an income perspective, it's key to pinpoint the sectors they predominantly contribute labor to. According to data from RAIS, presented in Table 2, the distribution of wages across different levels of education reveals a significant trend: the service sector dominates as the primary labor source across all educational tiers. However, this dominance is especially pronounced among the more educated individuals. Specifically, for those with education beyond secondary school, services account for more than 86% of their labor income, contrasting sharply with less than 42% for those with only primary schooling. For the latter group, farming stands out as a particularly important source of earnings. On the other hand, those with a middle-level education, including both lower and upper secondary schooling, show a slight preference towards the manufacturing and transportation sectors.

3 Environment

In this section we characterize our environment and lay down some key definitions that are necessary for our main theoretical results in the next section. The setup is a static general equilib-

Table 2: Sectoral Wage Mass Distribution by Schooling Level

	Primary	Lower Secondary	Upper Secondary	Post Secondary
Farming	12.2%	4.2%	1.3%	0.3%
Energy	2.6%	1.4%	1.3%	1.9%
Foods Beverage	7.1%	4.3%	2.8%	0.9%
Extractive Activities	0.2%	0.2%	0.8%	1.3%
Other Industries	26.4%	22.6%	15.3%	5.4%
Chemical Industry	1.2%	1.7%	2.1%	1.5%
Mineral Industry	3.1%	3.4%	2.7%	0.8%
Transports	5.6%	6.3%	4.4%	1.4%
Services	41.6%	55.9%	69.3%	86.4%

Note: the table displays the wage mass shares of workers within each schooling level in each sector (the columns sum up to 100%).
Source: RAIS.

rium model comprised of multiple competitive firms and heterogeneous workers interconnected by input output linkages. Specifically, workers differ in their consumption preferences, labor endowments and consumer tax rates that they face according to the type they are exogenously assigned to $h \in \mathcal{H} = \{1, \ldots, H\}$. Labor types should be interpreted according to the collection of workers characteristics, such as schooling, occupation, location, and other demographic aspects, that cannot easily be changed in the short run, and therefore generate labor market segmentation. In our baseline estimation, labor types are defined according to the pair of schooling level and occupation of a worker.

Workers are fully mobile across firms, thereby receiving the same type-specific wage, but cannot change their type, defined according to their preferences and to the labor type they are endowed with. Since workers within the same type have the same characteristics, but may differ based on the firm/sector to which they supply their labor to, we treat each type as a representative consumer that supplies a share of its labor factor endowment to which sector. Similarly, firms in different sectors employ distinct mixes of intermediate inputs, labor factors and fuel inputs and also face heterogeneous consumer tax rates, so that we consider sectors as representative firms.

A subset of the sectors are fuels producers, such that $\mathcal{N}$ can be partitioned into general materials and fuels sectors:

$$\mathcal{N} = E \cup \not{E}$$

Fuels and materials ultimately differ only because of the consumption of fuels generates GHGs emissions, with no economic implication at the initial steady state. The only role played by emissions in this economy is to parameterize the intensity of the carbon pricing policy, as will become clear at the end of this section.

We start by describing the supply and demand sides of the economy, then we define the equilibrium and finish by outlining some standard definitions in the networks literature.

3.1 Final Consumers

Any worker of type $h \in \mathcal{H}$ faces the same consumer-product-specific tax rates[19], receives the same tax revenue rebate from the government and spends all available income on consumption, thereby being subject to the budget constraint

$$\underbrace{\sum_{i \in \mathcal{N}} p_i \left(1 + \tau_{h,i}\right) c_{h,i}}_{consumption} = \underbrace{\sum_{i \in \mathcal{N}} l_{i,h} w_h + \mathcal{T}_h}_{income}, \tag{3}$$

where $c_{h,i}$ is the final consumption of the good i by h, $p_{h,i}$ is the price of product i, $\tau_{h,i}$ is the tax rate on top of the product price, $l_{i,h}$ is the supply of labor of type h to a firm of sector i, w_h is the type-specif wage, and $\mathcal{T}_h$ is the tax rebates revenue. To facilitate the notation, from now on we will denote the product unitary cost as $p_{h,i} \equiv p_i \left(1 + \tau_{h,i}\right)$.

To account for the two documented facts that consumers: can substitute within fuels more easily than across fuels and other products; and change their consumption pattern as their overall level of expenditure alternates (see subsection 2.2), we assume that the utility function that each final consumer maximizes has a nested CES formulation embedding two layers of nests and non-

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[19] Assuming tax rates specific to consumers and sellers is convenient because it allows us to implement more flexible carbon tax designs, and to study economies where distortive taxes on consumption such as these still exist, such as the Brazilian (Delalibera et al. 2024).

homothetic preferences

$$C_h = \left(\left(\sum_{j \in \not{E}} \left[\bar{\omega}_{h,j} C_h^{\epsilon_j(\sigma-1)} \right]^{\frac{1}{\sigma}} c_{h,j}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\pi-1}{\pi}} + \left(\left(\sum_{j \in E} \left[\bar{\omega}_{h,j} C_h^{\epsilon_j(\iota-1)} \right]^{\frac{1}{\iota}} c_{h,j}^{\frac{\iota-1}{\iota}} \right)^{\frac{\iota}{\iota-1}} \right)^{\frac{\pi-1}{\pi}}, \tag{4}$$

where C_h is h's real consumption level and $\bar{\omega}_{h,j}$ are fixed taste parameters.

The parameter π denotes the elasticity of substitution across non-fuels and fuel products, while σ and ι determine the elasticities within the two categories of goods, respectively. To get a better grasp of this, consider the Allen-Uzawa elasticity of substitution of a consumer h between a pair of products (i, j): [20]

$$\theta_{i,j}^h \equiv \frac{1}{\tilde{\omega}_{h,j}} \frac{d \log c_{h,i}}{d \log p_{h,j}} = \begin{cases} \pi - \frac{\sigma \times \mathbb{1}_{\{i=j\}}}{\tilde{\omega}_{h,j}} + \frac{(\sigma-\pi)}{\left(\sum_{n \in \not{E}} \tilde{\omega}_{h,n}\right)}, & \forall i, j \in \not{E} \\ \pi - \frac{\iota \times \mathbb{1}_{\{i=j\}}}{\tilde{\omega}_{h,j}} + \frac{(\iota-\pi)}{\left(\sum_{n \in E} \tilde{\omega}_{h,n}\right)}, & \forall i, j \in E \end{cases} \tag{5}$$

where $\mathbb{1}$ is the index function and $\tilde{\omega}_{h,j}$ is the share of j in h's total expenditure. If i and j are a fuel and a non-fuel products, $\theta_{i,j}^h = \pi$. Otherwise, assuming inputs within the same classification are more easily substitutable (i.e. $\pi < \sigma, \iota$), will translate into elasticities of substitution within bundles that are larger than the cross bundle one: $\theta_{i,j}^h > \pi$. Moreover, this formulation is flexible enough so that we can incoporporate the empirical finding that fuels are, in general, substitutes between then ($\iota > 1$) (D. Stern 2012), while inputs in general are complements ($\sigma < 1$) (Atalay 2017). Finally, if all the elasticities are the same, this formulation collapses into the CES with a single nest.

The terms inside the brackets capture the non-homotheticity of preferences, as introduced by Comin et al. (2021)[21]. The utility elasticity of each sectoral product ($\epsilon_j < 1$) governs how much households change expenditure share allocations towards/away j according the variation in their real consumption. To see that, notice the income elasticity equations:

$$\psi_{h,j} \equiv \left\{ \begin{array}{ll} 1 + \frac{(1-\sigma)(\bar{\epsilon}_h^{\not{E}} - \epsilon_j) + (1-\pi)(\bar{\epsilon}_h - \bar{\epsilon}_h^{\not{E}})}{1-\bar{\epsilon}_h}, & \text{if} \quad j \in \not{E} \\ 1 + \frac{(1-\iota)(\bar{\epsilon}_h^{E} - \epsilon_j) + (1-\pi)(\bar{\epsilon}_h - \bar{\epsilon}_h^{E})}{1-\bar{\epsilon}_h}, & \text{if} \quad j \in E \end{array} \right\}, \tag{6}$$

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[20] See Appendix B for the demonstrations.

[21] Similar to Borusyak and Jaravel (2021), this formulation corresponds to a version of equation (1) of Comin et al. (2021) with $g_j(U) = U^{1-\epsilon_j}$

where $\bar{\epsilon}_h \equiv \sum_{i \in \mathcal{N}} \bar{\omega}_{h,i} \epsilon_j$ are the weighted arithmetic average income elasticities, and $\bar{\epsilon}_h^{\not{E}}$ and $\bar{\epsilon}_h^{E}$ are the weighted averages within fuels and non-fuel inputs. When real consumption increases, so does the expenditure share of j if $\psi_{h,j} > 1$.

According to Equation 6, for a pair (i, j) of a fuel and a non-fuel, respectively, j will in general be more income-elastic as long as the non-fuels products are, on average, sufficiently more income-elastic

$$\psi_{h,i} - \psi_{h,j} = \underbrace{\frac{1}{1 - \bar{\epsilon}_h}}_{>0} \left[\underbrace{(\sigma - \pi)}_{>0} \bar{\epsilon}_h^{E} - \underbrace{(\iota - \pi)}_{>0} \bar{\epsilon}_h^{\not{E}} \underbrace{- (\iota - 1) \epsilon_i - (1 - \sigma) \epsilon_j}_{<0} \right]$$

Equation 6 also implies that, for a pair (i, j) of fuels, the one with higher ϵ is always more income-elastic $\psi_{h,j} - \psi_{h,i} = \frac{(\iota - 1)(\epsilon_j - \epsilon_i)}{1 - \bar{\epsilon}_h}$, and the opposite holds for non-fuels. Moreover, if $\epsilon_j = 0, \forall j \in \mathcal{N}$ we collapse back to the homothetic case where $\psi_{h,j} = 0$.

3.2 Producers

On the production side, each firm in a sector $i \in \mathcal{N} = \{1, \ldots, N\}$ produces an undifferentiated product by combining factors, clean inputs and combustion fuels according to the same constant returns to scale technology, thereby allowing us to consider a representative firm in each sector, indexed by i, that employs the production function

$$y_i = \left(\bar{\omega}_{i,E} \mathbf{X}_E^{\frac{\pi - 1}{\pi}} + \bar{\omega}_{i,\not{E} \cup \mathcal{F}} \mathbf{X}_{\not{E} \cup \mathcal{F}}^{\frac{\pi - 1}{\pi}} \right)^{\frac{\pi}{\pi - 1}}, \tag{7}$$

where y_i is i's output and π is again the elasticity of substitution across fuels and non-fuel products and $\mathbf{X}_E$ is defined analogously to an homothetic version of the utility function (Equation 4)

$$\mathbf{X}_E \equiv \left(\sum_{j \in E} \bar{\omega}_{i,j}^{\frac{1}{\iota}} x_{i,j}^{\frac{\iota - 1}{\iota}} \right)^{\frac{\iota}{\iota - 1}}$$

and the non-product bundle aggregates materials that are not fuels and factors

$$\mathbf{X}_{\not{E} \cup \mathcal{F}} = \left(\bar{\omega}_{i,\mathcal{F}} \mathbf{X}_{\mathcal{F}}^{\frac{\phi - 1}{\phi}} + \bar{\omega}_{i,\not{E}} \mathbf{X}_{\not{E}}^{\frac{\phi - 1}{\phi}} \right)^{\frac{\phi}{\phi - 1}}$$

$$\mathbf{X}_{\not{E}} \equiv \left(\sum_{j \in \not{E}} \bar{\omega}_{i,j}^{\frac{1}{\xi}} x_{i,j}^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}} \qquad \mathbf{X}_{\mathcal{F}} \equiv \left(\sum_{f \in \mathcal{F}} \bar{\omega}_{i,f}^{\frac{1}{\gamma}} l_{i,f}^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where $l_{i,f}$ and $x_{i,j}$ are the demands of producer i for the labor type f and input k.

Each representative producer i operates competitively, minimizing costs and taking prices and consumption tax rates as given

$$p_i = \min_{\{l_{i,f} x_{i,n}, x_{i,k}\}} \left(\sum_{f \in \mathcal{F}} l_{i,f} w_f + \sum_{n \in E} x_{i,n} p_{i,n} + \sum_{k \in \not{E}} x_{i,k} p_{i,k} \right), \tag{8}$$

subject to the production function (Equation 7), where $p_{i,k} \equiv p_k(1+\tau_{i,k})$ and $p_{i,n} \equiv p_n(1+\tau_{i,n})$ are the unitary costs of k, n and f, and $\tau_{i,j}$ is a tariff specific to consumer i and seller j.

This production formulation allows us to account for the heterogeneous substitutabilities within and across bundles, similar to the argument made by the Allen-Uzawa equations for the households (Equation 5).

3.3 Equilibrium

Definition 1 *Given a matrix of tax rates τ, an equilibrium is a set of consumer prices p, wages w, intermediates allocations x, labor allocations l, outputs y, and final demands c, such that:*

1. *Each producer chooses labor factors, inputs and fuels to minimize its costs taking prices and tax rates as given;*
2. *Workers maximize their utility subject to their budget constraints;*
3. *Tax revenues are rebated lump sum to the consumers; and*
4. *Markets for all intermediates and factors clear:*

$$\sum_{j \in \mathcal{N}} x_{j,i} + \sum_{h \in \mathcal{H}} c_{h,i} = y_i, \quad \forall i \in \mathcal{N} \tag{9}$$

$$\sum_{j \in \mathcal{N}} l_{j,f} = L_f, \quad \forall f \in \mathcal{F} \tag{10}$$

3.4 Definitions

In order to state the main analytical results of this paper in the following section we must introduce some definitions and notation that are standard in the production networks literature.[22]

As usual in the literature (Baqaee and Farhi 2020), we denote the cost-based objects with tilde, in order to differentiate them from revenue-based objects. The distinction between them is key to encode different patterns of propagation, as will become clear later.

Heterogeneous-Agents Input-Output Matrix (HA-IO)

The cost-based HA-IO ($\tilde{\Omega}$) is defined as the $(C+N+F)X(C+N+F)$ matrix which ijth element is consumer i's marginal costs elasticity of i's relatively to the price of input j, which in equilibrium is equivalent to j's share of i' expenditure, according to the Shephard's lemma[23]

$$\tilde{\omega}_{i.j} = \frac{p_i x_{i,j}}{\sum_k p_k x_{i,k}}, \quad \forall i \in \mathcal{H} \cup \mathcal{N}, \text{ and } \forall j \in \mathcal{F} \cup \mathcal{N}$$

Intuitively, $\tilde{\omega}_{i.j}$ captures the *direct* exposition of i's costs to input j. Throughout this paper we follow Baqaee and Farhi (2019) and assume that columns and rows of the HA-IO follow the order: consumers, producers/goods and factors:[24]

$$\tilde{\Omega} = \left[\begin{array}{ccc|ccc|ccc}
0 & \cdots & 0 & \tilde{\omega}_{11} & \cdots & \tilde{\omega}_{1N} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \tilde{\omega}_{C1} & \cdots & \tilde{\omega}_{CN} & 0 & \cdots & 0 \\
\hline
0 & \cdots & 0 & \tilde{\omega}_{11} & \cdots & \tilde{\omega}_{1N} & \tilde{\omega}_{11} & \cdots & \tilde{\omega}_{1F} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \tilde{\omega}_{N1} & & \tilde{\omega}_{NN} & \tilde{\omega}_{N1} & & \tilde{\omega}_{NF} \\
\hline
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0
\end{array}\right]$$

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[22] See Carvalho and Tahbaz-Salehi (2019) and Baqaee and Rubbo (2023) for reviews of this literature.

[23] The definition of cost-based IO matrices, contrasting with revenue-based IO matrices, was first introduced by Baqaee and Farhi (2020), while the the concept of HA-IO was introduced by Baqaee and Farhi (2019).

[24] As by definition firms do not purchase from consumers and consumers do not purchase factors, $\omega_{i,j} = \tilde{\omega}_{i,j} = 0$, $\forall i \in \mathcal{H}$, $\forall j \in \mathcal{F}$.

On the other hand, the revenue-based counterpart of $\tilde{\Omega}$, henceforth Ω, is composed of consumers' inflow of inputs as shares of their total revenue

$$\omega_{i,j} = \frac{p_j x_{i,j}}{p_i y_i}$$

The relationship between these two objects is straightforward: $\tilde{\omega}_{i,j} = (1 + \tau_{i,j})\omega_{i,j}$.

Leontief Inverse

The revenue and cost-based Leontief inverses are defined as:

$$\tilde{\Psi} = \left(I - \tilde{\Omega}\right)^{-1} = I + \tilde{\Omega} + \tilde{\Omega}^2 + \dots \qquad \Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \dots \tag{11}$$

and their typical elements $(\Psi_{i,j}, \tilde{\Psi}_{i,j})$ capture the network adjusted expositions between i and j. Specifically, $\Psi_{i,j}$ captures how much of each dollar made by sector/worker i finds its way through the production chain to product/factor j via backward linkages, while $\tilde{\Psi}_{i,j}$ captures how variations on j' price affects i's costs via forward linkages, direct and indirectly.

GDP and Domar Weights

The nominal GDP is equivalently calculated as the sum of all expenditures on final consumption and the total income earned by all consumers

$$GDP = \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} p_j c_{h,j} = \sum_{h \in \mathcal{H}} \sum_{j \in \mathcal{N}} l_{j,h} w_h + \sum_{h \in \mathcal{H}} \Theta_h \underbrace{\sum_{i \in \mathcal{N} \cup \mathcal{H}} \sum_{j \in \mathcal{N}} \omega_{i,j} \lambda_i \tau_{i,j}}_{\mathcal{T}_h},$$

where Θ_h denotes the share of total tax revenues accruing to the representative final consumer h. Additionally, we assume that the nominal GDP is the numeraire of this economy, so that household h's share of aggregate income coincides with its nominal income

$$\mathcal{S}_h \equiv \frac{\sum_{j \in \mathcal{N}} p_j c_{h,j}}{GDP}$$

The cost-based Domar weight of a input or factor i is defined as the average network-adjusted cost exposure towards i, and a similar definition applies for revenue-based Domar weights.

$$\tilde{\lambda}_i \equiv \sum_{h \in \mathcal{H}} \mathcal{S}_h \, \tilde{\Psi}_{h,i} \qquad\qquad \lambda_i \equiv \sum_{h \in \mathcal{H}} \mathcal{S}_h \, \Psi_{h,i}$$

The Domar weights encode how the economy as a whole is exposed to sectoral shocks. On equilibrium, they comprise the $N+F$ vector of products and factors sales as shares of the GDP

$$\lambda_i \equiv \frac{p_i y_i}{GDP} \qquad\qquad \Lambda_f \equiv \frac{w_f L_f}{GDP},$$

where we interchangeably denote the sales shares of factors with Λ instead of λ because they will have a special role in our analysis.

3.5 Real GDP and Welfare

Real consumption variations are computed by deflating nominal incomes by Divisia price indices. This allows us to disentangle the price effect from the income effect and analytically characterize the impact of these two channels in section 4. Furthermore, the evolution of real GDP is calculated as the weighted sum of real consumption, using the GDP shares of final consumers as weights.

Definition 2 (Real Consumption, Welfare and Real GDP) *The infinitesimal variation in the real consumption and welfare of the final consumer h, and its corresponding deflator are defined as:*

$$d\log C_h = \underbrace{d\log \mathcal{S}_h}_{income\ effect} - \underbrace{d\log P_h}_{price\ effect}, \qquad\qquad d\log P_h = \sum_{i \in \mathcal{N}} \tilde{\omega}_{h,i} d\log p_{h,i} \tag{12}$$

Moreover, the infinitesimal variations in the real GDP around the steady state are defined by:

$$d\log rGDP = \sum_{h \in \mathcal{H}} \mathcal{S}_h \, d\log C_h \tag{13}$$

4 Log-linearized Model Solution

In this section we analytically characterize the first-order elasticities of economic variables to the carbon pricing shocks. The expressions pinpoint infinitesimal changes from an initial steady state equilibrium according to microeconomic parametric primitives. Using these results, we can outline the effects of price and income channels on final consumers separately. Together, they provide a comprehensive characterization of the welfare impact of climate policy implementation on final consumers.

Before exposing the elasticities, we define the carbon pricing reform in terms of a ”reduced form” tax rate shock. We remain agnostic regarding the precise CPI scheme. The reader can think of it as either a tax rate exogenously introduced by the tax authority, or as a price arising from market interactions in alternative auction-based carbon pricing schemes.

We use dX to denote the differential of an endogenous variable X, i.e. the infinitesimal change of X in response to infinitesimal shocks to tariffs, and $d\log X = \frac{dX}{\hat{X}}$ to denote the percentage variation relative to the initial equilibrium $(\hat{X})$. To solve the model locally, we identify all the equilibrium endogenous variables in the model from the log-linearize system of equations formed by the factor sales changes, in terms of microeconomic primitives. To compute global comparative statics, we solve this system iteratively and integrate the local effects. [25]

4.1 The Carbon Pricing Reform Shock

Let p_{CO_2e} be the price induced by the climate policy for each ton of CO$_2$e emitted.[26] Furthermore, let EM_i denote the observed total CO$_2$e emission generated by the consumption of a fuel i, summed across all its intermediate and final consumers. Then, assuming a linear relationship between the consumption of fuel i and emissions in terms of values,[27] so that the additional carbon tax revenue from a flow (k,i) is $x_{k,i}p_i\left(\frac{p_{CO_2e}EM_i}{\sum_{j\in\mathcal{H}\cup\mathcal{N}}x_{j,i}p_i}\right)$, the carbon pricing shock is defined as the variation in the consumption tax of any fuel i

$$d\tau_i = p_{CO_2e}\left(\frac{EM_i}{\sum_{j\in\mathcal{H}\cup\mathcal{N}}x_{j,i}p_i}\right), \quad \forall k \in \mathcal{N}\cup\mathcal{H},\ \forall i \in E \tag{14}$$

Additionally, as there are preexisting distortive consumption tax rates in the initial steady state

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[25] All the mathematical proofs are detailed in Appendix B.

[26] Carbon equivalent tonnes are a unit of measurement used to compare the climate impact of different greenhouse gases based on their global warming potential (GWP) over a given time horizon. GWP-100 is a commonly used metric that compares the warming impact of a greenhouse gas to that of CO$_2$ over a 100-year period.

[27] This is a common hypothesis in the literature when quantities of products consumed are not observed (e.g. King et al. (2019); Devulder and Lisack (2020))

of the economy ($\hat{\tau}_{k,i}$), the *relative* increase in the tax rate will be different for each flow (k, i):

$$d\log(1+\tau_{k,i}) = \frac{d\tau_i}{(1+\hat{\tau}_{k,i})} \tag{15}$$

4.2 Prices and marginal costs: the price effect Channel

The tax shocks propagates throughout the supply chains, impacting the production costs of firms located immediately downstream to the targeted fuels in the network. The marginal costs of these firms increase proportionally to their reliance on the taxed fuels, measured by their cost-based *direct* exposure ($\tilde{\omega}_{i,j}$). These producers, in turn, pass on the increased marginal costs to the prices of their products, thereby transmitting the shock to producers reliant on them.

The total impact on the marginal cost of a producer i hinges on its reliance, both *directly and indirectly* through the supply chain, on dirtier fuels that are more targeted by the tax, as measured by the cost-based Leontief inverse parameters $\tilde{\Psi}_{i,j}$. Additionally, the effect is influenced by fluctuations in factor prices, which, in turn, are affected by shifts in the aggregate demand for these factors following the implementation of the reform:

$$d\log MC_i = \underbrace{\sum_{j\in\mathcal{N}} \tilde{\Psi}_{i,j}\left[\sum_{n\in E} \tilde{\omega}_{j,n}\, d\log(1+\tau_{j,n})\right]}_{direct\ and\ indirect\ effect\ of\ tariffs} + \underbrace{\sum_{f\in\mathcal{F}} \tilde{\Psi}_{i,f}\, d\log w_f}_{factor\ reallocation\ effect} \quad , \quad \forall i \in \mathcal{N} \tag{16}$$

Notice that, in Equation 16, only the producers j directly reliant on fuels n are subject to the direct effect of the policy in the first iteration of the propagation. However, as j passes on the tax shock to its marginal cost, all producers indirectly exposed via the network are affected, even they do not directly consume fuels at all.

On top of the marginal cost variation, the consumer price of a fuel i faced by buyer k changes according to the direct tariff shock

$$d\log p_{k,i} = d\log MC_i + \mathbb{1}_{\{i\in E\}} \times \left[d\log(1+\tau_{k,i})\right], \quad \forall k \in \mathcal{N}\cup\mathcal{H},\ \forall i \in \cancel{E}\cup E \tag{17}$$

where $\mathbb{1}_{\{i\in E\}}$ is an indicator function.

Applying the definition of price effect (Definition 2), the resulting decrease in the welfare of a worker of type h from the variation in the cost of the consumption cost is:

Proposition 1 *The price channel effects on households of type h, given a vector of carbon taxes shocks $d\tau_n$ $\forall n \in E$ is, to a first order:*

$$d \log P_h = \sum_{i \in \mathcal{N}} \tilde{\omega}_{h,i} \left(\sum_{j \in \mathcal{N}} \tilde{\Psi}_{i,j} \left[\sum_{n \in E} \tilde{\omega}_{j,n} d \log(1 + \tau_{j,n}) \right] + \sum_{f \in \mathcal{F}} \tilde{\Psi}_{i,f} d \log w_f + d \log(1 + \tau_{h,i}) \right)$$

4.3 Sales and Expenditure Shares

In response to changes in relative consumer prices, agents optimally adjust their expenditure patterns, substituting across inputs, fuels, and factors. Equation 18 in Proposition 2 shows at what rate consumer l alters expenditures towards (or away from) good/factor j following a price increase in intermediate or factor j according to both substitution and income effects (if l is a final consumer).

The direction of the substitution effect is governed by the magnitude of the price variations and complementarity among inputs. For instance, if the price of input j for consumer l rises, and/or if j is complementary to other inputs ($\theta^l_{j,k} < 1$) that became more expensive for l, then the expenditure on j tends to decrease. Conversely, if the price of j falls, and/or if j is substitutable for other inputs that became costier ($\theta^l_{j,k} > 1$), the expenditure on j is likely to increase. The overall substitution effect represents the net impact of these forces. Finally, the direction of the Engel effect is determined by the income elasticities of the inputs. As a worker's real income increases, it allocates a larger portion of its total expenditure to sectoral goods that have higher income elasticity (see section 3).

Proposition 2 (Expenditure Shares) *For a matrix of perturbations to tariffs rates, $d \log(1+\tau)$, the change in the share of a good or factor $j \in \mathcal{N} \cup \mathcal{F}$ in consumer $l \in \mathcal{H} \cup \mathcal{N}$'s revenue is, to a* *first-order:*[28]

$$d\log\omega_{l,j} = \underbrace{d\log p_{l,j} - \sum_{k\in\mathcal{N}\cup\mathcal{F}} \tilde{\omega}_{l,k}\left(1-\theta^{l}_{j,k}\right) d\log p_{l,k}}_{Substitution\ Effect} + \mathbb{1}_{\{l\in\mathcal{H}\}} * \underbrace{\left[\left(\psi_{l,j}-1\right) d\log C_l\right]}_{Engel\ effect} - d\log(1+\tau_{l,j}),$$

$$j \in \mathcal{N}\cup\mathcal{F} \quad (18)$$

where $\psi_{l,j}$ is the income elasticity of sectoral good j for household l, $d\log C_l$ is the real income index of household l and $\theta^{l}_{j,k}$ is the Allen-Uzawa elasticity of substitution between inputs j and k for consumer l, defined according to Equation 5.

Equation 19 in Proposition 3, summarises the effect on the sales of a input or factor k stemming from the carbon pricing shock. In short, this equation shows how much of the streams of demand generated by the changes in relative prices is channeled to k through the production network. The elements inside the bracket can be broken down in a distributive effect and a expenditure switching effect. The first captures how resources reallocations across households impacts demand for input i, holding fixed the initial expenditure shares. Intuitively, if a household h which allocates a particularly high share of its resources to i, becomes relatively wealthier than the other households, demand for i will increase. The latter elements encode the degree to which both intermediate and final consumers switch their demand, measured by expenditure shares, towards input i, while holding initial sales/GDP shares fixed, as a result of the expenditure adjustments driven by the substitution and income effects as detailed in Proposition 2.

Proposition 3 (Sales) *For a matrix of perturbations to tariffs rates, $d\log(1+\tau)$, the change in the sales of a good or factor k is, to a first-order:*

$$d\lambda_k = \sum_{i\in\mathcal{N}} \Psi_{i,k}\left[\underbrace{\sum_{h\in\mathcal{H}} \omega_{h,i} d\mathcal{S}_h}_{distributive\ effect} + \underbrace{\sum_{h\in\mathcal{H}} \mathcal{S}_h\, d\omega_{h,i} + \sum_{j\in\mathcal{N}} \lambda_j d\omega_{j,i}}_{expenditure\ switching\ effect}\right], \quad k\in\mathcal{N}\cup\mathcal{F} \qquad (19)$$

Hence, the terms inside the brackets capture the demand channeling towards a product $i \in \mathcal{N}$. Finally, the element outside the bracket ($\Psi_{i,k}$) is the revenue-based Leontief element that maps how

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[28] Remember that, by assumption, $\forall h \in \mathcal{H}$, $\tilde{\omega}_{h,f} = 0$, $\forall f \in \mathcal{F}$.

much of the increase in the revenue stream going to i makes its way to the sales of k through the network. An example of this mechanism would be that and increase in the demand for a cleaner fuel can cause a surge in the demand for a labor type of a sector upstream to it. In other words, (Equation 19) is aggregating all the network-adjusted stream of resources channeled towards the demand of sectoral product or factor k.

4.4 Nominal Income and Price Deflators: the income Channel

The increase in the nominal income of a household h is the summation of the variations in their respective labor factor sales (Equation 19) and of the tax revenue rebates distributed by the government

$$d\mathcal{S}_h = d\Lambda_h + d\mathcal{T}_h,\ \forall h \in \mathcal{H}$$

Changes in the aggregate tax revenue happens through two channels: i) holding fixed the initial tax rates, by how much the taxed financial flows varied (first bracket); and ii) holding fixed the financial flows, by how much the tax rates changed because of the carbon tax (second bracket).

$$d\mathcal{T}_h = \Delta_h \left[\sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} \tau_{i,j} d\omega_{i,j}\lambda_i + \sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} \tau_{i,j}\omega_{i,j} d\lambda_i\right] + \Delta_h \left[\sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} d\tau_{i,j}\omega_{i,j}\lambda_i\right],$$

where $\sum_h \Delta_h = 1$.

Applying the definition of income effect (Definition 2) we get our last proposition.

Proposition 4 *The income channel effects on households of type h, given a vector of carbon taxes shocks $d\tau_n$ $\forall n \in E$ is, to a first order:*

$$d\log\mathcal{S}_h = \sum_{i\in\mathcal{N}} \frac{\Psi_{i,h}}{\mathcal{S}_h} \left[\sum_{h\in\mathcal{H}} \omega_{h,i} d\mathcal{S}_h + \sum_{h\in\mathcal{H}} \mathcal{S}_h\, d\omega_{h,i} + \sum_{j\in\mathcal{N}} \lambda_j d\omega_{j,i}\right] \tag{20}$$
$$+ \frac{\Delta_h}{\mathcal{S}_h} \left[\sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} \tau_{i,j} d\omega_{i,j}\lambda_i + \sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} \tau_{i,j}\omega_{i,j} d\lambda_i\right] + \frac{\Delta_h}{\mathcal{S}_h} \left[\sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} d\tau_{i,j}\omega_{i,j}\lambda_i\right]$$

5 Quantitative Exercise: Carbon Tax Reform in Brazil

In this section we use our framework to quantitatively evaluate the effects of the implementation of a carbon tax in the Brazilian economy. Table 3 summarizes the parameters we calibrate and their sources. We combine three cross-sectional micro datasets to parameterize the HA-IO, initial distortions and GDP shares; one environmental datasets to parameterize fuel emissions; and estimations from other works to parameterize the elasticities.[29] For clarity, we divide the parameterization of the HA-IO into three parts: the input consumption pattern of producers (*Input Consumption Shares*); the factor consumption pattern of producers (*Factor Consumption Shares*); and the consumption pattern of consumers (*Final Consumption Shares*). In order to assess the role of distributive policies, we conjecture two tax revenue distribution schemes, which are further detailed later on.

Table 3: List of Parameters Disciplined and Sources

Description	**Parameters**	**Data Source**
Input Consumption Shares	$\tilde{\Omega}_{i,k} = \frac{p_k x_{i,k}}{\sum_{j\in\{N\cup\mathcal{F}\}} p_j x_{i,j}}$	Brazilian I-O Tables
Factor Consumption Shares	$\tilde{\Omega}_{i,f} = \frac{w_f l_{i,f}}{\sum_{j\in\{N\cup\mathcal{F}\}} p_j x_{i,j}}$	RAIS
Final Consumption Shares	$\tilde{\Omega}_{c,i} = \frac{p_i c_{c,i}}{\sum_{j\in\mathcal{N}} p_j c_{c,j}}$	POF
Total Income	$\mathcal{S}_h$	RAIS
Initial Tax Rates	$\tau_{i,j} = \frac{\tau_{i,j} p_j x_{i,j}}{p_j x_{i,j}}$	Brazilian I-O Tables
Elasticities of Substitution	$\sigma,\ \phi,\ \gamma,\ \xi,\ \pi,\ \iota$	Atalay (2017); Boehm et al. (2019); Oberfield and Raval (2021); D. Stern (2012)
Utility Elasticities	ϵ_j	Comin et al. (2021)
Sectoral Emissions	EM_i	SEEG
CO$_2$e ton Price	p_{CO_2}	Stiglitz et al. (2017)
Carbon Tax Rebate Revenue Shares	Δ_h	Authors' Choice
Preexisting Tax Rebate Scheme	Θ_h	Authors' Choice

To calibrate the intermediate consumptions shares of inputs and respective preexisting buyer-

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[29] For the substitution elasticities, which are fundamental priors of the model, we additionally consider a number of robustness options that are considerably more conservative.

supplier-specific consumption tariffs, we use the latest Brazilian input-output matrix, from 2015. It contains information on intermediate and final consumption flows for 64 sectors in its most disaggregated version.[30] Additionally, the national accounts inform us on the sectoral value added, breaking down across compensations to workers, revenue taxes, and capital employed plus profits. We use this information to calibrate the factor expenditure shares by disentangling each sector's payroll according to RAIS administrative data on worker compensations covering the universe of formal employees.[31] We segment the labor types of workers according to their occupations and schooling levels. There are four schooling levels defined according to the International Standard Classification of Education (ISCED): *Primary Education or Lower*, *Lower Secondary*, *Upper Secondary*, *Post-secondary or Higher*; and nine occupations defined according to the least granular aggregation of the Brazilian Classification of Occupations,[32] making up 36 pair of schooling and occupations types. Finally, we aggregate the remaining value added components as a sector-specific capital employment.

To parameterize the final consumption shares of each type of worker we combine data from POF, RAIS and the national accounts. First, we reconcile POF's data with the sectoral breakdown in the national accounts by matching POF's product categories to final consumption industries in the input-output matrix.[33] This yields a matrix of expenditure shares across the sectors for each worker type. Multiplying the expenditure shares by the total compensation income accrued to each labor type (compute from RAIS), we obtain a matrix of final consumption flows, in BRL. Lastly, following Borusyak and Jaravel (2021), we normalize the total final consumption of each sectoral good to match the aggregate final demand demand in the national accounts. By performing these normalizations, we ensure that the GDP shares accruing to each worker type and the sales shares of GDP of all sectors match the national accounts. Moreover, we show in Appendix F, in the

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[30] Originally the data covers 67 sectors and 127 products, but we discard the *Domestic Services* sector and product from the sample since it has only zeros in its entries and aggregate the *Public Health* and *Private Health* into a single *Health* sector, as well as the *Public Education* and *Private Education* sectors into a single *Education*.

[31] See subsection 2.2 for more information on RAIS.

[32] *Executives and Managers*, *Sciences and Arts*, *Mid-Level Technicians*, *Administrative Services*, *Commercial Services*, *Agricultural Workers*, *Extractive and Transformation Manufacturing*, *Steel and Construction Manufacturing*, *Repair and Maintenance*. Originally there were ten occupation categories, but we drop *Policemen, Firefighter and Military* from our sample.

[33] The sectoral aggregation in the POF is not compatible with the I-OM. To reconcile these two aggregation we use the official translator developed by IBGE. However, the translator considers the products classification of the 2008-2009 POF, therefore demanding manually adjustments as part of the matching. This process generated some small losses that are described in detail at Table 6 in Appendix C.

Appendix, that the resulting expenditure shares, which need not to be consistent with the ones firstly calculated from POF in the first step, do not deviate significantly from these moments.

Our parameterization of elasticities is in line with the estimations commonly used in the literature. For intermediate consumption, the elasticity of substitution across intermediates is set to $\xi = 0.2$ (Atalay 2017), between the value added bundle and the intermediates bundle is set to $\phi = 0.5$ (Atalay 2017), between the aggregate fuels bundle and the non-fuel bundle is set to $\pi = 0.02$ (Hassler et al. 2021), and between fuels is set to $\iota = 1.3$ (D. Stern 2012). For the final consumer, π and ι are the same, and across other consumption goods is set to $\sigma = 0.9$ (Atalay 2017; Oberfield and Raval 2021). In addition, we estimate the model taking into consideration other elasticities values as robustness.

Lastly, to compute the total emissions stemming from the consumption of each fuel, we leverage on total annual emissions per fuel for 2019 from the SEEG database, which reports Brazilian annual CO_2e emissions according to Intergovernmental Panel on Climate Change (IPCC) guidelines for a 31 fuels aggregation (De Azevedo et al. 2018). We manually reconcile SEEG fuel emissions data with five products classification of the national accounts.[34] The price charged per ton of CO_2e released in the atmosphere (p_{CO_2}) is set to 315 BRL (80 USD)[35] according to the estimation of Stiglitz et al. (2017).

5.1 Results

The tax rate shocks resulting from our calibration at the sectoral level are as follows. Intuitively, the most polluting intense sectors were more heavily taxed, starting with *Refined petroleum products*, which includes all petroleum-based fuels, having its tax rate increased by 15.5%. Next, *Forestry, fishing & aquaculture*, which includes firewood and lye, had its tax rate increased by 9.4%. *Electricity, gas & other utilities* experienced a tax rate increase of 3.7%, while *Coal & nonmetallic minerals* saw an increase of 2.0%.[36] Additionally, *Sugar manufacturing & refining* and *Biofuels* both had their tax rates increased by 0.9%. The resulting tax rate increase in the total *Energy*

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[34] See Table 12 in the appendix for details.

[35] We use a exchange rate of 3.944 for 2019, according to OECD estimations.

[36] *Coal & nonmetallic minerals* is not highly affected because the Brazilian national accounts aggregations do not differentiate nonmetallic fuels from coal, which makes the output intensity not as high (although coal is one of the most polluting fuel sources).

sector, taking into consideration the weighted average according to the sales shares, was 3.7%.[37]

5.1.1 Baseline

The tax rate shock is transmitted throughout the production chain, generating widespread changes in relative prices. Figure Figure 4 illustrates the variations in marginal costs for all sectoral products.[38] and reveals substantial heterogeneity in sectoral reactions, with the majority of producers experiencing a decrease in marginal costs. Conversely, a minority of producers that are heavily dependent on pollutant-intensive goods, particularly those in the air and land transportation, petroleum, and electricity generation sectors, witnessed a substantial increase in their marginal costs.

The reduction in marginal costs experienced by numerous producers is attributed to a decline in competitive wages paid to labor factors (**??**). Given the model assumption of fixed supply of factors, this downward pressure on wages is solely driven by a decrease in demand for labor that stems from two channels. First, in our calibration, factors are complementary to both energy inputs and non-energy materials. As the prices of these inputs increase due to the direct and indirect effects of the policy, firms substitute away from factors. Table 4 summarizes the intermediate consumption expenditure share shifts that resulted from the policy and confirms this mechanism across all consumers. Producers substantially increase their expenditure shares on energy and transports, at the cost of milder reduction of all other inputs shares. The shift away from labor is stronger for sectors more reliant on taxed inputs, such as transports and energy, and almost negligible for services and food sectors.

Second, contractions in sales across sectors that employ various labor forces also diminish demand for these workers due to the reduced scale of production. Table 5 presents the aggregate changes in sectoral sales and prices. Energy sales and output experienced a sharp drop, despite shifts in consumption expenditure (Table 4), and the most significant losses were observed in extractive activities, which are major suppliers to the energy sector. However, sales declined across all industries, except for transport, in spite of the reduction in prices.

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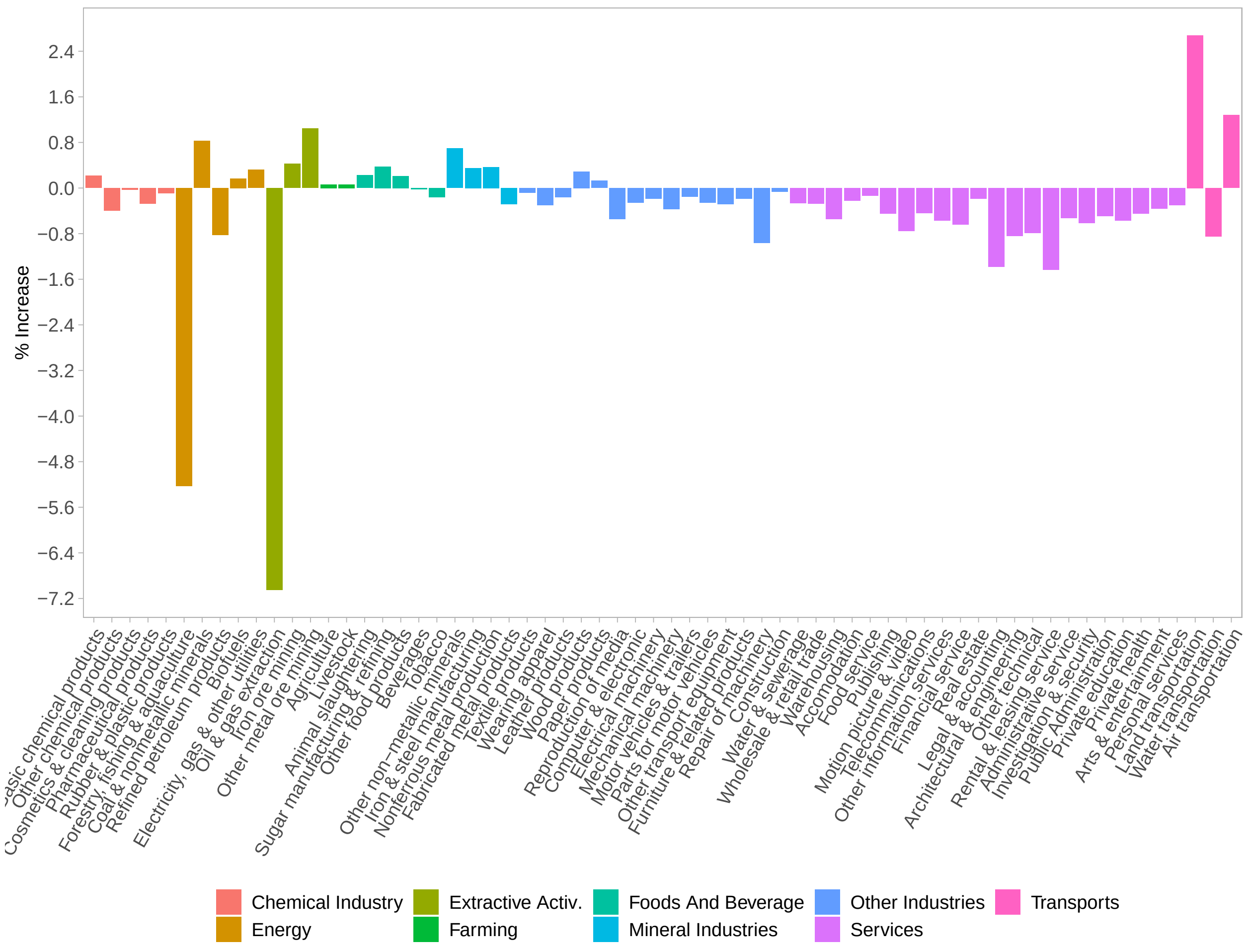
[37] See Appendix G for a discussion of the difference in the tax rate shock targeting when we also consider emissions other than fuel combustion.

[38] When combined with the initial tax perturbation (**??**), this represents the total price fluctuation observed for each sectoral product.

Figure 4: Changes in Sectoral Marginal Cost



Notes: The figure shows the percentage variation in the marginal cost of production of each sector.

Table 4: Intermediate Consumers Expenditure Shares Changes

	Farming	Energy	Foods Beverage	Extractive Activ.	Other Industries	Chemical Industry	Mineral Industries	Transports	Services	Factors
Primary	-0.22	-7.68	0.06	-0.70	-0.16	-0.19	-0.12	0.10	-0.19	0.00
Lower Secondary	-0.19	-7.76	0.08	-0.67	-0.13	-0.17	-0.10	0.12	-0.16	0.00
Upper Secondary	-0.17	-7.76	0.09	-0.65	-0.12	-0.15	-0.08	0.14	-0.14	0.00
Post Secondary	-0.13	-7.71	0.13	-0.61	-0.08	-0.12	-0.05	0.17	-0.12	0.00
Farming	-0.45	-2.06	-0.07	-4.45	-0.62	-0.54	-0.27	1.41	-0.86	-0.56
Energy	-0.95	-2.96	-0.62	-4.94	-1.13	-1.04	-0.77	0.91	-1.36	-5.78
Foods Beverage	-0.21	-2.78	0.06	-4.24	-0.38	-0.30	-0.03	1.65	-0.62	-0.52
Extractive Activ.	2.55	1.14	2.98	-1.49	2.37	2.45	2.72	4.40	2.14	-1.95
Other Industries	0.03	-1.84	0.37	-4.00	-0.14	-0.06	0.21	1.89	-0.38	-0.28
Chemical Industry	-0.25	-2.55	0.06	-4.29	-0.43	-0.34	-0.07	1.61	-0.66	-0.71
Mineral Industry	-0.43	-2.36	-0.08	-4.47	-0.61	-0.53	-0.26	1.42	-0.84	-0.70
Transports	-2.84	-4.91	-2.56	-6.35	-3.02	-2.93	-2.67	-0.99	-3.25	-1.63
Services	0.27	-1.43	0.59	-3.82	0.10	0.18	0.45	2.13	-0.14	-0.15

Notes: The table shows the percentage variations relative to initial steady state levels of expenditure share of the representative firms in each sector. Results were calculated for the initial 64 aggregation and then further aggregated using sales shares as weights. There is only one *Factors* columns to save space as the expenditure share variation for any factor is the same for each producer.

Table 5: Sectors Results (%)

	Output	Sales
Farming	-0.42%	-0.36%
Energy	-5.80%	-6.30%
Foods Beverage	-0.33%	-0.14%
Extractive Activities	-3.55%	-8.54%
Other Industries	-0.12%	-0.29%
Chemical Industry	-0.65%	-0.71%
Mineral Industry	-0.54%	-0.27%
Transports	-1.94%	0.43%
Services	-0.02%	-0.48%

Notes: The table breaks shows sectoral results according to authors aggregation. Results are generated for the 64 national accounts sectoral aggregation and then further aggregated using sales as weights.

?? summarizes the aggregate and distributional impacts at the baseline scenario policy. The price effect has increased the cost of consumption for all types of occupations and education levels in a highly heterogeneous manner. Consistent with the observation that low-schooling individuals have consumption baskets more exposed to more pollutant goods,[39] these individuals are more significantly impacted: Individuals with primary education experience an increase of 0.47% in the cost of their consumption basket, compared to an increase of 0.20% for those with post-secondary education.

Conversely, results suggest a progressive impact stemming from the income effect. Factor income fell across all types, but more pronouncedly for more educated individuals. Agents with post-secondary education experienced a significant wage decrease of 0.84%, while those with primary education had a loss of 0.79%.

The tax revenue rebates, allocated in proportion to the GDP shares in the baseline scenario, play a crucial role in shaping the outcomes of income effects. These rebates were substantial enough to offset the losses in factor income across all levels of education, with the sole exception of individuals with post-secondary education, thereby engendering positive income effects for these groups. These income effects proved robust enough to mitigate the regressive implications of the price effect, leading to a reduction in the Gini Index. This reduction, however, stems from a disproportionately greater welfare loss among post-secondary educated individuals compared to those with lower educational attainments. Despite these adjustments, the net welfare impact across all categories remained negative, indicating that the adverse consequences of the price effect prevailed over the benefits derived from the income effect.

The aggregate outcomes of the carbon pricing policy are a 0.13% reduction in GDP, a slightly larger impact on aggregate welfare, reflecting the fact that complementarities push expenditure shares towards the inputs that became more expensive, a 0.06% reduction in the Gini index, which indicates an overall progressive impact of the policy, and an 8.45% reduction in total fuel combustion emissions.

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[39] See Table 1 and Figure 5 in subsection 2.2.

6 Concluding Remarks

In this paper, we have developed a model of production networks with heterogeneous firms, workers, and factors to analyze the distributive implications of implementing a carbon tax on fossil fuel emissions. We show an analytical characterization of the effects emanating from each worker's income and price channels, grounded in microeconomic priors and structural assumptions.

Subsequently, we applied our analytical framework to empirical data, using the Brazilian economy as a testing ground for quantitative analysis. The findings suggest moderately regressive impacts resulting from the policy, yet there is notable heterogeneity in its effects across different workers and firms.

This manuscript represents an early stage of our research and, as such, possesses significant potential for further refinement. Quantitatively, a key priority lies in evaluating the effects of adopting the most conservative elasticity parameters as detailed in **??**. We also intend to examine the efficacy of distributive policies in mitigating the regressive effects of our proposed policy measures and to assess the differences in outcomes when the model incorporates versus excludes input-output networks. Another significant aim is to refine our model's parameterization through the use of microdata from a variety of countries, thereby enhancing the precision of our analysis. Theoretically, incorporating endogenous factor supply stands as a key objective in our short-term agenda.

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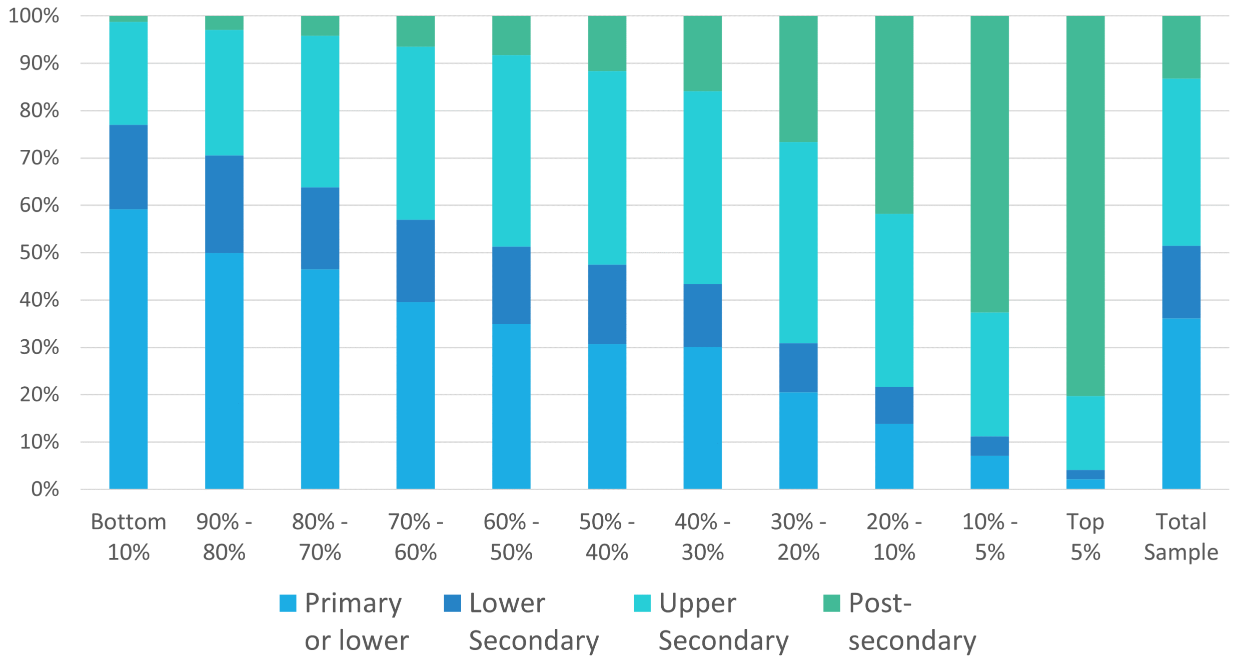
A Additional Information About the Brazilian Economy

A.1 Schooling and per capita Income

Using schooling information in the POF, we characterize four distinct types of labor according to schooling attainment levels, following the International Standard Classification of Education (ISCED): "Primary," includes individuals who have not completed any educational stage; the two intermediate categories, "Lower and Upper Secondary," cover individuals who have completed elementary and high school, respectively; while "Post-Secondary" includes those who have completed more advanced stages.[40] Figure 5 depicts the share of labor income accruing to each income group that stems from compensation to workers within the each schooling attainment bracket.

Figure 5: Labor Income Shares According to Schooling Attainment Levels



Notes: The figure shows the percentage share of the labor income of households of each per capita income decile that accrues from workers with each schooling attainment.

There is a clear inverse relationship between the proportion of income derived from "Primary or lower" education and income levels, with the highest concentration observed in the "Bottom 10%" at 59.2%, compared to just 2.1% in the "Top 5%." Conversely, the "Post-Secondary" education group

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[40] See Table 11 for more details.

displays an opposing trend: the share escalates from a mere 1.3% in the lowest decile to 80.3% in the highest. The intermediate categories display less pronounced yet discernible shifts across income groups. For example, the "Upper Secondary" income share peaks at 42.5% in the "30%-20%" bracket before declining. These patterns indicate a significant correlation between educational attainment and income distribution, and consequently, emphasize the need to carefully consider the heterogeneous effects on different labor types when assessing the policy's implications.

B Proofs

B.1 Households

From the utility maximization problem of the worker h (Equation 4) the final demand for each product $j \in \mathcal{N}$ is[41]

$$c_{h,j} = \begin{cases} \bar{\omega}_{h,j} \left(\frac{p_{h,j}}{P_{h,E}}\right)^{-\iota} \left(\frac{P_{h,E}}{P_h}\right)^{-\pi} C_h^{(\epsilon_j-1)(\iota-1)}, & \text{if } j \in E \\ \bar{\omega}_{h,j} \left(\frac{p_{h,j}}{P_{h,\not{E}}}\right)^{-\sigma} \left(\frac{P_{h,\not{E}}}{P_h}\right)^{-\pi} C_h^{(\epsilon_j-1)(\sigma-1)}, & \text{if } j \in \not{E}. \end{cases} \tag{21}$$

It follows that the final expenditure shares satisfy

$$\tilde{\omega}_{h,j} = \begin{cases} \bar{\omega}_{h,j} \left(\frac{p_{h,j}}{P_{h,E}}\right)^{1-\iota} \left(\frac{P_{h,E}}{P_h}\right)^{1-\pi} C_h^{\epsilon_j(\iota-1)}, & \text{if } j \in E \\ \bar{\omega}_{h,j} \left(\frac{p_{h,j}}{P_{h,\not{E}}}\right)^{1-\sigma} \left(\frac{P_{h,\not{E}}}{P_h}\right)^{1-\pi} C_h^{\epsilon_j(\sigma-1)}, & \text{if } j \in \not{E} \end{cases} \tag{22}$$

where $\tilde{\omega}_{h,j}$ is the expenditure shares of consumer h on good j,

$$\tilde{\omega}_{i.j} \equiv \frac{p_i c_{i,j}}{\sum_k p_k c_{i,k}}, \tag{23}$$

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[41] See Comin et al. (2021) for the solution within the nests. The overall solution follows from the usual strong separability condition (Sato 1967).

and $\left(P_{h,E},\ P_{h,\cancel{E}}\right)$ are the price indices of the fuels, non-fuel and aggregate bundles, respectively:[42]

$$P_{h,E} = \left(\sum_{j \in E} \frac{\tilde{\omega}_{h,j}}{\sum_{j \in E} \tilde{\omega}_{h,j}} U^{\epsilon_j (\iota - 1)} p_{h,j}^{1-\iota} \right)^{\frac{1}{1-\iota}}, \tag{24}$$

$$P_{h,\cancel{E}} = \left(\sum_{j \in \{\cancel{E}\}} \frac{\tilde{\omega}_{h,j}}{1 - \sum_{j \in E} \tilde{\omega}_{h,j}} U^{\epsilon_j (\sigma - 1)} p_{h,j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{25}$$

$$P_h = \left[P_{h,E}^{1-\pi} + P_{h}^{1-\pi} \right]^{\frac{1}{1-\pi}}, \tag{26}$$

where the price indices are defined according to:

$$C_h = \mathcal{S}_h \, / P_h \tag{27}$$

It is straightforward from the definition that the Allen-Uzawa elasticities of substitution for household h are:

$$\theta_{i,j}^{h} \equiv \frac{1}{\tilde{\omega}_{h,j}} \frac{d \log x_{h,i}}{d \log p_{h,j}} = \begin{cases} \pi - \frac{\sigma \times \mathbb{1}_{\{i=j\}}}{\tilde{\omega}_{h,j}} + \frac{(\sigma - \pi) \times \mathbb{1}_{\{j \in \cancel{E}\}}}{\sum_{n \in \cancel{E}} \tilde{\omega}_{h,n}}, & \text{if } i \in \cancel{E} \\ \pi - \frac{\iota \times \mathbb{1}_{\{i=j\}}}{\tilde{\omega}_{h,j}} + \frac{(\iota - \pi) \times \mathbb{1}_{\{j \in E\}}}{\sum_{n \in E} \tilde{\omega}_{h,n}}, & \text{if } i \in E \end{cases}$$

Log-differentiating Equation 24, Equation 25 and Equation 26 we get

$$d \log P_{h,\cancel{E}} = \sum_{j \in \cancel{E}} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,\cancel{E}}} \left(d \log p_{h,j} - \epsilon_j d \log C_h \right), \quad d \log P_{h,E} = \sum_{j \in E} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,E}} \left(d \log p_{h,j} - \epsilon_j d \log C_h \right)$$

$$d \log P_h = \sum_{i \in \mathcal{N}} \bar{\omega}_{h,i} \left(d \log p_{h,i} - \epsilon_j d \log C_h \right)$$

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[42] The price indices are defined as usual according to $P_h \equiv E_h / C_h$, and similarly for the inner nests.

which simplifies to

$$d\log P_{h,\not{E}} = \sum_{j\in\not{E}} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,\not{E}}} d\log p_{h,j} - d\log C_h \underbrace{\sum_{j\in\not{E}} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,\not{E}}} \epsilon_j}_{\bar{\epsilon}_h^{\not{E}}}$$

$$d\log P_{h,E} = \sum_{j\in E} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,E}} d\log p_{h,j} - d\log C_h \underbrace{\sum_{j\in E} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,E}} \epsilon_j}_{\bar{\epsilon}_h^{E}}$$

$$d\log P_h = \sum_{i\in\mathcal{N}} \bar{\omega}_{h,i} d\log p_{h,i} - d\log C_h \underbrace{\sum_{i\in\mathcal{N}} \bar{\omega}_{h,i}\epsilon_j}_{\bar{\epsilon}_h}$$

From 27 it follows that

$$d\log C_h = d\log \mathcal{S}_h - d\log P_h = \frac{d\log \mathcal{S}_h - \sum_{i\in\mathcal{N}} \bar{\omega}_{h,i} d\log p_{h,i}}{1-\bar{\epsilon}_h}$$

Furthermore, log-differentiating Equation 21 we get

$$d\log \tilde{\omega}_{h,j} = \left(\epsilon_j(\sigma-1)\right) d\log C_h + (1-\sigma)\left(d\log p_{h,j} - \sum_{j\in\not{E}} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,\not{E}}} d\log p_{h,j} + d\log C_h \bar{\epsilon}_h^{\not{E}}\right) +$$
$$(1-\pi)\left(\sum_{j\in\not{E}} \frac{\bar{\omega}_{h,j}}{\bar{\omega}_{h,\not{E}}} d\log p_{h,j} - d\log C_h \bar{\epsilon}_h^{\not{E}} - \sum_{i\in\mathcal{N}} \bar{\omega}_{h,i} p_{h,i} + d\log C_h \bar{\epsilon}_h\right)$$

for $j\in\not{E}$, and a symmetrical version of the same equation for $j\in E$.

We therefore finally get to the real income elasticity of the expenditure shares

$$\psi_{h,j} \equiv \left\{ \begin{array}{ll} 1+\frac{(1-\sigma)(\bar{\epsilon}_h^{\not{E}}-\epsilon_j)+(1-\pi)(\bar{\epsilon}_h-\bar{\epsilon}_h^{\not{E}})}{1-\bar{\epsilon}_h}, & \text{if} \quad j\in\not{E} \\ 1+\frac{(1-\iota)(\bar{\epsilon}_h^{E}-\epsilon_j)+(1-\pi)(\bar{\epsilon}_h-\bar{\epsilon}_h^{E})}{1-\bar{\epsilon}_h}, & \text{if} \quad j\in E \end{array} \right\}$$

B.2 Firms

From the solution of firm i's cost minimization problem we arrive at:

$$x_{i,j} = \left\{ \begin{array}{cc} \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,N}}\right)^{-\xi} \left(\frac{P_{i,N}}{P_{i,\not E}}\right)^{-\phi} \left(\frac{P_{i,\not E}}{P_i}\right)^{-\pi}, & \text{if} \quad j \in \not E \\ \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,F}}\right)^{-\gamma} \left(\frac{P_{i,F}}{P_{i,\not E}}\right)^{-\phi} \left(\frac{P_{i,\not E}}{P_i}\right)^{-\pi}, & \text{if} \quad j \in \mathcal{F} \\ \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,E}}\right)^{-\iota} \left(\frac{P_{i,E}}{P_i}\right)^{-\pi}, & \text{if} \quad j \in E \end{array} \right\}$$

Log-differentiating the equations above we get producer i's elasticities:

$$\theta^i_{j,k} = \frac{1}{\tilde{\omega}_{i,k}} \frac{d\log x_{i,j}}{d\log p_{i,k}} = \left\{ \begin{array}{cc} \pi - \frac{\xi \times \mathrm{I}_{\{j=k\}}}{\tilde{\omega}_{i,k}} + \frac{(\phi-\pi)\times \mathrm{I}_{\{k\notin E\}}}{\left(\sum_{n\notin E}\tilde{\omega}_{i,n}\right)} + \frac{(\xi-\phi)\times \mathrm{I}_{\left\{k\in \not E\right\}}}{\left(\sum_{n\in\mathcal{N}-E}\tilde{\omega}_{i,n}\right)}, & \text{if} \quad j \in \not E \\ \pi - \frac{\iota \times \mathrm{I}_{\{j=k\}}}{\tilde{\omega}_{i,k}} + \frac{(\iota-\pi)\times \mathrm{I}_{\{k\in\mathcal{N}\}}}{\left(\sum_{n\in E}\tilde{\omega}_{i,n}\right)}, & \text{if} \quad j \in E \\ \pi - \frac{\gamma \times \mathrm{I}_{\{j=k\}}}{\tilde{\omega}_{i,k}} + \frac{(\phi-\pi)\times \mathrm{I}_{\{k\notin E\}}}{\left(\sum_{n\notin E}\tilde{\omega}_{i,n}\right)} + \frac{(\gamma-\phi)\times \mathrm{I}_{\{k\in\mathcal{F}\}}}{\left(\sum_{n\in\mathcal{F}}\tilde{\omega}_{i,n}\right)}, & \text{if} \quad j \in \mathcal{F} \end{array} \right\}$$

$$\tilde{\omega}_{i,j} = \left\{ \begin{array}{cc} \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,N}}\right)^{1-\xi} \left(\frac{P_{i,N}}{P_{i,\not E}}\right)^{1-\phi} \left(\frac{P_{i,\not E}}{P_i}\right)^{1-\pi}, & \text{if} \quad j \in \not E \\ \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,F}}\right)^{1-\gamma} \left(\frac{P_{i,F}}{P_{i,\not E}}\right)^{1-\phi} \left(\frac{P_{i,\not E}}{P_i}\right)^{1-\pi}, & \text{if} \quad j \in \mathcal{F} \\ \tilde{\omega}_{i,j} \left(\frac{p_{i,j}}{P_{i,E}}\right)^{1-\iota} \left(\frac{P_{i,E}}{P_i}\right)^{1-\pi}, & \text{if} \quad j \in E \end{array} \right\}$$

B.3 Solving the Log-linearized Model

In this section we show the proofs for the theorems in the main text. We begin by log-linearizing key equations derived from agents' cost minimization behaviors and from definitions laid down in section 4. This process yields a system of differential equations that allows us to identify the equilibrium vector of sales shares variations. Finally, the vector of sales shares variations identifies all the remaining variations and elasticities desired.

From firms' cost minimization problem and Shephard's lemma we have that,

$$d\log p_i = \sum_{j\in\mathcal{N}} \tilde{\omega}_{i,j} \left(d\log p_j + d\log(1+\tau_{i,j})\right) + \sum_{f\in\mathcal{F}} \tilde{\omega}_{i,f} d\log w_f,$$

and further solving for the vector of prices we arrive at: [43]

$$d\log p_i = \sum_{j\in\mathcal{N}} \tilde{\Psi}_{i,j}\left[\sum_{k\in\mathcal{N}} \tilde{\omega}_{j,k} d\log(1+\tau_{j,k})\right] + \sum_{f\in\mathcal{F}} \tilde{\Psi}_{i,f} d\log\Lambda_f, \tag{A.1}$$

These equations pinpoint prices as a function of sales shares changes and priors, essentially demonstrating that a change in the price of $k \in \mathcal{N}\cup\mathcal{F}$ affects the marginal cost, and thereby the price of i via the direct and indirect exposure of its expenditures to k: $\tilde{\Psi}_{i,k}$.

Differentiating firms' intermediary expenditure shares we get

$$d\tilde{\omega}_{i,j} = \tilde{\omega}_{i,j}\left[d\log p_j + d\log(1+\tau_{i,j}) + \sum_{k\in\mathcal{N}\cup\mathcal{F}} \left(\theta^i_{j,k}-1\right)\tilde{\omega}_{i,k}\left[d\log p_k + d\log(1+\tau_{i,k})\right]\right], \quad j\in\mathcal{N}\cup\mathcal{F} \tag{A.2}$$

where we again denote by $\theta^i_{j,k}$ the Allen-Uzawa elasticities of substitution between the pair of inputs and factors $j,k \in \{\mathcal{N}\cup\mathcal{F}\}$ for firm i.

For every household $h\in\mathcal{H}$,

$$\frac{d\tilde{\omega}_{h,j}}{\tilde{\omega}_{h,j}} = d\log p_j + d\log(1+\tau_{h,j}) + \sum_{k\in\mathcal{N}} \tilde{\omega}_{h,k}\left(\theta_{j,k,h}-1\right)\left[d\log p_k + d\log(1+\tau_{i,k})\right] \tag{A.3}$$

$$+\left(\psi_{h,j}-1\right)\left[d\log\mathcal{S}_h - \sum_{k\in\mathcal{N}} \tilde{\omega}_{h,k} d\log p_{h,k}\right], \quad j\in\mathcal{N} \tag{A.4}$$

From the identity $\tilde{\omega}_{i,j} = (1+\tau_{i,j})\omega_{i,j}$ we have that:

$$d\omega_{i,j} = \frac{\omega_{i,j}}{\tilde{\omega}_{i,j}} d\tilde{\omega}_{i,j} - \omega_{i,j} d\log(1+\tau_{i,j}), \tag{A.5}$$

and, by differentiating the consumers' total income in terms of sales shares

$$\mathcal{S}_h \equiv \frac{\sum_{j\in\mathcal{N}} p_j c_{h,j}}{GDP} = \Lambda_h + \Theta_h \sum_{i\in\mathcal{N}\cup\mathcal{H}} \sum_{j\in\mathcal{N}} \omega_{i,j}\lambda_i\tau_{i,j}, \tag{A.6}$$

---

[43] Note that, since factor quantities are fixed, $d\log\Lambda_f = d\log w_f$, $\forall f \in \mathcal{F}$.

we get:

$$d\,\mathcal{S}_h = d\Lambda_h + \Theta_h \left(\sum_{i \in \mathcal{N} \cup \mathcal{H}} \sum_{j \in \mathcal{N}} d\omega_{i,j} \lambda_i \tau_{i,j} \right) + \Delta_h \left(\sum_{i \in \mathcal{N} \cup \mathcal{H}} \sum_{j \in \mathcal{N}} \omega_{i,j} \lambda_i d\tau_{i,j} \right) \tag{A.7}$$

$$+ \Theta_h \underbrace{\left(\sum_{i \in \mathcal{N} \cup \mathcal{H}} \sum_{j \in \mathcal{N}} \omega_{i,j} d\lambda_i \tau_{i,j} \right)}_{d\lambda_h^*} \tag{A.8}$$

where we use a different distribution scheme for tax income accruing from previous taxation, Θ and from carbon taxation Δ.

From multiplying both sides of the market clearing condition (9), the accounting identify[44]

$$p_i y_i = \sum_{h \in \mathcal{H}} c_{h,i} p_i + \sum_{j \in \mathcal{N}} x_{j,i} p_i = \sum_{h \in \mathcal{H}} \underbrace{\omega_{h,i}\,\mathcal{S}_h}_{\lambda_{c,i}} + \sum_{j \in \mathcal{N}} \underbrace{\omega_{j,i} \lambda_j}_{\lambda_{j,i}} \tag{A.10}$$

implies that

$$\lambda_i = \sum_{h \in \mathcal{H}} \omega_{h,i}\,\mathcal{S}_h + \sum_{j \in \mathcal{N}} \omega_{j,i} \lambda_j$$

which results, in matrix form, in

$$\lambda' = \mathcal{S}'\,\Psi \tag{A.11}$$

and thus:

$$\lambda_i d\log\lambda_i = \sum_{h \in \mathcal{H}} \omega_{h,i}\,\mathcal{S}_h\,d\log\mathcal{S}_h + \sum_{h \in \mathcal{H}} \mathcal{S}_h\,\omega_{h,i} d\log\omega_{h,i} + \sum_{j \in \mathcal{N}} \omega_{j,i}\lambda_j d\log\lambda_j + \sum_{j \in \mathcal{N}} \lambda_j \omega_{j,i} d\log\omega_{j,i}, \tag{A.12}$$

---

[44] Note that we can write:

$$d\Psi = \Psi d\omega \Psi \tag{A.9}$$

which, because of the definition of the Leontief inverse (11), can be restated as

$$\lambda_k d\log\lambda_k = \sum_i \Psi_{i,k}\sum_{h\in\mathcal{H}}\omega_{h,i}\,\mathcal{S}_h\,d\log\mathcal{S}_h + \sum_i \Psi_{i,k}\sum_{h\in\mathcal{H}}\mathcal{S}_h\,\omega_{h,i}d\log\omega_{h,i} + \sum_i \Psi_{i,k}\sum_{j\in\mathcal{N}}\lambda_j\omega_{j,i}d\log\omega_{j,i},$$

We now rewrite the equations delineated above in matrix form, preserving the numbering:

$$d\log p = A_2 d\lambda^F + B_2 \tag{A.1}$$

$$\begin{aligned} vec(d\tilde{\Omega}) &= A_3 d\log p + C_3 d\,\mathcal{S} + B_3 \\ &= A_3(A_2 d\lambda^F + B_2) + C_3 d\,\mathcal{S} + B_3 \end{aligned} \tag{A.2}$$

$$\begin{aligned} vec(d\Omega) &= A_1 vec(d\tilde{\Omega}) + B_1 \\ &= A_1(A_3(A_2 d\lambda^F + B_2) + C_3 d\,\mathcal{S} + B_3) + B_1 \\ &= \underbrace{A_1A_3A_2}_{A_\Omega} d\lambda_F + \underbrace{A_1C_3}_{C_\Omega} d\,\mathcal{S} + \underbrace{A_1A_3B_2 + A_1B_3 + B_1}_{B_\Omega} \end{aligned} \tag{A.3}$$

$$\begin{aligned} d\,\mathcal{S} &= A_4 d\lambda^F + C_4 vec(d\Omega) + Ed\lambda^* + B_4 \\ d\,\mathcal{S} &= A_4 d\lambda^F + C_4(A_\Omega d\lambda_F + C_\Omega d\,\mathcal{S} + B_\Omega) + Ed\lambda^* + B_4 \\ d\,\mathcal{S} - C_4C_\Omega d\,\mathcal{S} &= (A_4 + C_4A_\Omega)d\lambda^F + Ed\lambda^* + B_4 + C_4B_\Omega \\ d\,\mathcal{S} &= (\mathbf{I} - C_4C_\Omega)^{-1}[(A_4 + C_4A_\Omega)d\lambda^F + Ed\lambda^* + B_4 + C_4B_\Omega] \\ &= \underbrace{(\mathbf{I} - C_4C_\Omega)^{-1}(A_4 + C_4A_\Omega)}_{A_\mathcal{S}} d\lambda^F + \underbrace{(\mathbf{I} - C_4C_\Omega)^{-1}E}_{C_\mathcal{S}} d\lambda^* + \underbrace{(\mathbf{I} - C_4C_\Omega)^{-1}(B_4 + C_4B_\Omega)}_{B_\mathcal{S}} \\ vec(d\Omega) &= A_\Omega d\lambda_F + C_\Omega(\mathbf{I} - C_4C_\Omega)^{-1}\left[(A_4 + C_4A_\Omega)d\lambda^F + Ed\lambda^* + B_4 + C_4B_\Omega\right] + B_\Omega \\ vec(d\Omega) &= \underbrace{\left(A_\Omega + C_\Omega(\mathbf{I} - C_4C_\Omega)^{-1}(A_4 + C_4A_\Omega)\right)}_{A^*} d\lambda^F + \underbrace{C_\Omega(\mathbf{I} - C_4C_\Omega)^{-1}E}_{C^*} d\lambda^* \\ &\quad + \underbrace{\left(C_\Omega(\mathbf{I} - C_4C_\Omega)^{-1}(B_4 + C_4B_\Omega) + B_\Omega\right)}_{B^*} \end{aligned} \tag{A.4}$$

$$\begin{aligned} vec(d\Psi) &= (\Psi' \otimes \Psi)vec(d\Omega) \\ &= \underbrace{(\Psi' \otimes \Psi)A^*}_{A_\Psi} d\lambda^F + \underbrace{(\Psi' \otimes \Psi)C^*}_{C_\Psi} d\lambda^* + \underbrace{(\Psi' \otimes \Psi)B^*}_{B_\Psi} \end{aligned} \tag{A.5}$$

Differentiating A.11 we get:

$$d\lambda' = d\,\mathcal{S}'\,\Psi + \mathcal{S}'\,d\Psi \tag{A.6}$$

To solve the model our goal is to find matrices A and B such that

$$\begin{bmatrix} d\lambda_F \\ d\lambda^* \end{bmatrix} = A \begin{bmatrix} d\lambda_F \\ d\lambda^* \end{bmatrix} + B \tag{A.7}$$

This is accomplished in two parts. First we find the upper part of A and B:

$$\begin{aligned} d\lambda' &= d\,\mathcal{S}'\,\Psi + \mathcal{S}'\,d\Psi \\ &= (A_{\mathcal{S}} d\lambda_F + C_{\mathcal{S}} d\lambda^* + B_{\mathcal{S}})'\Psi + [(\mathbf{I}_n \otimes \mathcal{S}')(A_\Psi d\lambda_F + C_\Psi d\lambda^* + B_\Psi)]' \\ d\lambda &= \left[\Psi' A_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')A_\Psi\right] d\lambda_F + \left[\Psi' C_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')C_\Psi\right] d\lambda^* + \left[\Psi' B_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')B_\Psi\right] \\ &= \underbrace{\left[\Psi' A_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')A_\Psi \quad \Psi' C_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')C_\Psi\right]}_{A_{Up}} \begin{bmatrix} d\lambda_F \\ d\lambda^* \end{bmatrix} + \underbrace{\left[\Psi' B_{\mathcal{S}} + (\mathbf{I} \otimes \mathcal{S}')B_\Psi\right]}_{B_{Up}}, \end{aligned} \tag{A.8}$$

where

$$(\mathcal{S}'\,d\Psi)' = vec(\mathcal{S}'\,d\Psi) = (\mathbf{I}_n \otimes \mathcal{S}')(A_\Psi d\lambda^F + C_\Psi d\lambda^* + B_\Psi)$$

Selecting the equations corresponding to $d\lambda^F$ we arrive at:

$$d\lambda^F = A_{Up} \begin{bmatrix} d\lambda_F \\ d\lambda^* \end{bmatrix} + B_{Up} \tag{A.9}$$

Note that since we set the nominal GDP as numeraire one of these equations is redundant as must be replaced by Walras law:

$$\begin{aligned} &\mathbf{1}' d\,\mathcal{S} = 0 \\ &\mathbf{1}'(A_{\mathcal{S}} d\lambda^F + C_{\mathcal{S}} d\lambda^* + B_{\mathcal{S}}) = 0 \end{aligned}$$

Second, we can find the lower part of A and B in very similar fashion by noting that, from A.7, $d\lambda^*$ can be written as a linear function of $d\lambda$:

$$d\lambda^* = Zd\lambda = Z\left(d\,\mathcal{S}'\,\Psi + \mathcal{S}'\,d\Psi\right)$$

C POF Sector Aggregation Translation Losses

Table 6: POF Sector Aggregation Translation Losses C Authors' Sector Aggregation

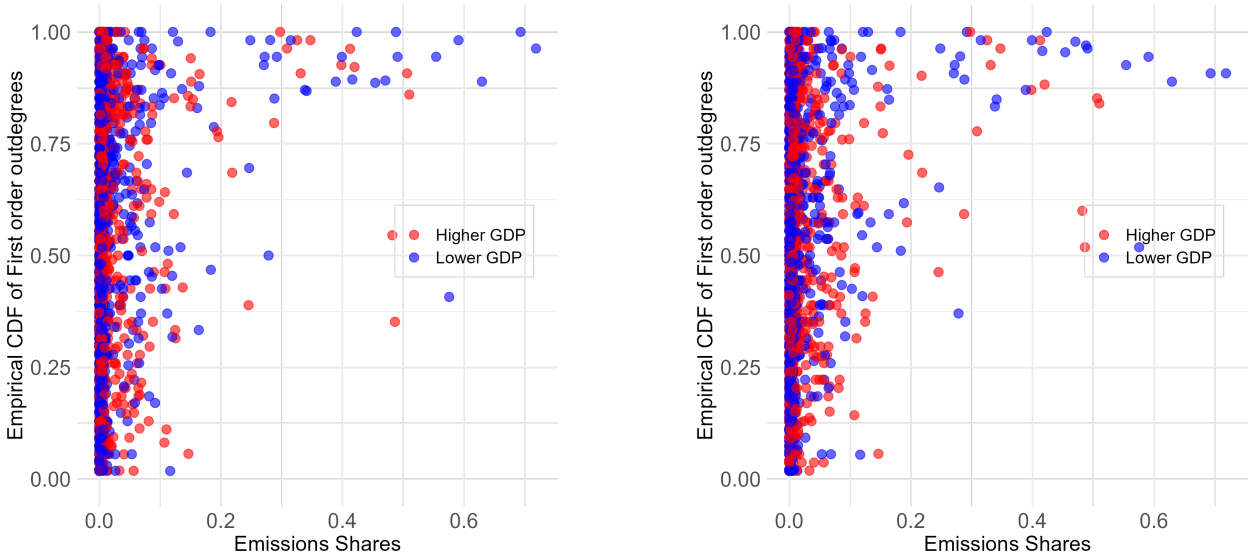
	POF	Reconciled	Loss (%)
Food	R$ 658	R$ 651	-1.1
Housing	R$ 1,377	R$ 748	-45.7
Clothing	R$ 160	R$ 156	-3.0
Transport	R$ 680	R$ 676	-0.6
Personal Care	R$ 137	R$ 137	-0.2
Health Care	R$ 302	R$ 301	-0.5
Education	R$ 176	R$ 168	-4.5
Recreation Culture	R$ 96	R$ 91	-5.4
Smoking	R$ 17	R$ 17	-1.0
Personal Services	R$ 49	R$ 48	-0.5
Miscellaneous	R$ 113	R$ 102	-9.8

D Authors' Sector Aggregation

Table 7: Authors' Sector Aggregation

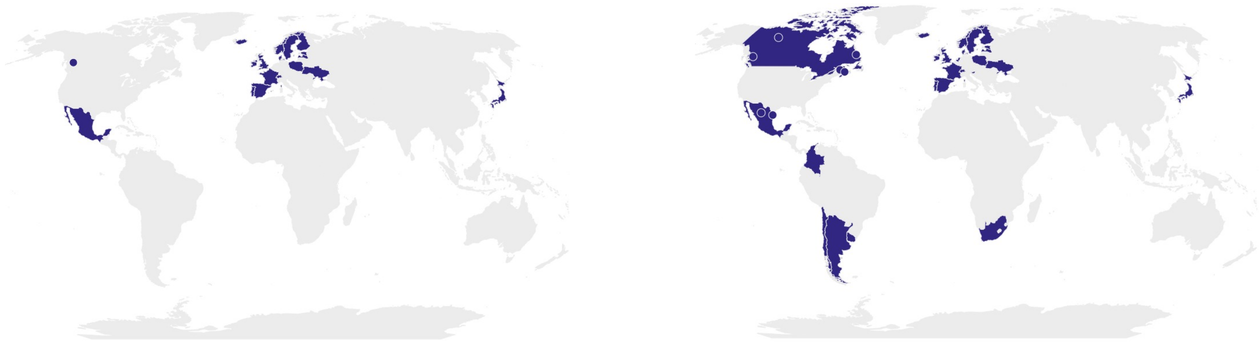
I-OM Sectors	Aggregated Sectors	I-OM Sectors	Aggregated Sectors
Agriculture	Farming	Parts and accessories for motor vehicles	Other Industries
Cattle raising	Farming	Other transport equipment	Other Industries
Forestry, fishing, and aquaculture	Energy	Furniture and related products	Other Industries
Coal and nonmetallic mineral mining	Energy	Repair and installation of machinery	Other Industries
Oil and gas extraction	Extractive Activities	Electricity, gas and other utilities	Energy
Iron ore mining	Extractive Activities	Water, sewerage, and waste collection	Public
Other metal ore mining	Extractive Activities	Construction	Construction
Animal slaughtering and rendering	Foods & Beverage	Wholesale and retail trade	Traditional Services
Sugar manufacturing and refining	Foods & Beverage	Land transportation	Transports
Other food products	Foods & Beverage	Water transportation	Transports
Beverages	Foods & Beverage	Air transportation	Transports
Tobacco	Foods & Beverage	Warehousing and support activities	Modern Services
Textile products	Other Industries	Accommodation activities	Traditional Services
Wearing apparel products	Other Industries	Food service activities	Traditional Services
Leather products	Other Industries	Publishing activities	Modern Services
Wood products	Other Industries	Motion picture, video, etc.	Modern Services
Paper products	Other Industries	Telecommunications	Modern Services
Reproduction of recorded media	Other Industries	Other information service activities	Modern Services
Coke and refined petroleum products	Energy	Financial service activities	Modern Services
Biofuels	Energy	Real estate activities	Modern Services
Basic chemical products	Chemical Industry	Legal and accounting activities	Modern Services
Other chemical products	Chemical Industry	Architectural and engineering activities	Modern Services
Cosmetics and cleaning products	Chemical Industry	Other technical activities	Modern Services
Pharmaceutical products	Chemical Industry	Rental and leasing service	Modern Services
Rubber and plastic products	Chemical Industry	Administrative and support service	Traditional Services
Other nonmetallic minerals	Mineral Industries	Investigation and security services	Traditional Services
Iron and steel manufacturing	Mineral Industries	Public administration	Public
Nonferrous metal production	Mineral Industries	Public education	Public
Fabricated metal products	Mineral Industries	Private education	Traditional Services
Computer and electronic	Other Industries	Public health	Public
Electrical machinery	Other Industries	Private health	Traditional Services
Mechanical machinery	Other Industries	Arts, entertainment and recreation	Modern Services
Motor vehicles and trailers	Other Industries	Personal services	Traditional Services

E Supplementary Tables and Figures



(a) First-order weighted outdegrees (b) Tenth-order weighted outdegrees

Figure 6: Largest emitters and downstream propagation potential



(a) 2016 (b) 2021

Figure 7: Carbon tax adoption map

Source: World Bank

Occupation	Schooling	Avg hour wage	
Public Power Seniors	Primary or lower	R$	54
	Lower secondary	R$	63
	Upper secondary	R$	72
	Post-secondary	R$	208
Sciences and Arts	Primary or lower	R$	54
	Lower secondary	R$	51
	Upper secondary	R$	74
	Post-secondary	R$	178
Mid-Level Technicians	Primary or lower	R$	51
	Lower secondary	R$	57
	Upper secondary	R$	62
	Post-secondary	R$	120
Administrative Services	Primary or lower	R$	37
	Lower secondary	R$	35
	Upper secondary	R$	39
	Post-secondary	R$	109
Commercial Services	Primary or lower	R$	31
	Lower secondary	R$	30
	Upper secondary	R$	33
	Post-secondary	R$	81
Agricultural Workers	Primary or lower	R$	29
	Lower secondary	R$	31
	Upper secondary	R$	33
	Post-secondary	R$	81
Extractive and Transformation Manufacturing	Primary or lower	R$	37
	Lower secondary	R$	37
	Upper secondary	R$	40
	Post-secondary	R$	85
Steel and Construction Manufacturing	Primary or lower	R$	34
	Lower secondary	R$	36
	Upper secondary	R$	44
	Post-secondary	R$	216
Repair and Maintenance	Primary or lower	R$	44
	Lower secondary	R$	43
	Upper secondary	R$	53
	Post-secondary	R$	120

Table 8: Average Hourly Wage by Occupation and Schooling Level

Table 9: Expenditure Shares by Income Deciles in Brazil

Income Bin	Energy	Transport	Food	Clothing	Self-Care	Healthcare	Education	Housing	Other
1st Decile	9.8%	7.3%	25.6%	5.2%	6.2%	5.2%	2.9%	29.7%	8.2%
2nd Decile	9.4%	8.7%	22.4%	4.9%	5.5%	5.9%	3.2%	29.4%	10.5%
3rd Decile	9.1%	9.2%	20.5%	4.8%	5.2%	6.4%	3.3%	30.0%	11.7%
4th Decile	8.8%	10.4%	19.7%	4.7%	4.6%	6.3%	3.4%	29.5%	12.6%
5th Decile	9.1%	10.7%	18.3%	4.4%	4.3%	7.0%	3.4%	29.1%	13.6%
6th Decile	8.3%	10.9%	17.0%	4.3%	3.8%	7.4%	4.0%	30.0%	14.2%
7th Decile	8.3%	12.2%	16.7%	4.0%	3.6%	7.1%	3.7%	28.4%	15.9%
8th Decile	7.8%	12.8%	15.2%	3.6%	3.2%	7.0%	4.5%	28.2%	17.5%
9th Decile	7.2%	12.6%	14.0%	3.4%	2.5%	7.7%	4.3%	28.1%	20.1%
10th Decile	4.8%	14.0%	10.3%	2.8%	1.6%	7.1%	4.6%	27.2%	27.6%
Total Sample	7.3%	12.1%	15.3%	3.7%	3.2%	7.0%	4.1%	28.4%	19.0%

Notes: Each row represents the average expenditure shares of households according to their percapita income deciles. Every row adds up to 100%. "*Other*" is mainly comprised of taxes and assets variation.

F Details on Parameterization

G Other Quantitative Results and Robustness

G.1 Alternative Calibration

As discussed in ??, since there the final demand aggregate in the national accounts has additional components other than households' final demand, the total final expenditure shares of the calibrated economy will not reflect the true proportions, thereby altering the Domar weights. To account for that, in this section we offer another parameterization strategy that accounts for that by adding a representative consumer that encodes the combined total final demand of exports, investment, fixed capital formation and government expenditure.

G.2 Other Elasticities of Substitution

G.3 The Role of Heterogeneous Agents

G.4 The Role of Income Elasticities

G.5 Freezing Labor Mobility

Work in Progress

Table 10: Share o Workers with Formal Working Permits According to Sector of Employment

Sector	Formal	Informal
Public Administration	43.6%	56.4%
Farming	67.9%	32.1%
Water Management	89.5%	10.5%
Hospitality	75.5%	24.5%
Arts Recreation	59.4%	40.6%
Administrative Services	91.1%	8.9%
Financial Services	95.7%	4.3%
Real Estate	79.8%	20.2%
Undefined Activities	98.6%	1.4%
Professional Services	69.8%	30.2%
Commerce Repair	83.8%	16.2%
Construction	70.8%	29.2%
Education	76.4%	23.6%
Electricity Gas	96.1%	3.9%
Manufacturing	90.0%	10.0%
Mining	93.2%	6.8%
Communications	91.1%	8.9%
International Bodies	100.0%	0.0%
Other Services	63.9%	36.1%
Healthcare	74.4%	25.6%
Domestic Services	38.0%	62.0%
Transportation	85.8%	14.2%

Notes: The table shows the share of workers in each sectors that have a booklet (*carteira de trabalho*).
Source: POF.

Table 11: Details on Schooling Levels

Classification RAIS	Classification POF	Classification ISCED	Description ISCED
1,2,3,4	1,2	0,1	Primary Education or Lower
5,6	3, 4	2	Lower Secondary
7,8	5, 6	3	Upper Secondary
9,10,11	7	4,5,6,7,8	Post-secondary or Higher

Table 12: Reconciled Fuel Products

SEEG Fuels	Input-Output Matrix Products	CO_2e Emissions ($10^5 t$)
Tar	Other petroleum refining products	3.098
Sugarcane bagasse	Sugar	27.483
Biodiesel	Ethanol and other biofuels	2.932
Coal steam 3100	Coal	0.038
Coal steam 3300	Coal	20.727
Coal steam 3700	Coal	0.164
Coal steam 4200	Coal	1.019
Coal steam 4500	Coal	50.263
Coal steam 4700	Coal	6.498
Coal steam 5200	Coal	13.280
Coal steam 6000	Coal	77.717
Unspecified steam coal	Coal	0.307
Coal coke	Other petroleum refining products	3.650
Petroleum coke	Other petroleum refining products	148.354
Petroleum Diesel	Diesel	1410.264
LPG	Other petroleum refining products	216.107
Automotive gasoline	Gasoalcohol	641.493
Aviation gasoline	Aviation fuels	0.972
Coke oven gas	Other petroleum refining products	30.988
Refinery gas	Other petroleum refining products	94.933
Dry natural gas	Electricity, gas and other utilities	585.567
Wet natural gas	Electricity, gas and other utilities	120.983
Firewood	Products of forestry and forestry	108.238
Lye	Products of forestry and forestry	1.778
Other non-renewable	Other petroleum refining products	36.869
Other petroleum energy sources	Other petroleum refining products	68.751
Aviation kerosene	Aviation fuels	100.165
Kerosene fo ilumination	Other petroleum refining products	0.083
Anhydrous alcohol	Ethanol and other biofuels	7.219
Hydrated alcohol	Ethanol and other biofuels	10.416
Fuel oil	Fuel oil	97.415

Notes: Data refer to the year 2019. We added emissions from ”other biomass” and ”biogas” to those from ”biofuels”.

Table 13: Robustness Scenarios of Elasticities of Substitution

	Baseline	High π	Low ξ	Low σ	High ι
γ	1	1	1	1	1
ϕ	0.2	0.2	0.2	0.2	0.2
σ	0.9	0.9	0.9	0.05	0.9
ξ	0.5	0.5	0.05	0.05	0.5
ι	1.3	1.3	1.3	1.3	2
π	0.02	0.5	0.02	0.02	0.02

G.6 The Role of Input-Output Linkages

Work in Progress

G.7 Removing Pre-existing Consumption Tariffs

Work in Progress

G.8 Robustness Results to Different Grid Sizes

G.8.1 Taxing Other Production Emissions

On the other hand, if emissions from Industrial Processes and Fugitive emissions were also covered, Mineral Industries would become the most affected aggregate sector. A closer look into the sectoral tariff increases shows that the Iron & Steel industry, as well as the nonmetallic minerals share of Coal are the main drivers of this this. The resulting sales share weighted increase in Minerals Industries is 13.4%, while the Energy index also increases up to almost 9%.

Table 14: Aggregate Tax Rate Increase (%)

	FC	FC+FE+IP	FC+FE+IP+AG
Farming	0.1%	0.1%	100.8%
Energy	6.6%	8.9%	8.9%
Foods Beverage	0.0%	0.0%	0.0%
Extractive Activities	0.0%	1.8%	1.8%
Other Industries	0.0%	0.0%	0.0%
Chemical Industry	0.0%	0.0%	0.0%
Mineral Industries	0.0%	13.4%	13.4%
Transports	0.0%	0.0%	0.0%
Services	0.0%	0.0%	0.0%

Notes: The table shows the tax rate increases for the aggregate sectors. Results are weighted means using sales shares as weights. The column *FC* represents the baseline case where only fuel combustion emissions are taxed, in *FC+FE+IP* fugitive emissions and emissions from industrial processes are also taxed and in *FC+FE+IP+AG* agricultural emissions are also included.